# To Do List

December 4, 2023

This chapter contains some material about relations and constructions with them. Notably, we discuss and explore:

## Contents



# 1 Notes to Self

## 1.1 Things To Ask On MO/Zulip

**Remark 1.1.1.1.** Here is a list of things to be asked on MO/Zulip.

1. What are
   (a) Cartesian bicategories
   (b) Double categories of relations (https://arxiv.org/abs/2107.07621)
   (c) Categories of relations
   (d) Allegories
   (e) 1-Category equipped with relations (https://ncatlab.org/nlab/show/1-category+equipped+with+relations)

   good for? What have these notions been developed for, why are they important, and what have they lead to?

## 1.2 Things To Explore

**Remark 1.2.1.1.** Here is a list of things to be explored.

1. internal adjunctions in $\mathsf{Mod}$ as in [JY21, Section 6.3]; see [JY21, Example 6.2.6].
2. write the "profunctors" equivalent of the relations chapter
3. change $\chi_B$ notation throughout the notes
4. maybe note that skew monoidal structures on $\mathbf{Rel}(A, B)$ satisfy coherence trivially since the 2-morphisms are inclusions
5. reconsider notation $\mathsf{FreeAlg}_{\mathcal{P}}$ in Relations
6. Constructions With Sets: Isbell duality for powersets
7. Categories: comma category notation as in https://mathoverflow.net/questions/455630
8. Universal property of the bicategory of spans, https://ncatlab.org/nlab/show/span
9. Codensity monad $\mathrm{Ran}_J(J)$ of a relation (What about $\mathrm{Rift}_J(J)$?)
10. Relative *co*monads in $\mathbf{Rel}$.
11. Write proper sections on straightening for lax functors from sets to Rel or Span (displayed sets) when I study the corresponding notions for categories
12. Write about cospans.
13. CoCartesian fibration classifying $\mathsf{Fun}(F, G)$, https://mathoverflow.net/questions/457533/cocartesian-fibration-classifying-mathrmfunf-g

## 1.3 Omitted Proofs To Add

Не так благотворна истина, как зловредна ее видимость.

*Даниил Данковский*

Truth does not do as much good in the world as the appearance of truth does evil.

*Daniil Dankovsky*

There's a very large number of omitted proofs throughout these notes; here I list some of the ones that I really want to add to the notes at some point.

**Remark 1.3.1.1.** Here is a list of omitted proofs that I want to eventually write up or add a reference to.

- Relations, Item 1 of Proposition 2.5.1.1
- Relations, Item 2 of Proposition 2.5.1.1
- Relations, Item 9 of Proposition 2.5.1.1
- Relations, Item 10 of Proposition 2.5.1.1

# Appendices

## A Other Chapters

**Set Theory**

1. Sets
2. Constructions With Sets
3. Pointed Sets
4. Tensor Products of Pointed Sets
5. Indexed and Fibred Sets
6. Relations
7. Spans
8. Posets

**Category Theory**

9. Categories
10. Constructions With Categories
11. Kan Extensions

**Bicategories**

12. Bicategories
13. Internal Adjunctions

**Internal Category Theory**

14. Internal Categories

**Cyclic Stuff**

15. The Cycle Category

**Cubical Stuff**

16. The Cube Category

**Globular Stuff**

17. The Globe Category

**Cellular Stuff**

18. The Cell Category

**Monoids**

19. Monoids