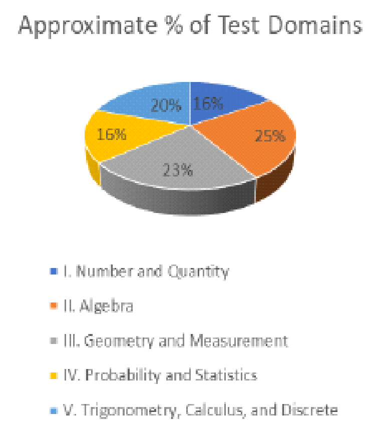
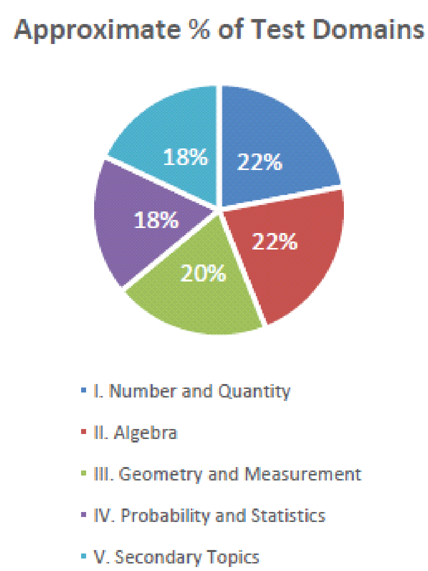
PRACTICE QUESTIONS-SET-1 ANSWERS

Content Domain	Approximate Percentage of Test	Approximate Number of Questions
I. Number and Quantity	16%	16
II. Algebra	25%	25
III. Geometry and Measurement	23%	23
IV. Probability and Statistics	16%	16
V. Trigonometry, Calculus, and Discrete	20%	20



Content Domain	Approximate Percentage of Test	Approximate Number of Questions
I. Number and Quantity	22%	22
II. Algebra	22%	22
III. Geometry and Measurement	20%	20
IV. Probability and Statistics	18%	18
V. Secondary Topics	18%	18



1. Two dice are tossed. What is the probability that the sum of the two dice is greater than 3?

Solution

When two dice are tossed, there are 6 * 6 = 36 possible outcomes.	(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6) (2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6) (3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6) (4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6) (5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6) (6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)
The outcomes that give a sum of 3 or less are	(1,1) , (1 , 2) and (2,1)
There are	36 - 3 = 33 possible outcomes that give a sum that is greater than 3
The probability that the sum of the two dice is greater than 3	33 / 36 = 11 / 12

2. If L is a line through the points (2,5) and (4,6), what is the value of k so that the point of coordinates (7,k) is on the line L?

Solution

The slope of the line through (2,5) and (4,6) is given by	(6 - 5) / (4 - 2) = 1 / 2
The slope of the line through (4,6) and (7,k) is given by	(k - 6) / (7 - 4) = (k - 6) / 3
The three points are on the same line and therefore the slopes found above must be equal	1 / 2 = (k - 6) / 3
Solve the above equation for k to find	k = 15/2

3. Find a negative value of k so that the graph of $y = x^2 - 2x + 7$ and the graph of $y = kx + 5$ are tangent?

Solution

To find points of intersection of the graphs of $y = x^2 - 2x + 7$ and $y = kx + 5$, we need to solve the system of the two equations	$y = x^2 - 2x + 7$ $y = kx + 5$
Eliminate y between the two equations to obtain	$x^2 - 2x + 7 = kx + 5$
The above equation may be written as	$x^2 - x(2 + k) + 2 = 0$
This is a quadratic equation which may have no solutions meaning that the two graphs have no points of intersection, two solutions which means the two graphs have two points of intersection or one solution which means the two graphs are tangent to each other. A quadratic equation have one solution if its discriminant is equal to 0. Hence	$(2 + k)^2 - 4(1)(2) = 0$
Solve for k and select the negative solution	$k = -2 - 2\sqrt{2}$

QUESTION NO 4.

Solution

Function $f(x) = |4 - x^2| / (x + 2)$ is undefined for $x = -2$ and the only graph that is undefined for x negative is graph B.

QUESTION NO 5.

