1.

$$\lim_{h \to 0} \frac{e^4 e^h - e^4}{h}$$ =

Solution

As h ---> 0

lim $(e^4 e^h - e^4) / h = (e^4 e^0 - e^4) / 0$

= 0 / 0 , indeterminate form

Another approach is needed. Let $f(x) = e^x$. The given limit may be written as follows: as x ---

h

lim $(e^4 e^h - e^4) / h$ = lim $(e^{4+h} - e^4) / h$ = limit [f(4+h) - f(4)] / h

which is the definition of the first derivative of $f(x) = e^x$ at x = 4. Hence as x ---> h

lim $(e^4 e^h - e^4) / h = e^4$

2.

The graph of function g defined by

$$g(x) = \frac{x^3 + 2x^2 - 3x}{x^2 + 2x - 3}$$

will have vertical asymptotes at

Solution

Let us first simplify, if possible, the given rational function

$g(x) = (x^3 + 2x^2 - 3x) / (x^2 + 2x - 3)$

$= x (x^2 + 2x - 3) / (x^2 + 2x - 3) = x$

Function g has no vertical asymptotes

3.

Given that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

find

$$\lim_{x \to 0} \frac{x + 4x^2 + \sin x}{3x}$$

Solution

Using the theorem that states that the limit of a sum is equal to the sum of the limits. Hence

x ----> 0,

lim $(x + 4x^2 + \sin x) / 3x$ = lim $(x / 3x)$ + lim $(4x^2 / 3x) + (1/3) \sin x / x$

Simplify

= lim $(1/3)$ + lim $(4x) + (1/3) \sin x / x$

= $1/3 + 0 + (1/3)*1 = 2/3$

4.

Function f is defined by

$$f(x) = 2x^3 \sin(x) + \frac{1}{x}\tan(x) + x\sec(x) + 2$$

Find df(x) / dx.

Solution

Using the theorem that states that the derivative of a sum of functions is the sum of the derivatives, we can write

$d/dx\,[\,2x^3\sin(x) + (1/x)\tan(x) + x\sec(x) + 2\,]$

$= d/dx\,[\,2x^3\sin(x)\,] + d/dx\,[\,(1/x)\tan(x)\,] + d/dx\,[\,x\sec(x)\,] + d/dx\,[\,2\,]$

we now calculate the derivative of each term above

$d/dx\,[\,2x^3\sin(x)\,] = 2\,[\,3x^2\sin(x) + x^3\cos(x)\,] = 6x^2\sin(x) + 2x^3\cos(x)$

$d/dx\,[\,(1/x)\tan(x)\,] = -(1/x^2)\tan(x) + (1/x)\sec^2(x))$

$d/dx\,[\,x\sec(x)\,] = \sec(x) + x\sin(x)\sec^2(x)$

$d/dx\,[\,2\,] = 0$

Hence

$df/dx = 6x^2\sin(x) + 2x^3\cos(x) - (1/x^2)\tan(x) + (1/x\sec^2(x)) + \sec(x) + x\sin(x)\sec^2(x)$

Let us calculate the first derivative. Differentiate both sides of the given equation

