The Associative Property: Definition and Examples

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What is the associative property? Learn the associative property definition and see specific associative property examples of addition, and multiplication.

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Principle Properties in Mathematical Operations

In mathematics, there are three basic principles that govern how numbers can be manipulated to solve equations. These properties are:

- The Distributive Property
- The Commutative Property
- The Associative Property

Distributive Property

This property states that a number multiplying a sum or a difference can be distributed among the numbers being added or subtracted without affecting the result.

Algebraically:

{eq}a(b+c) = ab+ac {/eq}

OR

{eq}a(b-c)=ab-ac {/eq}

Commutative Property

This property states that numbers in an addition or multiplication problem can be rearranged without affecting the result.

Algebraically:

{eq}a \times b\times c=b\times c\times a {/eq}

OR

{eq}a+b+c=b+c+a {/eq}

Associative Property

This property states that the order in which numbers are added or multiplied does not affect the outcome of the operation performed. To further explore what the associative property means, we'll go over it in more detail in this lesson, and review a few associative property examples as we go.

What is the Associative Property?

In mathematics, the **associative property** of addition (or multiplication) states that when adding (multiplying) three or more numbers, the sum (product) remains the same regardless of how the numbers are grouped to be added (multiplied).

This definition can also be stated algebraically.

Associative Property of Addition

{eq}(a + b) + c = a + (b+c) {/eq}

Associative Property of Multiplication

{eq}(a\times b)\times c=a\times (b\times c) {/eq}

Associative Property of Addition

When adding three or more numbers, the associative property of addition is a convenient way to simplify the operation without affecting the sum. This simplification can happen because the way in which the numbers are grouped will not affect the sum.

The rule is:

{eq}(a + b) + c = a + (b+c) {/eq}

Now that we've gone over the basics, one might be left wondering what an example of the associative property looks like. Let's go over a few problems to see this property in action.

Example 1 - Numeric:

{eq}16 + 12 + 13 {/eq}

This can be grouped as:

{eq}(16 + 12)+ 13 {/eq}

{eq}=28 + 13 {/eq}

{eq}=41 {/eq}

OR

{eq}16 + (12+ 13) {/eq}

{eq}=16 + 25 {/eq}

{eq}=41 {/eq}

Example 2 - Worded:

The table below shows the tips earned by a worker. The worker wishes to determine the sum of tips earned in this week.

Day	**Tip**

Day	**Tip**
Monday	55
Tuesday	22
Wednesday	28
Thursday	25
Friday	30

To find the sum, the worker needs to perform the operation below.

{eq}55+22+28+25+30 {/eq}

At first glance it may seem tedious to add the numbers in the order they are currently presented. However, the worker can apply the associative property as shown below.

{eq}55+(22+28)+(25+30) {/eq}

In this way, the worker could possibly even perform the operation mentally!

{eq}55+(22+28)+(25+30) {/eq}

{eq}=(55+50)+55 {/eq}

{eq}=105+55 {/eq}

{eq}=160 {/eq}

The important thing to remember is that the sum of these five numbers will still be 160, regardless of how the numbers are grouped.

Another option for grouping is shown below.

{eq}(55+22)+28+(25+30) {/eq}

{eq}=77+(28+55) {/eq}

{eq}=77+83 {/eq}

{eq}=160 {/eq}

Even more ways of grouping can be explored to show that the sum remains unchanged.

Associative Property of Multiplication

The associative property also applies to multiplication. The product of a multiplication problem will remain unchanged regardless of now the numbers are grouped. The rule is:

{eq}(a\times b)\times c=a\times (b\times c) {/eq}

Example 1 - Numeric:

{eq}(2\times 5)\times 8 {/eq}

{eq}=10\times8 {/eq}

{eq}=80 {/eq}

OR

{eq}2\times (5\times 8) {/eq}

{eq}=2\times40 {/eq}

{eq}=80 {/eq}

Example 2 - Worded:

Using the conventions below, the associative property can be applied to the process of determining how many hours are in a year.

24 hours = 1 day
7 days = 1 week
4 weeks = 1 month
12 months = 1 year

To convert, the product below must be found.

