The Axiomatic System: Definition & Properties

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In mathematics, the axiomatic system refers to the statements and rules used to develop and prove theorems. Explore the definition and properties of the axiomatic system, including consistency, independence, and completeness. Understand how an axiom compares to an axiomatic system.

The Axiomatic System

What exactly is an axiomatic system? I know it sounds like a big word for a complicated system, but it's actually not all that complicated. Defined, an **axiomatic system** is a set of axioms used to derive theorems. What this means is that for every theorem in math, there exists an axiomatic system that contains all the axioms needed to prove that theorem. An **axiom** is a statement that is considered true and does not require a proof. It is considered the starting point of reasoning. Axioms are used to prove other statements. They are basic truths. For example, the statement that all right angles are equal to each other is an axiom and does not require a proof. We know that all right angles are equal to each other and we do not argue that point. Instead, we use this information to prove other things. A collection of these basic, true statements forms an axiomatic system.

The subject that you are studying right now, geometry, is actually based on an axiomatic system known as Euclidean geometry. This system has only five axioms or basic truths that form the basis for all the theorems that you are learning. Everything can be traced back to these five axioms. What are they? Let me tell you.

1. A straight line can be drawn from any one point to any other point.

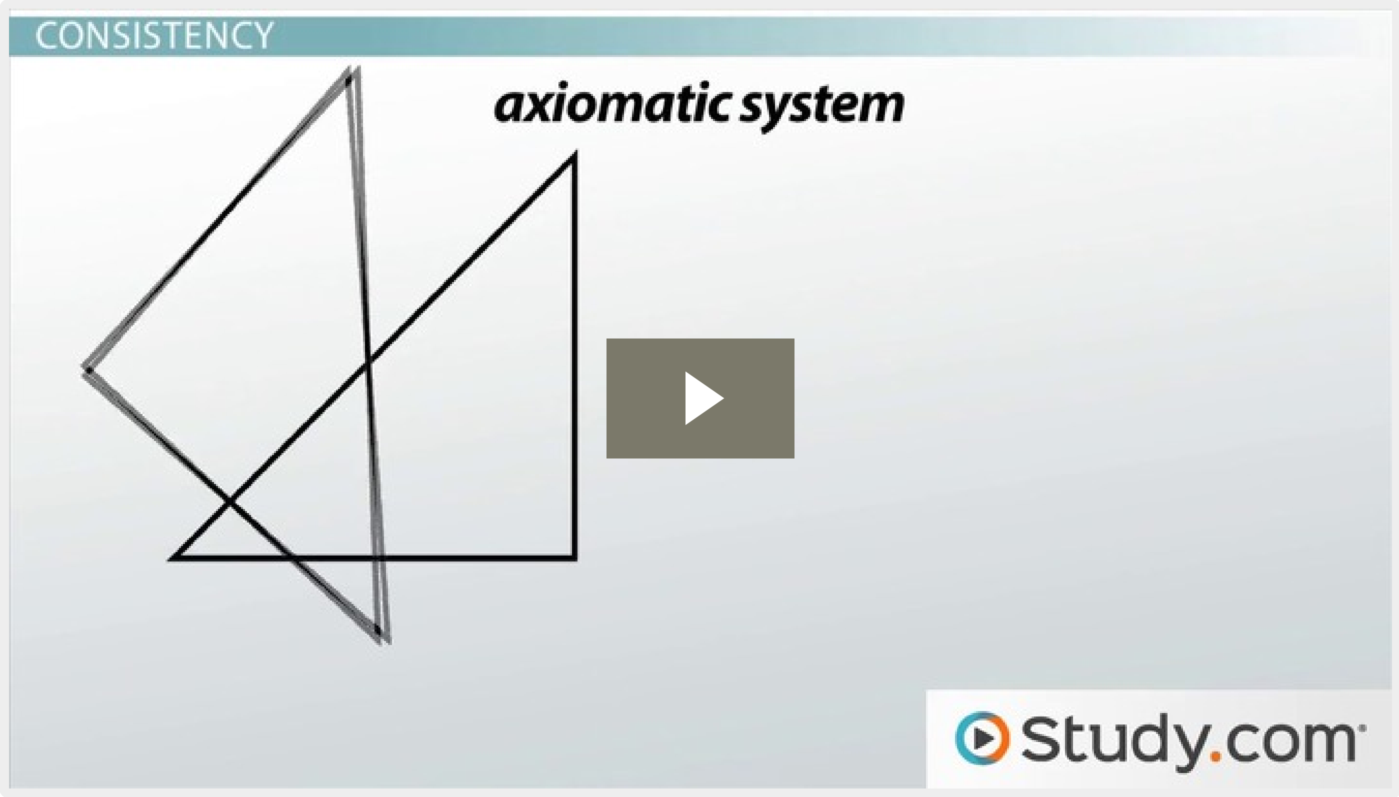
2. A line segment can be extended infinitely in both directions.

3. A circle can be described with a center and radius.

4. All right angles are equal to each other.

5. If a line intersecting two lines forms interior angles less than 90 degrees, then the two lines will intersect on the same side as the angles that are less than 90 degrees. The fifth axiom is also known as the parallel postulate.

Axiomatic systems also have three different properties.



Practice Quiz   Course Info

Consistency

The first property is called **consistency**. When an axiomatic system is consistent, then the system will NOT be able to prove both a statement and its negation. The consistent system will prove either the statement or its negative, but not both. If it did, then it would contradict itself. For example, if an axiomatic system was able to prove the statement 'squares are made from two triangles' as well as the statement 'squares are not made from two triangles,' then the system is not consistent. The system actually contradicts itself. You can't rely on the system. Because of this, this property is a requirement for an axiomatic system.

Independence

The next property is **independence**. The axioms in an axiomatic system are said to be independent if the axiom cannot be derived from the other axioms in the system. If you can use some of the axioms to prove another axiom in the system, then the system is not independent because one of the statements depends on the other statements. Look back at the five axioms of Euclidean geometry, for example, and you will see that this particular axiomatic system is independent since none of the five axioms can be proved by the other four. An axiomatic system does not have to be independent. It can be either dependent or independent, so this property, unlike the property for consistency, is not a requirement for an axiomatic system.

Completeness

The third property is that of **completeness**. A complete axiomatic system is a system where for any statement, either the statement or its negative can be proved using the system. If there is any statement the system cannot prove or disprove, then the system is not complete. As you can see, this is a pretty big property to fill. This is why completeness is also not a required property. This property is a tough one to fulfill for any axiomatic system.

Lesson Summary

So, what have we learned? We've learned that an **axiomatic system** is a set of axioms used to derive theorems where an **axiom** is a statement that is considered true and does not require a proof, a basic truth. Euclidean geometry with its five axioms makes up an axiomatic system. The three properties of axiomatic systems are consistency, independence, and completeness. A **consistent** system is a system that will not be able to prove both a statement and its negation. A consistent system will not contradict itself. An **independent** axiom in a system is an axiom that cannot be derived or proved from the other axioms in the system. A **complete** system is a system that can prove or disprove any statement. Out of the three properties, only the property of consistency is a requirement of axiomatic systems.

Learning Outcomes

When this has been thoroughly studied, you might be able to:

- Compare an axiom to an axiomatic system
- Remember the properties of an axiomatic system

The Axiomatic System: Definition & Properties Related Study Materials