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AE461 - Prelab 6

4/8/2020

1) a) Find Bounds of E_x, E_y, ν_{xy}, and G_{xy}

$V_f = 0.60 \pm 0.05 \rightarrow V_f = \begin{cases} 0.65 \\ 0.55 \end{cases}, \quad V_m = \begin{cases} 0.35 \\ 0.45 \end{cases}$

$E_x = V_f E_f + V_m E_m, \quad \frac{1}{E_y} = \frac{V_f}{E_f} + \frac{V_m}{E_m}, \quad \nu_{xy} = \nu_f V_f + \nu_m V_m, \quad \frac{1}{G_{xy}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$

using upper bound of V_f:

$E_x = 0.65 \cdot 120 + 0.35 \cdot 2.4 = 78.84 \text{ GPa}$

$\frac{1}{E_y} = \frac{0.65}{120} + \frac{0.35}{2.4} \longrightarrow E_y = 6.61157 \text{ GPa}$

$\nu_{xy} = 0.65 \cdot 0.32 + 0.35 \cdot 0.35 = 0.3305$

$G_f = \frac{E_f}{2(1+\nu_f)} = \frac{120}{2(1+0.32)} = 45.4545 \text{ GPa}$

$G_m = \frac{E_m}{2(1+\nu_m)} = \frac{2.4}{2(1+0.35)} = 0.8889 \text{ GPa}$

$\frac{1}{G_{xy}} = \frac{0.65}{45.4545} + \frac{0.35}{0.8889} \rightarrow G_{xy} = 0.40805 \text{ GPa}$

using lower bound of V_f:

$E_x = 0.55 \cdot 120 + 0.45 \cdot 2.4 = 67.08 \text{ GPa}$

$E_y = \frac{1}{\frac{0.55}{120} + \frac{0.45}{2.4}} = 5.20607 \text{ GPa}$

$\nu_{xy} = 0.55 \cdot 0.32 + 0.45 \cdot 0.35 = 0.3335$

$G_{xy} = \frac{1}{\frac{0.55}{45.4545} + \frac{0.45}{0.8889}} = 0.51835 \text{ GPa}$

$\therefore \boxed{E_x = \begin{cases} 78.84 \text{ GPa} \\ 67.08 \text{ GPa} \end{cases}, \quad E_y = \begin{cases} 6.61157 \text{ GPa} \\ 5.20607 \text{ GPa} \end{cases}, \quad \nu_{xy} = \begin{cases} 0.3305 \\ 0.3335 \end{cases}, \quad G_{xy} = \begin{cases} 0.40805 \text{ GPa} \\ 0.51835 \text{ GPa} \end{cases}}$

b) Determine average of E_x, E_y, ν_{xy}, and G_{xy}, also St dev

$\bar{E}_x = \frac{78.84 + 67.08}{2} = 72.96 \text{ GPa}$

$\bar{E}_y = \frac{6.61157 + 5.20607}{2} = 5.90882 \text{ GPa}$

$\bar{\nu}_{xy} = \frac{0.3305 + 0.3335}{2} = 0.332$

$\bar{G}_{xy} = \frac{0.40805 + 0.51835}{2} = 0.4264 \text{ GPa}$

$\rightarrow \therefore \boxed{\begin{aligned} \bar{E}_x &= 72.96 \text{ GPa} \\ \bar{E}_y &= 5.90882 \text{ GPa} \\ \bar{\nu}_{xy} &= 0.3320 \\ \bar{G}_{xy} &= 0.4264 \text{ GPa} \end{aligned}}$

$\sigma_{std} \approx \frac{max - min}{4}$

$\sigma_{E_x} = \frac{78.84 - 67.08}{4} = 2.94 \text{ GPa}$

$\sigma_{E_y} = \frac{6.61157 - 5.20607}{4} = 0.351 \text{ GPa}$

$\sigma_{\nu_{xy}} = \frac{0.3335 - 0.3305}{4} = 0.00075$

$\sigma_{G_{xy}} = \frac{0.51835 - 0.40805}{4} = 0.027575 \text{ GPa}$

$\rightarrow \therefore \boxed{\begin{aligned} \sigma_{E_x} &= 2.94 \text{ GPa} \\ \sigma_{E_y} &= 0.351 \text{ GPa} \\ \sigma_{\nu_{xy}} &= 0.00075 \\ \sigma_{G_{xy}} &= 0.0276 \text{ GPa} \end{aligned}}$

See code

2) a) Q_{11} @ $\theta = 0° = 133.0027$ GPa

$\frac{1}{2} Q_{11}$ @ $\theta = 0° = 66.5013$ GPa → $\theta = 36.3276°$

$\frac{1}{3} Q_{11}$ @ $\theta = 0° = 44.3342$ GPa → $\theta = 45.4365°$

b) $Q_{11} \cdot 90\% = 119.7024$ GPa → $\theta = 15.0315°$

↳ See plot attached @ end.

