Exploring the Muon g-2 Anomaly

Topical Report for PH503

Master of Science

by

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Abstract

The recent Muon g-2 experiment conducted at Fermi National Accelerator Laboratory (Fermilab) has revealed a compelling deviation in the magnetic moment of the muon from the theoretically predicted framework of the Standard Model (SM) of particle physics. The muon's magnetic anomaly given by,

$$a_\mu = \frac{g_\mu - 2}{2} \tag{1}$$

plays a crucial role in the history of the Standard Model (SM), representing one of its most precisely calculable quantities. The significant discrepancy between the experimental measurement and the theoretical prediction implies the potential existence of physics Beyond the Standard Model (BSM). In this report, I have studied the origins of the anomaly and briefly discussed about methodology employed at Fermilab to measure the muon's magnetic anomaly, and explored the implications of this deviation.

Declaration

This is to declare that the project report I submitted to the Department of Physics, Indian Institute of Technology, Guwahati, under the title ”Exploring the Muon g-2 Anomaly,” partially fulfills the requirements for the coursework of PH503. I hereby confirm that this report is entirely based on my own understanding and research, and I've given credit to all the materials and resources I that I have used.

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I would like to thank my course instructor, Prof. Bipul Bhuyan, for giving me an opportunity to work on this exciting project. Second, I would like to my family for always being there and to my friends for some useful constructive discussion on the topic.

Contents

Chapter 1

Introduction

1.1 The Standard Model of Particle Physics

The Standard Model of Particle Physics and General Relativity are two of the most successful frameworks in the history of Physics. With the discovery of the Higgs Boson at the Large Hadron Collider in 2012 and defending a multitude of experimental tests over the past decades, the Standard Model of particle physics remains an incredibly precise framework for understanding phenomena at the smallest scales

Particle	**Symbol**	**Charge**	**Mass (GeV/c^2)**
Leptons			
Electron	e^-	$-e$	0.511
Electron Neutrino	ν_e	0	$< 2 \times 10^{-9}$
Muon	μ^-	$-e$	105.66
Muon Neutrino	ν_μ	0	< 0.17
Tau	τ^-	$-e$	1.77686
Tau Neutrino	ν_τ	0	< 18.2
Quarks			
Up	u	$+\ 2/3e$	2.2 ± 0.6
Down	d	$-\ 1/3e$	4.7 ± 0.5
Charm	c	$+\ 2/3e$	1.27 ± 0.03
Strange	s	$-\ 1/3e$	93 ± 11
Top	t	$+\ 2/3e$	173.21 ± 0.51
Bottom	b	$-\ 1/3e$	4.18 ± 0.03
Force Carriers			
Photon	γ	0	0
Gluon	g	0	0
Z Boson	Z^0	0	91.1876 ± 0.0021
W Boson	$W^\pm$	$\pm e$	80.379 ± 0.012
Higgs Boson	H^0	0	125.09 ± 0.24

Table 1.1 The Standard Model of Particle Physics [1]

The standard model of particle physics as shown in table 1.1 describes elementary particles and their force carriers that govern their interactions. The quarks and leptons are called fermions with half integer spin while the force carriers are bosons with integer spin.

Experiments in particle physics aim to explore the unanswered questions and validate the predictions of the standard model. Experiments like the Large Hadron Collider (LHC) at CERN, Fermilab, German Electron Synchrotron (DESY), KEK, Brookhaven National Laboratory and IceCube Neutrino Observatory are some current ongoing research facilities studying these fundamental particles by measuring their properties and possibly looking for physics beyond the standard model.

Experiment	Years	Particle	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11620000(50000)	4300
CERN II	1962-1968	μ^+	11661600(3100)	270
CERN III	1974-1976	μ^+	11659110(110)	10
CERN III	1975-1976	μ^-	11659370(120)	10
BNL	2000	μ^+	11659204(9)	0.73
BNL	2001	μ^-	11659214(9)	0.72
Average			11659208.0(6.3)	0.54

Fig. 1.1 Timline of the Muon g-2 experiments. This figure is taken from [2]

In the following literature I've discussed about the idea for getting these numbers. In appendix, I briefly discuss about the importance of σ or 5 σ a standard for a discovery. But before going to the anomalous value of muon magnetic moment. We ask ourselves, "Why Muon?" which is the topic of discussion next.

1.2 Why Muons?

In table 1.1 I've shown the properties of the particles in the Standard Moddel. Let's narrow down to leptons and talk about their properties.

