GRADE 9-12 - 16 QUESTIONS

I. Number and Quantity

1. **Structure of Numerical Systems:**
   a. Place value
   b. Order relationships
   c. Relationships between operations
   d. Multiple forms of numbers
   e. Absolute value
   f. Signed numbers
   g. Integers and rational numbers
   h. Ratios and proportion

2. **Real and Complex Number Systems:**
   a. Rational and irrational numbers
   b. Multiple forms of complex numbers
   c. Properties of the real and complex number systems
   d. Operations with complex numbers
   e. Laws of exponents
   f. Roots and powers of real and complex numbers
   g. Scientific notation

3. **Elementary Number Theory:**
   a. Factors and divisibility
   b. Prime and composite
   c. Prime Factorization
   d. Euclid's Algorithm
   e. Congruence classes and modular arithmetic
   f. Mersenne primes and perfect numbers
   g. Fermat's Last Theorem
   h. Fundamental Theorem of Arithmetic

GRADE 3-8 – 22 QUESTIONS

I. Number and Quantity

1. **Structure of Numerical Systems:**
   a. Place value
   b. Order relationships
   c. Relationships between operations
   d. Multiple forms of numbers
   e. Factors and divisibility
   f. Prime and composite
   g. Prime factorization
   h. Properties of numerical systems

2. **Operations of integers, rational numbers, decimals, percentage, ratio and proportional relationships:**
   a. Order of operations
   b. Identity and inverse elements
   c. Associative, commutative, and distributive properties
   d. Absolute value
   e. Operations of signed numbers
   f. Multiple representations of numerical operations
   g. Analyzing algorithms for addition, subtraction, multiplication, and division of integers and rational numbers
   h. Number operations and their inverses

3. **Application of integers, rational numbers, decimals, percentage, ratio and proportional relationships:**
   a. Application problems using numerical systems
   b. Average rate of change
   c. Using estimation for verifying reasonableness of solutions

I. Number and Quantity cont.

4. **Structure of Real Number System:**
   a. Rational and irrational numbers and operations
   b. Properties of the real number system
   c. Operations and their inverses
   d. The real number line
   e. Roots and powers
   f. Laws of exponents
   g. Scientific notation
   h. Using number properties to prove theorems

Q.NO 1

Ans. (d)

SOLUTION We have,

$$i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n (1 + i + i^2 + i^3) = i^n (1 + i - 1 - i) = 0$$

QNO 2

Choice D is correct. It is given that the number of students surveyed was 336. Finding $\frac{1}{4}$ of 336 yields $\left(\frac{1}{4}\right)(336) = 84$, the number of freshmen, and finding $\frac{1}{3}$ of 336 yields $\left(\frac{1}{3}\right)(336) = 112$, the number of sophomores. Subtracting these numbers from the total number of selected students results in $336 - 84 - 112 = 140$, the number of juniors and seniors combined. Finding half of this total yields $\left(\frac{1}{2}\right)(140) = 70$, the number of juniors. Subtracting this number from the number of juniors and seniors combined yields $140 - 70 = 70$, the number of seniors.

Choices A and C are incorrect and may result from calculation errors. Choice B is incorrect. This is the total number of juniors and seniors.

QNO 3

Choice A is correct. It's given that the ratio of the heights of Plant A to Plant B is 20 to 12 and that the height of Plant C is 54 centimeters. Let x be the height of Plant D. The proportion $\frac{20}{12} = \frac{54}{x}$ can be used to solve for the value of x. Multiplying both sides of this equation by x yields $\frac{20x}{12} = 54$ and then multiplying both sides of this equation by 12 yields $20x = 648$. Dividing both sides of this equation by 20 yields $x = 32.4$ centimeters.

QNO 4

Choice D is correct. It's given that 1 kilometer is approximately equivalent to 0.6214 miles. Let x be the number of kilometers equivalent to 3.1 miles. The proportion $\frac{1 \text{ kilometer}}{0.6214 \text{ miles}} = \frac{x \text{ kilometers}}{3.1 \text{ miles}}$ can be used to solve for the value of x. Multiplying both sides of this equation by 3.1 yields $\frac{3.1}{0.6214} = x$, or $x \approx 4.99$. This is approximately 5 kilometers.