Solution

One way to find the coordinates of N is to find the components of vector ON.	Vector ON = Vector OC + Vector CN
The equation of the circle gives the coordinates of the center C which are	C (3 , 2)
The components of vector OC are	< 3 , 2 >
Distance CN is equal to the radius of the circle which is 1. Points C and M have the same y coordinates and therefore CM is parallel to the x axis, hence the components of vector CN are given by	$<1 \cdot \cos(30^o) , 1 \cdot \sin(30^o)> = < \sqrt{3} / 2 , 1 / 2>$
Hence the components of vector ON are given by	$< 3 , 2 > + < (1 / 2)\sqrt{3} , 1 / 2 > = < 3 + \sqrt{3} / 2 , 5/2 >$
The coordinates of point N are given by	$(3 + \sqrt{3} / 2 , 5/2)$

by The coordinates of point N are given by	(3 + √3 / 2 , 5/2)

6. Vectors **u** and **v** are given by **u** = < 2 , 0 > and **v** = < -3 , 1 >. What is the length of vector **w** given by **w** = -**u** - 2**v**?

Solution

We first find **w** = -**u** - 2**v** as follows	**w** = -**u** - 2**v** = - < 2 , 0 > -2 < - 3 , 1 >
	= < - 2 , 0> + < 6 , - 2 >
	= < 4 , - 2 >
The length of W is given by	$\sqrt{4^2 + (-2)^2} = 2\sqrt{5}$

7. What is the smallest distance between the point (-2,-2) and a point on the circumference of the circle given by

$$(x - 1)^2 + (y - 2)^2 = 4?$$

Solution

Let us find the center and radius of the circle	The equation of the circle is in standard form and its center has coordinates (1 , 2) and its radius is equal to 2.
The distance between the center of the circle and point (-2, -2) is given by	$\sqrt{(-2-1)^2 + (-2-2)^2} = 5$
Point (-2,-2) is outside the circle. The shortest distance between point (-2,-2) and a point on the circle is given by	5 - 2 = 3

8. What is the equation of the horizontal asymptote of function

$$f(x) = 2 / (x + 2) - (x + 3) / (x + 4)?$$

Solution

If x increases indefinitely, 2 / (x + 2) will approach 0 and - (x + 3) / (x + 4) will approach - 1. If x decreases indefinitely, 2 / (x + 2) will approach 0 and - (x + 3) / (x + 4) will approach - 1. Hence the equation of the horizontal asymptote is y = - 1.

QUESTION NO 9.

Solution

$$3y = - x + 2$$

Solution

	$3y = -x + 2$
Let us find the slope of the line with equation $x + 3y = 2$	$y = -(1/3)x + 2/3$
	slope $= -1/3$
	$k y = 2x + 5$
The slope of the line with equation $-2x + ky = 5$ is calculated as follows	$y = (2/k)x + 5/k$
	slope $= 2/k$
For two lines to be perpendicular, the product of their slopes must be equal to - 1. Hence	$(-1/3)*(2/k) = -1$
Solve the above equation for k to find	$k = 2/3$

10. If $f(x) = (x - 1)^2$ and $g(x) = \sqrt{x}$, then $(g_o f)(x) =$

Solution

$(g_o f)(x)$ is the composition of functions defined as follows	$(g_o f)(x) = g(f(x))$
	$(g_o f)(x) = g(f(x))$
	$= g((x - 1)^2)$
Substitute f (x) by $(x - 1)^2$	$= \sqrt{(x - 1)^2}$
	$= \lvert x - 1 \rvert$

NOTE THAT $\sqrt{x^2} = \lvert x \rvert$ not x

11. The domain of $f(x) = \sqrt{(4 - x^2)} / \sqrt{(x^2 - 1)}$ is given by the interval

Solution

Remember that the domain of a function is the set of all values of x that make f(x) a real number. The given function has square roots which must be positive or zero because the square root of a negative number is not a real number. The expression in the denominator cannot be zero because we cannot divide by zero. Hence the two conditions to satisfy

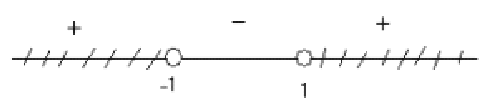
1) $(x^2 - 1) > 0$

2) $(4 - x^2) \geq 0$

Factor $(x^2 - 1)$ and solve the inequality 1)

$(x - 1)(x + 1) > 0$

The sign chart of $(x - 1)(x + 1)$ is shown below



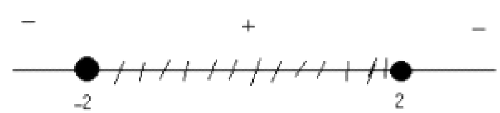
and the solution set of $(x - 1)(x + 1) > 0$ is given by the interval

[-1 , 0) U (0 , +1]

We now factor $(4 - x^2)$ and solve the inequality 2)

