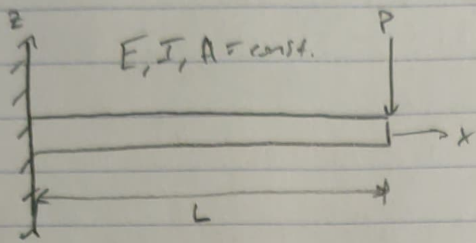
Chris [illegible]

AE 461 - Prelab 3

4/22/20

1)



GDE: $u(x) = 0$, $v(x) = 0$

$E_0 I_{yy} w'''' = f_z + m_y'$

$EIw'''' = 0 + 0$

BC's: @ $x = 0 \rightarrow w(0) = 0$ ①, $w'(0) = 0$ ②

@ $x = L \rightarrow EIw'''(L) = P$ ③, $EIw''(L) = 0$ ④

$EIw'''' = 0$

$EIw''' = C_1 = P$ ③

$EIw'' = Px + C_2 \rightarrow C_2 = -PL$ ④

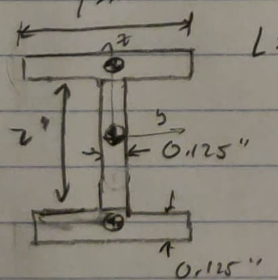
$EIw' = P\frac{x^2}{2} - PLx + C_3 \rightarrow C_3 = 0$ ②

$EIw = P\frac{x^3}{6} - PL\frac{x^2}{2} + C_4 \rightarrow C_4 = 0$ ①

$\hookrightarrow \frac{EIw(x)}{PL^3} = \frac{1}{6}\left(\frac{x}{L}\right)^3 - \frac{1}{2}\left(\frac{x}{L}\right)^2$

@ $x = L$: $\frac{EIw(L)}{PL^3} = \frac{1}{6}\left(\frac{L}{L}\right)^3 - \frac{1}{2}\left(\frac{L}{L}\right)^2 \rightarrow \boxed{w(L) = \frac{-PL^3}{3EI} \text{ (down)}}$

2)



$L = 32"$ $\quad E = \frac{PL^3}{3Iw(L)}$, $\frac{L^3}{3I} = \frac{(32\,in)^3}{3 \cdot 0.366\,in^4} \rightarrow E = \frac{P}{w(L)} \cdot 29852.7\,in^{-1}$

$I_{yy} = 2 \cdot \left[\frac{1\,in\,(0.125\,in)^3}{12} + (1.0625\,in)^2 \cdot 0.125\,in \cdot 1\,in\right] + \frac{0.125\,in\,(2\,in)^3}{12}$

$I_{yy} = 2 \cdot [0.000163\,in^4 + 0.141113\,in^4] + 0.08333\,in^4$

$\therefore I_{yy} = 0.365885 \rightarrow \underline{I_{yy} = 0.366\,in^4}$

@ $P = 0$, $w(L) = 0$

$\underline{E_1 = 0}$

@ $P = 5\,lb$, $w(L) = 0.011"$

$E_2 = \frac{5\,lb}{0.011\,in} \cdot 29852.7\,\frac{1}{in}$

$\underline{E_2 = 1.357 \times 10^7\,\frac{lb}{in^2}}$

@ $P = 10\,lb$, $w(L) = 0.021"$

$E_3 = \frac{10\,lb}{0.021"} \cdot 29852.7\,\frac{1}{in}$

$\underline{E_3 = 1.422 \times 10^7\,\frac{lb}{in^2}}$

@ $P = 15\,lb$, $w(L) = 0.033"$

$E_4 = \frac{15\,lb}{0.033\,in} \cdot 29852.7\,\frac{1}{in}$

$\underline{E_4 = 1.357 \times 10^7\,\frac{lb}{in^2}}$

@ $P = 20\,lb$, $w(L) = 0.042$

$E_5 = \frac{20\,lb}{0.042\,in} \cdot 29852.7\,\frac{1}{in}$

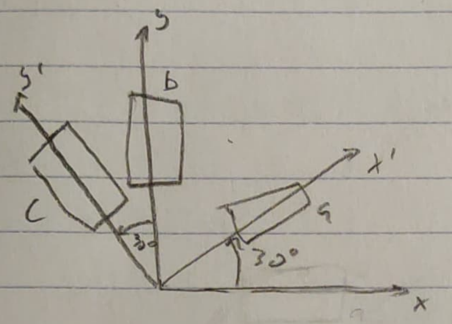
$\underline{E_5 = 1.422 \times 10^7\,\frac{lb}{in^2}}$

