AE353: Design Problem 03

Glider Control

March 27, 2019

1 Goal

`DesignProblem03` simulates an unpowered glider controllable only by its elevator. An actuator allows you to specify the angular rate of this elevator. Sensors allow measurements of both the glider pitch angle and elevator relative angle. The goal is to optimize glide distance with initial conditions $h_i = 2\ m$, and $v_i = 6\ \frac{m}{s}$ with a pitch angle of $\theta = -2°$.

1.1 Requirements

The glider must achieve a glide distance of at least 25 meters, reaching and oscillating within the angle of attack of $\theta = -2° \pm 1°$ within 1 second and last for the duration of flight.

1.2 Verification

`DesignProblem02` will run for 1000 trials and the procured data will be analyzed to check if $x_{final} \geq 25$ and `sensors.theta` is fluctuating around $-2°$ with a tolerance of $\pm 1°$. A histogram will then be plotted to show the overall performance and statistics of the trials.

2 State Space Model

The motion of the glider is governed by ordinary differential equations with the form

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix} = f(\theta, \phi, \dot{x}, \dot{y}, \dot{\theta}, \dot{\phi}), \quad x = \begin{bmatrix} \theta - \theta_e \\ \phi - \phi_e \\ \dot{x} - \dot{x}_e \\ \dot{y} - \dot{y}_e \\ \dot{\theta} - \dot{\theta}_e \end{bmatrix}, \quad u = \begin{bmatrix} \dot{\phi} - \dot{\phi}_e \end{bmatrix}, \quad y = \begin{bmatrix} \theta - \theta_e \\ \phi - \phi_e \end{bmatrix} \tag{1}$$

With the following definitions:

θ = pitch angle	$\dot{x}$ = horizontal velocity	$\dot{\theta}$ = pitch angular velocity
ϕ = elevator angle	$\dot{y}$ = vertical velocity	$\dot{\phi}$ = elevator angular velocity

These equations were derived by applying a flat-plate model of lift c_L and drag c_D as a function of angle of attack α, for both the wing and elevator:

$$c_L = 2\sin\alpha\cos\alpha \qquad\qquad c_D = 2\sin^2\alpha$$

By use of the `fsolve(f_numeric, v_guess)` function, the equilibrium points may be found for state space model. Note, v_{guess} values were chosen to match realistic equilibrium values for our simulation:

$\theta_e = 0.0047$ for $v_{guess}(\theta) = -2°$	$\dot{x}_e = 5.9629$ for $v_{guess}(\dot{x}) = 6$	$\dot{\theta}_e = -0.0221$ for $v_{guess}(\dot{\theta}) = 0$
$\phi_e = -0.0674$ for $v_{guess}(\phi) = -1°$	$\dot{y}_e = -0.5609$ for $v_{guess}(\dot{y}) = -1$	$\dot{\phi}_e = -0.0041$ for $v_{guess}(\dot{\phi}) = 0$

The resulting coefficients for the state space model were found by use of the `jacobian()` function and resulted in the following matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 0 & 0 \\ -8.76891 & 0.801336 & -0.0162797 & -0.173063 & 0.101591 \\ 115.354 & 23.1587 & 1.45768 & -19.4823 & 1.07654 \\ -53.9872 & -77.1851 & 0.844208 & 8.97444 & -5.26814 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1.0 \\ 0.0124201 \\ 0.193223 \\ -0.644061 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$