Identifying the Constant of Proportionality

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Learn the constant of proportionality definition and equation, as well as how to find the constant of proportionality in graphs, tables and word problems.

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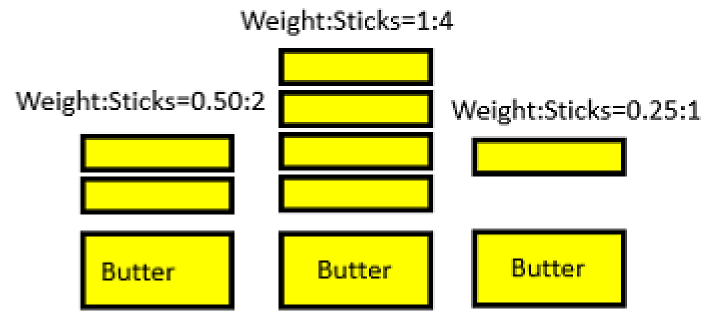
What Is the Constant of Proportionality?

When comparing two or more ratios, the **constant of proportionality** is a fixed number that indicates the rate at which ratios increase or decrease. When a ratio is written as a fraction, the constant of proportionality is the number gotten by dividing the denominator by the numerator.

Consider an example with 4 sticks of butter from a 1-lb carton of butter. Experiment with the ratio of *weight of sticks* to the *number of sticks* of butter taken out of the carton.



Example 1

Take out 2 sticks of butter from a 1-lb butter carton. Consider the ratio of weight of sticks to number of sticks. This ratio can be written as 0.5:2 or 0.5/2. The constant of proportionality is gotten by dividing the denominator (number of sticks) by the numerator (weight of sticks). In this case the constant of proportionality is 2 ÷ 0.5 = 4.

Example 2

Now take out 4 sticks of butter from a 1-lb butter carton. Consider the ratio of weight of sticks to number of sticks. This ratio can be written as 1:4 or 1/4. The constant of proportionality is gotten by dividing the denominator (number of sticks) by the numerator (weight of sticks). In this case the constant of proportionality is 4 ÷ 1 = 4.

Example 3

Next, take out 1 stick of butter from a 1-lb butter carton. Consider the ratio of weight of sticks to number of sticks. This ratio can be written as 0.25:1 or 0.25/1. The constant of proportionality is gotten by dividing the denominator (number of sticks) by the numerator (weight of sticks). In this case the constant of proportionality is 1 ÷ 0.25 = 4.

Constant of Proportionality Definition

'The constant or fixed rate at which ratios increase and decrease is known as the constant of proportionality. If two ratios have the same constant of proportionality, the ratios are said to be **directly proportional**.

In the example with the butter, the ratio of weight to the number of sticks of butter increased or decreased by a factor of 4. Or, the rate of increase or decrease is 4. This constant rate of increase or decrease is known as the constant of proportionality.

The three ratios - 0.5/2, 1/4 and 0.25/1 - can be said to be directly proportional because:

- The ratio of weight to number of sticks has the same constant of proportionality, 4.
- Multiplying the weight by 4 equals the number of sticks taken out of the carton.
- Dividing the number of sticks taken out of the carton by 4 equals the weight.
- When the weight increases, the number of sticks also increases by a factor of 4.
- When the weight decreases, the number of sticks also decreases by a factor of 4.

Constant of Proportionality Equation

The constant of proportionality of a ratio (when written as a fraction) is obtained by dividing the denominator by the numerator. If *x* is the numerator of a ratio and *y* is the denominator of a ratio, the constant of proportionality, *k*, can be derived using the equation:

$$k = \frac{y}{x}$$

the constant of proportionality k equals denominator divided by the numerator

This equation can also be written as:

$$y = kx$$

To use the equation to find the constant of proportionality of a ratio:

- Write the ratio as a fraction.
- Divide the denominator by the numerator to get the constant of proportionality.

Example 4

Cut out a paper rectangle that is 2 cm wide and 6 cm long. The ratio of width to length of this rectangle is 2:6. Use the equation to find k, the constant of proportionality, as $k = y/x$ or $k = 6/2$. In this example, $k = 3$.

