Massachusetts
Tests for Educator Licensure® (MTEL®)

Middle School Mathematics (47)

PRACTICE TEST

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Test policies and materials, including but not limited to tests, item types, and item formats, are subject to change at the discretion of the Massachusetts Department of Elementary and Secondary Education.

INTRODUCTION

This document is a printable version of the Massachusetts Tests for Educator Licensure® (MTEL®) Middle School Mathematics (47) Online Practice Test. This practice test is a sample test consisting of 100 multiple-choice questions and 2 open-response item assignments.

To assist you in recording and evaluating your responses on the practice test, a Multiple-Choice Answer Sheet, an Answer Key Worksheet, and an Evaluation Chart by test objective are included for the multiple-choice questions. A blank Response Sheet, Evaluation Information, and Sample Responses and Analyses, as well as a Scoring Rubric, are included for the open-response items. Lastly, there is a Practice Test Score Calculation worksheet.

PURPOSE OF THE PRACTICE TEST

The practice test is designed to provide an additional resource to help you effectively prepare for the MTEL Middle School Mathematics (47) test. The primary purpose of the practice test is to help you become familiar with the structure and content of the test. It is also intended to help you identify areas in which to focus your studies. Education faculty and administrators of teacher preparation programs may also find this practice test useful as they help students prepare for the official test.

TAKING THE PRACTICE TEST

In order to maximize the benefits of the practice test, it is recommended that you take this test under conditions similar to the conditions under which the official MTEL tests are administered. Try to take the practice test in a quiet atmosphere with few interruptions and limit yourself to the four-hour time period* allotted for the official test administration. You will find your results to be more useful if you refer to the answer key only after you have completed the practice test. Note that use of a calculator for the MTEL Middle School Mathematics (47) test is prohibited at the test administration.

INCORPORATING THE PRACTICE TEST IN YOUR STUDY PLAN

Although the primary means of preparing for the MTEL is your college education, adequate preparation prior to taking or retaking the MTEL test is strongly recommended. How much preparation and study you need depends on how comfortable and knowledgeable you are with the content of the test.

The first step in preparing to take the MTEL is to identify what information the test will address by reviewing the objectives for your field. A complete, up-to-date list of the Test Objectives is included in the Test Information Booklet for each test field. The test objectives are the core of the testing program and a helpful study tool. Before taking or retaking the official test, focus your study time on those objectives for which you wish to strengthen your knowledge.

This practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test. However, because of potential differences in format and difficulty between the practice test and an official MTEL Middle School Mathematics (47) test, it is not possible to predict precisely how you might score on an official MTEL Middle School Mathematics (47) test. Keep in mind that the subareas for which the test weighting is greatest will receive emphasis on this test. Refer to the Test Information Booklet for additional information about how to prepare for the test.

* For the Communication and Literacy Skills and General Curriculum tests, candidates may take one or both subtests during the four-hour session.

MIDDLE SCHOOL MATHEMATICS
PRACTICE TEST

Candidates taking the Middle School Mathematics test (field 47) will be provided with the formulas shown below at the test administration.

FORMULAS

Description	Formula
Sum of the measures of the interior angles in a polygon	$S = (n - 2) \times 180$
Circumference of a circle	$C = 2\pi r$
Area of a circle	$A = \pi r^2$
Area of a triangle	$A = \frac{1}{2}bh$
Surface area of a sphere	$A = 4\pi r^2$
Lateral surface area of a right circular cone	$A = \pi r\sqrt{r^2 + h^2}$
Surface area of a cylinder	$A = 2\pi rh + 2\pi r^2$
Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Volume of a right circular cone and a pyramid	$V = \frac{1}{3}Bh$
Volume of a cylinder	$V = \pi r^2 h$
Sum of an arithmetic series	$S_n = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$
Sum of a geometric series	$S_n = \frac{a(1 - r^n)}{1 - r}$
Sum of an infinite geometric series	$\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r},\ \lvert r \rvert < 1$
Distance formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint formula	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Slope	$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

FORMULAS (continued)

Description	Formula
Law of sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Law of cosines	$c^2 = a^2 + b^2 - 2ab \cos C$
Arc length	$s = r\theta$
Density of an object	$D = \frac{m}{V}$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

GENERAL TEST DIRECTIONS

This practice test consists of two sections: (1) a multiple-choice question section and (2) an open-response item assignment section. Each multiple-choice question on the practice test has four answer choices. Read each question carefully and choose the ONE best answer. Record each answer on the answer sheet provided.

Sample Question: 1. What is the capital of Massachusetts?

A. Worcester
B. New Bedford
C. Boston
D. Springfield

The correct answer to this question is C. You would indicate that on the answer sheet.

The open-response section of this practice test requires written responses. Directions for the open-response item assignments appear immediately before those assignments.

You may work on the multiple-choice questions and open-response item assignments in any order that you choose. You may wish to monitor how long it takes you to complete the practice test. When taking the actual MTEL Middle School Mathematics (47) test, you will have one four-hour test session in which to complete the test.

MULTIPLE-CHOICE ANSWER SHEET

Question Number	Your Response
1	
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Question Number	Your Response
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Question Number	Your Response
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99	
100	

MULTIPLE-CHOICE QUESTIONS

1. **Use the expression below to answer the question that follows.**

   37% ___?___ $\frac{3}{7}$ ___?___ 0.45

   Which of the following symbols, when inserted in the expression above from left to right, will correctly complete the number sentence?

   A. = and ≤

   B. = and ≥

   C. ≥ and >

   D. ≤ and <

2. What is the number 212_{three} in base-two form?

   A. 101

   B. 10111

   C. 100011

   D. 1101001

3. **Use the equations below to answer the question that follows.**

   $A = 1 \times 10^{-1} + 6 \times 10^{-2}$
   $B = 2 \times 10^{0} + 7 \times 10^{-2}$

   Two numbers, *A* and *B,* are shown above in expanded notation. What is the sum of *A* and *B*?

   A. 0.223

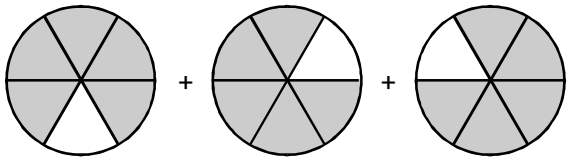
   B. 0.286

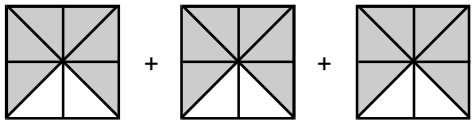
   C. 2.23

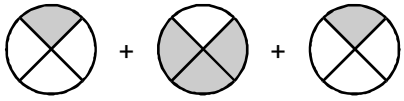
   D. 2.86

4. Which of the following figures describes mixed number $A\frac{b}{c}$ in simplest form such that $Abc = 8$?

   A.
   

   B.
   

   C.
   

   D.
   

5. The prime factorization of 27,360 can be expressed as $a^w \cdot b^x \cdot c^y \cdot d^z$. What is $w + x + y + z$?

   A. 7

   B. 8

   C. 9

   D. 10

6. What is the value of the expression $4 + 6 \times 3^2 - 6 \div 2 + 4$?

   A. 46

   B. 59

   C. 89

   D. 91

7. Which of the following equations best illustrates the concept that multiplication and division are inverse operations?

   A. $\frac{1}{4} \times \frac{-4}{1} = -1$

   B. $4 \div 1 = 4 \times 1$

   C. $(-4) \times (-4) = 16 \div 4$

   D. $\frac{1}{4} \div \frac{1}{16} = \frac{1}{4} \times \frac{16}{1}$

8. **Use the information below to answer the question that follows.**

Given: $-6x + 3x - 4 = 3y + 2$
Step 1: $-3x - 4 = 3y + 2$
Step 2: $-3x = 3y + 6$
Step 3: $-x = y + 2$
Step 4: $x = -y - 2$

What number concept or property is being applied in going from Step 1 to Step 2?

A. additive inverse

B. commutative property

C. transitive property of equality

D. multiplicative identity element

9. Which of the following statements is true for all integers?

A. $|m + n| \geq |m| + |n|$

B. $|m + n| \leq |m| + |n|$

C. $|m + n| < |m| + |n|$

D. $|m + n| = |m| + |n|$

10. **Use the information below to answer the question that follows.**

Given: $2x + 3 = 4y + 1$
Step 1: $2x = 4y - 2$
Step 2: $x = 2y - 1$

What number concept or property is being applied in going from Step 1 to Step 2?

A. multiplicative identity

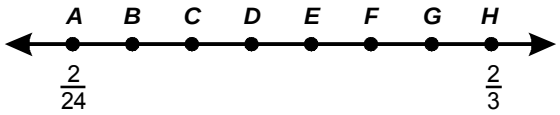
B. commutative property of multiplication

C. multiplicative inverse

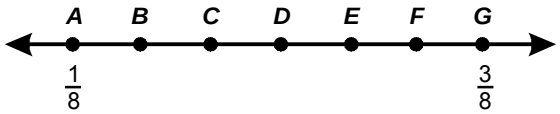
D. distributive property of multiplication

11. Given that the points on each of the number lines shown below are equally spaced, on which of the following number lines does point *D* correspond with the fraction $\frac{1}{4}$?

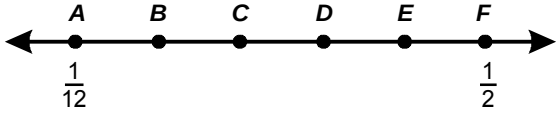
A.



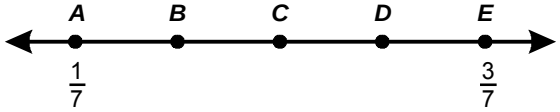
B.



C.



D.



12. Two rectangular house lots are for sale. Although the width is different for the two lots, they are both 110 feet long. The area of lot *A* is 4,950 feet and the area of lot *B* is 7,920 feet. What is the ratio of the width of lot *A* to the width of lot *B*?

A. 5:8

B. 5:13

C. 8:5

D. 8:13

13. The average distance between the earth and the sun is 149,598,000 kilometers. When Mars and the earth are on the same side of the sun, the distance between them is approximately 0.5 times the distance from the earth to the sun. At that time it takes between 7 and 11 months to travel from the earth to Mars. Which of the following intervals is a reasonable estimate for the average speed of a spacecraft traveling from the earth to Mars?

    A. 100,000 km/hour to 150,000 km/hour

    B. 10,000 km/hour to 15,000 km/hour

    C. 1,000 km/hour to 2,000 km/hour

    D. 100 km/hour to 200 km/hour

14. A land developer sells $\frac{1}{4}$ of $8\frac{2}{5}$ acres of land for $5000 an acre. From the proceeds of this land sale, the developer will then pay taxes of 2% on the value of the remaining land, which is valued at $4500 per acre. How much money will the developer have left after completing both transactions?

    A. $1092

    B. $3780

    C. $5040

    D. $9933

15. If $a = (-32)^{\frac{1}{5}}$ and $b = 2^{-3}$, what is the product of a and b?

A. $-\frac{\sqrt[3]{2}}{2}$

B. $-\frac{1}{4}$

C. $\frac{2}{\sqrt[3]{2}}$

D. $\frac{1}{4}$

16. **Use the table below to answer the question that follows.**

◘	**1**	**2**	**3**	**4**
1	1	3	2	4
2	2	4	3	1
3	4	2	1	3
4	3	1	4	2

Which of the following number properties holds under operation ◘ illustrated in the table?

A. associative

B. closure

C. commutative

D. identity

17. Which of the following numbers is the approximate value of $\frac{\sqrt[3]{64}\sqrt{22}}{\sqrt{2}}$?

    A. 13

    B. 20

    C. 27

    D. 38

18. A submarine is traveling at 375 feet below sea level. It rises 183 feet and then dives 228 feet. If the submarine can come safely to the surface at 30 feet per second, approximately how long will it take to reach the surface?

    A. 9 seconds

    B. 11 seconds

    C. 12 seconds

    D. 14 seconds

19. Which of the following expressions is a result of completely simplifying the expression $\frac{\left(\frac{x^4}{y^6}\right)^{-\frac{1}{2}}\left(\frac{x^{\frac{1}{3}}}{y}\right)^3}{x^{-3}y^2}$?

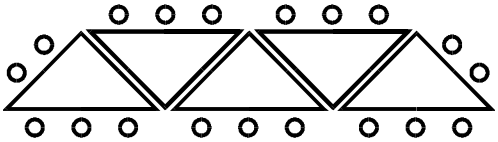
    A. x

    B. $\frac{1}{xy^7}$

    C. $\frac{y^4}{x^5}$

    D. $\frac{x^2}{y^2}$

20. **Use the diagram below to answer the question that follows.**



The diagram above shows the arrangement of tables and chairs in an elementary classroom. For any number of tables linked together in the above pattern, which of the following equations represents the maximum number of students, N, that can be seated at T tables?

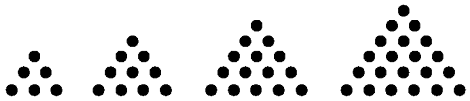
A. $N = 5T + 2$

B. $N = 4T + 3$

C. $N = 3T + 4$

D. $N = 2T + 5$

21. **Use the diagram below to answer the question that follows.**



In the diagram above, which of the following equations represents the relationship between the number of dots, d, in a triangle and the number of rows, R, in that triangle?

A. $d = \frac{2(R + 3)}{2}$

B. $d = \frac{R(R + 1)}{2}$

C. $d = \frac{2(R + 6)}{3}$

D. $d = \frac{2(R^2)}{3}$

22. Which of the following patterns of change is best represented by a quadratic function?

    A. the changing temperature of a gas as the pressure is increased

    B. the rate at which a radioactive element decays into different elements

    C. the changing area of a circle as the circle's radius increases

    D. the rate at which a population of mice grows when resources are unlimited

23. **Use the equation below to answer the question that follows.**

    $^{\circ}F = \frac{9(^{\circ}C)}{5} + 32$

    The equation shows the mathematical relationship between degrees Celsius and degrees Fahrenheit. This mathematical relationship is best described as:

    A. inverse.

    B. quadratic.

    C. periodic.

    D. linear.

24. **Use the table below to answer the question that follows.**

Distance Traveled (meters)	2	0.8	0.32	0.128	0.0512	. . .
Time Period (seconds)	0–30	30–60	60–90	90–120	120–150	. . .

A ball is rolled across a surface. The table above gives the distance the ball rolls every 30 seconds up until 150 seconds have elapsed. Which of the following infinite series represents the sum of the distances covered by the ball in each 30-second interval?

A. $2[1+ 0.8 + (0.8)^2 + (0.8)^3 + (0.8)^4 + \ldots]$

B. $2[1+ 0.4 + (0.4)^2 + (0.4)^3 + (0.4)^4 + \ldots]$

C. $2[0.8 + (0.8)^2 + (0.8)^3 + (0.8)^4 + \ldots]$

D. $2[0.4 + (0.4)^2 + (0.4)^3 + (0.4)^4 + \ldots]$

25. A particular tractor-trailer carrying no cargo can reach a maximum speed of 40 miles per hour when ascending a steep hill. The same tractor-trailer carrying a load weighing 12 tons ascends a hill of the same steepness at a maximum speed of 20 miles per hour. Which of the following linear models could be used to predict the weight that the tractor-trailer could carry up a similar hill?

