AE353: Design Problem 02

Controlling Robotics

February 28, 2019

1 Goal

The code provided in `DesignProblem02` simulates a "gravity-assisted underactuated robot arm." This robot arm has two joints, but only one motor. The second joint spins freely. Optical encoders measure joint angles and joint velocities. The goal is to make the tip of the robot arm move to a sequence of points in space. Equivalently, the goal is to make the second joint angle track a piecewise-constant reference trajectory.

2 Model

The motion of the robot is governed by ordinary differential equations with the form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q,\dot{q}) = \tau \text{ where,} \tag{1}$$

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \dot{q} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \tau = \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix}$$

q is a matrix of joint angles, $\dot{q}$ is a matrix of joint velocities, τ is a matrix of applied torques, and $M(q), C(q,\dot{q}), N(q,\dot{q})$, are matrix-valued functions of q and/or $\dot{q}$. These functions depend on a number of parameters.

3 Linearization

$$\dot{x} = Ax + Bu, \;\; y = Cx + Du \tag{2}$$

In order to linearize the system, the `Jacobian()` function will be utilized to find the coefficients for the state space system, equation(2), A, B, C, D. Due to the large space **A** and **B** takes up, the code for linearizing the system by use of `Jacobian()` will be displayed below as a temporary measure (EDIT!):

$$\mathbf{C} = [y]_{(q)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \;\; \mathbf{D} = [y]_{(u)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

```
clear all, clc

load('DesignProblem02_EOMs.mat')

M = symEOM.M;
C = symEOM.C;
N = symEOM.N;
tau = symEOM.tau;

syms q1 q2 qD qDD v1 v2 tau1 tau2 a1 a2 real
qD = [v1; v2];
qDD = [a1; a2];
u = [tau1; tau2];
y = [v1; v2];
eqn = u == M*qDD + C*qD + N;
qDD = [solve(eqn, qDD)];
qDD = [qDD.a1; qDD.a2];

A = vpa(jacobian(qDD, qD),3)
B = vpa(jacobian(qDD, u),3)
C = vpa(jacobian(y, qD),3)
D = jacobian(y, u)
```

4 Requirements

A *requirement* is a property that the system must have in order to solve the problem. As the goal is to control the tip of the robotic arm, it is important to provide a quantifiable and detailed requirement to ensure precision.

The second joint angle, q_2 shall reach within $\pm 2^\circ$ of the desired constant reference value of $\theta = 60^\circ$ in under 5 seconds and shall hold for a duration of at least 15 seconds.

5 Verification

A *verification* is a test that is performed to verify the system meets the requirements set in section (4).

`DesignProblem02('Controller', 'datafile', 'data.mat')` will be run $i = \mathbf{5}$ times with randomly generated initial values to check whether the requirements are satisfied under different initial conditions. The data points generated by the `'datafile', 'data.mat'` will then be imported into a dedicated verification script which will calculate the error between the second joint angle, q_2, and reference value, θ, at each time-step value of $\Delta t = 0.1[s]$ by the following computation:

$$E = \max E_i = |q_2 - \theta|, \text{ s.t. } \theta = 60^\circ \text{ for } i \in [1, 5]$$

If $E \leq 2°$, then the requirement is satisfied. The error from each iteration will be saved to provide an average E value in order to provide a more detailed understanding of the simulation.