Use the value of k to create a second rectangle which is directly proportional in ratio to this rectangle. To create a rectangle with directly proportional width and length, multiply the width and length of the first rectangle by the same constant, 3, to get the new rectangle. The new rectangle will have width = 6 (2*3) and length = 18 (6*3). The constant of proportionality of this new rectangle is:

- $k = y/x$
- $k = 18/6$
- $k = 3$

Make a table to compare the values.

Rectangle	Width	Height	Constant of Proportionality
Rectangle 1	2	6	$6 \div 2 = 3$
Rectangle 2	6	18	$18 \div 6 = 3$

The two rectangles have the same constant of proportionality, 3. This means that:

- The width and length of both rectangles changed in the same proportion by a constant factor of 3.
- The ratios of width to length for both rectangles are directly proportional.

How to Find the Constant of Proportionality

There are four different methods to find the constant of proportionality:

- From a table
- Using a graph - counting the squares
- Using a graph - slope of the line
- In a word problem

Let's take a look at these methods.

How to Find the Constant of Proportionality from a Table

Example 5

Given the table of values for *x* and *y*, find the constant of proportionality.

	Ratio 1	Ratio 2	Ratio 3	Ratio 4
x	5	20	10	40
y	10	40	20	80

To find the constant of proportionality, divide the denominator by the numerator.

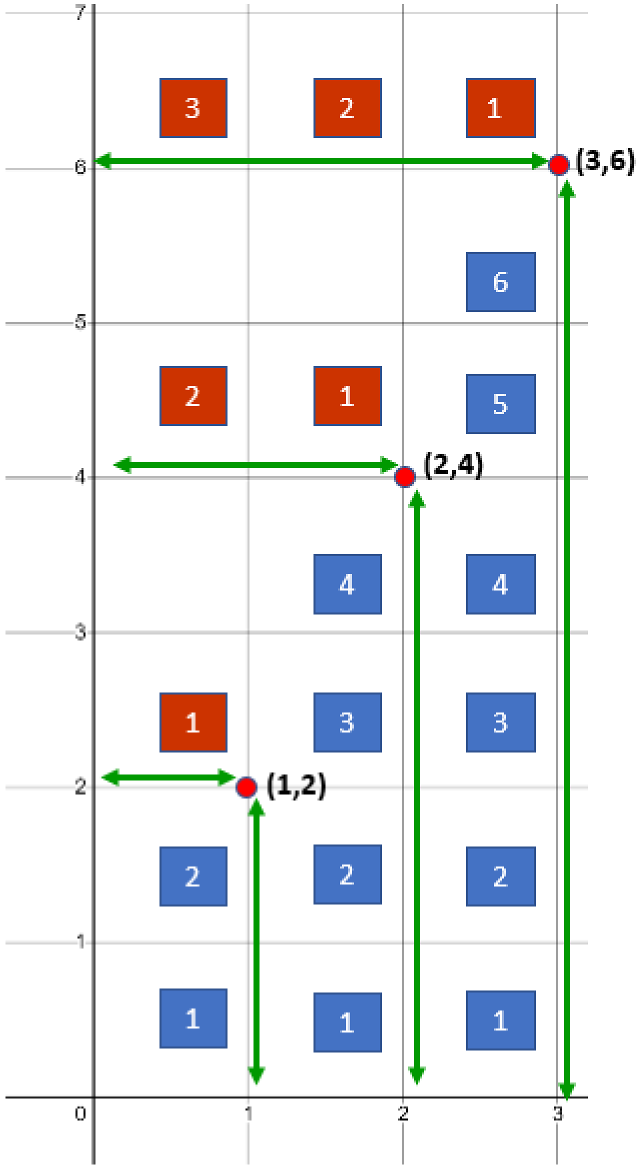
	Ratio 1	Ratio 2	Ratio 3	Ratio 4
$k = y/x$	$k = 10 \div 5 = 2$	$k = 40 \div 20 = 2$	$k = 20 \div 10 = 2$	$k = 80 \div 40 = 2$

The constant of proportionality for all the ratios is the same number, 2, so the ratios are directly proportional.

How to Find the Constant of Proportionality from a Graph

Example 6

Given three ratios - 1:2, 2:4 and 3:6 - find the constant of proportionality using a graph.



constant of proportionality using a graph

Answer:

Step 1:

- Take the first ratio, 1/2.

- Write this as coordinate point (1,2).
- Plot this point on a graph.

