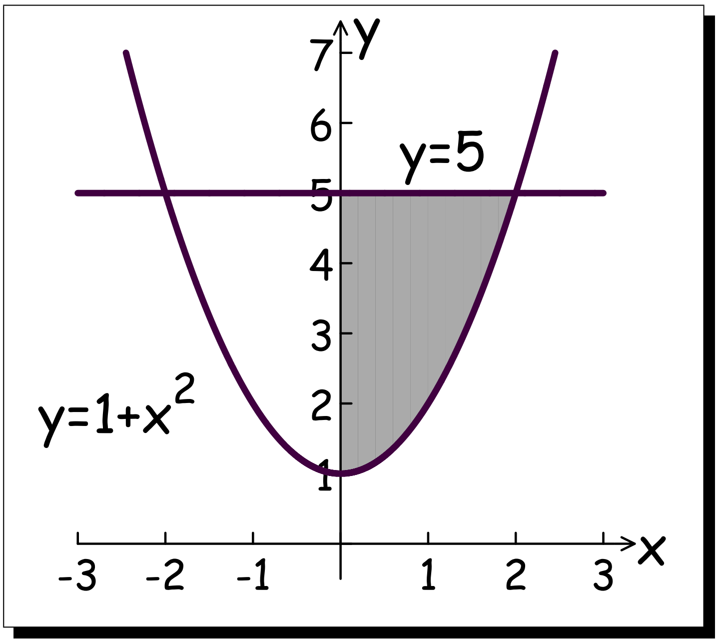
AP Calculus Chapter 5 Testbank

Part I. Multiple-Choice Questions

1. Which of the following integrals correctly corresponds to the area of the shaded region in the figure to the right?



   (A) $\int_0^2 (x^2 - 4)\,dx$

   (B) $\boxed{\int_0^2 (4 - x^2)\,dx}$

   (C) $\int_0^5 (x^2 - 4)\,dx$

   (D) $\int_0^5 (x^2 + 4)\,dx$

   (E) $\int_0^5 (4 - x^2)\,dx$

2. $\dfrac{d}{dx}\displaystyle\int_{-2}^{\sqrt{x}} t^2\sqrt{4-t^2}\,dt =$

   (A) $t\sqrt{4-t}$

   (B) $x\sqrt{4-x}$

   (C) $\boxed{\frac{1}{2}\sqrt{x}\sqrt{4-x}}$

   (D) $\dfrac{\sqrt{4-x}}{2\sqrt{x}}$

   (E) $x\sqrt{4-x^2}$

3. $\displaystyle\int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{-\frac{\pi}{4}}^{0} \cos x\, dx =$

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) $\boxed{1}$ (E) $\sqrt{2}$

4. If $\displaystyle\int_{30}^{100} f(x)\, dx = A$ and $\displaystyle\int_{50}^{100} f(x)\, dx = B$, then $\displaystyle\int_{30}^{50} f(x)\, dx =$

(A) $A + B$ (B) $\boxed{A - B}$ (C) 0 (D) $B - A$ (E) 20

5. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\dfrac{16}{3}$ (B) $\boxed{\dfrac{16}{3}}$ (C) $\dfrac{64}{3}$ (D) $\dfrac{66}{3}$ (E) $\dfrac{256}{3}$

6. The average value of the function $f(x) = \ln^2 x$ on the interval $[2, 4]$ is

(A) -1.204 (B) $\boxed{1.204}$ (C) 2.159 (D) 2.408 (E) 8.636

7. $\displaystyle\frac{d}{dx}\int_0^{3x} \cos t\, dt =$

(A) $\sin 3x$ (B) $-3\sin 3x$ (C) $\cos 3x$ (D) $3\sin 3x$ (E) $\boxed{3\cos 3x}$

8. If the definite integral $\int_1^3 (x^2+1)\,dx$ is approximated by using the Trapezoid Rule with $n=4$, the error is

(A) 0 (B) $\frac{7}{3}$ (C) $\boxed{\frac{1}{12}}$ (D) $\frac{65}{6}$ (E) $\frac{97}{3}$

9. $\int \ln 2x\,dx =$

(A) $\frac{\ln 2x}{x} + C$

(B) $\frac{\ln 2x}{2x} + C$

(C) $x\ln x - x + C$

(D) $\boxed{x\ln 2x - x + C}$

(E) $2x\ln 2x - 2x + C$

10. $\int x\sqrt{3x}\,dx =$

(A) $\boxed{\frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C}$

(B) $\frac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$

(C) $\frac{5\sqrt{3}}{2}x^{\frac{1}{2}} + C$

(D) $2\sqrt{3x} + C$

(E) $\frac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$

11. The average value of $f(x) = e^{4x^2}$ on the interval $[-\frac{1}{4}, \frac{1}{4}]$ is

(A) $\boxed{0.272}$ (B) 0.545 (C) 1.090 (D) 2.180 (E) 4.360

12. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$, where t stands for time.