Particle	**Mean Life Time [s]**	**Relative Mass (wrt e^-)**
Electron (e^-)	*Stable*	1
Muon (μ^-)	2.2×10^{-6}	206.87
Tau (τ^-)	290.3×10^{-15}	3483.56

Table 1.2 Leptons [2]

Theoretically one could calculate the anomalous magnetic moment for all leptons, however experimental point of view it's not that straightforward. Measuring a_e is much easier than measuring a_μ or a_τ. This problem can be seen in table 1.2. As lifetime of μ and τ particles are much smaller, it is hard to measure their properties experimentally. The value of a_e is one of the most precise value measured and it matches significantly to the value predicted by the standard model. However, contributions to a_l at high energy scales are proportional to $\Delta a_l \approx \frac{m_l^2}{m_x^2}$, where m_l is the lepton mass and m_x is the mass of a particle beyond the standard model. As we can see the muon mass with respect to electron is about 200 and the square of the numbers would be 40000. Looking at the

Δa_l expression again, it tells us that a_μ is a better parameter to measure to search for beyond the standard model particles. Measuring a_τ would be even better to look for new particles as it's ratio is nearly 3500 but it comes with challenges. The short lifetime of the τ particle would require a notable Lorentz Boost in the accelerators and as τ decays into hadronic states it is easier to work with μ whose decay channel consists of an electron and two neutrions. Thus muon falls into the *Goldilock's Zone* in order to search for physics beyond the standard model.

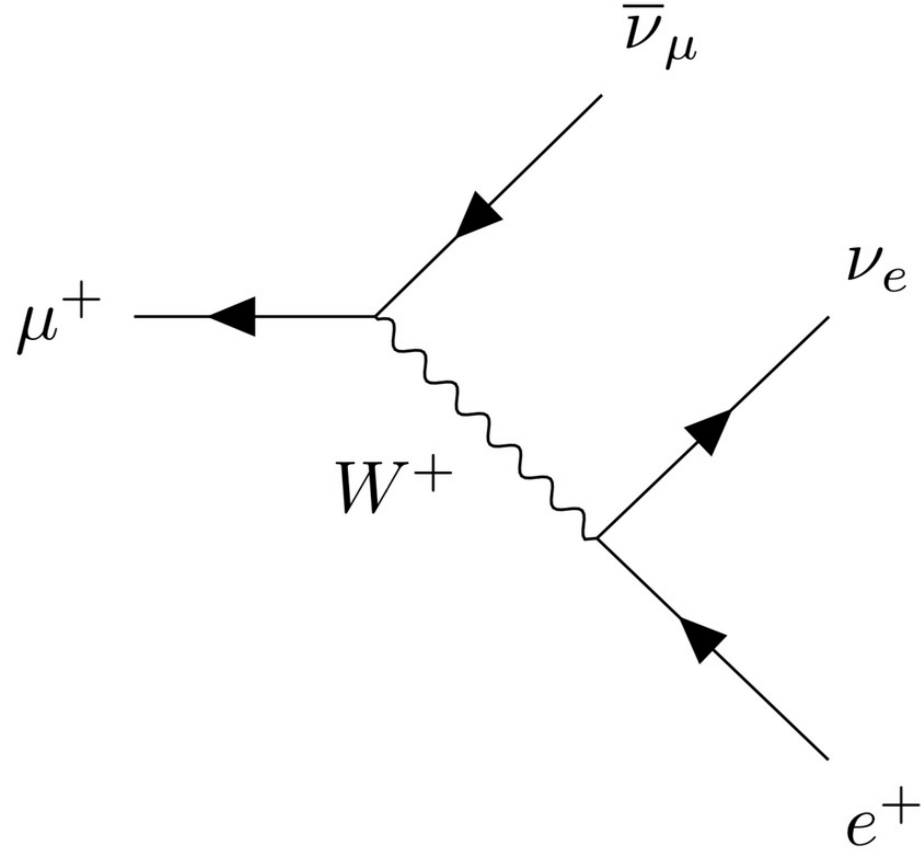


Fig. 1.2 A positive muon decaying to a positron and two neutrinos [3]

Chapter 2

The Classical Picture

Consider a charged particle (q) moving in a closed circular loop of radius r with a speed v. We are interested in finding the relationship between the magnetic moment $\vec{\mu}$ and the angular momentum $\vec{L}$ of the particle. The electric current, $I = \frac{q}{T}$, where T is the time period of the revolution given by, $T = \frac{2\pi r}{v}$. The general expression to calculate the magnetic moment is given as follows,

$$\mu \equiv \frac{1}{2} \int (\mathbf{r} \times \mathbf{J}(\mathbf{r}))\, d^3\mathbf{r}$$