$(2 - x)(2 + x) \geq 0$

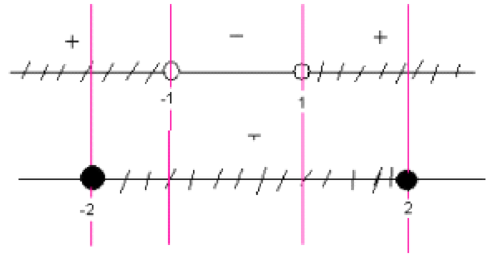
The sign chart of $(2 - x)(2 + x)$ is shown below



and the solution set of $(2 - x)(2 + x) \geq 0$ is given by the interval

[-2 , 2]

For a given value of x to be in the domain of f it must satisfy both inequalities above which means the domain is the set of x values belonging to the intersection of the intervals [-1 , 0) U (0 , +1] and [-2 , 2] as shown below



The domain of f is given by

[-2, -1) U (1, 2]

12. The area of the circle $x^2 + y^2 - 8y - 48 = 0$ is

Solution

We first need to write the given equation in standard form by completing square

$$x^2 + y^2 - 8y - 48 = 0$$

Write the term $(y^2 - 8y)$ as a square plus a constant : add and subtract 16

$$y^2 - 8y = y^2 - 8y + 16 - 16$$

$$= (y - 4)^2 - 16$$

Substitute by in the given equation and write the equation of the circle in standard form

$$x^2 + (y - 4)^2 - 16 - 48 = 0$$

or

$$x^2 + (y - 4)^2 = 64$$

Hence the radius R of the circle is equal to 8. The area is given

$$\text{Pi}\, R^2 = 64\, \text{Pi}$$

13. The y coordinates of all the points of intersection of the parabola $y^2 = x + 2$ and the circle $x^2 + y^2 = 4$ are given by

Solution

The points of intersections of the parabola and the circle are solutions to both equations simultaneously and therefore to find these points we need to solve the system of equations defined by	$y^2 = x + 2$ $x^2 + y^2 = 4$
Substitute y^2 by $x + 2$ in the second equation and rewrite the equation obtained in standard form	$x^2 + x + 2 = 4$ $x^2 + x - 2 = 0$
Solve the quadratic equation obtained	$x = 1$ and $x = -2$
We now substitute the values of x obtained in the equation $y^2 = x + 2$ to find y	For $x = 1$, and $y^2 = 1 + 2 = 3$ $y = +$ or $- \sqrt{3}$ for $x = -2$, and $y^2 = -2 + 2 = 0$ hence $y = 0$
The y coordinates of the points of intersection are	$- \sqrt{3}$, 0 and $\sqrt{3}$

14. What is the smallest positive zero of function f(x) = 1/2 - sin(3x + Pi/3)?

Solution

The zeros of function f are found by solving the equation	1/2 - sin(3x + Pi/3) = 0
Rearrange the equation to obtain	sin(3x + Pi/3) = 1 / 2
In order to solve the trigonometric equation above, we need to solve the two equations below. Note that because sine functions are periodic, there is an infinite number of solutions.	3x + Pi/3 = Pi / 6 + 2 k Pi and 3x + Pi/3 = 5 Pi / 6 + 2 k Pi for k any integer
We now solve the above equations.	First equation: 3x + Pi/3 = Pi / 6 + 2 k Pi 3x = - Pi / 6 + 2 k Pi x = - Pi / 18 + 2 k Pi / 3 Second equation: 3x + Pi/3 = 5 Pi / 6 + 2 k Pi 3x = Pi / 2 + 2 k Pi x = Pi / 6 + 2 K Pi / 3
We now need to find the smallest positive solution.	For k = 0 we have two solutions: x = - Pi / 6 and Pi / 6 and hence the smallest zero is equal to Pi / 6

15. If x - 1, x - 3 and x + 1 are all factors of a polynomial P(x) of degree 3, which of the following must also be a factor of P(x)?

I) $x^2 + 1$

II) $x^2 - 1$

III) $x^2 - 4x + 3$

Solution

We first use the factors to write polynomial P as follows

$P(x) = k (x - 1)(x - 3)(x + 1)$, k is a nonzero real number

$(x - 1)(x - 3) = x^2 - 4 x + 3$

Other possible factors of P(x) are

$(x - 1)(x + 1) = x^2 - 1$

and $(x - 3)(x + 1) = x^2 - 2x - 3$

Of those listed in I, II and III, II and III are also factors of P(x).

16. A cylinder of radius 5 cm is inserted within a cylinder of radius 10 cm. The two cylinders have the same height of 20 cm. What is the volume of the region between the two cylinders?

