



How to Solve Complex Fractions

Did you know fractions can nest within fractions? In this lesson, we'll learn about complex fractions and look at two different methods for solving them.

Getting Complex

Have you ever seen Russian nesting dolls? You know, those wooden figures that you can open up and there's a smaller figure inside. Then you open that one and there's an even smaller figure inside, and then another, and another, and so on.

When you look at the outermost doll, it's just an ordinary figure. But when you open it, you realize that it has these cool layers just below the surface.

Complex Fractions

Complex fractions are just like nesting dolls. They're like regular fractions but then, whoa, there's another fraction inside that fraction.

A **complex fraction** is a fraction that contains fractions in the numerator, denominator or both.

Here's an example:

$$(3/4)/(1/2)$$

Our *numerator*, the number on top, is $3/4$. And our *denominator*, the number on the bottom, is $1/2$.

Method One

When you put a nesting doll back together, you start with the innermost doll and work outwards. You solve complex fractions in much the same way.

There's usually only one way to put a nesting doll back together. But, there are actually two ways to solve a complex fraction.

The first way involves handling the top and bottom separately.

Step 1: *Simplify the numerator.* Here's an example:

$$(3/4 + 2/5)/(1/2 - 1/6)$$

The top is $3/4 + 2/5$. We need to make the denominators the same by finding the *least common denominator*, or the smallest shared multiple of the denominators. Here, that's 20. To make the denominator of $3/4$ 20, we multiply it by $5/5$, which gets us $15/20$. We multiply $2/5$ by $4/4$ to get $8/20$. $15/20 + 8/20$ is $23/20$.

Now for Step 2: *Simplify the denominator.* That's $1/2 - 1/6$. What's the least common denominator? 6. So, $1/6$ is fine. We multiply $1/2$ by $3/3$ to get $3/6$. $3/6 - 1/6$ is $2/6$, or $1/3$.

Now we have $(23/20)/(1/3)$, and we're ready for the final step:

Step 3: *Divide the Fractions.* $(23/20)/(1/3)$ is really $23/20$ divided by $1/3$. That's all the line in the middle means. To divide fractions, we first flip the second one, giving us its reciprocal. So, $1/3$ becomes $3/1$. Then, we multiply them together. So, $23/20 * 3/1$ is $69/20$. That can be simplified to 3 and $9/20$. That's almost as pretty as a nesting doll!

Method Two

That's a perfectly fine way to handle complex fractions. But, maybe you get a little impatient dealing with the numerator, then the denominator. Maybe you just want to throw all those dolls together in one big flurry.

That's kind of like the second method, which just has two steps.

Step 1: *Find the least common denominator for all fractions.* Let's try an example with variables this time: $(x + 1/x)/(1 + 4/2x)$. Remember that x is the same as $x/1$, and 1 is the same as x/x or $1/1$. Our least common denominator here is $2x$.

Now, Step 2: *Multiply all fractions by the common denominator.* $x * 2x$ is $2x^2$. $1/x * 2x$ is $2x/x$, which is 2. So, our new numerator is $2x^2 + 2$.

$1 * 2x$ is $2x$.

$4/2x * 2x$ is $8x/2x$, or just 4. So, our new denominator is $2x + 4$.

Now we have $(2x^2 + 2)/(2x + 4)$. We can pull a 2 out of both parts, which cancels out, leaving us with $(x^2 + 1)/(x + 2)$.

That's not the prettiest nesting doll you've ever seen, but it is simplified.

So, which method is better? It's really more of a personal preference. Sometimes, one method will be simpler than another. But, most people just find that they like using one method more than another. We could have solved both examples we looked at using either method and gotten the same answers.

Lesson Summary

In summary, a **complex fraction** is just a fraction that contains fractions in the numerator, denominator or both.

There are two methods of solving them. The first one treats the numerator and denominator separately. You simplify each by finding the least common denominator. Then, you divide the fractions, which involves multiplying the numerator by the reciprocal of the denominator.

The second method involves finding the least common denominator for all fractions. Then, you multiply all terms by that common denominator, and simplify as far as you can.

In the end, you'll have taken a bunch of separate fractions and turned them into a much simpler, single fraction, just like putting nesting dolls back together.

Learning Outcomes

Upon completing this lesson, you will be able to:

- Define complex fraction
- Explain two methods to solve problems involving complex fractions