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What are Rational Numbers? - Definition & Examples

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What is a rational number? Learn about rational numbers, rational numbers examples, irrational numbers, and their use in math. Also learn about ratios.

Table of Contents

Rational Number Definition

A **rational number** is a number that can be be expressed as a ratio of two integers, meaning in the form {eq}\dfrac{p}{q} {/eq}. In other words, rational numbers are fractions. The set of all possible rational numbers is represented by the symbol {eq}\mathbb{Q} {/eq}, for "quotient".

Since any integer, *x*, is equal to itself divided by 1 {eq}\left(\dfrac{x}{1} \right) {/eq}, we can see that all the integers are also rational numbers, because we can represent them as fractions. This includes the number zero, since zero divided by any denominator is still zero. In mathematical terms, the set of integers, denoted by {eq}\mathbb{Z} {/eq}, is a subset of the rational numbers, and we represent this symbolically as {eq}\mathbb{Z} \subset \mathbb{Q} {/eq}.

Whether we realize it or not, rational numbers are everywhere in daily life. In addition to all the uses of whole-valued integers, rational numbers appear in any situation that involves a fraction, a proportion, or a percentage. For example, your grade on a math test might be recorded as a fraction out of the total score, or converted to a percentage: a fraction out of 100. Some other examples of fractions, or rational numbers, in everyday life are the proportions of different ingredients in a recipe or how we divide slices of pizza or pie among our friends.

Forms of Rational Numbers

While we can immediately recognize a rational number when it is expressed as a **fraction**, fractional values can also be expressed in **decimal** form. A decimal number is rational if either:

1. The decimal expansion **terminates**, or
2. The decimal expansion doesn't terminate, but takes on a **repeating** sequence of digits that goes on forever past its decimal point.

Since integers, fractions, and decimals can be positive or negative, rational numbers (whether written as integers, fractions, or decimals) can be positive or negative numbers.

Rational Numbers Examples

The numbers {eq}\dfrac{1}{4} {/eq}, {eq}\dfrac{-2}{3} {/eq}, and {eq}\dfrac{4}{2} {/eq} are all rational numbers since they are fractions (proper or improper), where the numerator and denominator are both integers (positive or negative). Notice that after a little simplification, we can see that {eq}\dfrac{4}{2} {/eq} is an integer as well:

$$\dfrac{4}{2} = \dfrac{2}{1} = 2 $$

Thus, we can represent the rational number of {eq}\dfrac{4}{2} {/eq} as a fraction or as the integer 2. This illustrates the fact that not all rational numbers are given in fraction form. However, regardless of the form they are given in, we can identify various rational numbers as rational using the definition of rational numbers and their properties. Consider the following examples.

The numbers {eq}0.2 {/eq}, {eq}-0.45 {/eq}, and {eq}0.123 {/eq} are terminating decimals, and are therefore rational numbers. Each one of them can be expressed as a ratio of integers. Using a power of 10 as the denominator allows us to quickly find an appropriate fraction representation of each of these numbers, which can then be reduced to lowest terms:

$$0.2 = \frac{2}{10} = \frac{1}{5} \quad\qquad -0.45 = \frac{-45}{100} = \frac{-9}{20} \quad\qquad 0.123 = \frac{123}{1000} $$

The numbers {eq}\dfrac{1}{3} {/eq}, {eq}\dfrac{17}{99} {/eq}, and {eq}\dfrac{5}{7} {/eq} are clearly rational numbers, but their decimal expansions do not terminate. Instead, they take on an endless repeating pattern of one or more digits past their decimal point:

$$\frac{1}{3} = 0.333... = 0.\overline{3} \quad\qquad \frac{17}{99} = 0.1717... = 0.\overline{17} \quad\qquad \frac{5}{7} = 0.714285714285... = 0.\overline{714285} $$

Anytime that a decimal number eventually takes on a repeating pattern of digits that continues forever past its decimal point, it can be written as a fraction, and is hence a rational number.

Irrational Numbers

Some numbers simply cannot be expressed as a ratio of two integers. These numbers are said to be **irrational**, meaning ''not rational''. In decimal form, irrational numbers can be recognized by the fact that their decimal expansion never terminates and never repeats. We've already seen that rational numbers are those whose decimal expansions *do* either terminate or infinitely repeat, so the irrational numbers are those that do the opposite of this.

Together, the rational and irrational numbers compose a complete set, known as the **real numbers**, that is denoted by {eq}\mathbb{R} {/eq}. We can visualize {eq}\mathbb{R} {/eq} as the set of all points on a number line, extending infinitely in both the positive and negative directions. Scattered along the line is the subset of points corresponding to rational numbers. That is, the rational numbers are a subset of the real numbers, and we write this in symbols as: {eq}\mathbb{Q} \subset \mathbb{R} {/eq}. We can summarize the relationship between the integers, rational numbers, and real numbers in a circle diagram.

