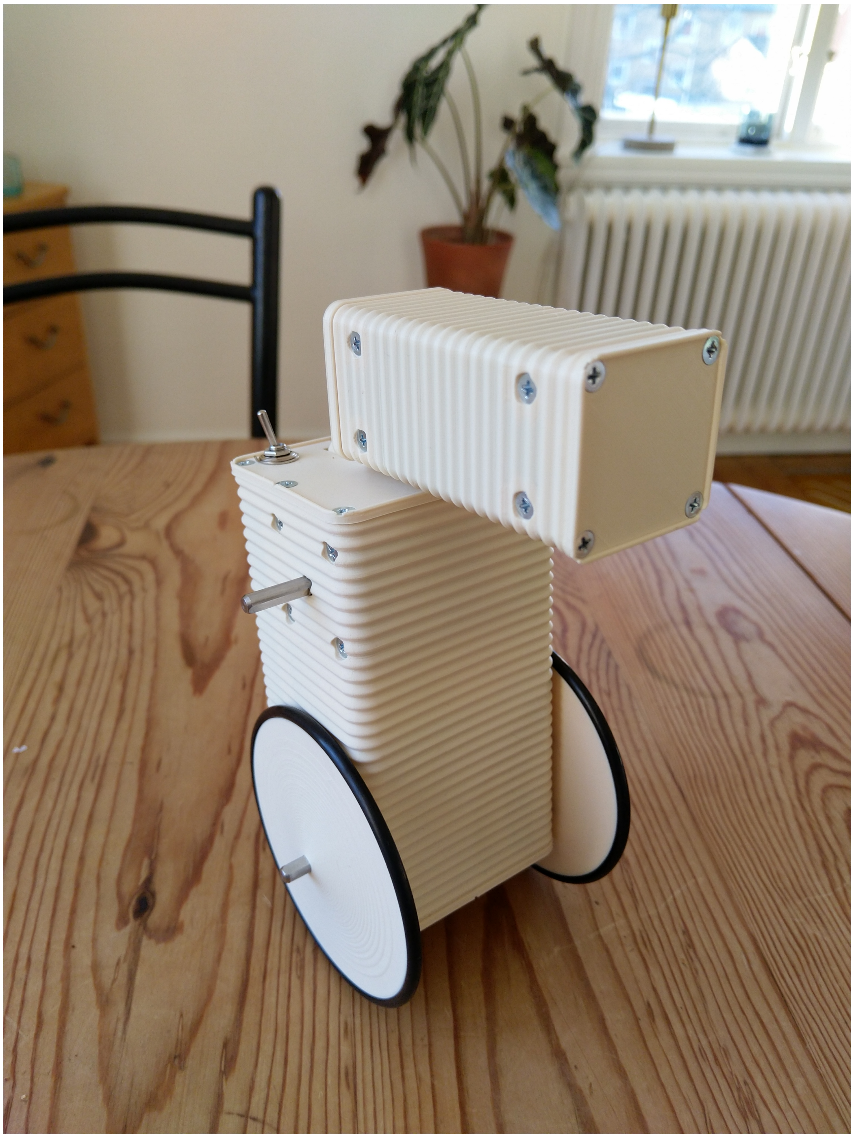


Figure 1: Ropsu looks good and has wheels and head

# How to make Ropsu Balance

Topsu 20/05/2020

## 1 Overview

You need to keep Ropsu upright. Ropsu is very unstable!

You can only acces the wheel angular velocity, under no slip this means velocity of Ropsus hip.

Lagrange of Ropsu reads (See. A):

$$\mathcal{L} = -Fx(t) + \frac{J}{2}\left(\frac{d}{dt}\theta(t)\right)^2 - L_{cm}gm\cos\left(\theta(t)\right) + L_{cm}m\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)\frac{d}{dt}x(t) + \frac{m}{2}\left(\frac{d}{dt}x(t)\right)^2 \tag{1}$$

From which we yield Eqs. of motion:

$$F - L_{cm} m \sin(\theta(t)) \left(\frac{d}{dt}\theta(t)\right)^2 + L_{cm} m \cos(\theta(t)) \frac{d^2}{dt^2}\theta(t) + m\frac{d^2}{dt^2}x(t) = 0 \quad (2)$$

$$J\frac{d^2}{dt^2}\theta(t) - L_{cm} g m \sin(\theta(t)) + L_{cm} m \cos(\theta(t)) \frac{d^2}{dt^2}x(t) = 0 \quad (3)$$