{eq}24\times7\times4\times12 {/eq}

Below are two of the different ways the associative property may be applied to this problem to find the product.

Option 1:

{eq}(24\times7)\times(4\times12) {/eq}

{eq}=168\times48 {/eq}

{eq}=8,064 {/eq}

Option 2:

{eq}24\times(7\times4)\times12 {/eq}

{eq}=24\times(28\times12) {/eq}

{eq}=24\times336 {/eq}

{eq}=8,064 {/eq}

Why Not Subtraction or Division

It has been shown how the associative property applies to addition and multiplication. This may raise the question of whether or not the property applies to the other two of the four basic arithmetic operations, subtraction and division.

To determine the answer, examine the numerical examples below.

Example 1 - Subtraction:

This example demonstrates that there is no associative property of subtraction.

$25-12-10$		$25-12-10$
$\Rightarrow (25-12)-10$	NOT EQUAL TO	$\Rightarrow 25-(12-10)$
$\Rightarrow 13-10$		$\Rightarrow 25-2$
$\Rightarrow 3$		$\Rightarrow 23$

A Caution!

Though the associative property definition cannot be widely applied to subtraction in the general sense, it is possible to manipulate subtraction problems by rewriting them as the addition of negative numbers in order to use the associative property. Examine the example below.

{eq}6-3-1 {/eq} can be rewritten as:

{eq}6+(-3)+(-1) {/eq}

The associative property definition may then be applied as:

{eq}[6+(-3)]+(-1) = 6+[(-3)+(-1)] {/eq}

{eq}3+(-1) = 6+-4 {/eq}

{eq}2 = 2 {/eq}

To be sure, examine the contrast when the problem is not rewritten as an addition problem.

{eq}(6-3)-1\neq6-(3-1) {/eq}

{eq}3-1\neq6-2 {/eq}

{eq}2\neq4 {/eq}

Example 2 - Division:

$$130 \div 5 \div 13 \qquad\qquad 130 \div 5 \div 13$$

$$\Rightarrow (130 \div 5) \div 13 \quad \text{NOT EQUAL TO} \quad \Rightarrow 130 \div (5 \div 13)$$

$$\Rightarrow 26 \div 13 \qquad\qquad \Rightarrow 130 \div \frac{5}{13}$$

$$\Rightarrow 2 \qquad\qquad \Rightarrow 130 \times \frac{13}{5}$$

$$\Rightarrow 338$$

This example demonstrates that the associative property does not apply to division.

From these examples, we can see that changing the order in which numbers are subtracted or divided changes the answer. This is why the associative property does not work in these cases.

Lesson Summary

A property common to addition and multiplication is the **associative property** which states that numbers can be grouped in any order without affecting the outcome of the operation. The **associative property of addition** is written algebraically as:

{eq}(a+b)+c=a+(b+c) {/eq}

The **associative property of multiplication** is written as:

{eq}(a \times b) \times c=a \times (b \times c) {/eq}

- Though the associative property does not apply to subtraction, subtraction problems can be manipulated in order to use the property by taking the sum of the negative numbers.
- The associative property does not apply to division.

Video Transcript

I Call Shotgun

How many times have you been out with friends and heard, or yelled, that statement: 'I call shotgun!'? The unquestioned right to sit in the front seat and have room to stretch your legs and de facto control of the radio. At the next stop, someone else may call out for their turn in the front seat.

I bet the thought never crossed your mind that sitting in different seats in the vehicle did not change the makeup of the people that were with you. They are all the same, no matter where they are sitting. Bob is still Bob wherever he is riding, and Jody is still herself, even if she's no longer in control of what music you are listening to.

Who's in Charge of the Math Radio?

So, who's in charge of the math radio? There is a mathematical rule governing that very question! OK, it's not presented in those terms, but it means the same. And the answer is the same as when you are out with your friends. But first, let's do a bit of background.

In mathematics, there are three basic principles for how equations work. They form the backbone of all higher math. These properties are:

• The commutative property

• The associative property

• The distributive property

They all govern different aspects of how you can manipulate and solve mathematical equations correctly.

