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Notation for Rational Numbers, Fractions & Decimals

How do fractions become decimals? And how do decimals become fractions? In this lesson, we'll learn about the notation of rational numbers, fractions and decimals and learn how they're related.

Rational Numbers

I have one dog and three cats in my house (yes, three). One of the cats seems to think she's a dog. She eats dog food, plays with the dog, and I'm pretty sure she'd try to catch a tennis ball if you threw one.

Like that weird cat, some numbers just aren't comfortable in their own skin. Maybe a number was born as a fraction but feels more like a decimal. Or maybe it's a decimal but believes its destiny was to be a fraction. I can't make my weird cat into a dog, can I? No. I can't. But we can make fractions into decimals and vice versa. Let's learn how.

It all starts with rational numbers. I'm not talking about rational like cats that act like cats. I mean rational as in 'ratio.' A **rational number** is a number that can be written as a fraction or ratio of integers. Remember that a fraction is a ratio. And that's where the name 'rational number' comes from; see the word 'ratio' in there?

2 is a rational number. We could write it as a fraction: 2/1. Likewise, 7/8 is a rational number. And 12 and 82/135 and 300 billion and... Well, let's not mention them all. That would take an infinite amount of time - and, well, infinity is not a rational number. It's not a number at all, really.

Fractions

Again, a rational number can be written as a fraction. Whole integers, like 4 or 11, are simplified fractions. Those two are 4/1 and 11/1.

How would we convert them to decimals? Well, 4 is a whole number, so it doesn't have any decimals. We could say 4 is 4.0, but that's not what we're after here.

But what about 1/5? That's a rational number. How would we convert that to a decimal?

Remember that that line in a fraction is the same as a division symbol. 1 over 5 is the same as 1 divided by 5. So, one method is to divide the numerator by the denominator. 1 divided by 5 is 0.2. And that's our decimal.

There's a second method that's a bit more hands-on. It's like what we'd need to do if we wanted to turn that cat into a dog. No. That's still a really bad idea. Let's stick with fractions and decimals.

To use the second method, you need to find the number that converts the denominator to 10, 100, 1000, or any subsequent power of ten. Why does this work? Think about 0.2. We call that 2 *tenths*. What about 0.02? That's 2 *one-hundredths*. 0.002 is 2 *one-thousandths*. And, well, it goes on from there. Decimals are numbers where, as a fraction, the denominator is a power of ten.

Let's say we have 3/4. How can we make that 4 into a power of ten? 4 * 25 is 100, which is a power of ten.

That gets us to step two: multiply the numerator and denominator by this number. We know 4 * 25 is 100. What's 3 * 25? 75. That gets us 75/100.

And then there's step three: write the numerator as a decimal, moving the decimal point left one place for each 0 in the denominator. Here, we write 75 and move the decimal how many places? There are 2 zeros in 100, so two places. That gives us 0.75, which is 75 one-hundredths. So 3/4 is the same as 0.75. And we didn't have to harm any cats to figure it out.

Decimals

Converting decimals to fractions follows the same logic as this second method. Let's try it with 0.125.

To start, put the decimal in the numerator and 1 in the denominator. That looks like this: the decimal over 1. With our example of 0.125, we'd have 0.125/1.

Next, multiply the top and bottom of the fraction by a power of ten to eliminate the decimal. In other words, count how many numbers are after the decimal. For 1, multiply by 10. For 2, multiply by 100, and so on. 0.125 has 3 numbers after the decimal, so that's 1000.

You can also figure this out by reading the decimal aloud. We have 125 one-thousandths, so we want to multiply the fraction by 1000.

What's 0.125/1 times 1000/1000? 125/1000.

The third and final step is to simplify the fraction. Can we simplify this fraction? The first thing to do is see if 1000 is divisible by 125. Guess what? It is! This fraction simplifies to 1/8. So 0.125 is the same as 1/8. If 0.125 is our cat, in another life it's 1/8.

Lesson Summary

To summarize, fraction, decimals, cats and dogs - it's okay to wish you were something else. We focused on rational numbers, or numbers that are a fraction, or ratio, of integers.

We started by turning fractions into decimals. You can just divide the numerator by the denominator. The second method involves three steps. First, find the number that converts the denominator to a power of ten, like 10, 100 or 1000. Then multiply both parts by this number. Finally, take the numerator and write it as a decimal, moving the decimal point left one place for each 0 in the denominator.

Then, we turned decimals into fractions. To do this, we put the decimal in the numerator and 1 in the denominator. Then we multiply both parts by a power of ten that eliminates the decimal. Finally, we simplify the fraction.

And now I have to go. I think I hear my cat barking.

Learning Outcomes

By working through this lesson to its completion, you could prepare to:

- Define a rational number
- Write rational numbers as fractions
- Convert a fraction to a decimal
- Change a decimal into a fraction