Solution

Let V1 be the volume of the small cylinder and V2 the volume of the large cylinder where

$V1 = (5^2 \text{ Pi}) * 20$

$V2 = (10^2 \text{ Pi}) * 20$

The volume V of the region bewteen the two cylinders in the difference between the two volumes

$V = V2 - V1 = 1500 \text{ Pi cm}^3$

17. A data set has a standard deviation equal to 1. If each data value in the data set is multiplied by 4, then the value of the standard deviation of the new data set is equal to

Solution

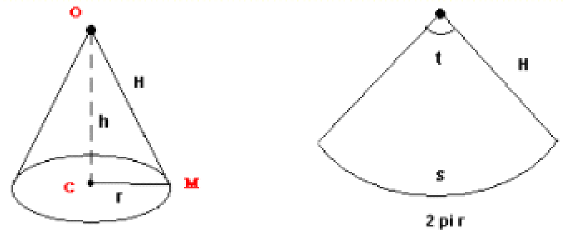
and therefore the new standard deviation will be k times the old standard deviation. In this case k = 4 and the old standard deviation is one. The standard deviation after multiplication will be equal to 4.

The standard deviation involves the square root of the sum of$(x_i - m)^2$ where xi are the data values and m is the mean. If each data value is multiplied by k then the mean will be k times. Hence the new standard deviation will involve the square root of the sum of $(k x_i - k m)^2$

18.

A cone made of cardboard has a vertical height of 8 cm and a radius of 6 cm. If this cone is cut along the slanted height to make a sector, what is the central angle, in degrees, of the sector?

When cut open along the slant height H and flattened, the cone becomes a sector of a circle as shown in the figure below right and the perimeter of the base of the cone is equal to arc length of the sector shown on the right. The perimeter P of the base of the cone is given by:



P = 2 Pi r = 2 Pi 6 = 12 Pi

The slanted height H is calculated using Pythagora theorem

$H = \sqrt{(r^2 + h^2)} = \sqrt{(6^2 + 8^2)} = 10$

The arc length of S the sector is given by

S = H * t = 10 t , where t is the sector angle to find.

S = P gives 12 Pi = 10 t

t = 1.2 Pi = 216 degrees.

19. If $\sin(x) = -1/3$ and $\pi \le x \le 3\pi/2$, then $\cot(2x) =$

Solution

We first express cot(2x) as follows

$\cot(2x) = 1 / \tan(2x) = (1 - \tan^2) / (2 \tan(x))$

We now need to find tan(x) using the identity

$\tan(x) = \sin(x) / \cos(x)$

We know the value of sin(x), we need to find the value of cos(x) using the identity $\sin^2(x) + \cos^2(x) = 1$ as follows

$\cos^2(x) = 1 - \sin^2(x)$

Solving the above for cos(x), we obtain two solutions

$\cos(x) = \sqrt{1 - \sin^2(x)}$ and $\cos(x) = - \sqrt{1 - \sin^2(x)}$

Angle x satisfies the condition $\pi \le x \le 3\pi/2$ for which cos(x) is negative. Hence

$\cos(x) = - \sqrt{1 - \sin^2(x)}$

We now substitute and determine cot(2 x) as follows

$\cos(x) = - \sqrt{1 - (-1/3)^2}$

$= - \sqrt{8/9} = - 2 \sqrt{2} / 3$

$\tan(x) = \sin(x) / \cos(x) = (-1/3) / (- 2 \sqrt{2} / 3) = 1 / 2 \sqrt{2}$

$\cot(2 x) = (1 - \tan^2) / (2 \tan(x)) = (1 - 1/8) / (1 / \sqrt{2}) = 7 \sqrt{2} / 8 = 7 / (4 \sqrt{2})$

20. Which of the following functions satisfy the condition $f(x) = f^{-1}(x)$?

I) $f(x) = - x$

II) $f(x) = \sqrt{x}$

III) $f(x) = - 1/x$

Solution

Let us find the inverse of $f(x) = - x$

$y = - x$, write given function as equation

$x = - y$, change x to y and y to x

$y = - x = f^{-1}(x)$, solve for y to find inverse

The given function satisfies $f(x) = f^{-1}(x)$.