$0.25 (2x) + 2 y y' = 0$

$y' = - 0.5 x / (2 y)$

We now solve the given equation $0.25 x^2 + y^2 = 9$ for x

$x = + \text{ or } - \text{sqrt} [(9 - y^2) / 0.25]$

Substitute x in $y' = - 0.5 x / (2 y)$ by + or - sqrt $[(9 - y^2) / 0.25]$

$y' = - 0.5 (+ \text{ or } - \text{sqrt} [(9 - y^2) / 0.25] / (2 y)$

The slope of the tangent is equal to 1. Hence

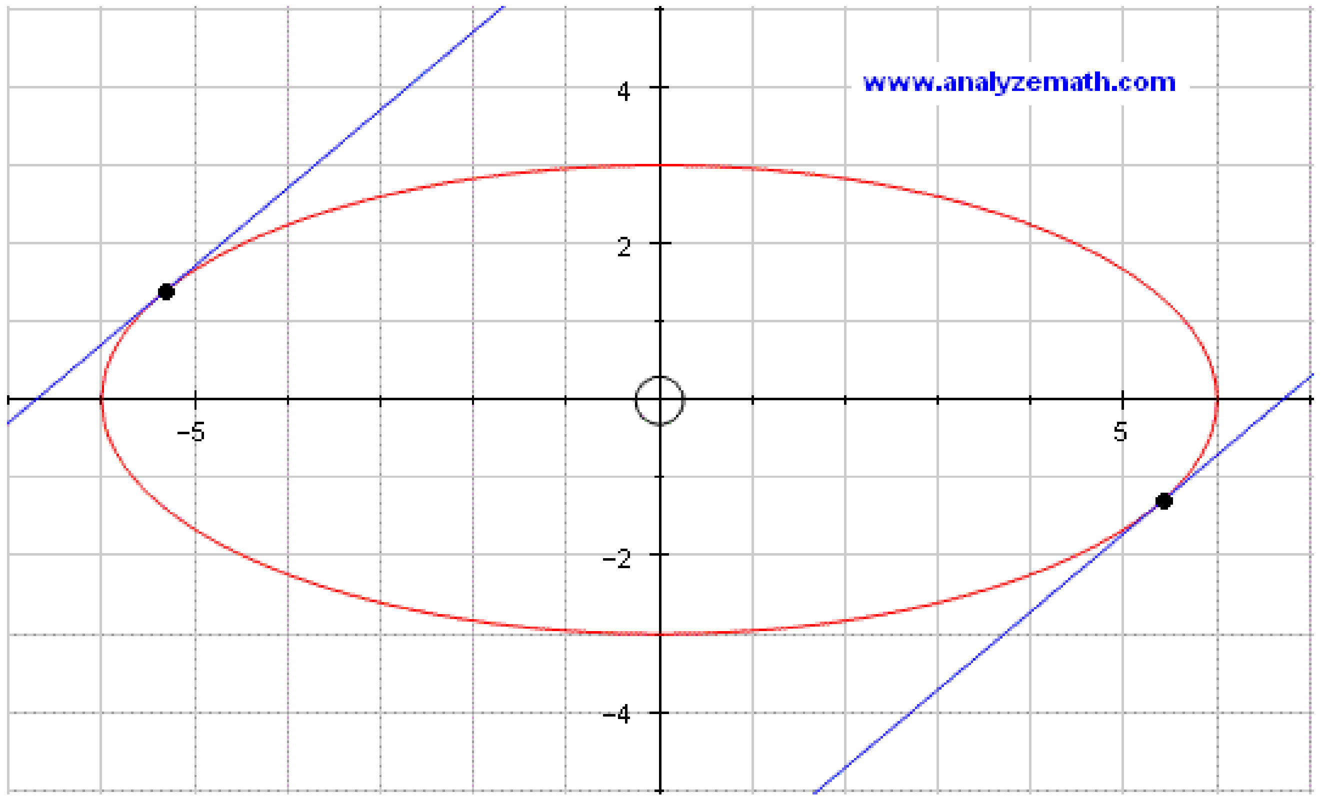
$- 0.5 (+ \text{ or } - \text{sqrt} [(9 - y^2) / 0.25])/ (2 y) = 1$

Solve the above for y. Two solutions

$y = 3 \text{sqrt}(5) / 5$, $y = - 3 \text{sqrt}(5) / 5$

The graph of $0.25 x^2 + y^2 = 9$ is shown below with the two tangent lines.

Q.NO 5



Solution

The variable in the given differential equation may be separated as follows

$y^2\,dy = \cos(x)\,dx$

Integrate both sides

$\int y^2\,dy = \int \cos(x)\,dx$

$(1/3)\,y^3 = \sin(x) + C$, constant of integration

We now use the condition $y(\pi/2) = 0$ to find the constant C

$(1/3)\,y^3(\pi/2) = \sin(\pi/2) + C$

$0 = 1 + C$

$C = -1$

Q.NO 6

Substitute C by -1 in $(1/3)\,y^3 = \sin(x) + C$ and solve for y

$(1/3)\,y^3 = \sin(x) - 1$

$y^3 = 3(\sin(x) - 1)$

$y = (3\sin(x) - 3)^{1/3}$

7.

