Computational Approaches to Ramsey Problems

Alex Weinstock-Collins
& Ethan Mark

REU Week 2 Report

June 14, 2013

Outline

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Ramsey Theory

Review of definitions
Particularly Ramsey, bipartite Ramsey, Zarankiewicz ($z(n, m)$)

Nauty

Explain nauty, how we use it

Limits of nauty

How long nauty took to generate C_4-free bipartite graphs.

Vertices	Edges	Graphs Generated	Time Taken
16	24	4[1]	1.11 seconds
18	29	1	15.90 seconds
20	34	1	$\approx$ 5.5 minutes
22	39	2[2]	$\approx$ 3 hours

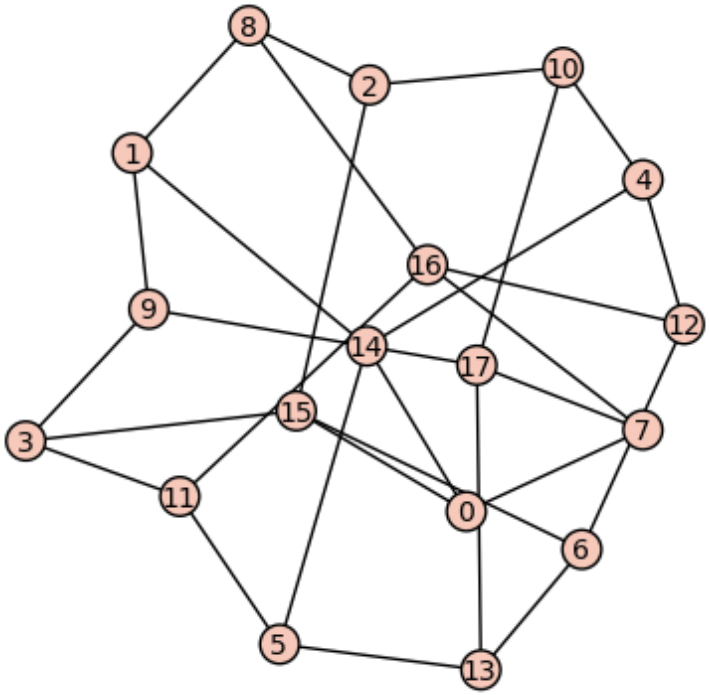
This technique will only be useful for diagonal Zarankiewicz numbers.

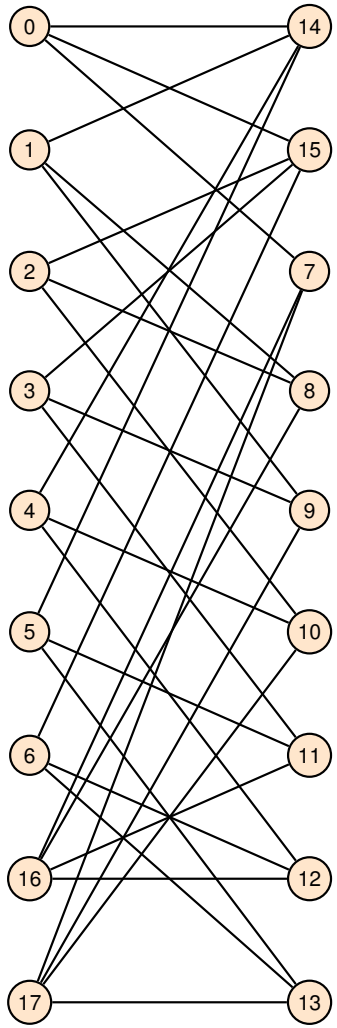
[1] Includes three subgraphs of $K_{8,8}$ and one subgraph of $K_{7,9}$, since $z(8,8) = z(7,9) = 24$ and nauty does not allow specification of vertex allocation.

[2] One subgraph of $K_{11,11}$, one of $K_{10,12}$, $z(11,11) = z(10,12) = 39$.