Choice A is incorrect and may result from misidentifying the ratio of kilometers to miles as miles to kilometers. Choice B is incorrect and may result from calculation errors. Choice C is incorrect and may result from calculation and rounding errors.

QNO 5

Choice C is correct. Let a equal the number of 120-pound packages, and let b equal the number of 100-pound packages. It's given that the total weight of the packages can be at most 1,100 pounds: the inequality $120a + 100b \leq 1{,}100$ represents this situation. It's also given that the helicopter must carry at least 10 packages: the inequality $a + b \geq 10$ represents this situation. Values of a and b that satisfy these two inequalities represent the allowable numbers of 120-pound packages and 100-pound packages the helicopter can transport. To maximize the number of 120-pound packages, a, in the helicopter, the number of 100-pound packages, b, in the helicopter needs to be minimized. Expressing b in terms of a in the second inequality yields $b \geq 10 - a$, so the minimum value of b is equal to $10 - a$. Substituting $10 - a$ for b in the first inequality results in $120a + 100(10 - a) \leq 1{,}100$. Using the distributive property to rewrite this inequality yields $120a + 1{,}000 - 100a \leq 1{,}100$, or $20a + 1{,}000 \leq 1{,}100$. Subtracting 1,000 from both sides of this inequality yields $20a \leq 100$. Dividing both sides of this inequality by 20 results in $a \leq 5$. This means that the maximum number of 120-pound packages that the helicopter can carry per trip is 5.

Choices A, B, and D are incorrect and may result from incorrectly creating or solving the system of inequalities.

QNO 6

Choice B is correct. The difference between the machine's starting value and its value after 10 years can be found by subtracting \$30,000 from \$120,000: $120{,}000 - 30{,}000 = 90{,}000$. It's given that the value of the machine depreciates by the same amount each year for 10 years. Dividing \$90,000 by 10 gives \$9,000, which is the amount by which the value depreciates each year. Therefore, over a period of t years, the value of the machine depreciates by a total of $9{,}000t$ dollars. The value v of the machine, in dollars, t years after it was purchased is the starting value minus the amount of depreciation after t years, or $v = 120{,}000 - 9{,}000t$.

Choice A is incorrect and may result from using the value of the machine after 10 years as the machine's starting value. Choice C is incorrect. This equation shows the amount the machine's value changes each year being added to, rather than subtracted from, the starting value. Choice D is incorrect and may result from multiplying the machine's value after 10 years by t instead of multiplying the amount the machine depreciates each year by t.

QNO 7

Choice B is correct. It's given that a $\frac{3}{4}$-cup serving of Crunchy Grain cereal provides 210 calories. The total number of calories per cup can be found by dividing 210 by $\frac{3}{4}$, which gives $210 \div \frac{3}{4} = 280$ calories per cup. Let c be the number of cups of Crunchy Grain cereal and s be the number of cups of Super Grain cereal. The expression $280c$ represents the number of calories in c cups of Crunchy Grain cereal, and $240s$ represents the number of calories in s cups of Super Grain cereal. The equation $280c + 240s = 270$ gives the total number of calories in one cup of the mixture. Since $c + s = 1$ cup, $c = 1 - s$. Substituting $1 - s$ for c in the equation $280c + 240s = 270$ yields $280(1 - s) + 240s = 270$, or $280 - 280s + 240s = 270$. Simplifying this equation yields $280 - 40s = 270$. Subtracting 280 from both sides results in $-40s = -10$. Dividing both sides of the equation by -40 results in $s = \frac{1}{4}$, so there is $\frac{1}{4}$ cup of Super Grain cereal in one cup of the mixture.

Choices A, C, and D are incorrect and may result from incorrectly creating or solving the system of equations.