Step 2:

- Count the number of squares on the *y* axis - there are 2 blue squares along the *y* axis.
- Count the number of squares on the *x* axis - there is 1 red square along the *x* axis.
- Divide the number of squares along the *y* axis (2) by the number of squares along the *x* axis (1).
- The constant of proportionality is 2/1 = 2.

Step 3:

- Repeat with the 2/4 ratio: plot the point (2,4).
- Divide the number of blue squares along the *y* axis (4) by the number of red squares along the *x* axis (2).
- The constant of proportionality is 4/2 = 2.

Step 4:

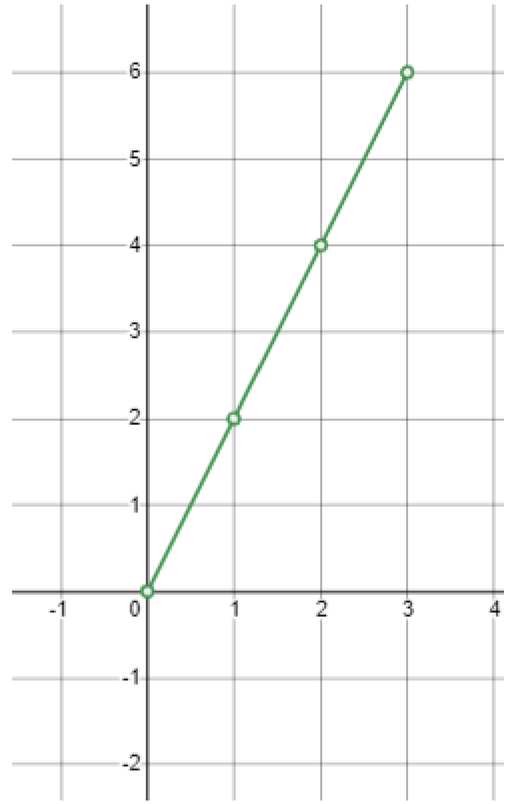
- For the 3/6 ratio, plot the point (3,6) on the graph.
- Divide the number of blue squares along the *y* axis (6) by the number of red squares along the *x* axis (3).
- The constant of proportionality is 6/3 = 2.

Constant of Proportionality and Slope of a Line Graph

Example 7

For the three ratios - 1:2, 2:4 and 3:6 - find the constant of proportionality using slope of a line graph.

1. Convert the ratio into coordinate points: (1,2), (2,4) and (3,6).
2. Plot the points on a graph.
3. Draw a line connecting all of the points.
4. Extend the line - check to see whether the line passes through the origin (0,0).



constant of proportionality using a line graph

Answer:

If the graph of the line passes through the origin, the ratios are proportional. The equation for the constant of proportionality is $k = y/x$. This is the same as the equation of the slope of a straight line that passes through the origin, $m = y/x$. Using the equation for the slope of the line, one can find the value of m, which will be the same as the value of k, or the constant or proportionality.

Ratio	***y* Coordinate**	***x* Coordinate**	***m* = *y*/*x***	***k* = *m***
1:2	2	1	$m = 2 \div 1 = 2$	2
2:4	4	2	$m = 4 \div 2 = 2$	2

3:6	6	3	$m = 6 \div 3 = 2$	2

Note: The slope of the line is the same as the constant of proportionality in Example 6 found by dividing the number of squares along the *y* axis (2) by the number of squares along the *x* axis.

Constant of Proportionality - Diagrams and Tables

Example 8

The first example of ratio of weight to sticks taken out of a 1-lb butter carton to number of sticks is in the form of a diagram. The same information can be in the form of a table. To find the constant of proportionality, first identify the *x* and *y* variables. To find the ratio of weight to number of sticks, *x* will be the weight and *y* will be the number of sticks of butter. Use the equation $k = y \div x$ to find the constant of proportionality.

	Option 1	**Option 2**	**Option 3**
Weight (*x*)	0.5	1	0.25
Number of sticks (*y*)	2	4	1
$k = y \div x$	$2 \div 0.5 = 4$	$4 \div 1 = 4$	$1 \div 0.25 = 4$

How to Find the Constant of Proportionality in a Word Problem

Example 9

Kira plays 3 different video games in 12 days.

1. What is the constant of proportionality that relates the number of games to the number of days?
2. What is the constant of proportionality that relates the number of days to the number of games?

