### **Constraint Based Typing**

$$\Gamma \vdash e:T \mid \{C\}$$

It means "expression e has type T under assumptions whenever the constraints C are satisfied".

$$\Gamma \vdash s \mid \{C\}$$

It means "statement s typechecks under assumptions whenever the constraints C are satisfied".

#### Note:

- **1)** PICOInfer assumes the input program is a well-typed in Freedom-Before-Committment(FBC) type system and initialization qualifiers are given already.
- **2)** PICOInfer assumes that input program is well-formed in terms of assignability on fields(meaning one and only one assignability qualifier is used on a field; no @RDA is used on static fields) and assignability qualifiers are given already.
- **3)** Therefore, PICOInfer **only infers solution in mutability hierarchy**, not the initialization hierarchy or assignability dimension.
- **4)** In PICOInfer formalization, we only write initialization and assignability qualifiers in assumptions that affect how we generate mutability constraints, but not the in the conclusions(because it doesn't contribute to contraint generation and those two dimensions are assumed to be valid already).

# **Typing Rules (Constraint Based)**

$$\begin{array}{c}
x \in \Gamma_{q} \\
\hline
\Gamma_{q} \vdash x : \Gamma_{q}(x) \mid \{\}
\end{array}$$
(T-VAR)

\* Note :  $\Gamma_q$  is the type environment that only stores mutability qualifiers of variables.

$$\Gamma_{q}(x) = q_{x} \qquad \text{fType}(f) = q_{f}$$

$$\Gamma_{q} \vdash x.f : q \mid \{q = q_{x} \triangleright q_{f}\}$$
(T-FLD)

Figure 2 Expression typing

$$\Gamma_{q} \vdash e = q_{e}$$

$$\Gamma_{q} \vdash x = e \mid \{q_{e} <: \Gamma_{q}(x)\}$$
(T-VARASS)

$$\Gamma(x) = k_x q_x \quad \Gamma_q(y) = q_y \quad typeof(f) = q_f a_f$$

$$\Gamma_q \vdash x.f = y \mid \{q_v <: q_x \rhd q_f, fieldWrite(k_x, q_x, a_f)\}$$
(T-FLDASS)

Red V means CFI currently doesn't support this constraint(disjunction)

$$\Gamma_{q}(x) = q_{x} \qquad \Gamma_{q}(y) = q_{y} \qquad \Gamma_{q}(\overline{z}) = \overline{q}_{z} \qquad typeof(m) = q_{this}, \overline{q}_{p} \rightarrow q_{ret}$$

$$\Gamma_{q} \vdash x = y.m(\overline{z}) \mid \{q_{this-vp} = q_{y} \triangleright q_{this}, \overline{q}_{p-vp} = q_{y} \triangleright \overline{q}_{p}, q_{ret-vp} = q_{y} \triangleright q_{ret}, q_{y} <: q_{this-vp}, \overline{q}_{z} <: q_{p-vp}, q_{ret-vp} <: q_{x}\} \qquad (T-CALL)$$

**TODO**: inference of polymutable methods will be implemented later.

kd in C C <: D typeof(D) = 
$$\overline{q}_{p-D} \rightarrow q_{ret-D}$$
 typeof(kd) =  $\overline{\phantom{q}}_{p-D} \rightarrow q_{ret-C}$ 

$$\Gamma_q(z) = q_z$$

$$\Gamma_q \vdash \text{super}(\overline{z}) \text{ in kd} \mid \{q_{ret-C} <: q_{ret-C} \rhd q_{ret-D}, \overline{q}_z <: q_{ret-C} \rhd \overline{q}_{p-D}\}$$
(T-SUPER)

(T-THIS) (omitted)

\* *Note:* As the type rule in the typechecking side, the constraint based type rule T-THIS is very much the same as T-SUPER except that the constructor invoked by "this(..., ...)" comes from the same class.

$$\Gamma_{q}(x) = q_{x} \qquad \Gamma_{q}(y) = q_{y} \qquad typeof(C) = q_{p} \rightarrow q_{ret}$$

$$\Gamma_{q} \vdash x = new \ q \ C(y) \mid \{q_{y} <: q \triangleright q_{p}, \ q <: q \triangleright q_{ret}, \ q <: q_{x}, \ q \neq readonly\}$$

$$(T-NEW)$$

$$\Gamma_{q} \vdash s_{1} \mid \{C1\} \quad \Gamma_{q} \vdash s_{2} \mid \{C2\} \\
\Gamma_{q} \vdash s_{1}; s_{2} \mid \{C1, C2\}$$
(T-SEQ)

