Computational Approaches to Ramsey Problems

Alex Weinstock-Collins
& Ethan Mark

REU Week 2 Report

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Outline

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Ramsey Theory Review

Recall:.

Ramsey Number $R(n_1, ..., n_c) = r$ is the minimum order of a complete graph in c colors that must contain a complete subgraph of order n_i whose edges are all color i.

Bipartite Ramsey Number: $b(n_1, ..., n_k)$ is the smallest number b such that any coloring of the edges of $K_{b,b}$, with k colors guarantees a monochromatic copy of K_{n_i,n_i}, in the i-th color, for some i, $1 \leq i \leq k$.

Zarankiewicz Numbers: $z(m, n; s, t)$ is the maximum number of edges in a subgraph of $K_{m,n}$ that does not contain $K_{s,t}$ as a subgraph.

Nauty

nauty created in 1984 is a program for creating and computing graphs.

Examples:

Generating complete graphs or bipartite graphs.

Counting graphs with certain properties.

Limits of nauty

How long nauty took to generate C_4-free bipartite graphs.

Vertices	Edges	Graphs Generated	Time Taken
16	24	4[1]	1.11 seconds
18	29	1	15.90 seconds
20	34	1	$\approx$ 5.5 minutes
22	39	2[2]	$\approx$ 3 hours

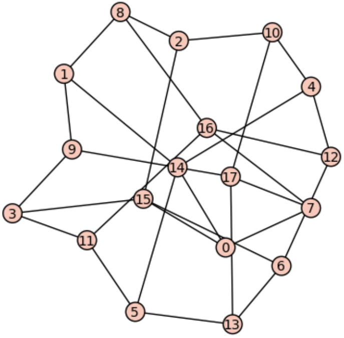
This technique will only be useful for diagonal Zarankiewicz numbers.

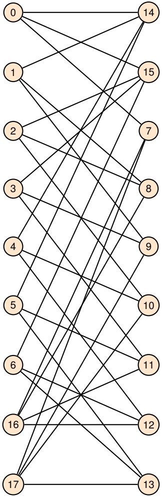
[1] Includes three subgraphs of $K_{8,8}$ and one subgraph of $K_{7,9}$, since $z(8,8) = z(7,9) = 24$ and nauty does not allow specification of vertex allocation.

[2] One subgraph of $K_{11,11}$, one of $K_{10,12}$, $z(11,11) = z(10,12) = 39$.

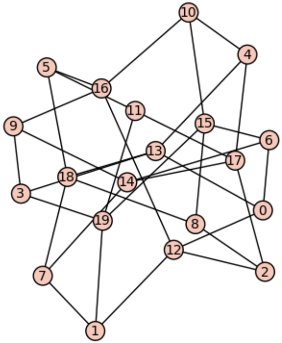
Outline

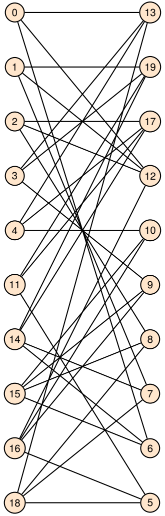
The unique witness for $z(9,9) = 29$





The unique witness for $z(10, 10) = 34$





Outline

Zarankiewicz numbers to check

m \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
3		7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
4			10	11	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
5				13	15	16	18	19	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
6					17	19	20	22	23	25	26	28	29	31	32	33	34	35	36	37	38	39	40	41	42	43	44
7						22	23	25	26	28	29	31	32	34	35	37	38	40	41	43	44	45	46	47	48	49	50
8							25	27	29	31	33	34	36	37	39	40	42	43	45	46	48	49	51	52	54	55	57
9								30	32	34	37	38	40	41	43	44	46	47	49	50	52	53	55	56	58	59	61
10									35	37	40	41	43	45	47	48	50	52	53	55	56	58	59	61	62	64	65
11										40	43	45	46	48	51	52	54	56	58	60	61	63	64	66	67	69	70
12											46	49	50	52	54	56	58	61	62	64	66	67	69	71	73	74	76
13												53	54	56	58	60	62	65	67	68	70	72	74	76	79	80	82
14													57	59	61	64	66	69	71	73	74	76	78	80	82	84	86
15														61	64	67	70	73	76	78							
16																			81								

TABLE 1. $k_2(m,n)$

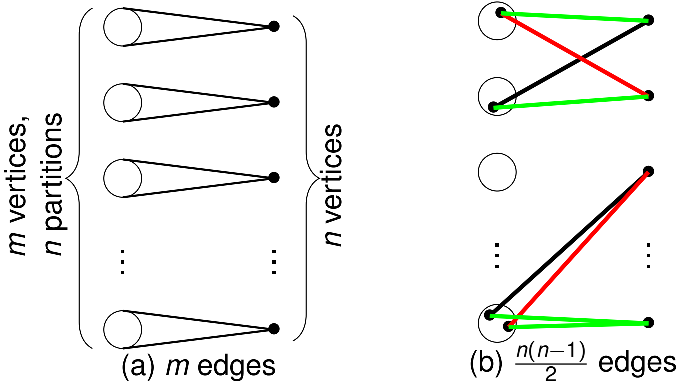
Zarankiewicz numbers to check

m \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
3		7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
4			10	11	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
5				13	15	16	18	19	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
6					17	19	20	22	23	25	26	28	29	31	32	33	34	35	36	37	38	39	40	41	42	43	44
7						22	23	25	26	28	29	31	32	34	35	37	38	40	41	43	44	45	46	47	48	49	50
8							25	27	29	31	33	34	36	37	39	40	42	43	45	46	48	49	51	52	54	55	57
9								30	32	34	37	38	40	41	43	44	46	47	49	50	52	53	55	56	58	59	61
10									35	37	40	41	43	45	47	48	50	52	53	55	56	58	59	61	62	64	65
11										40	43	45	46	48	51	52	54	56	58	60	61	63	64	66	67	69	70
12											46	49	50	52	54	56	58	61	62	64	66	67	69	71	73	74	76
13												53	54	56	58	60	62	65	67	68	70	72	74	76	79	80	82
14													57	59	61	64	66	69	71	73	74	76	78	80	82	84	86
15														61	64	67	70	73	76	78							
16																			81								

TABLE 1. $k_2(m,n)$

- From Guy, 1969
- Values are $z(n, m) + 1$ due to alternate definition
- Values above top line determined by easily verifiable theorem
- Values between lines determined by other theorem (proof missing)
- Other values determined by individual argument
- Circled value does not match newer paper (Dybizbański, Dzido, Radziszowski, 2013)
- Values beyond blue line are not feasibly checkable by nauty

Zarankiewicz numbers to check



Theorem (Guy, 1968): $z(n, m) = m + \binom{n}{2}$ for $m > \binom{n}{2}$.

Alternative methods

Algorithm for checking Zarankiewicz numbers:

- Begin with n, m ($n \leq m$) and z, a guess at $z(n, m)$ to check
- Generate all possible degree sequences of the n vertices on the left which add up to z.
- Eliminate all theoretically impossible sequences
  - Denote the degree sequence $S = (s_0, s_1, \ldots, s_{n-1})$
  - Let $\Delta_k = \sum_{i=0}^{k-1} s_i$
  - If $\Delta_k > m + \binom{k}{2}$, then the degree sequence can be eliminated (generalization of prior theorem)
- Check all remaining (theoretically possible) degree sequence

Note: Still working on last step

Outline

Projective Planes

What are they?
Why are they useful?

Readings

Questions

Questions?