(A) 0.976 (B) $\boxed{6.204}$ (C) 6.359 (D) 12.720 (E) 7.000

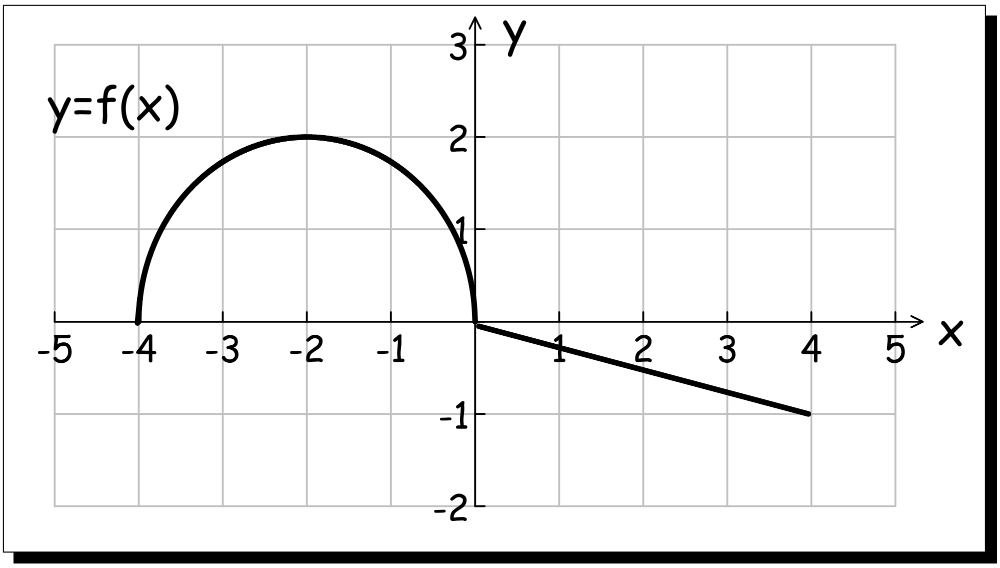
13. Approximate $\int_0^1 \sin^2 x\, dx$ using the Trapezoid Rule with $n = 4$ to three decimal places.

(A) $\boxed{0.277}$ (B) 0.270 (C) 0.555 (D) 1.109 (E) 2.219

14. Use the Trapezoid Rule with $n = 4$ to approximate the area between the curve $y = x^3 - x^2$ and the x-axis over the interval $[3, 4]$.

(A) 35.266 (B) 27.766 (C) 63.061 (D) $\boxed{31.516}$ (E) 25.125

15. The graph of the function f shown below consists of a semicircle and a straight line segment. Then $\int_{-4}^{4} f(x)\, dx =$



(A) $2\pi + 1$ (B) $\boxed{2(\pi - 1)}$ (C) $\pi + 2$ (D) $2(2\pi + 1)$ (E) $\pi + 1$

16. $\dfrac{d}{dx}\displaystyle\int_{2x}^{5x} \cos t \, dt =$

(A) $\boxed{5\cos 5x - 2\cos 2x}$

(B) $5\sin 5x - 2\sin 2x$

(C) $\cos 5x - \cos 2x$

(D) $\sin 5 \sin 2x$

(E) $\frac{1}{5}\cos 5x - \frac{1}{2}\sin 2x$

17. $\dfrac{d}{dx}\displaystyle\int_{2x}^{5x} \sqrt{2-\cos t} \, dt =$

(A) $2\sqrt{2-\cos 5x} - 5\sqrt{2-\cos 2x}$

(B) $5\sqrt{2-\cos 2x} - 2\sqrt{2-\cos 5x}$

(C) $2\sqrt{2-\cos 2x} - 5\sqrt{2-\cos 5x}$

(D) $\boxed{5\sqrt{2-\cos 5x} - 2\sqrt{2-\cos 2x}}$

(E) $5\sqrt{2-\cos 5x} + 2\sqrt{2-\cos 2x}$

18. Given that the function f is continuous on the interval $[1, \infty)$, and that $\displaystyle\int_1^x f(t)\,dt = \sqrt{x}$, then $\displaystyle\int_1^x f^2(t)\,dt =$