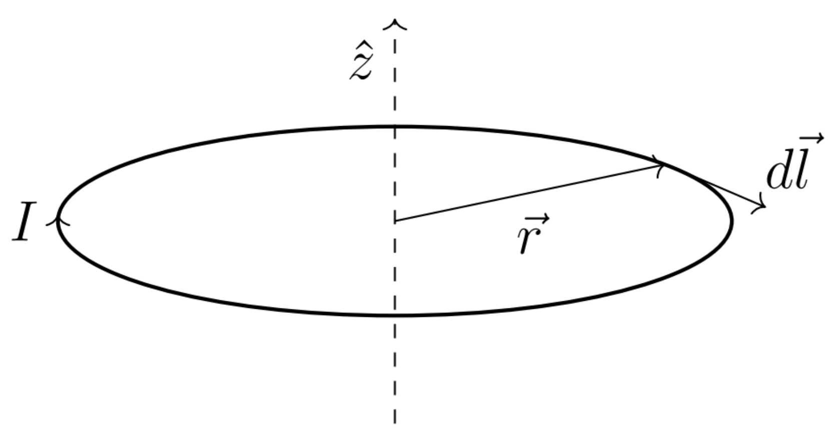


Fig. 2.1 A charged particle moving in cricle around the z - axis [2]

For a constant current and from the figure we can write $dr = dl = rd\theta$. Therefore,

$$\begin{aligned}\vec{\mu} &= \frac{I}{2}\mathbf{r^2} \int_0^{2\pi} \mathbf{d}\theta \hat{z} \\ &= I\pi r^2 \hat{z} \\ &= IA\hat{z}\end{aligned}$$

From fig 2.1 we can write, $I = \frac{qv}{2\pi r}$ and the angular momentum $\vec{L} = \vec{r} \times \vec{P} = mvr\hat{z}$. Thus, we can write the following,

$$\vec{\mu} = \frac{qvr}{2}\hat{z} = \frac{qmvr}{2m}\hat{z} = \frac{q}{2m}\vec{L} \tag{2.1}$$

When the loop shown in fig 2.1 is tilted about its plane and placed in a constant magnetic field, $\vec{B} = B\hat{z}$, the charged particle experiences a torque and the energy of the configuration is given by,

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

Going back in 1922, it was the time when Stern and Gerlack [4] performed an experiment that would question the calculations given by classical mechanics. It was seen that when a beam of neutral silver atoms were passed in a magnetic field, the beam split into two bands along the axis of applied field. This implies that the torque experienced by two bunches were different. We can conclude that the silver atoms have two different magnetic moments. In addition to this the spectroscopy experiments suggested the addition of a fourth quantum number s along with n, l and m in order to obey the Pauli's exclusion principle. Considering this into account in 1925, two physicists Samuel Goudsmit and George Uhlenbeck introduced the concept of spin and that it has m_s eigenvalues of $\pm\frac{1}{2}$. Considering this into account the Landè g factor was introduced and the equation 2.2 was written with g = 2 in the case of electrons as suggested from the experiments. However, it can also be derived from the Dira'c equation which is suggested in the next chapter. The generic expression for magnetic moment of a charged particle with spin, S is given as,

$$\vec{\mu} = \frac{gq}{2m}\vec{S} \tag{2.2}$$

Where q is the charge of the particle, m is the mass of the particle, S is the spin vector and g is called the Landè g factor. From equation (2.1) and (2.2) we can conclude that, $\mathbf{g = 1}$

Chapter 3

The Dirac Picture

In order to incorporate special relativity into quantum mechanics that is placing x and t on equal footing, Paul Dirac in 1928 [5] introduced a wave equation given by,

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0 \tag{3.1}$$

Where, γ^{μ} are 4×4 matrices with special properties [2]. The equation (3.1) includes the concept of spin and antiparticles. One could obtain the value of Landè g factor, g = 2 by considering the evolution of the dirac Hamiltonian for an electron interacting with an electromagnetic field [2]. Here, I'm interested in knowing the deviation of g value from 2 which is what I talk about in the following literature.

3.1 The Deviation

One of the experiments that showed deviation from the g value predicted by Dirac was performed by Kusch and Foley in 1947 [6]. They studied spectral lines of Gallium due to Weak Zeeman effect as $B_{external} = 380$ Gauss which is less than the internal magnetic field. They studied the g_j factors of $2p_{\frac{3}{2}}$ and $2p_{\frac{1}{2}}$ which is calculated below.