A. $3w + 5s = 120$

B. $5w + 3s = 120$

C. $5s + 7w = 200$

D. $7s + 5w = 200$

26. Which of the following expressions represents the evaluation of $\frac{(a + c)^b}{b(a^2 - c^2)}$ when $a = 5$, $b = 4$, and $c = -2$?

A. $\frac{1}{28}$

B. $\frac{1}{2}$

C. $\frac{81}{116}$

D. $\frac{27}{28}$

27. The equation $S = \frac{a_1 - a_1 r^n}{1 - r}$ computes the sum of n terms of a geometric sequence. Which of the following equations expresses this equation in terms of a_1?

A. $a_1 = Sr^{n-1}$

B. $a_1 = \frac{S - r}{1 - r^n}$

C. $a_1 = S(1 - r) + r^n$

D. $a_1 = \frac{S - Sr}{1 - r^n}$

28. Which of the following binomials is a factor of $m^4 - 5m^2 - 36$?

A. $m^3 - 12$

B. $m - 3$

C. $m^2 + 9$

D. $m + 2$

29. The quantity p varies directly with the cube of q and inversely with the square of r. If $p = \frac{3}{2}$, $q = \frac{1}{2}$, and $r = \frac{1}{3}$, which of the following equations represents the relationship between p, q, and r?

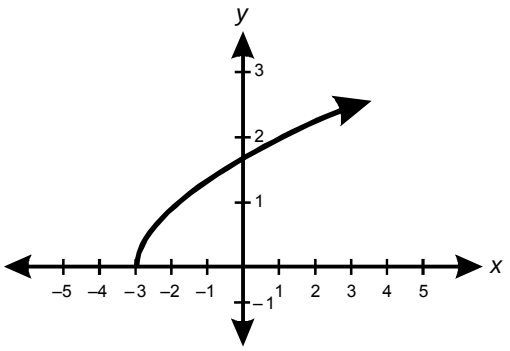
A. $p = \frac{4q^3}{3r^2}$

B. $p = \frac{2q^3}{9r^2}$

C. $p = \frac{3r^2}{2q^3}$

D. $p = \frac{27r^2}{16q^3}$

30. **Use the graph below to answer the question that follows.**



The graph of the function f(x) = $\sqrt{x + 3}$ is shown. Which of the following functions, g(x), represents a translation of the graph of f(x) four units to the left on the x-axis?

A. g(x) = $\sqrt{x + 7}$

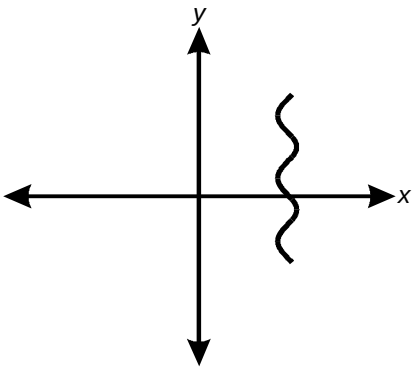
B. g(x) = $\sqrt{x - 1}$

C. g(x) = $\sqrt{x + 3} + 4$

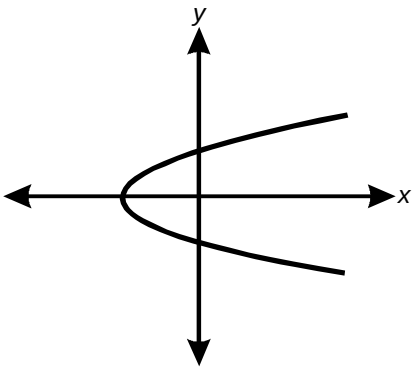
D. g(x) = $\sqrt{x + 3} - 4$

31. Which of the following graphs represents a function?

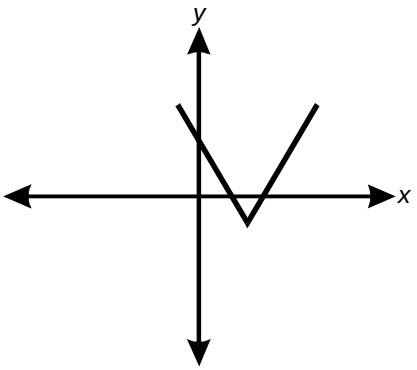
A.



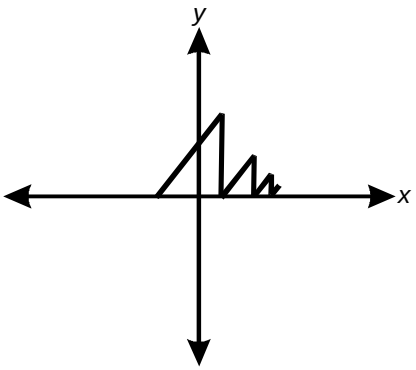
B.



C.

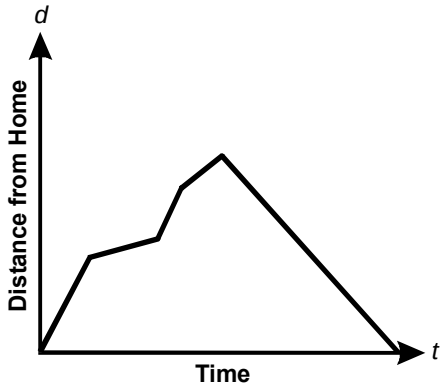


D.

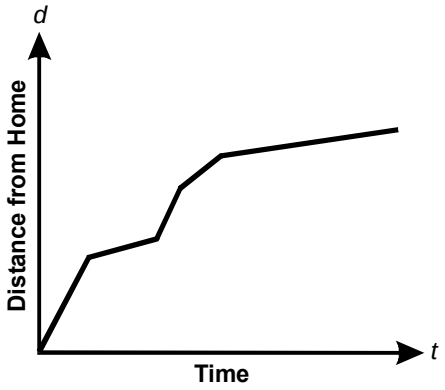


32. The first part of a long car trip is made at highway speed. The driver then reduces the speed of the car in traffic, later resuming highway speed before exiting the highway and following local roads to the destination. The return home is at a steady rate on local roads. Which of the following graphs represents the relationship of distance from home, *d*, to time, *t*, for the round trip?

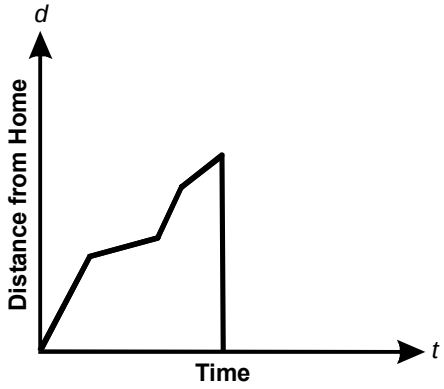
A.



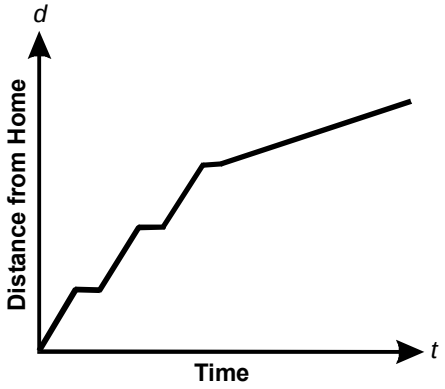
B.



C.



D.



33. **Use the table below to answer the question that follows.**

x	***y***
1	$3\frac{1}{3}$
2	$2\frac{2}{3}$
3	2

Which of the following equations represents the function described by the points in the table?

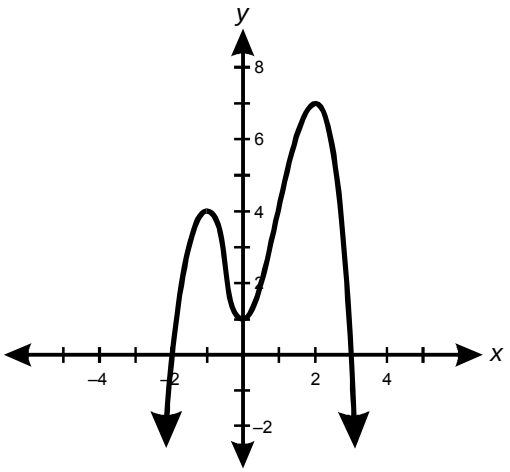
A. $x - 3y = 6$

B. $3y - x = 6$

C. $2x + 3y = 12$

D. $2y + 3x = 12$

34. **Use the graph below to answer the question that follows.**



Which of the following intervals describes the range of the polynomial function in the graph?

A. $(-\infty, \infty)$

B. $(-\infty, 7]$

C. $[4, 7]$

D. $[-2, 3]$

35. If $3x + 4y = -20$ and $2x - 3y = -2$, then what is the value of x?

A. $x = -4$

B. $x = -2$

C. $x = -\frac{28}{17}$

D. $x = -\frac{22}{17}$

36. If line *A* has a slope of $-\frac{5}{8}$, what is the equation of line *B* that contains point *P* = (40, 54) and is perpendicular to line *A*?

A. $y = \frac{8}{5}x + 118$

B. $y = \frac{5}{8}x + 79$

C. $y = \frac{5}{8}x + 29$

D. $y = \frac{8}{5}x - 10$

37. A customer is choosing between two cellular phone plans. One plan has a monthly fee of $50 for an allowance of 500 minutes per month. If the customer uses more than 500 minutes, the charge is $0.35 per additional minute used. The other plan has a monthly fee of $75 for an allowance of 1000 minutes per month. If the customer uses more than 1000 minutes, the charge is $0.40 per additional minute. After how many minutes used are the monthly costs of the plans equal?

A. 4000 minutes

B. 4250 minutes

C. 5000 minutes

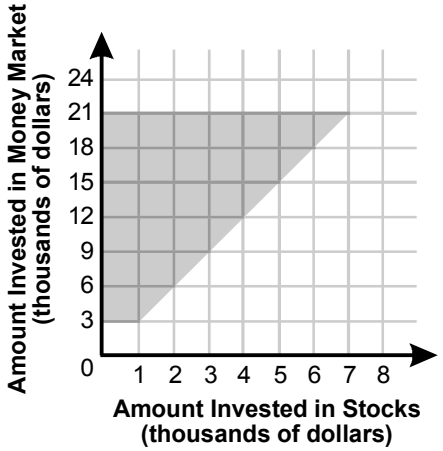
D. 9500 minutes

38. **Use the problem below to answer the question that follows.**

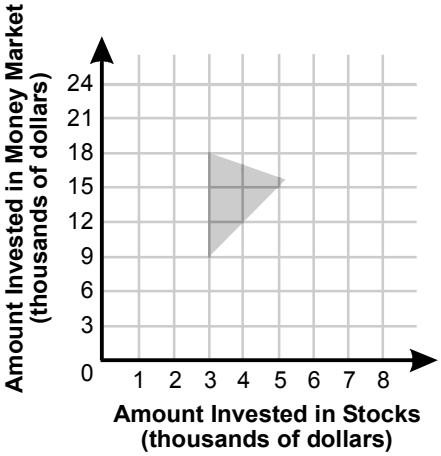
A town manager plans to invest a total of no more than $21,000 in the stock market and in a money market fund. The amount invested in the money market should be at least three times the amount invested in the stock market, and at least $3,000 should go into the stock market. Given these constraints, what are the possible ways the manager can invest the money?

The shaded portion of which of the following graphs represents the solution to this problem?

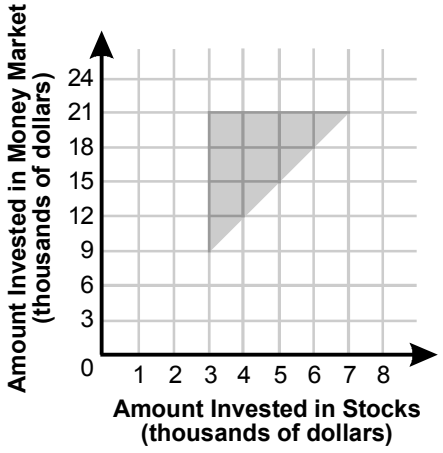
A.



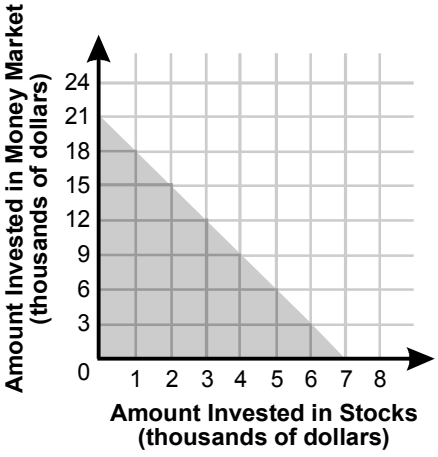
B.



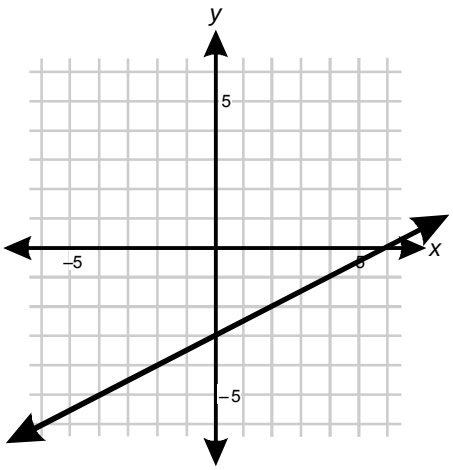
C.



D.



39. **Use the graph below to answer the question that follows.**



The graph represents the equation $2y - ax + 6 = 0$. What is the value of a?

A. –3

B. –2

C. 1

D. 6

40. **Use the system of equations below to answer the question that follows.**

$y = 2x - 4$
$y = 2x^2 - 4x - 7$

Which of the following expressions represents the x-coordinate of a point in the solution set of the above system of equations?

A. $\frac{1 + 2\sqrt{23}}{2}$

B. $\frac{2 - \sqrt{26}}{4}$

C. $\frac{3 + \sqrt{15}}{2}$

D. $\frac{3 - \sqrt{3}}{4}$

41. The graph of a parabola of the form $y = x^2 + bx + c$ crosses the y-axis at -36 and the x-axis at $x = 3$. At what other point does the graph cross the x-axis?

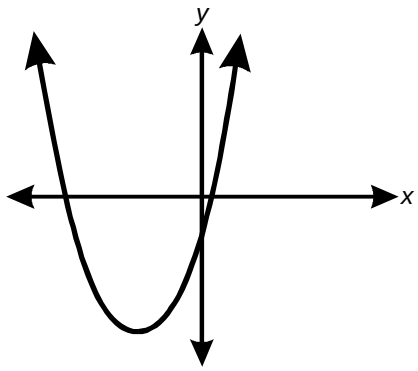
A. $x = -12$

B. $x = -9$

C. $x = 9$

D. $x = 12$

42. **Use the graph below to answer the question that follows.**



If the equation representing the graph above is written as $y = ax^2 + bx + c$, which of the following statements about the signs of a and b must be true?

