

AE353: Design Problem 04

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1 Goal

The code provided in `DesignProblem04` simulates a two-wheeled robot that is similar to the segway robotic mobility platform¹, which has been considered for use with the NASA robonaut (Figure 1). Actuators allow you to specify the torque applied to each wheel. Sensors allow you to measure the forward speed and the turning rate. Sensors also allow you to measure the turning radius of a road along which the robot can drive, as well as the error in lateral position and the error in heading relative to this road. The goal is to make the robot race along this road—without crashing—as fast as possible.

2 Model

2.1 Robot

The motion of the robot is governed by ordinary differential equations with the form

$$\begin{bmatrix} \ddot{\phi} \\ \dot{v} \\ \dot{w} \end{bmatrix} = f(\phi, \dot{\phi}, v, w, \tau_R, \tau_L) \quad (1)$$

where ϕ is the pitch angle of the chassis, $\dot{\phi}$ is the pitch angular velocity, v is the forward speed, w is the turning rate, and τ_R and τ_L are the torques applied by the chassis to the right and left wheel, respectively [1, 2]. The function f in (1) depends on a number of parameters (e.g., lengths, masses, moments of inertia). You can access a symbolic description of this function either within your controller code (`Controller.m`) as `parameters.symEOM.f`, or in a separate piece of code that you write for the purpose of control design. In particular, the



Figure 1: The NASA robonaut on top of a segway robotic mobility platform (<http://spaceflight.nasa.gov/gallery/images/station/eva/html/jsc2005e11678.html>).

¹See also <http://www.segwayrobotics.com>.

first time you run **DesignProblem04**, it will create a file that you can load to access the equations of motion. Here is an example:

```

1 % Load the equations of motion.
2 load('DesignProblem04_EOMs.mat');
3 % Parse the equations of motion.
4 f = symEOM.f;
5 % Define symbolic variables that appear in the equations of motion.
6 syms phi phidot v w tauR tauL real
7 % Proceed to linearize or do whatever else you like...
8 % (see: help sym/jacobian, help sym/diff, help sym/subs, etc.)

```

The simulator integrates three more equations of motion that relate the forward speed v and turning rate w to the position (x, y) and orientation θ of the robot:

$$\begin{aligned}\dot{x} &= v \cos \theta \\ \dot{y} &= v \sin \theta \\ \dot{\theta} &= w.\end{aligned}$$

However, for the purpose of control design to follow a road, it is probably more useful to keep track of the position and orientation *relative to the road* and not with respect to an absolute reference frame. In particular, suppose you know the radius of curvature r_{road} of the road along which the robot is currently moving (where $r_{\text{road}} > 0$ means turning left, $r_{\text{road}} < 0$ means turning right, $r_{\text{road}} = 0$ means turning in place, and $r_{\text{road}} = \infty$ means going straight). Then, given a speed v_{road} at which you would like to travel along the road, the turning rate w_{road} necessary to follow its centerline can be computed as

$$w_{\text{road}} = \frac{v_{\text{road}}}{r_{\text{road}}}. \quad (2)$$

Now, define e_{lateral} as the perpendicular distance from the centerline of the road to the position (x, y) of the robot (where $e_{\text{lateral}} > 0$ means being too far to the right and $e_{\text{lateral}} < 0$ means being too far to the left), and define e_{heading} as the difference between the orientation θ of the robot and the direction of the road. It is possible to show that

$$\begin{aligned}\dot{e}_{\text{lateral}} &= -v \sin(e_{\text{heading}}) \\ \dot{e}_{\text{heading}} &= w - \left(\frac{v \cos(e_{\text{heading}})}{v_{\text{road}} + w_{\text{road}} e_{\text{lateral}}} \right) w_{\text{road}}.\end{aligned} \quad (3)$$

A complete set of nonlinear equations of motion for the purpose of control design can be obtained from (1) and (3)—with states $e_{\text{lateral}}, e_{\text{heading}}, \phi, \dot{\phi}, v, w$ and inputs τ_R, τ_L —augmented as usual with a differential equation that describes $d(\phi)/dt$.

2.2 Road

The road is defined by its centerline, which has piecewise-constant curvature. You can create a new, random road (saved by default to the file '**road.mat**') by calling the function

MakeRoad. This road will be a sequence of circular arcs, placed one after another. The array `road.s` stores the length of each arc. The array `road.w` stores the curvature of each arc. You need not worry about these data structures (unless you want to hand-design your own roads, for the purpose of testing)—your controller will not have access to them. Instead, as described in Section 2.1, your controller will know the lateral error (e_{lateral}), the heading error (e_{heading}), and the radius of curvature (r_{road}) at the point on the road that is closest to the robot at any given time. This information is available in `sensors`, as usual.

