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Course  
Notes

专业课笔记

Professional Course Notes

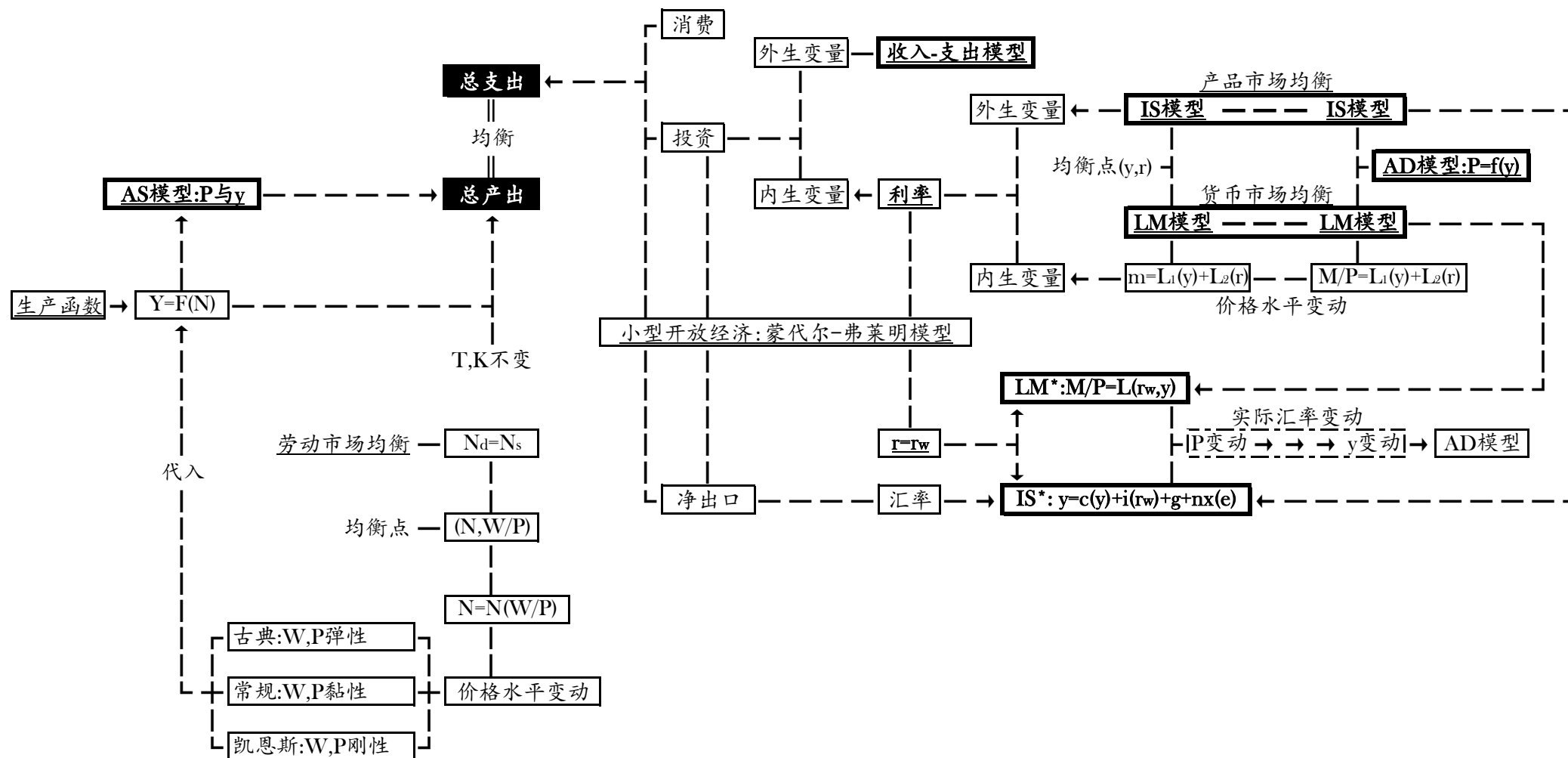
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## 自然选择-专业课笔记-内容清单

第一版  
第二次分享

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# 宏观经济的基本指标及其衡量(2-1)

## 宏观经济学的特点

### 研究对象

研究社会总体的经济行为及其后果

#### 研究国民收入

如何使国民收入稳定以较合适的速度增长

经济波动⇒就业与失业问题

价格水平⇒通胀问题

经济增长问题

### 重要指标

#### 国民收入及其增长率

国内生产总值; 国内生产净值;

国民生产总值; 国民生产净值;

国民收入(狭义); 个人收入;

个人可支配收入

失业率, 物价水平及其变动

### 微观经济学

以价格为中心⇒价格理论

### 宏观经济学

以国民收入为中心⇒收入理论

## 基本理论框架

三大部门: 家庭, 企业, 政府

三大市场: 产品, 货币, 劳动

国民收入决定的收入-支出模型

产品市场供求关系⇒国民收入决定

国民收入决定的IS-LM模型

加入货币市场供求关系⇒国民收入决定

国民收入决定的AD-AS模型

再加入劳动市场供求关系⇒国民收入决定

## 与微观经济学的异同

### 相同点

有着相同的供求曲线形状, 交点决定价格和产量

经济政策都是政府运用一定经济手段引导和规范微观

经济主体行为, 使之趋向有利于改善社会的经济福利

### 不同点

#### 微观经济学

#### 宏观经济学

个体经济活动参与者行为及其后果

社会总体的经济行为及其后果

单个商品的价格和成交量

整个社会的价格水平和产出水平

经济政策目标

解决资源优化配置的问题

解决资源利用的问题

## 国内生产总值GDP及其核算方法

### 国内生产总值概念

国民收入核算(核心指标GDP)

研究定义和计量总产出or总收入的方法

最终产品

中间商品

在一定时期内生产的并由其最后使用者购买的产品和劳务

用于再出售而供生产别种产品用的产品

### 国内生产总值

经济社会(一国或一地区)在一定时期内运用生产要素所生产的全部

最终产品(物品和劳务)的市场价值

- 1.市场价值; 2.最终产品; 3.生产(非销售);
- 4.流量(非存量); 5.地域概念; 6.市场活动

产出=收入=支出

### 国民生产总值

某国国民所拥有的全部生产要素在一定时期内所生产的最终产品的市场价值

## GDP核算方法

### 支出法 $GDP=C+I+G+(X-M)$

核算在一定时期内整个社会购买最终产品的总支出

#### 消费支出

包括购买耐用消费品、非耐用消费品和劳务的支出

投资: 增加或更换资本资产的支出

折旧: 资本物品由于损耗造成的价值减少

生产中资本物品的有形磨损&资本老化的无形磨损

固定资产投资+存货投资

总投资=净投资+重置投资

政府购买(政府购买=政府支出)

各级政府购买物品和劳务的支出

转移支付&公债利息=政府支出, 不计入GDP

#### 净出口

## 名义GDP和实际GDP

$\Delta GDP = \Delta Q \times \Delta P$

### 名义GDP(货币GDP)

用生产物品和劳务当年价格计算的全部最终产品的市场价值

### 实际GDP

用从前某一年作为基期价格计算出来的全部最终产品的市场价值

GDP折算指数=名义GDP/实际GDP

名义GDP不反映实际产出的变动

不作说明, 本书GDP均为实际  
小写字母表示实际量

## 收入法

用要素收入(企业生产成本)核算GDP

包括间接税+折旧+未分配利润等

### 包含项目

- 1.工资+利息+租金等生产要素的报酬
- 2.非公司企业主收入
- 3.公司税前利润  
公司所得税+社会保险税+div.+R/E
- 4.企业转移支付及企业间接税  
间接税: 货物税+销售税+周转税
- 5.资本折旧

与支出法计算理论相等

实际中加统计误差

## 加总法(注意事项)

宏观分析中有些总量变化可以从微观分析的个量直接加总

有时微观经济学一些个体变量可以加总, 但达不到研究社会经济行为的目的

有时微观经济个体的行为根本不能直接加总

## 宏观经济的基本指标及其衡量(2-2)

### 国民收入的其他衡量指标

#### 国民收入衡量

##### 国内生产总值GDP

未扣除当期的资本耗费即折旧

若扣除→国内生产净值

##### 国内生产净值NDP

$NDP = GDP - \text{资本折旧}$

扣除资本消耗或者说重置投资部分

##### 国民收入NI

$NI = NDP - \text{间接税} - \text{企业转移支付} + \text{政府补助金}$

此处指按生产要素报酬计算的国民收入

一国生产要素在一定时期内提供生产性服务  
所得到报酬意义上的国民收入

工资+利息+租金+利润

##### 个人收入PI

$PI = NI - \text{公司未分配利润} - \text{公司所得税}$   
 $- \text{社会保险税(费)} + \text{政府对个人的转移支付}$

##### 个人可支配收入DPI

$DPI = PI - \text{个人所得税}$

#### 几点说明

$GNP = GDP + \text{国外要素所得净额}$

国外要素所得净额(国外要素净支付)

=本国生产要素在他国获得收入 -

本国付给外国生产要素在本国获得的收入

本国在国外要素所得的净额, 而非外国的要素

国民收入并不会都给个人

减去公司留存收益和社保缴款

净利息=企业付给家庭部门的利息 -

家庭部门付给企业的利息(消费信贷)

#### GDP的意义上

$\text{总收入} = \text{总支出} = \text{总产量}$

收入  $\Rightarrow NI = \text{工资} + \text{利息} + \text{租金} + \text{利润} = \text{消费} + \text{储蓄}$

支出  $\Rightarrow GDP = C + I + G + (X - M)$

产量  $\Rightarrow Y$

### 国民收入的基本公式

#### 两部门经济(家庭+企业)

无企业间接税, 无折旧

$\Rightarrow Y = GDP = NDP = NI$

定义:  $GDP = C + I$ ;  $NI = C + S$

$\Rightarrow I = S$

恒等式由储蓄和投资的定义得出

支出角度  $GDP (\Delta \text{库存} = \text{存货投资})$

$\hookrightarrow GDP = C + I \quad NI = C + S \quad \uparrow$

收入角度  $NI (\text{按收入去向计算})$

#### 三部门经济(家庭+企业+政府)

政府收入=税收

政府支出=政府购买+转移支付

支出角度  $Y = GDP = C + I + G$

转移支付已包含在  $C + I$

收入角度  $Y = NI = C + S + T$

政府净收入  $T = T_0 - t_r$

= 税收 - 转移支付

$I + G = S + T$ , or  $I = S + (T - G)$

投资=私人储蓄+政府储蓄

$T - G = \text{政府储蓄}$

#### 四部门经济(家庭+企业+政府+国外)

支出角度  $Y = GDP = C + I + G + (X - M)$

收入角度  $Y = NI = C + S + T + K_r$

$K_r$ : 本国居民对外国的转移支付

$I + G + (X - M) = S + T + K_r$

$I = S + (T - G) + (M - X + K_r)$

投资=储蓄(私人+政府+海外)

加入企业间接税和折旧, 等式仍成立

$Y = GDP$ : 包含折旧的总投资和总储蓄

$Y = NDP$ : 不含折旧的净投资和净储蓄

$Y = NI$ : 两边同减间接税的量值

#### 名义值增加部分

目前数量  $\times (\text{目前价格} - \text{基期价格})$

扣除后为实际增量  $\rightarrow GDP \text{ 折算指数用帕氏}$

基期交易量易获得  $\rightarrow CPI \text{ 用拉氏}$

### 失业与物价水平的衡量

#### 失业的衡量

美国劳工统计局人口分类

未满16岁+现役军人+精神病人和劳教人员

非劳动力: 操持家务者+在校学习者+年老退休者+病残者

劳动力: 就业者+失业者

劳动力=就业人数+失业人数

失业率=(失业人数/劳动力数量) $\times 100\%$

劳动力参与率=(劳动力数量/成年人口总数) $\times 100\%$

失业率的局限性

不能明确区分全日制工作和打短工

存在劳动者未能充分利用其技能的问题

劳动者或由于各种原因虚报、谎报就业状况以谋取好处

经济增长与失业率变动关系

注意是劳动密集型还是资本密集型

#### 物价水平的衡量

物价总水平(一般物价水平/价格指数)

所有商品和劳务交易价格总额的加权平均值

#### 3种价格指数

消费价格指数CPI(生活费用价格指数)

通过计算城市居民日常消费的生活用品和劳务的  
价格水平变动而得到的指数

$CPI = (\text{现期价格指数} / \text{基期价格指数}) \times 100\%$

生产者价格指数PPI

通过计算生产者在生产过程中所有阶段上所获得  
的产品的价格水平变动而得到的指数

GDP的价格指数: GDP折算指数

#### GDP折算指数与CPI的区别

全部最终产品与劳务	消费品, 不含生产性物品
只包括国内产品价格	进口产品价格影响CPI
各种产品权重变动	各种产品权重固定

#### 几点说明

CPI计算未把居民所购消费品全部计入

只挑选对居民生活影响较大的

CPI只能大致反映居民生活成本(未考虑产品性能&质量)

价格指数统计方法的区别(交易量是基期/当期)

基期加权价格指数/拉氏价格指数/拉斯拜尔公式

$L_P = \Sigma P_1 Q_0 / \Sigma P_0 Q_0$  夸大成本上升(未考虑替代品)

计算期加权价格指数/帕氏价格指数/帕煦公式

$P_P = \Sigma P_1 Q_1 / \Sigma P_0 Q_1$  低估生活质量的下降(替代品)

# 国民收入的决定：收入-支出模型(2-1)

## 均衡产出

### 最简单的经济关系

#### 假设

##### 二部门经济

只有家户部门和企业部门  
消费和储蓄都发生在家户部门  
生产和投资都发生在企业部门  
企业投资是外生的,不随利率&产量变动

##### 凯恩斯定律

不论需求量是多少,经济社会均能以不变的价格提供相等的供给量  
总需求变动影响产量和收入以达到供求相等,不影响价格  
定律适用于短期,价格具有黏性

##### 折旧和公司未分配利润为零

$$GDP=NDP=NI=PI$$

### 均衡产出的概念

#### 均衡产出或收入:和总需求相等的产出

产出=需求  $\Rightarrow y=c+i \Rightarrow$  均衡条件

$c+i$ :居民和企业实际想要有的消费和投资

意愿(计划)消费和投资的数量  
不是国民收入构成公式中的实际发生

#### 区别

##### 国民收入核算:

实际产出=计划支出+非计划存货支出

##### 国民收入决定理论:

均衡产出=计划需求(支出)