Using 3 we get:

$$\frac{d^2}{dt^2}\theta = \frac{L_{cm} m}{J}\left(g\sin(\theta(t)) - \cos(\theta(t))\frac{d^2}{dt^2}x(t)\right) \quad (4)$$

$$a = -\frac{J\frac{d^2}{dt^2}\theta(t)}{L_{cm} m \cos(\theta(t))} + g\tan(\theta(t)) \quad (5)$$

Equipped with Eq. 4 we can see how to apply $a$ in each situation $\theta$ to achieve given $\frac{d^2}{dt^2}\theta$. Control is then based in deciding desired $\frac{d^2}{dt^2}\theta$ using Eq. 4 get $a$.

# 2 Model Fit

We always record $a, \theta, \frac{d^2}{dt^2}\theta$ so using Eq. 4 we can fit a model and obtain the factor $\frac{L_{cm} m}{J}$. This is further fed to the Kalman filter.

# 3 Control

We use PID to get:

$$\frac{d^2}{dt^2}\theta = \mathsf{P}\Delta\theta + \mathsf{I}\int \Delta\theta dt + \mathsf{D}\frac{d}{dt}\Delta\theta, \quad (6)$$

where $\Delta\theta$ is the difference from targeted $\theta$. Note that using $\mathsf{I} = 0$ this is damped harmonic oscillator.

## 3.1 Issue

We get particularly crappy and noisy values for the $\frac{d^2}{dt^2}\theta$ (see Fig. 2). I suppose also the values $\frac{d}{dt}\theta$ are messed and hence the obtained values $a$ become also very noisy. This leads to all sorts of issues. <++>

# 4 Hardware

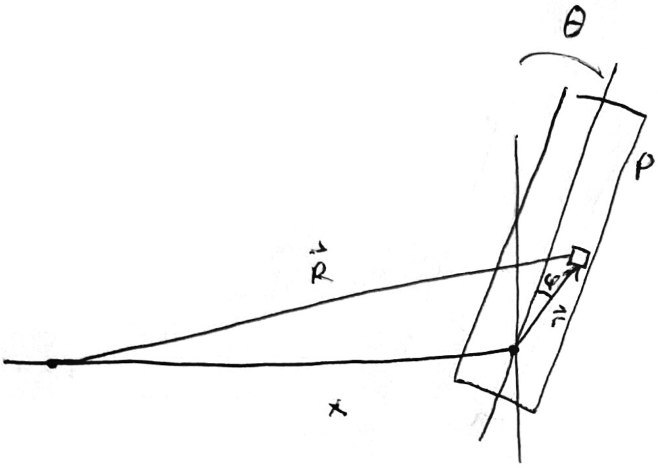
We use RaspberryPi Zero and an arduino due communicating via serila connection. Arduino takes care of readin values from IMU (inertial measurement unit) and controlling the wheels/-head/hands (as of now only wheels and head). We use stepper motors to have precise control over the position. Raspberry is used to take care of all control logic since I am not very comfortable with c++. Moreover I can easily ssh into the Raspberry and do the development without need to flash the program into arduino etc..

## A　Lagrange

$$\hat{e}_\theta = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$

$$\vec{R} = \vec{x} + \vec{r}$$

$$\dot{\vec{R}} = \dot{\vec{x}} + \dot{\vec{r}}$$




$$K = \frac{1}{2}\int_P \dot{\vec{R}}^2\,dm$$

$$= \frac{1}{2}\int_P \dot{\vec{x}}^2 + \dot{\vec{r}}^2 + 2\dot{\vec{x}}\cdot\dot{\vec{r}}\,dm \quad , \qquad \dot{\vec{r}} = r\omega\,\hat{e}_\theta$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\omega^2\underbrace{\int_P r^2\,dm}_{J} + \omega\dot{x}\underbrace{\int_P \cos\theta\, r\,dm}_{L_{cm}m} \quad * $$

see p. 3.