2.3 Variables

This section provides extra information about how variables are named in `DesignProblem04`, to avoid confusion. The function f in (1) is provided in MATLAB as `parameters.symEOM.f`. The inputs to this function are:

- ϕ , the chassis angle, called `phi` in MATLAB
- $\dot{\phi}$, the time derivative of the chassis angle, called `phidot` in MATLAB
- v , the forward speed, called `v` in MATLAB
- w , the turning rate, called `w` in MATLAB
- τ_R and τ_L , the right and left motor torques, called `tauR` and `tauL` in MATLAB

You can initialize these variables as follows:

```
1 syms phi phidot v w tauR tauL real
```

The extra two equations of motion in (3) are not provided in MATLAB. You will have to define them. The extra variables in these equations are:

- e_{lateral} , the lateral error in following the road, called `e_lateral` in MATLAB
- e_{heading} , the heading error in following the road, called `e_heading` in MATLAB
- v_{road} and w_{road} , the forward speed and turning rate of a trajectory that would follow the road centerline, called `v_road` and `w_road` in MATLAB—remember that w_{road} can be computed as in (2), given a choice of v_{road} and given the radius of curvature r_{road} , called `r_road` in MATLAB

You can initialize these extra variables as follows:

```
1 syms e_lateral e_heading v_road w_road real
```

Note that v_{road} and w_{road} are not states—they are parameters on which your model might depend. Also note that you have choice over the names you use to describe variables in the equations of motion (3), since you'll be defining these equations yourself (they are not given to you in `DesignProblem04`). However, the names listed above are recommended.

3 Tasks

3.1 Analysis

The focus of your analysis this time will be on identifying, diagnosing, and eliminating failure modes. “Failure” for the robot is a crash—either running off the road or dropping the chassis. There are many reasons why a particular control design might lead to failure, on certain roads or in certain situations. Your job as a control engineer is to uncover and address these sources of failure. At minimum, you should consider at least three different control designs, and for each design, you should do the following:

- Identify at least one road that causes failure.

(Remember—a failed experiment is a blessing! Then you can address the source of the failure. There are always failure modes, and sometimes these are very hard to find.)

- Say why the failure occurred, providing evidence to support your argument.

(Was there a large error in the state estimate? Did your control design exceed the maximum torque in each wheel? Is there a limit on how tight a turn your control design can handle, and can you quantify this limit? Thinking of the road as a source of disturbance in the closed-loop system, do failures have something to do with characteristics of the frequency response? These are examples of questions you might ask when thinking about causes of failure.)

- Suggest a change to the control design that would eliminate the failure, and verify in simulation that it does.

(Sometimes, it is impossible to eliminate a failure mode. If you believe that this is the case, and that you have reached a fundamental limit in the performance of the robot, provide evidence for this claim.)

Remember that you have a lot of practice in doing rigorous data collection, analysis, and visualization—this was the focus of the third design problem, and can help a lot in identifying and diagnosing failure modes.

3.2 Presentation

The focus of your presentation this time will be on citations. For example, suppose you want to quote a result that appears in the first edition of the textbook “Feedback Systems: An Introduction for Scientists and Engineers,” which was written by Karl Aström and Richard Murray. To do this in L^AT_EX you would do the following:

- Create a file called `references.bib` and add it, for example, to the list of files in your overleaf project (you’ll note that a “bib file” like this one is being used to generate this document—you can use it as a template).
- Add the following text to this file:

```
@book{Astrom2010,
  title={Feedback Systems: An Introduction for Scientists and Engineers},
  author={K. J. Astr{\o}m and R. M. Murray},
  isbn={9781400828739},
  edition={First},
  url={http://www.cds.caltech.edu/~murray/amwiki/index.php/First_Edition},
  year={2010},
  publisher={Princeton University Press}
}
```

- Add the following text in your main document, where you want to cite the book:

```
\cite{Astrom2010}
```

For example, you may want to say something like the following:

```
Robustness of the closed-loop system to time delay was verified
by analysis of the Nyquist plot \cite{Astrom2010}.
```

- Add the following text to your main document, just before the call to `\end{document}`:

```
% Display list of references in IEEE format.
\bibliographystyle{IEEEtran}
\bibliography{IEEEabrv,references}
```

- Make sure the following line appears near the top of your main document (it makes citations look nicer):

```
\usepackage{cite}
```

When you compile your document, all calls to `\cite{...}` will be replaced by numbered references [...], which correspond to items in the list that will appear at the end of your document. For example, here is a citation to Aström and Murray, exactly as described [3]. There is a lot more information online about how to create, manage, and use bibliographies with latex documents². Do not hesitate to ask for help. At minimum, you should include at least one citation (other than to Aström and Murray) in your report.