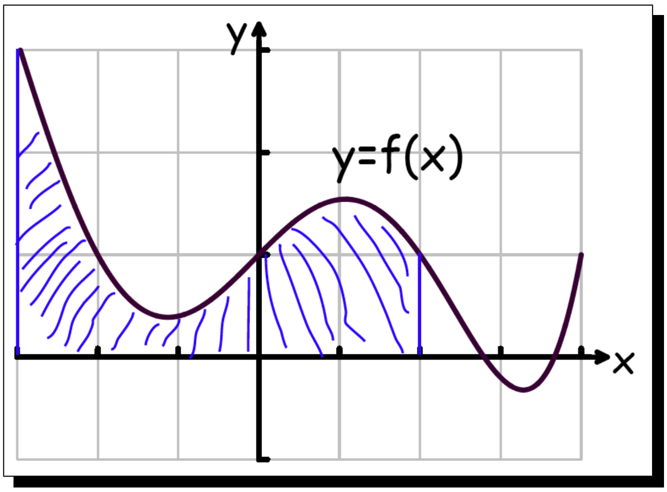
(A) x (B) $x/4$ (C) $\boxed{\frac{1}{4}\ln x}$ (D) $\frac{1}{4}(\ln x - 1)$ (E) Can't be determined

19. Given that the function f is continuous on the interval $[1, \infty)$, and that $\displaystyle\int_1^x \sqrt{f(t)}\,dt = \sqrt{x}$, then $\displaystyle\int_1^\infty f^2(t)\,dt =$

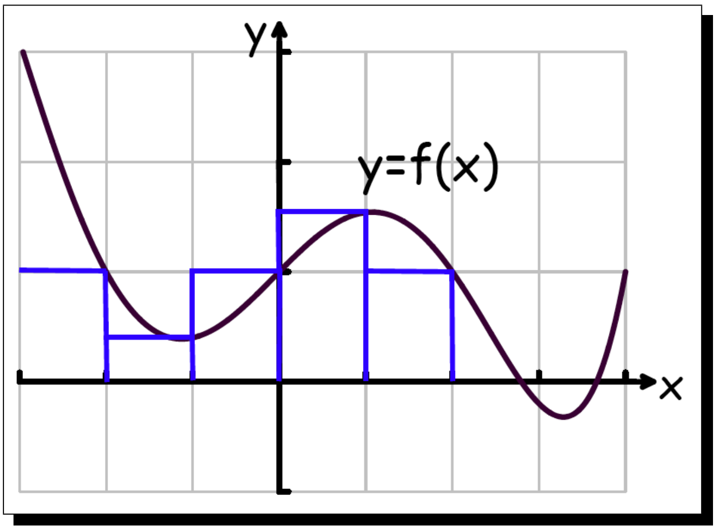
(A) 0 (B) $\boxed{\frac{1}{16}}$ (C) $\frac{1}{4}$ (D) 1 (E) ∞

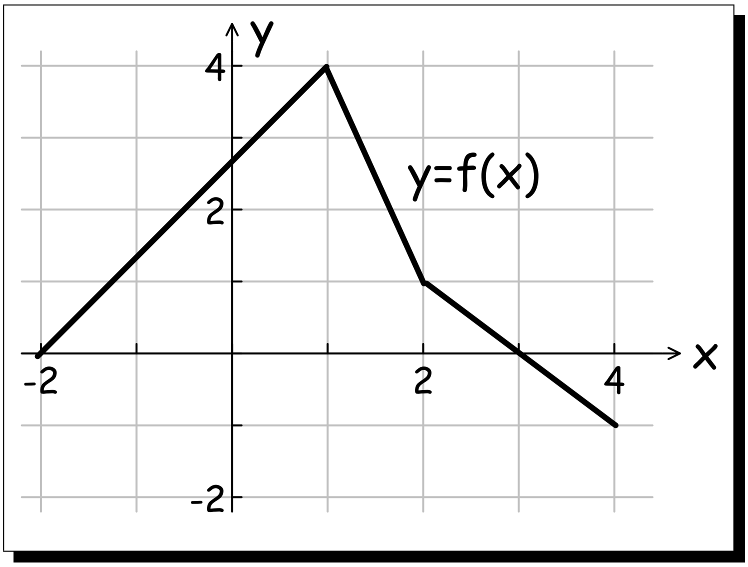
Part II. Free-Response Questions

20. On the graph below, shade in the appropriate area indicated by the integral $\int_{-3}^{2} f(x)dx$



21. In the graph to the right, indicate the rectangles that would used in computing a RRAM (right rectangular approximation) to $\int_{-3}^{2} f(x)dx$ using five rectangles, each of width 1. (Don't try to compute anything; just draw the relevant picture.)[1] Does it appear the result gives an **overestimate** or an **underestimate** of the true area?





22. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$. $g(4) = \int_1^4 f(t)\,dt = \frac{5}{2}$; $g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^1 f(t)\,dt = -6$.

