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Proportional Relationships Between Two Quantities

When two values always maintain the same ratio, forming the same fraction when you divide them, they have a proportional relationship. In this lesson, you can learn about proportional relationships between two quantities.

Proportional Relationships

Do you have a friend that can affect your mood? Whenever she's up, you're up. Whenever she's down, you're down. A proportional relationship between quantities is a lot like that. For example, imagine a beehive that has a lot of bees in it. Each of those bees has six legs. If we take away half the bees, there will only be half as many legs left in the hive as well. There is a proportional relationship between the number of bees and the number of bee legs in that hive.

A **proportional relationship** exists between two values x and y when they can be expressed in the general form $y = kx$, where k is the constant of proportionality.

Our beehive example could be represented by $y = 6x$, where x is the number of bees, y is the number of legs, and k is 6, since each bee has 6 legs. If we double x, then y will also double, and if we divide y by x, then y/x should always be 6.

Another way to say that is if two values are proportional, then dividing them by each other will always produce the same ratio. This ratio will be the constant k, which can be expressed as a fraction or a decimal.

Taste Test for Proportionality

Say you're making lemonade, and you decide to do a taste test to determine if the amount of sugar you need for the perfect lemonade is proportional to the amount of lemon juice. One recipe calls for 5 cups of sugar for the first cup of lemon juice, but does that mean that you should keep adding 5 cups of sugar for each additional cup of lemon juice? In other words, should the ratio of sugar to lemon juice always be the same, no matter how much lemonade you're making?

You're using 2 cups of lemon juice, so if the amounts listed in the recipe are proportional, you'd need to add another 5 cups of sugar for that second cup of lemon juice, so 10 cups of sugar in total. You decide to follow the recipe for the first cup then add just 1 cup of sugar for the second cup of lemon juice, so 6 cups of sugar in total. You mix it all together, and the taste test almost leaves your lips in a permanent pucker. Finally, and after many pucker tests, you realize that, yes, you really need to use 10 cups of sugar for 2 cups of lemon juice. The sugar and the lemon juice are proportional.

You can express the recipe's proportional relationship as $s = 5l$, where s is sugar and l is lemon juice. Notice that your proportionality constant is 5. This just means that for every cup of lemon juice you should need 5 cups of sugar. For example, you'd use 25 cups of sugar for 5 cups of lemon juice.

Bike Test for Proportionality

If two values are proportional, then there's a constant rate at which they both change. Say you're riding a bike with a speedometer and want to know if your stopping distance is proportional to how fast you were going before you applied the brakes. In other words, if you're going twice as fast, will it also take twice as far to stop?

You set up the test in a parking lot. You put a line of tape on the pavement, about in the middle, and ride to the far end. For each run, you hit the brakes right when you come up even with the tape. Here are the results for each of your runs appearing in this table:

Speed	**Stopping Distance**	**Ratio**
5 mph	9 feet	5/9
10 mph	20 feet	10/20
15 mph	37 feet	15/37

So do speed and stopping distance have a proportional relationship? To figure this out, just compare the ratios for each run, using the bike's speed for your numerator and the stopping distance for your denominator. If the two quantities are proportional, their ratio should never change.

Your first ratio is 5/9, or about 0.55. The second one is 10/20, or 0.5. The third one is 15/37, or about 0.41. Whoa, the ratio is definitely changing, which means these two quantities are definitely not proportional.

Finding the Other Value

If you know that two quantities are proportional, then you can find the constant of proportionality by dividing one quantity by the other. If you know the value of one quantity and the constant of proportionality, you can always find the value of the other quantity.

Let's go back to our beehive example, where the number of bees and the number of bee legs follows the proportional relationship $y = 6x$ (each bee has 6 legs).

Say that you know there are 45 bees in the hive. How many bee legs are scuttling around that hive? The number of legs y is always 6 times the number of bees x, so 45 * 6 = 270 legs.

You can work it the other way too. If there are only 180 legs present in the hive, how many bees are there? Just divide the number of legs by the proportionality constant of 6 to get 30 bees.

What if you don't know the constant? All you need is one pair of values. Take the number of bee legs, divide by the number of bees, and (voila!) you'll have your constant. Both 270/45 and 180/30 will give us that constant proportionality of 6.

Lesson Summary

Two quantities have a **proportional relationship** if they can be expressed in the general form $y = kx$, where k is the constant of proportionality. In other words, these quantities always maintain the same ratio. That is, when you divide any pair of the two values, you always get the same number k. Both quantities also change at the same rate: if x doubles, then y will double, and so on. As long as you know a value and the constant of proportionality or a pair of values, you can always find the other value.