Prof. Marco Panesi
AE410/CSE461, Spring 2020

Final Project

Due Wednesday, May 6, 2020

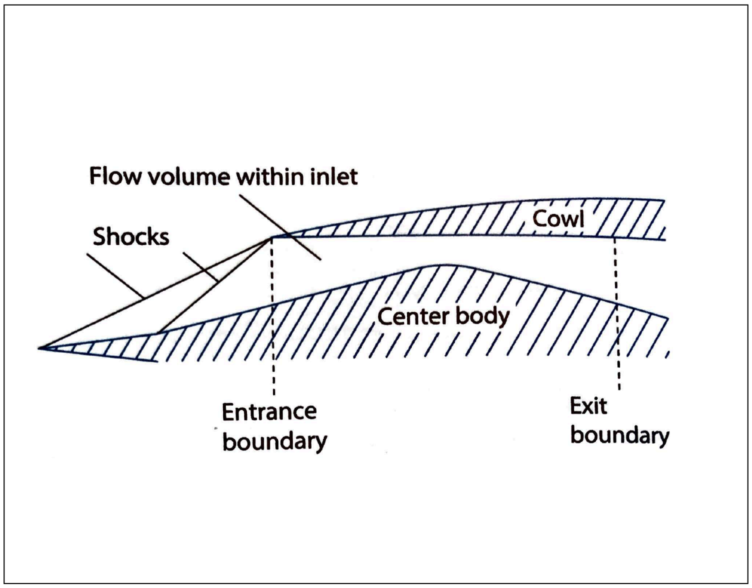

Figure 1: **Flow within a supersonic inlet**

Background:
The non-linear, quasi-one-dimensional Euler equations for the flow through a variable-area stream tube $A(x)$ are

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = -\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}x}\rho u \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + p) = -\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}x}\rho u^2 \tag{2}$$

$$\frac{\partial \rho E}{\partial t} + \frac{\partial}{\partial x}[(\rho E + p)u] = -\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}x}(\rho E + p)u \tag{3}$$

where ρ is the fluid density, u is the fluid velocity, p is the thermodynamic pressure, and

$$\rho E = \rho e + \frac{1}{2}\rho u^2$$

is the total energy density. The pressure is related to ρE by the equation of state

$$p = (\gamma - 1)\left(\rho E - \frac{1}{2}\rho u^2\right)$$

for a perfect gas with ratio of specific heats $\gamma = 1.4$.

Problem 1 Calculate the flow within a supersonic inlet shown in Figure (1). Solve the Euler equations written above for your convenience using Roe's method. At the entrance and the exit S(x) = 0.2 [m^2]. The lower surface of the channel is formed by two straight lines inclined at angle θ given by $\tan(\theta) = 0.25$, with their intersection at the channel midpoint rounded out with a radius equal to 0.5 [m]. See Figure (2). Use equally spaced volumes located along the line y=0 as shown in Figure (2), to span the flow volume.

(a) Develop a Finite Volume solver (in Matlab) to solve numerically the same problem with the following general program layout:

- Input parameters:
  - Length of the physical domain, $x \in [0, 1]$.
  - Number of volumes $N_x = 40$ (the number of nodes is 41), used to discretize the x-domain.
- Initial solution using the exact solution.
- Time Integrate (e.g., ode45 routine of Matlab)
  - Develop a general purpose Matlab function to compute the numerical flux functions using Roe scheme.
  - Compute ($\boldsymbol{F}_{i+\frac{1}{2}}$ - $\boldsymbol{F}_{i-\frac{1}{2}}$) / dx.
  - Implement two different sets of boundary conditions using the injection method:
    * Supersonic inlet on the left and subsonic outlet.
    * Supersonic inlet on the left and supersonic outlet.
  - Implement the source terms to account for the area change. Make sure you do **NOT** add to the source term to the boundaries.

(b) Four cases, each progressively more intensive, are given here. Run each case for 5 milliseconds ($t_{\text{Final}} = 5 \times 10^{-3}$ seconds).

