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Constructing Proportions to Solve Real World Problems

In this lesson, we will concentrate on proportions. Through definitions and examples, we will learn how to construct proportions and how to use proportions to solve real-world problems.

Proportions

Suppose you recently got a job, and you just received your first paycheck. You worked 22 hours, and your paycheck is $223. Next week, you are scheduled to work 31 hours, and you are wondering how much your paycheck for that week will be. The answer to this question can be found using proportions.

A **proportion** is an equation that sets two ratios equal to each other, with a **ratio** being a fraction comparing two different values. Let's take a look at how to find an unknown in a proportion.

Cross Multiplication

In a proportion, if one of the numbers is unknown, we can use a process called cross multiplication to solve for that unknown. To perform **cross multiplication**, we multiply the numerator of the left-hand ratio by the denominator of the right-hand ratio, and we multiply the denominator of the left-hand ratio by the numerator of the right-hand ratio. Then we set the two products equal to each other and solve for the unknown.

For example, suppose we have the following proportion:

$$\frac{2}{3} = \frac{6}{x}$$

Let's use cross multiplication to solve for the unknown quantity in the proportion:

$$\frac{2}{3} = \frac{6}{x} \; Cross\, multiply$$

$$2x = 3 \cdot 6 \; Simplify$$
$$2x = 18 \; Divide\, both\, sides\, by\, 2$$
$$x = 9$$

As you can see, we have 2*x* = 3 * 6, which, simplified, gives us 2*x* = 18. We then divide both sides by 2 and we get *x* = 9. Simple, right?

Solving Real-World Problems

Let's look back at our paycheck example. We can use proportions to solve this problem, but first we have to construct the proportion that represents this problem. To construct a proportion, we just need to set up two ratios comparing the same quantities and then set them equal. In our example, we have the number of hours you work and the amount of your paycheck as our quantities.

We know that when you work 22 hours, you make $223. This information is enough to set up one ratio comparing the number of hours worked to the amount of money made:

$$\frac{22\; hours}{223\; dollars}$$

The rest of the information that we have is that next week you will be working 31 hours, and you want to know how much money you'll make for that many hours. The unknown here is the amount of money you'll make for 31 hours of work. Let's call the unknown *x* and set up another ratio comparing these two quantities:

$$\frac{31\; hours}{x\; dollars}$$

We now have two ratios comparing number of hours worked to the amount of money made. All we have to do is set them equal, and we have our proportion:

$$\frac{22\ hours}{223\ dollars} = \frac{31\ hours}{x\ dollars}$$

It's important to note that you want your quantities in the numerator and denominator to be consistent in both ratios of the proportion. We see that we did this with our example, since hours is in the numerator in both ratios and dollars is in the denominator in both ratios. Lastly, we can use the proportion to solve for the unknown:

$$\frac{22\ hours}{223\ dollars} = \frac{31\ hours}{x\ dollars}$$

$$\frac{22}{223} = \frac{31}{x}\ \ Cross\ Multiply$$

$$22x = 223 \cdot 31\ \ Simpify$$
$$22x = 6913\ \ Divide\ both\ sides\ by\ 22$$
$$x = 314.23$$

We see that you will get a paycheck of $314.22 for your 31 hours of work next week. Aren't these proportions handy when it comes to real-world applications? Let's look at one more example.

Quick Example

Suppose you are baking cookies for an upcoming event. You have a lot of cookies to make, and you can make 120 cookies in 2 hours. You're going to be able to bake for 7 hours, and you want to know how many cookies you'll be able to make in that amount of time if you continue to bake at this rate.

Once again, we have a real-world problem that we can use proportions to solve. First, we need to construct our proportion, so we need two ratios. We know you can make 120 cookies in 2 hours. This gives us two quantities to put together in a ratio:

$$\frac{120\ cookies}{2\ hours}$$

We want to know how many cookies you will be able to make in 7 hours, so the unknown quantity is the number of cookies. Let's call it *c*. Once again, we can set up a ratio comparing the number of cookies to hours:

$$\frac{c\ cookies}{7\ hours}$$

We now have our two ratios, so we set them equal and use the proportion to solve for the unknown. This gives us 120 / 2 = *c* / 7. We cross multiply and that gives 120 * 7 = 2*c*. Simplifying, we get 840 = 2*c*. Then we divide both sides by 2 and we get 420.

So, therefore, you're going to be able to make 420 cookies. Talk about sweetness overload!

Lesson Summary

All right, let's now take a moment or two to review. In this lesson, we learned that a **proportion** is an equation in which two ratios are set equal to one another, and a **ratio** is a fraction comparing two quantities. We can use proportions to solve real-world problems by using the following steps:

1. Use the information in the problem to set up two ratios comparing the same quantities. One of your ratios will contain the unknown.
2. Set the ratios equal creating a proportion.
3. Use cross multiplication to solve for the unknown in the proportion.

Proportions are great tools to add to our mathematical toolbox since they can be used in many instances of our everyday lives!