Consider an electron orbiting around the nucleus placed in an external magnetic field. There will be two contributions to the magnetic moment. One due to spin of the electron ($\vec{S}$) and the other due to orbital motion of the electron ($\vec{L}$).

$$\begin{aligned} \vec{\mu_s} &= -\frac{ge}{2m}\vec{S} \\ \vec{\mu_l} &= -\frac{e}{2m}\vec{L} \end{aligned} \tag{3.2}$$

Where, g is the Landè g factor and m is the mass of the electron. Now the Hamiltonian can be written as,

$$\begin{aligned} H &= -(\vec{\mu_s} + \vec{\mu_l}) \cdot \vec{B_{ext}} \\ &= \frac{e}{2m}(\vec{L} + g\vec{S}) \cdot \vec{B_{ext}} \end{aligned} \tag{3.3}$$

We're working in week Zeeman approximation as $B_{ext} << B_{int}$. Thus my good quantum numbers are $\mathbf{n}, \mathbf{l}, \mathbf{j}, \mathbf{m_j}$. Now I'm interested in calculating the energy eigen value corresponding to the Hamiltonian (3.3)

$$
\begin{aligned}
E &= \langle nljm_j|H|nljm_j\rangle \\
&= \frac{e}{2m}\vec{B_{\text{ext}}}\cdot\langle\vec{L}+\vec{S}\rangle
\end{aligned} \tag{3.4}
$$

Here $\vec{J}=\vec{L}+\vec{S}$. I can use the projection theorem and write, $\vec{S_{avg}}=\frac{(\vec{S}\cdot\vec{J})\vec{J}}{|J^2|}$

$$
\begin{aligned}
\vec{S_{avg}} &= \frac{(\vec{S}\cdot\vec{J})\vec{J}}{|J^2|} \\
\vec{L}+g\vec{S} &= \vec{J}+(g-1)\vec{S}
\end{aligned}
$$

Using these results and we know the energy eigenvalues of L^2, S^2 and J^2 operators,

$$
E = \frac{e}{2m}\vec{B_{\text{ext}}}\cdot\langle(1+(g-1)\frac{(\vec{S}\cdot\vec{J})}{|J^2|}))\vec{J}\rangle \tag{3.5}
$$

$$
\begin{aligned}
\vec{J} &= \vec{L}+\vec{S} \\
L^2 &= J^2+S^2-2\vec{J}\cdot\vec{S} \\
\vec{J}\cdot\vec{S} &= \frac{1}{2}(J^2+S^2-L^2)
\end{aligned}
$$

Therefore,

$$
\vec{J}\cdot\vec{S} = \frac{1}{2}(j(j+1)+s(s+1)-l(l+1)) \tag{3.6}
$$

Substituting equation (3.6) in equation (3.5) we get,

$$
E = \frac{e}{2m}\vec{B_{\text{ext}}}\cdot\langle(1+(g-1)\frac{(j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}))\vec{J}\rangle \tag{3.7}
$$

We define the quantity in brackets as g_j and considering magnetic field along the z axis as shown in fig 1.1,

$$
g_j = 1+(g-1)\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)} \tag{3.8}
$$

$$
E = \frac{e}{2m}B_{ext}g_jm_j \tag{3.9}
$$

Thus by measuring the the energy eigen value of $2P_{3/2}$ and $2P_{1/2}$ one could measure the g_j value which is exactly what Kusch and Foly did in 1947. According to Dirac, putting $g=2$ the expected ratio would be $\frac{g_{3/2}}{g_{1/2}}=2$. But the measured value was slightly greater than 2 which provided an indirect measure for the g value of electron: $g_e=2.00229\pm0.00008$. This is where Quantum Field Theory comes into picture.

3.2 Radiative Corrections

The Dirac equation (3.1) tells us about a single particle interaction with the field. However, as many particle interactions are possible in the framework of QFT. It considers all the possible interaction of a particle with the field. The so called Feynman diagrams calculates the possible interactions with virtual particles which are not detectable but have significant contribution to the observables (radiative corrections). Considering the framework of QFT, the g factor for electron calculated is slightly greater than 2 and matching with the experiment with very high precision. The magnetic anaomly of electron is defined as,

$$a_e = \frac{ge - 2}{2} \tag{3.10}$$

The current best measured value of a_e is listed below.

