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Recursive Sequence: Formula & Overview

One of the most famous recursive sequences is the Fibonacci sequence. In this lesson, learn what makes the Fibonacci sequence a recursive sequence, and discover how you can recognize and create your own.

Recursive Sequence: Definition

A **recursive sequence** is a sequence where the next terms use the previous terms. Let's take a look at the famous Fibonacci sequence to see what that means.

This famous sequence is recursive because each term after the second term is the sum of the previous two terms. Our first two terms are 1 and 1. The third term is the previous two terms added together, or 1 + 1 = 2. The next term is the addition of the two prior terms, or 1 + 2 = 3. And this pattern continues indefinitely.

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$$

The famous Fibonacci sequence.

Let's see what this looks like in formula form.

Recursive Sequence: Formulas

The Fibonacci sequence will look like this in formula form:

$$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = a_{n-2} + a_{n-1} \end{cases}$$

The famous Fibonacci sequence in recursive sequence formula form.

Each term is labeled as the lowercase letter *a* with a subscript denoting which number in the sequence the term is. Lower case a_1 is the first number in the sequence. Lower case a_2 is the second number in the sequence and so on.

Recursive sequences do not have one common formula. You will have one formula for each unique type of recursive sequence.

There is one thing that recursive formulas will have in common, though: that is that each formula will have a sub *n* equal to a formula involving a sub some previous term. It doesn't have to be the *n* - 1 term or the previous term. It could be the third previous term. It could be any previous term. But the formula has to have a previous term in it to make it recursive.

Using the Formulas

Using formulas requires you to know the starting terms and to plug these terms into the formula to calculate the next terms. Let's see how this works out with the following recursive formula appearing here:

$$\begin{cases} a_1 = 0 \\ a_2 = 1 \\ a_3 = 3 \\ a_n = a_{n-1} + (n-1) \end{cases}$$

A recursive sequence formula.

This formula is a bit tricky because we have *n* - 1 in our formula. What does this mean? We know what the $a_{(n-1)}$ means. It means the previous term.

So, what could *n* - 1 possibly mean? Hmmm. What would happen if we simply plugged in my value for *n* and evaluated it like that? Yes! That is exactly what the *n* - 1 means. It wants you to take your current position in the sequence and subtract 1 from it.

If you are figuring out the fourth term in the sequence, it is asking you to take 4 and subtract 1 from it to get 3. This means we are adding 3 to the previous term, the third term. Our fourth term is then 6. Looking at our first three terms, we can also ask ourselves if these numbers follow the same pattern. The second number in the sequence is a 1. Our position at this point is 2, and if we subtract 1 from it we get 1. If we add this to our previous term, we get 1. Does this match our second term? Yes, it does, actually. Looking at the third term and asking the same question, we see that it also works with our pattern.

Here is a simpler formula to try out:

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + 3 \end{cases}$$

A simpler recursive sequence formula.

This formula is telling us to add 3 to our previous term. Our first term is 1, so our next term will be 1 + 3 = 4. The third term will be 4 + 3 = 7. Our sequence will look like this:

Recursive sequences all have patterns that use a previous term in the sequence. The pattern may add previous terms together or may add a constant to previous terms. It could be anything, but a pattern will emerge that will involve previous terms.

$$1, 4, 7, 10, 13, \ldots$$

Our simple recursive sequence.

Finding Patterns

We can find patterns to sequences to determine whether a sequence is a recursive sequence or not. Some sequences may take longer to figure out than others, but with practice, your number sense will become finely tuned to sequences.

Let's look at a couple of sequences to see if we can spot patterns:

$$Sequence\ 1: 1, 3, 9, 27, 81, 243$$
$$Sequence\ 2: 31, 28, 31, 30, 31$$

Two sequences. Which is recursive and which is not?

Looking at both sequences, it looks like the first one has a pattern to it. The second one looks like it might have a pattern because every other number is 31. Looking more carefully at the first sequence, I see that each successive number is the previous term multiplied by 3.

The second sequence, though, doesn't seem to have any kind of pattern that is related to previous terms. The second term is 3 less than the first. The third term is 3 more than the second. But the fourth term is only 1 less than the third. There doesn't seem to be a pattern.

Oh, but wait, these numbers do look somewhat familiar. What do I know that has 31 every other number? Isn't it the number of days in a month? January has 31, February has 28, and March has 31, etc. Hmmm. That's not a recursive sequence, though, because it doesn't use previous terms. So I can say that Sequence 1 is recursive while Sequence 2 is not.

Lesson Summary

All right, let's take a moment to review. In this lesson, we learned that **recursive sequences** are sequences or strings of numbers that are formed by using previous terms to come up with the next terms. The Fibonacci sequence is a famous one (probably the most famous, if we're being honest). Each successive term in that sequence is the addition of the two previous terms. Sequences can have formulas to show you the pattern, or they can have a pattern for you to figure out. Not all sequences are recursive.