Irrational Number Examples

There are infinitely many irrational numbers, but several specific ones appear so frequently in fields ranging from geometry to finance as to deserve their own unique symbols:

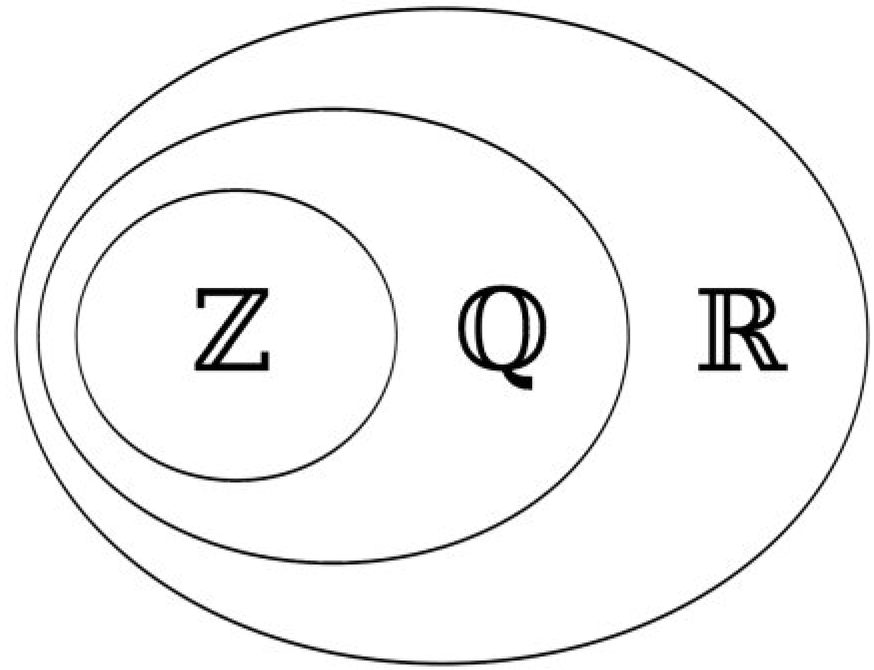
- {eq}\pi \approx 3.14159... {/eq} represents the ratio of a circle's circumference to its diameter.

- {eq}e\approx 2.71828... {/eq}, known as Euler's number, is the base of the natural logarithm. First discovered in connection with compound interest, {eq}e {/eq} appears in many other areas of mathematics.

- {eq}\sqrt 2 \approx 1.41421... {/eq} was found to be irrational by the Ancient Greeks. A famous proof of this fact appears in Euclid's *Elements*.

Relationship between Ratio and Rational Numbers

Rational numbers are precisely those that can be expressed as **ratios**. While it may seem that the term "rational" must have been chosen for exactly this reason, the opposite is actually closer to the truth. Both terms derive from the Latin word *ratio*, which can mean specifically a calculation, or reasoning in general, and the corresponding word *logos* was similarly used in Ancient Greece. The Greeks were the first to conceive of ''illogical'', irrational numbers, which couldn't be precisely measured geometrically. Through time and translation, "rational" was eventually adopted in English as a label for those numbers that could be precisely computed, and finally the expression of these numbers became known as simply a "ratio".



The integers are a subset of the rational numbers, which are a subset of the real numbers.

Lesson Summary

In this lesson, we've learned:

- **Rational numbers** are numbers which can be represented as the ratio of two integers.
- Rational numbers can be written as **fractions** or as **decimals**.
- The decimal expansion of a rational number will either **terminate** or **repeat** a fixed sequence of digits forever.

The Greek philosopher Hippasus may have identified the first irrational number. According to legend, he was murdered for his disturbing discovery!

- Decimals which don't terminate and don't repeat represent **irrational numbers.**
- All integers (positive, negative, or zero) are also rational numbers. The rationals combined with the irrationals make up the set of **real numbers**.
- While rational numbers are **ratios**, both terms actually came into use separately, long after the Ancient Greeks first named what we call irrational numbers.

Video Transcript

Definition

Mathematicians have taken all the numbers in the world and sorted them into categories, based on their characteristics. Generally, the categories, or sets, go from most to least complicated: complex numbers, imaginary numbers, real numbers, rational numbers, integers, whole numbers and natural numbers. Most numbers belong to more than one category.

Here, we'll talk specifically about the category, or set, of rational numbers. The set of **rational numbers**:

- Consist of positive numbers, negative numbers and zero
- Can be written as a fraction

The name **rational** is based on the word 'ratio.' A **ratio** is a comparison of two or more numbers and is often written as a fraction. A number is considered a **rational number** if it can be written as one integer divided by another integer. Sometimes this is referred to as a simple fraction.

The number 1/2 is a rational number because it is written as the integer 1 divided by the integer 2. The number 5 is a rational number because we can write it as 5/1. We can also write it as 15/3 or 50/10 because 15 divided by 3 or 50 divided by 10 both equal 5. The mixed number 1 ½ is also a rational number because we can write it as 3/2.

Any number that can be rewritten as a simple fraction is a rational number. This means that natural numbers, whole numbers and integers, like 5, are all part of the set of rational numbers as well because they can be written as fractions, as are mixed numbers like 1 ½.