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J\omega^2 + \dot{x}\,\omega\cos\theta\, L_{cm}m$$

$$U = g\int_P r_y\,dm = g\int_P r\sin\left(\frac{\pi}{2} - (\theta+\varphi)\right)dm$$

$$= g\int_P r\cos(-\theta-\varphi)\,dm$$

$$= g\int_P r\cos(\theta+\varphi)\,dm$$

$$= g\int_P r\cos\theta\cos\varphi - r\sin\theta\sin\varphi\,dm$$

$0$ Ei $L_{cm} \Rightarrow$

$$= g\overset{\cos\theta}{\vee}\int_P \cancel{r\cos\varphi}\,dm - g\sin\theta\int \cancel{r\sin\varphi}^{\,0}\,dm$$

$$= g \cos\theta \, L_{cm} \, m$$

$$\Rightarrow \quad L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 + \dot{x}\dot{\theta} L_{cm} \cos\theta$$

$$- g m L_{cm} \cos\theta$$

$$\vec{r} = r\cos\left(\frac{\pi}{2} - (\theta+\varphi)\right)\hat{i} + r\sin\left(\frac{\pi}{2} - (\theta+\varphi)\right)\hat{j}$$

$$\dot{\vec{r}} = -r\sin\left(\frac{\pi}{2} - (\theta+\varphi)\right)(-\dot{\theta})\,\hat{i} + r\cos\left(\frac{\pi}{2} - (\theta+\varphi)\right)(-\dot{\theta})\,\hat{j}$$

$$\dot{\vec{x}} \cdot \dot{\vec{r}} = \dot{x}\, r\, \dot{\theta}\, \sin\left(\frac{\pi}{2} - (\theta+\varphi)\right)$$

$$= r\dot{x}\dot{\theta}\cos(\theta+\varphi)$$

$$= r\dot{x}\dot{\theta}(\cos\theta\cos\varphi - \sin\theta\sin\varphi)$$

$$\int \dot{\vec{x}} \cdot \dot{\vec{r}}\, dm$$

$$= \dot{x}\dot{\theta}\overbrace{\ }^{\cos\theta}\underbrace{\int_P r\cos\varphi\, dm}_{L_{cm}\cdot m} - \dot{x}\dot{\theta}\sin\theta\overbrace{\int_P r\sin\varphi\, dm}^{=0 \quad \text{cm at line } \varphi=0}$$

$$= L_{cm}\cdot m\,\dot{x}\dot{\theta}\cos\theta$$

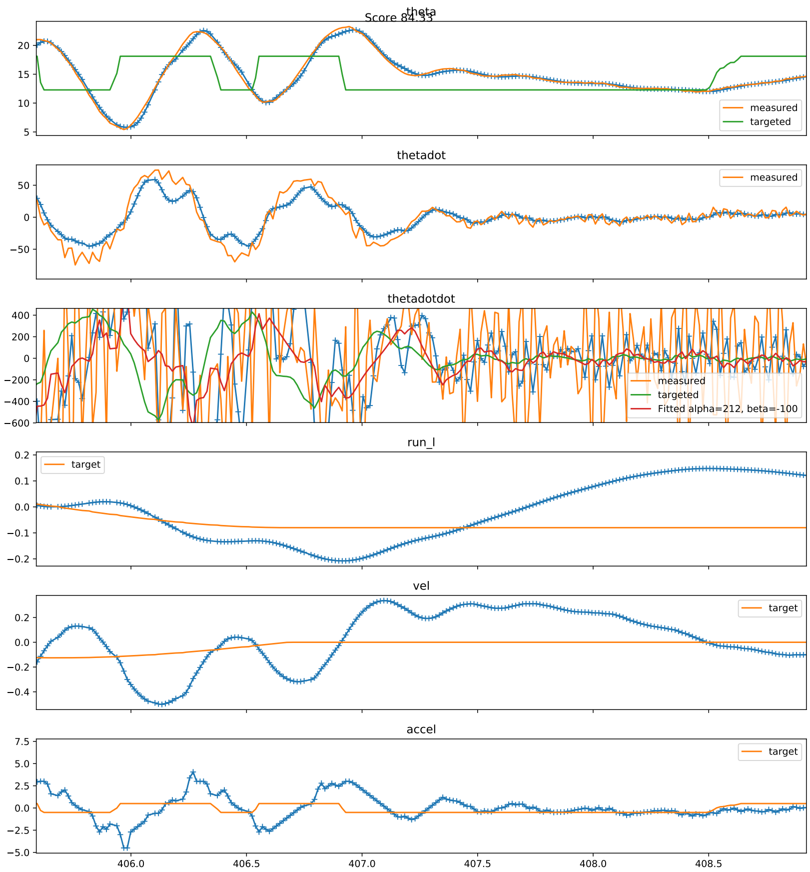



Figure 2: Recorded run. In $\theta$ blue lines are Kalman filtered, while orange is the measurement values.