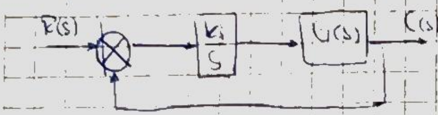
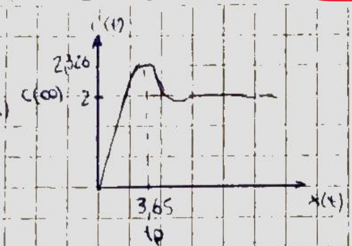
1er Parcial - Sistemas de control 2020 — 60.5% — Hoja 1-4

Legajo 41500 Mercado Nicolás — Tema 2

Ejercicio N°1





$$G(s) = \frac{K\,\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

a) Determinar la función de transferencia $M(s) \to G(s)$ — 10p

$$M_p = \frac{c(max) - c(\infty)}{c(\infty)} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \longrightarrow \xi = \frac{|\ln(M_p)|}{\sqrt{\pi^2 + |\ln(M_p)|^2}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \longrightarrow \omega_n = \frac{\pi}{t_p\sqrt{1-\xi^2}} = 0{,}99 \approx 1$$

$$M_p = \frac{2{,}326 - 2}{2} = 0{,}163 \longrightarrow \xi = 0{,}5$$

$$K = \frac{C(\infty)}{U(t)} = \frac{2}{1} = 2 \quad \therefore \quad G(s) = \frac{2 \cdot 0{,}99^2}{s^2 + 2 \cdot 0{,}5 \cdot 0{,}99 s + 0{,}99^2} \approx \boxed{\frac{2}{s^2 + s + 1}}$$

b) $$M(s) = \frac{\frac{K_i}{s}G(s)}{1 + \frac{K_i}{s}G(s)} = \frac{\frac{K_i\, 2}{s(s^2+s+1)}}{1 + \frac{K_i}{s}\frac{2}{s^2+s+1}} = \frac{2K_i}{s^3 + s^2 + s + 2K_i}$$

La EC es $s^3 + s^2 + s + 2K_i = 0$

Armando la tabla de Routh-Hurwitz

s^3	1	1
s^2	1	$2K_i$
s^1	$1-2K_i$	0
s^0	$2K_i$	

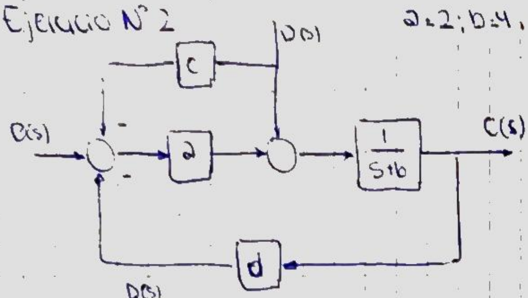
$$b_1 = \frac{1.1 - 1.2K_i}{1} = 1 - 2K_i \qquad 1 - 2K_i > 0 \to K_i < \frac{1}{2}$$

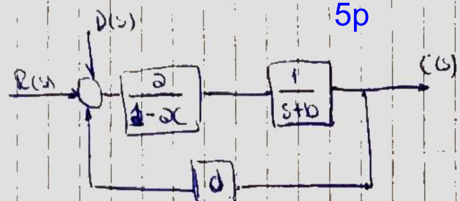
$$c_1 = \frac{\cancel{(1-2K_i)}(2K_i) - 0}{\cancel{(1-2K_i)}} = 2K_i \qquad 2K_i > 0 \to K_i > 0$$

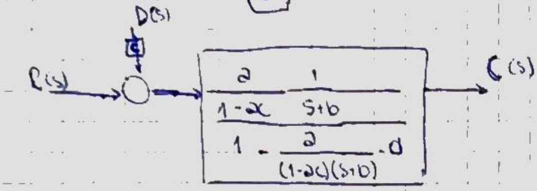
Para valores entre $0 < K_i < \frac{1}{2}$ $M(s)$ será estable