QNO 8

The correct answer is 8. In this group, $\frac{1}{9}$th of the people who are rhesus negative have blood type B. The total number of people who are rhesus negative in the group is $7 + 2 + 1 + x$, and there are 2 people who are rhesus negative with blood type B. Therefore, $\frac{2}{(7 + 2 + 1 + x)} = \frac{1}{9}$. Combining like terms on the left-hand side of the equation yields $\frac{2}{(10 + x)} = \frac{1}{9}$. Multiplying both sides of this equation by 9 yields $\frac{18}{(10 + x)} = 1$, and multiplying both sides of this equation by $(10 + x)$ yields $18 = 10 + x$. Subtracting 10 from both sides of this equation yields $8 = x$.

QNO 9

The correct answer is 15. It's given that the deductions reduce the original amount of taxes owed by \$2,325.00. Since the deductions reduce the original amount of taxes owed by d%, the equation $\frac{2,325}{15,500} = \frac{d}{100}$ can be used to find this percent decrease, d.

Multiplying both sides of this equation by 100 yields $\frac{232,500}{15,500} = d$, or $15 = d$. Thus, the tax deductions reduce the original amount of taxes owed by 15%.

QNO 10

The correct answer is 3. Let y be the number of international tourist arrivals in Russia in 2012, and let x be the number of these arrivals in 2011. It's given that y is 13.5% greater than x, or $y = 1.135x$. The table gives that $y = 24.7$, so $24.7 = 1.135x$. Dividing both sides of this equation by 1.135 yields $\frac{24.7}{1.135} = x$, or $x \approx 21.8$ million arrivals. The difference in the number of tourist arrivals between these two years is 24.7 million − 21.8 million = 2.9 million. Therefore, the value of k is 3 when rounded to the nearest integer.

ANSWER C

QNO 12

408, 1032

H.C.F. = 24

```
408)1032(2
     816
    ----------
     216 )408( 1
           216
          ----------
          192 )216( 1
                192
               ---------
                 24 ) 192 ( 8
                      192
                     -----
                       ×
                     -----
```

$\therefore\ 24 = (216 - 192)$

$= 216 - (408 - 216)$

$= 216 - 408 + 216$

$= 216 \times 2 - 408$

$= [1032 - 408 \times 2] \times 2 - 408$

$= 1032 \times 2 - 408 \times 4 - 408$

$= 1032 \times 2 - 408 \times 5$

$= 1032 \times 2 - 408 \times 5$

Which is in the form of 1032m – 408 x 5 comparing, we get m = 2

QNO 13

The required number of animals will be the H.C.F. of 105 goats, 140 donkeys, 175 cows

$$
\begin{array}{r|r|r|l}
4 & 140 & 175 & 1 \\
 & 140 & 140 & \\
\hline
 & 0 & 35 &
\end{array}
$$

H.C.F. of 175 and 140 = 35

and H.C.F. of 35 and 105 = 35

$$
\begin{array}{|r|r|l}
35 & 105 & 3 \\
 & 105 & \\
\cline{2-2}
 & 0 & \\
\text{H.C.F.} & \text{Remainder} &
\end{array}
$$

The required number of animals = 35

QNO 14

L.C.M. of 520 and 468

2	520, 468
2	260, 234
13	130, 117
	10, 9

= 2 x 2 x 9 x 10 x 13 = 4680

The number which is increased = 17

Required number 4680 – 17 = 4663

QNO 15

Circumference of a circular field = 360 km

Three cyclist start together who can cycle 48, 60 and 72 km per day round the field

L.C.M. of 48, 60, 72

$$
\begin{array}{r|l}
2 & 48, 60, 72 \\ \hline
2 & 24, 30, 36 \\ \hline
2 & 12, 15, 18 \\ \hline
3 & 6, 15, 9 \\ \hline
 & 2, 5, 3
\end{array}
$$

$$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 720$$

They will meet again after 720 km distance

QNO 16

Let $x = 6.\overline{12}$ ...(i)

$100x = 612.\overline{12}$ ...(ii)

...[Multiplying both sides by 100]

Subtracting (i) from (ii),

$99x = 606$

$x = \frac{606}{99} = \frac{202}{33}$

Denominator = 33

Prime factorisation = 3×11

QNO 17

Solution:

$$\frac{43}{2^4 \times 5^3} = \frac{43}{2^4 \times 5^3} \times \frac{5^1}{5^1} = \frac{43 \times 5}{2^4 \times 5^4} = \frac{215}{10^4}$$

After 4 decimal places.