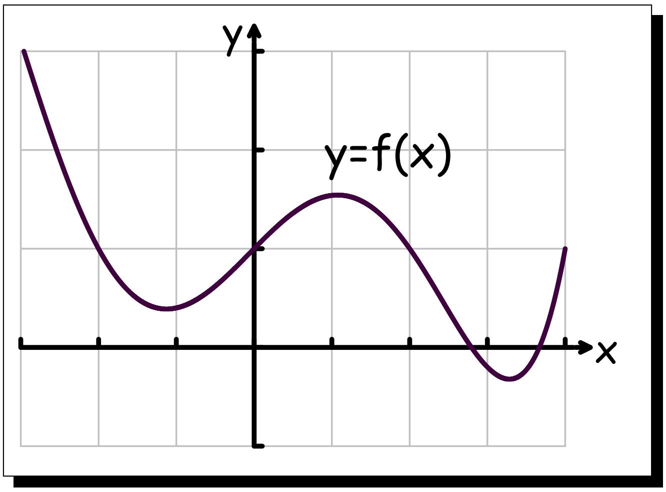
(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$. By the Fundamental Theorem of Calculus, we have

$$g'(1) = f(1) = 4.$$

(c) Find the minimum value of g on the closed interval $[-2, 4]$. Justify your answer. We have $g'(x) = f(x) = 0$ when $x = -2$ or when $x = 3$. Comparing with endpoints we have $g(-2) = -6$ (from part (a)), $g(3) = \int_1^3 f(t)\,dt = 3$, and $g(4) = \frac{5}{2}$ (from part (a)). Therefore, the minimum value of g on the closed interval $[-2, 4]$ is -6 (which happens at $x = -2$).

23. Let f be the function depicted in the graph to the right. Arrange the following numbers in increasing order:

$$\int_{-1}^{2} f(x)\,dx, \quad \int_{-1}^{3} f(x)\,dx, \quad \int_{1}^{3} f'(x)\,dx.$$



$$\int_{1}^{3} f'(x)\,dx < \int_{-1}^{2} f(x)\,dx < \int_{-1}^{3} f(x)\,dx.$$

24. Assume that the average value of a function f over the interval $[-1, 3]$ is 3.25. Compute $\int_{-1}^{3} f(x)dx$.

$\int_{-1}^{3} f(x)dx$ is equal to 4 times its average value over this interval. Therefore,

$$\int_{-1}^{3} f(x)dx = 4 \times 3.25 = 13.$$

25. Express the area of a circle of radius r as a definite integral.

This is clearly $2\int_{-r}^{r} \sqrt{r^2 - x^2}\,dx = 4\int_{0}^{r} \sqrt{r^2 - x^2}\,dx.$

26. Find an antiderivative for each function below:

(a) $f(x) = x^2 - x + 2$ $\int f(x)\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + C.$

(b) $g(x) = 2 - e^{-x}$ $\int g(x)\,dx = 2x + e^{-x} + C.$

(c) $v(t) = gt + v_0$ (here, v_0 is just a constant)

$$\int v(t)\,dt = \frac{gt^2}{2} + v_0 t + C$$

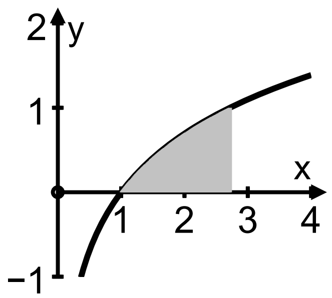
(d) $h(x) = 1/x$, where $x < 0$ (This is not hard, you just need to be careful!) $\int h(x)\,dx = \ln(-x) + C$ (This says that, in general, $\int \frac{dx}{x} = \ln|x| + C$.)

(e) $k(x) = (1+x)/x$ $\int k(x)\,dx = \ln x + x + C$.

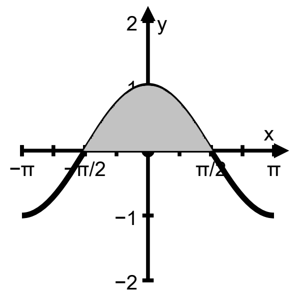
(f) $\theta(x) = 1/(1+x)$ $\int \theta(x)\,dx = \ln|1+x| + C$.

27. For each definite integral below, sketch the area represented. (Don't try to compute anything, just draw the relevant picture.)

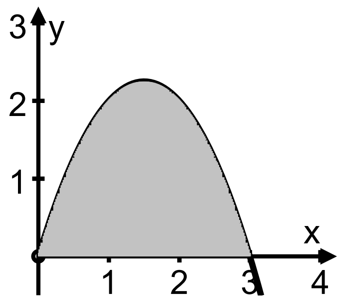
(a) $\int_1^e \ln x\,dx$



(b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$

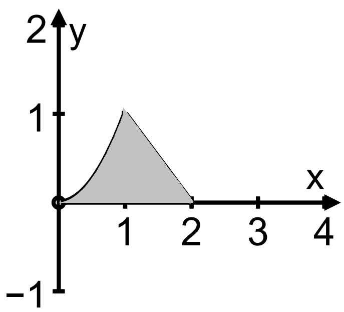


(c) $\int_0^3 t(3-t)\,dt$



(d) $\int_0^2 f(x)\,dx$, where

$$f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 2-x & \text{if } x > 1. \end{cases}$$



