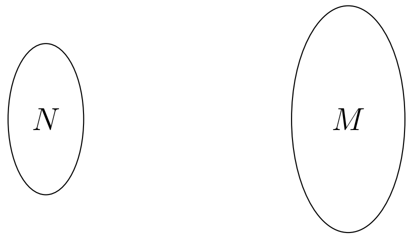
Problem:
Determine a general form for the maximum sum of the degrees of $k+1$ vertices in one partition of a bipartite graph. The purpose is to prove an easy method for weeding out invalid degree sequences for the Zarankiewicz number checker script.



$$|N| = n, \ |M| = m, \ n \leq m$$

Denote the degree sequence S of N as $s_0, s_1, \ldots, s_{n-1}$.

Lemma:

$$\sum_{i=0}^{k} s_i \leq m + \binom{k+1}{2}$$

In addition, equality is theoretically achievable if $m \geq \binom{k+1}{2}$.

Proof:
Generalize Theorem 2 given in Guy. Alternatively:

Let Δ_k denote $\sum_{i=0}^{k} s_i$.
Suppose

$$\Delta_k = m + \binom{k+1}{2} + 1.$$

Partition the set M into $k+1$ subsets.
We must add $\binom{k+1}{2} + 1$ edges to the $k+1$ vertices in N in order for the sum of the degree sequences to reach $m + \binom{k+1}{2} + 1$.
We can have a maximum of $\binom{k+1}{2}$ intersections of two of the $k+1$ subsets.
By the pigeon-hole principle, at least one such intersection must contain at least two vertices.
But this would constitute a C_4. Contradiction.

A simple construction based on this proof shows that $\Delta_k = m + \binom{k+1}{2}$ is theoretically achievable if $m \geq \binom{k+1}{2}$.

Solution:
Thus we can eliminate all degree sequences in which the sum of the largest k degrees exceeds $m + \binom{k}{2}$.