$\mathbf{a_e(theory) = 1159652181.72(77) \times 10 - 12}$ [7]

$\mathbf{a_e(experiment) = 1159652180.73(28) \times 10 - 12}$ [7]

where the number in brackets is the uncertainty multiplies by the decimal points of the actual number. Thus this gives us an uncertainty of $\mathbf{0.66ppb}$ for theory and $\mathbf{0.25ppb}$ for experiment, where $1ppb = 10 - 9$. [7] Thus the difference between theory and experiment is,

$\mathbf{a_e(theory) - a_e(experiment) = 0.99(82) \times 10 - 12}$ (The experimental and theoretical uncertainties add in quadrature.)

However, we're interested in calculating the magnetic anomaly of muon. The principal behind measuring the g-2 value at FERMILAB is discussed in the next chapter.

Chapter 4

a_μ at FNAL

4.1 Measuring Principle

Almost 22 years ago, the E821 experiment at Brookhaven National Laboratory (BNL) announced the value of a_μ with 0.73 ppm precision. (Fig 1.1). The new E989 experiment at Fermi National Accelerator Laboratory (FNAL) is designed to measure the a_μ value with a precision of 0.14 ppm. Here, I'll briefly discuss about the principle in measuring the g-2 value without going into experimental details.

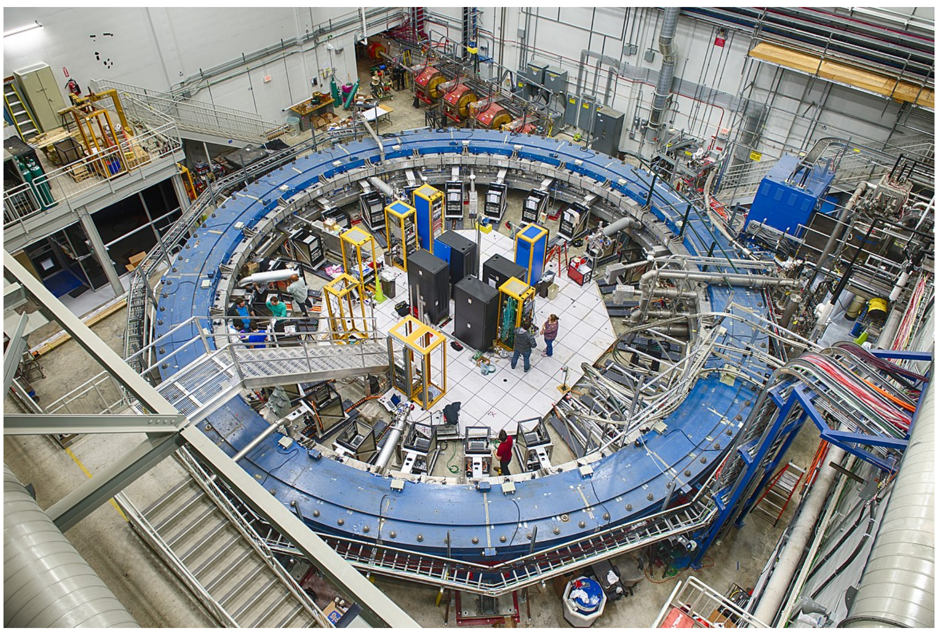


Fig. 4.1 Storage Ring at FNAL[8]

Polarized positive muons are injected into the storage ring at FNAL, which are produced from pion decays. The muons spin precises in presence of the orthogonal magnetic field to the ring plane. The frequency at which the spin vector rotates is called the Larmor Frequency and is given by,

$$\omega_S = \frac{ge}{2m}B + (1-\gamma)\frac{e}{m\gamma}B \tag{4.1}$$

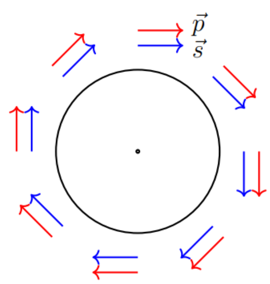
Where, γ is the Lorentz factor and B is the applied field. Also, as the positive muons are orbiting in the cyclotron. They have a cyclotron frequency given by,

$$\omega_C = \frac{e}{m\gamma}B \tag{4.2}$$

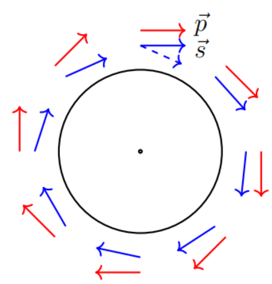
Now taking the difference of equation (4.2) and (4.1), we get the "anomalous precision frequency (ω_a)" i.e. the rate at which muon spin rotates with respect to its orbital momentum in the lab frame. [2]Therefore,

$$\omega_a \equiv \omega_S - \omega_C = a_\mu \frac{e}{m}B \tag{4.3}$$

One can extract a_μ by measuring ω_a and B at very high precision.