QNO 18

3, 8, 13, 18, 23, 28, 33, 38, ...

4, 10, 16, 22, 28, 34, 40, ...

QNO 19

ANSWER C- NO SOLUTION

Existence of a solution

Consider modulus = 11.

Squares: 1,3,4,5,9

Non-squares: 2,6,7,8,10

For non-squares, a solution for x^2 = a mod p does not exist.

Thus there is no value of x, which satisfies x^2 = 6 mod 11.

x	1	2	3	4	5	6	7	8	9	10
x^2 mod 11	1	4	9	5	3	3	5	9	4	1

Quadratic Residues and Nonresidue

$x^2 \equiv a \pmod p$

- a is called quadratic residue (QR) if the equation has two solutions
- a is called quadratic nonresidue(QNR) if the equation has no solutions
- In Z_{p^*} with p-1 elements, exactly (p-1)/2 elements are quadratic residues and (p-1)/2 are quadratic nonresidues

Euler's Criterion

- Used to tell whether an integer is a QR or NQR
- If $a^{(p-1)/2} \equiv 1 \pmod p$, a is a quadratic residue modulo p
- If $a^{(p-1)/2} \equiv -1 \pmod p$, a is a quadratic nonresidue modulo p
- Example 9.42
  - find out if 14 or 16 is a QR in Z_{23^*},
  - $14^{(23-1)/2}$ mod 23 → 22 mod 23 → -1 mod 23 **nonresidue**
  - $16^{(23-1)/2}$ mod 23 → 1 mod 23 **residue**

Solving Quadratic Equation Modulo a Prime

- Solve the following quadratic equations:

a. $x^2 \equiv 3 \pmod{23}$

b. $x^2 \equiv 2 \pmod{11}$

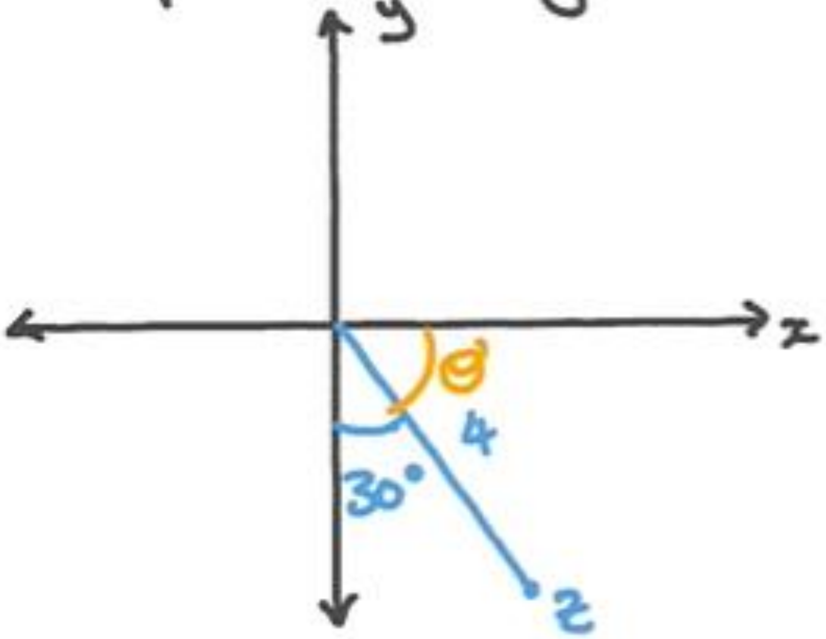
c. $x^2 \equiv 7 \pmod{19}$

a. $x \equiv \pm 16 \pmod{23}$ $\sqrt{3} \equiv \pm 16 \pmod{23}$.

b. There is no solution for $\sqrt{2}$ in Z11.

c. $x \equiv \pm 11 \pmod{19}$. $\sqrt{7} \equiv \pm 11 \pmod{19}$.

