Detecting emergent continuous symmetries at quantum criticality

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MY, Bram Vanhecke, Norbert Schuch, *Phys. Rev. Lett.* 131, 036505 (2023)

topo23

What is emergent symmetry?

$$H = \boxed{H_0} + g\boxed{H_1}$$

Has some symmetry Break the symmetry

- H_0 is the fixed point Hamiltonian.

- If H_1 is irrelevant under RG, the symmetry of H_0 will emerge in the low-energy and long-distance limit in the system governed by H.

- Emergent space-time symmetry: e.g. conformal invariance

- Emergent **internal** symmetry + **continuous**

Examples

- 1D quantum critical Hamiltonian with microscopic internal Lie group symmetry G, whose low-energy effective theory is a 1+1d non-chiral conformal field theory (CFT). In the low-energy and long-distance limit, the symmetry of the system will be extended to $G \otimes G$.

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Critical Behavior of Two-Dimensional Systems with Continuous Symmetries

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(Received 18 June 1985)

Conformal invariance allows a complete classification of critical theories in two-dimensional systems with continuous symmetries. We study the spin-$\frac{1}{2}$ chain by non-Abelian bosonization to show how it fits into this classification.

$$\bar{\partial} J = 0$$

$$\partial \bar{J} = 0$$

$$J^{\mu} = (J + \bar{J}, J - \bar{J})$$

$$\partial_{\mu} J^{\mu} = 0$$

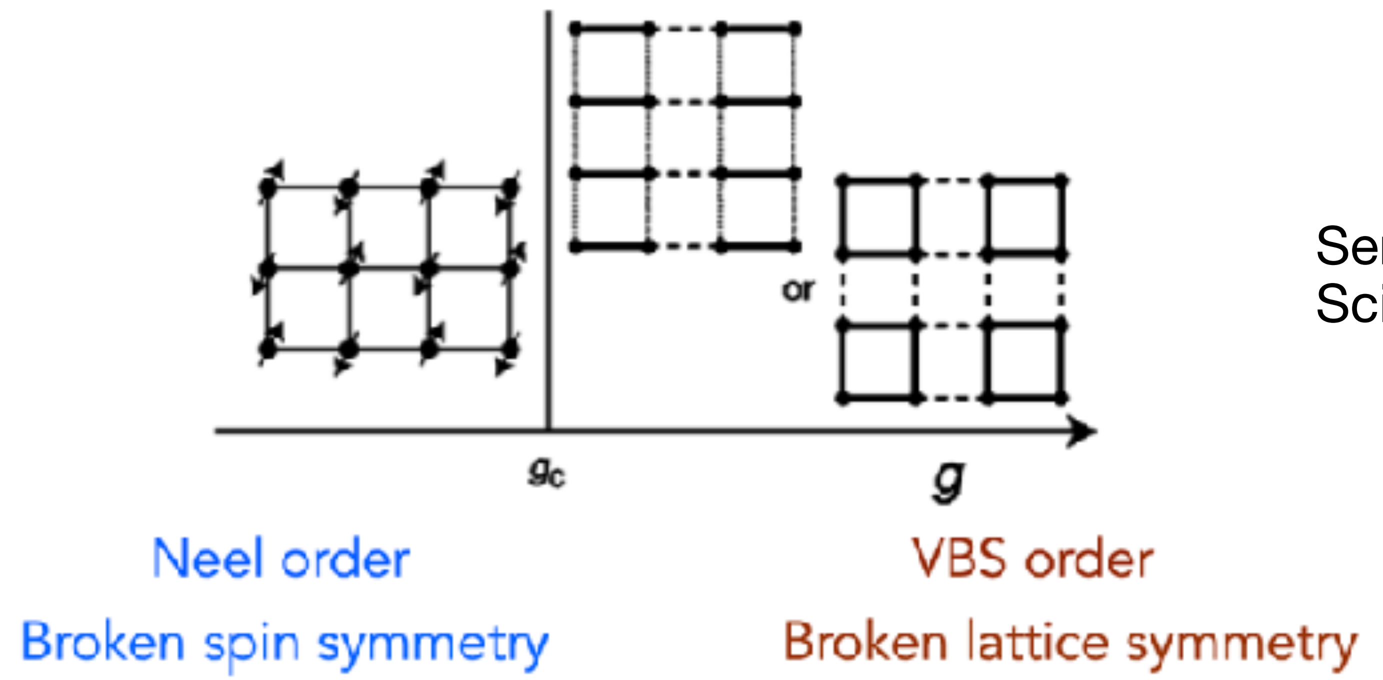
$$\partial_{\mu}(\epsilon^{\mu\nu} J_{\nu}) = 0$$

$$Q = \int \mathrm{d}x J^{0}(x) = \int \mathrm{d}x \left[J(x) + \bar{J}(x) \right]$$

$$M = \int \mathrm{d}x J^{1}(x) = \int \mathrm{d}x \left[J(x) - \bar{J}(x) \right]$$