3) See code: $Q_{16\,max} = 54.2471$ GPa, → $\theta = 30.2860°$

a) No, θ @ $Q_{16\,max}$ does NOT depend on material properties

b) Q_{16} is the stiffness in the XZ plane

Q_{26} is the stiffness in the YZ plane

c) Yes, isotropic materials have equal material properties in all directions meaning there is a full Q matrix with non-zero Q_{16} and Q_{26} values

4) $F_1\sigma_1 + \cancel{F_2\sigma_2}^{0} + F_{11}\sigma_1^2 + \cancel{F_{22}\sigma_2^2}^{0} + \cancel{F_{66}\tau^2}^{0} + 2\cancel{F_{12}\sigma_1\sigma_2}^{0} = 1$

$X = 1500$ MPa $\quad Y = 40$ MPa $\quad S = 68$ MPa

$X' = 1500$ MPa $\quad Y' = 246$ MPa $\quad m = \cos(\theta), n = \sin(\theta) \quad \bar{F}_{66} = \frac{1}{S^2}$

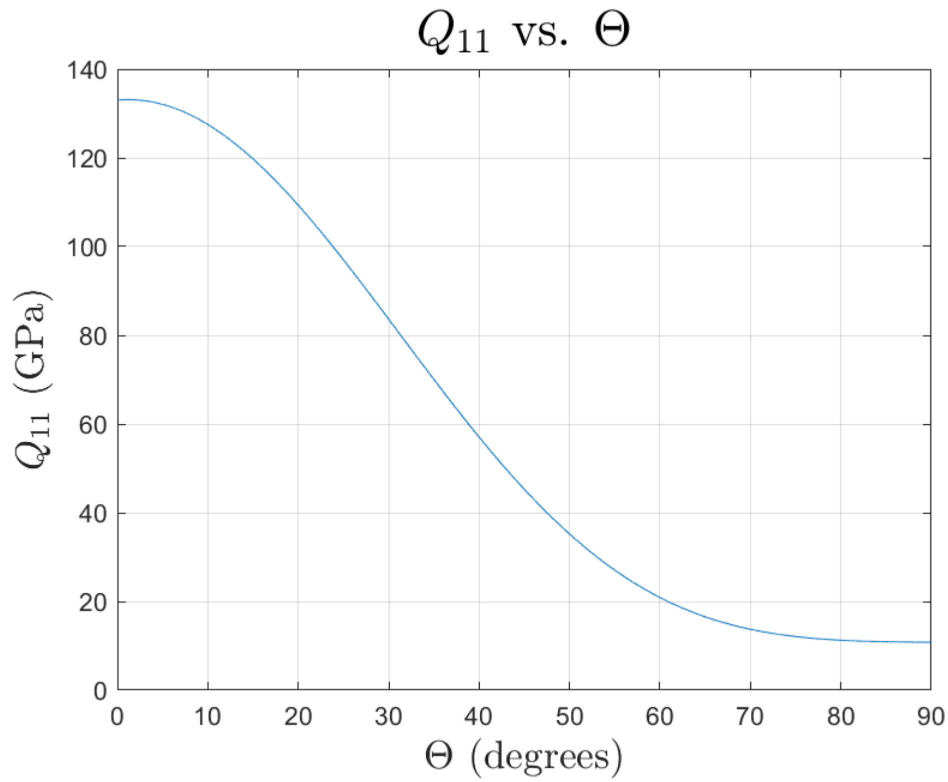
$F_1 = \cos^2(\theta)\bar{F}_1 + \sin^2(\theta)\bar{F}_2, \bar{F}_1 = \frac{1}{X} - \frac{1}{X'}, \bar{F}_2 = \frac{1}{Y} - \frac{1}{Y'}, \bar{F}_{11} = \frac{1}{XX'}, \bar{F}_{22} = \frac{1}{YY'}, \bar{F}_{12} = \frac{-1}{2}\sqrt{\bar{F}_{11}\bar{F}_{22}}$