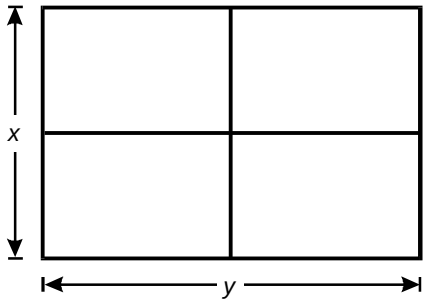
A. $a > 0$ and $b > 0$

B. $a > 0$ and $b < 0$

C. $a < 0$ and $b > 0$

D. $a < 0$ and $b < 0$

43. **Use the diagram below to answer the question that follows.**



An artist is designing the rectangular wire structure shown above. The artist has a 12-meter length of wire. If the entire area of the figure is to be maximized, what should be the length of *x*? (Assume that all of the wire is used.)

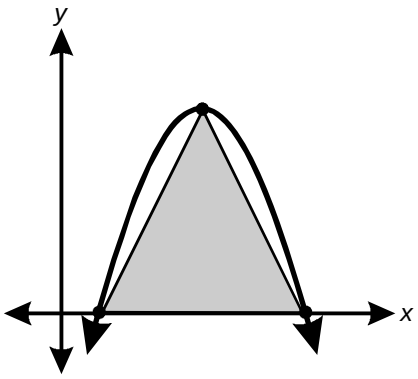
A. 1.5 m

B. 2 m

C. 4 m

D. 6 m

44. **Use the diagram below to answer the question that follows.**



The diagram above shows a design consisting of an isosceles triangle drawn under a parabola so that the top of the triangle coincides with the vertex of the parabola. If the equation of the parabola is $y = -3x^2 + 18x - 15$, what is the area of the triangle?

A. 6 square units

B. 12 square units

C. 18 square units

D. 24 square units

45. The final step in solving a quadratic equation by factoring is based on which of the following properties of the real numbers?

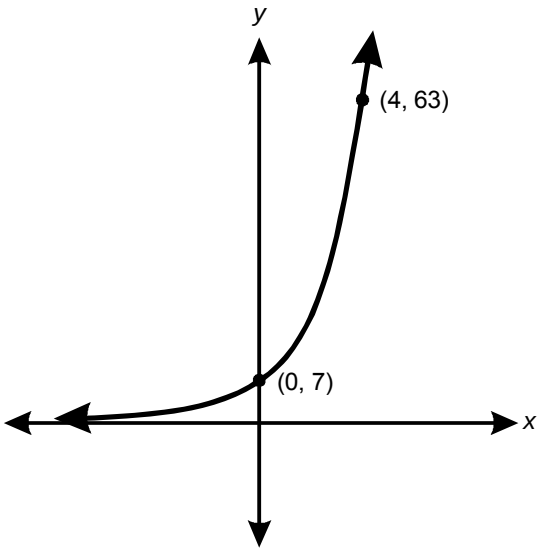
A. If $a \neq 0$, then $a \bullet \frac{1}{a} = 1$.

B. If a divides both b and c, then a divides $b - c$.

C. If a and b are both ≥ 0, then $\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$.

D. If $ab = 0$, then $a = 0$ or $b = 0$ or they both equal zero.

46. **Use the graph below to answer the question that follows.**



The graph above shows an exponential function represented by $y = a3^{bx}$, where a and b are constants. What is the value of b?

A. 0.25

B. 0.5

C. 2

D. 9

47. Each year, the profits at a company increase by 7%. If the profits at the end of the first year are $4000, which of the following expressions represents the profits at the end of the 10th year?

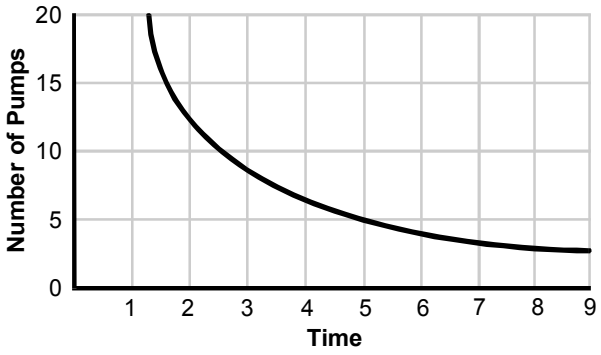
A. 4000(9)(0.7)

B. 4000(10)(1.07)

C. $4000 + 10(1.07)^{10}$

D. $4000(1.07)^{9}$

48. **Use the graph below to answer the question that follows.**



The graph above shows an inverse relationship between the amount of time required to drain a large tank and the number of pumps draining the tank. What is the constant of proportionality for this relationship?

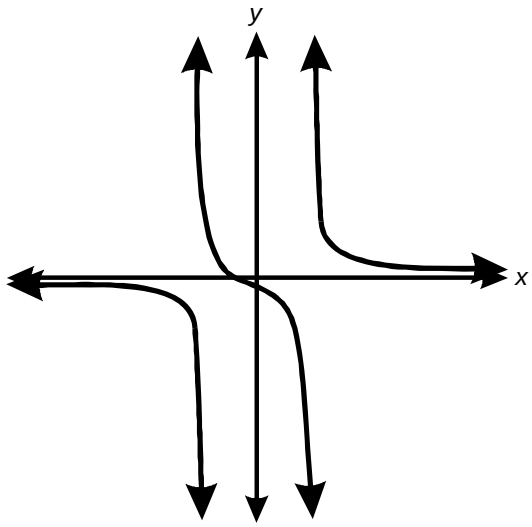
A. $\frac{1}{20}$

B. $\frac{1}{5}$

C. 5

D. 25

49. **Use the graph below to answer the question that follows.**



The graph shown above represents a function of the form $y = \frac{ax + b}{cx^2 + d}$. How many asymptotes does the graph have?

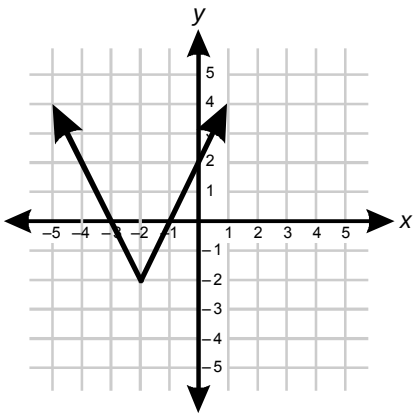
A. 0

B. 2

C. 3

D. 4

50. **Use the graph below to answer the question that follows.**



Which of the following equations represents this graph?

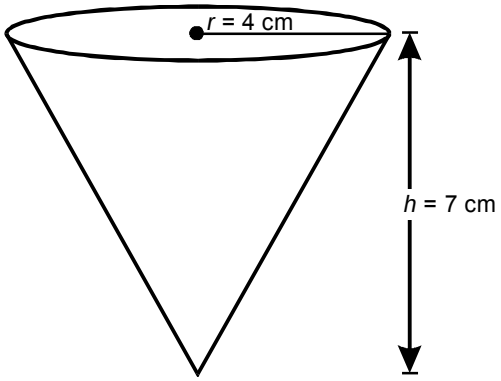
A. $y = |2x + 4| - 2$

B. $y = |2x + 4|$

C. $y = |2x - 4| - 2$

D. $y = |2x - 4|$

51. **Use the diagram below to answer the question that follows.**



A toy manufacturer packages 25 of its marbles in a plastic cone-shaped cup with the dimensions shown in the diagram. If the radius of a marble is 1 centimeter, approximately what percent of a full cone is empty space?

A. 5%

B. 10%

C. 15%

D. 20%

52. **Use the information below to answer the question that follows.**

1 U.S. dollar = 1.7 Brazilian real 1 U.S. dollar = 44 Indian rupees 1 Indian rupee = 0.016 euros

A journalist has 400 Brazilian reals left from a recent trip and wants to convert them to euros before leaving on an assignment in Europe. Based on the currency exchange rates shown, how many euros can be purchased with the Brazilian money?

A. 166

B. 334

C. 479

D. 966

53. A spherical balloon with a surface area of 36π square inches is inflated further until its diameter has increased by 50%. The surface area of the balloon is now:

A. 54π square inches.

B. 81π square inches.

C. 122π square inches.

D. 216π square inches.

54. The dimensions of a rug are 4.2 meters × 2.8 meters. Based on the greatest degree of error in the measurements provided, the actual area of the rug is within which of the following ranges?

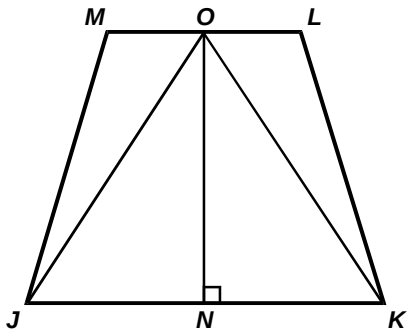
    A. (4 m × 2 m) to (5 m × 3 m)

    B. (4.1 m × 2.7 m) to (4.3 m × 2.9 m)

    C. (4.15 m × 2.75 m) to (4.24 m × 2.84 m)

    D. (4.199 m × 2.799 m) to (4.211 m × 2.811)

55. **Use the diagram below to answer the question that follows.**



If quadrilateral *JKLM* is an isosceles trapezoid and *O* is the midpoint of $\overline{LM}$, the most direct proof that $\triangle OJM \cong \triangle OKL$ is based on which of the following triangle congruence theorems?

A. AAA

B. ASA

C. SAS

D. SSS

56. Which of the following pairs of sets could be the lengths of the sides of two similar triangles?

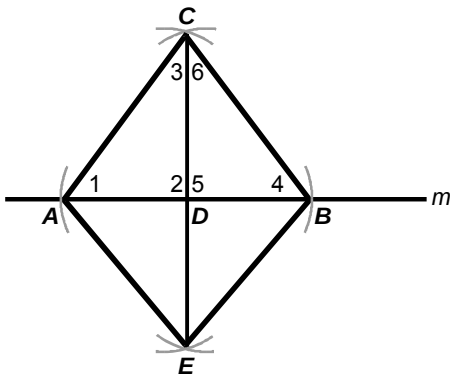
A. {0.64, 1.44, 1.69} and {0.8, 1.2, 1.3}

B. $\left\{\frac{3}{2}, 4, 5\right\}$ and $\left\{3, 2, \frac{5}{2}\right\}$

C. {5, 10, 12} and {7, 12, 14}

D. $\left\{\frac{8}{3}, 4, 5\right\}$ and $\left\{2, 3, \frac{15}{4}\right\}$

57. **Use the diagram and construction below to answer the question that follows.**



Step 1: Use a compass, and with *D* as the center of a circle construct arcs that intersect line *m* at points *A* and *B*.

Step 2: Use a radius larger than that used in Step 1, and with *A* and *B* as centers, construct arcs that intersect at points *C* and *E*.

Step 3: Use a straightedge to draw a line through points *C*, *D*, and *E*.

Which of the following relationships allows the conclusion that the constructed line is perpendicular to line *m*?

A. $\angle 2 \cong \angle 5$

B. $\overline{AD} \cong \overline{BD}$

C. $\angle 1 \cong \angle 4$

D. $\overline{CD} \cong \overline{CD}$

58. A mathematical challenge confronting the Hindu civilization as far back as 800 B.C.E. was the construction of a particular altar in the shape of a circle. It was essential that the area of this altar be exactly equal to that of another altar that was square-shaped. Mathematicians now recognize that this "squaring the circle" problem was not solved by the Hindus because:

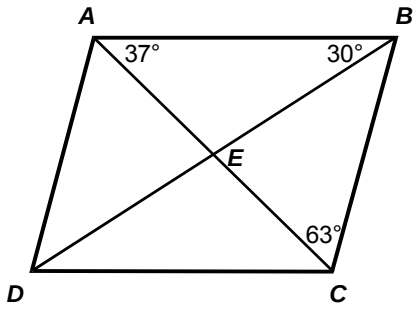
A. axioms of Euclidean geometry had not yet been recognized.

B. pi is a transcendental rather than an algebraic number.

C. techniques of differential calculus are required.

D. zero as a placeholder in number representation was as yet unknown.

59. **Use the diagram below to answer the question that follows.**



In parallelogram *ABCD*, what is the measure of $\angle BEC$?

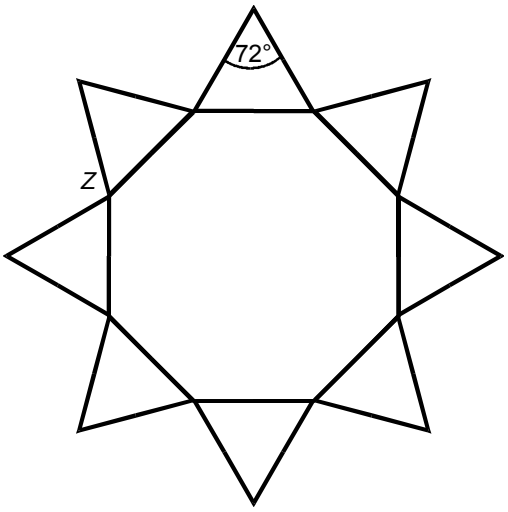
A. 60°

B. 63°

C. 67°

D. 70°

60. **Use the diagram below to answer the question that follows.**



The diagram above was created by adding isosceles triangles to the sides of a regular octagon. What is the measure of angle *Z*?

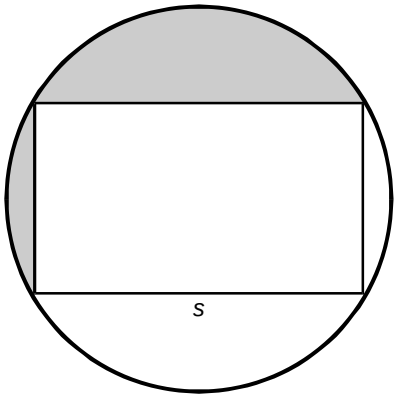
A. 135°

B. 126°

C. 117°

D. 108°

61. **Use the diagram below to answer the question that follows.**



In the diagram above, a rectangle is inscribed in a circle with a radius of 5 units, and the sum of the shaded areas is $12.5\pi - 24$ square units. What is s, the length of the rectangle?

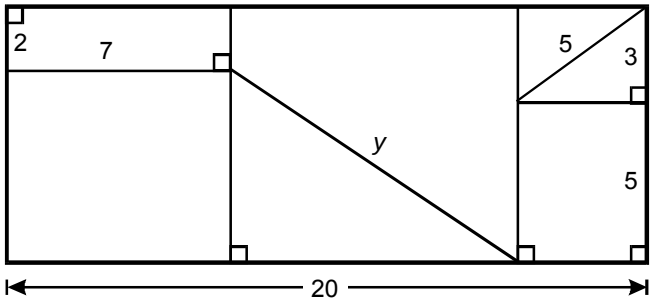
A. 5 units

B. 6 units

C. 8 units

D. 10 units

62. **Use the diagram below to answer the question that follows.**



What is the length of *y* in the diagram above?