均衡产出是和总需求相一致的产出,也就是经济社会的收入正好等于全体居民和企业想要有的支出

$E$ 代表支出,  $y$ 代表收入: $E=y$

均衡产出水平决定于总需求或总支出水平

### 投资等于储蓄(均衡条件)

$$E=c+i, y=c+s$$

计划支出=计划消费+计划投资

收入=计划消费+计划储蓄

$i=s$  均衡条件

经济达到均衡,计划投资必须等于计划储蓄

区别:此处为均衡条件,国民收入核算为

根据定义得出的实际数字必然相等

## 凯恩斯的消费理论

### 消费函数

影响消费最重要的因素是家户收入(凯恩斯)

消费函数/消费倾向:消费和收入的关系

边际消费倾向MPC ( $\Delta c/\Delta y, dc/dy$ )

增加的消费与增加的收入的比率

边际消费倾向递减

平均消费倾向APC ( $c/y$ )

任一收入水平上消费支出在收入中的比率

平均消费倾向递减,但始终大于MPC

### 凯恩斯绝对收入消费理论

$$c=\alpha+\beta y$$

$\alpha$ :自发消费部分

$\beta$ :边际消费倾向MPC

随收入增加,APC趋向于MPC

### 储蓄函数

储蓄函数:储蓄与收入的关系

边际储蓄倾向MPS ( $\Delta s/\Delta y, ds/dy$ )

储蓄增量对收入增量的比率

平均储蓄倾向APS ( $s/y$ )

任一收入水平上储蓄在收入中所占的比率

### 消费函数和储蓄函数的关系

定义  $s=y-c, s=-\alpha+(1-\beta)y$

APC&MPC 都随收入增加而递减,  $APC>MPC$

APS&MPS 都随收入增加而递增,  $APS<MPS$

$APC+APS \equiv 1, MPC+MPS \equiv 1$

### 家庭消费函数和社会消费函数

#### 前者推导后者需考虑的限制条件

国民收入的分配

不均等 $\Rightarrow$ 社会消费曲线向下移动

政府税收政策

累进个人所得税 $\Rightarrow$ 消费多

公司未分配利润在利润中所占比例

未分配利润占比大 $\Rightarrow$ 消费少

### 影响消费的其他因素

#### 利率

替代效应(低收入):利率提高 $\Rightarrow$ 储蓄增加

收入效应(高收入):利率 $\uparrow \Rightarrow$ 未来收入 $\uparrow$   
 $\Rightarrow$ 认为自己富有 $\Rightarrow$ 消费增加,储蓄减少

#### 价格水平

通过实际收入改变影响消费

物价 $\uparrow \Rightarrow$ 实际收入 $\downarrow \Rightarrow$ 保持生活水平 $\Rightarrow MPC \uparrow$

货币幻觉

物价&货币收入 $\uparrow \Rightarrow$

消费者误以为实际收入 $\uparrow \Rightarrow APC \uparrow$

#### 收入分配

国民收入分配平均 $\Rightarrow APC \uparrow$

#### 社会保障制度

### 两部门经济中国民收入的决定及变动

#### 使用消费函数决定收入

假设:计划净投资是外生变量

$$y=c+i \Rightarrow y=\frac{\alpha+i}{1-\beta}$$

消费曲线: $c$

消费投资曲线: $c+i \Rightarrow$ 总支出曲线

#### 使用储蓄函数决定收入

$$i=s=y-c \Rightarrow y=\frac{\alpha+i}{1-\beta}$$

### 国民收入决定·小节

	均衡条件	函数	
①	产出需求(支出)等式	消费函数	联立
②	投资储蓄等式	储蓄函数	联立

### 乘数论

#### 投资乘数( $k=\Delta y/\Delta i$ )

收入变化与带来该变化的投资支出的变化的比率

增加投资购买的是最终产品

最终产品价值=国民收入

$$k=\frac{1}{1-MPC}=\frac{1}{1-\beta}=\frac{1}{MPS}$$



# 国民收入的决定：收入-支出模型(2-2)

## 三部门经济的收入决定

支出角度  $Y = GDP = C + I + G$

转移支付已包含在  $C + I$

收入角度  $Y = NI = C + S + T$

政府净收入  $T = T_g - t_r$

宏观均衡条件

$C + I + G = C + S + T$ ;  $I + G = S + T$

税收

定量税：税收量不随收入而改变

比例所得税：随收入增加而增加

定量税下：

外生变量： $t, i, g$

$$\begin{cases} I + G = S + T \\ S = Y - C \end{cases} \rightarrow Y = \frac{\alpha + I + G - \beta T}{1 - \beta}$$

$$C = \alpha + \beta Y_d$$

$$Y_d = Y - T$$

$$S + T = -\alpha + (1 + \beta)T + (1 - \beta)Y$$

$Y_d$ ：可支配收入

## 四部门经济中国民收入的决定

收入决定

总支出(总需求)

$$Y = C + I + G + NX; NX = X - M$$

$Y$ ：需求(家庭+企业+政府+外国(净))

$X$ ：由外国购买力和购买需求决定

$M$ ：随收入提高而增加(独立的消费函数)

$$M = M_0 + \gamma Y \quad \text{国民收入} \uparrow \Rightarrow M \uparrow \Rightarrow NX \downarrow$$

$M_0$ ：自发性进口； $\gamma$ ：边际进口倾向

$$Y = C + I + G + X - M \quad \begin{cases} \text{外生变量} \\ t = t \quad i = i \\ g = g \quad t_r = t_r \\ x = x \end{cases}$$

$$C = \alpha + \beta Y_d$$

$$Y_d = Y - T$$

$$M = M_0 + \gamma Y$$

↓

$$Y = \frac{1}{1 - \beta + \gamma} (\alpha + I + G - \beta T + \beta T_r + X - M)$$

乘数

对外贸易乘数  $dy/dx$

出口增加1单位引起国民收入变动的多少

$$\frac{dy}{dx} = \frac{1}{1 - \beta + \gamma} < \frac{1}{1 - \beta} \quad \because 1 > \gamma > 0$$

投资乘数、政府购买乘数均变小

$$k = k_g = \frac{1}{1 - \beta + \gamma} < \frac{1}{1 - \beta}$$

增加的收入一部分购买进口商品

结束语

关于乘数理论的限制

社会中过剩生产能力的大小

投资和储蓄决定的相互独立性

货币供给量增加能否适应支出增加的需要

增加的收入不能用于购买进口货物

## 三部门经济中各种乘数

$$Y = \frac{\alpha + I + G - \beta T}{1 - \beta}$$

政府购买支出乘数

收入变动对引起这种变动的政府购买支出变动的比率

$$k_g = \frac{\Delta Y}{\Delta G} = \frac{1}{1 - \beta}$$

税收乘数

收入变动对引起这种变动的税收变动的比率

$$k_t = \frac{\Delta Y}{\Delta T} = \frac{-\beta}{1 - \beta}$$

$t \uparrow \Rightarrow$  可支配收入  $\downarrow \Rightarrow$  消费  $\downarrow \Rightarrow$  总支出  $\downarrow$

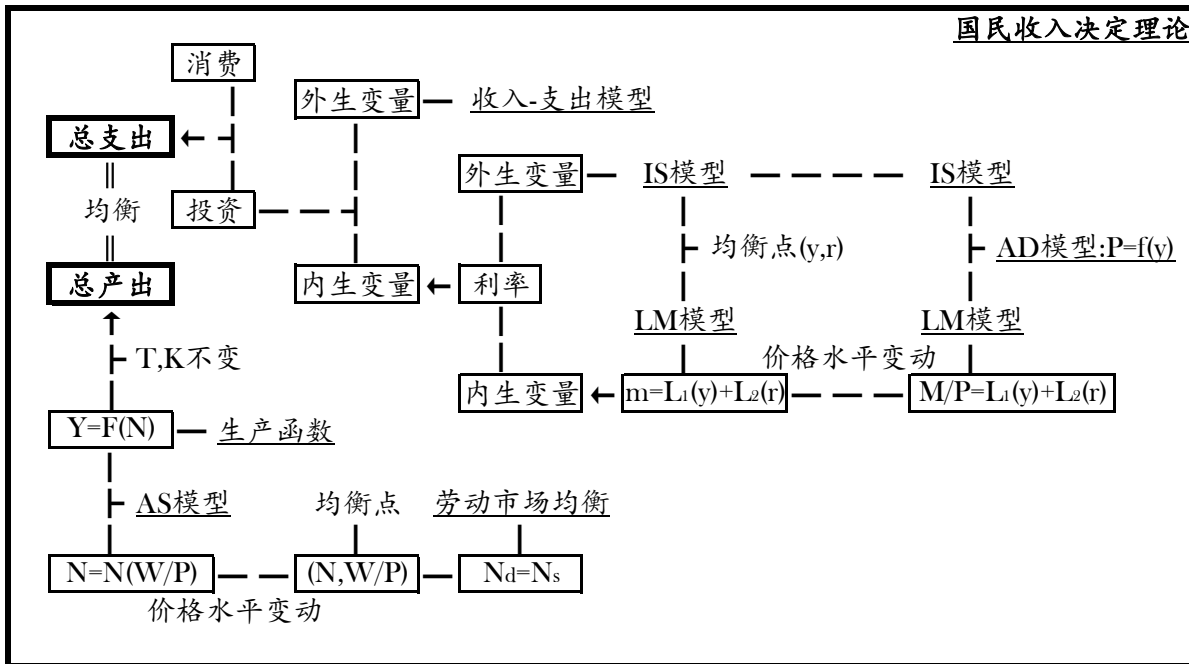
政府转移支付乘数

收入变动对引起这种变动的政府转移支付变动的比率

$$Y = \frac{\alpha + I + G + \beta T_r - \beta T}{1 - \beta}; \quad k_{t_r} = \frac{\Delta Y}{\Delta T_r} = \frac{\beta}{1 - \beta}$$

平衡预算乘数(税收和政府购买)  $k_b = \Delta Y / \Delta G = 1$

政府收入和支出同时以相等数量增加或减少时,国民收入变动与政府收支变动的比率



国民收入的决定: IS-LM模型(3-1) IS: 由利率解释投资; LM: 由供求解释利率

投资的决定

打破假设: 将投资变为内生变量

投资: 资本的形成, 社会实际资本的增加

厂房、设备等的增加

决定投资的因素

实际利率, 预期收益率, 投资风险

实际利率与投资

凯恩斯: 投资取决于(预期利润率vs利率)

实际利率=名义利率-通胀率

$i=i(r)=e-dr$ ;  $e$ : 自主投资

$d$ : 投资需求对于利率变动的反应程度

资本边际效率MEC

一种贴现率, 使得一项资本物品的使用期内各项预期收益的现值之和等于这项资本品的供给价格或重置成本(IRR)

$$R = \sum_{i=1}^n \frac{R_i}{(1+r)^i} + \frac{J}{(1+r)^n}; r: MEC$$

资本边际效率曲线

预期收益( $R_i, J$ )既定时,  $R \uparrow \Rightarrow r \downarrow$

即: 投资量 $\uparrow$ 资本边际效率 $\downarrow$

投资量和利率反向变动

投资边际效率曲线MEI

由于 $R$ 上升而被缩小的 $r$ 的数值

int.rate  $\downarrow \Rightarrow$  投资 $\uparrow \Rightarrow$  需求(资本品) $\uparrow$

$\Rightarrow$  投资量 $R \uparrow \Rightarrow r \downarrow$

更精确地表示投资与利率的关系

预期收益与投资(影响预期收益的因素)

对投资项目的产出的需求预期

产品成本

特别是劳动者的工资成本

① 工资 $\uparrow \Rightarrow$  投资需求 $\downarrow$

② 工资 $\uparrow \Rightarrow$  需求(购买机器替代人力) $\uparrow$

投资税抵免

新古典经济学

政府规定, 投资的厂商可从其所得税单中扣除其投资总值的一定百分比

取决于政策是临时的or长期的

风险与投资(经济繁荣, risk  $\downarrow$ )

凯恩斯: 投资需求与投资者情绪大有关系

托宾的q说

$q$  = 企业的股票市值/新建企业的成本

IS曲线

IS曲线及其推导

产品市场均衡

产品市场上总供给等于总需求

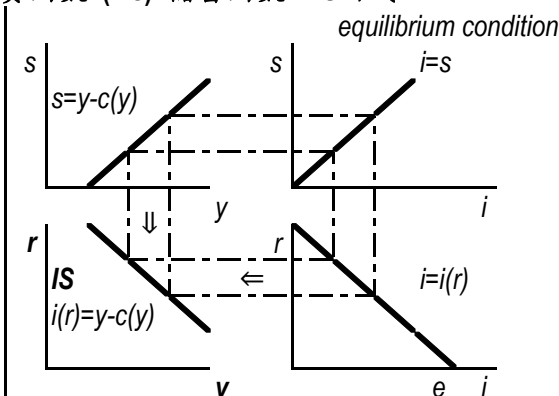
$$\uparrow i = e - dr \downarrow$$

$$y = \frac{\alpha + i}{1 - \beta} \Rightarrow y = \frac{\alpha + e - dr}{1 - \beta} \quad \star$$

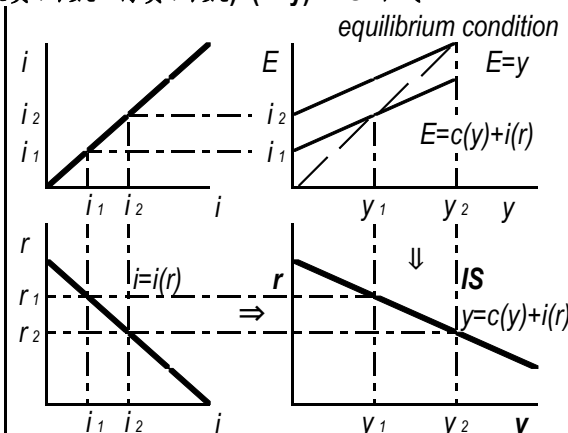
IS曲线

$\star$ 式是一条反映利率和收入间相互关系的曲线; 该曲线上任何一点所代表的利率与收入的组合下,  $i=s$ , 从而产品市场是均衡的

投资函数( $i=s$ )=储蓄函数 $\Rightarrow$ IS曲线



(投资函数+消费函数)=( $E=y$ ) $\Rightarrow$ IS曲线



IS曲线的斜率

$$y = \frac{\alpha + e - dr}{1 - \beta}; r = \frac{\alpha + e}{d} - \frac{1 - \beta}{d} y \quad \star$$

