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theorems from axioms.[1]

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A **formal system** is an abstract structure and formalization of an axiomatic system used for deducing, using rules of inference,

In 1921, David Hilbert proposed to use formal systems as the foundation of knowledge in mathematics.<sup>[2]</sup> However, in 1931 Kurt

Gödel proved that any consistent formal system sufficiently powerful to express basic arithmetic cannot prove its own completeness. This effectively showed that Hilbert's program was impossible as stated. The term *formalism* is sometimes a rough synonym for *formal system*, but it also refers to a given style of notation, for example, Paul

Dirac's bra-ket notation.

## A formal system has the following components, as a minimum: [3][4][5]

Concepts [edit]

 Formal language, which is a set of well-formed formulas, which are strings of symbols from an alphabet, formed by a formal grammar (consisting of production

- rules or formation rules). Deductive system, deductive apparatus, or proof system, which has rules of inference that take axioms and infers theorems, both of which are part of the formal
- language. In some cases an inductive system, used to derive a proof by first establishing a simple case, then generalizing it. A formal system is said to be recursive (i.e. effective) or recursively enumerable if the
- set of axioms and the set of inference rules are decidable sets or semidecidable sets, respectively.

Formal language [edit] Main articles: Formal language, Formal grammar, Syntax (logic), and Logical form

## specific alphabet, and operations used to form sentences from them. . Like languages

in linguistics, formal languages generally have two aspects:

A formal language is a language uses a set of strings whose symbols are taken from a

 the syntax is what the language looks like (more formally: the set of possible expressions that are valid utterances in the language) the semantics are what the utterances of the language mean (which is formalized in

- various ways, depending on the type of language in question) Usually only the syntax of a formal language is considered via the notion of a formal grammar. The two main categories of formal grammar are that of generative grammars,
- which are sets of rules for how strings in a language can be written, and that of analytic grammars (or reductive grammar<sup>[6]</sup>[unreliable source?][7]</sup>), which are sets of rules for how a

string can be analyzed to determine whether it is a member of the language. **Deductive system** [edit] Main articles: Inference, Logical consequence, and Deductive reasoning This section may **require cleanup** to meet Wikipedia's quality standards. The specific

Well-formed formulas Theorems This diagram shows the syntactic entities that may be constructed from formal languages. The symbols and strings of symbols may be broadly divided into nonsense and well-formed formulas. A formal language can be thought of as identical to the set of its

well-formed formulas, which may be broadly

Part of a series on Formal languages

divided into theorems and non-theorems.

Symbols and

strings of symbols

**Key concepts** [hide] Formal system · Alphabet · Syntax · Formal semantics • Semantics (programming languages) • Formal grammar · Formation rule · Well-formed formula · Automata theory · Regular expression · Production · Ground expression · Atomic formula show **Applications** V • T • E

Systems science

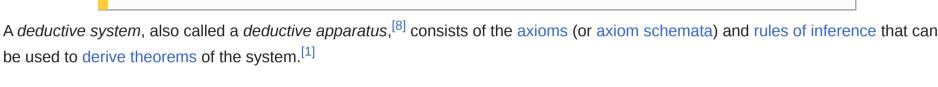
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portal

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be used to derive theorems of the system.<sup>[1]</sup> In order to sustain its deductive integrity, a *deductive apparatus* must be definable without reference to any intended interpretation of

The logical consequence (or entailment) of the system by its logical foundation is what distinguishes a formal system from others

the language. The aim is to ensure that each line of a derivation is merely a logical consequence of the lines that precede it. There

The two main types of deductive systems are proof systems and formal semantics. [8][9]

Formal proofs are sequences of well-formed formulas (or WFF for short) that might either be an axiom or be the product of applying

that there will be a decision procedure for deciding whether a given WFF is a theorem or not. The point of view that generating formal proofs is all there is to mathematics is often called *formalism*. David Hilbert founded

metamathematics as a discipline for discussing formal systems. Any language that one uses to talk about a formal system is called a *metalanguage*. The metalanguage may be a natural language, or it may be partially formalized itself, but it is generally less completely formalized than the formal language component of the formal system under examination, which is then called the *object language*, that is, the object of the discussion in question. The notion of theorem just defined should not be confused with theorems about the