Let us find the inverse of $f(x) = \sqrt{x}$

$y = \sqrt{x}$

$x = \sqrt{y}$

$y = x^2$ and is different from $f(x) = \sqrt{x}$

The given function does not satisfy the condition $f(x) = f^{-1}(x)$

Let us find the inverse of $f(x) = - 1/x$

$y = - 1/x$

$x = - 1/y$

$x y = - 1$

$y = - 1/x = f^{-1}(x)$

The given function satisfies the condition $f(x) = f^{-1}(x)$

QUESTION NO 21

Solution

A function of the form y = | f(x) | take positive values or zero. The graphs in A) and B) have negative values and therefore neither can be the graph of y = | f(x) |. In fact B) is not the graph of a function. Also f(x) is undefined at x = 2 and also has a vertical asymptote at x = 2. The only graph that seems to have a vertical asymptote at x = 2 is graph C) and is therefore the closet to the graph of | f(x) |.

22. If in a triangle ABC, sin(A) = 1/5, cos(B) = 2/7, then cos(C) =

Solution

The sum of all three angles in a triangle is equal to 180^0. Hence

$A + B + C = 180^0$

or $A + B = 180^0 - C$

The above gives

$\cos(180^0 - C) = \cos (A + B)$

Use trigonometry formula to rewrite above as

$\cos(180^0) \cos(C) + \sin(180^0) \sin(C) = \cos (A) \cos (B) - \sin(A) \sin(B)$

Simplify and rewrite the above as

$\cos (C) = \sin(A) \sin(B) - \cos(A) \cos(B)$

Given that sin(A) = 1/5 and cos(B) = 2/7, We now need to find cos (A) and sin (B) using identities as follows:

$\cos (A) = \sqrt{(1 - (1/5)^2)} = \sqrt{24} / 5$

$\sin (B) = \sqrt{(1 - (2/7)^2)} = \sqrt{45} / 7$

Substitute the values of sin(A), sin(B), cos(A) and cos(B) into cos (C) = sin(A) sin(B) - cos(A) cos(B) we obtain

$\cos(C) = (\sqrt{45} - 2\sqrt{24}) / 35$

23. Find the sum

$$\sum_{k=1}^{100} (3 + k)$$

Solution

Rewriting the sum explicitly, we have

$$\sum_{k=1}^{100} (3 + k) = 4 + 5 + 6 + ... + 103$$

The terms in the sum are those of an arithmetic sequence with common difference 1. Hence the sum is equal to the average the first term and the last term multiplied by the number of terms which is 100. Hence

$$\sum_{k=1}^{100} (3 + k) = 100*(4 + 103)/2 = 5350$$

24. What value of x makes the three terms x, x / (x + 1) and 3x / [(x + 1)(x + 2)] those of a geometric sequence?

Solution

Let r be the common ratio of the geometric sequence. Hence the relationship between the first and second term and the second and third term are as follows

x / (x + 1) = r x and 3x / [(x + 1)(x + 2)] = r x / (x + 1)

Solve both equations above for x to obtain

r = 1 / (x + 1) and r = 3 / (x + 2)

r is constant, hence

1 / (x + 1) = 3 / (x + 2)

Solve the above for x to obtain

x = -1 / 2

25. As x increases from Pi/4 to 3 Pi/4, |sin(2x)|

A) always increases

B) always decreases

C) increases then decreases

D) decreases then increases

E) stay constant

Solution

As x changes g(x) = |sin(2x)| takes either positive values or zero.

For x = Pi / 4, g(Pi/4) = | sin(2 Pi/4) | = 1

For x = Pi / 2, g(Pi/2) = | sin(2 Pi/2) | = 0

For x = 3 Pi / 4, g(3 Pi/4) = | sin(3 Pi/2) | = 1

As x increases from Pi/4 to 3Pi/4, |sin(2x)| decreases then increases.

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26. If $ax^3 + bx^2 + c\,x + d$ is divided by x - 2, then the remainder is equal

Solution

Let $P(x) = a\,x^3 + b\,x^2 + c\,x + d$. According to the remainder theorem, the remainder r when dividing P(x) by x - 2 is equal to P(2). Hence

$r = P(2) = a\,2^3 + b\,2^2 + 2\,c + d$

= 8a + 4b + 2c + d

27. A committee of 6 teachers is to be formed from 5 male teachers and 8 female teachers. If the committee is selected at random, what is the probability that it has an equal number of male and female teachers?

Solution

Let C(n,r) be the number of samples of r elements to be selected from a set of n elements (combinations). Then

C(n,r) = n! / r! (n - r)!