28. Compute the definite integrals

(a) $\int_{-3}^{3}(9-x^2)dx$

$= 2\int_0^3 (9-x^2)\,dx = 9x - \frac{x^3}{3}\Big|_0^3 = 27-9 = 18.$

(b) $\int_0^{\pi} \sin x\,dx$

$= -\cos x\Big|_0^{\pi} = 2$

(c) $\int_0^4 e^{-x}dx$

$= -e^{-x}\Big|_0^4 = 1-e^{-4}.$

(d) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$= \sin^{-1} x\Big|_0^1 = \pi/2$

(e) $\int_0^1 \frac{dx}{1+x^2}$

$= \tan^{-1} x\Big|_0^1 = \pi/4$

(f) $\int_0^1 \frac{dx}{1+x}$

$= \ln(1+x)\Big|_0^1 = \ln 2$

29. Solve for a in each of the below:

(a) $\int_0^a (1-x)dx = \frac{1}{2} = \left(x-\frac{x^2}{2}\right)\Big|_0^a = a-\frac{a^2}{2} \Rightarrow a=1$

(b) $\int_0^a \frac{dx}{1+x^2} = \frac{\pi}{4} = \tan^{-1}\Big|_0^a = \tan^{-1} a \Rightarrow a=1$

(c) $\int_1^a \frac{dx}{x} = 1 = \ln a\Big|_1^a = \ln a \Rightarrow a=e.$

30. Suppose that f is an even function and that $\int_0^a f(x)dx = 10,$ where $a>0$. Compute $\int_{-a}^{a} f(x)dx.$

Since f is an even function, we have

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx = 2\cdot 10 = 20.$$

31. Suppose that f is an odd function and that $a>0$. Compute $\int_{-a}^{a} f(x)dx.$ This is $=0$. To prove this, note that $\int_{-a}^{0} f(x)\,dx =$

$-\int_0^{-a} f(x)\,dx \overset{u=-x}{=} \int_0^a f(-u)\,du = -\int_0^a f(u)\,du$. That is to say, $\int_{-a}^0 f(x)dx = -\int_0^a f(x)\,dx$, and so

$$\int_{-a}^a f(x)dx = \int_{-a}^0 f(x)dx + \int_0^a f(x)dx = -\int_0^a f(x)\,dx + \int_0^a f(x)\,dx = 0.$$

32. Solve the differential equations:

    (a) $y' = 3x^2 - 1,\ y(0) = 0$. $y = \int (3x^2 - 1)\,dx = x^3 - x + C$. Also, $0 = y(0) = 0^3 - 0 + C \Rightarrow C = 0$. Therefore, $y = x^3 - x$.

    (b) $y' = 1 - e^{-x},\ y(0) = 1$. $y = \int (1 - e^{-x})\,dx = x + e^{-x} + C$. Also, $1 = y(0) = 0 + e^0 + C \Rightarrow C = 0$. Therefore, $y = x + e^{-x}$.

    (c) $(x+1)y' = 1,\ y(0) = 0$. $y = \int \frac{dx}{x+1} = \ln|x+1| + C$. Also, $0 = y(0) = \ln 1 + C \Rightarrow C = 0$. Therefore, $y = \ln|x+1|$.

    (d) $y' = y,\ y(0) = 2$. Write this as $\frac{dy}{dx} = y$, which separates as $\frac{dy}{y} = \int dx \Rightarrow \ln|y| = x + C$. Therefore, $|y| = e^{x+C}$; it's better to write this as $y = Ke^x$, where K is a (possibly negative) arbitrary constant. From $2 = y(0) = Ke^0 = K$, get $K = 2$ and so $y = 2e^x$.

    (e) $y' = 2y,\ y(0) = 1$. Argue almost exactly as in (d) to arrive at the solution $y = e^{2x}$.

33. Suppose that you know that $f''(x) = 0$. What can you say about $f(x)$? Write down the **most general form** that $f(x)$ could have. Clearly the most general solution must be of the form $f(x) = A + Bx$, where A and B are arbitrary constants.

34. Suppose that the velocity of a particle is given by $v(t) = 1 + \sin t,\ t \geq 0$, where v is given in units of cm/sec. How far has this particle traveled after 10 sec?

We know that the TOTAL DISTANCE the particle travels is given by the integral $\int_0^{10} s(t)\,dt$, where $s(t)$ is the speed of the particle. Next, since $s(t) = |v(t)|$, we have that the total distance traveled is $\int_0^{10} |v(t)|\,dt = \int_0^{10} |1+\sin t|\,dt = \int_0^{10} (1+\sin t)\,dt = t-\cos t\Big|_0^{10} = 11 - \cos 10 \approx 11.84$ cm.

