Discussion Questions for the Algebra of Sets:

Key Reminders:

- The union of two sets, $A \cup B$, consists of all of the elements in either A or B.
- The intersection of two sets, $A \cap B$, consists of all of the elements in both A and B.

Questions:

1. The formula for the number of elements in $A \cup B$, denoted as $n(A \cup B)$, is $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. If $A \cup B$ consists of all of the elements in either A or B, why do we need to subtract the number of elements in $A \cap B$ from the number of elements in A plus the number of elements in B to get the number of elements in $A \cup B$?
2. How can we use the formula for the number of elements in $A \cup B$ to find a formula for the number of elements in $A \cap B$? Find this formula.
3. Suppose Brian made 22 cookies with chocolate chips and 35 cookies with raisins. Of these, 14 have raisins and chocolate chips. How many cookies did he make all together?

Answers:

1. If we add the number of elements in A and the number of elements in B, we are adding the number of elements in A and B, or $n(A \cap B)$ twice. Therefore, we must subtract one copy of $n(A \cap B)$ from this sum in order to find the correct number of elements in either A or B, or $n(A \cup B)$.
2. We can solve for $n(A \cap B)$ in $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ to find a formula for $n(A \cap B)$. To do this, we add $n(A \cap B)$ to both sides, and we subtract $n(A \cup B)$ from both sides. This gives $n(A \cap B) = n(A) + n(B) - n(A \cup B)$.
3. If we let A = chocolate chip cookies and B = raisin cookies, then we want to find the number of cookies in $A \cup B$, or $n(A \cup B)$. Thus, we can use our formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. The number of chocolate chip cookies is given as 22, so $n(A) = 22$. The number of raisin cookies is given as 35, so $n(B) = 35$. The number of cookies that have both raisins and chocolate chips is given as 14, so $n(A \cap B) = 14$. Plugging these in gives $(A \cup B) = 22 + 35 - 14$, and simplifying gives $n(A \cup B) = 43$. Thus, Brian made 43 cookies all together.