The committee has a total of 6 members. If the number of males and females in this committee are equal then the committee has 3 males and 3 females. There are C(5,3) ways to select 3 males from 5, C(8,3) ways to select 3 females from 8 and C(13,6) ways to select 6 persons from 13. The probability is equal to

P = C(5,3)*C(8,3) / C(13,6) = (5! / 3!2!)*(8! / 3!5!) / (13! / 6!7!) = 140 / 429

28. The range of the function $f(x) = -|x - 2| - 3$ is

Solution

The absolute value is either positive or zero. Hence

$|x - 2| \geq 0$

Multiply both sides of the inequality by -1 and reverse the symbol of inequality to obtain

$- |x - 2| \leq 0$

Subtract 3 from both sides of the inequality to obtain

$- |x - 2| - 3 \leq - 3$

The right side of the above inequality is equal to f(x). Hence the inequality gives

$f(x) \leq - 3$

The above inequality states that the range of y = f(x) is given by

$y \leq -3$

29. What is the period of the function $f(x) = 3 \sin^2(2x + Pi/4)$?

Solution

The period P is defined for sine and cosine functions of the form A sin (Bx + C) + D or A cos (BX + C) + D as follows

P = 2 Pi / | B |

Hence we need to reduce the power of the given function using the identity $\sin^2(x) = 1/2 - 1/2 \cos(2x)$ as follows

$f(x) = 3 \sin^2(2x + Pi/4) = 3 [1/2 - 1/2 \cos 2((2x + Pi/4))]$

$= -3/2 \cos (4x + Pi/2) + 3/2$

Hence the period P of f(x) is given by

P = 2 Pi / | 4 | = Pi / 2

30. It is known that 3 out of 10 television sets are defective. If 2 television sets are selected at random from the 10, what is the probability that 1 of them is defective?

Solution

Out of the 10 tv sets, 3 are defective and therefore 7 are non defective. If you select 2 out of 10 tv and 1 is defective the second one is non defective. There are C(3,1) ways of selecting 1 defective out of 3 , C(7,1) ways of selecting 1 non defective out of 7 and C(10,2) ways of selecting 2 tv sets out of 10. Hence the probability P is given by

P = C(3,1)*C(7,1) / C(10,2) where C(n,r) is the combination of n terms taken r at the time.

= 7/15

31. In a triangle ABC, angle B has a size of 50^o, angle A has a size of 32^o and the length of side BC is 150 units. The length of side AB is

Solution

We first find the size of the third angle C

$C = 180^o - (50^o + 32^o) = 98^o$

We now use the sine law to find the length of side AB

AB / sin(C) = BC / sin(A)

Solve the above for AB, substitute and evaluate

AB = sin(C) * BC / sin(A) = sin(98) * 150 / sin(32) = 280 (rounded to the nearest unit)

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32. For the remainder of the division of $x^3 - 2x^2 + 3kx + 18$ by x - 6 to be equal to zero, k must be equal to

Solution

Let $P(x) = x^3 - 2x^2 + 3kx + 18$. Using the remainder theorem, the remainder R of the division of P(x) by x - 6 is equal to P(6). Hence

$R = P(6) = 6^3 - 2(6)^2 + 3(6) k + 18$

= 216 - 72 + 18 k + 18 = 162 + 18 k

Set R = 0 and solve for k

162 + 18 k = 0 , gives k = - 9

33. It takes pump (A) 4 hours to empty a swimming pool. It takes pump (B) 6 hours to empty the same swimming pool. If the two pumps are started together, at what time will the two pumps have emptied 50% of the water in the swimming pool?

Solution

The rates RA and RA of pumps A and B are given by

RA = 1/4 and RB = 1/6

Let t be the time that the two pumps will take to empty 50% of the pool. Hence

t(1/4 + 1/6) = 0.5

Multiply all terms in the above equation by the lcm of 4 and 6 which 12, simplify and solve

t(3 + 2) = 6 t = 6/5 = 1.2 hours = 1 hour 12 minutes

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34. The graph of $r = 10 \cos(\Theta)$, where r and Θ are the polar coordinates, is

A) a circle

B) an ellipse

C) a horizontal line

D) a hyperbola

E) a vertical line

Solution

If r and Θ are the polar coordinates then the x and y coordinates are given by

$x = r \cos(\Theta)$ and $y = r \sin(\Theta)$ and $x^2 + y^2 = r^2\cos^2(\Theta) + r^2\sin^2(\Theta) = r^2$

Multiply both sides of the given equation $r = 10 \cos(\Theta)$ by r

$r^2 = 10 r \cos(\Theta)$

Substitute r^2 by $x^2 + y^2$ and $r \cos(\Theta)$ by x in the above equation to obtain

$x^2 + y^2 = 10 x$

The above equation may be written as

$x^2 - 10 x + y^2 = 0$

$(x - 5)^2 + y^2 = 25$

The above equation is that of a circle of center (5,0) and radius 5. Hence the given equation $r = 10 \cos(\Theta)$ is that of a circle.