Examples

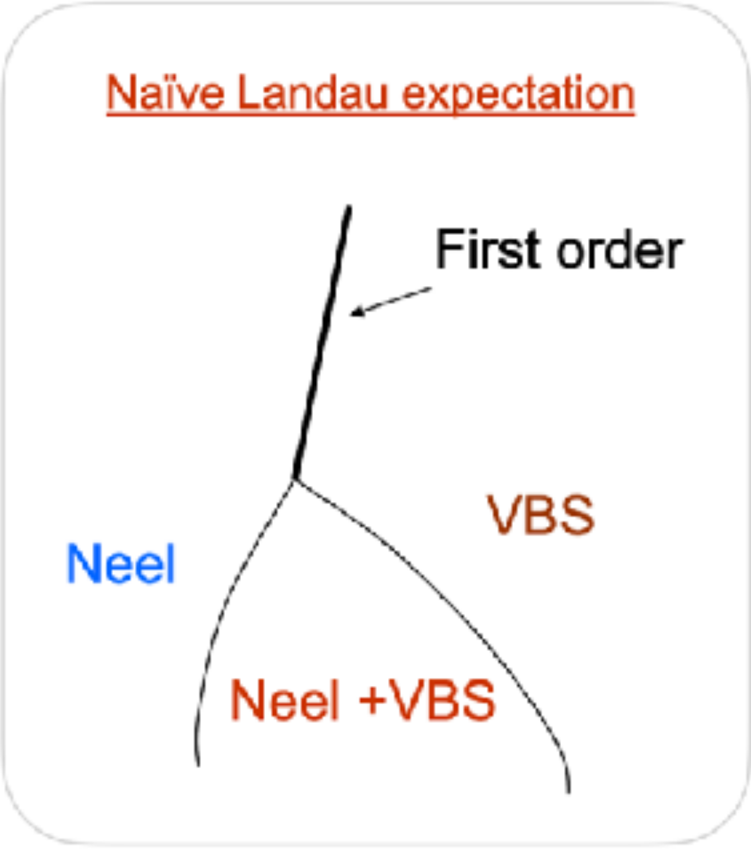
- Deconfined quantum critical points (DQCP)



Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 303, 1490 (2004)

Landau-Ginzburg-Wilson: two independent order parameter
- No generic direct second order transition
- Either first order or phase coexistence

DQCP: direct second order phase transition is allowed, but not in terms of the natural order parameters in each side
- Fractionalized degrees of freedom
- Emergent, topological, global conservation law



Motivation

- To find the emergent symmetries, one usually relies on doing field theory analysis.

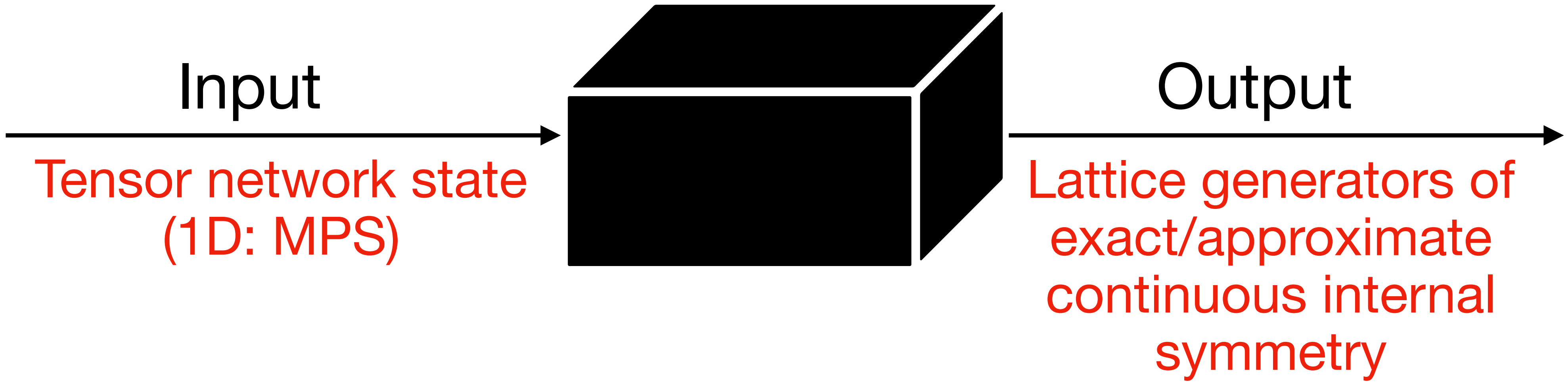
  Previous work:
  Measure correlation functions of order parameters or effective lattice operators of emergent conserved currents derived from field theory and symmetry analysis, and check the scaling dimensions.

  e.g. in 1D $\langle J(0)J(r)\rangle \sim 1/r^2$

Sandvik, Phys. Rev. Lett. 98, 227202 (2007).
Patil, Katz, Sandvik, Phys. Rev. B 100, 125137 (2019).
Ma, You, Meng, Phys. Rev. Lett. 122, 175701 (2019).
Huang, Lu, You, Meng, Xiang, Phys. Rev. B 100, 125137 (2019).
… …