## Figure 3 Statement typing

#### **Well-formdness Rules**

(WF-CONS-OBJECT)

 $\vdash$  object kd is OK | {}

*Note*:  $\vdash_{C \text{ kd}}$  reads "constructor kd in class C is well-formed".

$$\begin{array}{ll} mBody(md) = \stackrel{-}{s;} return \ \underline{z} & typeof(md) = q_{this}, \ \stackrel{-}{q_p} \rightarrow q_{ret} \\ \Gamma_q = (this: q_{this}, \ \stackrel{-}{p}: \stackrel{-}{q_p}, \ \stackrel{-}{y}: q_{local}) & \Gamma_q \ \vdash \stackrel{-}{s} \mid \{C1\} \quad \Gamma_q(z) <: q_{ret} \mid \{C2\} \\ Standard \ method \ overriding \ rule \ holds \mid \{C3\} \\ & \vdash_{C} md \ is \ OK \mid \{C1, C2, C3\} \end{array} \tag{WF-METH}$$

$$\vdash_{\mathsf{C}} \overline{\mathsf{fd}} \; \mathsf{is} \; \mathsf{OK} \; | \; \{\mathsf{C1}\} \qquad \vdash_{\mathsf{C}} \mathsf{kd} \; \mathsf{is} \; \mathsf{OK} \; | \; \{\mathsf{C2}\} \qquad \vdash_{\mathsf{C}} \overline{\mathsf{md}} \; \mathsf{is} \; \mathsf{OK} \; | \; \{\mathsf{C3}\} \qquad \qquad \mathsf{(WF-CLASS)}$$

Figure 4 Well-formdness typing

### **Extension to real Java**

In real Java, there are static fields, static methods, initialization blocks.

## **Helper Method**

```
usedQualifiers(s) returns all mutability qualifiers used in s recursively
```

```
cd := class C extends D \{ \overline{sfd} \overline{fd}; \overline{sib} \overline{ib} \overline{kd} \overline{smd} \overline{md} \}
                                                                                                                                               class
sfd ::= static q C sf
                                                                                                                                     static field
smd ::= static t C sm ( t C x ) { \overline{t C y} \overline{s}; return z; }
                                                                                                                                 static method
sib ::= static{s;}
                                                                                                             static initialization block
ib ::= \{\bar{s};\}
                                                                                                                        initialization block
                                      fType(sfd) = q
                                                                                                                      (WF-STATIC-FLD)
\vdash sfd is OK | {q \neq polymutable, q \neq receiverdependantmutable}
\begin{array}{ll} mBod\underline{y}(\underline{smd}) = \underline{\overset{-}{s}}; return\ z & type\underline{of}(\underline{smd}) = \overline{\overset{-}{q}}_p \rightarrow q_{ret} \\ \Gamma_q = (\underline{p}: \underline{q}_p,\ \underline{y}: \underline{q}_{local}) & \Gamma_q \ \vdash \ \underline{s}\ |\ \{C1\} \end{array}
                                                                                           \Gamma_{q}(z) \leq q_{ret} | \{C2\}
  \vdash smd is OK | {C1 , C2 , \overset{-}{q_p} \neq receiver dependant mutable} , q_{ret} \neq
receiverdependantmutable, foreach q in usedQualifiers(\bar{s};return z): q \neq \bar{s}
receiverdependantmutable}
                                                                                                                  (WF-STATIC-METH)
                                      \Gamma_{\sigma} \vdash \bar{s} \mid \{C\}
                                                                                                                     (WF-STATIC-BLK)
\vdash sib is OK | {C, foreach q in usedQualifiers(\bar{s}) : q \neq receiverdependantmutable }
                                         \Gamma_q \vdash \overline{s} \mid \{C\}
                                                                                                                                    (WF-BLK)
                                     \vdash_{\mathsf{C}} ib is \mathsf{OK} \mid \{\mathsf{c}\}
\vdash \overline{sfd} is OK | {C1} \vdash_{C} \overline{fd} is OK | {C2} \vdash_{C} kd is OK | {C3} \vdash \overline{smd} is OK | {C4}
\vdash_{C} \overline{\mathrm{md}} is OK | {C5} \vdash_{C} \overline{\mathrm{sib}} is OK | {C6} \vdash_{C} \overline{\mathrm{ib}} is OK | {C7}
                                                                                                                                (WF-CLASS)
                     \vdash C is OK | {C1, C2, C3, C4, C5, C6, C7}
```