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How to Solve Problems with Time

It's amazing how much time can be saved when people work together. In this lesson, we'll practice figuring out just how much time we're talking about as we solve problems involving time.

Let's Work Together

Many hands make light work. This is an old saying that means big jobs get easier when more people help. Imagine you're painting a house. That's a big job for one person. But what if you had 5 helpers? It'll go much faster.

There's another saying: *Many hands make light pizza.* This one notes how those same people who help you paint the house also eat the pizza that would've lasted you for days.

If we don't worry about the pizza, we can focus on how working together saves time. Let's consider a simple example. Allison is working on a jigsaw puzzle. Alone, it would take her 3 hours to complete. If her friend Beth did the same puzzle, it would take her 6 hours to complete. What if they worked together?

We wouldn't say it's 3 + 6. Those numbers represent the total time each person takes when working alone. And, obviously, it wouldn't take Allison and Beth nine hours to do the puzzle together. Well, it would if they're really bad at teamwork.

Work Rate Formula

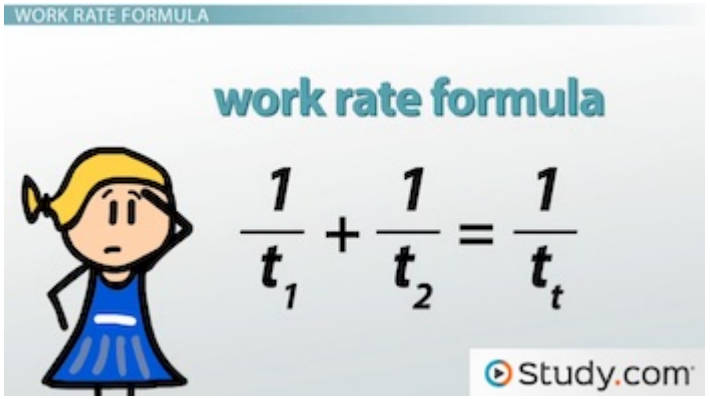
We need a formula. The formula needs to be built on the rate each person takes.

Allison's rate is 1/3. It takes her 3 hours to complete the puzzle, so she completes 1/3 of the puzzle in an hour. Beth's rate is 1/6. Let's call x the time it would take them to work together. By the same logic, their rate when working together is $1/x$.

So we can state that $1/3 + 1/6 = 1/x$. In other words, this is our **work rate formula**:

In this formula, T1 is the time it takes person 1, T2 is the time it takes person 2, and Tt is the time it takes them when working together.

This formula works no matter how many people are involved. You can just add a 1 / T3, T4, etc. And the numerator is always 1 to represent the one task.



Work rate formula

Solving for Time

So we have our equation for Allison and Beth: *1/3 + 1/6 = 1/x*. How do we solve it? All those pesky denominators make it a little tricky. We can get rid of the fractions by finding the least common multiple. Here, it's 6*x*. So let's multiply both sides by 6*x*.

That gets us 6*x*/3 + 6*x*/6 = 6*x*/*x*. This simplifies to 2*x* + *x* = 6. 2*x* + *x* is 3*x*. And 6/3 is 2. So *x* = 2

That means that if Allison and Beth join forces, this dynamic duo of puzzle building can complete the job in just 2 hours, as opposed to 3 hours for Allison or 6 hours for Beth.

More Than Two People

Let's try one with multiple people. Actually, let's do one that doesn't even involve people. *A garden hose can fill a swimming pool with water in 15 hours. A larger hose can do the job in 10 hours. A fire hose can do the job in 6 hours. If we use all three hoses together, how long will it take to fill the pool?*

Sometimes, you just need to go for a swim and you aren't willing to wait around for 15 hours. Besides, if you happen to have a fire hose at the ready, why not use it?

This problem doesn't involve people, but the concept of how we solve it is the same. The garden hose fills the pool at a rate of 1/15. The larger hose does it at 1/10. And rate of the fire hose is 1/6. So our equation looks like this: *1/15 + 1/10 + 1/6 = 1/x*.

There are three hoses working together, but we still just find the least common multiple. Here, it's 30*x*. So let's multiply both sides by 30*x*.

We get 30*x*/15 + 30*x*/10 + 30*x*/6 = 30*x*/*x*. That simplifies to 2*x* + 3*x* + 5*x* = 30. 2*x* + 3*x* + 5*x* is 10*x*. And 30 divided by 10 is 3. So *x* = 3. If we use all three hoses, we'll get the job done in just 3 hours! That's hose-based teamwork!

Solving for Individuals

Sometimes we know the time it takes when people work together, but not the time it takes when they work alone. Here's another problem: *Kyle can mow a lawn 1.25 times faster than Steve. If they work together, the job takes 5 hours. How long would it take Steve to mow the lawn by himself?*

First of all, that must be one big lawn. 5 hours? Are they cutting it with nail clippers? Anyway, we start this one by defining our variables. Let's say Kyle takes *x* hours to mow the lawn by himself. Steve is 1.25 times slower than Kyle, so it takes him 1.25*x*.

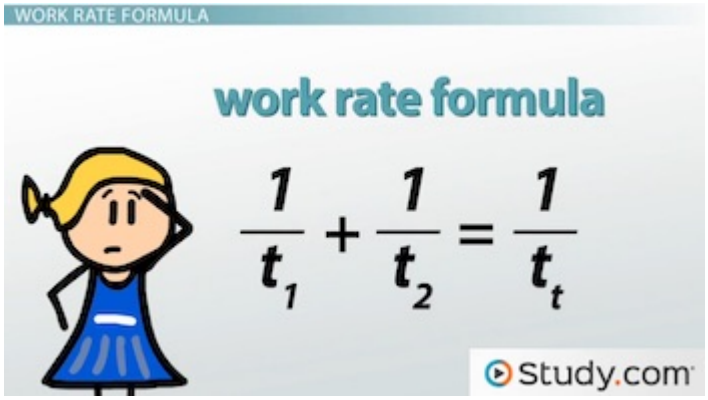
Now we can set up our equation like this: *1/*x + *1/1.25*x = *1/5*. So it's the same equation, we're just using the variables differently. To solve this, we again find the least common multiple. That's 5*x*. Let's multiply both sides by 5*x*.

We get 5*x*/*x* + 5*x*/1.25*x* = 5*x*/5. That simplifies to 5 + 5/1.25 = *x*. 5/1.25 simplifies to 4. So 5 + 4 = *x*. So *x* = 9. What is *x* again? That's how long Kyle takes to mow the lawn. We want to know about Steve. Steve is 1.25 slower. So what's 1.25 * 9? 11.25. So it takes Steve 11 and a quarter hours to do the job by himself. Somebody get this guy a riding mower.

Lesson Summary

To summarize, we learned about teamwork. Specifically, we focused on the work rate formula. This formula helps us understand the relationship between the time it takes different individuals to perform a task and the time it takes if they work together.

We can write the formula like this:



Work rate formula

We can have as many 1 / T-whatevers we need, depending on how many people are involved in the problem.

Learning Outcomes

After you've completed this lesson, you should be able to:

- Identify the work rate formula
- Use this formula to solve problems involving the time it takes individuals to perform a task when working together