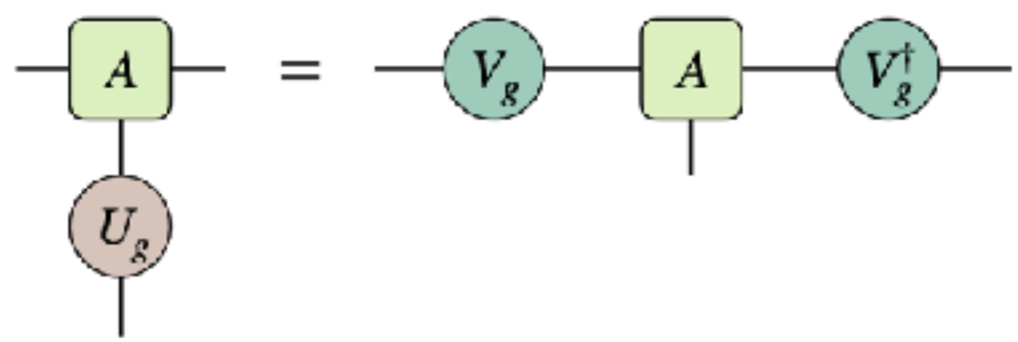
- **Suppose we do not know what kind of emergent symmetries the ground state should have, how do we know if it exists and obtain what it is without human intelligence?**

Algorithm



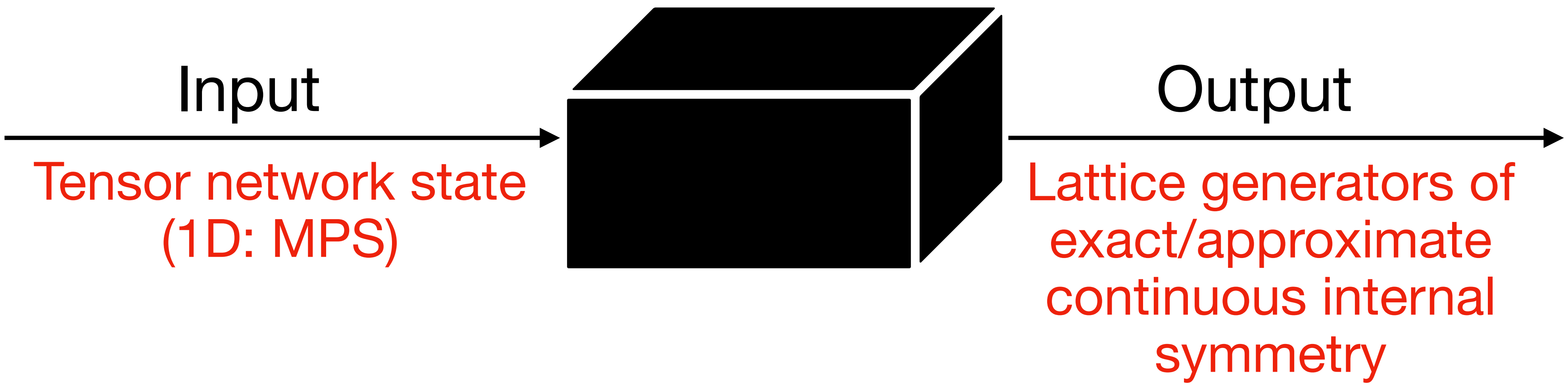
- Two ways to solve the problem:
  - MPS fundamental theorem based

$U_g^{\otimes N}|\psi(A)\rangle = |\psi(A)\rangle \quad \rightarrow$



  - Variational

Algorithm

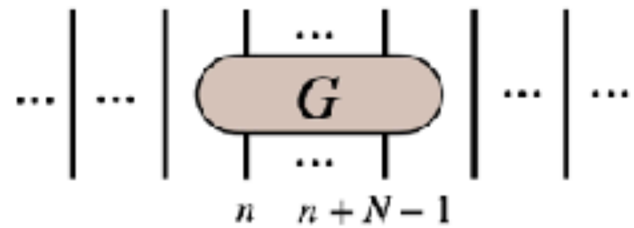


- For critical system:
  - MPS fundamental theorem based ❌
  - Variational ✅

Algorithm

$$e^{i\varepsilon O}|\psi\rangle = |\psi\rangle \quad \Leftrightarrow \quad O|\psi\rangle = 0 \quad \Leftrightarrow \quad \langle\psi|O^\dagger O|\psi\rangle = 0$$

where $O = \sum_n G_{n,\cdots,n+N-1} = \sum_n$



Therefore, we can consider the optimization problem with the normalization constraint $\|G_{n,\cdots,n+N-1}\|_2 = 1$.

$$\min_G f(G, G^\dagger) = \min_G \frac{\langle\psi|O^\dagger O|\psi\rangle}{V\mathrm{Tr}G^\dagger G}$$

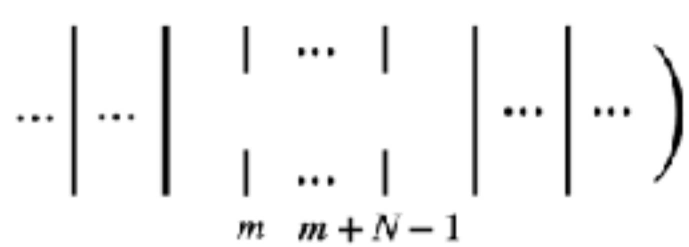
The optimum is reached when $\dfrac{\partial f(G,G^\dagger)}{\partial G^\dagger} = 0$, i.e.