Answer (1):

Identify the *x* and *y* variables. The constant of proportionality is asking for the number of games to the number of days. The first quantity, number of games, is *x*. The second quantity, number of days, is *y*. This can be written as a ratio:

- Number of games to number of days = 3:12 or 3/12

- Constant of proportionality = 12 ÷ 3 = 4

Answer (2)

Identify the *x* and *y* variables. The constant of proportionality is asking for the number of days to the number of games. The first quantity, number of days, is *x*. The second quantity, number of games, is *y*. This can be written as a ratio:

- Number of days to Number of games = 12:3 or 12/3
- Constant of proportionality = 3 ÷ 12 = 0.25

Lesson Summary

In a ratio the **constant of proportionality** is a fixed number that shows the relationship between the first quantity and the second quantity. When the ratio is written as a fraction, the constant of proportionality is the number by which the numerator is multiplied to get the denominator.

When two or more ratios have the same constant of proportionality, it can be said that the ratios are proportional. This is because both ratios increase or decrease at the same rate. The constant rate by which the ratios increase or decrease is the constant of proportionality. When ratios increase or decrease at the same rate, by a constant value, they are said to be **directly proportional**.

The equation to find the constant of proportionality is written as $k = y/x$. Here, *k* is the constant, *y* is the denominator, and *x* is the numerator of the ratio.

There are four ways to find the constant of proportionality:

- Using a table
  - Write the ratios in a table so that the first quantity - the numerator - is in one cell, and the second quantity - the denominator - is in the cell below.
  - Divide the denominator by the numerator to find the constant of proportionality.
- From a graph
  - Write the ratios as ordered pairs so that the first quantity is on the *x* axis and the second quantity is on the *y* axis.
  - For each ordered pair, count the number of squares along the *y* axis and the number of squares along the *x* axis.
  - Divide the number of *y* axis squares by the number of *x* axis squares to find the constant of proportionality.

- From a graph using slope of the line
  - Plot the points and draw a line through the points.
  - If the line is a straight line that passes through the origin, calculate the constant of proportionality using the formula for slope of the line, $y = mx$.
- In a word problem
  - Identify the first quantity in the ratio as x and the second quantity as y.
  - Divide y by x to get the constant of proportionality, k.

Video Transcript

Identifying the Constant of Proportionality

One Saturday morning, you find yourself at the local grocery store helping out with a little shopping for your family. You've been asked to go pick up a can of tomatoes. When you get to the aisle, you see the brand you want on sale: two for $3.00. In your head, you divide three by two and know the tomato can will cost $1.50. What you just did, without even knowing it, is find the constant of proportionality for the cost of a tomato can.

The **constant of proportionality** is the ratio between two directly proportional quantities. In our tomato example, that ratio is $3.00/2, which equals $1.50. Two quantities are **directly proportional** when they increase and decrease at the same rate. When you buy more or less, the total price of the tomato cans in this example goes up or down correspondingly.

Even though the amount of tomato cans bought and the total price can change, the price for a single tomato can, our constant of proportionality, remains the same. This is why we call the constant of proportionality a constant, a number with a fixed value.

In this lesson, we're going to look at multiple ways to identify the constant of proportionality. We'll see how to identify it in equations, word problems, diagrams, tables, and graphs.

Constant of Proportionality Equation

Earlier we mentioned that the ratio for the constant of proportionality in our example was $3.00/2. We can use this ratio as an example to find a general equation for any constant of proportionality. The constant of proportionality (*k*) equals the total price (*y*) divided by the number of cans (*x*).

$k = y/x$

Or, the total price equals the constant of proportionality multiplied by the number of cans:

$y = kx$

The second formula is a common alternate form of the constant of proportionality formula. While working with the equations, just replace the total price and number of cans with whatever two other directly proportional quantities you have in your problem.

Word Problems

When trying to find the constant of proportionality, you're not always going to begin with an equation. This is especially true when it comes to speech. In a math class, we replicate verbal descriptions with word problems. Let's look at an example of this.

Chris mows 3 acres of his parent's farmland in 12 hours. Over the next several days, he mows 8 more acres in a total of 32 hours. How many hours does Chris spend per acre mowed?

Here, the constant of proportionality ratio we're trying to find is how many hours it takes to mow an acre. In other words, hours divided by acres mowed. We have two situations to test: 3 acres in 12 hours and 8 acres in 32 hours.