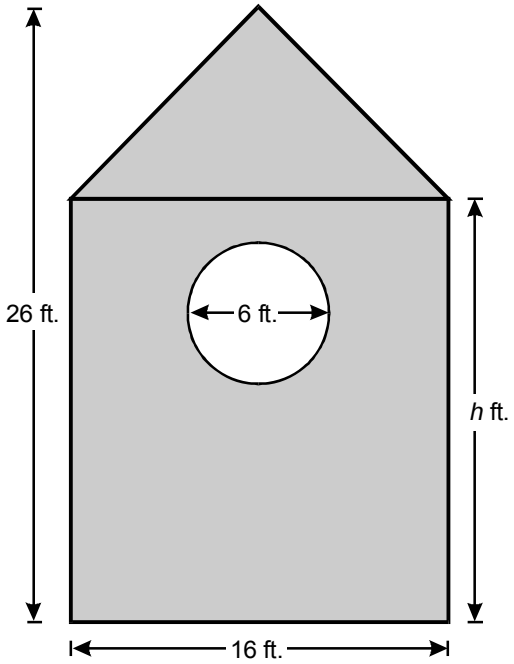
A. 13

B. $\sqrt{117}$

C. 10

D. $\sqrt{85}$

63. **Use the diagram below to answer the question that follows.**



The front of a house with the dimensions indicated is shown in the diagram. The shaded portion of the diagram represents the 324 square feet that will be painted. What is *h*, the height of the house rounded to the nearest foot, excluding the attic and eaves?

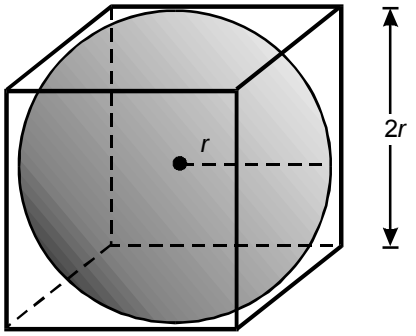
A. 15

B. 16

C. 17

D. 18

64. **Use the diagram below to answer the question that follows.**



A sphere is inscribed in a cube as shown in the diagram above. If the volume of the cube is 64 cubic inches, what is the approximate volume of the sphere?

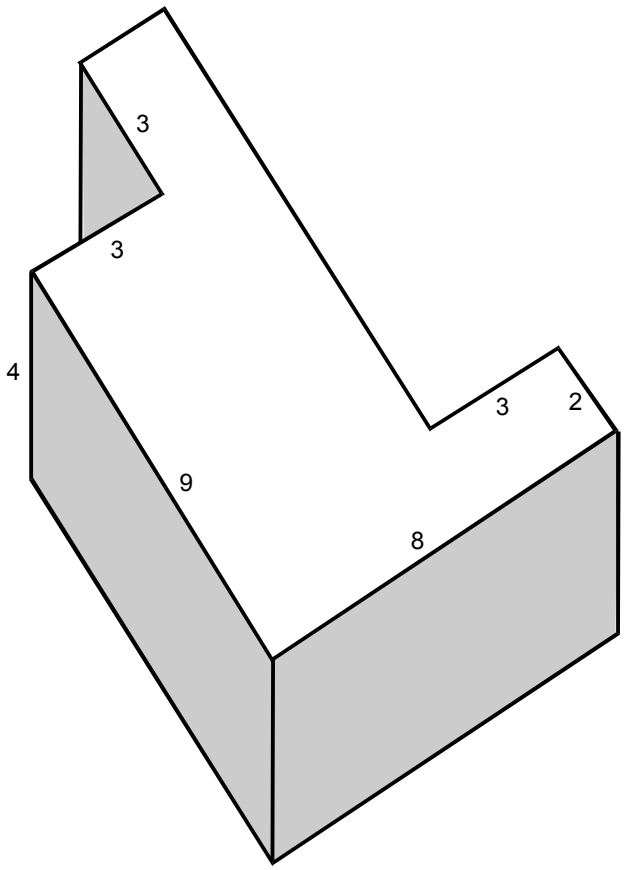
A. 33 cubic inches

B. 41 cubic inches

C. 50 cubic inches

D. 61 cubic inches

65. **Use the figure below to answer the question that follows.**



In the figure above, all edges meet at right angles. What is the volume of the figure?

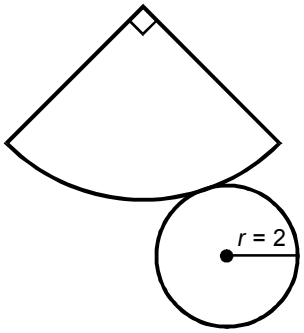
A. 188 cubic units

B. 228 cubic units

C. 252 cubic units

D. 268 cubic units

66. **Use the diagram below to answer the question that follows.**



The diagram above shows the net for a right circular cone that has a base with a radius of 2 units. What is the total surface area of the cone, including the base, in square units?

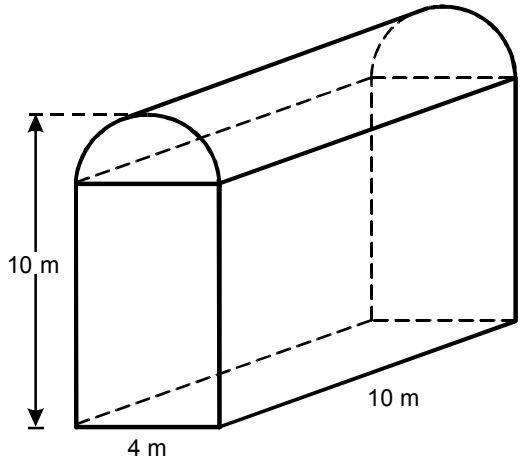
A. 6π square units

B. 8π square units

C. 16π square units

D. 20π square units

67. **Use the diagram below to answer the question that follows.**



A carpenter builds the structure in the diagram above with a rectangle for its base, a half cylinder for a roof, and the dimensions shown. Which of the following estimates best approximates the total volume of the structure?

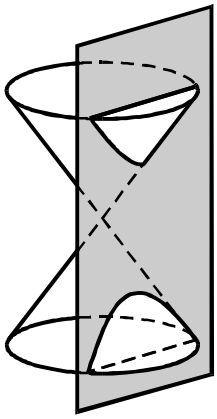
A. 303 cubic meters

B. 366 cubic meters

C. 383 cubic meters

D. 420 cubic meters

68. **Use the diagram below to answer the question that follows.**



A plane intersects a double-napped cone as shown in the diagram above. If a, b, and c are positive real numbers, which of the following types of equations could be used to describe the intersection?

A. $ax^2 + by = c$

B. $ax - by^2 = c$

C. $ax^2 + by^2 = c$

D. $ax^2 - by^2 = c$

69. A circle with a circumference of 16π is constructed on a coordinate grid. The center of the circle is located at (5, 5). Which of the following coordinates could be a point on the circle?

A. (5, –3)

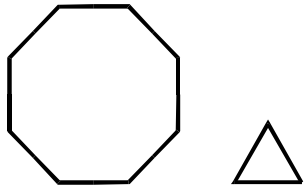
B. (5, 1)

C. (7, 5)

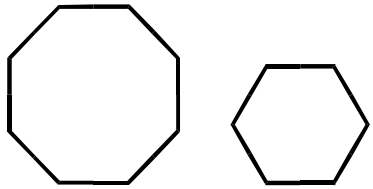
D. (21, 5)

70. Which of the following pairs of shapes can be used to create a semi-regular tessellation?

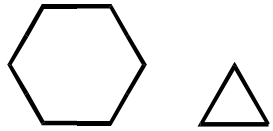
A.



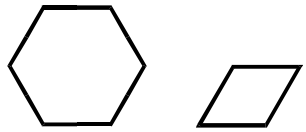
B.



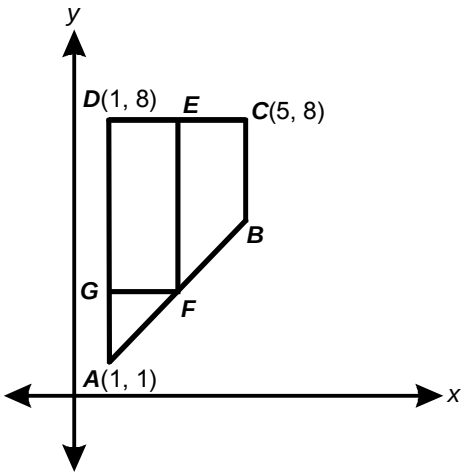
C.



D.



71. **Use the diagram below to answer the question that follows.**



Quadrilateral *ABCD* is constructed on a coordinate plane, as shown in the diagram. $\overline{EF}$ bisects $\overline{DC}$ and $\overline{AB}$ and triangle *AGF* is isosceles. What is the area of the rectangle *DEFG*?

A. 6 square units

B. 7 square units

C. 10 square units

D. 12 square units

72. Rectangle *RSTU* with an area of 108 square units in the coordinate plane undergoes a dilation with a scale factor of $\frac{2}{3}$.

What is the area of the new rectangle, *R'S'T'U'*?

A. 48 square units

B. 64 square units

C. 162 square units

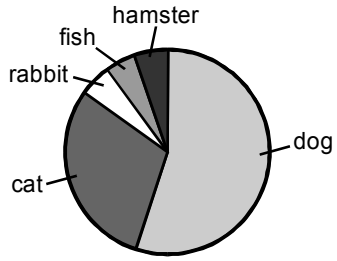
D. 243 square units

73. **Use the table below to answer the question that follows.**

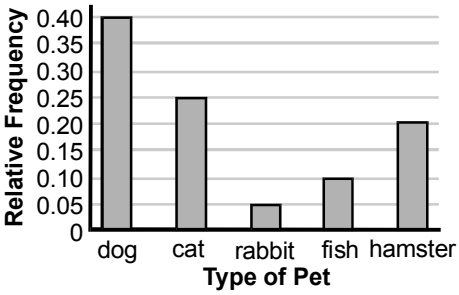
Type of Pet	Frequency
dog	8
cat	5
rabbit	1
fish	2
hamster	4

The table above shows the results of a survey asking a classroom of seventh graders to name their favorite type of pet. Which of the following graphs most accurately illustrates the percentage of students in the class preferring each type of pet?

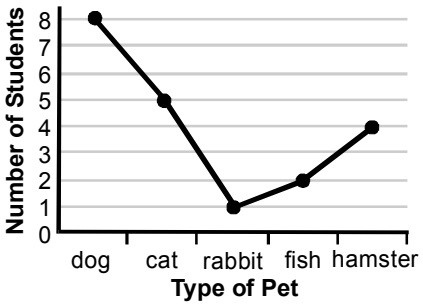
A.



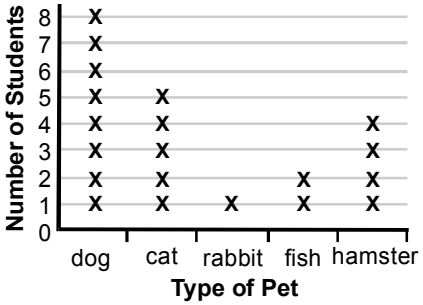
B.



C.



D.



74. **Use the information below to answer the question that follows.**

0, 0, 1, 1, 1, 2, 4, 4, 4, 4, 5, 6, 6, 7, 8

A human resources manager is compiling data on the number of sick days taken by employees during the previous year. The data are shown in the box above. Which of the following statistics will be *least* affected if an employee who took 15 sick days is included in the data set?

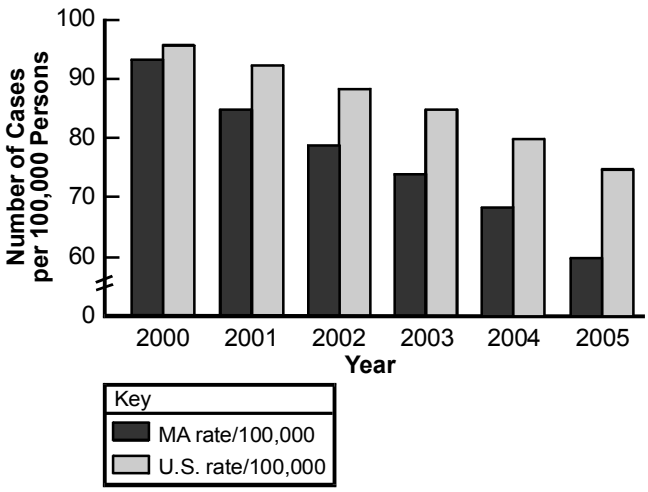
A. midrange

B. mean

C. median

D. range

75. **Use the graph below to answer the question that follows.**



The graph above compares the incidence rate for a disease in Massachusetts with the overall incidence rate for the disease in the United States. The data summarized in the graph suggest that:

A. the disease is likely to be eradicated in the state of Massachusetts by 2015.

B. in 2005, the Massachusetts incidence rate was less than half the incidence rate in the United States.

C. the incidence of the disease appears to be decreasing at a faster rate in Massachusetts than in the United States.

D. in 2003, the total number of cases in the United States was approximately 8.5 million.

76. **Use the table below to answer the question that follows.**

Vehicle Type	Miles per Gallon
1	15, 14, 16, 13, 17, 14, 16, 14, 15, 14
2	19, 19, 18, 17, 18, 19, 18, 20, 18, 19
3	29, 30, 30, 30, 29, 31, 30, 29, 28, 30
4	11, 10, 12, 15, 11, 13, 14, 10, 12, 15

The table above shows highway gas mileage data compiled by a consumer product testing company for four different types of vehicles. Based on the data shown, which type of vehicle demonstrates the greatest standard deviation in highway gas mileage?

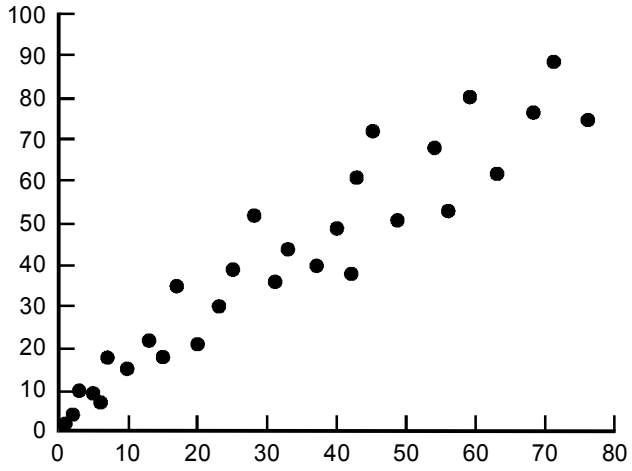
A. Type 1

B. Type 2

C. Type 3

D. Type 4

77. **Use the graph below to answer the question that follows.**



The data shown in the scatter plot above could be used to generate a linear regression equation. Which of the following statements best describes the effect on this equation of adding an additional data point with the coordinates (120, 50) to the analysis?

