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The Algebra of Sets: Properties & Laws of Set Theory

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The algebra of sets is an analysis of values. This lesson provides an overview of the properties of sets and laws of set theory and illustrates them with real-life examples.

Sets in Real Life

Do you have a favorite meal? Maybe it's a cheeseburger meal from your favorite hamburger restaurant. This meal probably includes a cheeseburger, French fries, a drink, some ketchup packets, and napkins. In real life, this is what we call a set.

The technical definition of a **set** is a collection of very specific objects. Let's go through the properties and laws of set theory in general.

Set Theory

A set of anything has to have specific criteria and be well defined. For example, one person may think that a cheeseburger dinner from a fast food restaurant is amazing, while someone else might be repulsed by its taste, making the criteria invalid. An example of a valid set would be edible foods that include bread, so the cheeseburger dinner would qualify. Let's make a list of foods and determine which ones are eligible for a set of edible items that include bread; we'll call our set ''S.''

S = {sandwich, hamburger, cheeseburger, toast, bread pudding}

The symbol ∈ indicates that something is part of a set. For example, grilled cheese ∈ S means that grilled cheese is part of set S. This is a true statement because grilled cheese is a sandwich. Ice cream ∉ S means that ice cream is not part of set S because it doesn't include bread.

Let's take a look at the properties of sets. The order of items in a set doesn't matter. In our set of edible foods that include bread, we could list toast first and sandwich last. If an item in a set is repeated, count it once. For example, let's say we have a set W that represents the letters in the word ''cheeseburger''. The example is here: W = {c, h, e, e, s, e, b, u, r, g, e, r}. As there are four e's and two r's, we can rewrite the set as W = {c, h, e, s, b, u, r, g}.

The Laws of Sets

Let's take a look at the different laws of sets one at a time.

1. Union of Sets

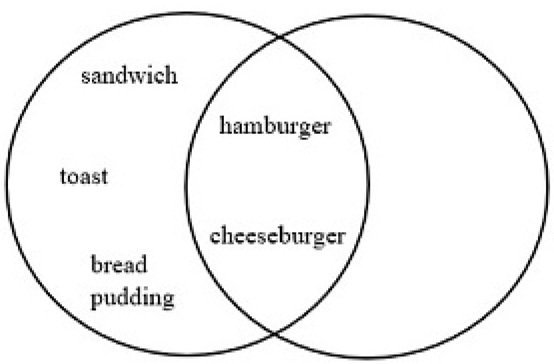
Let's say that we have two sets: S = {sandwich, hamburger, cheeseburger, toast, bread pudding} and B = {hamburger, cheeseburger}. We'll refer back to these sets throughout the rest of the lesson. The **union** of these sets is all items that are part of both sets, or ∪. The union of sets S and B is written as A ∪ B = {sandwich, hamburger, cheeseburger, toast, bread pudding}, which includes all of the items in both sets, but only one of each item if there are multiples.

2. Intersection of Sets

The **intersection of sets** defines what is common to both sets. For instance, in sets S and B, the hamburger and cheeseburger are common to both sets. The intersection of these sets is S ∩ B = {hamburger, cheeseburger}. This notation is similar to a Venn diagram of the two sets.

3. Commutative Law

Addition is a commutative property because 4 + 3 = 7 and 3 + 4 = 7; the order in which the numbers are added doesn't matter. This is true of the **commutative law** of sets too. For instance, S ∪ B is the same as B ∪ S. Likewise, S ∩ B is the same as B ∩ S.



4. Distributive Law

When solving the equation 3(x + y), we distribute the "3" to both the x and the y = 3x + 3y. The same **distributive law** applies to sets.

Hamburger and cheeseburger are included in both sets; therefore, they are the intersection of the two sets.

Let's go back to our original sets S and B and add a new set D:

D ∪ (S ∩ B) = (D ∪ S) ∩ (D ∪ B)

5. Associative Law

The **associative law** of sets is similar to the commutative law and states that with either unions or intersections of sets, the order doesn't matter. For example, if A ∪ (B ∪ D), it's also true that (A ∪ B) ∪ D.

6. Difference of Sets

To illustrate the **difference of sets**, which involves items that aren't shared between sets, let's determine that S - B, which refers to the elements of set S that are not part of set B; here, S - B = {sandwich, toast, bread pudding}. Since the hamburger and cheeseburger are found in both sets, they aren't included in the difference between sets S and B.

7. Complement of Sets

A **complement** of a set is everything that is not part of the set. For instance, in our set B, any food that's not a cheeseburger or a hamburger would be a complement. In this case, chicken livers would be a complement of set B! The complement of a set that includes everything (a universal set) is nothing, while the complement of a set with nothing in it is everything.

Lesson Summary

Let's take a few moments to review what we've learned. A **set** is a collection of specific values or items. This symbol ∈ indicates that something *is* part of a set, while this symbol ∉ indicates that something is not part of a set.

When listing items in a set, the order doesn't matter. Any item that is repeated is only counted once.

The laws of sets are:

- **Union** of sets: all items that are part of both sets (∪)
- **Intersection** of sets: items that are common to multiple sets (∩)
- **Commutative law**: no order required for items in a set

- **Distributive law**: distribution of a set through another union or an intersection of sets
- **Associative law**: no order required in unions or intersections of sets
- **Difference of sets**: items that aren't shared between sets
- **Complement of sets**: everything that isn't included in a set. The complement of a universal set is nothing, while the complement of a set of nothing is everything.

Additional Activities

Discussion Questions for the Algebra of Sets:

Key Reminders:

- The union of two sets, $A \cup B$, consists of all of the elements in either A or B.
- The intersection of two sets, $A \cap B$, consists of all of the elements in both A and B.

Questions:

1. The formula for the number of elements in $A \cup B$, denoted as $n(A \cup B)$, is $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. If $A \cup B$ consists of all of the elements in either A or B, why do we need to subtract the number of elements in $A \cap B$ from the number of elements in A plus the number of elements in B to get the number of elements in $A \cup B$?
2. How can we use the formula for the number of elements in $A \cup B$ to find a formula for the number of elements in $A \cap B$? Find this formula.
3. Suppose Brian made 22 cookies with chocolate chips and 35 cookies with raisins. Of these, 14 have raisins and chocolate chips. How many cookies did he make all together?

Answers:

1. If we add the number of elements in A and the number of elements in B, we are adding the number of elements in A and B, or $n(A \cap B)$ twice. Therefore, we must subtract one copy of $n(A \cap B)$ from this sum in order to find the correct number of elements in either A or B, or $n(A \cup B)$.
2. We can solve for $n(A \cap B)$ in $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ to find a formula for $n(A \cap B)$. To do this, we add $n(A \cap B)$ to both sides, and we subtract $n(A \cup B)$ from both sides. This gives $n(A \cap B) = n(A) + n(B) - n(A \cup B)$.
3. If we let A = chocolate chip cookies and B = raisin cookies, then we want to find the number of cookies in $A \cup B$, or $n(A \cup B)$. Thus, we can use our formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. The number of chocolate chip cookies is given as 22, so $n(A) = 22$. The number of raisin cookies is given as 35, so $n(B) = 35$. The number of cookies that have both raisins and chocolate chips is given as 14, so $n(A \cap B) = 14$. Plugging these in gives $(A \cup B) = 22 + 35 - 14$, and simplifying gives $n(A \cup B) = 43$. Thus, Brian made 43 cookies all together.