1) *No Flow Case* – $M = 0$, total pressure p_{t0}, $= 10^6$ N/m^2, static temperature $T = 300$ [K] everywhere. The purpose of this Case is to check that in the early development of the code that no acceleration is produced within a zero pressure-gradient field.
2) *No Shock Wave Case* – $M = 2.5$, total pressure $p_{t0} = 10^6$ N/m^2, static temperature $T_0 = 300$ [K] at the *entrance*.
3) *Stationary Shock Wave Case* – Use the same conditions as in Case 2), but place a shock wave at $x = 0.85$ m.
4) *Moving Shock Case* – Start from the exact solution with the shock as in Case 2. Solve the problem at constant outlet pressure p_e at first, then increase the exit pressure by 67% ($p_e = 1.67\ p_e$) and thereafter hold it fixed again. The details of the boundary conditions are given hereafter:

$$\begin{cases} \frac{dp_e}{dt} = 0 & t < 10^{-3}\,\text{s} \\ \frac{dp_e}{dt} = \frac{\Delta p}{\Delta t} & 1 \times 10^{-3} < t < 1.1 \times 10^{-3}\,\text{s} \\ \frac{dp_e}{dt} = 0 & t > 1.1 \times 10^{-3}\,\text{s} \end{cases}$$

where Δp is ($0.67 \times p_e$) and Δt is (1×10^{-4})s.

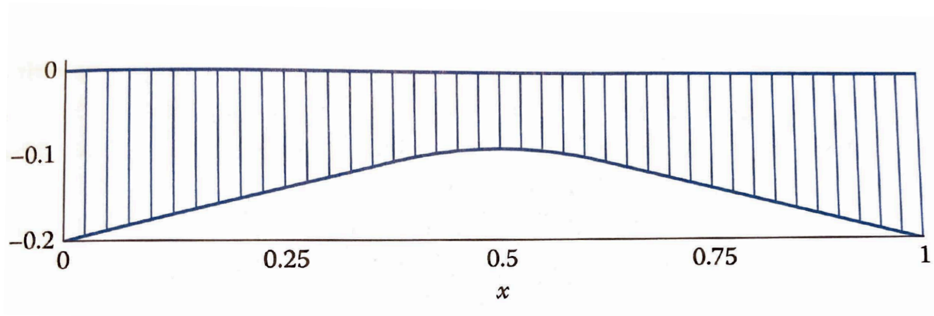

Figure 2: **Computational Mesh and Geometry. The computational domain is indicated by the presence of the volumes and it is bounded from the top by the horizontal wall, and from the bottom by the curved shape.**

Show the flow-field results for:

(a) the initial / **exact solutions** for Cases 1-3. Generate three plots, one for each case, and plot in the same graph: p/p_0, ρ/ρ_0 and u/u_0 in function of x for cases 1-3. Here the index sub 0 indicates the quantity evaluated in correspondence of the inlet. Comment what you see.

(b) the **numerical solutions** for Cases 1-2. Generate two separate plots for Case 1 and 2: for each plot show p/p_0 in function of x. Here the index sub 0 indicates the quantity evaluated in correspondence of the inlet. Comment what you see.

(c) the **numerical solutions** for Case 3. Generate two separate plots of the solution for Case 3 at different times: $[t_{\text{Initial}}, t_{\text{Intermediate}}, t_{\text{Final}}]$: i) plot p/p_0 in function of x. ii) plot the Mach number. Here the index sub 0 indicates the quantity evaluated in correspondence of the inlet. Select $t_{\text{Intermediate}}$ so that the solution sits approximately between t_{Initial} and t_{Final}. Comment what you see.

(d) the **numerical solutions** for Case 4. Generate one plot p/p_0 for Case 3 at time intervals of $\Delta t = 5 \times 10^{-4}$ seconds. Here the index sub 0 indicates the quantity evaluated in correspondence of the inlet. Comment what you see.