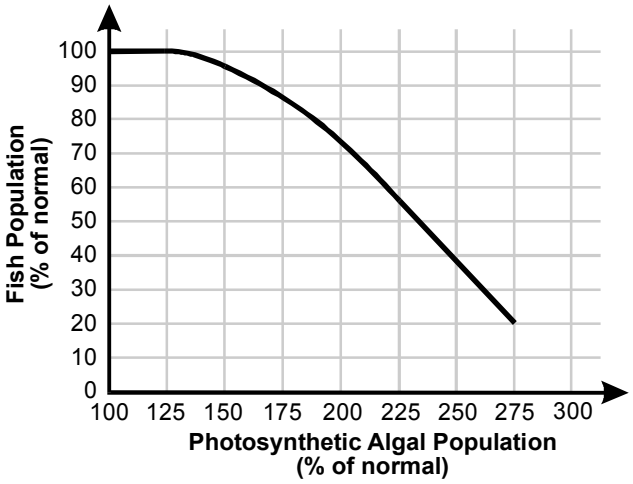
A. It would have a minimal effect because it is outside the range of the other data points.

B. It would have a minimal effect because of the number of other data points.

C. It would be discarded because it is an outlier with respect to the other data points.

D. It would have a significant effect because it is an outlier with respect to the other data points.

78. **Use the graph below to answer the question that follows.**



The graph above shows the relationship between the size of the algal population and the numbers of fish in a pond. Which of the following conclusions can be drawn from the data in the graph?

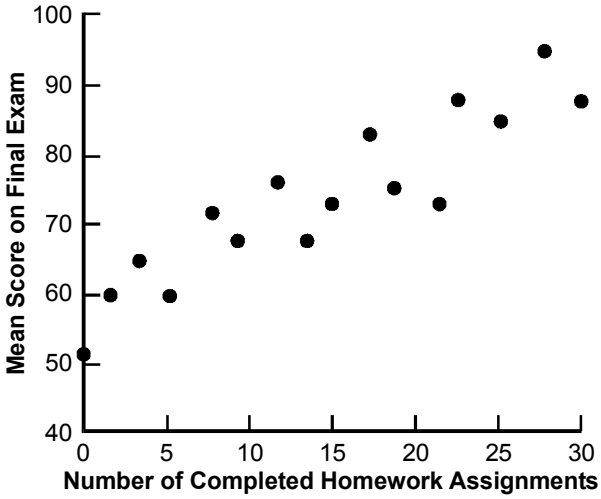
A. The fish population will have decreased by more than 85% when the algal population reaches 300% of normal.

B. The algae in the pond are responsible for the decline in the fish population.

C. The algal population is dependent on the presence of a large and healthy population of this fish species.

D. The number of fish in the pond could be increased to greater than 100% of normal by removing all the algae.

79. **Use the graph below to answer the question that follows.**



The scatter plot above is based on data from 100 math students and compares their success on a final exam with the number of homework assignments they completed during the semester. Based on these data, it can be concluded that:

A. students who complete 30 homework assignments in a semester will score a grade of 85 or better on the final exam.

B. there is a strong positive correlation between completing homework assignments and performance on the final exam.

C. scoring poorly on the final exam is a result of failing to complete homework assignments during the semester.

D. completion of homework assignments during the semester is a poor predictor of how students will perform on the final exam.

80. Two boxes of tickets will be used to select the winners in a raffle drawing. Each box contains 50 raffle tickets with unique identifying numbers. If one ticket is withdrawn from each box, how many outcomes are in the sample space for the drawing?

A. 50

B. 100

C. 1500

D. 2500

81. **Use the table below to answer the question that follows.**

	Students' Ages (years)					Totals
	9	**10**	**11**	**12**	**13**	
Boys	1	4	5	3	2	15
Girls	2	5	4	3	4	18
Totals	3	9	9	6	6	33

The table above shows the ages and genders of students in a drama club. If one student is selected randomly from the club members, what is the probability that the student will be a girl or be 11 years old?

A. 0.303

B. 0.545

C. 0.697

D. 0.818

82. A statistician is interested in determining the probability of getting at least one vowel in random five-letter samplings of the English alphabet. Which of the following simulations would generate data to answer this question?

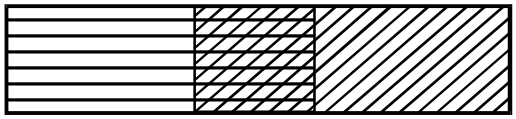
    A. using a computer to randomly select 1000 five-letter words from a dictionary

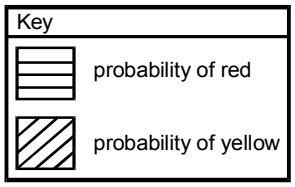
    B. assigning the numbers 1–26 to the letters A–Z and using a calculator or computer to generate 100 random sets of five numbers

    C. designating each side of five coins as either vowel or consonant and flipping the set of five coins 100 times

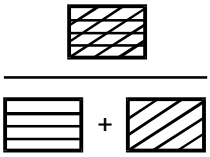
    D. writing the letters of the alphabet on pieces of paper and randomly drawing five-letter samples until a sample is drawn with only one vowel

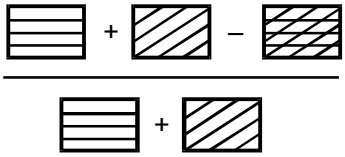
83. **Use the area model below to answer the question that follows.**

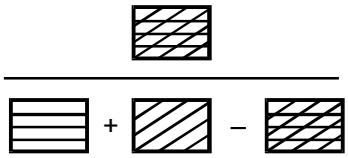


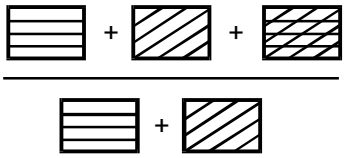


Two sets of colored blocks are placed in a bag. The six red blocks are numbered 1–6 and the six yellow blocks are numbered 1–6. Based on the area model shown, which of the following expressions best represents the probability that a block randomly selected from the bag will be red or an odd number?

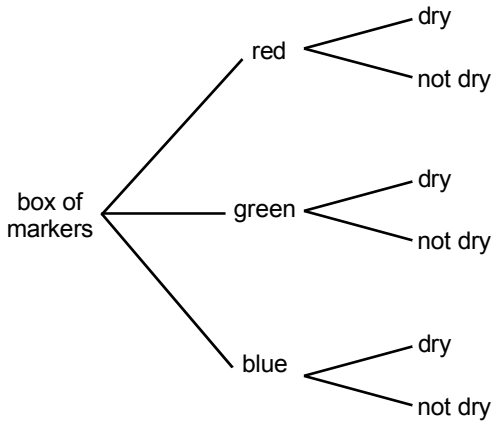
A. 

B. 

C. 

D. 

84. **Use the diagram below to answer the question that follows.**



The tree diagram above shows the composition of a box of used color markers in a classroom. There are 2 red markers, 4 green markers, and 6 blue markers. One-half of the red markers have dried out, 25% of the green markers are dry, and 50% of the blue markers are dry. What is the approximate probability that a marker randomly selected from the box will be dried out?

A. 0.063

B. 0.333

C. 0.417

D. 0.538

85. A number cube has 6 faces, numbered from 1 to 6. If the cube is rolled six times, what is the probability that a face with a 6 comes up exactly once?

A. $\frac{1}{6}$

B. $\left(\frac{5}{6}\right)^5$

C. $1 - \left(\frac{5}{6}\right)^5$

D. $\frac{5}{6}$

86. A student is flipping four pennies at the same time and recording the total number of heads that come up when the coins land on the ground. How many outcomes are possible in the sample space for this activity?

A. 2

B. 4

C. 5

D. 16

87. Three red and three blue flags are arranged randomly along a wire. What is the probability that the six flags alternate in color?

A. $\frac{1}{20}$

B. $\frac{1}{10}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

88. Which of the following expressions is equal to the area of an isosceles triangle with a base angle θ, a base of length 16, and legs of length 12?

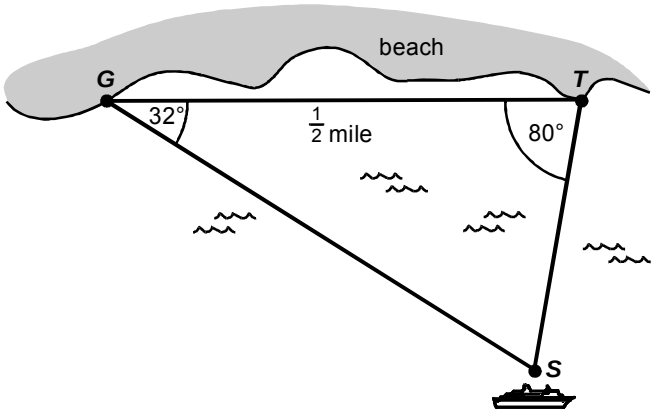
A. $48 \cos \theta$

B. $96 \cos \theta$

C. $48 \sin \theta$

D. $96 \sin \theta$

89. **Use the diagram below to answer the question that follows.**



Two people are located $\frac{1}{2}$ mile apart at points *G* and *T* on a beach when they see a ship at point *S*. They determine that m∠*TGS* = 32° and m∠*GTS* = 80°. Which of the following expressions equals *ST*?

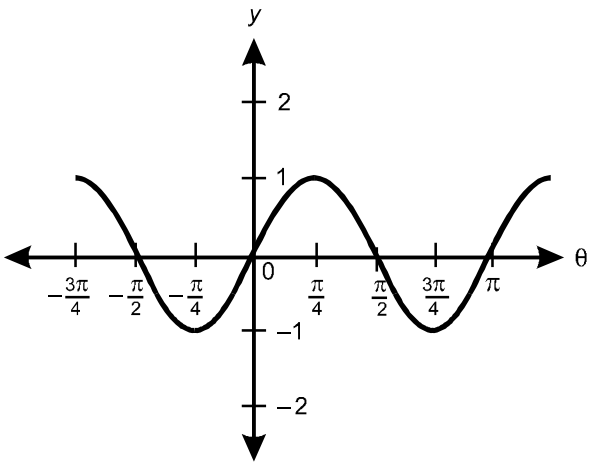
A. $\frac{\sin 32°}{2 \sin 68°}$

B. $\frac{\sin 80°}{2 \sin 68°}$

C. $\frac{\sin 32°}{2 \sin 80°}$

D. $\frac{\sin 80°}{2 \sin 32°}$

90. **Use the graph below to answer the question that follows.**



The graph above represents which of the following trigonometric functions?

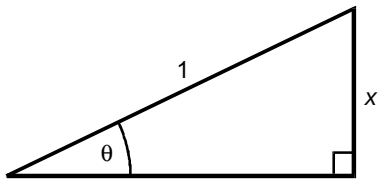
A. $y = \cos\left(\frac{1}{2}\theta\right)$

B. $y = \cos(2\theta)$

C. $y = \sin\left(\frac{1}{2}\theta\right)$

D. $y = \sin(2\theta)$

91. **Use the diagram below to answer the question that follows.**



Which of the following expressions is equal to tan θ?

A. $\sqrt{1 - x^2}$

B. $\frac{1}{\sqrt{1 - x^2}}$

C. $\frac{x}{\sqrt{1 - x^2}}$

D. $\frac{\sqrt{1 - x^2}}{x}$

92. **Use the table below to answer the question that follows.**

t	*h*
–3	1.1
–2	–1.9
–1	1.5
0	0
1	–1.5
2	1.9
3	–1.1

Which of the following types of functions best models the data in the table?

A. exponential

B. quadratic

C. rational

D. trigonometric

93. What is $\lim_{x \to \infty} \frac{3x^2 - 5x - 2}{4x^2 + 3}$?

A. $-\frac{4}{7}$

B. 0

C. $\frac{3}{4}$

D. ∞

94. A farmer plans to use 120 feet of fencing to build a rectangular animal pen with the side of a barn serving as one of the longer sides of the pen. If x represents the width of the pen and the techniques of differential calculus are used, solving which of the following equations is a step in finding the maximum possible area of the animal pen?

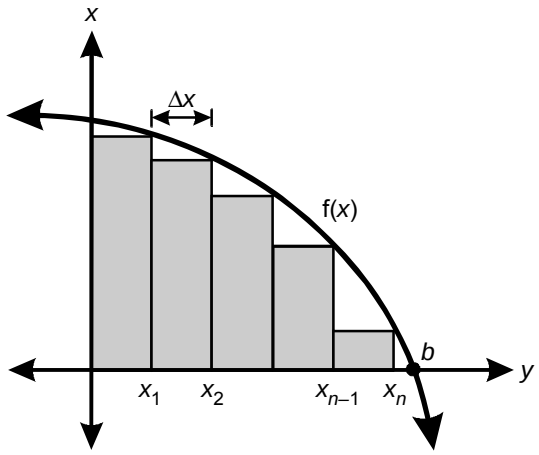
A. $900 - x^2 = 0$

B. $120 - 4x = 0$

C. $x^2 - 120 = 780$

D. $9x - 120 = 150$

95. **Use the graph below to answer the question that follows.**



The top right-hand corner of each rectangle shown under the curve lies on the graph of $y = f(x)$ and the base of each rectangle has length Δx. The sum of the areas of the rectangles is an approximation for which of the following expressions?

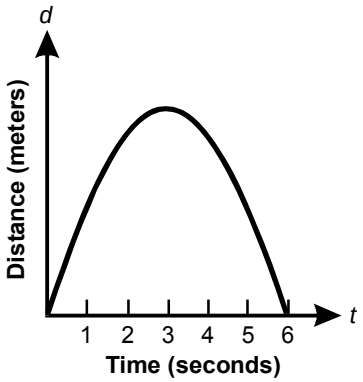
A. $\dfrac{f(b) - f(0)}{b}$

B. $\dfrac{df(x)}{dx}$

C. $\dfrac{f(b) + f(0)}{b}$

D. $\displaystyle\int_0^b f(x)\,dx$

96. **Use the graph below to answer the question that follows.**



The graph shows the distance, d, in meters from the origin of an object moving in a straight line as a function of time, t, in seconds. If the graph is described by $d = -t^2 + 6t$, what is the average velocity of the object between 1 second and 3 seconds?

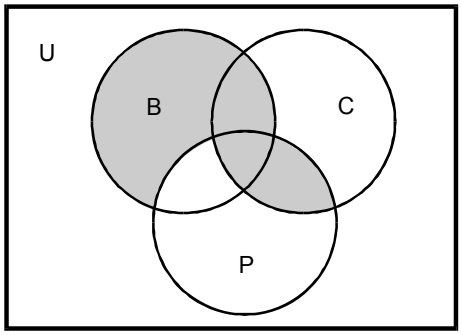
A. 2 m/s

B. 7 m/s

C. 8 m/s

D. 14 m/s

97. **Use the diagram and information below to answer the question that follows.**



U = universal set of students who have taken science

B = subset of students who have taken biology

C = subset of students who have taken chemistry

P = subset of students who have taken physics

Which of the following expressions best represents the shaded region of the Venn diagram shown above?