$\star$ 式是IS曲线的代数表达式

斜率  $= -(1-\beta)/d = (1-\beta)/d$

$d$ : 投资需求对于利率变动的反应程度

$d \uparrow \Rightarrow$  投资对利率变化敏感  $\Rightarrow$  斜率小  $\Rightarrow$  IS平缓

$\beta$ : 边际消费倾向

$\beta \uparrow \Rightarrow$  支出乘数 $\uparrow \Rightarrow$  利率引发投资变动时,  $y$ 变动加剧

三部门经济

定量税下(斜率不变)

比例所得税下

$c = \alpha + \beta(1-t)y$ ;  $t$ : 边际税率

$$|\text{斜率}| = \left| \frac{1-\beta(1-t)}{-d} \right| = \frac{1-\beta(1-t)}{d}$$

$t \downarrow \Rightarrow$  投资乘数 $\uparrow \Rightarrow$  IS越平缓

西方观点

影响IS斜率的主要是 $d$ , 因为 $\beta$ 比较稳定;  $t$ 一般不变

以上分析均以预期不变为前提

加入预期因素, IS会更陡峭

利率 $\downarrow \Rightarrow$  企业预期未来实际利率不会如此低

$\Rightarrow$  企业不按照 $d$ 大幅改变投资计划

人们预期不会持久的收入变化对 $c+i$ 的影响很有限

$\beta \downarrow$ , 投资乘数 $\downarrow$

IS曲线的移动

一切自发支出量变动, 都会移动IS( $\alpha, e, g, t$ )

移动幅度:  $\Delta \times$  对应乘数

投资 $\uparrow \Rightarrow$  IS右移

$$\text{定量税下: } y = \frac{\alpha + e + g - \beta t - dr}{1 - \beta}$$

投资需求变动( $e$ 变动,  $k=1/(1-\beta)$ )

投资 $\Delta \uparrow \Rightarrow$  投资需求曲线右移 $\Delta \Rightarrow$  IS右移

储蓄变动( $\alpha$ 变动,  $k=1/(1-\beta)$ )

储蓄 $\uparrow \Rightarrow$  储蓄曲线左移  $\Rightarrow$  投资不变时, 较低  
IS左移  $\Rightarrow y \downarrow \Rightarrow$  收入即可满足投资

政府购买性支出变动( $g$ 变动,  $k=1/(1-\beta)$ )

政府购买性支出 $\uparrow \Rightarrow$  投资 $\uparrow \Rightarrow$  IS右移

政府税收变动( $t$ 变动,  $k=\beta/(1-\beta)$ )

税收 $\uparrow \Rightarrow$  投资or消费 $\downarrow \Rightarrow y \downarrow \Rightarrow$  IS左移

财政政策(扩张: IS右移)(紧缩: IS左移)



国民收入的决定: IS-LM模型(3-2) IS: 由利率解释投资; LM: 由供求解释利率

利率的决定

古典学派

投资和储蓄只与利率有关

投资/储蓄是利率的减/增函数

凯恩斯: 储蓄更收收入的影响

利率由货币供给量和需求量决定

货币实际供给量m由国家控制

流动性偏好

=对货币的需求=灵活偏好=流动偏好

由于货币具有使用上的灵活性, 人们宁肯牺牲利息收入而储存不生息的货币来保持财富的心理倾向

三大动机

$L_1 = L_1(y) = ky$ ; LM与y的联系桥梁

交易动机

个人和企业需要货币是为了进行正常的交易活动

主要决定于收入, 正相关

谨慎动机/预防性动机

为预防意外支出而持有一部分货币的动机

全社会看, 大体也和收入成正比

$L_2 = L_2(r) = -hr$ ; h: 货币投机需求的利率系数

投机动机

人们为抓住有利的购买有价证券的机会而持有一部分货币的动机

利率高  $\Rightarrow$  证券价格低  $\Rightarrow$  人们买入  $\Rightarrow$  投机动机货币需求量  $\downarrow$

流动偏好陷阱(凯恩斯陷阱)

人们不管有多少货币都愿意持在手中的情况

利率极高  $\Rightarrow$  证券价格极低  $\Rightarrow$  人们认为利

持有货币全部买证券  $\Rightarrow$  率不会更高

利率极低  $\Rightarrow$  证券价格极高  $\Rightarrow$  人们认为利

有了货币也  $\Rightarrow$  所有证券换成货币  $\Rightarrow$  率不会更低

不买证券  $\Rightarrow$  流动偏好趋向无穷大

货币需求关于利率的系数

也叫流动偏好的利率系数

货币需求函数

$L = L_1 + L_2 = ky - hr$

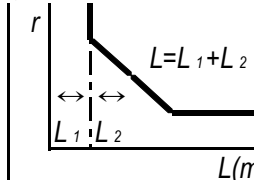
$L_1, L_2$ : 实际货币需求量

$m = M/P$ , m: 实际货币量

M: 名义货币量; P: 价格指数

名义货币需求函数:  $L = (ky - hr)P$

货币需求曲线



货币需求量与收入水平正向关系

ky是函数的截距,  $y \uparrow$  曲线右移

货币供求均衡和利率的决定

M1狭义的货币供给(以下只涉及M1)

流通中的硬币、纸币和银行活期存款

M2广义的货币供给 = M1 + 定期存款

M3更广泛的货币供给

M2 + 个人 & 企业持有的政府债券等流动资产

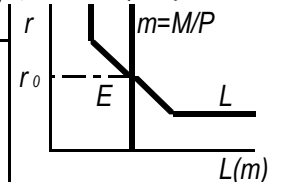
西方认为: 货币供给由国家决定, 是外生变量

与r无关, 货币供给曲线垂直于横轴

货币需求  $\uparrow$  货币需求曲线右移

货币供给  $\uparrow$  货币供给曲线右移

货币市场均衡



LM曲线的移动(截距 =  $-m/h$ )

不考虑转动斜率不变k&h不变

LM移动只受货币供给量m变动的影响

$m = M/P$

$m \uparrow \Rightarrow$  LM右移

名义货币供给量M变动

$M \uparrow \Rightarrow m \uparrow \Rightarrow -m/h \downarrow$  LM右下方移动

价格水平P的变动

$P \uparrow \Rightarrow m \downarrow \Rightarrow -m/h \uparrow \Rightarrow$  LM左上方移动

价格水平与收入反向  $\Rightarrow$  AD曲线

LM曲线

LM曲线及其推导

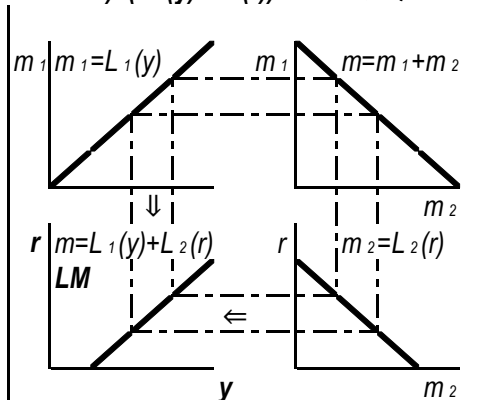
货币市场均衡(★式: LM曲线)

$$m = ky - hr; y = \frac{h}{k}r + \frac{m}{k}; r = \frac{k}{h}y - \frac{m}{h} \quad \star$$

LM曲线

线上任一点所代表的利率和收入的组合下, 货币需求和供给都是相等的, 即货币市场是均衡的

$(m = m_1 + m_2) = (L_1(y) + L_2(r)) \Rightarrow$  LM曲线

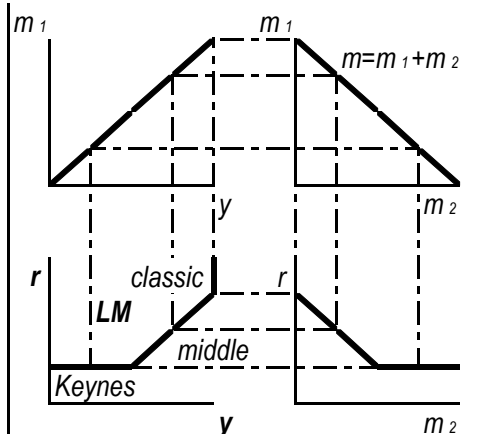


LM曲线的斜率

斜率 =  $k/h$

西方观点: 交易需求函数比较稳定, 斜率主要受h影响

LM曲线的三个区域

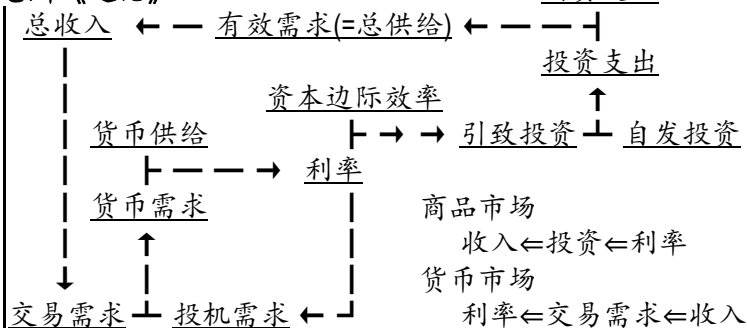


# 国民收入的决定：IS-LM模型(3-3) IS: 由利率解释投资 LM: 由供求解释利率

## IS-LM分析

两个市场同时均衡的利率和收入

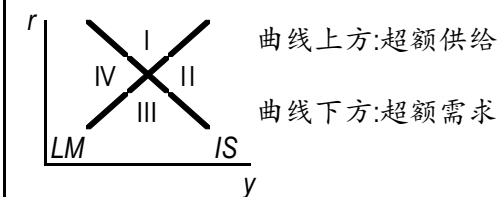
### 凯恩斯《通论》



### 希克斯: 联立产品市场和货币市场, 解决循环推论问题

IS 曲线:  $i(r) = s(y)$

LM 曲线:  $M = L_1(y) + L_2(r)$



## 均衡收入和利率的变动

IS 和 LM 的交点同时实现两个市场均衡

该均衡不一定是充分就业的均衡

可通过财政 & 货币政策调节交点位置

### 财政政策 (投资 ↑ IS 右移)

政府变动支出和税收调节国民收入

### 货币政策 ( $m \uparrow$ LM 右移)

央行变动货币供给量改变利率和收入

## IS-LM 模型的用处

可清楚直观地表示/表现出

经济短期波动原因

投资/消费/政府支出/税收/货币供给/货币需求

政府干预经济用的是财政政策还是货币政策

构成总需求诸因素对总需求变动影响的强弱程度

斜率  $\Rightarrow$  利率变动如何影响总需求

政府干预经济政策的效果 (与斜率相关)

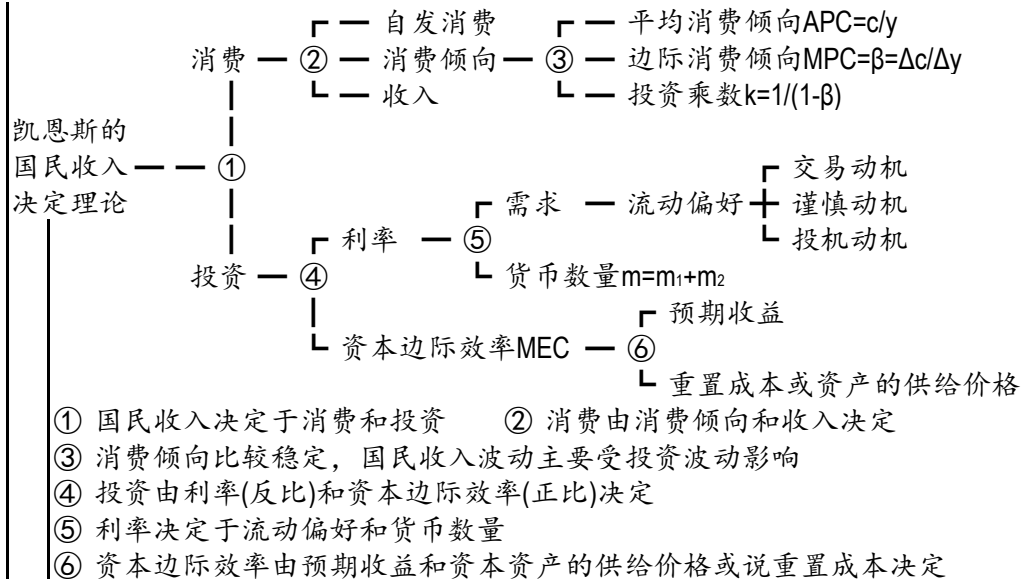
凯恩斯主义和货币主义为什么对财政政策和货币政策态度不同

总需求曲线的来历

挤出效应

其他不变:  $P \uparrow \Rightarrow m \downarrow \Rightarrow -m/h \uparrow \Rightarrow LM$  左上方移动

## 凯恩斯的基本理论框架



- ① 国民收入决定于消费和投资
- ② 消费由消费倾向和收入决定
- ③ 消费倾向比较稳定, 国民收入波动主要受投资波动影响
- ④ 投资由利率 (反比) 和资本边际效率 (正比) 决定
- ⑤ 利率决定于流动偏好和货币数量
- ⑥ 资本边际效率由预期收益和资本资产的供给价格或说重置成本决定

## 关于大萧条

凯恩斯

原因

需求不足

消费: 边际消费倾向  $< 1$

投资: 资本边际效率长期内递减

政策

财政政策有效, 货币政策无效 (流动偏好陷阱)