$k = y/x$

1) $k = 12/3 = 4$ hours per acre

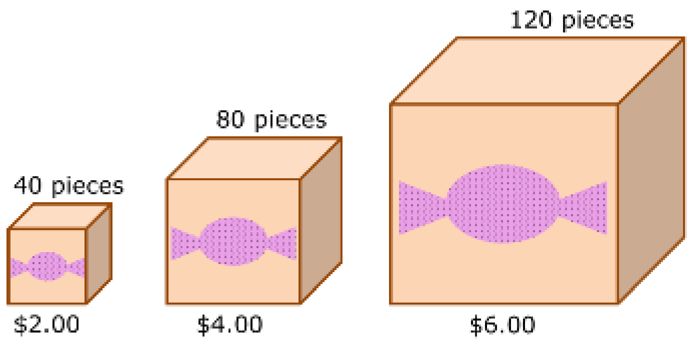
2) $k = 32/8 = 4$ hours per acre

We can see it takes Chris 4 hours to mow a single acre. This is our constant because no matter how many acres he mows, or how much time he spends mowing, it will always take 4 hours to mow a single acre.

Diagrams and Tables

We just used a word problem to find the constant of proportionality, but in other situations, you might be asked to find one through a visual diagram or a table.

Diagrams and tables like the ones below work exactly the same way for finding the constant of proportionality. If we compare the two, we can see that each column of the table is like one image in the diagram.



Candy Box 1	Candy Box 2	Candy Box 3
40 pieces	80 pieces	120 pieces
$2.00	$4.00	$6.00

Constant of Proportionality Diagram and Table

Let's use this diagram and table to find the number of pieces of candy you get per dollar. We have three different sizes of candy boxes. Each image in the diagram and each column in the table tells us the price of a box and how many pieces of candy are in it. If there's a constant of proportionality, we should get the same number of pieces of candy per dollar for each of the three boxes.

$k = y / x$

1) k = 40 / 2.00 = 20 pieces per dollar

2) k = 80 / 4.00 = 20 pieces per dollar

3) k = 120 / 6.00 = 20 pieces per dollar

Regardless of the size of the box you purchase, you're always getting 20 pieces of candy per dollar.

Graphs

The final way we can find a constant of proportionality is by using a line graph. A line graph that has a constant of proportionality in it will be a straight line that passes through the origin, e.g., the (0,0) point.

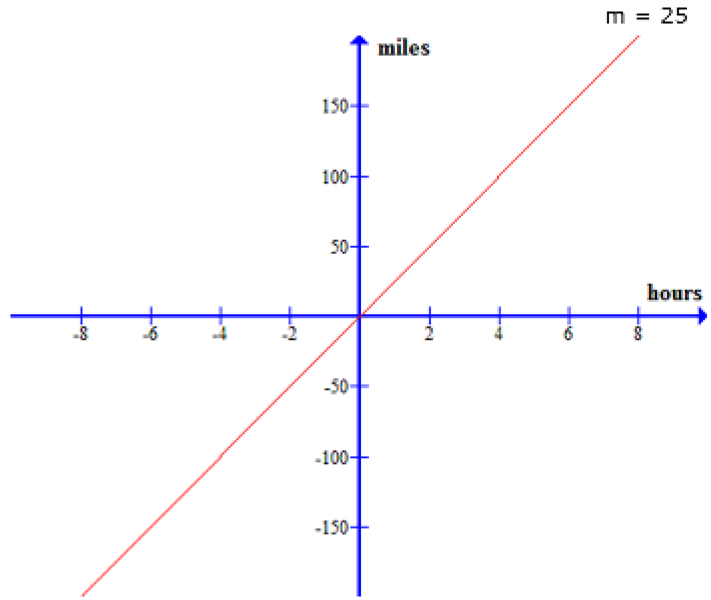
To find the constant of proportionality here let's look at the equation of a line for a straight graph that passes through the origin.

$y = mx$

y is your vertical y-value for any given point on the line, x is your horizontal x-value for any given point on the line, and m is the slope. Does this equation look familiar to you? It looks exactly like the alternate form of the equation for the constant of proportionality we went over earlier in the lesson.

y = *kx*

In a graph like this, the constant of proportionality is equal to the graph's slope. Our graph has a slope, and therefore a constant of proportionality of 25 miles per hour.