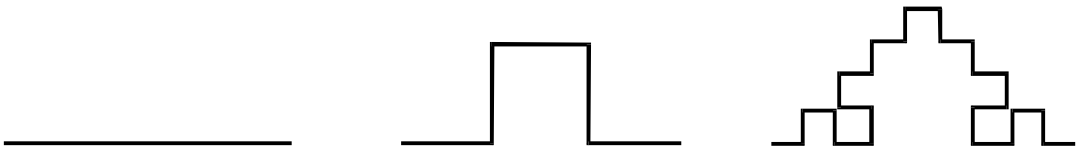
A. $[(B) – (C \cup P)] + (C \cap P)$

B. $[(B) + (C \cap P)] – (C \cup P)$

C. $[(B) – (C) – (P)] + (B \cap C \cap P)$

D. $(B) + (C \cap P)$

98. **Use the information below to answer the question that follows.**



Starting Line Segment **1st Application** **2nd Application**

The diagram above shows a line segment and the first two figures that result from the execution of the following procedure:

1. Begin with a line segment.
2. Divide the line segment into three equal sections.
3. Remove the middle section of the segment and replace it with a square while removing the square's base.
4. Repeat this procedure with each segment in the new figure, then again with the next figure, and so on.

If the length of the original line segment is 3, what is the combined length of the segments in the figure that results from the third application of the procedure?

A. $\frac{625}{3}$

B. $\frac{625}{9}$

C. $\frac{125}{3}$

D. $\frac{125}{9}$

99. **Use the figure below to answer the question that follows.**

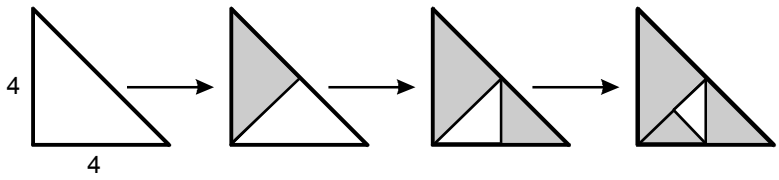


Figure 1 **Figure 2** **Figure 3** **Figure 4**

If the pattern above continues, what is the combined area of the shaded regions in the *sixth* figure?

A. $7\frac{15}{16}$ square units

B. $7\frac{7}{8}$ square units

C. $7\frac{3}{4}$ square units

D. $7\frac{3}{8}$ square units

100. **Use the matrices below to answer the question that follows.**

$$\begin{bmatrix} 1 & 0 & 4 \\ 3 & -2 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 \\ 2 & 4 \\ 3 & 2 \end{bmatrix}$$

A **B**

Which of the following matrices represents the product of matrix A and matrix B?

A. $\begin{bmatrix} 9 & 6 \\ 11 & 8 \end{bmatrix}$

B. $\begin{bmatrix} 11 & 8 \\ -4 & -6 \end{bmatrix}$

C. $\begin{bmatrix} 9 & 11 \\ 6 & 8 \end{bmatrix}$

D. $\begin{bmatrix} 11 & -4 \\ 8 & -6 \end{bmatrix}$

DIRECTIONS FOR THE OPEN-RESPONSE ITEM ASSIGNMENTS

This section of the test consists of two open-response item assignments that appear on the following pages. You will be asked to prepare a written response of approximately 150–300 words (1–2 pages) for each assignment. You should use your time to plan, write, review, and edit your response for each assignment.

For each assignment, read the topic and directions carefully before you begin to work. Think about how you will organize your response. You may use any blank space in this test booklet to make notes, write an outline, or otherwise prepare your response.

As a whole, your response to each assignment must demonstrate an understanding of the knowledge of the field. In your response to each assignment, you are expected to demonstrate the depth of your understanding of the subject area by applying your knowledge rather than by merely reciting factual information.

Your response to each assignment will be evaluated based on the following criteria.

- **PURPOSE:** the extent to which the response achieves the purpose of the assignment
- **SUBJECT KNOWLEDGE:** appropriateness and accuracy in the application of subject knowledge
- **SUPPORT:** quality and relevance of supporting evidence
- **RATIONALE:** soundness of argument and degree of understanding of the subject area

The open-response item assignments are intended to assess subject knowledge. Your responses must be communicated clearly enough to permit valid judgment of the evaluation criteria by scorers. Your responses should be written for an audience of educators in this field. The final version of each response should conform to the conventions of edited American English. Your responses should be your original work, written in your own words, and not copied or paraphrased from some other work.

Be sure to write about the assigned topics. Please write legibly. You may not use any reference materials during the test. Remember to review your work and make any changes you think will improve your responses.

Write or print your response in the space provided following the assignment.

OPEN-RESPONSE ITEM ASSIGNMENT #1

Use the information below to complete the exercise that follows.

A football on the ground is kicked into the air so that it follows a parabolic path. At time equal to three seconds, the ball reaches a height of twelve feet. At time equal to six seconds, the ball hits the ground. At the time that the ball is kicked, a bug leaves the ground from the same spot and flies in the direction of the ball. The equation representing the bug's path is $h(t) = \frac{4}{3}t$, where t is time in seconds and h is height in feet. Use your knowledge of quadratic and linear functions to develop a response in which you analyze the height-time relationship for the ball and the bug. In your response:

- find an equation for the height, h, of the ball in terms of time, t;
- sketch the graphs of the functions representing the path of the ball and the path of the bug on the same coordinate grid; and
- use the equations to calculate the time and the height at which the paths of the ball and the bug intersect.

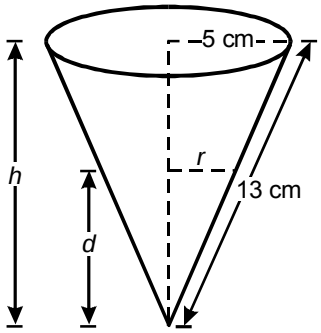
Be sure to show your work and explain the reasoning you use in analyzing and solving this problem.

OPEN-RESPONSE SHEET—ASSIGNMENT #1

OPEN-RESPONSE SHEET—ASSIGNMENT #1

OPEN-RESPONSE ITEM ASSIGNMENT #2

Use the diagram and information below to complete the exercise that follows.



A coffee filter has the shape of a right circular cone with a radius of 5 cm and a slant height of 13 cm. Use your knowledge of two-dimensional and three-dimensional geometry to develop a response in which you analyze properties of the cone. In your response:

- find the height, *h*, of the cone;
- calculate the volume of liquid that the cone holds when it is filled completely;
- use similar triangles to find a formula for any radius, *r*, in terms of its distance, *d*, from the vertex of the cone; and
- calculate the height of the liquid in the cone when the volume of the liquid is half the volume of the cone.

Be sure to show your work and explain the reasoning you use in analyzing and solving this problem.

OPEN-RESPONSE SHEET—ASSIGNMENT #2

OPEN-RESPONSE SHEET—ASSIGNMENT #2

PRACTICE TEST RESULTS

PRACTICE TEST RESULTS OVERVIEW

The practice test provides valuable information regarding your preparedness for the MTEL Middle School Mathematics (47) test. In this section, you will find information and tools to help you determine your preparedness on the various sections of the test.

Multiple-Choice Questions

A Multiple-Choice Question Answer Key Worksheet is provided to assist you in evaluating your multiple-choice responses. The worksheet contains five columns. The first column indicates the multiple-choice question number, the second column indicates the objective to which the test question was written, and the third column indicates the correct response. The remaining columns are for your use in calculating the number of multiple-choice questions you answered correctly or incorrectly.

An Evaluation Chart for the multiple-choice questions is also provided to help you assess which content covered by the test objectives may require additional study.

Open-Response Items

Evaluation Information, Sample Responses and Analyses, as well as a Scoring Rubric are provided for these items. You may wish to refer to this information when evaluating your practice test responses.

Total Test

Practice Test Score Calculation information is provided to help you estimate your score on the practice test. Although you cannot use this practice test to precisely predict how you might score on an official MTEL Middle School Mathematics (47) test, you may be able to determine your degree of readiness to take an MTEL test at an operational administration. No passing score has been determined for the practice test.

MULTIPLE-CHOICE QUESTION ANSWER KEY WORKSHEET

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
1	0001	D		
2	0001	B		
3	0001	C		
4	0001	C		
5	0001	C		
6	0002	B		
7	0002	D		
8	0002	A		
9	0002	B		
10	0002	C		
11	0003	B		
12	0003	A		
13	0003	B		
14	0003	D		
15	0004	B		
16	0004	B		
17	0004	A		
18	0004	D		
19	0004	D		
20	0005	C		
21	0005	B		
22	0005	C		
23	0005	D		
24	0005	B		
25	0006	B		
26	0006	D		
27	0006	D		
28	0006	B		
29	0006	A		
30	0007	A		
31	0007	C		
32	0007	A		
33	0007	C		
34	0007	B		

MULTIPLE-CHOICE QUESTION ANSWER KEY WORKSHEET (continued)

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
35	0008	A		
36	0008	D		
37	0008	A		
38	0008	B		
39	0008	C		
40	0009	C		
41	0009	A		
42	0009	A		
43	0009	B		
44	0009	D		
45	0009	D		
46	0010	B		
47	0010	D		
48	0010	D		
49	0010	C		
50	0010	A		
51	0011	B		
52	0011	A		
53	0011	B		
54	0011	C		
55	0012	C		
56	0012	D		
57	0012	A		
58	0012	B		
59	0013	C		
60	0013	C		
61	0013	C		
62	0013	B		
63	0013	D		
64	0014	A		
65	0014	B		
66	0014	D		
67	0014	C		
68	0014	D		

MULTIPLE-CHOICE QUESTION ANSWER KEY WORKSHEET (continued)

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
69	0015	A		
70	0015	C		
71	0015	C		
72	0015	A		
73	0016	B		
74	0016	C		
75	0016	C		
76	0016	D		
77	0016	D		
78	0016	A		
79	0016	B		
80	0017	D		
81	0017	C		
82	0017	B		
83	0017	B		
84	0017	C		
85	0017	B		
86	0017	C		
87	0017	B		
88	0018	D		
89	0018	A		
90	0018	D		
91	0018	C		
92	0018	D		
93	0019	C		
94	0019	B		
95	0019	D		
96	0019	A		
97	0020	A		
98	0020	D		
99	0020	C		
100	0020	B		

Count the number of multiple-choice questions you answered correctly:

_____ of 100 multiple-choice questions

MULTIPLE-CHOICE QUESTION PRACTICE TEST EVALUATION CHART

In the evaluation chart that follows, the multiple-choice questions are arranged in numerical order and by test objective. Check your responses against the correct responses provided to determine how many questions within each objective you answered correctly.

Subarea I: Number Sense and Operations

Objective 0001: Understand the structure of numeration systems and multiple representations of numbers.	
1D____ 2B_____ 3C_____ 4C_____ 5C_____	**_____/5**

Objective 0002: Understand principles and operations related to integers, fractions, decimals, percents, ratios, and proportions.	
6B____ 7D_____ 8A_____ 9B_____ 10C_____	**_____/5**

Objective 0003: Understand and solve problems involving integers, fractions, decimals, percents, ratios, and proportions.	
11B____ 12A____ 13B_____ 14D_____	**_____/4**

Objective 0004: Understand the properties of real numbers and the real number system.	
15B____ 16B____ 17A_____ 18D_____ 19D_____	**_____/5**

Subarea I (Objectives 0001–0004) Total _____/19

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea II: Patterns, Relations, and Algebra

Objective 0005: Understand and use patterns to model and solve problems.	
20C____ 21B_____ 22C_____ 23D_____ 24B_____	_____/5

Objective 0006: Understand how to manipulate and simplify algebraic expressions and translate problems into algebraic notation.	
25B____ 26D_____ 27D_____ 28B_____ 29A_____	_____/5

Objective 0007: Understand properties of functions and relations.	
30A____ 31C_____ 32A_____ 33C_____ 34B_____	_____/5

Objective 0008: Understand properties and applications of linear relations and functions.	
35A____ 36D_____ 37A_____ 38B_____ 39C_____	_____/5

Objective 0009: Understand properties and applications of quadratic relations and functions.	
40C____ 41A_____ 42A_____ 43B_____ 44D_____ 45D_____	_____/6

Objective 0010: Understand properties and applications of exponential, polynomial, rational, and absolute value functions and relations.	
46B____ 47D_____ 48D_____ 49C_____ 50A_____	_____/5

Subarea II (Objectives 0005–0010) Total _____/31

MULTIPLE-CHOICE QUESTION PRACTICE TEST EVALUATION CHART (continued)

Subarea III: Geometry and Measurement

Objective 0011: Understand principles, concepts, and procedures related to measurement.	
51B____ 52A_____ 53B_____ 54C_____	**_____/4**

Objective 0012: Understand the principles of Euclidean geometry and use them to prove theorems.	
55C____ 56D_____ 57A_____ 58B_____	**_____/4**

Objective 0013: Apply Euclidean geometry to analyze the properties of two-dimensional figures and to solve problems.	
59C____ 60C____ 61C_____ 62B_____ 63D_____	**_____/5**

Objective 0014: Solve problems involving three-dimensional shapes.	
64A____ 65B_____ 66D_____ 67C_____ 68D_____	**_____/5**

Objective 0015: Understand the principles and properties of coordinate and transformational geometry.	
69A____ 70C_____ 71C_____ 72A_____	**_____/4**

Subarea III (Objectives 0011–0015) Total _____/22

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea IV: Data Analysis, Statistics, and Probability

Objective 0016: Understand descriptive statistics and the methods used in collecting, organizing, reporting, and analyzing data.	
73B____ 74C_____ 75C_____ 76D_____ 77D_____ 78A_____ 79B_____	_____/7

Objective 0017: Understand the fundamental principles of probability.	
80D____ 81C____ 82B_____ 83B_____ 84C_____ 85B_____ 86C_____ 87B_____	_____/8

Subarea IV (Objectives 0016–0017) Total _____/15

Subarea V: Trigonometry, Calculus, and Discrete Mathematics

Objective 0018: Understand the properties of trigonometric functions and identities.	
88D____ 89A_____ 90D_____ 91C_____ 92D_____	_____/5

Objective 0019: Understand the conceptual basis of calculus.	
93C____ 94B_____ 95D_____ 96A_____	_____/4

Objective 0020: Understand the principles of discrete/finite mathematics.	
97A____ 98D_____ 99C_____ 100B_____	_____/4

Subarea V (Objectives 0018–0020) Total _____/13

OPEN-RESPONSE ITEM EVALUATION INFORMATION

How Open-Response Items Are Scored

Open-response items are scored through a process called focused holistic scoring. Scorers judge the overall effectiveness of the response rather than individual aspects considered in isolation. Scorer judgments are based on the quality of the response, not on length or neatness. Responses must be long enough to cover the topic adequately and scorers must be able to read what is written.