The Associative Property

The **associative property** is the focus for this lesson. It states that terms in an addition or multiplication problem can be grouped in different ways, and the answer remains the same. In other words, it doesn't matter which terms are in the back seat and which are in the front - the makeup of the equation is the same, just like it was in the car in the introduction.

Let's look a little bit at how that works. If you have an addition problem such as (3 + 6) + 13, you can also write it as 3 + (6 + 13), and, when you solve the problem, the answer will be the same either way: 22. Remember that the parenthesis signify the portion of the problem that should be completed first.

So, the associative property states that it doesn't matter which portion of the problem you do first, the answer will be the same. Again, this only works with addition and multiplication problems, and not if they are mixed. With mixed operations, you need to always follow the order of operations, which is: multiplication and division then addition and subtraction.

However, if your problem contains only addition or only multiplication, you can group them in any way and still get the same answer.

Look at this next example: (2 * 5) * 7 is the same as 2 * (5 * 7). It doesn't matter if you multiply the 2 and 5 first or the 5 and 7 first, the answer is still 70.

Let's try it both ways:

(2 * 5) * 7 = 10 * 7, or 70.

2 * (5 * 7) = 2 * 35, which is also 70.

The associative property can work with subtraction, but only if you convert your subtraction problem to an addition problem. If you remember, subtraction is the opposite of addition. Because of this, you can turn any subtraction problem into an addition problem. This means that 6 - 3 is equal to 6 + (-3).

When you turn your subtraction problem into an addition problem, you can use the associative property to rearrange the groupings.

Why is This Important?

With simple problems such as these, you might be scratching your head, wondering why mathematicians go to all this trouble; why is the associative property so important?

While these examples might seem simplistic, the associative property can be very useful when working with more complicated problems. When mathematicians or scientists or engineers are working with a complex equation, it can help them with the solution if they are sure that they can regroup the terms without negatively affecting the outcome of the problem.

Lesson Summary

The associative property states that in addition and multiplication problems, the grouping of the terms does not matter to the final outcome of the problem. You can group the terms in any order and still obtain the correct answer. The property is also true for subtraction if you convert your subtraction problem to an addition problem and are very careful to keep the negative with the correct number.

Learning Outcomes

After you've completed this lesson, you should be able to:

- Understand what the associative property states and when you can use it
- Remember the order of operations
- Use the associative property with subtraction problems
- Determine when the associative property can be helpful

Activities FAQs

Associative Property Practice Problems

In this activity, you will put what you have learned about the associative property into practice.

Key Terms

- Operations: The four basic operations are addition, subtraction, multiplication, and division.

- Associative property: Application of this property involves combining terms in expression based on order of appearance (only works for addition and multiplication)

Materials Needed

- Paper
- Pencil

Example

Demonstrate the associate property by adding and multiplying the numbers 2, 3 and 5.

Associate Property of Addition

When adding 2, 3 and 5, we perform the operation inside the parentheses first to get:

(2 + 3) + 5 = 5 + 5 = 10

Then:

2 + (3 + 5) = 2 + 8 = 10

Hence, from above:

(2 + 3) + 5 = 2 + (3 + 5)

We have shown the associative property is true for addition of these three numbers.

Associate Property of Multiplication

When multiplying 2, 3 and 5, we perform the operation inside the parentheses first to get:

(2 * 3) * 5 = 6 * 5 = 30

Then:

2 * (3 * 5) = 2 * 15 = 30

Hence, from above:

(2 * 3) * 5 = 2 * (3 * 5)

We have shown the associative property is true for multiplication of these three numbers.

Problems

Practice using the associative property using the procedure outlined above (show your work).

1. Demonstrate the associate property of addition by adding the three numbers 4, -3 and 11 (in that order). *Hint: you can add 4 and -3 as 4 + (-3).*
2. Demonstrate the associate property of multiplication by multiplying the three numbers -4, -3 and -5 (in that order).
3. Does the associate property hold for division? Try it out using the numbers, 8, 4 and 2 (in that order).

Solutions

1. The sum should be 12 for both associations of addition.
2. The product should be -60 for both associations of multiplication.
3. No. Since (8/4)/2 = 2/2 = 1 and 8/(4/2) = 8/2 = 4.