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Using the Closure Property for Addition of Whole Numbers & Integers

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We are always looking for ways to better understand numbers. In this lesson, we'll explore closure, which is a mathematical idea relating an operation, like addition and subtraction, to a set of numbers.

Closure Property

Model trains are great fun! You can capture the excitement of this venerable mode of cargo movement and transportation in the comfort of your own home. Attention to detail is the key. This includes the spacing of the miniature railroad ties. The pattern of the ties is regular, just like the pattern in a set of whole numbers and integer numbers. There's no supporting structure between the ties on a railroad track. Similarly, we won't include the fraction and decimal numbers in between any two whole numbers or integers.

In this lesson, we'll explore what it means to say whole numbers and integers are closed under the mathematical operation of addition.

Closed Under Addition

A **set of whole numbers (W)** contains all the positive numbers including zero but does not include decimals or fractions. We write this as W = { 0, 1, 2, 3, 4, . . . } Those three dots mean that the numbers keep going and going, just like an endless railroad track. One way to visualize whole numbers is with dots on a number line:

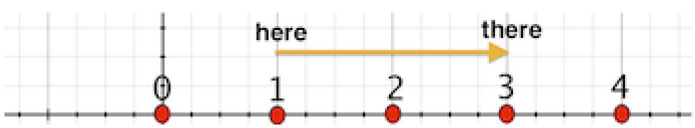
Location of whole numbers on the number line.

Looking at a number line is similar to looking at a railroad track from the side and seeing the ends of the ties. Remember, whole numbers don't stop at 4 but keep going to the right.

The **elements** are the numbers in the set. A set of whole numbers is **closed under addition** if the addition of any two elements produces another element in the set. If an element outside of the set is produced, then the set of whole numbers is not closed under addition.

Addition of Two Numbers: Example

Let's look at the addition of two numbers as a way to get from 'here' to 'there' on the railroad track.



Adding whole numbers.

For example, let's add 1 and 2. The 'here' is the location of the first number, or 1, while the 'there' is the location of the result, or 3.

When we add a positive number, we move our location to the right or we stay at the 'here' (if the number added is a zero). Thus, we get from the 1 to the 3 by adding 2 to the 1.

Here we moved two units to the right. No matter what whole number we add, the 'here' will never move to the left. Even if the 'here' is zero, we can never end up to the left of zero. For example, there is no way to get -1 by adding two whole numbers.

Also, when the 'here' is a whole number and the number we're adding is a whole number, we never end up in between two numbers. For example, there is no way to get 2.5 by adding two whole numbers.

The addition of any two whole numbers always produces another whole number. Therefore, the set of whole numbers is closed under addition.

Adding of Integer Numbers

What if we have a set containing negative numbers as well as positive numbers?

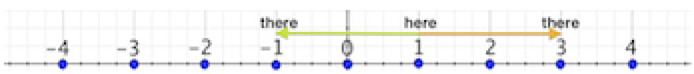
A **set of integer numbers** contains all of the positive and negative numbers including zero, but does not include decimals or fractions. We write this as I = {. . . , -4, -3, -2, -1, 0, 1, 2, 3, 4, . . .}

The location of the integer numbers on a number line can be shown as follows:

Location of integer numbers.

A set of integer numbers is closed under addition if the addition of any two elements of the set produces another element in the set. If an element outside the set is produced, then the set of integers is not closed under addition.

As with whole numbers, when we add a positive number we move to the right. But now the set has negative numbers as well. When we add a negative number, we move from 'here' to the left.



Adding integers.

If the 'here' is 1, adding +2 moves us to the right by two units. The result, 3, is still in the set of integers. If the 'here' is 1, adding -2 moves us to the left by two units. The result is -1. But the -1 is still in the set of integers. The numbers in between the integers, like 2.5 and -2.5 are not in the set of integers. Adding two integers will never result in fractional numbers and decimals. Therefore, the set of integers is closed under addition.

Are you wondering what would happen if our model track was circular? Most likely, we should save this train of thought for another time.

Lesson Summary

Let's review. **Closure** is based on a particular mathematical operation conducted with the elements in a designated set of numbers. A **set of whole numbers (W)** contains all of the positive numbers, including zero, but does not include fractions and decimals.

A **set of integers** contains all of the positive and negative numbers including zero but does not include fractions and decimals. A set is **closed under addition** if adding any two numbers from a set produces a number that is still in the set. In this lesson, we showed that:

1. A set of whole numbers is closed under addition
2. A set of integers is closed under addition

Additional Activities

True or False Activity

This activity will help you assess your knowledge of using the closure property for the addition of whole numbers and integers.

Guidelines

For this activity, print or copy this page on a blank piece of paper. Read each statement carefully. Write TRUE if the statement is valid and FALSE if otherwise. Neatly write your answers on the appropriate blank space provided.

__________ 1. All of the positive numbers, excluding zero, fractions, and decimals are contained in a set of whole numbers.

__________ 2. The numbers in a set are referred to as elements.

__________ 3. When adding positive numbers, you need to move your location to the right on the number line.

__________ 4. 2.5 is an element in the set of integers.

__________ 5. The addition of whole numbers 4 and 5 is a way to demonstrate that the set of whole numbers is closed under addition.

__________ 6. Prime numbers are a set of numbers that are not closed under addition.

__________ 7. A fractional or decimal number may be a result of adding two integers.

__________ 8. When adding a negative number to a positive number, you need to move your location to the left on the number line.

Answer Key

1. FALSE

Zero is included in a set of whole numbers.

2. TRUE

3. TRUE

4. FALSE

Decimals are not included in the set of integers.

5. TRUE

6. FALSE

Prime numbers are whole numbers, therefore, they are closed under addition.

7. FALSE

Adding two integers will never result in fractional or decimal numbers.

8. TRUE