$$\frac{\partial f(G,G^\dagger)}{\partial G^\dagger} = \frac{\partial}{\partial G^\dagger}\frac{\langle\psi|O^\dagger O|\psi\rangle}{V\mathrm{Tr}G^\dagger G} = \frac{\langle\psi|\frac{\partial O^\dagger}{\partial G^\dagger}O|\psi\rangle\cdot\mathrm{Tr}G^\dagger G - \langle\psi|O^\dagger O|\psi\rangle\cdot G}{V(\mathrm{Tr}G^\dagger G)^2} = 0$$

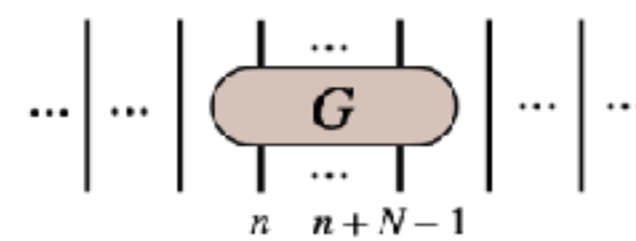
$$\Updownarrow$$

$$\langle\psi|\frac{\partial O^\dagger}{\partial G^\dagger}O|\psi\rangle = \frac{\langle\psi|O^\dagger O|\psi\rangle}{\mathrm{Tr}G^\dagger G}G \quad \Leftrightarrow \quad \mathscr{F}\cdot\mathbf{g} = \lambda_{\min}\mathbf{g}$$

where $\dfrac{\partial O^\dagger}{\partial G^\dagger}O = \Big(\dfrac{1}{V}\sum_m$

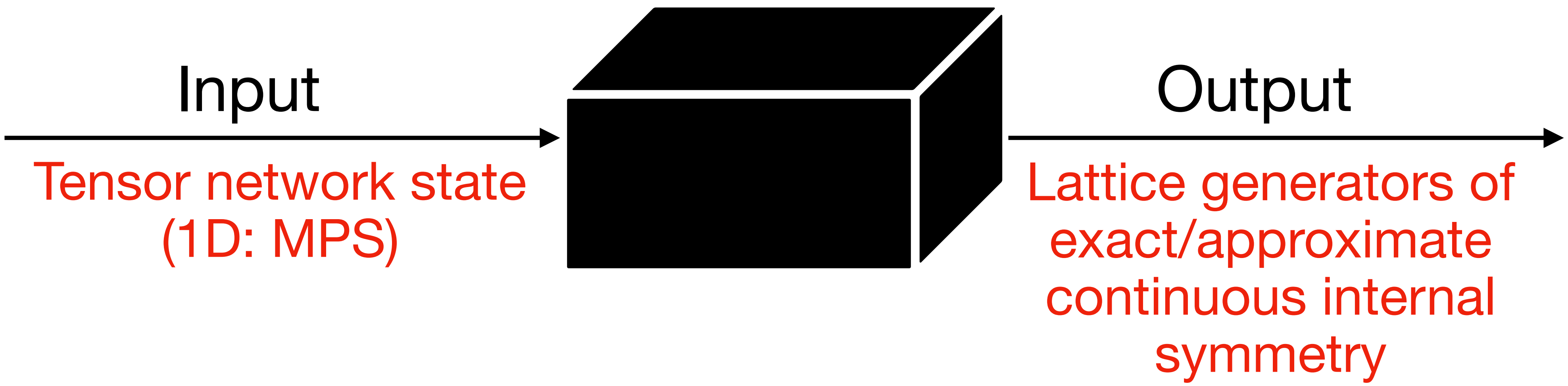


$\Big)\Big(\sum_n$



$\Big)$

Variational algorithm



1) Given a uMPS, e.g. variationally optimized already by iDMRG/iTEBD/VUMPS.
2) Get its correlation length by $\xi = -\dfrac{1}{\log|\lambda_2/\lambda_1|}$.
3) Feed the MPS to our algorithm.
4) Our algorithm solves an eigenvalue problem $(\mathscr{F} + \mathscr{F}^T) \cdot G = 2\lambda G$, the eigenvalue λ characterize how good the eigenvector G is a symmetry. (Remove trivial solutions, imposing microscopic symmetries of the Hamiltonian…)
5) Do the above procedure for several different bond dimensions of uMPS, get the finite entanglement scaling.