How to Evaluate Your Practice Responses

On the following pages, you will find two "strong" and two "weak" sample responses. PLEASE DO NOT REVIEW THE SAMPLE RESPONSES UNTIL AFTER YOU HAVE WRITTEN YOUR OWN RESPONSE. When you do review the two "strong" and "weak" sample responses and analyses included here, please note the following points:

- ✓ For the purposes of the practice test, responses are identified as "strong" or "weak" rather than given a score point of 1–4.
- ✓ The responses identified as "strong" may contain flaws; however, these responses do demonstrate the performance characteristics of a "strong response."
- ✓ The two "strong" responses demonstrate the examinees' appropriate understanding and application of the subject matter knowledge. However, these responses do not necessarily reflect the full range of "correct answers" that would demonstrate an understanding of the subject matter.
- ✓ The "Analysis" accompanying each "strong" and "weak" response discusses the main attributes of the responses, but does not identify all flaws or strengths that may be present.

Compare your practice responses to the Sample Responses to determine whether your responses are more similar to the strong or weak responses. Also review the Analyses on those pages and the Scoring Rubric to help you better understand the characteristics of strong and weak responses. This evaluation will help you identify specific problems or weaknesses in your practice responses. Further information on scoring can be found in the Test Information Booklet and Faculty Guide at www.mtel.nesinc.com and at www.doe.mass.edu/mtel; select "FAQ," then "After the Test."

OPEN-RESPONSE ITEM
SCORING RUBRIC, SAMPLE RESPONSES, AND ANALYSES

Massachusetts Tests for Educator Licensure®
SCORING RUBRIC FOR SUBJECT TESTS

Performance Characteristics:

Purpose	The extent to which the response achieves the purpose of the assignment.
Subject Matter Knowledge	Accuracy and appropriateness in the application of subject matter knowledge.
Support	Quality and relevance of supporting details.
Rationale	Soundness of argument and degree of understanding of the subject matter.

Scoring Scale:

Score Point	Score Point Description
4	**The "4" response reflects a thorough knowledge and understanding of the subject matter.** • The purpose of the assignment is fully achieved. • There is a substantial, accurate, and appropriate application of subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects an ably reasoned, comprehensive understanding of the topic.
3	**The "3" response reflects an adequate knowledge and understanding of the subject matter.** • The purpose of the assignment is largely achieved. • There is a generally accurate and appropriate application of subject matter knowledge. • The supporting evidence is adequate; there are some acceptable, relevant examples. • The response reflects an adequately reasoned understanding of the topic.
2	**The "2" response reflects a limited knowledge and understanding of the subject matter.** • The purpose of the assignment is partially achieved. • There is a limited, possibly inaccurate or inappropriate, application of subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited, poorly reasoned understanding of the topic.
1	**The "1" response reflects a weak knowledge and understanding of the subject matter.** • The purpose of the assignment is not achieved. • There is little or no appropriate or accurate application of subject matter knowledge. • The supporting evidence, if present, is weak; there are few or no relevant examples. • The response reflects little or no reasoning about or understanding of the topic.

U	**The response is unrelated to the assigned topic, illegible, primarily in a language other than English, not of sufficient length to score, or merely a repetition of the assignment.**
B	**There is no response to the assignment.**

FIRST SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

Find an equation for the height, h, of the football in terms of the time, t.

$y = ax^2 + bx + c$

$h(t) = at^2 + bt + c$

$t = 0, h = 0$: $h(0) = 0 = c$

$t = 6, h = 0$: $h(6) = 0 = 36a + 6b$

$t = 3, h = 12$: $h(3) = 12 = 9a + 3b$

$0 = 36a + 6b$

$-24 = -18a - 6b$

$24 = 18a$

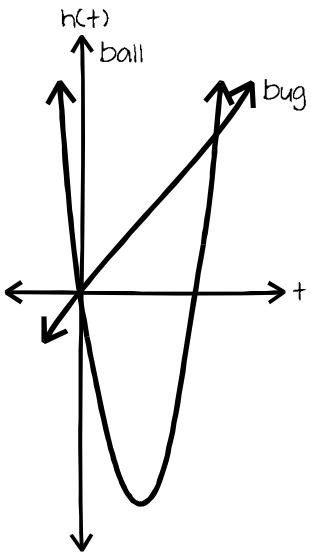
$\frac{4}{3} = a$

Substitute $\frac{4}{3}$ for a. $0 = 36(\frac{4}{3}) + 6b$

$-8 = b$

$h(t) = \frac{4}{3}t^2 - 8t$

Sketch the graphs of the functions representing the path of the ball and the path of the bug on the same coordinate grid.



Use the equations to calculate the time and the height at which the paths of the ball and the bug intersect.

$$\frac{4}{3}t^2 - 8t = \frac{4}{3}t$$

$$\frac{4}{3}t^2 - \frac{20}{3}t = 0$$

$$\frac{4}{3}t(t - 5) = 0$$

$$t = 5$$

$$\frac{4}{3}(5^2) - 8(5) = \frac{4}{3}(25) - 8(5) = \frac{100 - 120}{3} = \frac{-20}{3} = -6\frac{2}{3}$$

ANALYSIS FOR FIRST WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a weak response because it is characterized by the following:

Purpose: The purpose of the assignment is partially achieved. The candidate responds adequately from a computation perspective. But the candidate's response indicates limited analytical skills.

Subject Matter Knowledge: The candidate recognizes the quadratic relationship and applies the points available about the football's path to a generic form of a quadratic equation. There is a sign error in the first part of the assignment that results in a sign error in the third part of the assignment. The final number sentence weakens the response, especially since the candidate doesn't recognize its significance while sketching the graph. When $t = 5$, the height of the ball calculates to $-6\frac{2}{3}$ feet. This is an unreasonable response to a question that asks for the height at which the ball and the bug intersect. Thus, the candidate shows little awareness of the connection between the mathematics and the real-world situation to which the mathematics is being applied.

Support: One of the errors may result from skipping steps. For example, when the candidate calculates the time that the paths intersect, the candidate correctly sets the two equations equal to each other. The solution to $\frac{4}{3}t(t - 5) = 0$ is $t = 0$ or $t = 5$. This becomes apparent when the step $\frac{4}{3}t = 0$ and $t - 5 = 0$ is included in the solution. The candidate's sketch of the graph is correct for the equations graphed. More significantly, the candidate does not recognize that the shape of the graph and the intersection of the two curves at a height of $-6\frac{2}{3}$ indicates an inconsistency in the entire solution.

Rationale: The candidate does not recognize the unreasonableness of a negative answer and consequently that the graph and the calculations do not model the situation. This suggests a poor understanding of the value of modeling in mathematics.

SECOND SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

Find an equation for the height, h, of the football in terms of the time, t.

- Football: (3, 12) and (6, 0)

  $\frac{12-0}{3-6} = \frac{12}{-3} = -4$

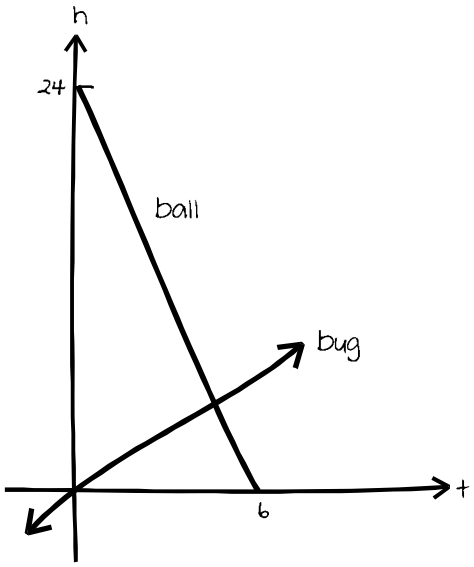
  $h = -4t + b$

  $12 = -4(3) + b$

  $24 = b$

  $h = -4t + 24$

Sketch the graphs of the functions representing the path of the ball and the path of the bug on the same coordinate grid.



Use the equations to calculate the time and the height at which the paths of the ball and the bug intersect.

- $\frac{4}{3}t = -4t + 24$

  $\frac{4}{3}t + 4t = 24$

  $\frac{16}{3}t = 24$

  $t = 24(\frac{3}{16})$

  $t = 4.5$
- $h = -4(4.5) + 24$

  $h = 6$ feet

ANALYSIS FOR SECOND WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a weak response because it is characterized by the following:

Purpose: The candidate's response partially achieves the purpose of the assignment. A misconception in the first part of the assignment means that the response cannot be completely accurate.

Subject Matter Knowledge: The modeling of the parabolic path of the football with a linear equation results in a poor representation of the football's path. The graphs of the paths of the ball and the bug are correct, given the equations used. The candidate properly sets equations equal to each other to find the intersection point. This indicates some understanding of algebraic methods.

Support: The sketch of the graph of the two equations used is correct, though the candidate's equation of the path of the football is incorrect. The identifiers in the coordinate plane on which the candidate draws the graphs are sparse. The equations for the ball and the bug should be part of the graph. The vertical axis of the graph is $h(t)$, not just h.

Rationale: The candidate demonstrates little understanding of the connection between the shape of a parabolic path and the graph of a quadratic equation. By leaving the final part of the response as "$h = 6$ feet," the candidate does not connect the mathematics with the situation and thus bring together the totality of the assignment. The candidate's statement should reflect the fact that at a height of 6 feet, the paths of the ball and the bug intersect.

FIRST SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

Find an equation for the height, h, of the football in terms of the time, t.

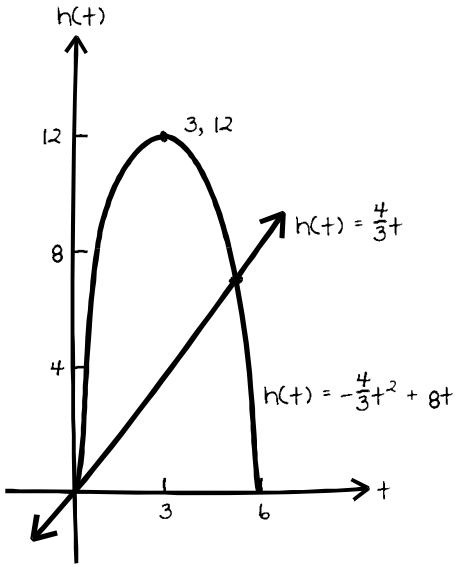
- The football's path is parabolic so its equation is quadratic, with the general form $y = at^2 + bt + c$, with $c = 0$ in this case. If $h(t) = 0$ when $t = 0$ and $t = 6$, then $h(t) = a(t - 0)(t - 6)$.

  To find a, use the fact that at the time 3 seconds, the height = 12.

  $h(3) = 12$, so $12 = a(3 - 0)(3 - 6)$, and $a = -\frac{4}{3}$

  The path of the football is described by $h(t) = -\frac{4}{3}(t)(t - 6) = -\frac{4}{3}t^2 + 8t$

Sketch the graphs of the functions representing the path of the ball and the path of the bug on the same coordinate grid.



Use the equations to calculate the time and the height at which the paths of the ball and the bug intersect.

- Let the equations for both the ball and the bug equal each other and solve for t.

  $\frac{4}{3}t = -\frac{4}{3}t^2 + 8t$

  $4t = -4t^2 + 24t$

  $t = -t^2 + 6t$

$t^2 - 5t = 0$

$t(t - 5) = 0$

$t = 0$ or $t = 5$

At $t = 0$ seconds, both the bug and the ball are on the ground.
Let $t = 5$ seconds.

$h = \frac{4}{3}(5)$

$h = \frac{20}{3} = 6\frac{2}{3}$ feet

At $t = 5$ seconds, the paths of the ball and the bug intersect at a height of $6\frac{2}{3}$ feet.

ANALYSIS FOR FIRST STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a strong response because it is characterized by the following:

Purpose: The candidate fully achieves the purpose of the assignment. The response is accurate and thorough.

Subject Matter Knowledge: The candidate recognizes that a quadratic equation will model the path of the football, is familiar with the general form of a quadratic equation, and knows how to use the information presented to derive the specific equation needed. The sketch accurately portrays the graphs of the equations that represent the paths of the football and the bug. Analysis of points using the model then allows the prediction of the intersection of the two paths. The candidate's response reflects mastery of the skill sets required to model real-life situations using linear and quadratic equations.

Support: The organization and flow of the candidate's response require very little in the way of additional support. The candidate draws together the work in the final sentence, "At $t = 5$ seconds, the paths of the ball and the bug intersect at a height of $6\frac{2}{3}$ feet," thereby strengthening the entire response. Little additional explanation is necessary as the numerical solution is so well organized.

Rationale: The organization of the response is logical, flows reasonably, and indicates a comprehensive understanding of how algebra can be used to model real-life situations.

SECOND SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

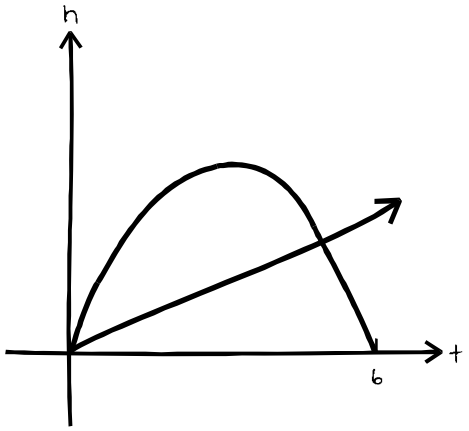
Find an equation for the height, h, of the football in terms of the time, t.

- The height of the football can be modeled using: $h = at^2 + bt + c$
  When $t = 0$, then $h = 0$, so $c = 0$ and $h = at^2 + bt$
  When $t = 6$, then $h = 0$, so $0 = a(36) + 6b \Rightarrow b = -6a$
  When $t = 3$, then $h = 12$, so $12 = a(9) + 3b$
  Recall that $b = -6a$, so $12 = a(9) + 3(-6a)$

$$12 = 9a - 18a$$
$$12 = -9a$$
$$a = -\frac{4}{3}$$
$$b = -6a$$
$$b = -6\left(-\frac{4}{3}\right)$$
$$b = 8$$

So the height of the football can be modeled using $h = -\frac{4}{3}t^2 + 8t$.

Sketch the graphs of the functions representing the path of the ball and the path of the bug on the same coordinate grid.



Use the equations to calculate the time and the height at which the paths of the ball and the bug intersect.

- Let the equation for the path of the football and the path of the bug equal each other.

$$-\frac{4}{3}t^2 + 8t = \frac{4}{3}t$$

$$-\frac{4}{3}t^2 = -\frac{20}{3}t$$

$$t^2 = 5t$$

$$t = 5$$

- Substitute 5 for t in the equation, $h = -\frac{4}{3}t^2 + 8t$

$$h = -\frac{4}{3}(25) + 8(5)$$

$$h = -\frac{100}{3} + \frac{120}{3}$$

$$h = \frac{20}{3} \text{ feet}$$

ANALYSIS FOR SECOND STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a strong response because it is characterized by the following:

Purpose: The candidate completely and correctly solves each element of the problem as assigned. The purpose of the assignment is fully achieved.