$F_{11} = m^4\bar{F}_{11} + n^4\bar{F}_{22} + 2m^2n^2\bar{F}_{12} + 4m^2n^2\bar{F}_{66}$

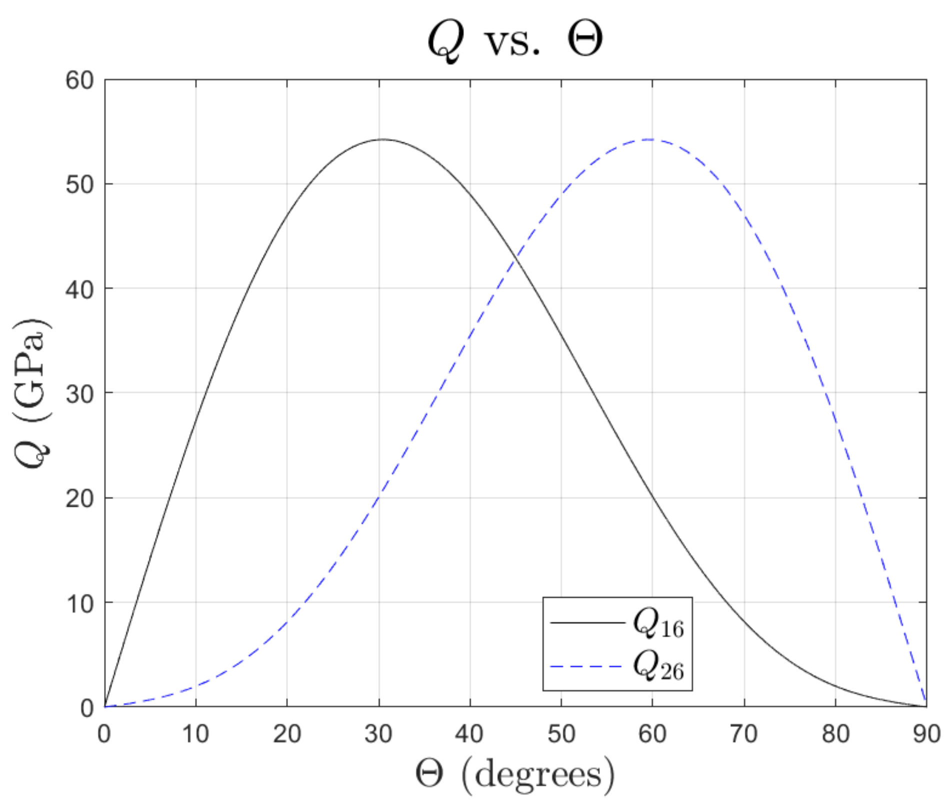
∴ from calc: $\boxed{\begin{matrix}\sigma_1 = -148.95 \text{ MPa} \\ \sigma_1 = 123.21 \text{ MPa}\end{matrix}}$

5) The rule of $0 < \nu_{xy} \leq 0.5$ is also used with orthotropic materials because the materials which make orthotropic materials are either isotropic or anisotropic, which follow the rule.

Problem 2 Plot:



Problem 3 Plot:



```
%% Prelab 6
clc;
clear all;
close all;

%% Problem 2
% For T300/5208 graphite/epoxy composite, construct a plot of Q_11 versus
% laminate orientation angle theta
Ex = 132.38;     % GPa
Ey = 10.76;      % GPa
Gxy = 5.65;      % GPa
vxy = 0.24;
c = (1-vxy^2*(Ey/Ex))^-1;
Qm11 = c*Ex;
Qm22 = c*Ey;
Qm12 = c*vxy*Ey;
Qm66 = Gxy;

theta = linspace(0, pi/2, 10000);
Q_11 = zeros(1, 10000);
theta_d = zeros(1, 10000);
figure(1)
for i = 1:length(theta)
    m = cos(theta(i));
    n = sin(theta(i));
    Q_11_i = m^4*Qm11 + n^4*Qm22 + 2*m^2*n*2*Qm12 + 4*m^2*n^2*Qm66;
    Q_11(1,i) = Q_11_i;
    theta_d(:,i) = 180/pi * theta(i);
end
plot(theta_d, Q_11)
grid on
xlabel('$$\Theta$$ (degrees)', 'interpreter', 'latex', 'fontsize', 16);
ylabel('$$Q_{11}$$ (GPa)', 'interpreter', 'latex', 'fontsize', 16)
title('$$Q_{11}$$ vs. $$\Theta$$', 'interpreter', 'latex', 'fontsize', 20)
saveas(1, 'Problem2.png')

%a: For what theta will the composite longitudinal stiffness be at half the
%theta = 0 degrees value? 1/3 the value?
Q_11_0 = Q_11(1,1);
Q_11_0_half = Q_11_0/2;
Q_11_0_third = Q_11_0/3;
half = false; third = false;
theta_half = 0; theta_third = 0;
for i = 1:length(Q_11)
    Q = Q_11(1,i);
    if half == false
        if Q <= Q_11_0_half
            theta_half = theta_d(i)
            half = true;
        end
    end
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    if third == false
        if Q <= Q_11_0_third
            theta_third = theta_d(i)
            third = true;
        end
    end
end

%b: For what value of theta with the composite longitudinal stiffness drop
%by 10%?
Q_11_0_90 = Q_11_0 * .9;
ninety = false; theta_ninety = 0;
for i = 1:length(Q_11)
    Q = Q_11(1,i);
    if ninety == false
        if Q <= Q_11_0_90
            theta_ninety = theta_d(i)
            ninety = true;
        end
    end
end

%% Problem 3
% What lamina orientation angle, theta, if any, yields a maximum value of
% Q_16 for T300/5208 graphite/epoxy?
Ex = 181;    % GPa
Ey = 10.3;     % GPa
Gxy = 7.17;     % GPa
vxy = 0.28;
c = (1-vxy^2*(Ey/Ex))^-1;
Qm11 = c*Ex;
Qm22 = c*Ey;
Qm12 = c*vxy*Ey;
Qm66 = Gxy;

theta = linspace(0, pi/2, 10000);
Q_16 = zeros(1, 10000);
Q_26 = zeros(1, 10000);
theta_d = zeros(1, 10000);
figure(2)
for i = 1:length(theta)
    m = cos(theta(i));
    n = sin(theta(i));
    Q_16_i = m^3*n*Qm11 - m*n^3*Qm22 + (m*n^3-m^3*n)*Qm12 + 2*(m*n^3-m^3*n)*Qm66;
    Q_16(1,i) = Q_16_i;
    Q_26_i = m*n^3*Qm11 - m^3*n*Qm22 + (-m*n^3+m^3*n)*Qm12 + 2*(-m*n^3+m^3*n)*Qm66;
    Q_26(1,i) = Q_26_i;
    theta_d(:,i) = 180/pi * theta(i);
end
plot(theta_d, Q_16, 'k'); hold on;
plot(theta_d, Q_26, 'b--');
```

```
grid on
legend({'$$Q_{16}$$', '$$Q_{26}$$'}, 'interpreter', 'latex', 'location', 'best',↙
'fontsize', 14)
xlabel('$$\Theta$$ (degrees)', 'interpreter', 'latex', 'fontsize', 16);
ylabel('$$Q$$ (GPa)', 'interpreter', 'latex', 'fontsize', 16)
title('$$Q$$ vs. $$\Theta$$', 'interpreter', 'latex', 'fontsize', 20)
saveas(2, 'Problem3.png')

% Find max values of Q16 and the associated theta value
Q16_max = max(Q_16)
Q16_max_index = find(Q_16==Q16_max);
Q16_max_theta = theta_d(1, Q16_max_index)

%% Problem 4
% From table F.2
X = 1500;
Xp = 1500;
Y = 40;
Yp = 246;
S = 68;
m = cos(15*pi/180);
n = sin(15*pi/180);

F1_l = 1/X - 1/Xp;
F2_l = 1/Y - 1/Yp;
F11_l = 1/(X*Xp);
F22_l = 1/(Y*Yp);
F12_l = -.5*sqrt(F11_l*F22_l);
F66_l = 1/(S^2);

F1 = n^2 * F2_l;
F11 = m^4*F11_l + n^4*F22_l + 2*m^2*n^2*F12_l + 4*m^2*n^2*F66_l;

% Solve for failure stress for compressive and tensile loads
syms o real
eq1 = 1 == F1*o + F11*o^2;
o_solve = vpa(solve(eq1, o), 8)
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