Constant of Proportionality Graph

Lesson Summary

We call the ratio between two directly proportional quantities the **constant of proportionality**. When two quantities are **directly proportional**, they increase and decrease at the same rate. While these two quantities may increase or decrease, the constant of proportionality always remains the same. This is why we call it a constant.

The constant of proportionality (*k*) ratio is shown in this formula:

k = *y* / *x*

Here, *y* and *x* are two quantities that are directly proportional to each other. Often we see the constant of proportionality equation in this form as well:

y = *kx*

We can use these equations to find the constant of proportionality in word problems, diagrams, tables, and graphs. Finally, it's worth noting that when looking at a straight-line graph that goes through the origin, (0,0) point, the constant of proportionality will always be equal to that graph's slope.

Additional Activities

Using the Constant of Proportionality to Solve Word Problems

The constant of proportionality k is given by k=y/x where y and x are two quantities that are directly proportional to each other. Once you know the constant of proportionality you can find an equation representing the directly proportional relationship between x and y, namely y=kx, with your specific k. This equation can then be used to solve for other unknown quantities.

Examples

1. The number of gallons of gas, y, a car uses is directly proportional to the number of miles, x, driven.

- Find the constant of proportionality if a car uses 20 gallons of gas to drive 500 miles
- Write an equation showing the directly proportional relationship using your constant of proportionality
- How many miles were driven if 100 gallons of gas were used?

2. Circulation time is the amount of time it takes for blood to circulate through the entire body. The circulation time of a mammal is proportional to the fourth root of the body mass of the mammal. The circulation time for an elephant of body mass 5230 kg is 148 seconds.

- Find the constant of proportionality (rounded to the nearest tenth)
- Write an equation showing the directly proportional relationship using your constant of proportionality.
- What is the body mass of a human whose circulation time is 50.3 seconds? Round to the nearest tenth of a kilogram.

Solution

1.

- To find the constant of proportionality, we use the formula k=y/x. So we have k=20/500 = 0.04
- The equation representing this situation will look like y=kx. Substituting in our value of k that we found, we have the equation y=0.04x
- We can use our equation to find the answer to this problem. We know y=0.04x and we are given that 100 gallons of gas were used, so y=100. Substituting this in, we have 100=0.04x. To solve for x, we just have to divide both sides of the equation by 0.04 to get x=2500. So 2500 miles were driven.

2.

- To find the constant of proportionality, we use the formula k=y/x - except this problem is different. We don't just have one quantity proportional to another quantity - we have one quantity proportional to the fourth root of another quantity. So instead of k=y/x, we will use

$$k = y/\sqrt[4]{x}$$

where y is the circulation time and x is the body mass. Using the data for the elephant, we can find the constant of proportionality.

$$k = y/\sqrt[4]{x}$$

$$k = 148/\sqrt[4]{5230}$$

$$k = 17.4$$

- Now that we know the constant of proportionality, we can write an equation of the form

$$y = k\sqrt[4]{x}$$

$$y = 17.4\sqrt[4]{x}$$

- To find the body mass of the human, we can substitute y=50.3 into our equation and solve for x.

$$y=17.4\sqrt[4]{x}$$

$$50.3=17.4\sqrt[4]{x}$$

$$50.3/17.4=\sqrt[4]{x}$$

$$(50.3/17.4)^4=x$$

$$69.8=x$$

So a human of mass 69.8 kg will have a circulation time of 50.3 seconds.

Discussion

You are given the set of data below and are told that the data shows that the number of hours spent studying is directly proportional to the percentage received on an exam. How can you find if you were told the correct information? Is the relationship actually a direct proportion?

Time Studying (hours)	Percentage on Exame
1	60
2	70
3	80
4	90
5	100

Guide to Discussion

Students may observe that there is a pattern in the table - each hour of studying resulting in 10% more on the exam. However - in order for a relationship to be directly proportional the constant of proportionality must be a constant. One possibility to find out if the table shows a directly proportional relationship is to calculate k=y/x for multiple instances in the table and see if the result is always the same.

k=1/60=0.0167

k=2/70=0.0286

k=3/80=0.0375

k=4/90=0.0444

k=5/100=0.05

None of the values for k are equal - so this is not a direct proportion.