Formal semantics of logical system [edit] Main articles: Semantics of logic, Interpretation (logic), and Model theory A logical system is a deductive system (most commonly first order logic) together with additional non-logical axioms. According to model theory, a logical system may be given interpretations which describe whether a given structure - the mapping of formulas to a

• Sound, if each well-formed formula that can be inferred from the axioms is satisfied by every model of the logical system.

nonnegative integers and gives the symbols their usual meaning.<sup>[10]</sup> There are also non-standard models of arithmetic.

An example of a logical system is Peano arithmetic. The standard model of arithmetic sets the domain of discourse to be the

History [edit]

Early logic systems includes Indian logic of Pāṇini, syllogistic logic of Aristotle, propositional logic of Stoicism, and Chinese logic of

Gongsun Long (c. 325–250 BCE). In more recent times, contributors include George Boole, Augustus De Morgan, and Gottlob Frege.

• Semantically complete, if each well-formed formula that is satisfied by every model of the logical system can be inferred from the

Mathematical logic was developed in 19th century Europe. David Hilbert instigated a formalist movement called Hilbert's program as a proposed solution to the foundational crisis of mathematics, that was eventually tempered by Gödel's incompleteness theorems.<sup>[2]</sup> The QED manifesto represented a subsequent,

 Formal method – Mathematical program specifications Formal science – Study of abstract structures described by formal systems Logic translation – Translation of a text into a logical system

axioms.

References [edit] 1. ^ a b Hunter 1996, p. 7.

Sources [edit]

2. ^ a b Zach, Richard (31 July 2003). "Hilbert's Program" . Hilbert's Program, Stanford Encyclopedia of Philosophy. Metaphysics Research Lab, Stanford University.

4. ^ Rapaport, William J. (25 March 2010). "Syntax & Semantics of Formal Systems" ∠. University of Buffalo.

Rewriting system – Replacing subterm in a formula with another term

as yet unsuccessful, effort at formalization of known mathematics.

5. ^ "Formal System" ∠. Pr∞fWiki. 6. ^ "Reductive grammar" . Dictionary of Scientific and Technical Terms (6th ed.). McGraw-Hill. "Reductive grammar: (computer science) A set of syntactic rules for the analysis of strings to determine whether the strings exist in a language."

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Ray Taol, Formal Systems

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3. ^ "Formal System" ∠. PlanetMath.

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- ₩ 
   Wiktionary, the free Encyclopædia Britannica, Formal system 

  definition, 2007. ★ 维 型 dictionary. Daniel Richardson, Formal systems, logic and semantics
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V • T • E

- **Predicate** Monadic predicate calculus
- operations: (intersection · union · complement · Cartesian product · power set · identities) Countable · Uncountable · Empty · Inhabited · Singleton · Finite · Infinite · Transitive · Ultrafilter Types of sets

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Formal systems (list), language and syntax

**Example axiomatic** systems (list) Principia Mathematica Formal proof · Natural deduction · Logical consequence · Rule of inference · Sequent calculus · Theorem · Systems (axiomatic · deductive · Hilbert (list)) · Complete theory · Independence (from ZFC) · Proof of impossibility · **Proof theory** 

> Type · Ultraproduct · Validity Primitive recursive function · Recursion · Recursive set · Turing machine · Type theory Abstract logic · Algebraic logic · Automated theorem proving · Category theory · Concrete/Abstract category ·

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> Ordinal analysis · Reverse mathematics · Self-verifying theories Interpretation (function · of models) · Model (equivalence · finite · saturated · spectrum · submodel) · Non-standard model

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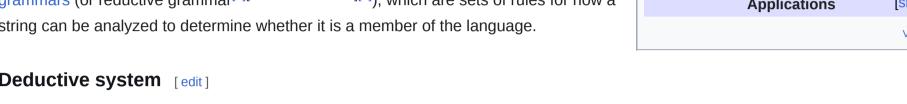
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should be no element of any interpretation of the language that gets involved with the deductive nature of the system. which may have some basis in an abstract model. Often the formal system will be the basis for or even identified with a larger theory

or field (e.g. Euclidean geometry) consistent with the usage in modern mathematics such as model theory. [clarification needed]

An example of a deductive system would be the rules of inference and axioms regarding equality used in first order logic.

an inference rule on previous WFFs in the proof sequence. The last WFF in the sequence is recognized as a theorem.