$E = (E_2 + E_3 + E_4 + E_5)\frac{1}{4}$

$E = (1.357 \times 10^7 + 1.422 \times 10^7 + 1.357 \times 10^7 + 1.422 \times 10^7)\frac{1}{4}$

$\therefore \boxed{E = 1.38925 \times 10^7\,\frac{lb}{in^2}}$

3)



$\varepsilon_{x'} = 10.00 \frac{\mu in}{in}$, $\varepsilon_{y'} = 30.00 \frac{\mu in}{in}$

$\varepsilon_y = 20.00 \frac{\mu in}{in}$

a) Determine γ_{xy} & ε_x as function of ε_x', ε_y', & ε_y

$$\boxed{\varepsilon_x = \varepsilon_x' + \varepsilon_y' - \varepsilon_y}$$

$$\varepsilon_x' = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\cdot 30^\circ) + 2\tfrac{1}{2}\gamma_{xy}\sin(2\cdot 30^\circ)$$

$$= \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{4} + \gamma_{xy}\frac{\sqrt{3}}{2} \longrightarrow \gamma_{xy} = \left(\varepsilon_x' - \frac{\varepsilon_x' + \varepsilon_y' - \cancel{\varepsilon_y} + \cancel{\varepsilon_y}}{2} - \frac{\varepsilon_x' + \varepsilon_y' - \varepsilon_y - \varepsilon_y}{4}\right)\frac{2}{\sqrt{3}}$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}\left[\varepsilon_x' - \frac{3}{4}\left[\varepsilon_x' + \varepsilon_y'\right] + \frac{\varepsilon_y}{2}\right] \longrightarrow \boxed{\gamma_{xy} = \frac{1}{\sqrt{3}}\left[\frac{1}{2}\varepsilon_x' - \frac{3}{2}\varepsilon_y' + \varepsilon_y\right]}$$

b) Determine σ_{xy} & σ_x

$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$, $\sigma_{xy} = G\gamma_{xy}$, $G = \frac{E}{2(1+\nu)}$

For titanium: $E = 15.229 \times 10^6 \frac{lb}{in^2}$
$\nu = 0.37$
$G = 5.558 \times 10^6 \frac{lb}{in^2}$
} from Google (Azom, pure Ti)

$$\sigma_x = \frac{15.229 \times 10^6 \frac{lb}{in^2}}{1-(0.37)^2}\left(10\times10^{-6} + 30\times10^{-6} - 20\times10^{-6} + 0.37 \cdot 20\times10^{-6}\right)$$

$$\therefore \boxed{\sigma_x = 483.46 \frac{lb}{in^2}}$$

$$\sigma_{xy} = 5.558\times10^6 \frac{lb}{in^2}\left(\frac{1}{\sqrt{3}}\left[\frac{1}{2}(10\times10^{-6}) - \frac{3}{2}(30\times10^{-6}) + 20\times10^{-6}\right]\right)$$

$$\therefore \boxed{\sigma_{xy} = -64.18 \frac{lb}{in^2}}$$

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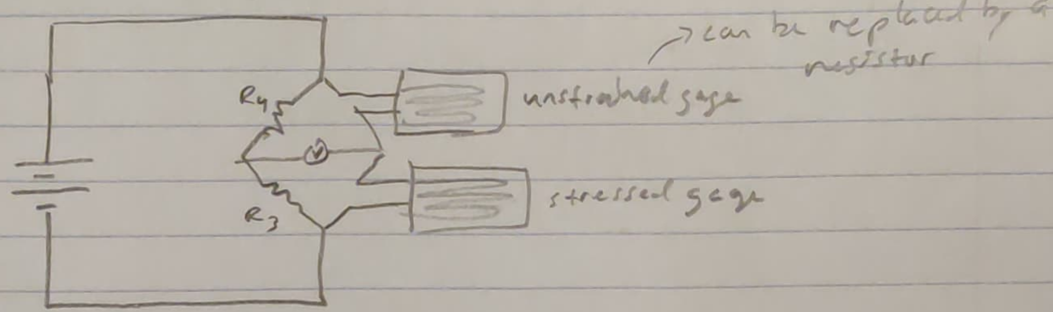
4) A strain gage measures strain by measuring the change in resistance through a thin wire as it stretches due to strain on the surface it is attached to. This is done by taking measurements through a Wheatstone Bridge, seen in figure below:



$$\varepsilon = \frac{\Delta R_4}{R_4} \cdot \frac{1}{gf}$$, where gf is the gage factor