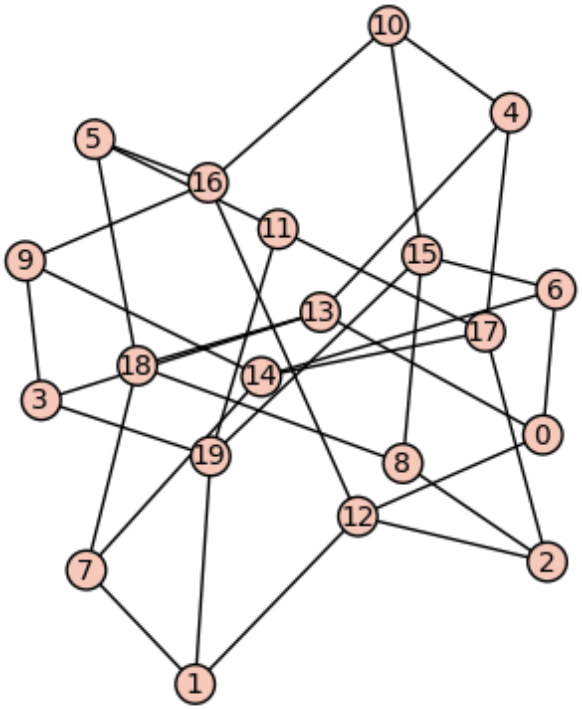
Outline

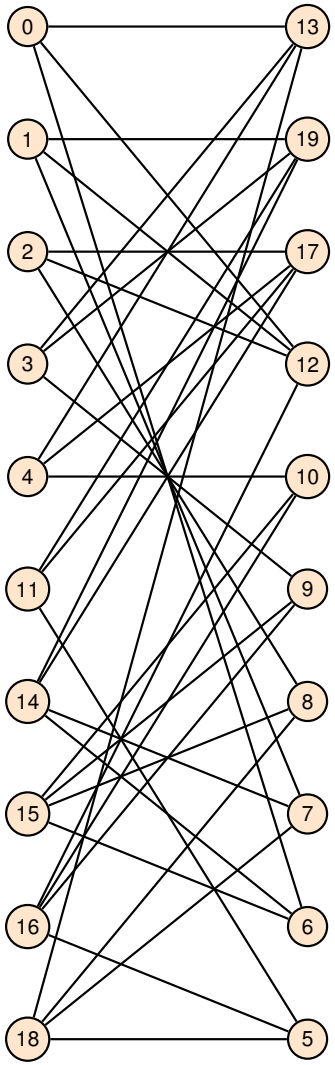
The unique witness for $z(9,9) = 29$





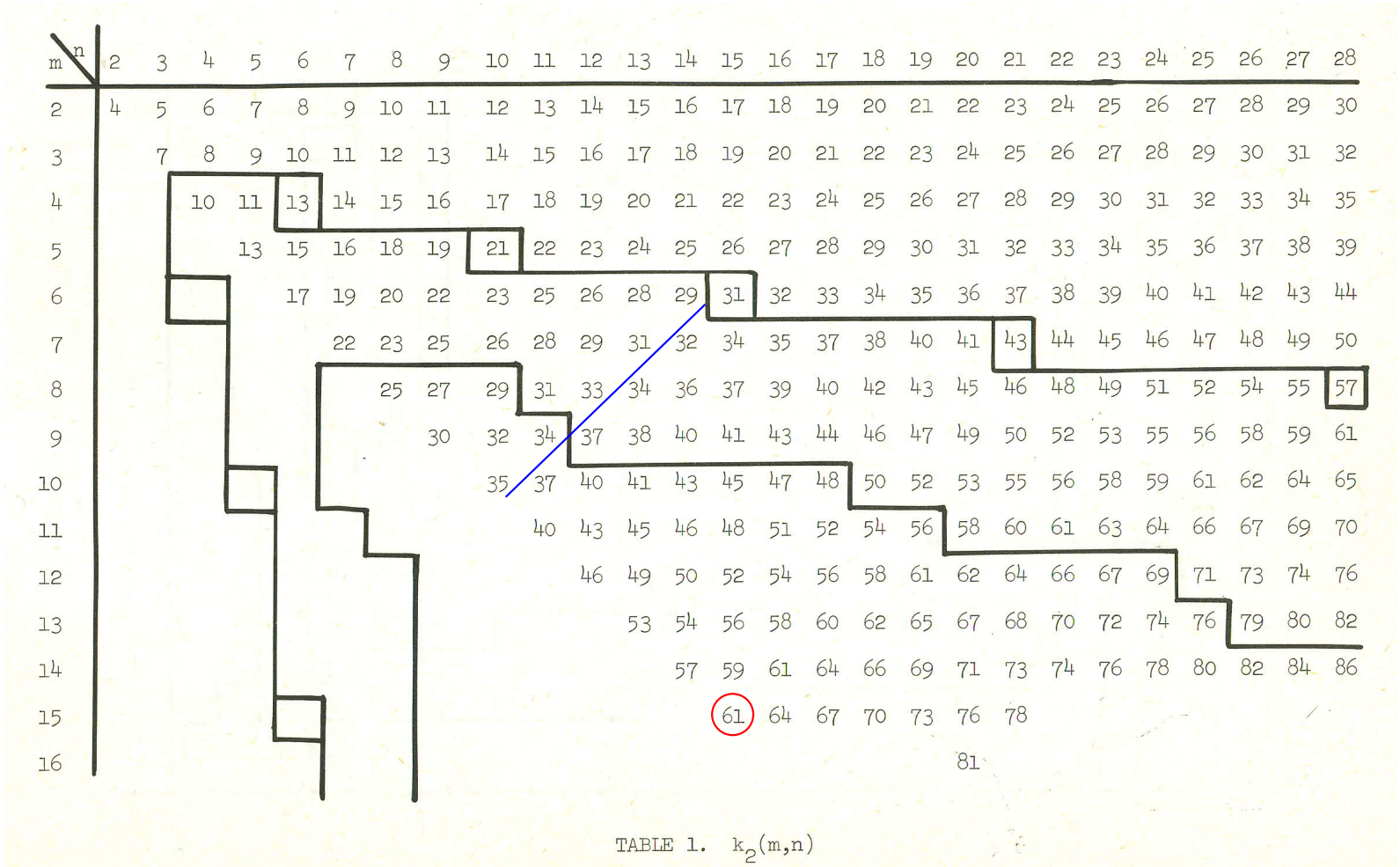
The unique witness for $z(10, 10) = 34$





Outline

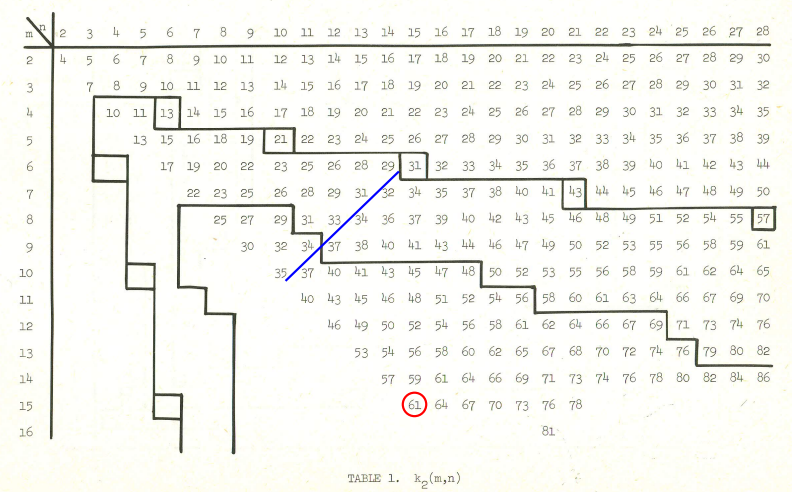
Zarankiewicz numbers to check



m \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
3		7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
4			10	11	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
5				13	15	16	18	19	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
6					17	19	20	22	23	25	26	28	29	31	32	33	34	35	36	37	38	39	40	41	42	43	44
7						22	23	25	26	28	29	31	32	34	35	37	38	40	41	43	44	45	46	47	48	49	50
8							25	27	29	31	33	34	36	37	39	40	42	43	45	46	48	49	51	52	54	55	57
9								30	32	34	37	38	40	41	43	44	46	47	49	50	52	53	55	56	58	59	61
10									35	37	40	41	43	45	47	48	50	52	53	55	56	58	59	61	62	64	65
11										40	43	45	46	48	51	52	54	56	58	60	61	63	64	66	67	69	70
12											46	49	50	52	54	56	58	61	62	64	66	67	69	71	73	74	76
13												53	54	56	58	60	62	65	67	68	70	72	74	76	79	80	82
14													57	59	61	64	66	69	71	73	74	76	78	80	82	84	86
15														61	64	67	70	73	76	78							
16																			81								

TABLE 1. $k_2(m,n)$

Zarankiewicz numbers to check



- From Guy, 1969
- Values are $z(n, m) + 1$ due to alternate definition
- Values above top line determined by easily verifiable theorem
- Values between lines determined by other theorem (proof missing)
- Other values determined by individual argument
- Circled value does not match newer paper (Dybizbański, Dzido, Radziszowski, 2013)
- Values beyond blue line are not feasibly checkable by nauty

Alternative methods

Talk about algorithm I was working on yesterday

Outline

Projective Planes

What are they?
Why are they useful?

Readings

Questions

Questions?