MATH 2352 Problem Sheet 06

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[Problems] 5.5: 6, 11; 5.6: 10, 18;

5.5 - 6. For the equation

$$x^2\,y'' + x\,y' + (x-2)\,y \;=\; 0.$$

(a) Show that the given differential equation has a regular singular point at $x=0$.

(b) Determine the indicial equation, the recurrence relation, and the roots of the indicial equation.

(c) Fine the series solution ($x>0$) corresponding to the larger root.

(d) If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.

5.5 - 11. The Legendre equation of order α is

$$(1-x^2)\,y'' - 2\,x\,y' + \alpha\,(\alpha+1)\,y \;=\; 0.$$

The solution of the equation near the ordinary point $x=0$ was discussed in Problem 22 and 23 of Section 5.3. In Example 4 of Section 5.4, it was also shown that $x=\pm 1$ are regular singular points.

(a) Determine the indicial equation and its roots for the point $x=1$.

(b) Find a series solution in powers of $x-1$ for $x-1>0$.

Hint: Write $1+x=2+(x-1)$. Alternatively, make the change of variable $x-1=t$ and determine a series solution in power of t.

5.6 - 10. For the equation

$$(x-2)^2\,(x+2)\,y'' + 4x\,y' + 3(x-2)\,y \;=\; 0.$$

(a) Find all the regular points of the given differential equation.

(b) Determine the indicial equation and the exponents at the singularity for each regular singular point.

5.6 - 18.

(a) Show that

$$(\ln x)\,y'' + \frac{1}{2}\,y' + y \;=\; 0$$

has a regular singular point at $x=1$.

(b) Determine the roots of the indicial equation at $x=1$.

(c) Determine the first three nonzero terms in the series $\sum_{n=0}^{\infty} a_n\,(x-1)^{r+n}$ corresponding to the larger root. Take $x-1>0$.

(d) What would you expect the radius of convergence of the series to be?