Subject Matter Knowledge: The candidate demonstrates substantial knowledge of how to model real-world situations using mathematical methods. The constructed model and graph accurately reflect the movement of the football and the bug. However, the candidate's calculation of the time and height of the intersection of the paths of the ball and bug is incomplete, when $t^2 = 5t$, then $t^2 - 5t = 0$ and $t = 0$ or $t = 5$. The candidate divides both sides of the equation by t and that is not a defined action, since t could equal 0. The response shows mastery of most of the concepts and procedures required to correctly complete the assignment.

Support: The graph of the two paths is a useful sketch, but the axes on which the candidate has drawn the graph should be scaled and the curves should be labeled. It should be made clear precisely what is meant by the final sentence, "$h = \frac{20}{3}$ feet." The intersection of the paths of the football and the bug occurs $6\frac{2}{3}$ feet above the ground. Despite some oversights, the candidate solves the assignment in a way that is clear and easy to follow.

Rationale: The development of the response is clear and logical and smoothly attains the solution to the assignment. The candidate reveals a solid understanding of modeling with quadratic functions.

FIRST SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

Find the height, h, of the cone.

- $h^2 + 5^2 = 13^2$

  $h^2 + 25 = 169$

  $h^2 = 144$

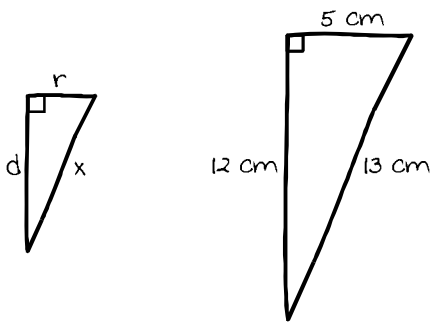
  $h = 12$

Calculate the volume of liquid the cone holds when it is filled completely.

- $V = \frac{1}{3}\pi r^2 h$

  $V = \frac{1}{3}\pi \cdot 5^2 \cdot 12 = 100\pi = 314.2$ cubic centimeters

Use similar triangles to find a formula for any radius, r, in terms of its distance, d, from the vertex of the cone.



- $\frac{5}{r} = \frac{d}{12}$

  $rd = 60$

  $r = \frac{60}{d}$

Calculate the height of the liquid in the cone when the volume of liquid is half the volume of the cone.

- $(\frac{1}{2})(\frac{1}{3}\pi r^2 h) = (\frac{1}{2})(100\pi)$

  $(\frac{1}{3})(\frac{60}{d})^2\pi(d) = 100\pi$

  $(\frac{1}{3})\frac{3600d}{d^2} = 100$

  $\frac{1200}{d} = 100$

  $100d = 1200$

  $d = 12$ centimeters

ANALYSIS FOR FIRST WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a weak response because it is characterized by the following:

Purpose: The candidate correctly applies the Pythagorean theorem and formula to find the volume of a completely filled cone. However, the candidate inverts ratios in the third part of the solution and makes a conceptual error in the final part. The purpose of the assignment is partially achieved.

Subject Matter Knowledge: The candidate applies the Pythagorean theorem correctly to find the height of the cone. The candidate makes the proper substitutions for height and radius into the correct formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$. However, the proportion describing the relationship between the sides of the similar triangles is incorrect. The candidate makes a conceptual error when calculating the height of the liquid when the volume of the liquid is half the volume of the cone. Half of the volume of the liquid can be stated as "$(\frac{1}{2})(100\pi)$." The misconception stems from taking one half of the formula for the volume of a cone, "$(\frac{1}{2})(\frac{1}{3}\pi r^2 h)$," when calculating the height of the liquid.

Support: The candidate gives no verbal direction in this response. The illustration the candidate uses to analyze the properties of similar triangles is correctly drawn. The candidate's solution to the final part of the assignment, "d = 12 centimeters," should have indicated that errors were made. Since the height of the cone was determined to be 12 centimeters in the first part of the problem, the height of the liquid when the cone is half-filled clearly must be less than 12 centimeters. This implies a disconnection between the computations and the application of the mathematical concepts to a real-world problem.

Rationale: The candidate's response is well organized and applies elemental skills accurately to the problem. The response falls short at the conceptual level, implying a limited understanding of the skills required to address this assignment.

SECOND SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

Find the height, h, of the cone.

- $h^2 = 5^2 + 13^2$

  $h^2 = 25 + 169 = 194$

  $h = 13.9 \approx 14$

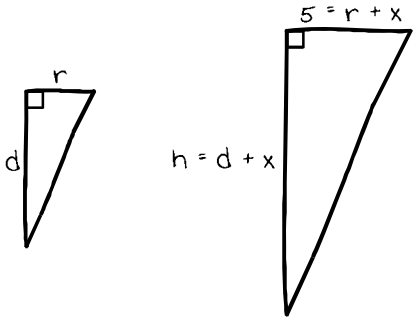
Calculate the volume of liquid the cone holds when it is filled completely.

- $V = \frac{1}{3}\pi r^2 h$

  $V = \frac{1}{3}\pi \cdot 25 \cdot 14$

  $V = 117\pi \approx 368$

Use similar triangles to find a formula for any radius, r, in terms of its distance, d, from the vertex of the cone.



- $r + x = 5$ and $x = 5 - r$
- $d + x = h$ and $d + (5 - r) = h$

  $-r = 14 - d - 5$

  radius: $r = d - 9$

Calculate the height of the liquid in the cone when the volume of liquid is half the volume of the cone.

- $\frac{1}{2}V = \frac{1}{2}(368) = 184$

  $\frac{1}{3}\pi(d - 9)^2(14) = 184$

  $(d - 9)^2 = \frac{(184)(3)}{(14)(\pi)} \approx 13$

  $(d - 9)^2 = 13$

  $d - 9 = \sqrt{13} \approx 4$

  $d = 4 + 9 = 13$

ANALYSIS FOR SECOND WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a weak response because it is characterized by the following:

Purpose: The candidate's response includes both fundamental and conceptual errors. The purpose of the assignment is nominally achieved.

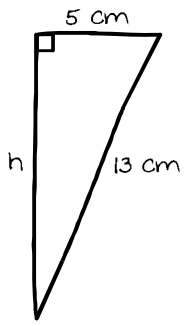
Subject Matter Knowledge: The candidate correctly recalls but incorrectly applies the Pythagorean theorem. The height of the cone is a leg of the right triangle, not the hypotenuse. Thus, $h^2 = 5^2 + 13^2$ does not solve for the correct height of the cone. The candidate has little conceptual knowledge of the properties of similar triangles. When solving for any radius, r, in terms of its distance, d, from the vertex of the cone, the candidate does not attempt to create the ratios needed to analyze similar triangles. The candidate correctly applies the formula for the volume of a cone and correctly analyzes the connection between the half-filled cone and the related volume. This response reflects a limited grasp of geometric concepts.

Support: The candidate does not include a sketch of the right triangle that is used to determine the height of the cone. A sketch might have clarified the identification of the needed height as a leg of the triangle rather than the hypotenuse. The candidate does include a sketch in the third part of the response. The sketch demonstrates, however, that the candidate is unaware of the properties of similar triangles. The response does not address what appears to be an inconsistency between two quantities. In the first part of the assignment, the height of the cone is calculated as 14 centimeters. In the last part, the candidate is asked to find the height of the liquid in the cone when the volume of the liquid is half the volume of the cone. The calculated value for this height is 13 centimeters. This should serve as evidence of an error in the model.

Rationale: The candidate executes the basic skills required to successfully complete the assignment but shows no recognition of the properties of similar triangles. The response does not include units for any quantities computed, and, more significantly, shows little grasp of reasoning skills.

FIRST SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

Find the height, h, of the cone.



- The height, h, is one of the legs of a right triangle so use the Pythagorean theorem to find the height of the cone.

  $h^2 + 5^2 = 13^2$

  $h^2 = 169 - 25 = 144$

  $h = 12$ centimeters

- $h = 12$ because 5, 12, 13 form a Pythagorean triple.

Calculate the volume of liquid the cone holds when it is filled completely.

- Substitute values for radius and height into the formula for the volume of a cone to find the amount of liquid.

  $V = \frac{1}{3}\pi r^2 h$

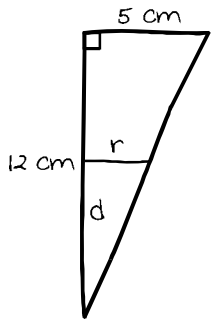
  $V = \frac{1}{3}\pi(5^2)12$

  $V = \frac{1}{3}\pi(25)12$

  $V = 100\pi$

  $V \approx 314.2$ cubic centimeters

Use similar triangles to find a formula for any radius, r, in terms of its distance, d, from the vertex of the cone.



- Because the three angles in each of the two triangles formed are equal, the two triangles are similar and the ratios of corresponding parts are equal.

  $\frac{5}{r} = \frac{12}{d}$

  $12r = 5d$

  $r = \frac{5}{12}d$

Calculate the height of the liquid in the cone when the volume of liquid is half the volume of the cone.

- Use the quantities found for volume and radius in parts 2 and 3. Since the volume of the completely filled cone is 100π, the volume of a half-filled cone is 50π.

  $\frac{1}{3}\pi(\frac{5}{12}d)^2(d) = 50\pi$

  $\frac{1}{3}(\frac{25}{144})d^3 = 50$

  $d^3 = 864$

  $d \approx 9.5$ centimeters

ANALYSIS FOR FIRST STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a strong response because it is characterized by the following:

Purpose: The purpose of the assignment is fully achieved. The height of the cone, the volume of a filled cone, a formula for a radius, and the volume of a half-filled cone are thoroughly investigated and values accurately obtained.

Subject Matter Knowledge: The candidate demonstrates appropriate application of the following: the Pythagorean theorem when finding the height of the cone, the formula for finding the volume of a cone, and the concept of similar triangles when finding the radius of a cone filled to an unknown depth. These correct applications indicate a firm grasp of the geometric concepts required to respond to this assignment.

Support: The verbal additions organize the response clearly and lead to a logical conclusion. The formulas are correct and accurately employed. The accompanying illustrations are correct and strengthen the response.

Rationale: Each step in the candidate's response is accurately executed and clearly justified. The candidate is careful to include units at each phase of the response. This indicates a comprehensive understanding of the underlying geometric concepts.

SECOND SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

Find the height, h, of the cone.

- $h^2 = 5^2 + 13^2$

  $h^2 = 25 + 169 = 194$

  $h = 13.9$ centimeters

Calculate the volume of liquid the cone holds when it is filled completely.

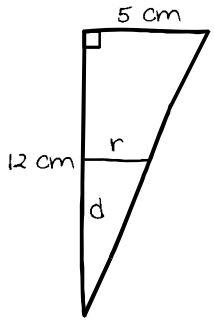
- $V = \frac{1}{3}\pi r^2 h$

  $V = \frac{1}{3}\pi(5^2)13.9$

  $V = \frac{1}{3}\pi(25)13.9$

  $V = 363.9$ cubic centimeters

Use similar triangles to find a formula for any radius, r, in terms of its distance, d, from the vertex of the cone.



$\frac{5}{r} = \frac{12}{d}$

$12r = 5d$

$r = \frac{5}{12}d$

Calculate the height of the liquid in the cone when the volume of liquid is half the volume of the cone.

- $V = \frac{363.9}{2} = 181.95 \approx 182$

  $\frac{1}{3}\pi r^2 d = 182$

  $\pi r^2 d = 3 \cdot 182$

  $\pi(\frac{5}{12}d)^2 \cdot d = 546$

  $\pi \cdot \frac{25}{144}d^3 = 546$

  $d^3 = \frac{546 \cdot 144}{25\pi} = 1001.1$

  $d \approx 10$ centimeters

ANALYSIS FOR SECOND STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a strong response because it is characterized by the following:

Purpose: The candidate's response is conceptually correct and well organized. The purpose of the assignment is greatly achieved.

Subject Matter Knowledge: The candidate incorrectly applies the Pythagorean theorem; the height of the cone is a leg of the right triangle, not the hypotenuse. Thus, $h^2 = 5^2 + 13^2$ does not solve for the correct height of the cone. The candidate approaches the remaining three parts of the assignment with appropriate application of formulas and concepts, suggesting a considerable mastery of the subject matter. Despite this knowledge, the candidate did not review what had been written and correct this error, suggesting a less-than-thorough approach that affects the quality of the final response.

Support: The candidate has applied the correct formula to find the volume of a cone, has used the concept of similarity between triangles appropriately, and has completed the associated calculations correctly. The candidate's response has few verbal explanations but is nonetheless a thorough and mostly accurate response.

Rationale: Despite the incorrect application of the Pythagorean theorem, the candidate presents a clear and thorough response to the assignment. The candidate's response includes units in most of the parts. Overall, the response adequately addresses the intent of the assignment.

PRACTICE TEST SCORE CALCULATION

The practice test score calculation is provided so that you may better gauge your performance and degree of readiness to take an MTEL test at an operational administration. Although the results of this practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test, it is not possible to predict precisely how you might score on an official MTEL test.

The Sample Responses and Analyses for the open-response items may help you determine whether your responses are more similar to the strong or weak samples. The Scoring Rubric can also assist in estimating a score for your open responses. You may also wish to ask a mentor or teacher to help evaluate your responses to the open-response questions prior to calculating your total estimated score.

How to Calculate Your Practice Test Score

Review the directions in the sample below and then use the blank practice test score calculation worksheet on the following page to calculate your estimated score.

SAMPLE

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly:	65	
Use Table 1 below to convert that number to the score and write your score in **Box A**:		**A:** 189

Open-Response Section

Enter the number of points (1 to 4) for your first open-response question:	3	
Enter the number of points (1 to 4) for your second open-response question:	3	
	======	
Add those two numbers (number of open-response question points):	6	
Use Table 2 below to convert that number to the score and write your score in **Box B**:		**B:** 52

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score:	**A + B =** 241

Practice Test Score Calculation Worksheet: Middle School Mathematics

Table 1:

Number of Multiple-Choice Questions Correct	Estimated MTEL Score		Number of Multiple-Choice Questions Correct	Estimated MTEL Score
0 to 25	134		61 to 65	189
26 to 30	141		66 to 70	196
31 to 35	148		71 to 75	203
36 to 40	155		76 to 80	210
41 to 45	162		81 to 85	217
46 to 50	169		86 to 90	224
51 to 55	176		91 to 95	230
56 to 60	182		96 to 100	237

Table 2:

Number of Open-Response Question Points	Estimated MTEL Score
2	36
3	40
4	44
5	48
6	52
7	56
8	60

Print the form below to calculate your estimated practice test score.

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly:

Use Table 1 above to convert that number to the score and write your score in **Box A**: **A:** ____

Open-Response Section

Enter the number of points (1 to 4) for your first open-response question:

Enter the number of points (1 to 4) for your second open-response question:

======

Add those two numbers (number of open-response question points):

Use Table 2 above to convert that number to the score and write your score in **Box B**: **B:** ____

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score: **A + B =** ____