(a) If $g = 2$. The muon spin and momentum vectors remain parallel.



(b) If $g > 2$. The dashed blue arrow is the spin vector after one turn.

Fig. 4.2 ω_s relative to ω_c[2]

As the positive moun would eventually decay into a positron (e^+) and two neutrinos, the calorimeters at FNAL detect these highly energetic positrons (above some threshold E_{th})and measure its energy. From the measured value of energy one can determine the spin of the muons as highly energetic positrons spin would be alligned preferentially in the dirction of muon spin.[2] Now as the rate at which muon spin preccese with respect to orbital motion is given by equation (4.2). The rate of detecting positrons above E_{th} would also have the same frequency. i.e. ω_a. The counts of positrons above the threshold energy is governed by the equation,

$$N_{e^+count}(t) = N_0 \exp\left(-\frac{t}{\gamma\tau_\mu}\right)[1 + A\cos(\omega_a t + \phi)] \tag{4.4}$$

Where, measuring the count rate of e^+would give us the measurement on ω_{mu}. Here, $\gamma\tau_{mu}$ is the life time of muon in the bossted frame.

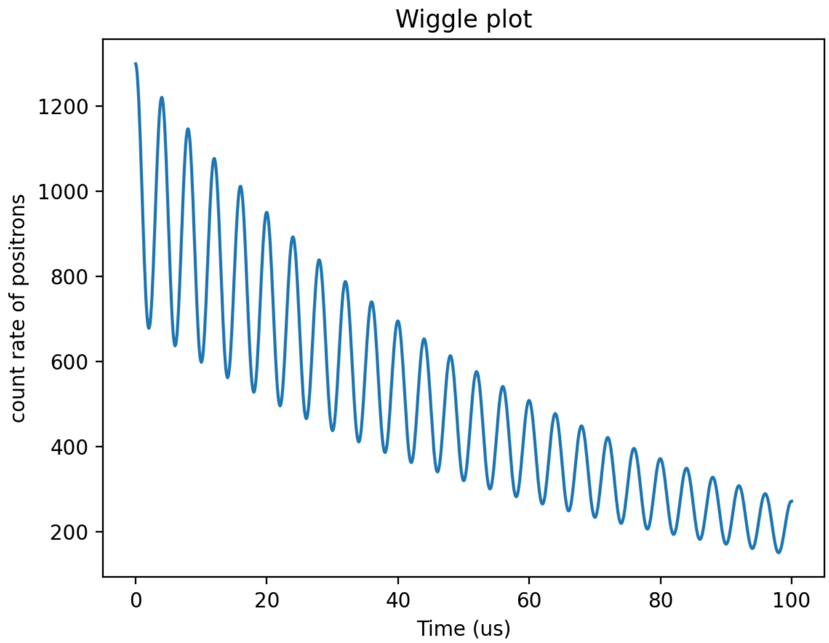


Fig. 4.3 A toy diagram plotted in python with $\omega_a = \pi/2, \phi = 0$

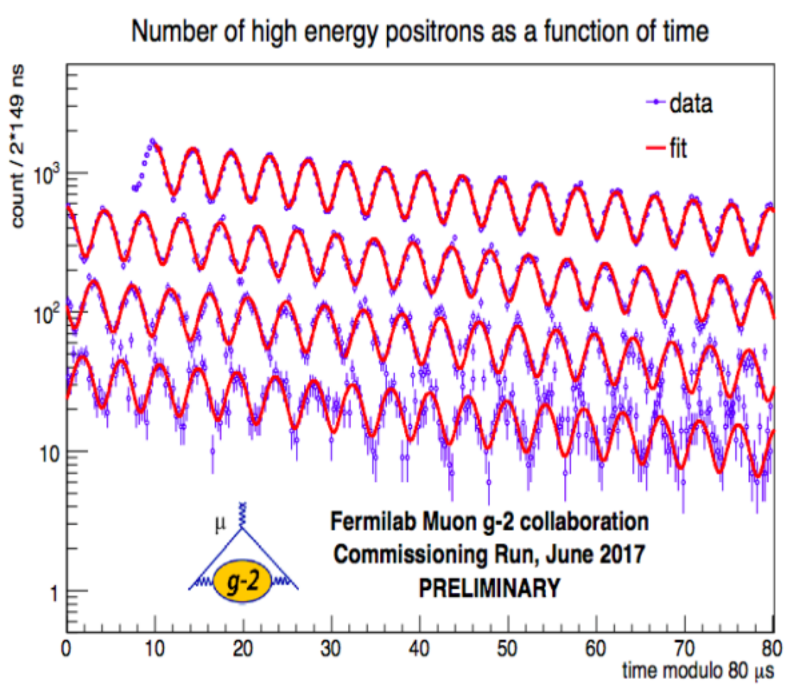


Fig. 4.4 Fermilab data plot. Taken from [9]

4.2 The Result

The combined result from run-1,2,3 at FNAL on a_μ is,

$\mathbf{a_\mu(FNAL) = 116592055(24) \times 10^{-11}}$ [10]

with the precision of 0.20 ppm and the combined BNL and FNAL average is with 0.19 ppm precision,