## IS-LM 模型的数学表达

产品市场均衡条件	$s = i$	$s = s(y)$ 储蓄函数 (与 $y$ 相连)	$i = i(r)$ 投资函数 (与 $r$ 相连)
货币市场均衡条件	$m = L$	$L = L_1 + L_2 = L_1(y) + L_2(r)$ 货币需求函数 (与 $y, r$ 相连)	$M/P = m = m_1 + m_2$ 货币供给函数

# 国民收入的决定：总需求-总供给模型(2-1) 取消价格不变假设

## 总需求曲线

含义(总需求(常以产出水平表示))

经济社会对产品和劳务的需求总量

总需求=需求(消费+投资+政府+国外)

不考虑国外需求下

价格,收入和其他经济变量在既定条件下,家庭,企业和政府将要支出的数额

衡量经济中各种行为主体的总支出

总需求函数

以产量(国民收入)所表示的需求总量和价格水平之间的关系

图形(从 $y=C+I+G+NX$ 解释)

向右下方倾斜,价格水平与总需求反向关系

P对家庭部门的消费的影响

$P \downarrow \Rightarrow$  货币真实价值  $\uparrow \Rightarrow$  消费者感觉富有  
 $\Rightarrow$  更多支出  $\Rightarrow$  需求量(物品和劳务)  $\uparrow$

$P \uparrow \Rightarrow$  货币真实价值  $\downarrow \Rightarrow$  消费者感觉贫穷  
 $\Rightarrow$  减少支出  $\Rightarrow$  需求量(物品和劳务)  $\downarrow$

P对企业部门的投资的影响

价格水平是货币需求量的决定因素

$$L = (ky - hr)P$$

$P \downarrow \Rightarrow$  购买物品&劳务的货币需求量  $\downarrow$   
 $\Rightarrow$  利率  $\downarrow \Rightarrow$  企业投资  $\uparrow$

利率效应:P变动通过影响利率

进而影响企业投资的效应

推导

总需求函数的数学推导

$$\begin{aligned} IS: & y = c(y-t) + i(r) + g \\ LM: & M/P = L_1(y) + L_2(r) \end{aligned} \quad \begin{aligned} & \rightarrow \text{联立消去 } r, \\ & \text{得到 } y = D(P) \end{aligned}$$

总需求曲线的图形推导

P对IS,LM的影响

IS曲线:无影响

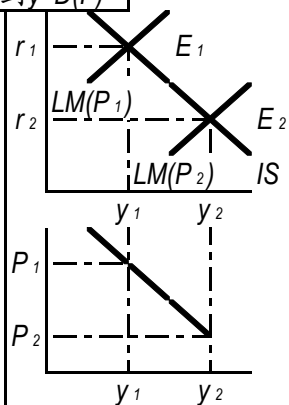
决定IS的变量被假定是实际量,非名义量

LM曲线

P改变  $\Rightarrow$  m改变  $\Rightarrow$  LM移动  $\Rightarrow$  新均衡的y

总需求曲线

表示社会的需求总量和P之间的反向关系



移动

价格水平既定下

总需求曲线右移

消费/投资/政府购买/净出口增加

	财政政策	货币政策	AD
扩张	IS右移	LM右移	右移
紧缩	IS左移	LM左移	左移

总供给的一般说明

总供给

经济社会所提供的总产量(或国民收入),即经济社会投入基本资源所生产的产量

基本资源

劳动,生产性资本存量,技术

生产函数

描述总产出与劳动,资本,技术间的关系

短期与长期宏观生产函数

宏观生产函数(总量生产函数)

假定经济社会在一定的技术水平下

$$y = f(N, K)$$

y总产出;K社会的资本存量;

N社会就业水平或就业量

产出主要取决于

就业量N、资本存量K、技术水平T

短期宏观生产函数(K,T不会有较大变化)

$$y = f(N, \bar{K})$$

$\bar{K}$ :不变的资本存量

一定技术水平和资本存量下,y取决于N

假定宏观经济函数(上式)有两条性质

总产出随总就业增加而增加

总就业增加,总产出按递减比率增加

T和K不变;边际报酬递减规律

长期生产函数(N,K,T均可变)

$$y_f = F(N^*, K^*)$$

$N^*$ 各个短期中的充分就业量

$K^*$ 各期的资本存量

$y_f$ 各期的充分就业时的产量(潜在产量)

总供给  $\leftarrow$  就业水平  $\leftarrow$  劳动市场(本书仅限短期生产函数)

劳动市场

总供给理论的争议  $\leftarrow$  劳动市场理论的争议

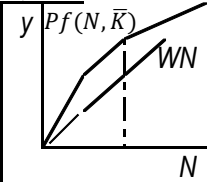
完全竞争的劳动市场

企业只能接受既定的市场工资和其产品的市场价格

企业选一个就业水平  $\rightarrow$  边际产品=实际工资

$$\text{实际工资} = \text{名义工资 } W / \text{价格水平 } P$$

劳动的边际产品随劳动投入增加而降低



$\Rightarrow$  劳动需求函数是实际工资的减函数

上述微观经济学意义上的关系在宏观也成立

劳动需求函数

$$N_d = N_d(W/P); N_d \text{ 与实际工资 } (W/P) \text{ 成反方向变动}$$

劳动供给函数

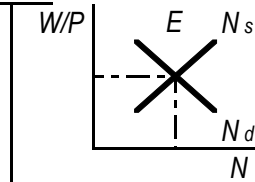
$$N_s = N_s(W/P); N_s \text{ 与实际工资 } (W/P) \text{ 成正方向变动}$$

劳动市场均衡

$$N_s\left(\frac{W}{P}\right) = N_d\left(\frac{W}{P}\right)$$

均衡的实际工资

均衡的就业量



两点说明

有伸缩性的工资和价格下

实际工资调整到劳动供求相等的水平;

从而劳动市场均衡  $\Rightarrow$  充分就业

充分就业并非每个愿意工作的人都就业

摩擦性失业,自愿失业等均可以在均衡状态下存在

劳动市场在经济的总供给方面处于主导地位

在任一时点,K都由以往的投资决策决定(K不变)

更进一步地

工资和价格具有完全伸缩性的情况下,经济中的产量始终等于充分就业时的产量(潜在产量)

国民收入的决定：总需求-总供给模型(2-2) 取消价格不变假设

两种极端的总供给曲线

按货币工资和价格水平进行调  
整所要求的时间长短划分

- 古典总供给曲线
- 凯恩斯总供给曲线
- 常规总供给曲线

古典总供给曲线

长期中

价格和货币工资具有伸缩性 $\Rightarrow$ 劳动市  
场均衡 $\Rightarrow$ 经济处于充分就业水平

经济的就业水平或产量不随价格水  
平而变,始终处于充分就业状态

经济的产量水平位于潜在产量或  
充分就业的水平上,不受价格影响

总供给曲线是一条位于经济潜在产量  
或充分就业产量水平上的垂直线

原因

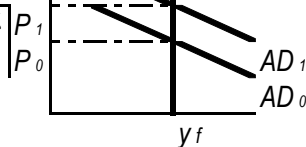
①古典学派假设货币工资和价格水  
平可以迅速调节,使实际工资总处  
于充分就业水平下

②古典学派一般研究经济的长期状  
态,长期中二者有充分时间调整

古典总供给曲线代表长期总供给曲线

政策含义

增加需求的政策不  
能改变产量,只能  
造成物价上涨,甚  
至通货膨胀



凯恩斯总供给曲线

假设:货币工资具有刚性

产量变动,价格和货币工资均不变化

原因

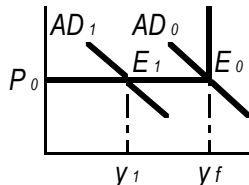
一条水平线

①货币工资具有刚性

②研究的是短期,二者来不及调整

政策含义

只要国民收入或产量  
处在小于充分就业的  
水平,就可以用增加需  
求的政策使经济达到  
充分就业状态



常规总供给曲线

图形

右上方延伸

价格水平越高,总产出越多

微观经济学

短期,工资和其他资源价格相  
对固定,企业产品价格提升,  
增产通常盈利

推导(从黏性价格模型推导)

价格黏性原因

价格由长期合约约定

更改价格有成本

个体企业的定价决策

基本讨论:有定价权企业

$$p = P + a(y - y_f)$$

两种成本

总价格水平P

总收入水平y产生需求

边际成本 $\uparrow$ (更高生产水平上)

两种企业

价格有弹性(占比 $(1-b)$ ,按上式)

价格有黏性(占比b)

按自己预期事先宣布价格

$$p = EP + a(Ey - Ey_f) \Rightarrow p = EP$$

假设预期产出=自然水平

价格总水平

P=黏性企业定价加权+弹性企业定价加权

$$P = bEP + (1-b)(P + a(y - y_f))$$

$$P = EP + \frac{(1-b)a}{b}(y - y_f)$$

$$y = y_f + \lambda(P - EP); \lambda = \frac{b}{(1-b)a}$$

常规总供给曲线方程

移动

可得到的劳动量

物质资本或人力资本 $\uparrow$

自然资源可获得性

技术知识/可得到的技术

$\Rightarrow$  AS右移 AS左移

预期价格水平 $\uparrow$

(要求高工资)

①投入品价格上升,短期AS曲线左移

②名义工资增加,短期AS曲线左移

总需求-总供给模型对现实的解释

宏观经济的短期目标

充分就业和物价稳定

不存在非自愿失业

总需求曲线移动的后果

低于充分就业水平时

AD左移:价格下降比例小于产量下降比例

AD右移:价格上升比例低于产量上升比例

高于充分就业水平时

AD左移:价格下降比例高于产量下降比例

AD右移:价格上升比例高于产量上升比例

总供给曲线移动的后果

左移

滞胀状态

失业和通货膨胀并存

二者变动比例相对关系不明确

右移十分少见

短期内生产技术可能提高,但难以快速得到成果

总需求-总供给模型的政策含义

需求管理政策

财政政策,货币政策

供给管理政策

总需求-总供给模型的数学小节

产品市场均衡条件  $y = c(y - t) + i(r) + g$

总需求函数  
联立消去r,得  
到y与P的关系

货币市场均衡条件  $\frac{M}{P} = L_1(y) + L_2(y)$

劳动市场均衡条件  $f(N) = \frac{W}{P}; h(N) = \frac{W}{P}$

总供给函数  
联立消去N,得  
到y与P的关系

短期总量生产函数  $y = y(N, \bar{K})$



# 失业与通货膨胀(3-1)

## 失业的描述

### 失业的数据

#### 失业率

劳动力中没有工作而又在寻找工作的人所占的比例  
其波动反映就业的波动

### 自然失业率和自然就业率

#### 自然失业率

经济社会在正常情况下的失业率,是劳动力市场处于供求稳定状态时的失业率

稳定状态:不会造成通胀或通缩的状态

劳动力市场处于稳定状态

$$fU=IE$$

N劳动力,E就业人数,  
↓ U失业人数,l离职率,f就职率

$$E=N-U$$

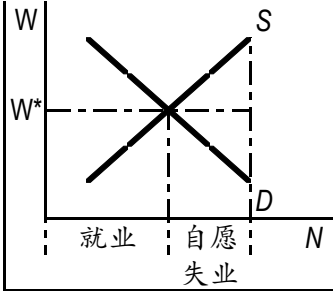
$$\text{自然失业率} = U/N = l/(l+f)$$

自然就业率(二者之和=100%)

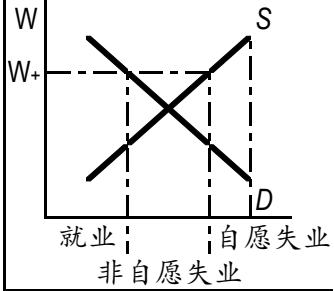
统称为自然率

### 失业的经济学解释

#### 市场出清



#### 市场非出清(工资无伸缩性,劳动力价格 $W_+$ )



## 失业的原因

### 分类

摩擦性失业;结构性失业;

#### 周期性失业

由于整体经济的支出和产出水平下降,即经济的需求下降造成的失业

### 摩擦性失业的原因

#### 原因

求职者和空缺职位的信息流动不完全  
工人在不同地区间流动不即刻  
部门转移总发生,工人改变部门需要时间  
部门转移:需求在不同行业和地区之间的构成变动

### 结构性失业的原因

劳动力的供给和需求不匹配所造成的失业

#### 原因

##### 工资刚性

工资不能调整到使劳动市场均衡从而消除失业的水平

##### 工资刚性的原因

###### 最低工资法

对不熟练的工人,最低工资法将他们的工资提到均衡水平之上

###### 效率工资

高工资使工人生产效率高  
减少劳动力的更替  
提高工人的努力程度  
即便有超额劳动供给,也不减工资

###### 工会的垄断力量

## 失业的影响与奥肯定律

### 失业的影响

社会影响(吸毒,离婚率,犯罪率)

#### 经济影响(机会成本)

失业者的收入总损失等于生产的损失  
丧失的产量:用来计量周期性失业损失

### 奥肯定律(产品市场和劳动市场间的关系)

失业率每高于自然失业率1个百分点,实际GDP将低于潜在GDP2个百分点

$$\frac{y - y_f}{y_f} = -\alpha(u - u^*); \alpha > 0$$

y实际产出; u实际失业率;  
yf潜在产出; u\*自然失业率

## 通货膨胀的描述

通货膨胀的数据(通货膨胀率)

### 通货膨胀的衡量

当一个经济中的大多数商品和劳务的价格连续在一段时间内普遍上涨时,称该经济经历着通货膨胀

#### 通货膨胀率

定义为从一个时期到另一个时期价格水平变动的百分比

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

### 价格指数

描述经济中各种商品和劳务价格的总体平均数

GDP折算指数

CPI,PPI

### 通货膨胀的分类

#### 按价格上升速度划分

温和的通货膨胀(通胀率 $<10\%$ )  
奔腾的通货膨胀( $10\% < \text{通胀率} < 100\%$ )  
超级通货膨胀( $100\% < \text{通胀率}$ )

#### 按对不同商品价格影响大小划分

平衡的通货膨胀  
每种商品的价格同比例上升(包括生产要素)  
非平衡的通货膨胀

#### 按人们的预期程度加以区分

未预期到的通货膨胀  
预期到的通货膨胀  
具有自我维持的特点  
又叫惯性的通货膨胀



三个方面解释

货币数量论;总需求与总供给;