35. Assume that a water pump is pumping water into a large tank at a variable rate: after t hours, water is being pumped at a rate of $v(t) = \dfrac{500t}{1+t}$ gallons/hour.

(a) Write down the definite integral that expresses how much water has been pumped into the vessel after 24 hours. (Don't compute this.) This is clearly $\displaystyle\int_0^{24} \frac{500t\,dt}{1+t}$.

(b) Compute how much water is in the tank after 24 hours, assuming that the tank was empty to begin with and that $F(t) = 500\,(t - \ln(1+t))$ is an antiderivative for $v(t)$.

The amount pumped into the tank over the first 24 hours is

$$\int_0^{24} \frac{500t\,dt}{1+t} = 500(t-\ln(1+t))\Big|_0^{24} = 500(24-\ln 25) \approx 10390.6 \text{ gallons.}$$

36. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) \;=\; \frac{15600}{t^2 - 24t + 150}.$$

the rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) \;=\; \frac{9890}{t^2 - 38t + 370}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number. This is the integral of the rate of people entering the park: $\displaystyle\int_0^{17} E(t)\,dt = \int_0^{17} \frac{15600\,dt}{t^2 - 24t + 150} \approx 12,746$ people, where the approximation was done via numerical integration on a TI-84 calculator.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admission to the park on the given day? Round your answer to the nearest whole number. The total in receipts would be the sum

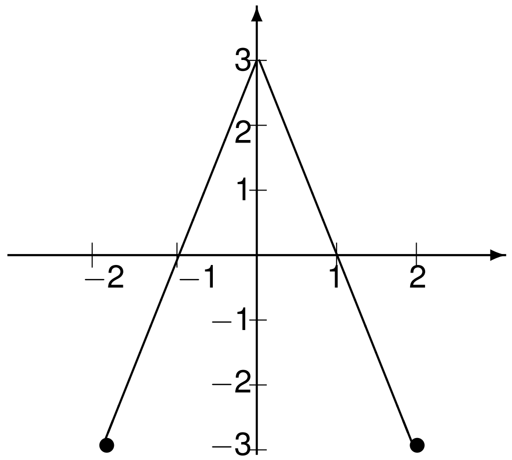
$$\$15 \times \int_0^{17} E(t)\,dt + \$11 \times \int_{17}^{23} E(t)\,dt$$

$$\begin{aligned}
&= 15 \times \int_0^{17} \frac{15600\,dt}{t^2 - 24t + 150} + \$11 \times \int_{17}^{23} \frac{15600\,dt}{t^2 - 24t + 150} \\
&= \$15 \times 12,746 + \$11 \times 1,506 \\
&= \$207,756
\end{aligned}$$

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park. The meaning of $H(17)$ is the total number of people having arrived in the park between 9 a.m. and 5 p.m. less the total number of people having left the park during this same time. In other words, $H(17)$ is the total number of people in the park at 5 p.m.

$H'(17) = E(17) - L(17)$ which is the net rate of people (in people per hour) entering the park at 5 p.m.

(d) At what time, t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum? As $H(t)$ gives the total number of people in the park at time t, the maximum number of people in the park will happen when $H'(t) = 0$, i.e., when $E(x) = L(x)$. Using a graphics calculator, one infers that this happens when $t \approx 16.37$, i.e. at roughly 4:20 p.m.



$(-2,-3)$ **Graph of f** $(2,-3)$

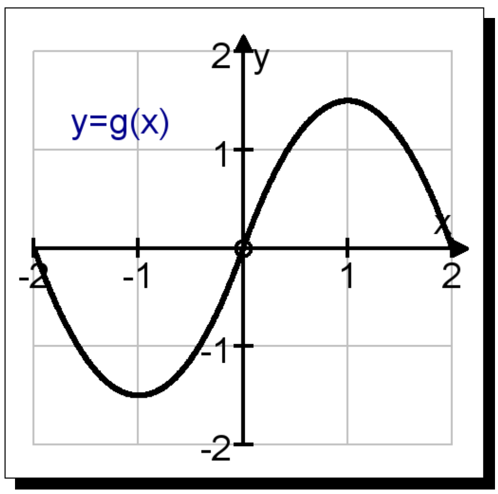
37. The graph of the function f shown above consists of two line segments. Let g be the funtion given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$. $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -1.5$. $g'(-1) = f(-1) = 0$; $g''(-1) = f'(-1) = 3$.

(b) Over which interval(s) within $(-2,2)$ is g increasing? Explain your reasoning. g is increasing where $g'(x) > 0$, i.e., where $f(x) > 0$. This latter condition is satisfied on the interval $-1 < x < 1$.

(c) Over which interval(s) within $(-2,2)$ is the graph of g concave down? Explain your reasoning. The graph of g is concave down where $g''(x) < 0$, i.e., where $f'(x) < 0$. This happens on the interval $0 < x < 2$.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2,2]$.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

38. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of t. A table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, is shown above.

