AE353: Design Problem 01

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January 29, 2019

1 Goal

The code provided in `DesignProblem01` simulates the rotational motion of a spacecraft. This spacecraft has actuators that can apply torque about two different axes. This spacecraft also has sensors—an inertial measurement unit (IMU)—to measure its angular velocity. The spacecraft starts with some random angular velocity. The goal is to achieve an angular velocity of $w = M = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ rad/s.

2 Model

The rotational motion of the spacecraft, if it is modeled as a single rigid body, is governed by the ordinary differential equations

$$\begin{aligned} \tau_1 &= J_1 \dot{w}_1 - (J_2 - J_3) w_2 w_3 \\ 0 &= J_2 \dot{w}_2 - (J_3 - J_1) w_3 w_1 \\ \tau_3 &= J_3 \dot{w}_3 - (J_1 - J_2) w_1 w_2, \end{aligned}$$

where w_1, w_2, w_3 are the components of angular velocity, J_1, J_2, J_3 are the principle moments of inertia, and τ_1, τ_3 are the two different torques that can be applied.

3 State Space Model Linearization

In order to determine the linear model, we first define the State, x, Input, u, and Output, y. Our goal is to simplify our equations of motion to the following:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

To begin, we define the state, input, and output as the following:

$$x = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}, \quad u = \begin{bmatrix} \tau_1 \\ 0 \\ \tau_3 \end{bmatrix}, \quad y = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Gathering all this information, we move to MatLab to utilize Jacobian() in order to find A, B, C, and D for the state space model above.

$$A = \begin{bmatrix} 0 & (\omega_3 \times \frac{J_2 - J_3}{J_1}) & (\omega_2 \times \frac{J_2 - J_3}{J_1}) \\ -(\omega_3 \times \frac{J_1 - J_3}{J_2}) & 0 & -(\omega_1 \times \frac{J_1 - J_3}{J_2}) \\ (\omega_2 \times \frac{J_1 - J_2}{J_3}) & (\omega_1 \times \frac{J_1 - J_2}{J_3}) & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{J_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{J_3} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$