5) $\alpha = 0.10 \frac{deg}{in}$, $\gamma_{ys} = 150\times10^{-6}$. Determine r_i + r_o. Brass

$\tau_{sy} = G\gamma_{ys}$, where $E = 15\times10^{6} \frac{lb}{in^2}$, $\nu = 0.37$ } from Google

$G = \frac{E}{2(1+\nu)}$

$\underline{G = 5.6391\times10^{6} \frac{lb}{in^2}}$

$$\tau_{sy} = 5.6391\times10^{6} \frac{lb}{in^2} \cdot 150\times10^{-6} \rightarrow \therefore \underline{\tau_{sy} = 845.865 \frac{lb}{in^2}}$$

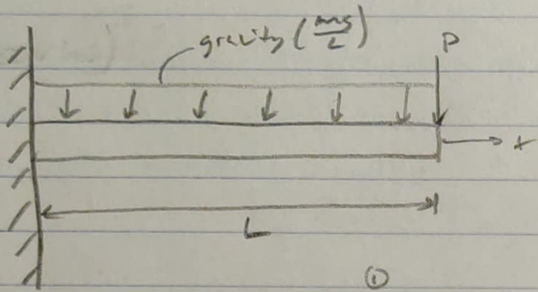
$$G\frac{\pi}{2}(r_o^4 - r_i^4)\alpha = 2\cdot t\cdot \tau_{sy} \cdot \frac{\pi}{4}(r_o + r_i)^2$$

$$\frac{(r_o^4 - r_i^4)}{(r_o + r_i)^2} = \frac{(r_o - r_i)\tau_{sy}}{G\alpha}$$

using MATLAB solve function (see attached code):

$$\boxed{\begin{array}{l} r_o = 0.1031 \text{ in} \\ r_i = 0.0519 \text{ in} \end{array}}$$

BONUS



E, I, A = const.

GDE: $u(x)=0,\ v(x)=0$

$E_o I_{33} w'''' = f_2 + m'g$

$EI w'''' = \frac{mg}{L}$

BCs: @ $x=0 \rightarrow w(0)=0$ ①, $w'(0)=0$ ②

@ $x=L \rightarrow EIw'''(L)=P$ ③, $EIw''(L)=0$ ④

a)

$EI w'''' = \frac{mg}{L}$

$EIw''' = \frac{mg}{L}x + C_1 \overset{③}{=} P \rightarrow C_1 = P - mg$

$EIw'' = \frac{mg}{2L}x^2 + C_1 x + C_2 \rightarrow C_2 \overset{④}{=} -\frac{mgL}{2} - PL + mgL = \frac{mgL}{2} - PL$

$EIw' = \frac{mg}{6L}x^3 + \frac{1}{2}C_1x^2 + C_2x + C_3 \rightarrow C_3 \overset{②}{=} 0$

$EIw = \frac{mg}{24L}x^4 + \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + \cancel{C_3x}^{\,0} + C_4 \rightarrow C_4 \overset{①}{=} 0$

$EIw = \frac{mg}{24L}x^4 + \frac{1}{6}(P-mg)x^3 + \frac{1}{2}\left(\frac{mgL}{2} - PL\right)x^2$

$\frac{EIw(L)}{L^3} = \frac{mg}{24}\cancel{L^3} + \frac{1}{6}(P-mg)\cancel{L^3} + \frac{1}{2}\left(\frac{mg}{2} - P\right)\cancel{L^3}$

$= \frac{1}{24}mg + \frac{4}{24}P - \frac{4}{24}mg + \frac{6}{24}mg - \frac{12}{24}P$

$= \frac{1}{8}mg - \frac{1}{3}P$

$$\underline{w(L) = \frac{PL^3}{3EI} - \frac{mgL^3}{8EI}}$$

$$\%\text{ error} = \frac{\cancel{\frac{PL^3}{3EI}} + \left(\cancel{\frac{-PL^3}{3EI}} + \frac{mgL^3}{8EI}\right)}{\frac{PL^3}{3EI} - \frac{mgL^3}{8EI}} \cdot 100 = \frac{\frac{mg}{8}}{\frac{P}{3} - \frac{mg}{8}} \cdot 100 \rightarrow \boxed{\%\text{ error} = \frac{300mg}{8P - 3mg}}$$

b) Parameters which influence the % error include the mass of the bar and the applied force P. As m increases, % error decreases, as P increases, % error increases.