经济结构因素的变动

作为货币现象的通货膨胀

货币数量论解释的基本思想

每一次通货膨胀背后都有货币供给的迅速增长

交易方程式(费雪)

$$MV=Py$$

MV:经济中的总支出

Py:名义收入水平

↓ M货币供给量;P价格水平;

V货币流通速度;y实际收入水平

V定义为名义收入与货币量之比

$$\pi = \hat{m} - \hat{y} + \hat{v}$$

↓  $\pi$ 通货膨胀率;m货币增长率;

v货币流通速度变化率;y产量增长率

$$\pi = \hat{m} - \hat{y} \quad \text{假定货币流通速度不变}$$

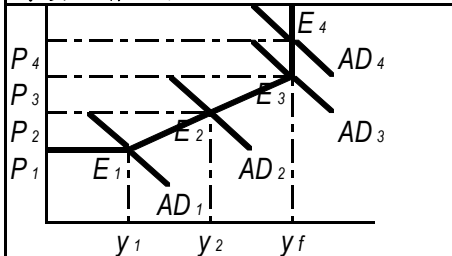
$$\pi = \hat{m} \quad \text{长期内,实际产量增长率不变}$$

归纳为宏观的基本共识

通货膨胀时时处处都是一种货币现象

需求拉动的通货膨胀

超额需求通货膨胀,指总需求超过总供给所引起的一般价格水平的持续显著上涨



瓶颈式的通货膨胀(价格水平从P<sub>1</sub>到P<sub>2</sub>、P<sub>3</sub>)

瓶颈现象:由于劳动/原料/生产设备等不足而使成本提高

需求拉动的通货膨胀(价格水平从P<sub>3</sub>到P<sub>4</sub>)

需求方面的原因或冲击

财政政策,货币政策,消费习惯的突然改变  
国际市场的需求变动

成本推动的通货膨胀

成本通货膨胀或供给通货膨胀

在没有超额需求的情况下由于供给方面成本的提高所引起的一般价格水平持续和显著的上涨

工资推动的通货膨胀

不完全竞争的劳动市场造成的高工资所导致的一般价格水平的上涨

工会⇒不再是竞争性的工资,而是工会与雇主集体议价的工资

工资-价格螺旋

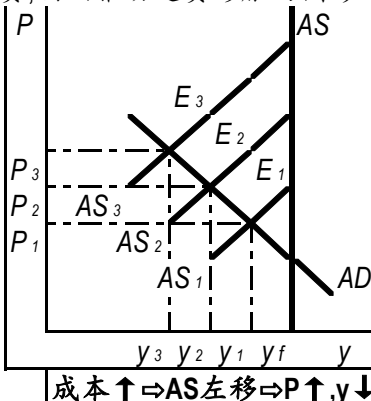
工资↑⇒价格↑

利润推动的通货膨胀

垄断企业和寡头企业利用市场势力谋取过高利润所导致的一般价格水平的上涨

前提:不完全竞争的产品市场

工资,利润推动通货膨胀的图形



成本↑⇒AS左移⇒P↑,y↓

混合通货膨胀理论

供给和需求两方面及相互影响

结构性通货膨胀

经济结构本身具有的特点(不同部门)

生产率提高的速度看:快vs慢

经济发展的过程看:迅速发展vs渐趋衰落

同世界的关系看:紧密(开放部门)vs不紧密(非开放部门)

结构性通胀原因

现代社会经济结构不容易使生产要素从后者转移到前者

后者在工资和价格问题上要求"公平",导致一般价格水平上涨

通货膨胀率=货币工资增长率-劳动生产增长率

微观:货币工资=劳动边际产品价值

$$W=P \times MP$$

[W平均货币工资,P价格水平,MP劳动边际产品]均为社会值

↓ 对t求微分

$$\dot{W} = \dot{P} \cdot MP + P \cdot \dot{MP}$$

↓ 同除W(=P×MP)

$$\frac{\dot{P}}{P} = \frac{\dot{W}}{W} - \frac{\dot{MP}}{MP} \Rightarrow \Delta\%P = \Delta\%W - \Delta\%MP$$

通货膨胀的持续

通货膨胀螺旋

多数情况下,通货膨胀有惯性

原因:预期

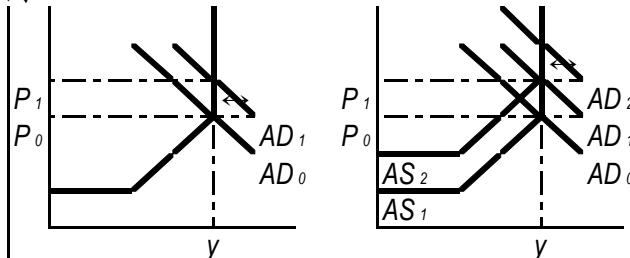
通货膨胀不是价格水平一次改变,而是持续上升

单纯用需求拉动或成本推动不足以

说明一般价格水平的持续上涨

只要通胀开始,需求拉动和成本推动几乎都发挥作用

举例



总需求↑⇒AD右移⇒超额需求⇒价格↑

↑

消费↑

⇔

货币收入↑

⇔

工资↑⇒AS上移

# 失业与通货膨胀(3-3)

## 通货膨胀的成本

### 预期到的通货膨胀成本

#### 菜单成本

调整价格的成本

#### 鞋底成本

为了使保留的钱少于没有通货膨胀时的数量,人们必须牺牲的时间与便利

#### 税收扭曲

一个经济的税率不会对预期到的通货膨胀作出充分调整,会对经济中当事人产生一种成本  
税收按名义值而非真实值征收,名义值>真实值

#### 相对价格变动导致的资源配置不当

通货膨胀扭曲相对价格(各资源间),消费者的决策被扭曲,市场资源配置效率下降

### 未预期到的通货膨胀成本

#### 不确定性的增加(经济效率/福利降低)

对经济当事人而言,与其预期相异的通货膨胀可能导致不正确的投资和储蓄决策  
扭曲关于工作多长时间和企业该雇用多少劳动的决策

工资与劳动时间成正比

#### 不合意的财富再分配

一种与才能,需要无关的方式在经济中重新分配财富  
通胀不利于靠固定货币收入生活的人  
靠变动收入的人获益  
通胀对储蓄者不利  
导致债务人和债权人收入再分配

#### 相对价格变动性的增加

经济的低效率和资源的不当配置  
eg.通胀率>企业预期通胀率  
企业按预期通胀率提价⇒实际价  
企业扩产⇐需求增加⇐格偏低  
高于预期的通胀或导致生产过剩

## 失业与通胀的关系:菲利普斯曲线

### 曲线的提出

#### 菲利普斯曲线(最初)

横轴表示失业率,纵轴表示货币工资增长率,向右下方倾斜的曲线

#### 新古典综合派对其改造的出发点

通胀率=货币工资增长率-劳动生产增长率

↓劳动生产增长率=0

通胀率=货币工资增长率

↓

$$\pi = -a(u - u^*); a > 0$$

a衡量价格对于失业率变动的反应程度  
失业率超过自然失业率,价格水平下降

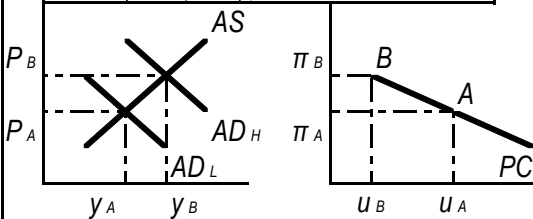
与奥肯定律(GDP vs 失业率)对比

$$\frac{y - y_f}{y_f} = -a(u - u^*); a > 0$$

### 曲线的推导

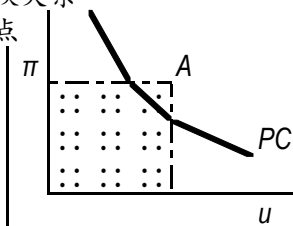
经济短期中出现的通胀与失业的组合

∴总需求冲击⇒总需求曲线的移动  
⇒经济沿着短期总供给曲线变动



### 政策含义

通胀率与失业率有替换关系  
经济社会选一个临界点  
组合区域内,不干预



### 牺牲率和痛苦指数

牺牲率(也可根据奥肯定律,用失业率表示)  
为了使通胀率降低一个百分点而必须放弃的一年实际GDP的百分点数

痛苦指数=失业率+通货膨胀率

### 附加预期的菲利普斯曲线

#### 工人对通货膨胀的预期

预期通胀率越高,名义工资增加越快

#### 短期菲利普斯曲线

预期通胀率保持不变时,表示通胀率与失业率之间关系的曲线

$$(\pi - \pi^e) = -a(u - u^*)$$

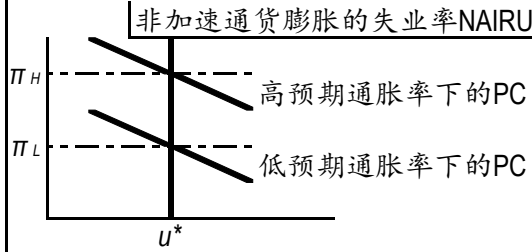
$$\pi = \pi^e - a(u - u^*)$$

∴预期短期不能及时改变

短期:从预期到需  
要根据通胀作出调  
整的时间间隔

现代菲利普斯曲线or附加预期的菲利普斯曲线

当实际通胀率=预期通胀率时,失业率=自然失业率  
自然失业率的又一定义



附加预期的菲利普斯曲线

短期中,预期通胀率<实际通胀率,  
替换关系仍存在

货币主义:调节总需求的政策短期内有效

### 长期菲利普斯曲线

长期中,预期通胀率=实际通胀率

无法通过调节工资来调节失业率

长期PC是一条垂直线

长期中,失业率与通胀率不存在替换关系

长期中,实现充分就业,失业率=自然失业率

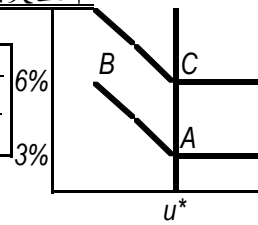
#### 推导

短期,起点A:扩张性政策⇒总需求↑  
⇒价格↑,通胀⇒π>π(e),实际工资↓  
⇒生产↑⇒就业↑,终点B

A:预期通胀率3%

B:实际通胀率6%>预期

长期,起点B:工人发现  
价格↑实际工资↓  
要求名义工资↑  
提高预期通胀率  
实际工资上升  
回归  
失业率  
回归



宏观经济政策(5-1)

宏观经济政策目标以及经济政策影响

宏观经济政策

政府为增进整体经济福利,改善整体经济运行状况,以达到一定的经济目标而对宏观经济领域进行的有意识的干预

目标

充分就业(通常指劳动)

(广泛意义上)一切生产要素都有机会以自己愿意的报酬参加生产的状态

失业率=失业人数/劳动力人数

摩擦性失业

在生产过程中由于难以避免的摩擦而造成的短期,局部性失业

劳动力流动性不足

工种转换的困难

自愿失业

工人不愿接受现行工资水平而形成的失业

非自愿失业

愿意接受现行工资但仍找不到工作的失业

结构性失业

由经济结构变化等原因造成的失业  
| 可视为摩擦性失业的极端形式

周期性失业

经济周期中衰退或萧条时因需求下降而造成的失业

奥肯定律

充分就业:仅有摩擦性和自愿失业(凯恩斯)

自然失业率(货币主义)

在没有货币因素干扰的情况下,让劳动市场和商品市场自发供求力量发挥作用时,总需求和总供给处于均衡状态的失业率

价格稳定

价格总水平的稳定(宏观经济概念)

经济持续均衡增长(常用 $\Delta\%$ GDP/年)

在一个特定时期内经济社会所生产的人均产量和人均收入的持续增长

国际收支平衡

作用和影响

宏观经济政策

需求管理政策

财政政策

政府变动税收和支出以便影响总需求进而影响就业和国民收入的政策

变动税收

税率

税率结构

eg. 所得税, 高/低收入者

变动政府支出

政府对商品和劳务的购买支出

转移支付

投资税收抵免, 加速折旧

货币政策

货币当局(央行)通过银行体系变动货币供给量来调节总需求的政策

均通过[利率, 消费, 投资]影响总需求

财政政策	利率	消费	投资	GDP
所得税↓	↑	↑	↓	↑
g↑, tr↑	↑	↑	↓	↑
投资津贴↑	↑	↑	↑	↑

货币政策	利率	消费	投资	GDP
货币供给↑	↓	↑	↑	↑

供给管理政策

人力政策, 收入政策

实现既定经济政策目标, 需要:

各种政策手段必须相互配合, 协调一致

制定目标时, 不能追求单一目标, 而要综合考虑

考虑到政策本身的协调性和对实际的把握程度

供给管理的政策

短期供给管理政策的含义

在短期内如何从供给方面采取一些政策措施来消除较大经济波动带来的失业和通胀

收入政策和人力政策

收入政策

用来限制垄断企业和工会对物价和工资操纵的一种重要政策

以管制工资-物价为主要内容的政策

包括措施

工资-物价'指导'线(对企业和工会)

政府根据长期劳动生产率增长趋势确定工资和物价增长标准(工资和物价限制在全社会劳动生产率平均增长幅度以内)

政府进行'权威性'劝说或施压来扭转局势

对某种具体的较快上涨的工资/物价形式

实施工资-物价的硬性管制(颁布法令)

以税收为基础的收入政策

以税收为奖惩, 限制工资增长

人力政策

用以改进劳动市场状况, 消除劳动市场不完全性, 以便克服失业

和通胀进退两难的困境

发展多吸收劳动力的服务部门

政府雇佣非熟练工人, 使之提升为正规工人

加强劳动力的重新训练

协助失业人员寻找工作

增加劳动力在地区或职业方面的流动性

供给学派的政策主张

强调激励的作用 认为凯恩斯的需求管理政策:

增强激励的措施 政府支出↑ ⇨ 为弥补赤字, 挫伤积极性 ⇨ 收&货币供给↑

减税, 降低高边际税率( $\Delta t/\Delta y$ )