35. If $(2 - i)*(a - bi) = 2 + 9i$, where i is the imaginary unit and a and b are real numbers, then a equals

Solution

We first expand the left side of the given equation.

$2 a - 2b i - i a - b = 2 + 9 i$

which may be written as

$2 a - b + i(-2 b - a) = 2 + 9 i$

Two complex numbers are equal if their real parts and imaginary are equal. Hence the above equation gives two other equations

$2 a - b = 2$ and $- 2 b - a = 9$

Solve the above system of equation to find $a = - 1$.

36. Lines L1 and L2 are perpendicular that intersect at the point (2 , 3). If L1 passes through the point (0 , 2), then line L2 must pass through the point

A) (0 , 3)

B) (1 , 1)

C) (3 , 1)

D) (5 , 0)

E) (6 , 7)

Solution

We first find the slope m1 of line L1 since we know two points on this line.

m1 = (2 - 3) / (0 - 2) = 1 / 2

Since L1 and L2 are perpendicular, the slope m2 of line L2 is related to m1 as follows

m1*m2 = -1 or (1/2) * m2 = -1 which gives m2 = -2

We now know a point and a slope of line L2, hence its equation is given by

y - 3 = -2 (x - 2)

We now substitute the coordinates in the equation above and determine which point is on the line L2. Point (3 , 1) in C) is on L2.

37.

Solution

The cube has 6 square faces. The total surface is 24 square cm and therefore the area of one square face of the cube is

24 / 6 = 4 square cm and the side of one square face is 2 cm

The volume of the pyramid is equal to the one third the area of the base which is 4 square cm times the height. Since the pyramid is inscribed in the cube, the height is equal to a side of the square.

Volume = (1/3) * 4 * 2 = 8/3 cubic cm

38. The graph defined by the parametric equations

$x = \cos^2 t$

$y = 3 \sin t - 1$

is

Solution

Let us solve the given equation $y = 3 \sin t - 1$ for $\sin t$

$\sin t = (y + 1) / 3$

Square both sides of the above equation

$\sin^2 t = (y + 1)^2 / 9$

The above equation may be written

$1 - \cos^2 t = (y + 1)^2 / 9$

We now substitute $\cos^2 t$ by x in the above equation to obtain

$1 - x = (y + 1)^2 / 9$

Solve the above for x

$x = - (y + 1)^2 / 9 + 1$

The above equation is that of a parabola with axis parallel to x axis. However there are restrictions on both x and y. Since $x = \cos^2 t$ and $y = 3 \sin t - 1$, x cannot be greater than 1 for example. So the graph defined by the parabolic equations is not a whole parabola but a part of a parabola.

39.

$$\lim_{x \to 2} \frac{(x^4 - 16)}{x - 2} =$$

Solution

Calculating the limit as is gives an undetermined form 0 / 0 . Rewrite the numerator in factored form and simplify

$(x^4 - 16) / (x - 2) = (x^2 - 4)(x^2 + 4) / (x - 2)$

$= (x - 2)(x + 2)(x^2 + 4) / (x - 2)$

$= (x + 2)(x^2 + 4)$, after simplification

The limit as x approaches 2 is given by

$(2 + 2)(2^2 + 4) = 32$

40. For x > 0 and x not equal to 1, $\log_{16}(x)$ =

Solution

Use the change of base formula and log with base to rewrite $\log_{16}(x)$ as follows

$\log_{16}(x) = \log_2(x) / \log_2(16)$

$= \log_2(x) / \log_2(2^4)$

$= \log_2(x) / 4$

$= 0.25 \log_2(x)$

41. The value of k that makes function f, defined below, continous is

$$f(x) = \begin{cases} \dfrac{2x^2 + 5x}{x} & ,when\ x \neq 0 \\ 3k - 1 & ,when\ x = 0 \end{cases}$$

Solution

We first find the limit of f(x) as x approaches 0.

$f(x) = (2x^2 + 5x) / x = 2x + 5$

The limit of the above as x approaches 0 is.

$2(0) + 5 = 5$

For a function to be continuous at x = 0, the following condition must be satisfied

lim f(x) as x approaches = f(0)

$5 = 3k - 1$

Solve the above for k.

