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AE 461 - Prelab 6

4/8/2020

1) a) Find Bounds of E_x, E_y, ν_{xy}, and G_{xy}

$V_f = 0.60 \pm 0.05 \rightarrow V_f = \begin{cases} 0.65 \\ 0.55 \end{cases}, \; V_m = \begin{cases} 0.35 \\ 0.45 \end{cases}$

$E_x = V_f E_f + V_m E_m, \quad \frac{1}{E_y} = \frac{V_f}{E_f} + \frac{V_m}{E_m}, \quad \nu_{xy} = \nu_f V_f + \nu_m V_m, \quad \frac{1}{G_{xy}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$

using upper bound of V_f:

$E_x = 0.65 \cdot 120 + 0.35 \cdot 2.4 = 78.84 \text{ GPa}$

$\frac{1}{E_y} = \frac{0.65}{120} + \frac{0.35}{2.4} \rightarrow E_y = 6.61157 \text{ GPa}$

$\nu_{xy} = 0.65 \cdot 0.32 + 0.35 \cdot 0.35 = 0.3305$

$G_f = \frac{E_f}{2(1+\nu_f)} = \frac{120}{2(1+0.32)} = 45.4545 \text{ GPa}$

$G_m = \frac{E_m}{2(1+\nu_m)} = \frac{2.4}{2(1+0.35)} = 0.8889 \text{ GPa}$

$\frac{1}{G_{xy}} = \frac{0.65}{45.4545} + \frac{0.35}{0.8889} \rightarrow G_{xy} = 0.40805 \text{ GPa}$

using lower bound of V_f:

$E_x = 0.55 \cdot 120 + 0.45 \cdot 2.4 = 67.08 \text{ GPa}$

$E_y = \frac{1}{\frac{0.55}{120} + \frac{0.45}{2.4}} = 5.20607 \text{ GPa}$

$\nu_{xy} = 0.55 \cdot 0.32 + 0.45 \cdot 0.35 = 0.3335$

$G_{xy} = \frac{1}{\frac{0.55}{45.4545} + \frac{0.45}{0.8889}} = 0.51835 \text{ GPa}$

$\therefore \boxed{E_x = \begin{cases} 78.84 \text{ GPa} \\ 67.08 \text{ GPa} \end{cases}, \; E_y = \begin{cases} 6.61157 \text{ GPa} \\ 5.20607 \text{ GPa} \end{cases}, \; \nu_{xy} = \begin{cases} 0.3305 \\ 0.3335 \end{cases}, \; G_{xy} = \begin{cases} 0.40805 \text{ GPa} \\ 0.51835 \text{ GPa} \end{cases}}$

b) Determine average of E_x, E_y, ν_{xy}, and G_{xy}, also Stdev

$\bar{E}_x = \frac{78.84 + 67.08}{2} = 72.96 \text{ GPa}$

$\bar{E}_y = \frac{6.61157 + 5.20607}{2} = 5.90882 \text{ GPa}$

$\bar{\nu}_{xy} = \frac{0.3305 + 0.3335}{2} = 0.332$

$\bar{G}_{xy} = \frac{0.40805 + 0.51835}{2} = 0.9264 \text{ GPa}$

$\therefore \boxed{\begin{aligned} \bar{E}_x &= 72.96 \text{ GPa} \\ \bar{E}_y &= 5.90882 \text{ GPa} \\ \bar{\nu}_{xy} &= 0.332 \\ \bar{G}_{xy} &= 0.9264 \text{ GPa} \end{aligned}}$

$\sigma_{std} \approx \frac{\text{max} - \text{min}}{4}$

$\sigma_{E_x} = \frac{78.84 - 67.08}{4} = 2.94 \text{ GPa}$

$\sigma_{E_y} = \frac{6.61157 - 5.20607}{4} = 0.351 \text{ GPa}$

$\sigma_{\nu_{xy}} = \frac{0.3335 - 0.3305}{4} = 0.00075$

$\sigma_{G_{xy}} = \frac{0.51835 - 0.40805}{4} = 0.027575 \text{ GPa}$

$\therefore \boxed{\begin{aligned} \sigma_{E_x} &= 2.94 \text{ GPa} \\ \sigma_{E_y} &= 0.351 \text{ GPa} \\ \sigma_{\nu_{xy}} &= 0.00075 \\ \sigma_{G_{xy}} &= 0.0276 \text{ GPa} \end{aligned}}$

See code

2) a) Q_{11} @ $\theta = 0° = 133.0027$ GPa

$\frac{1}{2} Q_{11}$ @ $\theta = 0° = 66.5013$ GPa $\rightarrow \theta = 36.3276°$

$\frac{1}{3} Q_{11}$ @ $\theta = 0° = 44.3342$ GPa $\rightarrow \theta = 45.4365°$

b) $Q_{11} \cdot 90\% = 119.7024$ GPa $\rightarrow \theta = 15.0315°$

↳ See plot attached @ end.

3) See code: $Q_{16\,max} = 54.2471$ GPa, $\rightarrow \theta = 30.2860°$

a) No, θ @ $Q_{16\,max}$ does NOT depend on material properties

b) Q_{16} is the stiffness in the XZ plane

Q_{26} is the stiffness in the YZ plane

c) Yes, isotropic materials have equal material properties in all directions meaning there is a full Q matrix with non-zero Q_{16} and Q_{26} values

4) $F_1\sigma_1 + F_2\cancel{\sigma_2}^{0} + F_{11}\sigma_1^2 + F_{22}\cancel{\sigma_2^2}^{0} + F_{66}\cancel{\tau^2}^{0} + 2F_{12}\cancel{\sigma_1\sigma_2}^{0} = 1$

$X = 1500$ MPa $\quad Y = 40$ MPa $\quad S = 68$ MPa

$X' = 1500$ MPa $\quad Y' = 246$ MPa $\quad m = \cos(\theta),\ n = \sin(\theta) \quad \bar{F}_{66} = \frac{1}{S^2}$

$\bar{F}_1 = \cos^2(\theta)\bar{F}_1 + \sin^2(\theta)\bar{F}_2,\ \bar{F}_1 = \frac{1}{X} - \frac{1}{X'},\ \bar{F}_2 = \frac{1}{Y} - \frac{1}{Y'},\ \bar{F}_{11} = \frac{1}{XX'},\ \bar{F}_{22} = \frac{1}{YY'},\ \bar{F}_{12} = \frac{-1}{2}\sqrt{\bar{F}_{11}\bar{F}_{22}}$

$\bar{F}_{11} = m^4\bar{F}_{11} + n^4\bar{F}_{22} + 2m^2n^2\bar{F}_{12} + 4m^2n^2\bar{F}_{66}$

$\therefore$ From code: $\boxed{\sigma_1 = -146.95 \text{ MPa}}$ $\quad \boxed{\sigma_1 = 123.21 \text{ MPa}}$

5) The rule of $0 < \nu_{xy} < 0.5$ is also used with orthotropic materials because the materials which make orthotropic materials are either isotropic or anisotropic, which follow the rule.