Benchmarks: exact symmetry

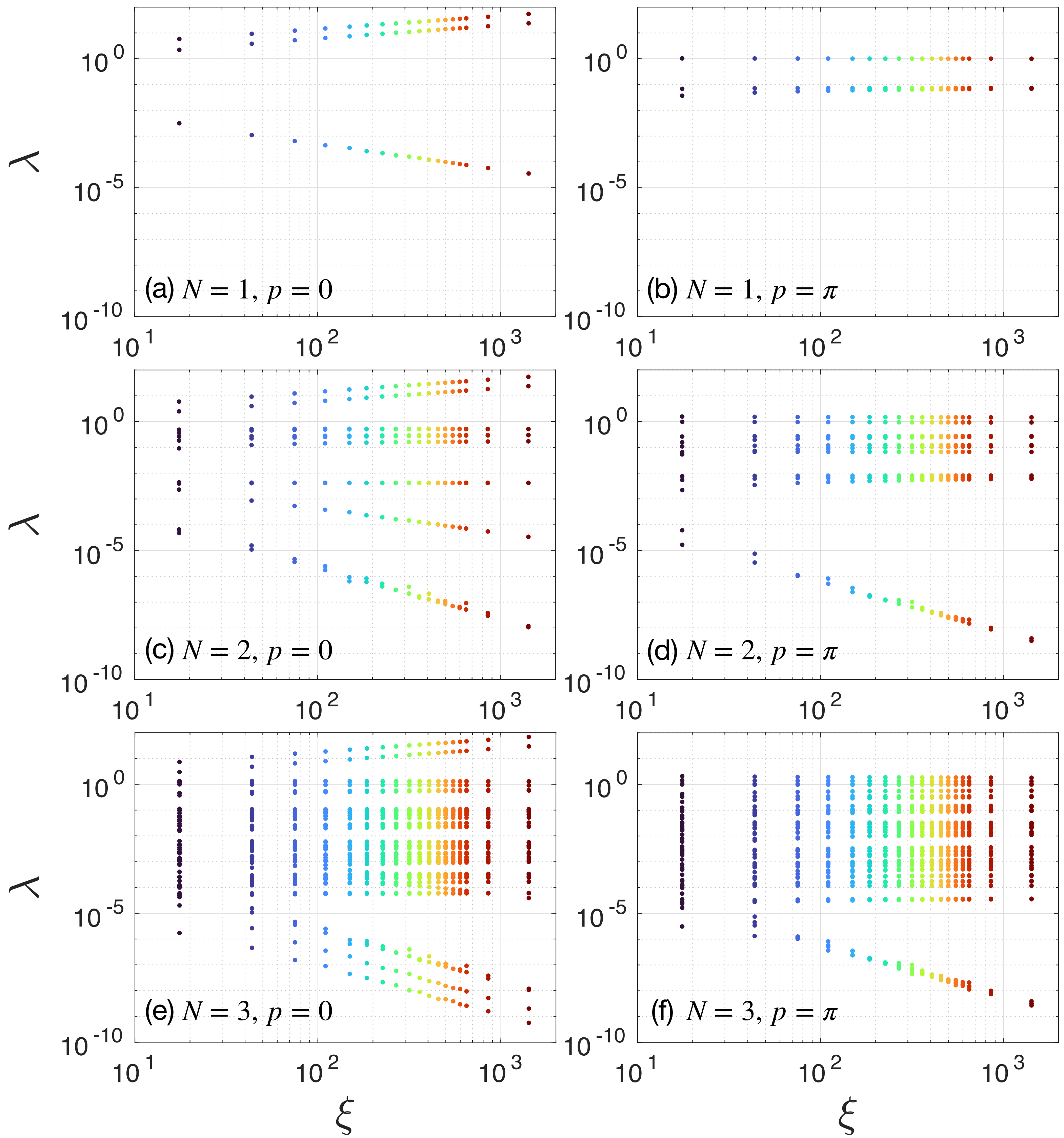
- 1D quantum isotropic XY model

$$H = -\sum_n (X_n X_{n+1} + Y_n Y_{n+1})$$

  - Exact U(1) symmetry
  - Integrable

$$O = \sum_n e^{\mathrm{i}pn} G_{n,\cdots,n+N-1}$$

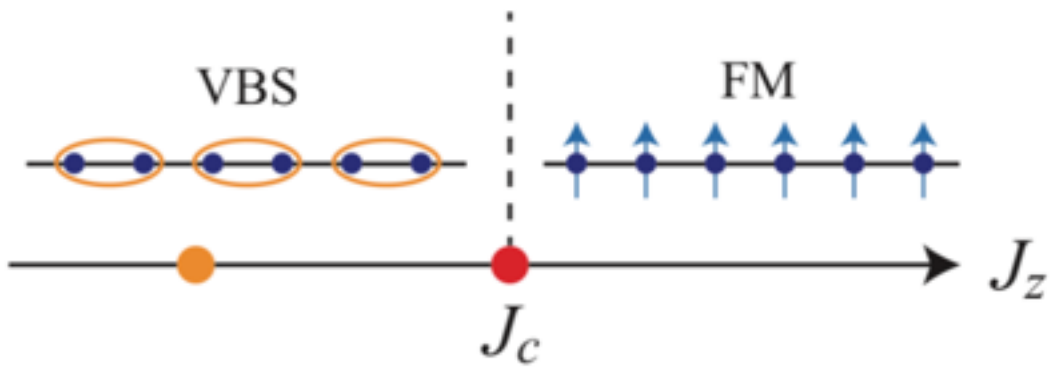
p	N	G	η'
0	1	Z	1.009
	2	$XX+YY$	1.985
		$XY-YX$	1.933
	3	$XZX+YZY$	1.008
		$XZY-YZX$	1.939
π	1	—	—
	2	$XX-YY$	2.005
		$XY+YX$	2.008
	3	$XZX-YZY$	2.046
		$XZY+YZX$	2.063