Ejercicio N° 2 $a=2;\ b=4,\ c=d=3$

5p







$$\frac{C(s)}{R(s)} = \frac{\frac{2}{(1-6)(s+4)}}{1-\frac{2\cdot 3}{(1-6)(s+4)}} = \frac{\frac{0{,}39}{s+4}}{1+\frac{1{,}2}{s+4}} = \frac{0{,}39}{s+4}\cdot\frac{1}{1+\frac{1{,}2}{s+4}}$$

$$\frac{C(s)}{R(s)} = \frac{0{,}39}{s+4+1{,}2} = \boxed{\frac{0{,}39}{s+5{,}2}}$$

a) $r(t)=u(t)$ y $d(t)=0$

$$c(\infty) = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot\frac{0{,}39}{s+5{,}2} = 0{,}075 \quad ; \quad c(0) = \lim_{s\to\infty} \cancel{s}\cdot\frac{1}{\cancel{s}}\cdot\frac{0{,}39}{s+5{,}2} = 0 \quad \to \text{Figura C}$$

✓

b) $r(t)=0$ y $d(t)=u(t)$

$$c(\infty) = \lim_{s\to 0} 3\cdot\frac{0{,}39}{s+5{,}2} = 1{,}17 \quad ; \quad c(0) = \lim_{s\to\infty} s\cdot\frac{1}{s}\cdot\frac{0{,}39}{s+5{,}2} = 0 \quad \to \text{Figura A}$$

✗

c) Figura B ✗

1er Parcial - Sistemas de Control 2020 — Hoja 2-4

Legajo 41500 — Mercado Nicolás — Tema 2

Ejercicio N° 3

10p

$$G(s) = \frac{50s+300}{(s^2+2s+10)(s^2+16s+73)} = \frac{50(s+6)}{(s+1-3j)(s+1+3j)(s+8-3j)(s+8+3j)} = \frac{50.6}{(s+1-3j)(s+1+3j)(8-3j)(8+3j)}$$

$$G(s) = \frac{300}{(s^2+2s+10)(64+\cancel{24j}-\cancel{24j}+9)} = \frac{300}{(s^2+2s+10)\,73} = \boxed{\frac{4{,}1}{s^2+2s+10}}$$

Función simplificada

$$G(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \rightarrow \omega_n^2 = 10 \rightarrow \omega_n = 3{,}16$$

$$2\xi\omega_n = 2 \rightarrow \xi = \frac{1}{\omega_n} = 0{,}31$$

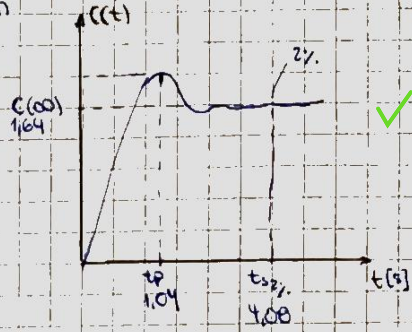
$$K\omega_n^2 = 4{,}1 \rightarrow K = \frac{4{,}1}{\omega_n^2} = \frac{4{,}1}{10} = 0{,}41$$

$$K = \frac{c(\infty)}{u(t)} \rightarrow c(\infty) = K\,u(t) = 0{,}41 \cdot 4 = \boxed{1{,}64}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{3{,}16\sqrt{1-(0{,}31)^2}} = 1{,}04\,s$$

$$t_{s_{2\%}} = \frac{4}{\xi\omega_n} = \frac{4}{0{,}31 \cdot 3{,}16} = 4{,}08\,s$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 0{,}359$$



Ejercicio N° 4

2p

a) G_1

b) G_1

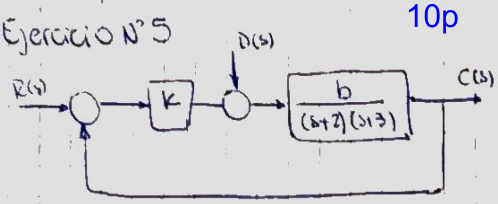
c) G_1 — G1 = G2 misma wd

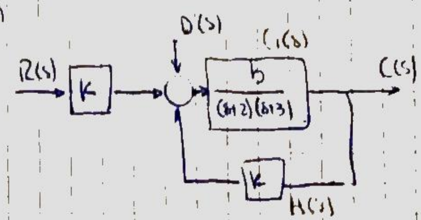
d) G_2 — G1=G2

e) G_2

Ejercicio N°5

10p





a) $M_D(s) = \frac{C(s)}{D(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{b}{(s+2)(s+3)}}{1+\frac{b}{(s+2)(s+3)}K} = \frac{b}{(s+2)(s+3)} \cdot \frac{1}{1+\frac{bK}{(s+2)(s+3)}} = \frac{b}{s^2+3s+2s+6+bK}$