$\mathbf{a_\mu(exp) = 116592059(22) \times 10^{-11}}$ [10]

From theory,
$\mathbf{a_\mu(theory) = 116591794(53) \times 10^{-11}}$ Taken from [11]

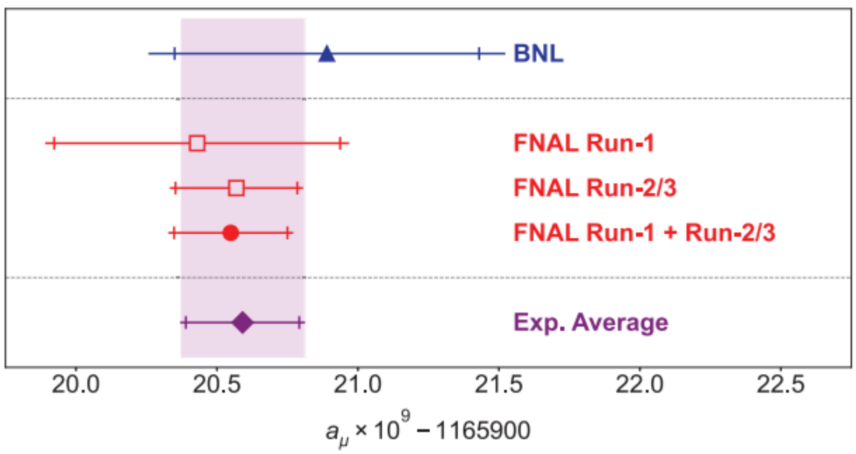


Fig. 4.5 Fermilab 2023 result [10]

$\mathbf{a_\mu(exp) - a_\mu(theory) = 265(57) \times 10^{-11}}$
This difference gives a 4.6σ deviation. However, the current deviation between theory and experiment is around 5σ. (The theory data of a_μ used here is from 2016 by [11])

Chapter 5

Summary

5.1 Conclusion

The anomaly found in measuring the g-2 of the muon can be a tantalizing hint for physics beyond the Standard Model. However, the current 5 σ discrepancy is not enough to announce it as a discovery as we're stepping into the unknown. One of the Beyond the Standard Model possibility is the inclusion of a Dark Matter particle and looking at its contribution to the muon g-2 value. However, run 4 and run 5 results are still awaited and to be announced by 2025 keeping the excitement alive. Below I've included the result from the previous run in 2021.

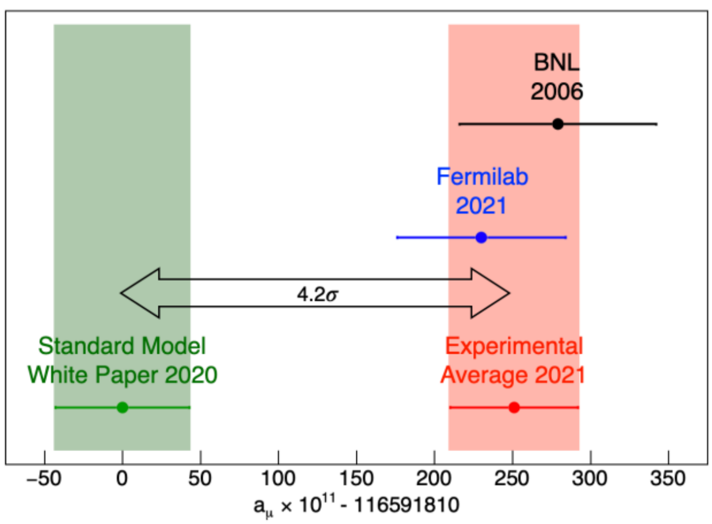


Fig. 5.1 Fermilab 2021 result [12]

5.2 Future Outlook

In this report I've explored the measurement principle from theoretical side. However, I'd also like to study the experimental side of the story. I'd also like to work on Beyond the Standard Model models, like the inclusion of a Dark Matter particle in order to resolve the anomaly.