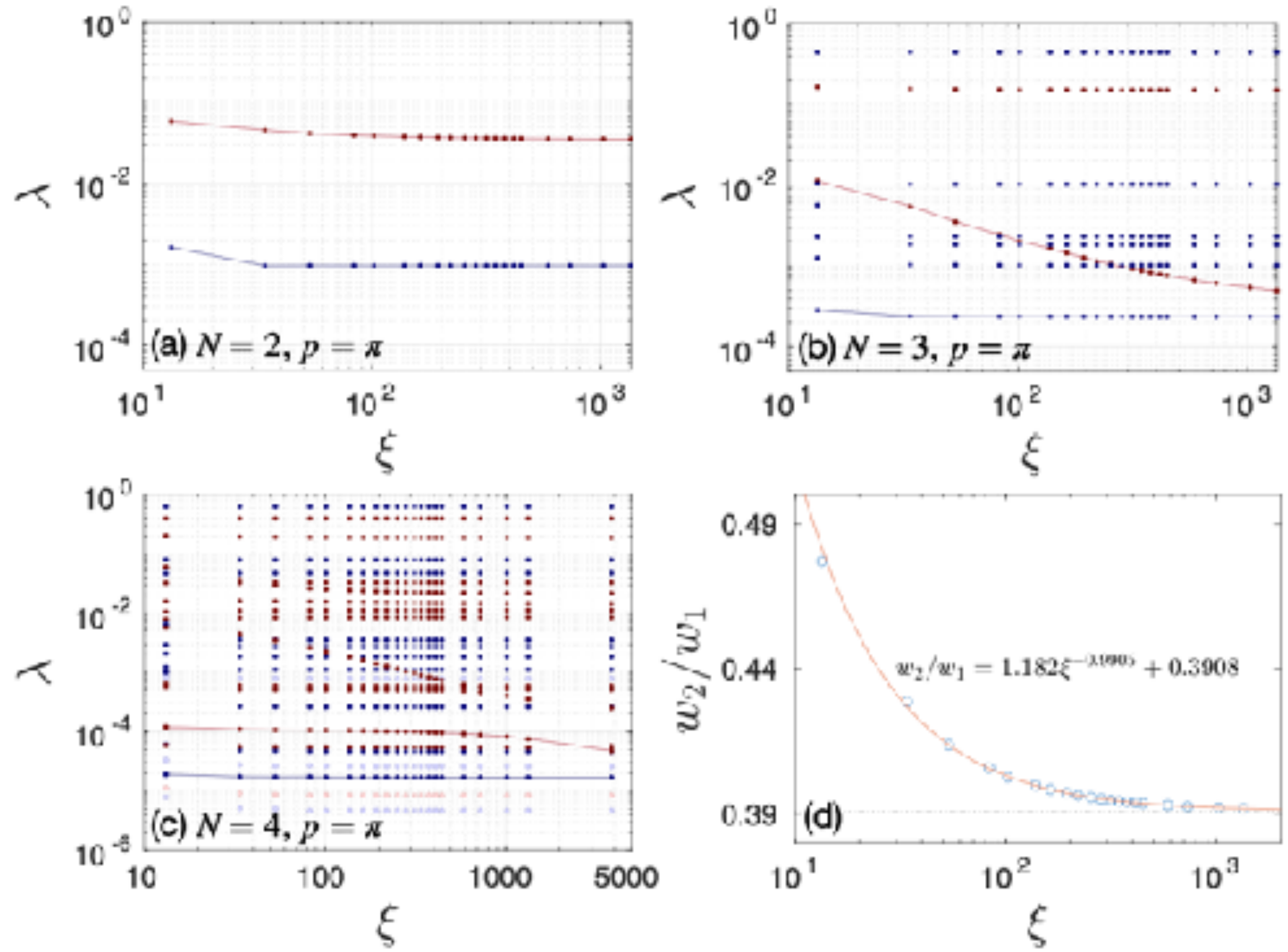


Results: Abelian symmetry

- A 1D spin-1/2 model with DQCP

$$H = \sum_i -S_i^x S_{i+1}^x - J_z S_i^z S_{i+1}^z + \frac{1}{2}(S_i^x S_{i+2}^x + S_i^z S_{i+2}^z)$$





- On-site spin flip $Z_2 \times Z_2$ symmetry
- Emergent $U(1) \times U(1)$ at J_c

Rotation between the VBS and z-FM fluctuations

Rotation between the x-FM and y-AFM fluctuations

$$G_1 = Z$$

$$G_2 = X\,Y + Y\,X$$
$$G_1 = Z\,I - I\,Z$$

$$G_2 = [(X\,Y + Y\,X)I - I(X\,Y + Y\,X)] + 2 \times 0.3908(X\,I\,Y - Y\,I\,X)$$

$$G_1 = \frac{1}{3}(-Z\,I\,I + I\,Z\,I - I\,I\,Z) + 0.1615 Z\,Z\,Z + 0.0988(Y\,Y\,Z + Z\,Y\,Y) + 0.0882(X\,X\,Z + Z\,X\,X) + 0.0410 X\,Z\,X - 0.1399 Y\,Z\,Y$$