$$\int \cos^4(x)\sin(x)\,dx =$$

Solution

Let u = cos x and therefore du/dx = - sin x. We now substitute cos x by u and sin x by -du/dx

the given integral. Hence

$$\int \cos^4(x)\sin(x)\,dx = \int u^4\,(-du/dx)\,dx$$

$$= -\int u^4 du$$

$$= (-1/5)u^5 + C \text{ , C constant of integration}$$

$$= (-1/5)\cos^5(x) + C$$

8.

$$\frac{d}{dx}\int_{3}^{2x}\sin(t^2+1)dt =$$

Solution

Let u = 2x and therefore du/dx = 2 or dx = du / 2. Hence the given integral becomes

$d/dx \int_3^{2x} \sin(t^2 + 1)\, dt = 2\, d/du \int_3^{u} \sin(t^2 + 1)\, dt$

using the fundamental theorem of calculus, we obtain

$= 2 \sin(u^2 + 1)$

Substitute u by 2x

$= 2 \sin(4x^2 + 1)$

9.

$$\int_0^{10} (|4 - x| + |2 - 2x|)\, dx =$$

Solution

We first to analyze the signs of the expressions 4 - x and 2 - 2x between the limits of integration 0 and 10. 4 - x changes sign at x = 4 and 2 - 2x changes sign at x = 1.

for x between 0 and 4: 4 - x is positive and hence |4 - x| = 4 - x

for x between 4 and 10: 4 - x is negative and hence |4 - x| = -(4 - x)

for x between 0 and 1: 2 - 2x is positive and hence |2 - 2x| = 2 - 2x

for x between 1 and 10: 2 - 2x is negative and hence |2 - 2x| = -(2 - 2x)

We now rewrite the given integral as a sum of two integrals as follws.

$$\int_0^{10} (|4 - x|+|2 - 2x|)\, dx =$$

$$\int_0^{10} (|4 - x|)\, dx + \int_0^{10} (|2 - 2x|)\, dx$$

We now calculate each of the individual integrals above as follows.

$$\int_0^{10} (|4 - x|)\, dx = \int_0^{4} (4 - x)\, dx + \int_4^{10} -(4 - x)\, dx = 8 + 18 = 26$$

and

$$\int_0^{10} (|2 - 2x|)\, dx = \int_0^{1} (2 - 2x)\, dx + \int_1^{10} -(2 - 2x)\, dx = 1 + 81 = 82$$

We now have

$$\int_0^{10} (|4 - x|)\, dx + \int_0^{10} (|2 - 2x|)\, dx = 26 + 82 = 108$$

10.

Evaluate the integral

$$\int \frac{(5 + x^{3/4})^9}{(x^{1/4})}\, dx$$

[Solution](#)

Let u = 5 + $x^{3/4}$ and therefore du/dx = (3/4) 1/$x^{1/4}$ and substitute in the given integral

$\int (5 + x^{3/4})^9 / (x^{1/4})\, dx = \int [(u^9) / (x^{1/4})] (4/3)\, x^{1/4}\, du$

$= (4/3) \int u^9\, du$

$= (4/3)\,(1/10)\, u^{10}$

$= (2/15)\,(5 + x^{3/4})^{10}$

11.

Given that function *h* is defined by

$$h(x) = (\arctan(x^3 + 1) + 2x)^4$$

find *h'(x)*.

Solution

Let u = arctan(x^3 + 1) + 2x. Hence function h can be written as

h(x) = u^4 h '(x) = 4 u^3 u'

We now let v = arctan(x^3 + 1) and calculate u '

u ' = (v ')(1 / (1 + v^2))

= ($3x^2$) / (1 + (x^3 + 1)^2)