$$\boxed{M_D(s) = \frac{b}{s^2+5s+6+bK}}$$

b) $S_D^M = \frac{\Delta M}{\Delta D} \cdot \frac{D}{M} = \frac{\partial M}{\partial D} \cdot \frac{D}{M} = \frac{(1+GH)-GH}{(1+GH)^2} \cdot \frac{G}{\frac{G}{1+GH}} \approx \frac{1}{1+GH} = \frac{1}{1+\frac{bK}{(s+2)(s+3)}} = \frac{1}{1+\frac{bK}{s^2+5s+6}}$

$$\boxed{S_D^M = \frac{s^2+5s+6}{s^2+5s+6+bK}}$$

c) $E(s) = \frac{R(s)}{1+M_D(s)} = \frac{K}{1+\frac{b}{s^2+5s+6}}$

Falta conclusion sobre valor de K

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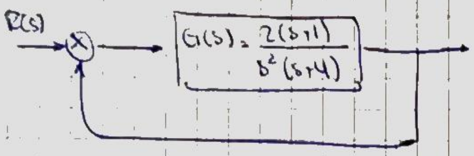
Legajo 41500 Mercado Nicolas — Tema 2

Ejercicio N°6

5p



$R(s) = \frac{1}{s} + \frac{2}{s^2}$

$E(s) = \frac{R(s)}{[1+G(s)]}$

Tipo \ entrada	$\frac{1}{s}$	$\frac{1}{s^2}$	$\frac{1}{s^3}$
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

El Tipo de sistema es 2

$e(\infty) = \lim_{s\to 0} \cancel{s}\, \frac{1}{1+\frac{2(s+1)}{s^2(s+4)}} \cdot \frac{1}{\cancel{s}} \xrightarrow{} 0 + \lim_{s\to 0} s\, \frac{1}{1+\frac{2(s+1)}{s^2(s+4)}} \cdot \frac{2}{s^2} \xrightarrow{} 0 = 0$ ✓

Ejercicio N°7 → error a una entrada escalón menor al 10%

10p

$G(s) = \frac{K}{(s+1)(s+2)(s+3)} = \frac{K}{(s^2+2s+s+2)(s+3)} = \frac{K}{s^3+3s^2+\cancel{6}+2s+3s^2+9s+6} = \frac{K}{s^3+6s^2+11s+6}$

La EC es $1+G(s) = 1+\frac{K}{s^3+6s^2+11s+6} = 0 \rightarrow s^3+6s^2+11s+6+K = 0$

s^3	1	11
s^2	6	6+K
s^1	$\frac{60-K}{6}$	0
s^0	6+K	

$b_1 = \frac{6\cdot 11-(6+K)}{6} = \frac{66-(6+K)}{6} = \frac{60-K}{6}$ $\qquad \frac{60-K}{6} > 0 \rightarrow 60-K>0 \Rightarrow K<60$

$c_1 = \frac{\frac{(60-K)}{6}(6+K)}{\frac{60-K}{6}} = 6+K$

El sistema es de tipo 0

$e(\infty) = \frac{1}{1+K_p} \quad K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{K}{s^3+6s^2+11s+6} = \frac{K}{6}$ ✓

El error debe ser de menos de 10%, entonces:

$\frac{1}{1+\frac{K}{6}} < 0{,}1 \rightarrow \frac{1}{0{,}1} < 1+\frac{K}{6} \rightarrow 10-1 < \frac{K}{6} \rightarrow 9.6 < K \rightarrow K > 54$

Los valores para el cual K hace estable y con un error menor al 10% es

$\underbrace{54 < K < 60}$ ✓

Ejercicio N°8

6p

a) $G(s)H(s) = \frac{K}{s(s^2+5s+6)} = \frac{K}{s^3+5s^2+6s} = \frac{K}{s(s+2)(s+3)}$