QNO 20



$z = a + bi$

real, imaginary

$a = x$ coordinate
$b = y$ coordinate

$z = (r, \theta)$

$x = r\cos\theta$
$y = r\sin\theta$

let $r = 4$, $\theta = -\pi/3$

$x = 4\cos\left(\frac{-\pi}{3}\right)$

$\therefore\ z = 4\cos\left(\frac{-\pi}{3}\right) + 4\sin\left(\frac{-\pi}{3}\right)i$

$\boxed{z = 4\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(-\pi/3\right)\right)}$

QNO 21

In 2007, the price of gasoline was \$2.84. In 2009, the cost was \$2.41. The average rate of change is

$$\begin{cases} \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} \\ = \dfrac{2.41 - 2.84}{2009 - 2007} \\ = \dfrac{-0.43}{2 \text{ years}} \\ = -0.22 \text{ per year} \end{cases}$$

QNO 22

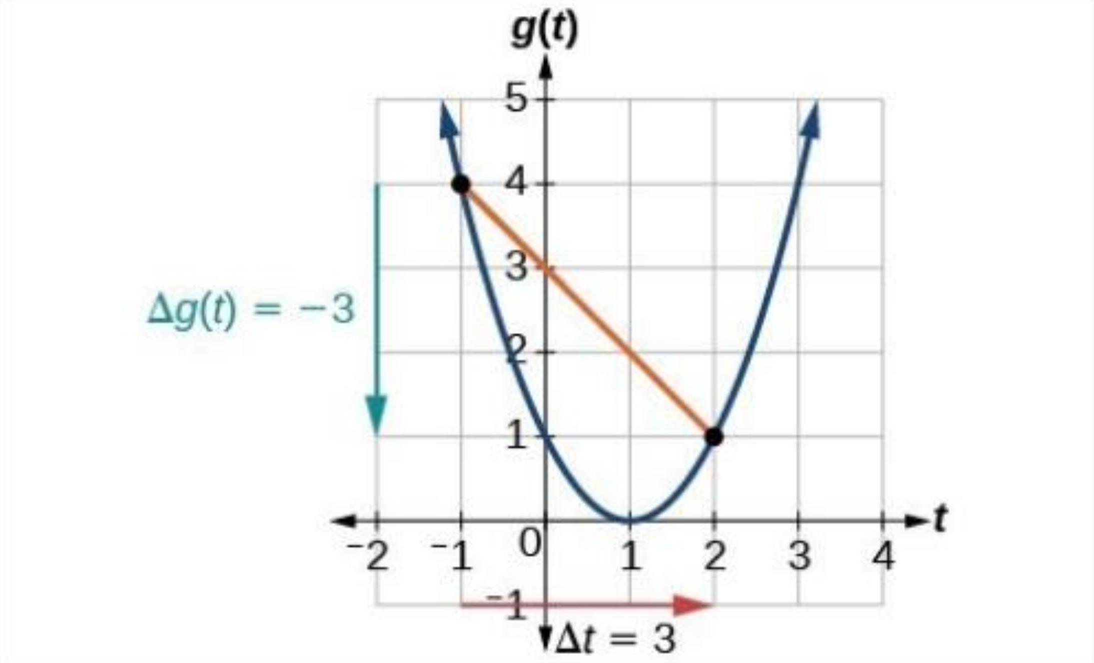


Figure 2

At $t = -1$, the graph shows $g(-1) = 4$. At $t = 2$, the graph shows $g(2) = 1$.

The horizontal change $\Delta t = 3$ is shown by the red arrow, and the vertical change $\Delta g(t) = -3$ is shown by the turquoise arrow. The output changes by –3 while the input changes by 3, giving an average rate of change of

$$\frac{1-4}{2-(-1)} = \frac{-3}{3} = -1$$