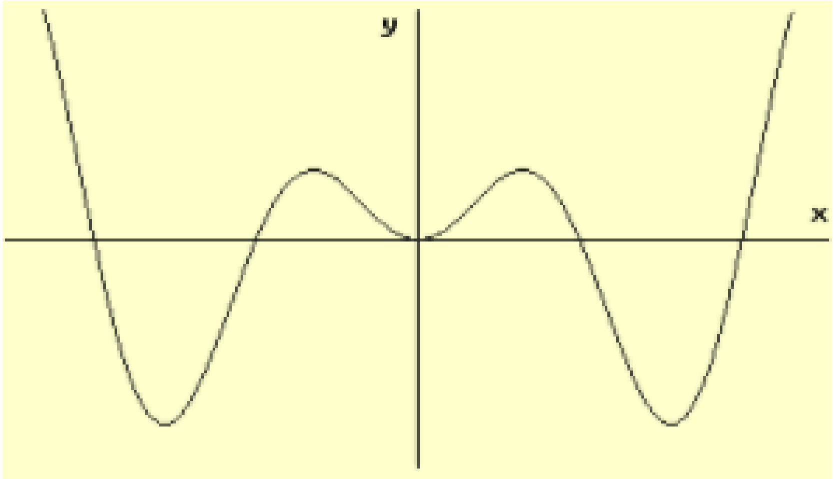
= ($3x^2$) / (x^6 + $2x^3$ + 2)

Hence

h '(x) = 4 (arctan(x^3 + 1) + 2x)^3 ($3x^2$) / (x^6 + $2x^3$ + 2)

12.

The graph of function h is shown below. How many zeros does the first derivative h' of h have?



[Solution](#)

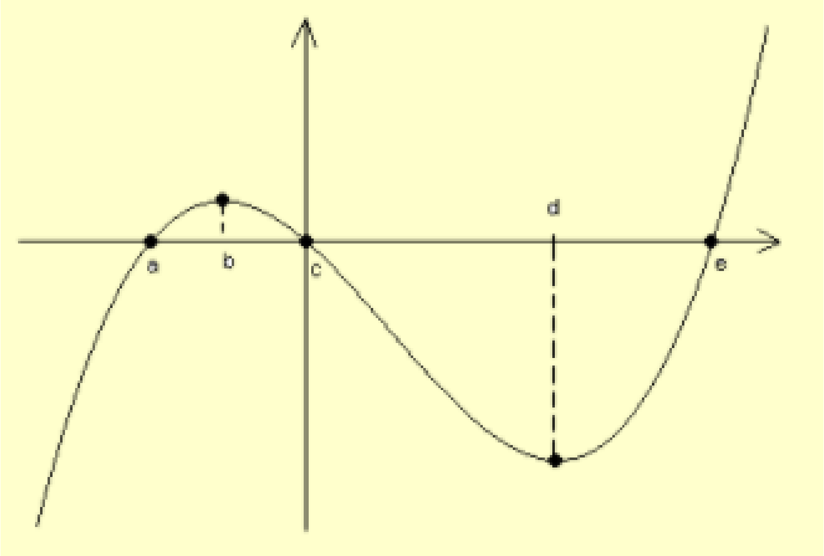
Whenever the graph of h has a local maximum or local minimum h '(x) is equal to 0. The giv

graph has 3 local minima and 2 local maxima and therefore h ' has 5 zeros.

13.

The graph of a polynomial f is shown below. If f' is the first derivative of f, then the remainder

of the division of f'(x) by x - b is more likely to be equal to



Solution

The graph of f has a local maximum at b and therefore f'(b) = 0. Since f is a polynomial then

is also a polynomial function such that f '(b) = 0 and according to the remainder theorem the

division of f '(x) by x - b is equal to o.

14.

The set of all points $(\ln(t - 2), 3t)$, where t is a real number greater than 2, is the graph of

Solution

The given parametric equations may written as

$x(t) = \ln(t - 2)$ and $y(t) = 3t$

Solve $y(t) = 3t$ for t

$t = y / 3$

Substitute t by $y / 3$ in $x(t) = \ln(t - 2)$

$x = \ln(y / 3 - 2)$

Solve for y

$y/3 - 2 = e^x$ $y = 3 (e^x + 2)$

15.

Let $P(x) = 2x^3 + Kx + 1$. Find K if the remainder of the division of P(x) by x - 2 is equal to 1(

Solution

The Remainder theorem states that the division of P(x) by x - 2 is equal to P(2). Hence

$P(2) = 2(2)^3 + K(2) + 1 = 10$

Solve for K

$K = -7/2$

16.

Function *f* is defined by

$$\begin{cases} f(x) = \dfrac{\sqrt{4x+4} - \sqrt{2x+4}}{2x} \\ f(0) = C \end{cases}$$

.

where C is a constant. What must the value of C be equal to for function *f* to be continuous a

x = 0?

