AE353: Design Problem 01

[redacted]

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1 Goal

The code provided in `DesignProblem01` simulates the rotational motion of a spacecraft. The goal of this project is to control the spacecraft to a specific chosen angular velocity given that it starts with a random angular velocity. Furthermore, the spacecraft has the ability to apply torque about two different axes. It is desired that the spacecraft rotate about two different axes, with one of the axis rotations to remain facing towards the Earth. Thus, the goal is to achieve an angular velocity w_e given below:

$$w_e = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix} rad/s$$

2 Model

The rotational motion of the spacecraft, if it is modeled as a single rigid body, is governed by the ordinary differential equations

$$\begin{aligned} \tau_1 &= J_1\dot{w}_1 - (J_2 - J_3)w_2w_3 \\ 0 &= J_2\dot{w}_2 - (J_3 - J_1)w_3w_1 \\ \tau_3 &= J_3\dot{w}_3 - (J_1 - J_2)w_1w_2, \end{aligned}$$

where w_1, w_2, w_3 are the components of angular velocity, J_1, J_2, J_3 are the principle moments of inertia, and τ_1, τ_3 are the two different torques that can be applied.

Now, rearranging the following equations to express them in terms of $\dot{w}_1, \dot{w}_2$ and $\dot{w}_3$

$$\dot{w}_1 = ((J_2 - J_3)/J_1)(w_2w_3) \tag{1}$$
$$\dot{w}_2 = ((J_2 - J_3)/J_1)(w_3w_1) + \tau_2/J_2 \tag{2}$$
$$\dot{w}_3 = ((J_1 - J_2)/J_3)(w_1w_2) + \tau_3/J_3 \tag{3}$$

Based on the value of w_e, we can calculate the value for τ_e using the equations for $\dot{w}$ above. This can be done by setting $\dot{w}$ to zero and plugging in the desired values for w_e. The computed τ_e is listed below.

$$\tau_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4}$$

The state x and input u for this system can be represented as:

$$x = \begin{bmatrix} w_1 - w_{e1} \\ w_2 - w_{e2} \\ w_3 - w_{e3} \end{bmatrix} \; u = \begin{bmatrix} \tau_1 - \tau_{e1} \\ \tau_3 - \tau_{e3} \end{bmatrix} \tag{5}$$

After linearizing the system by using the Jacobian of the given sets of equation with respect to w, the subsequent A, B, and C matrices are obtained and given below:

$$A = \begin{bmatrix} 0 & w_3(J_2 - J_3/J_1) & w_2(J_2 - J_3/J_1) \\ w_3(J_3 - J_1/J_2) & 0 & w_1(J_3 - J_1/J_2) \\ w_2(J_1 - J_2/J_3) & w_1(J_1 - J_2/J_3) & 0 \end{bmatrix} \; B = \begin{bmatrix} 1/J_1 & 0 \\ 0 & 0 \\ 0 & 1/J_3 \end{bmatrix} \tag{6}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

The linearized model is presented below:

$$\dot{x} = Ax + Bu \tag{8}$$
$$y = Cx \tag{9}$$

3 Application of Zero Input

If we apply the input $u = 0$, the resulting system is:

$$\dot{x} = Ax \tag{10}$$

Running the controller.m file on MATLAB and obtaining the J values of the satellite, the following A and B matrices can be calculated:

$$A = \begin{bmatrix} 0 & 8.6154 & 0 \\ -9.3208 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; B = \begin{bmatrix} 9.3208 & 0 \\ 0 & 0 \\ 0 & 3.1830 \end{bmatrix} \tag{11}$$

The computed real parts of the eigenvalues of the A matrix are $s_1 = 0$, $s_2 = 0$, $s_3 = 0$. Not all of the real part of the eigenvalues are negative, and thus it can be predicted that that the system is not asymptotically stable.

The plot of the zero input system is provided at the end of the section.

The plotted results confirm that the zero input system is indeed asymptotically unstable.

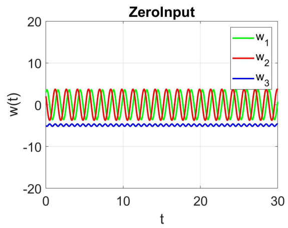

Figure 1: Application of Zero Input

4 Application of State Feedback

Applying state feedback means we apply input $u = -Kx$. The resulting system is:

$$\dot{x} = (A - BK)x \tag{12}$$

The gain matrix K selected is shown below:

$$K = \begin{bmatrix} 0.6498 & -0.2284 & -0.3736 \\ -0.45 & 0 & 0.6284 \end{bmatrix} \tag{13}$$

The real part of the eigenvalues of $A - BK$ are $s_1 = -2.9432$, $s_2 = -2.9432$, $s_3 = -2.1137$. All the eigenvalues have negative real parts, so it can be predicted that the system is asymptotically stable. This prediction was verified by implementing the controller with state feedback in MATLAB and plotting the results. The results are plotted below:

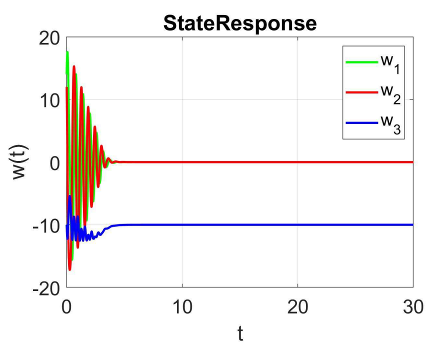

Figure 2: Application of State Feedback

The plotted results shows that w stabilizes and converges, albeit with some small steady-state error. This thus proves that the state feedback system is asymptotically stable.

5 Initial Conditions and Resulting Motion

The variation of initial conditions affect the initial resulting motion. One conclusion that can be drawn from the plots below is that the angular velocity oscillates in extreme frequency until it is able to stabilize with state response. Furthermore, the further the initial conditions vary from the desired angular velocities, the longer it takes the satellite to stabilize. However, despite how much initial conditions may vary from desired values, the steady state resulting motion is always stable.

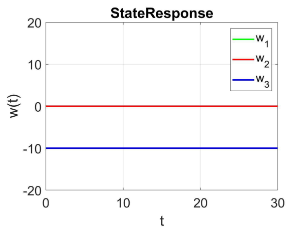

Figure 3: Initial conditions $w_i = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$

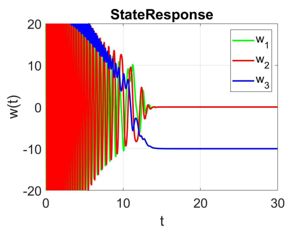

Figure 4: Initial conditions $w_i = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix}$

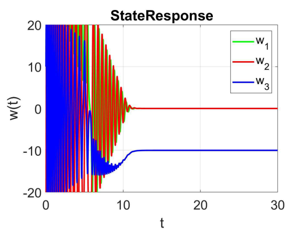

Figure 5: Initial conditions $w_i = \begin{bmatrix} 50 \\ -30 \\ 10 \end{bmatrix}$