**Proof system** [edit] Main articles: Proof system and Formal proof

Once a formal system is given, one can define the set of theorems which can be proved inside the formal system. This set consists of all WFFs for which there is a proof. Thus all axioms are considered theorems. Unlike the grammar for WFFs, there is no guarantee

formal system, which, in order to avoid confusion, are usually called metatheorems.

particular meaning - satisfies a well-formed formula. A structure that satisfies all the axioms of the formal system is known as a model of the logical system. A logical system is:

Main articles: Formalism (philosophy of mathematics) and Formal logical systems

See also [edit] List of formal systems

 Substitution instance – Concept in logic Theory (mathematical logic) – Set of sentences in a formal language

7. ^ Rulifson, Johns F. (April 1968). "A Tree Meta for the XDS 940" [a] (PDF). Augmentation Research Center. Retrieved 30 November 2024. "There are two classes of formal-language definition compiler-writing schemes. The productive grammar approach is the most common. A productive grammar consists primarrly of a set of rules that describe a method of generating all possible strings of the language. The

roughly divided into two parts: proof theory and formal semantics... The division is not exact; many questions have been dealt with from both points of view, and some proof-theoretic methods and results are indispensable in semantics." 10. ^ Kaye, Richard (1991). "1. The Standard Model". Models of Peano arithmetic. Oxford: Clarendon Press. p. 10. ISBN 9780198532132.

reductive or analytical grammar technique states a set of rules that describe a method of analyzing any string of characters and deciding

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• Peter Suber, Formal Systems and Machines: An Isomorphism & Archived 2011-05-24 at the Wayback Machine, 1997.

What is a Formal System? 

 Z: Some quotes from John Haugeland's `Artificial Intelligence: The Very Idea' (1985), pp. 48–64.

**Mathematical logic** 

Axiom (list) · Cardinality · First-order logic · Formal proof · Formal semantics · Foundations of mathematics ·

• Smullyan, Raymond M., 1961. Theory of Formal Systems: Annals of Mathematics Studies, Princeton University Press (April 1,

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- Classical logic · Logical truth · Tautology · Proposition · Inference · Logical equivalence · Consistency **Traditional** (Equiconsistency) · Argument · Soundness · Validity · Syllogism · Square of opposition · Venn diagram Boolean algebra · Boolean functions · Logical connectives · Propositional calculus · Logics **Propositional** Propositional formula · Truth tables · Many-valued logic (3 · finite · ∞) First-order (list) · Second-order (Monadic) · Higher-order · Fixed-point · Free · Quantifiers · Predicate ·
- · Recursive · Fuzzy · Universal · Universe (constructible · Grothendieck · Von Neumann) Function/Map (domain · codomain · image) · In/Sur/Bi-jection · Schröder-Bernstein theorem · **Set theory** Isomorphism · Gödel numbering · Enumeration · Large cardinal (inaccessible) · Aleph number · **Maps and cardinality** Operation (binary)
- · ∀ · rank) · Sentence (atomic · spectrum) · Signature · String · Substitution · Symbol (function · logical/constant · non-logical · variable) · Term · Theory (list) of arithmetic: (Peano · second-order · elementary function · primitive recursive · Robinson · Skolem) · of the real numbers (Tarski's axiomatization) · of Boolean algebras (canonical · minimal axioms) · of geometry: (Euclidean: (Elements · Hilbert's · Tarski's) · non-Euclidean)
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