$k = 2$

42. If $\log_b(a) = x$ and $\log_b(c) = y$, and $4x + 6y = 8$, then $\log_b(a^2 . c^3)$

Solution

We first divide all terms in the equation by 2 to obtain

$2x + 3y = 4$

We next substitute x by $\log_b(a)$ and y by $\log_b(c)$ in the above equation.

$2 \log_b(a) + 3 \log_b(c) = 4$

We next use the power formula to rewrite the above as .

$\log_b(a^2) + \log_b(c^3) = 4$

Using the product formula, the above is rewritten as

$\log_b(a^2 . c^3) = 4$

43. The point (0 , -2 , 5) lies on the

Solution

The x coordinate of the given point is the only coordinate equal to 0 and therefore the given point is on the yz plane

44. Curve C is defined by the equation $y = \sqrt{(9 - x^2)}$ with $x \geq 0$. The area bounded by curve C, the x axis and the y axis is

Solution

Let us square both sides of the given equation to obtain

$y^2 = 9 - x^2$

Rewrite as the equation of a circle in standard form.

$x^2 + y^2 = 9$

The above equation is that of a circle with radius 3 and center at (0,0). The region bounded by curve C, the x axis and the y axis and such that $x \geq 0$ is a quarter of this circle and therefore its area is the quarter of the area of the circle and is given by

$(1/4)\ \text{Pi}\ (3^2) = 9\ \text{Pi} / 4$

45. In a plane there are 6 points such that no three points are collinear. How many triangles do these points determine?

Solution

A triangle is defined by any 3 non collinear points. Hence the number of triangles that 6 points can determine is equal to the number of combinations of 6 points taken 3 at the time and is given by

C(6,3) = 6! / (3!(6-3)!) = 6*5*4*3! / (3!3!) = 20

46.

$$\begin{bmatrix} 3 & 2 \\ -1 & -2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 0 & a \\ b & c \end{bmatrix} = \begin{bmatrix} -4 & 9 \\ 4 & -7 \\ -10 & 19 \end{bmatrix}$$

Find a, b and c.

Solution

What is given is the multiplication of two matrices A.B = c. The product of the first row of A by the the first column of B gives

3(0) + 2(b) = - 4

Solve for b

b = - 2

The product of the first row of A by the the second column of B gives

3 a + 2 c = 9

The product of the second row of A by the the second column of B gives

- a - 2 c = - 7

Solve the last two equations for a and c simultaneously to obtain

a = 1 and c = 3

47. If the sum of the repeating decimals 0.353535... + 0.252525... is written as a fraction in lowest terms, the product of the numerator and denominator is

Solution

We first write 0.353535...as a sum as follows

0.353535... = 0.35 + 0.0035 + 0.000035 ....

The above is an infinite geometric series with common ration 0.001. Hence the sum

= 0.35 / (1 - 0.01) = 0.35 / 0.99 = 35 / 99

We proceed in the same way for 0.252525...

0.252525... = 0.25 + 0.0025 + 0.000025 +....

= 0.25 / (1 - 0.01) = 25 / 99

The sum is given by

0.353535... + 0.252525... = 35/99 + 25/99 = 60/99 = 20/33

The product of numerator and denominator

20 * 33 = 660

48. $\sin(\tan^{-1}\sqrt{2})$ =

Solution

Define arc t as follows

$t = \tan^{-1}\sqrt{2}$

tan t = opposite side / adjacent side = $\sqrt{2}$ = $\sqrt{2}$ / 1

Hypotenuse = $\sqrt{(1 + 2)}$ = $\sqrt{3}$

$\sin(\tan^{-1}\sqrt{2})$ = sin t

= opposite side / hypotenuse = $\sqrt{2}$ / $\sqrt{3}$ = 0.82 (approximated at two decimal places)

49. If $8^x = 2$ and $3^{x+y} = 81$, then y =

Solution

Equation $8^x = 2$ may be written as follows

$(2^3)^x = 2$

Hence

3x = 1 or x = 1/3

Equation $3^{x+y} = 81$ may be written as follows

$3^{x+y} = 81 = 3^4$

Hence

x + y = 4 , y = 4 - x = 4 - 1/3 = 11/3

50. Let $f(x) = -x^2 / 2$. If the graph of f(x) is translated 2 units up and 3 units left and the resulting graph is that of g(x), then g(1/2) =

Solution

Let us first find the equation for g(x).

When $- x^2 / 2$ is translated 2 units up it becomes $- x^2 / 2 + 2$.

$g(x) = - (x + 3)^2 / 2 + 2$, translated 3 units left

$g(1/2) = - (1/2 + 3)^2 / 2 + 2 = - 33/8$