   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure. $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{15}{10} = 1.5$ (gallons/min^2).

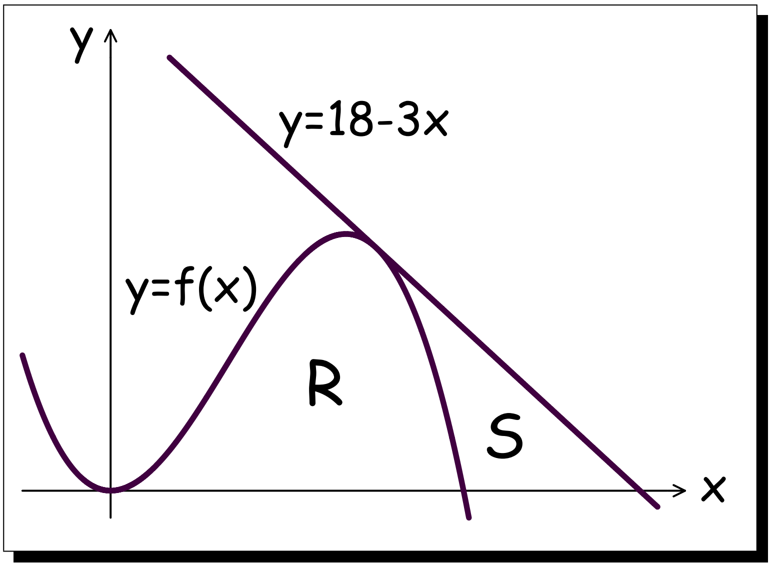
   (b) The rate of fuel consumption is increasing fastest at time $t = 45$. What is the value of $R''(45)$? Explain your reasoning. For the rate of fuel consumption to be increasing at its fastest, we must have that $R'(t)$ is a maximum. This happens (since R is twice-differentiable) when $R''(t) = 0$; therefore, we infer that $R''(45) = 0$.

   (c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning. We have

$$\int_0^{90} R(t)\,dt \approx 20{\cdot}30 + 30{\cdot}10 + 40{\cdot}10 + 55{\cdot}20 + 65{\cdot}20 = 3,700 \text{ gallons.}$$

Since R is a strictly-increasing function of t, we conclude that the left Riemann sum must give an underestimate of the total fuel consumption over the first 90 minutes.

(d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for this plane. Indicate units of measure in both answers. $\int_0^b R(t)\,dt$ is the total fuel consumption (in gallons) over the first b minutes, where $0 < b \leq 90$. The integral $\frac{1}{b}\int_0^b R(t)\,dt$ is the average fuel consumption (in gallons per minute) over the first b minutes.



39. Let f be the function given by $f(x) = 4x^2 - x^3$, and let L be the line $y = 18 - 3x$, where L is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line l, and the x-axis, as shown above.

(a) Show that L is tangent to the graph of $y = f(x)$ at the point $x = 3$. We have that $f'(3) = 8x - 3x^2\Big|_{x=3} = 24 - 27 = -3$, and so the line has the correct slope. Also, $f(3) = 4 \cdot 3^2 - 3^3 = 9 = 18 - 3 \cdot 3$, and so the line and the graph of $y = f(x)$ both intersect where $x = 3$. These two facts imply that the given straight line is tangent to the graph of $y = f(x)$ at the indicated point.

(b) Find the area of S.

$$
\begin{aligned}
\text{Area } (S) &= \int_3^6 (18 - 3x)\, dx - \int_3^4 (4x^2 - x^3)\, dx \\
&= \frac{27}{2} - \left(\frac{4}{3}x^3 - \frac{1}{4}x^4\right)\Big|_3^4 \\
&= \frac{27}{2} - \left(\frac{256}{3} - 64\right) + \left(36 - \frac{81}{4}\right) = \frac{95}{12}.
\end{aligned}
$$

40. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \quad \text{for } 0 \leq t \leq 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period? The total number of cars passing through the given intersection over the 30-minute period is the integral:

$$\int_0^{30} F(t)\,dt = \int_0^{30} \left(82 + 4\sin\left(\frac{t}{2}\right)\right) dt \approx 2,474 \text{ cars.}$$

