Proportional Relationships in Multistep Ratio & Percent Problems

This lesson introduces the concepts of ratios, rates, and proportions. It will show you how to use rates and ratios to develop proportional relationships. Using proportions to solve problems is very common in mathematics, because oftentimes many rates and ratios are constant.

Ratios

Suppose Mrs. Hernandez owns two cats and three dogs. We could express this as a **ratio**, a comparison of two values, objects, or groups. There are many ways to write ratios, but let's use the three most common ones in this case:

- Use the word 'to'. We can say that Mrs. Hernandez has a cat to dog ratio of 2 to 3. Notice that the order of the groups and numbers matters. Since cats comes before dogs, 2 must come before 3.
- Use a colon. We can say that Mrs. Hernandez has a cat to dog ratio of 2:3. Again, the order of the numbers in the ratio must match the order of the categories.
- Use a fraction. We can say that Mrs. Hernandez has a cat to dog ratio of 2/3. This form of ratios is important when comparing ratios and finding proportions.

Reducing Ratios

Suppose that Mr. Brown has a cat to dog ratio of 5:10. This means he owns 5 cats and 10 dogs. Another way to look think about this is that for every one cat Mr. Brown owns, he owns two dogs.

This means that the ratio 5:10 is the same as 1:2. Just like with fractions, ratios can be reduced. Just find the **greatest common factor**, in other words, the largest number that divides evenly into both values used in the ratio. Then divide each term in the ratio by the greatest common factor (GCF).

Suppose that Mr. Thomas has a cat to dog ratio of 14:35. How could this ratio be reduced?

- Find the largest number that divides evenly into 14 and 35: GCF = 7.

- Divide 14 and 35 each by 7
- 14 divided by 7 = 2
- 35 divided by 7 = 5
- 14:35 = 2:5
- Mr. Thomas has a cat to dog ratio of 2:5

Rates

A **rate** is a special type of ratio where the two terms are different units. A common rate is speed, which could be written in say, miles/hour or kilometers/hour. Typically, rates are written by expressing the ratio as a fraction. An example of a rate would be 50 miles/2 hours.

A **unit rate** is a rate that has a denominator of one. Therefore, 50 miles/2 hours is not a unit rate, but 25 miles/1 hour is. Though the number 1 is usually omitted in unit rates. This means we would write the rate as 25 miles/hour.

Ahmed drove 120 kilometers in 4 hours. What was his unit rate in kilometers per hour? Well 120 kilometers/4 hours is the same as 30 kilometers/1 hour. Therefore, the unit rate is 30 kilometers/hour.

Proportions

A **proportion** is a comparison of two rates or ratios. Proportions are used to find relationships when a value is unknown. Try this problem. If a plane can fly 350 miles in two hours, how many miles can it fly in three hours?

The most common way to set up a proportion is to write two equivalent ratios in fraction form using a variable for the missing value. Be careful that the units align in the numerators and denominators.

$$\frac{350\ \textit{miles}}{2\ \textit{hours}} = \frac{x\ \textit{miles}}{3\ \textit{hours}}$$

Solve for *x* to find the miles that this plane can fly in three hours. Use the cross-multiplication method to solve for *x*.

- 2 * x = 3 * 350
- 2x = 1050
- x = 525
- The plane can fly 525 miles in 3 hours.

The Percent Proportion

A common proportion that is used in mathematics is the **percent proportion**, which establishes a relationship between a percent, a part, and a whole. The equation is percent / 100 = part / whole.

For example, assume that we want to find 22% of 60. The percent is 22 / 100, and 60 is the whole (the word 'of' tells us the whole). We can set up the percent proportion: 22 / 100 = x / 60. Then use the cross-multiplying method to solve for the part.

- 100 * x = 22 * 60
- 100x = 1320
- x = 13.2

This percent proportion works when you are finding the part (as in this example), the whole, or the percent.

Try this problem. What is 12% of 703?

- percent /100 = part / whole
- 12 / 100 = x / 703
- 100 * x = 12 * 703
- 100x = 8436
- x = 84.36

91 is 34% of what number?

Here, the 'of what number' tells us that the whole is missing. We will put the variable in the whole position.

- percent /100 = part / whole

- 34 / 100 = 91 / *x*
- 34 * *x* = 91 * 100
- 34*x* = 9100
- *x* = 267.65

13 is what percent of 222?

- Here, the 'what percent' tells us that the percent is missing. We will put the variable in the percent position. 13 is the part and 222 is the whole.
- percent / 100 = part / whole
- *x* / 100 = 13 / 222
- 222 * *x* = 13 * 100
- 222*x* = 1300
- *x* = 5.86%

Lesson Summary

Ratios, rates, and proportions are used in solving mathematics problems. A **ratio** is a comparison of two values, objects, or groups. It can be expressed using the word 'to', a colon, or as a fraction. To reduce a ratio, find the **greatest common factor** (the largest number that divides evenly into both values) and then divide both terms by that number.

A **rate** is a specific type of ratio that has two different units. A **unit rate** is a rate with a denominator of one. We can use ratios and rates in mathematics to set up **proportions**, an equation that shows the relationship between two ratios or rates. If one value in a proportion is missing, we can solve the proportion for the missing value using algebraic techniques.

This is very helpful in **percent proportion** problems, which establishes a relationship between a percent, a part, and a whole.
percent / 100 = part / whole