Results: non-Abelian symmetry

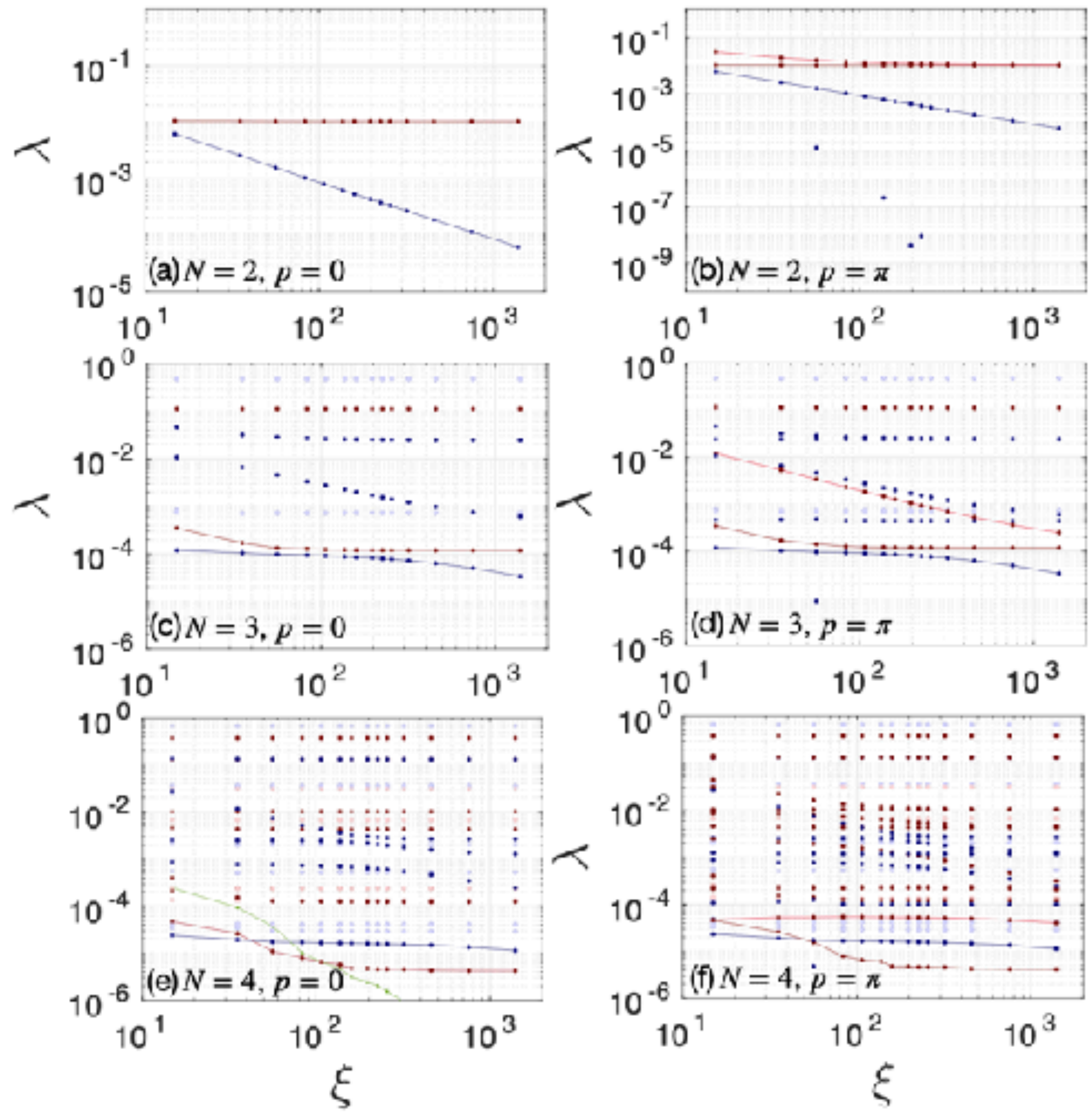
- 1D spin-1/2 J-Q model

$$H = -J\sum_n P_{n,n+1} - Q\sum_n P_{n,n+1}P_{n+2,n+3}$$

where $P_{n,n+1} = 1/4 - \mathbf{S}_n \cdot \mathbf{S}_{n+1}$.

critical dimer order

~ 0.84831 Q/J



$$Q = \sum_n S_n^\alpha,\quad \alpha = x, y, z \qquad S_n^\alpha \sim a\,(J_n^\alpha + \bar{J}_n^\alpha) + (-1)^n a\,\mathrm{Tr}[(g - g^\dagger)\sigma^\alpha]$$

$$M = \sum_n m_n^\alpha = \sum_n \epsilon^{\alpha\beta\gamma} S_n^\beta S_{n+1}^\gamma \qquad m_n^\alpha \sim J_n^\alpha - \bar{J}_n^\alpha$$

$$M = \sum_n m_n^\alpha = \sum_n \epsilon^{\alpha\beta\gamma}(S_n^\beta S_{n+1}^\gamma + 0.2253 S_n^\beta S_{n+2}^\gamma)$$