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer. The traffic flow is increasing or decreasing at $t = 7$ according as to whether $F'(7) > 0$ or $F'(7) < 0$. However,

$$F'(7) = 2\cos\left(\frac{7}{2}\right) \approx -1.87 < 0,$$

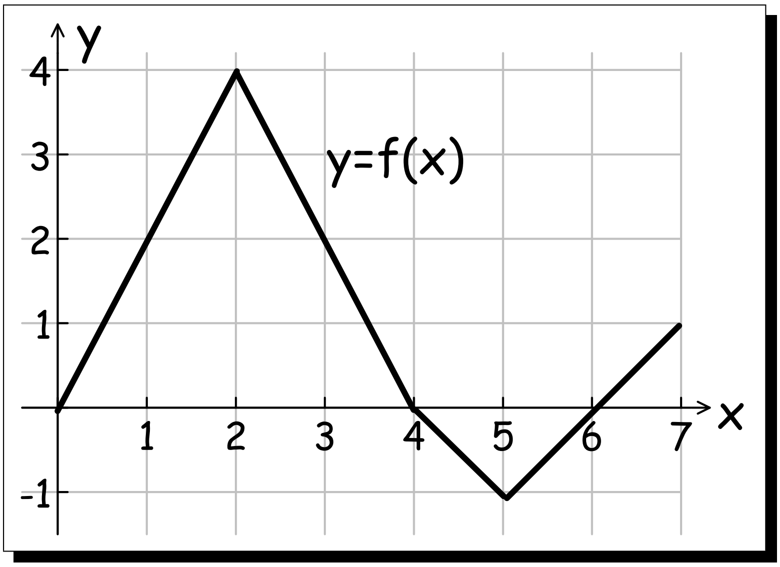
and so the traffic flow is decreasing at $t = 7$.

(c) What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure. The average traffic flow over the time interval $10 \leq t \leq 15$ is

$$\frac{1}{5}\int_{10}^{15} \left(82 + 4\sin\left(\frac{t}{2}\right)\right) dt \approx 81.9 \text{ cars/min}$$

(d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure. The average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$ is the difference quotient

$$\frac{F(15) - F(10)}{15 - 10} = \frac{1}{5}\left(4\sin\left(\frac{15}{2}\right) - 4\sin\left(\frac{10}{2}\right)\right) \approx 1.52 \text{ cars/min}^2$$



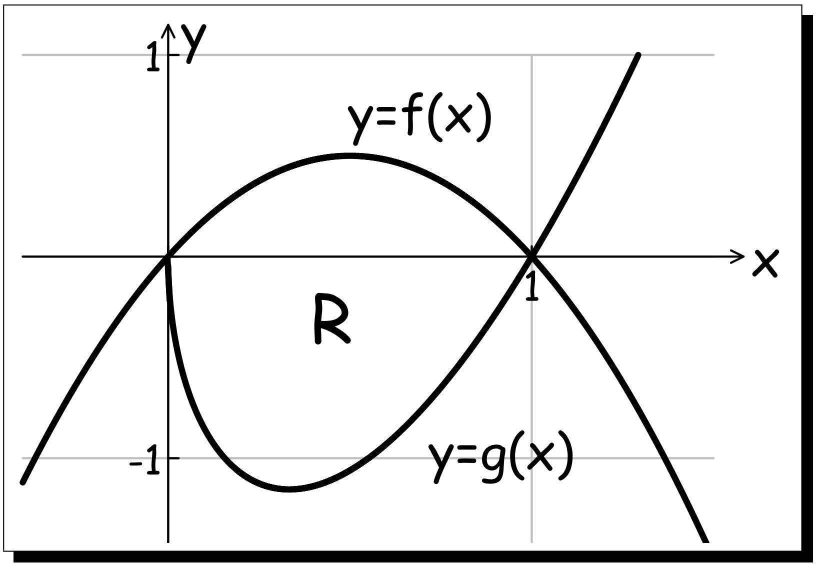
41. Let f be the function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$. $g(3) = \int_2^3 f(t)\,dt = 3,\ g'(3) = f(3) = 2,\ g''(3) = f'(3) = -2.$

(b) Find the average rate of change of g on the interval $0 \leq x \leq 3$. This is $\dfrac{g(3) - g(0)}{3 - 0} = \dfrac{3 - (-4)}{3} = \dfrac{7}{3}$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning. As $g'(c) = f(c)$, we are seeking the number of solutions of $f(c) = \dfrac{7}{3}$ on the interval $0 < c < 3$. An inspection of the graph reveals that there are two such solutions.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer. There are no solutions of $g''(x) = f'(x) = 0$ on the interval $0 < x < 7$. However, there are changes in the concavity of the graph of $y = g(x)$ at $x = 2$ and at $x = 5$ since the sign of $f'(x)$ changes sign at these two values of x.



42. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

Find the area of the region R enclosed by the graphs of f and g.
This area is given by the integral

$$\begin{aligned}
\int_0^1 (f(x) - g(x))\,dx &= \int_0^1 (2x(1-x) - 3(x-1)\sqrt{x})\,dx \\
&= \left(x^2 - \frac{2}{3}x^3 - \frac{6}{5}x^{5/2} + 2x^{3/2}\right)\Big|_0^1 \\
&= 1 - \frac{2}{3} - \frac{6}{5} + 2 = \frac{17}{15}.
\end{aligned}$$