3 polos en: 0; -2; -3 ; no tengo ceros

Como hay 3 polos en lazo abierto hay 3 ramas ~~de~~ entre 0 y -2 y entre -3 y -∞ ✓

b) $\sigma_A = \frac{0-2-3}{3} = -\frac{5}{3}$ centroide → -1,66 ✓

c) Angulo de los asintotas $\phi = \frac{(2k+1)180°}{n-m}$ $k=0,1,2$

$\phi_1 = \frac{180°}{3} = 60°$ ✓ ; $\phi_2 = \frac{3}{3} \cdot 180° = 180°$ ✓ ; $\phi_3 = \frac{5}{3} 180° = 300°$ ✓

d) Punto de alejamiento $\frac{dk}{ds} = 0$

$K = s^3+5s^2+6s \rightarrow \frac{dk}{ds} = 0 = 3s^2+10s+6$

$S_1 = -0{,}78$ → punto de alejamiento ✓

$S_2 = -2{,}54$

e) Punto de intersección en el eje imaginario

$1+G(s)H(s) = 1 + \frac{K}{s^3+5s^2+6s} = 0$

$s^3+5s^2+6s+K=0$

s^3	1	6
s^2	5	K
s^1	$\frac{30-K}{5}$	0
s^0	K	

$b_1 = \frac{5 \cdot 6 - 1 \cdot K}{5}$

$c_1 = \frac{\left(\frac{30-K}{5}\right)K - 0}{\frac{30-K}{5}} = K$

$\frac{30-K}{5} \geq 0 \rightarrow 6 \geq \frac{K}{5}$

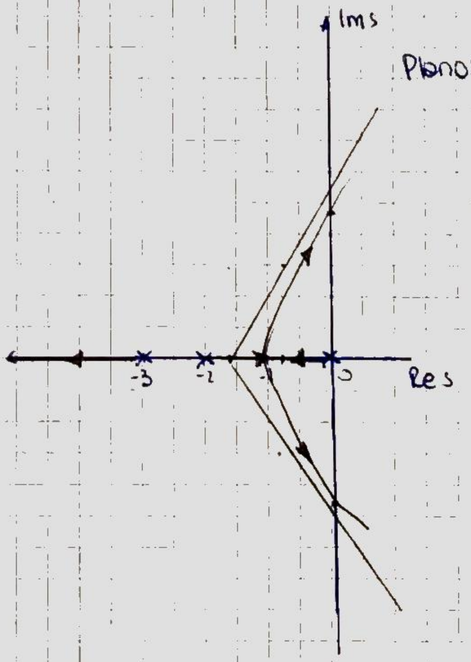
$K \leq 30$

$\therefore \boxed{K^* = 30 \rightarrow K \text{ crítico}}$ ✓

f) $5s^2 + K^* = 0 \rightarrow 5s^2+30=0$

$s = \sqrt{-\frac{30}{5}} = \pm j2{,}44$ ✓

Cruces en el eje imaginario



Nm

1er Parcial - Sistemas de control 2020 Hoja 4-4

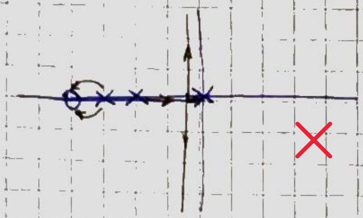
Legajo 41500 Mercado Nicolás Tema 2

Responder:

a) A las zonas cercanas al origen ✗ para valores altos de K

b) K* disminuye al agregar un polo en $s=-4$ ✗ Aumenta

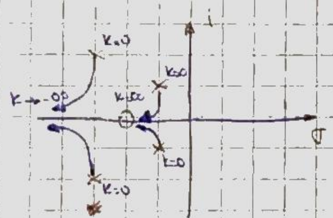
c) Al agregar el polo el sistema tiende más rápido a lo estable



✗ Un cero, hace al sistema estable para cualquier valor de K

d) Primero se obtiene la ecuación característica, luego se la ingresa en la tabla de Routh-Hurwitz y se obtiene el valor de k de los elementos de la 1era columna para que sean mayores a 0 y el sistema será estable ✗ No responde a la pregunta

Ejercicio N° 9



✗ Falta repasar un poco LdR

Ejercicio N° 10 2.5p

1) V ✓ 5) V ✗

2) V ✓ 6) V ✗

3) F ✗ 7) F ✗

4) F ✗ 8) V ✗