Rational numbers can be positive, negative or zero. When we write a negative rational number, we put the negative sign either out in front of the fraction or with the numerator. That's the standard mathematical notation. For example, we would write -5/7 as opposed to 5/-7.

Examples of Rational Numbers

We mentioned earlier that natural numbers, whole numbers and integers are also rational numbers because they can be written as fractions. The simplest way to do this is to put the number over 1. For example: We can write 7 as 7/1; we can write -3 as -3/1; and we can write 0 as 0/1. Therefore, all of these numbers are rational numbers.

Terminating decimals are rational numbers. A **terminating decimal** is a decimal that ends. All terminating decimals are rational numbers because they can be converted to fractions. We can write the decimal 1.2 as 12/10 or as 6/5. We can write 3.25 in a number of ways as a fraction, but one way is 325/100.

Repeating decimals are rational numbers. Repeating decimals are decimals that do not end, but instead eventually repeat digits. It is possible to rewrite all repeating decimals as fractions. A great example of this is .33333. . . We can write that as the fraction 1/3. Try it for yourself - divide 1 by 3! You'll quickly see how the 3 repeats.

Examples of Irrational Numbers

Just as numbers that can be written as one integer divided by another integer are rational numbers, there are also numbers that are irrational numbers. **Irrational numbers** are numbers that cannot be written as one integer divided by another integer. Irrational numbers cannot be written as fractions, cannot be written as terminating decimals, cannot be written as repeating decimals.

There are two types of numbers that are irrational numbers. Roots and radicals are the first type. If a number does not have a perfect root, its root is an irrational number. The square root of 5 is a good example. Written as a decimal, the value of the square root of 5 begins 2.23606. . . Square root of 5 = 2.23606. . .

However, the digits after the decimal will go on and on forever without a repeating pattern. We cannot find a way to write it as a fraction, so it is not a rational number. In contrast, the square root of 25 is a rational number because that has an exact value of 5, which can be written as a fraction. The square root of 25 = 5 = 5/1

The other type of irrational numbers are special numbers like pi and e. These special numbers, like many radicals, go on and on and do not repeat or end. Pi, the ratio of a circle's circumference to its diameter, is the value 3.14159265. . ., which continues without repeating or ending. The value of e is 2.71828. . . and also goes on forever without repeating or ending.

Lesson Summary

The **rational numbers** includes all positive numbers, negative numbers and zero that can be written as a ratio (fraction) of one number over another. Whole numbers, integers, fractions, terminating decimals and repeating decimals are all rational numbers. Roots and radicals, and special numbers, like pi and e, can be written only as decimals that go on forever without a repeating pattern of digits and so are not rational numbers.

Rational Number Notes

Rational Number
Any number that can be written as one integer over another
Includes positive numbers, negative numbers, zero, whole numbers, integers, fractions, terminating decimals, and repeating decimals
Ex: 1/4, 5, -9, 1.8, 1.33333

Learning Outcomes

When you've finished, you should be able to:

- State what numbers are included in rational numbers
- Recall what irrational numbers are

Activities FAQs

What Are Rational Numbers

Examples of Rational Numbers

The following are rational numbers because they are fractions made out of one integer divided by another integer:

1/3, -8/15, 6/31, 8 (or 8/1)

The following are also rational numbers because a decimal that stops (terminates) can be written as a rational number: 0.3, -0.25, 0.8976

The following are rational because every repeating decimal can be written as a fraction.

0.1111..., -0.254254254.., 0.837583758375...

The following are not rational numbers because the decimals go on forever without repeating any pattern:

$7 + \pi$, $-6e$, $\sqrt{(18)}$

Questions

Examine the following numbers. Use a calculator or a spreadsheet to help visualize the result.

1. Do you think π^2 is a rational number or not? Why?
2. Do you think e + e is a rational number or not? Why?
3. Is $\sqrt{(7)}^2$ a rational number or not? Why?
4. Is zero a rational number or not? Why?
5. Can two irrational numbers be combined using addition or subtraction to get a rational number?
6. Can two irrational numbers be combined using multiplication or division to get a rational number?
7. Can two rational numbers be combined using addition or subtraction to get an irrational number?
8. Can two rational numbers be combined using multiplication or division to get an irrational number?

Answers:

1. It is not a rational number. Every multiple of π is irrational.
2. It is not a rational number, since e added to itself is irrational.
3. This is a rational number. The square of a square root is the number inside the square root. So this would be 7, a rational number.
4. Zero is a rational number. It can be written as 0/1.
5. Two irrational numbers cannot be combined with addition or subtraction to get a rational number unless the irrationals cancel each other out as in π + -π.
6. This is a trick question. Two radicals, in some cases, can be multiplied to get a rational number. For example, $\sqrt{(8)} * \sqrt{(2)} = \sqrt{(16)} = 4$.
7. Two rational numbers cannot be combined using addition or subtraction to get an irrational number.
8. Two rational numbers cannot be combined using multiplication or division to get an irrational number.