肯尼迪时期的解释

减税增加储蓄, 提高资本形成能力

其他政策主张

减少政府开支, 削减福利支出

货币供给量增长和长期经济增长相适应

减少国家对经济的干预和控制



## 财政政策及其效果

## ① 财政的构成与财政政策工具

## 国家财政

## 政府收入

## 税收(政府收入最主要部分)

国家为实现其职能按照法律预先规定的标准,强制地、无偿地取得财政收入的一种手段

财政收入的主要来源

财政政策的重要手段

特点

强制性,无偿性,固定性

## 根据课税对象划分

财产税(对不动产等所征收的税)

包括遗产税

所得税(对个人和公司的所得征税)

西方政府中,比重很大,影响经济活动

流转税(eg.增值税)

对流通中商品和劳务买卖的总额征税

## 根据收入中被扣除的比例划分

累退税(税率随征税客体总量增加而递减)

比例税(税率不随征税客体总量变动而变动)

多用于流转税和财产税

累进税(税率随征税客体总量增加而增加)

所得税多属于累进税

## 乘数效应

税率的变动→总收入

主要改变所得税税率

税收绝对量的变动→总收入

## 公债

政府对公众的债务,公众对政府的债权

短(3m,6m,1y)中(1-5y)长(5y以上)

中长期债券

资本市场上最主要的交易品种之一

作用

增加财政收入,影响财政收支

属于财政政策

对金融市场(货币+资本)的扩张和紧缩

起重要作用,影响货币供求,调节总需求水平

## 政府支出

国家中各级政府支出总和

## 政府购买

政府对商品和劳务的购买

军需品,机关办公品,  
雇员薪酬,公共项目工程

① 是一种实质性支出

② 有商品和劳务的实际交易

③ 直接形成社会需求和购买力

④ 是国民收入的组成部分

## 政府转移支付

政府在社会福利保险,贫困救济和补助等方面的支出

eg.政府对农业的补贴

① 是一种货币性支出

② 并无相应的商品和劳务的交换

③ 一种不以取得本年生产出来的商品和劳务作为包场的支出

④ 不能算作国民收入的组成部分

通过转移支付乘数作用于国民收入

其乘数效应小于政府购买支出

## ③ 功能财政和预算盈余

## 功能财政(凯恩斯主义)

根据斟酌使用的财政政策,政府实施财政政策主要为了实现无通货膨胀的充分就业水平,目标实现时,预算可以盈余,也可赤字

预算赤字:政府支出大于收入的差额

预算盈余:政府收入超过支出的余额

实施难度

经济波动难以预测

决策需要时间,效果也滞后

年度平衡预算

要求每个财政年度的收支平衡

周期平衡预算

政府收支在一个经济周期中保持平衡

## ② 自动稳定与斟酌使用

## 自动稳定器(内在稳定器)

经济系统本身存在的一种会减少各种干扰对国民收入冲击的机制,能够在经济繁荣时期自动抑制通胀,在经济衰退时期自动减轻萧条,无需政府采取任何行动

① 政府税收的自动变化  $\Rightarrow \Rightarrow \Rightarrow$  乘数效应小于

实行累进税

自发性支出的

② 政府转移支付的自动变化  $\Rightarrow \Rightarrow$  乘数效应

③ 农产品价格维持制度

## 斟酌使用的财政政策(权衡性的财政政策)

为确保经济稳定,政府要审时度势,主动采取一些财政措施,变动支出水平或税收以稳定总需求水平,使之接近物价稳定的充分就业水平

刺激总需求:扩张性财政政策

抑制总需求:紧缩性财政政策

交替使用:补偿性财政政策

## 限制其发挥作用的因素

时滞

不确定性

挤出效应

① 乘数大小难以确定

② 政府需预测总需求通过财政政策作用达到预定目标的时间

③ 外在随机因素干扰

## ④ 赤字与公债

## 弥补赤字

借债

出售政府资产

向央行借债(央行增加基础货币,通胀)

向国内公众,外国举债 本质:通胀税→解决赤字

利率 $\uparrow \Rightarrow$ 央行出面稳定 $\Rightarrow$ 通胀

总赤字=非利息赤字+利息支出

非利息赤字=(政府支出-利息)-政府收入

赤字 $\uparrow \Rightarrow$ 债务 $\uparrow \Rightarrow$ 利息 $\uparrow \Rightarrow$ 赤字 $\uparrow$

公债利息支出赤字和公债 $\uparrow$ 的重要因素

## 债务-收入比率

一国债务/GDP, 取决于

公债的实际利率

实际GDP的增长率

解决财政赤字

非利息预算盈余的状况

增税(对富人);减少支出(对穷人);举债

## 财政政策及其效果

### 5 西方财政的分级管理模式

一国通过财政政策所进行的一切干预活动,必须以本国的财政管理体制为基础

#### 分级管理的财政体制

原则:财权和事权相一致

#### 中央财政预算

支出	中央	收入
	政府	(税收)
中央政府		中央税
支出		共享税

各自分开 ↓ 调节:  
自求平衡 ↓ 税收返还制度

#### 地方财政预算

支出	地方	收入
	政府	(税收)
地方政府		地方税
支出		共享税

共享税:中央与地方共享税

美国为例:

#### 联邦财政预算

支出	联邦政府	收入	⇒ 占60%
国防和国际		个人所得税	
关系, 社会保险		社会保险税	
		财产税	

地方财政预算 ↓ 预算支出的10%补助地方

支出	地方政府	收入	⇒ 占40%
教育,道路,		公共设施税	
公共福利,		财产税	
公共设施			

影响地方发展,促进全美经济平衡,  
调动地方理财积极性

#### 宏观调控手段

政府预算收支,失业保险,  
所定制的税制结构与累进所得税制

### 6 财政政策效果的IS-LM图形分析

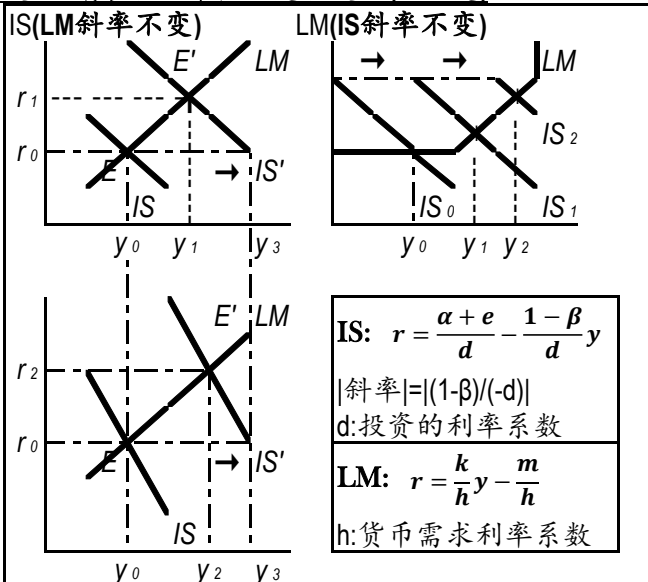
财政政策的效果

政府收支变化使IS曲线变动

对国民收入变动产生的影响

IS越陡峭,移动IS时收入变化越大(LM不变)

LM越陡峭,移动IS时收入变动越小(IS不变)



原理:

IS平坦	扩张性财政政策	LM陡峭
↓	↓	↓
d大	利率↑ ⇐ ⇐ 货币需求↑	h小
↓	↓	↓
投资对利率变动敏感	货币需求对Δ利率不敏感	
↓	↓	
私人投资下降很多	⇐ ⇐ ⇐ 利率↑↑	

挤出效应

政府支出增加所引起的私人  
消费或投资降低的效果

支出乘数的影响

支出乘数大,政策效果大

若d极大,h极小,  
支出乘数很大也难  
以奏效

按什么规则对经济进行干预(以货币政策为例)

稳定比率货币供应量增长的规则	自由	以真实利率为操作变量	以名义GDP为目标变量 (操作变量:
单一规则,增长率(国民收入+人口)	主义	泰勒规则,目标变量都是GDP,通胀	以名义通胀率为目标 货币供给)

### 7 财政政策案例:肯尼迪减税

背景(20世纪60年代初)

高失业率(最高7.1%),美元贬值

萨缪尔森:既保美元又刺激增长

#### 货币政策

短期贷款利率↑ ⇐ 阻止美元外流  
长期贷款利率↓ ⇐ 刺激国内投资

#### 财政政策

降低税收 ⇐ 增加收入 ⇐ 促进消费&投资

#### 政府措施

降低公司税收  
投资税优惠,提高加速折旧率  
降低个人所得税

效果(1964-1966)

GDP平均增速5.5%;失业率降至4.4%  
通货膨胀率低于3%

关于总需求管理政策的争论

要不要干预(反对干预)

政府预测能力有限

政策的时滞

内在时滞(财政政策,尤其是决策)

从经济发生不稳定到决策者  
制定出适当政策并付诸实践  
之间的时间间隔

认识/决策/实施时滞

外部时滞(货币政策)

从经济政策实施到对经济发生  
作用并达到预期目标所需要  
的时间过程(中间变量⇐目标变量)

公众对政策的反应(预期)

政策有效性的争论

凯恩斯主义

发公债比增加税收弥补赤字要好

巴罗-李嘉图等价

用公债和增税筹资对经济影响相同



## 货币政策及其效果

## ① 商业银行和中央银行

## 货币政策通过银行制度实现

## 金融机构

## 金融中介机构

## 商业银行

负债业务:吸收存款

资产业务

放款:为企业提供短期贷款

投资:购买有价证券获得利息收入

中间业务

代客户办理事项,收取手续费

保险公司,私人养老金等

## 中央银行

一国的最高金融当局,统

筹管理全国金融活动,实

施货币政策以影响经济

## 职能

发行的银行,发行国家的货币

银行的银行

为银行提供贷款,保管存款准备

金,为银行办理全国结算业务

国家的银行

代理国库

代收税款和公债,代拨经费

提供政府所需资金

贴现短期国债或购买国债

代表政府与外国发生金融业务关系

执行货币政策

监督管理全国金融市场的活动

## M1:流通中的硬币,纸币和银行存款

货币被定义为在商品和劳务的交换及债

务清偿中作为交换媒介或者支付工具而

被法定为普遍接受的物品

## 货币供给

$$M = \frac{r_c + 1}{r_c + r_d + r_e} \times H$$

影响货币供给的因素都可归结到准备金变动上

基础货币供给,法定准备率,贴现率,

利率,现金-存款比率

## ② 存款创造和货币供给

## 准备金

## 存款准备金

经常保留的供支付存款提取用的一定金额

## 法定准备率

由政府规定的准备金在存款中所占的比率的最低值

## 法定准备金

按该比率提留的准备金;

存放:库存现金+央行存款账户

## 超额准备金ER

超过法定准备金要求的准备金

实际准备金=法定准备金+超额准备金

## 存款创造

央行新增一笔原始货币供给将使活期

存款总和(货币供给量)扩大为这笔新

增原始货币供给量的(1/r<sub>d</sub>)倍

$$D = R/r_d$$

D存款总和;R原始存款;r<sub>d</sub>法定准备率

## 货币创造乘数k(把活期存款作为货币供给)

$$k = \frac{1}{r_d}$$

商业银行没有超额储备; 银行客户将一切货币收入存入银行

$$k = \frac{1}{r_d + r_e}$$

超额准备率r<sub>e</sub>=ER/D 利率↑  
实际准备率=r<sub>d</sub>+r<sub>e</sub> ⇨ r<sub>e</sub>↓ ⇨ k↑

$$k = \frac{1}{r_d + r_e + r_c}$$

r<sub>c</sub>现金在存款中的比率 再贴现率↑  
⇨ r<sub>e</sub>↑ ⇨ k↓

## 基础货币H(货币基础,高能货币,强力货币)

商业银行的准备金总额(法定+超额)+

非银行部门持有的通货

是存款扩张的基础,会派生出货币

基础货币H=C<sub>u</sub>+R<sub>d</sub>+R<sub>e</sub>C<sub>u</sub>非银行部门持有的通货R<sub>d</sub>法定准备金;R<sub>e</sub>超额准备金货币供给M=C<sub>u</sub>+D

M1角度:通货和活期存款之和

$$\frac{M}{H} = \frac{C_u + D}{C_u + R_d + R_e} = \frac{r_c + 1}{r_c + r_d + r_e} = k$$

把活期存款和通货作为货币供给

## ③ 债券价格与市场利息率的关系

二者反向变化,该关系适用于一切金融市场

## ④ 货币政策及其工具

## 货币政策(主流观点,凯恩斯主义观点)

央行通过控制货币供应量以及通过货币供应量来调节利率进而影响投资和整个经济以达到一定经济目标的行为

财政政策:直接影响总需求规模,没有中间变量

货币政策:通过利率变动影响总需求,间接作用

扩张性货币政策:增加货币供给以拉动总需求

紧缩性货币政策:削减货币供给以降低总需求

## 工具

## ① 再贴现率政策(重要性已削弱)

再贴现率:央行对商业银行及其他

金融机构的贷款利率(放款利率)

贴现率↑ ⇨ 准备金↓ ⇨ 货币供给量↓

贴现率↓ ⇨ 准备金↑ ⇨ 货币供给量↑

贴现窗口主要满足临时准备金不足

该政策往往作为补充手段搭配公开市场操作

## ② 公开市场操作(最重要最常用,∴灵活)

央行在金融市场上公开买卖政府债券

以控制货币供给和利率的行为

调节货币供给量(eg.联储公开市场购买政府债券)

向个人或公司等非银行机构买入

联储开支票→卖方→银行→联储(准备金)

向银行买入

直接按债券金额增加银行在联储的准备金

## 调节利率

央行买入债券⇨债券价格↑⇨利率↓

## ③ 变动法定准备率

一般不轻易使用

银行向央行报告准备金和存款状况有时滞;

该作用十分猛烈

常配合使用

公开市场出售债券⇨利率↑

调高再贴现率⇨防止银行增加贴现

## 道义劝告

央行运用自己在金融体系中的特殊地位和威望,通过对银行及其他金融机构的劝告,影响其贷款和投资方向,以达到控制信用的目的

## 货币政策及其效果

### 5 货币政策操作方式和泰勒规则

曾经选用的中介目标

利率,货币供应量

#### 泰勒规则

在各种影响物价水平和经济增长率的因素中,真实利率是唯一能与物价和经济增长率保持长期稳定关系的变量

货币政策 $\Rightarrow$ 真实利率 $\Rightarrow$ 物价经济增长率

央行货币政策应遵循

假定存在'真实'的均衡利率,就业率和物价均保持在自然法则水平下

$i=i^* + a(p-p^*) - b(u-u^*)$ ;  $a>0, b>0$

$p$ 实际通胀率; $p^*$ 目标通胀率;