$$\begin{aligned} M = \sum_n m_n^\alpha = &\sum_n \epsilon_{\alpha\beta\gamma}(S_n^\beta S_{n+1}^\gamma + 0.3557 S_n^\beta S_{n+2}^\gamma + 0.1467 S_n^\beta S_{n+3}^\gamma) \\ &+ \sum_n \epsilon_{\alpha\beta\gamma}[0.1577 S_n^\beta S_{n+3}^\gamma \mathbf{S}_{n+1}\cdot\mathbf{S}_{n+2} - 0.09690\,\mathbf{S}_n\cdot\mathbf{S}_{n+3} S_{n+1}^\beta S_{n+2}^\gamma \\ &\quad - 0.09141(S_n^\beta S_{n+1}^\gamma \mathbf{S}_{n+2}\cdot\mathbf{S}_{n+3} + \mathbf{S}_n\cdot\mathbf{S}_{n+1} S_{n+2}^\beta S_{n+3}^\gamma) \\ &\quad + 0.08169(S_n^\beta S_{n+2}^\gamma \mathbf{S}_{n+1}\cdot\mathbf{S}_{n+3} + \mathbf{S}_n\cdot\mathbf{S}_{n+2} S_{n+1}^\beta S_{n+3}^\gamma)] \end{aligned}$$

Conclusion

- **Take-home message:**

  We developed a variational method

  which can extract the local parent Hamiltonian, the local integrals of motions,

  and other exact/approximate/emergent conserved quantities from a tensor network ground state,

  no matter the problem is gapped or gapless,

  as long as the conserved quantity takes the form
  $$O = \sum_n e^{ipn} G_{n,\ldots,n+N-1}.$$

Outlook

- MPO symmetry?
- 2D?
- LIOM in MBL?

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Letter

Evidence for deconfined $U(1)$ gauge theory at the transition between toric code and double semion

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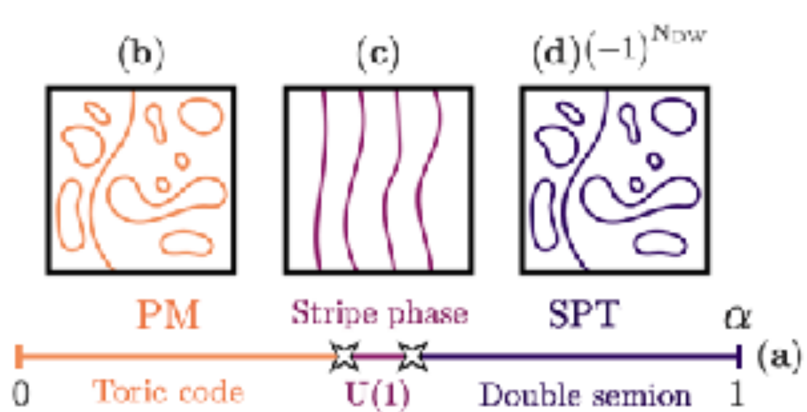
Building on quantum Monte Carlo simulations, we study the phase diagram of a one-parameter Hamiltonian interpolating between trivial and topological Ising paramagnets in two dimensions, which are dual to the toric code and the double semion. We discover an intermediate phase with stripe order which spontaneously breaks the protecting Ising symmetry. Remarkably, we find evidence that this intervening phase is gapless due to the incommensurability of the stripe pattern and that it is dual to a $U(1)$ gauge theory exhibiting Cantor deconfinement.



Uncovering Local Integrability in Quantum Many-Body Dynamics

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Interacting many-body quantum systems and their dynamics, while fundamental to modern science and technology, are formidable to simulate and understand. However, by discovering their symmetries, conservation laws, and integrability one can unravel their intricacies. Here, using up to 124 qubits of a fully programmable quantum computer, we uncover local conservation laws and integrability in one- and two-dimensional periodically-driven spin lattices in a regime previously inaccessible to such detailed analysis. We focus on the paradigmatic example of disorder-induced ergodicity breaking, where we first benchmark the system crossover into a localized regime through anomalies in the one-particle-density-matrix spectrum and other hallmark signatures. We then demonstrate that this regime stems from hidden local integrals of motion by faithfully reconstructing their quantum operators, thus providing a detailed portrait of the system's integrable dynamics. Our results demonstrate a versatile strategy for extracting hidden dynamical structure from noisy experiments on large-scale quantum computers.

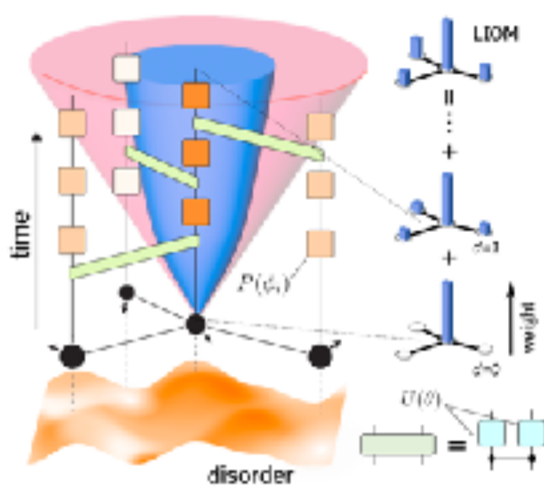

Fig. 1. Many-body dynamics: quantum simulation and integrals of motion. Four interacting spins (black circles) undergoing time-periodic dynamics in a modified kicked Ising model with random on-site disorder—modeled as a digital quantum circuit. Disorder is represented by single-qubit phase gates $P(\phi_i)$ with site-randomized angles ϕ_i (colored

Thanks.