Solution

For function f to be continuous at x = 0, lim f(x) as x approaches must be equal to f(0). We fi

find the limit of f(x) as x approaches 0. As x ---> 0,

lim [sqrt(4x + 4) - sqrt(2x + 4)] / 2x = 0 / 0 , indeterminate

Another approach is needed. Multiply numeartor and denominator by sqrt(4x + 4) - sqrt(2x +

4), simplify and find the limit. As x ---> 0,

lim [sqrt(4x + 4) - sqrt(2x + 4)] / 2x

= lim [sqrt(4x + 4) - sqrt(2x + 4)] [sqrt(4x + 4) + sqrt(2x + 4)]/ [2x [sqrt(4x + 4) + sqrt(2x +

]]

= lim [4x + 4 - 2x - 4] / [2x [sqrt(4x + 4) + sqrt(2x + 4)]]

= lim 2x / [2x [sqrt(4x + 4) + sqrt(2x + 4)]]

= lim 1 / [sqrt(4x + 4) + sqrt(2x + 4)]

= 1 / [2 + 2] = 1/4

In order for f to be continuous, we need to have

C = 1/4

Solution

First derivative of $(f \cdot g)(x)$

$(f \cdot g)' = f' g + f g'$

Second derivative of $(f \cdot g)(x)$

$(f \cdot g)'' = (f' g + f g')'$

$= f'' g + f' g' + f' g' + f g''$ (I)

Note that since $f'(x) = g(x)$ and $g'(x) = f(x)$, we have

$f'' = g'$ and $g'' = f'$

Substitute f'' and g'' in (I) above to obtain

$(f \cdot g)'' = g' g + f' g' + f' g' + f f'$

We now substitute g' by f and f' by g to obtain

$(f \cdot g)'' = fg + gf + fg + f g = 4 fg$

Q. NO 17

18.

The average rate of change of the function f defined by f(x) = sin(x) + x on the closed interva

[0 , pi] is equal to

Solution

The average rate of change of a function from a to b is defined by

(f(b) - f(a)) / (b - a)

Apply the above definition to the question above

(f(pi) - f(0)) / (pi - 0) = [(sin(pi) + pi) - (sin(0)+ 0)] / (pi - 0) = 1

[Solution](#)

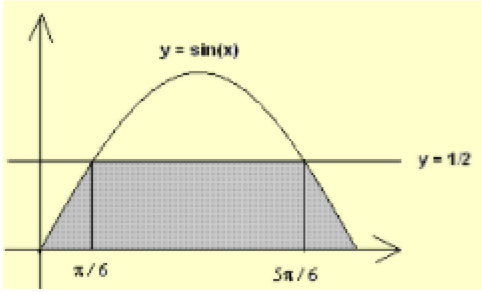
The x coordinates of the points of intersection the line y = 1/2 and the graph of f are found by

solving

$\sin(x) = 1/2$, for $0 \le x \le \pi$

solutions: $x = \pi / 6$ and $x = 5\pi / 6$

We first split the area to be calculated into 3 parts as shown below.



Calculate the area A adding the three parts (NOTING that the area in the middle is that of a rectangle):

$$A = \int_0^{pi/3} \sin(x)\, dx + 1/2(5Pi/6 - Pi/6) + \int_{5pi/6}^{pi} \sin(x)\, dx$$

$$= [- \cos(x)]_0^{pi/3} + pi / 3 + [- \cos(x)]_{5pi/6}^{pi}$$

$$= 2 + pi / 3 - \mathrm{sqrt}(3)$$

$g(x) = f(x^2) = h((x^2)^3 + 1)$

$= h(x^6 + 1)$

Let $u = x^6 + 1$. Hence

$g(x) = h(u)$, with $u = x^6 + 1$

Use chain rule to write

$g'(x) = (du/dx)\ (dh/du)$

If $h'(x) = 2x + 1$, then

$dh/du = h'(u) = 2u + 1$

Hence

$g'(x) = 6x^5 [2u + 1]$

Substitute u by $x^6 + 1$ in $g'(x)$

$g'(x) = 6x^5 [2(x^6 + 1) + 1] =$

$= 12 x^{11} + 18 x^5$

Q.NO 20