$u$ 实际失业率; $u^*$ 自然失业率;

$i$ 名义利率; $i^*$ 名义目标利率

选择通胀目标:

不仅考虑当前通胀,也考虑失业情况

与CFA对比

$R=r_n + \pi + a(\pi-\pi^*) + \beta(y-y^*)$

$R$ :central bank policy rate(implied by Taylor)

$r_n$ :netrual real policy int.rate

$\pi$ :current inflation rate

$\pi^*$ :central bank's target inflation rate

$y$ :log of current level of output

$y^*$ :log of central bank's target(sustainable) output

$a, \beta$ :policy response coefficient

real int.rate= $r=R-\pi=r_n+a(\pi-\pi^*)+\beta(y-y^*)$

### 7 货币政策的局限性

通胀时的紧缩效果好,衰退时的扩张效果不明显

反通胀主要表现于反需求拉动的通胀;

对成本推进的通胀,效果很小

看货币市场均衡:用货币供给影响利率,

必须保证货币流通速度不变

货币政策的外部时滞影响政策效果

变动货币供给量 $\Rightarrow$ 利率 $\Rightarrow$ 投资 $\Rightarrow$ 国民收入;

即使开始采用时间不长,但执行后到产生效果时间很长

开放经济中,还会因为资金在国际上流动而受到影响

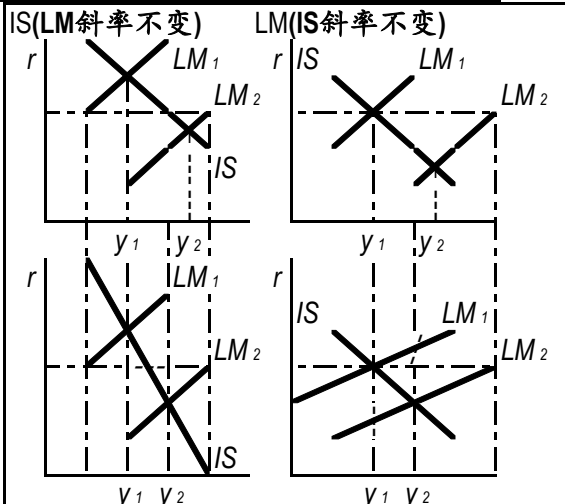
### 6 货币政策的IS-LM图形分析

货币政策的效果

变动货币供给量对总需求的影响

IS越平坦,LM移动时收入变化越大(LM不变)

LM越平坦,LM移动时收入变动越小(IS不变)



IS:  $r = \frac{\alpha + e}{d} - \frac{1 - \beta}{d} y$  LM:  $r = \frac{k}{h} y - \frac{m}{h}$

d:投资的利率系数 h:货币需求利率系数

原理:

IS陡峭	扩张性货币政策	LM平坦
$\downarrow$	$\downarrow$	$\downarrow$
$\downarrow$	货币供给量 $\uparrow$	$\downarrow$
d小	$\downarrow$	h大
$\downarrow$	利率 $\downarrow$	利率 $\downarrow$
$\downarrow$	$\downarrow$	$\downarrow$
投资对利率变动不敏感	货币需求对 $\Delta$ 利率敏感	
$\downarrow$		$\downarrow$
投资不会增加很多	利率下降幅度不大	货币需求 $\uparrow\uparrow$

### 8 货币政策案例:沃克尔紧缩货币治理通胀

背景:1979年,美国通胀率14.8%

措施:沃克尔出任联储主席

超常规紧缩银根;

80年代初联邦基金利率上升至20%

效果:1987年通胀率降至4.4%

美国经济进入30年代以来最大一次萧条

两种政策的混合使用

财政	货币	产出	利率	
扩张	紧缩	-	$\uparrow$	萧条但不严重
紧缩	紧缩	$\downarrow$	-	严重通胀
紧缩	扩张	-	$\downarrow$	不严重通胀
扩张	扩张	$\uparrow$	-	严重萧条

不仅看经济形势,还要考虑政治上的需要

不同政策可以对不同人群影响不同;

改变GDP组成比例

博弈论在宏观经济政策中的应用

货币政策的博弈论描述

政府(央行)vs工会

博弈规则

step.1.工会要求增加货币工资

增加名义工资or不增加名义工资

step.2.政府的相机抉择权

提高货币增长率or不提高货币增长率

货币政策的博弈模型

		政府			
		不增	增	不增	增
工会	不增	低通胀	高通胀	(5,5)	(1,7)
	增	低失业	低失业		
	增	低通胀	高通胀	(7,1)	(2,2)
		高失业	高失业		

时间不一致性

一项起初适合于今天的政策,随着时间的推移,就可能不再适合于明天

赢得信誉的可靠办法

借助于人人相信决策者必须遵从有约束力的

规则来消除政府可能改变政策的可能性

# 开放经济下的短期经济模型(2-1)

## 汇率和对外贸易

### 汇率及其标价

汇率  $A:B=2 \Leftrightarrow 2B/A$

一个国家货币折算成另

一个国家货币的比率

直接标价法(D/F,F/D)

用一单位外国货币作为标准,折算

为一定数额的本国货币的汇率

间接标价法(F/D,D:F)

用一单位本国货币作为标准,折算

为一定数额的外国货币的汇率

名义汇率

汇率高, 币值高

两个国家**通货**的相对价格

本章:e为名义汇率,间接标价F/D

### 汇率制度

固定汇率制

一国货币同他国货币的汇率基本固

定,其波动限于一定的幅度之内

浮动汇率制

一国不规定本国货币与他国货币的

官方汇率,听任汇率由外汇市场的

供求关系自发地决定

**自由浮动**:央行不采取任何措施

**管理浮动**:对外汇市场进行干预

### 自由浮动制度下汇率的决定(€为例)

使货币市场上对货币的供求达到均衡

横轴:€的数量;纵轴:€的价格(\$/€)

### 固定汇率制度的运行

联储宣布:2€/\$ 套利:2€ $\Rightarrow$ 1\$ $\Rightarrow$ 3€  $\Rightarrow$

均衡汇率:3€/ \$ 套利:1\$ $\Rightarrow$ 3€ $\Rightarrow$ 1.5\$

联储购买套利者的€ $\Rightarrow$ \$供给 $\uparrow$  $\Rightarrow$ 均衡汇率 $\downarrow$

### 实际汇率

两国产品的相对价格

实际汇率=  $\frac{\text{名义汇率} \times \text{国内产品价格}}{\text{外国产品价格}}$

实际汇率 $\epsilon$ =名义汇率 $\times$ 物价水平比率

$\epsilon=e \times (P/P_f)$ ; 物价水平比率:无单位

实际汇率 $\uparrow \Rightarrow$ 本币价值 $\uparrow$

$\Rightarrow$ 外国产品相对价值 $\downarrow$

### 净出口函数

出口:该国向其他国家销售产品和提供劳务

进口:该国从其他国家购买产品和劳务

净出口:出口额与进口额的差额

贸易顺差:净出口 $>0$ ;贸易逆差:净出口 $<0$

影响净出口的最主要因素(宏观经济学)

汇率

一国净出口反向地取决于实际汇率

国内收入水平

一国净出口反向地取决于一国的实际收入

出口不直接受一国实际收入的影响

### 净出口函数

$$nx = a - \gamma y - n\epsilon = a - \gamma y - ne \times \left( \frac{P}{P_f} \right)$$

$a, \gamma, n > 0, \gamma$ : 边际进口倾向

为强调汇率的影响,写作:  $nx = nx(\epsilon)$

### 货币市场与LM\*曲线(LM\*方程式)

用与IS-LM模型相似的方程:

LM:  $M/P = L(r, y)$

=: 实际货币供给  $M/P$  = 货币需求  $L(r, y)$

$r$ : 实际货币的需求反向地取决于利率

$\downarrow y$ : 正向地取决于收入  $y$

$M$ : 货币供给是由央行控制的外生变量

$P$ : MF模型分析短期波动

(令  $r=r_w$ ) **短期,假设物价水平是外生固定的**

LM\*:  $M/P = L(r_w, y)$

$r$

$r=r_w$

$y$

$e$

LM\*

$y$

LM连系  $r$  和  $y$ , 给定  $r_w \Rightarrow$

LM变为LM\* $\Rightarrow$ LM\*决定  $y$

汇率没有进入LM\*,不影响  $y$

## 蒙代尔-弗莱明模型

把对经济总需求的分析扩展到

开放经济环境中,建立的模型

**关键假设:资本完全流动的小型开放经济**

小型

所考察的经济是世界市场的一小部分,

从而其本身对世界某些方面,特别是利

率的影响微不足道

资本完全流动

该国居民可以完全进入世界金融市

场,该国政府**不阻止国际借贷**

$\Rightarrow$  利率该国=利率世界

( $r=r_w$ )

假设:国际资本流动之迅速足以使  $r=r_w$

### 开放经济的IS曲线(IS\*方程式)

在IS-LM模型上增加净出口:

IS:  $y = c(y) + i(r) + g + nx(\epsilon)$

消费正向取决于  $y$ ;

MF模型假设:

投资/净出口反向取决于  $r/\epsilon$

$\downarrow$

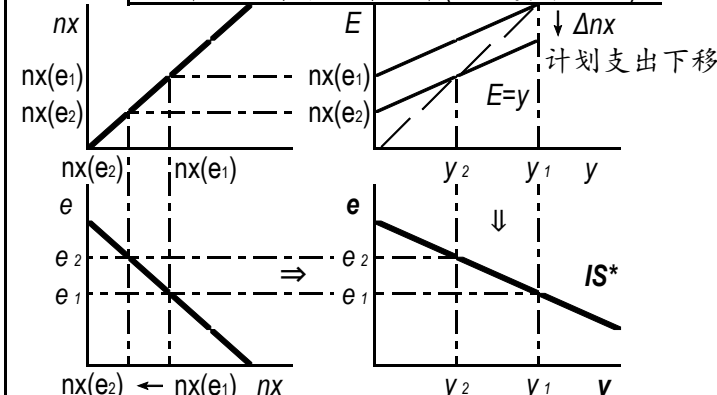
①国内物价水平和国外物价水平固定

②令  $r=r_w$

实际汇率与名义汇率是同比例的

IS\*:  $y = c(y) + i(r_w) + g + nx(\epsilon)$

\*:利率始终等于世界利率(完全资本流动)



### 蒙代尔-弗莱明模型

IS\*与LM\*联立

IS\*:  $y = c(y) + i(r_w) + g + nx(\epsilon)$

LM\*:  $M/P = L(r_w, y)$

产品市场均衡+货币市场均衡

$e$

LM\*

IS\*

$y$

外生变量:

财政变量  $g$ , 货币变量  $M$ ,

物价水平  $P$ , 世界利率  $r_w$

内生变量

收入  $y$ , 世界利率  $r_w$

IS\*曲线向右下方倾斜

高汇率净出口 $\downarrow$ 总收入 $\downarrow$



# 开放经济下的短期经济模型(2-2)

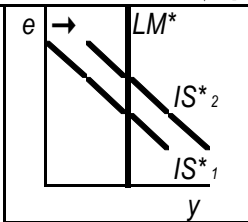
## 蒙代尔-弗莱明模型的应用

### 浮动汇率制下的财政政策和货币政策

允许汇率对经济状况的变动做出反应  
汇率调整  $\Rightarrow$  产品&货币市场同时均衡

#### 财政政策

政府购买  $\uparrow \Rightarrow$  计划支出  $\uparrow \Rightarrow$   
 $\Rightarrow$  IS\* 右移  $\Rightarrow$  汇率  $\uparrow$ , 收入水平不变  
 $\Rightarrow$  IS 右移  $\Rightarrow$  利率  $\uparrow$ , 收入水平不变



关键变量: 利率 & 汇率

#### ① 封闭经济中

收入  $\uparrow \Rightarrow L \uparrow \Rightarrow$  利率  $\uparrow$

#### ② 小型开放经济中

收入  $\uparrow \Rightarrow$  利率  $\uparrow \Rightarrow r > r_w \Rightarrow$  资本  
购买本币,  $\Leftarrow \Leftarrow \Leftarrow$  流入  $\Rightarrow r \downarrow$   
本币需求  $\uparrow \Rightarrow$  本币升值, 净出口  $\downarrow$

财政政策完全失效的原因:

LM:  $M/P = L(r, y)$

#### ①和②中

央行固定  $M \Rightarrow$  实际货币  
价格黏性  $\Rightarrow P$  固定  $\Rightarrow$  余额供给量  
LM 均衡: 需求 = 供给  $\Leftarrow$  固定

封闭: 财政扩张  $\Rightarrow$  均衡  $r \uparrow$  + 均衡  $y \uparrow$

$L$  不变  $\Leftarrow \Leftarrow \Leftarrow L \downarrow \Leftarrow \Leftarrow L \uparrow$

开放: 财政扩张  $\Rightarrow r \equiv r_w \Rightarrow$  收入不变

汇率和净出口变动

完全抵消财政政策的影响

① 非相依乘数越大, 相依乘数越大

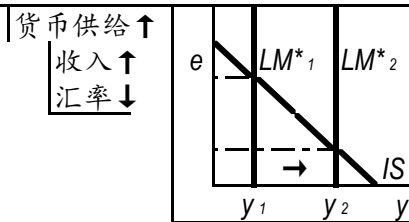
② 相互依赖提高每个地区乘数, 各地自主支出  $\uparrow$

③ 进口倾向低, 倾向具有较高收入(非相依乘数)

北方具有持久的收入优势:  $m_n$  低

#### 货币政策

央行增加  $M \uparrow \Rightarrow$  实际货币  
价格黏性  $\Rightarrow P$  固定  $\Rightarrow$  余额供给量  
LM\* 向右移动  $\Leftarrow \Leftarrow$  增加