3.3 Process

Your report will include the following four parts:

- Define requirements and verifications (at least one of each).
- Derive a model (i.e., linearize (1) and express your result in state-space form).
- Design a controller and an observer (remembering to check, first, that your linearized model is both controllable and observable).
- Implement and test your control design in simulation. Follow the instructions you wrote to verify that requirements are satisfied. Identify, diagnose, and eliminate at least one source of failure in at least three different designs, as described in Section 3.1.

As usual, you are encouraged to go beyond these requirements.

²For example: <https://www.overleaf.com/help/97-how-to-include-a-bibliography-using-bibtex>.

4 Contest

There will be an opportunity to race against your friends in a friendly competition on the last day of class (December 12). The code that will be used to run this race is `DesignProblem04Race.m`. The key difference between this and `DesignProblem04.m` is that, rather than requiring you to specify a single controller, the race code requires you to specify the name of a folder in which you can put any number of controllers. A robot will be created for each controller in the folder—all will race at the same time. To test your controller in race conditions, do the following:

- Put the controllers you want to test in the folder `DesignProblem04/racers`.
- Open MATLAB and make `DesignProblem04` your working directory.
- Seed the random number generator:

```
rng('shuffle');
```

- Make a road:

```
MakeRoad('road.mat');
```

- Run the simulation (there are some optional parameters to play with, if you like):

```
DesignProblem04Race('racers','road.mat');
```

- If desired, show the results:

```
ShowResults('racers/results.mat');
```

There are three small but important differences in your controllers for the race:

- The race code enforces a bound on computation time. If your “`runControlSystem`” function takes more than 0.02 seconds on my laptop to execute on five separate occasions, your controller will be turned off (disqualified). Note that my computer will likely run your code at least as fast as your computer, but please err on the safe side.
- The race code enforces a strict prohibition on printing anything to the command window (e.g., the result of forgetting to put a semicolon at the end of a line). If either your “`initControlSystem`” function or “`runControlSystem`” function prints anything to the command window, your controller will be turned off (disqualified). Please check carefully that it does not!
- The race code allows each controller to—if desired—specify its own value of “`iDelay`” (the number of time steps that sensor data will be delayed). Look at the example `racers/netid_controller.m` to see how this is done. If you make no change to your controller code, that’s fine—the default is `iDelay = 0`. If you choose to change your controller code to specify a non-zero delay, then your total time will be reduced by a number of seconds equal to $2 * iDelay$ (so, if `iDelay = 10`, then your time will start at -20 seconds).

To compete in the race, you must submit your code by the deadline (see below). All submitted controllers will race in a qualifying round. The top 9 finishers will advance to the semifinals (in 3 groups of 3). The top finishers in each semifinal round will advance to the finals (in 1 group of 3). The qualifying round will be done before class (on December 12)—the results will be revealed at the start of class. The semifinal and final round will be done in class. The winner of the final round will receive a prize. You **must** be present in class to compete in the semifinal/final round and to receive a prize.

Note that this race is just for fun. Although you are required to submit your code to the race, the results have no impact at all on the assessment of your fourth design report.

5 Deliverables

You must submit three things by 11:59PM on Tuesday, December 11:

- Code. This code will be written in MATLAB, using the template `Controller.m`. Please specify how to run your code (e.g., `DesignProblem04('Controller')`).
- Code for the race. This code will be written in MATLAB, using the template `racers/netid_controller.m`.
- Report. This report will be written in L^AT_EX. You will submit both a PDF document and the L^AT_EX source files that you used to produce this document. The PDF document must be exactly four pages, and is expected to include a method of approach and a description of results. *Your report must conclude with a section titled “Acknowledgements” that lists the colleagues with whom you worked (if any) and describes the nature of your collaboration (discussion, sharing code, etc.).*

You must also meet intermediate deadlines, the details of which will be posted to reddit. *Late submissions of either draft reports or the final report will not be accepted.*

6 Evaluation

Your work will be evaluated based on submission of race code that runs without being disqualified (10%), on completion of the required tasks (40%), on correctness of your code (10%), on presentation of results in your report (30%), and on meeting intermediate deadlines (10%). Our evaluation of your technical approach will place special emphasis on analysis of failure. Our evaluation of your report will place special emphasis on the use of citations—we will not look at style, grammar, or any other aspect of the text other than citations, as long as there is no barrier to understanding your work.

References

- [1] Y. X. Mak, “Realization of mini segway robot using ni myrio,” Essay (Bachelor), University of Twente, 2015. [Online]. Available: <http://purl.utwente.nl/essays/67004>

- [2] Z. Tobias and F. Matthias, “Tracking control of a balancing robot—a model-based approach,” *Archive of Mechanical Engineering*, vol. 61, no. 2, pp. 331–346, Apr. 2014. [Online]. Available: <http://dx.doi.org/10.2478/meceng-2014-0019>
- [3] K. J. Aström and R. M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 1st ed. Princeton University Press, 2010. [Online]. Available: http://www.cds.caltech.edu/~murray/amwiki/index.php/First_Edition