Chapter 6

Appendix

6.1 Understanding Sigma (σ)

Consider a random walk experiment where the events are not correlated, one can find that events follow a Gaussian distribution. The gaussian function can be written as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{6.1}$$

Where, μ is the mean and σ indicates the spread or uncertainty in the measurement. Below we can two Gaussians with same mean but different standard deviation.

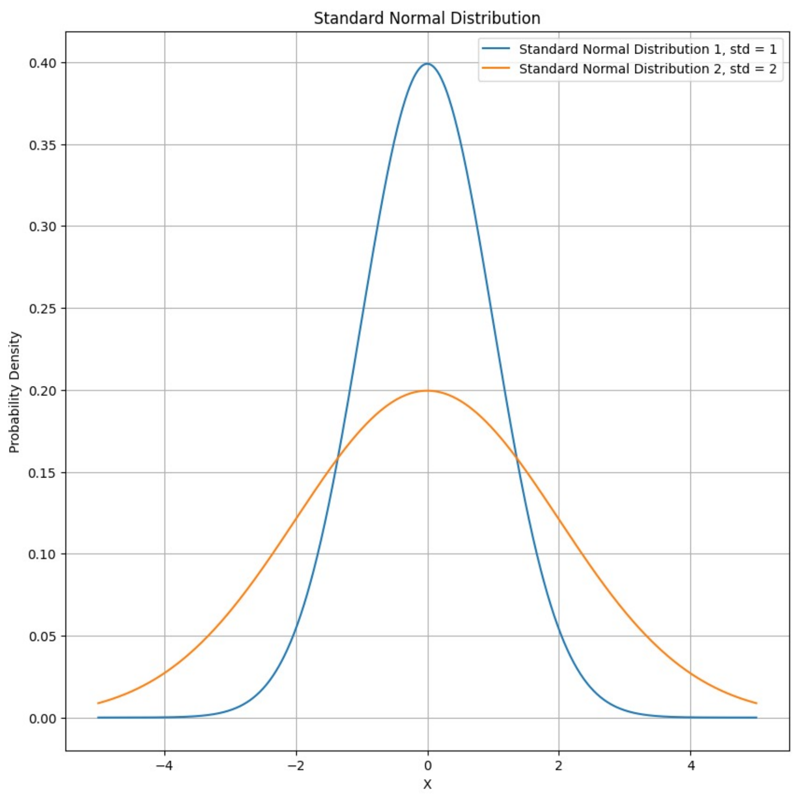


Fig. 6.1 Gaussian distribution. PLotted in python

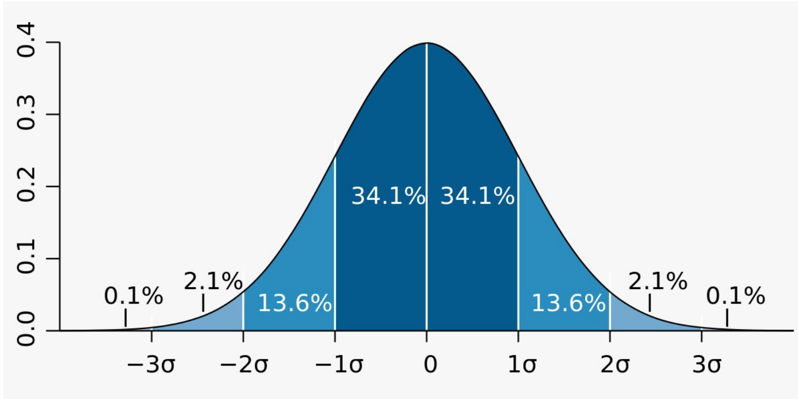


Fig. 6.2 Taken from [13]

Statistical significance is the extent to which a particular data point deviates from its mean value and is expressed in terms of standard deviations from the mean. In data with a gaussian distribution, the likelihood of a data point within one sigma , of the mean value is 68% for two sigma it is 95% and so on as indicated in figure (6.2).

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