关键变量: 利率 & 汇率

#### ① 封闭经济中

$M \uparrow \Rightarrow$  利率  $\downarrow \Rightarrow$  投资  $\uparrow$

#### ② 小型开放经济中(通过汇率而非利率影响 $y$ )

$M \uparrow \Rightarrow$  利率  $\downarrow \Rightarrow r < r_w \Rightarrow$  资本  
抛售本币,  $\Leftarrow \Leftarrow \Leftarrow$  流出  $\Rightarrow r \uparrow$   
本币供给  $\uparrow \Rightarrow$  本币贬值, 净出口  $\uparrow$

### 南-北关系的一种经济分析

北方: 工业化, 技术先进, 出口工业制成品

南方: 农业, 技术相对落后, 出口初级产品

两种思路 ① 对称方法; ② 非对称方法

① 在同一基本结构中按同一模型运行

② 各自具有根本区别的结构 具体参数不同

用对称方法分析南-北关系

北方收入恒等式  $y_n = \beta_n y_n + a_n + x_n - m_n y_n$

南方收入恒等式  $y_s = \beta_s y_s + a_s + x_s - m_s y_s$

$\downarrow$  互为进出口国:  $x_n = m_s y_s, x_s = m_n y_n$

$y_n = \beta_n y_n + a_n + m_s y_s - m_n y_n$

$y_s = \beta_s y_s + a_s + m_n y_n - m_s y_s$

未知量: 两地产出水平(相互依赖):

$$y_n^* = \left( \frac{k_n}{1 - k_n k_s m_s m_n} \right) (a_n + m_s k_s a_s)$$

$$y_s^* = \left( \frac{k_s}{1 - k_s k_n m_n m_s} \right) (a_s + m_n k_n a_n)$$

$y$ : 产出;  $\beta$  边际消费倾向;  
 $a$  投入在自己生产中的实际支出;  
 $x$  实际出口;  $m$  边际进口倾向

开放经济乘数  $k$  (非相依乘数):

$$k_n = \frac{1}{1 - \beta_n + m_n}$$

$$k_s = \frac{1}{1 - \beta_s + m_s}$$

划线部分为  
相依乘数;  
相依乘数  $>$   
非相依乘数

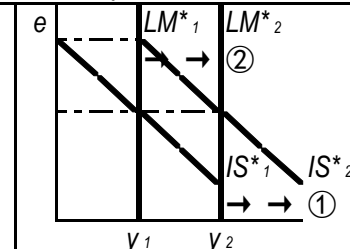
### 固定汇率制下的财政政策和货币政策

#### 财政政策

汇率  $\uparrow$ : 本币升值压力

政府购买  $\uparrow \Rightarrow$  IS\* 右移  $\Rightarrow$  汇率  $\uparrow \Rightarrow$  套利者卖出外汇

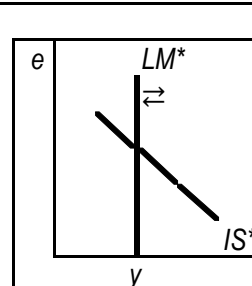
LM\* 右移,  $y$  上升  $\Leftarrow$  本币供给扩张  $\Leftarrow$  央行买入外汇



#### 货币政策(名义货币政策无效)

央行增加  $M \Rightarrow$  LM\* 右移  $\Rightarrow$  汇率  $\downarrow \Rightarrow$  套利者卖出本币

LM\* 左移,  $y$  不变  $\Leftarrow$  本币供给减少  $\Leftarrow$  央行买入本币



央行可以决定改变所固定的汇率水平

货币贬值: 通货的官方价值的下跌

货币升值: 通货的官方价值的上升

类似 货币贬值下:

LM\* 右移  $\Rightarrow$  本币贬值, 净出口  $\uparrow$

#### 小结

	浮动汇率			固定汇率		
	$y$	$e$	$nx$	$y$	$e$	$nx$
财政扩张	-	$\uparrow$	$\downarrow$	$\uparrow$	-	-
货币扩张	$\uparrow$	$\downarrow$	$\uparrow$	-	-	-

浮动汇率  
财政被汇率 & 净出口抵消  
固定汇率  
货币全部用于维持汇率

### 小型开放经济的总需求曲线(打开物价不变假设)

IS\*:  $y = c(y) + i(r_w) + g + nx(e)$

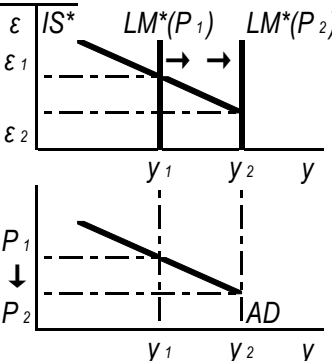
LM\*:  $M/P = L(r_w, y)$

实际汇率  $\varepsilon = eP/P_f$ ; 名义汇率  $e$

给定价格:

收入  $\uparrow \Rightarrow$  AD 右移

收入  $\downarrow \Rightarrow$  AD 左移



IS\* 不含  $P$ ,  $\therefore$  IS 固定, 移动 LM\*

# 经济增长(3-1)

## 国民收入长期趋势和波动

本章之前:短期宏观经济分析  
 解释和分析经济波动  
 本章:长期宏观经济分析  
 解释和分析经济体潜在产出的演进

## 经济增长的描述和事实

经济增长和经济发展  
**经济增长**:产量的增加(总量或人均)  
**经济发展**:反映一个社会经济总体发展水平的综合性概念  
 变量Z从t到t+Δt的增长率  

$$g_z = \frac{Z(t + \Delta t) - Z(t)}{\Delta t Z(t)}$$
**变量Z在时间t的瞬时增长率**  

$$g_z = \frac{\dot{Z}(t)}{Z(t)} = \frac{d \ln Z(t)}{dt}$$
**三个结论**  
 $Z(t) = X(t) \cdot Y(t) \Rightarrow g_z = g_x + g_y$   
 $Z(t) = X(t) / Y(t) \Rightarrow g_z = g_x - g_y$   
 $z(t) = [X(t)]^a \Rightarrow g_z = a \times g_x$   
 总产量意义下的增长率:  

$$g_Y = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$
 人均产量意义下的增长率:  

$$g_y = \frac{y_t - y_{t-1}}{y_{t-1}}$$

## 经济增长和发展的一些事实

### 经济增长的基本问题

- 基本问题
- ① 为什么一些国家富裕,一些贫穷?
  - ② 什么是影响经济增长的因素?
  - ③ 怎样理解一些国家的经济增长奇迹?

### 分析方法

#### 增长核算

把产量增长的不同决定因素的贡献程度数量化

#### 增长理论

把增长过程中生产要素供给,技术进步,储蓄和投资互动关系模型化

## 增长核算

### 经济增长的决定因素

#### 直接原因

与经济中的投入要素,如资本和劳动的积累有关,还与能够影响这些生产要素生产率的变量,如规模经济和技术变化有关

#### 基本原因

对一国积累生产要素的能力以及投资于知识生产的能力产生的变量  
 人口增长,金融部门影响力,宏观环境等

$$Y_t = f(K_t, R_t, N_t, A_t, B_t)$$

**Y**经济总产出,**K**资本存量,**R**自然资源,  
**N**劳动投入,**A**该经济应用知识的储量,  
**B**基本因素(社会文化环境/社会能力)

### 增长核算方程

经济的生产函数:

$$Y = AK^a N^{1-a}, 0 < a < 1$$

**Y**总产出,**A**技术状况,  
**N**投入劳动量,**K**投入资本量

### 增长核算方程

$$g_Y = g_A + a g_K + (1 - a) g_N$$

总产出增长率 **g<sub>Y</sub>**;  
 技术增长率 **g<sub>A</sub>**(全要素生产率);  
 资本增长率 **g<sub>K</sub>**; 劳动增长率 **g<sub>N</sub>**

产出增长=生产率增长的贡献+资本增长的贡献+劳动增长的贡献

全要素生产率(索洛余量):

$$g_A = g_Y - a g_K - (1 - a) g_N$$

### 增长的经验估算

### 经济增长因素分析(E.F.Denison)

#### 经济增长因素分类

生产要素投入量  
**劳动**(可变),**资本**(可变),**土地**(不变)  
 生产要素生产率(产出/投入量)  
**资源配置状况**,  
**规模经济**,  
**知识进展**,  
 技术知识,管理知识  
**其他因素**

## 促进经济增长的政策

根据增长核算方程,政府可决定:

**技术进步**; **资本形成**; **劳动投入**

### 投入量

#### 鼓励技术进步

专利制度;教育;  
 为进行研发的企业提供税收减免

#### 鼓励资本形成:

鼓励储蓄和投资  
 资本存量增长由储蓄和投资推动

#### 增加劳动供给

人力资本  
 教育,培训体系

### 生产效率

#### 建立适当的制度

各国指导稀缺资源配置的制度不同  
 制度:能支配个人和企业行为的一套规则,体制和惯例  
 影响经济增长最基础最根本的制度:产权  
 维护产权的基本方式:法制

### 总国民收入增长的源泉,1929-1982年

增长因素	增长率(%)	占比	举例或说明
总要素投入	1.90	65%	
劳动	★ 1.34	46%	
资本	0.56	19%	
单位投入的产量	1.02	35%	
知识进展	★ 0.66	23%	是发达资本主义国家最重要的劳动力从农村到城市就业小规模时技术的效率可能不高
资源配置	0.23	8%	
规模经济	★ 0.26	9%	
其他	-0.13	-4%	
国民收入	2.93	100%	



# 构建和分析新古典增长模型(20世纪50-60)

模型建立在新古典生产方程体系上,封闭且没有政府的经济中,储蓄&人口增长&技术进步对增长的作用

关注焦点:直接原因

## 1 基本假定

- ① 经济由一个部门组成,该部门生产一种既可用于投资也可用于消费的商品
- ② 生产的规模报酬不变
- ③ 技术进步&人口增长&资本折旧速度由外生因素决定
- ④ 社会储蓄函数 $S=sY$ , $s$ 为储蓄率

## 4 稳态及其条件

稳态:包括资本存量和产出在内的有关内生变量将不会随时间的推移而变化的一种状态

新古典增长模型稳态条件:  $\dot{k} = 0$

没有技术进步的情况:

$$sf(k) = (n + \delta)k$$

有技术进步的情况:

$$sf(\hat{k}) = (n + \delta + a)\hat{k}$$

图形表示:投入曲线与消耗曲线交点

新古典增长模型的稳态是稳定的

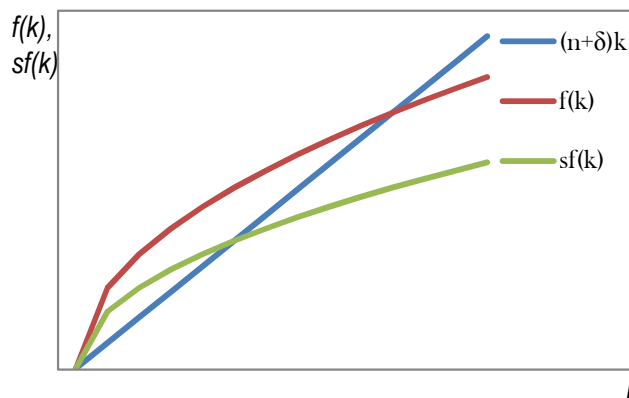
实际资本水平<稳态水平

实际水平在交点左边

$\Rightarrow s(f) > (n + \delta)k \Rightarrow$  资本存量  $\uparrow$

与资本的边际报酬递减密切相关

人均资本,人均产量,人均储蓄稳态值均为常量



$$dk/dt = \text{投入量(储蓄)} - \text{消耗量(折旧+人口} \uparrow \text{+技术} \uparrow \text{)}$$

## 2 没有技术进步的新古典增长模型

生产函数

$$Y = F(N, K)$$

总产出Y,总量劳动N,总量资本K

$\downarrow$  均随时间变化而变化

$$\lambda Y = F(\lambda N, \lambda K), \text{取} \lambda = 1/N$$

$$Y/N = F(1, K/N)$$

假定全部人口参与生产,则:

$\downarrow$  人均产量Y/N只依赖于K/N

人均产量 $y = Y/N$ ,人均资本 $k = K/N$

$y = f(k)$ ,生产函数人均形式

$$f(k) = F(1, K/N)$$

资本积累收到的影响&基本方程推导

投资 $I = S$ ;折旧(固定比率 $\delta \in (0, 1)$ )

$$\dot{K} = I - \delta K = S - \delta K = sY - \delta K$$

$\downarrow$  两边同除以N

$$\frac{\dot{K}}{N} = \frac{sY}{N} - \frac{\delta K}{N} = sy - \delta k = sf(k) - \delta k$$

$\downarrow$  对k关于时间变量t求导

$$\frac{\dot{K}}{N} = sf(k) - \delta k$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\dot{k} = \frac{dk}{dt} = \frac{NdK - KdN}{N^2 dt} = \frac{dK}{Ndt} - \frac{K}{N} \left( \frac{dN}{dt} \cdot \frac{1}{N} \right) = \frac{\dot{K}}{N} - nk$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\dot{k} = sf(k) - (n + \delta)k \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow$$

$$\frac{\dot{K}}{N} = \dot{k} + nk$$

新古典增长模型的基本方程

人均资本变化=人均储蓄 $-(n + \delta)k$

$|(n + \delta)k$ :必要的,临界的投资

劳动力增长 $nk$ +折旧 $\delta k$

资本深化=人均储蓄(投资)-资本广化

注意:假设中全部人口生产, $\therefore n$ =人口增长率

新古典增长模型的图形表示

(图中折线部分为作图所致,本应平滑)

横轴k;纵轴Y

直线: $(n + \delta)k$

下方曲线(投入量): $sf(k)$

上方曲线(消耗量): $f(k)$

## 3 具有技术进步的新古典增长模型

生产函数

$$Y = F(AN, K)$$

AN:有效劳动

$\downarrow$  更改假定:Y是K和AN的一次齐次函数

$$\lambda Y = F(\lambda AN, \lambda K)$$

按有效劳动平均的产量/资本

$$\downarrow \quad \hat{y} = \frac{Y}{AN}, \hat{k} = \frac{K}{AN}$$

$$\hat{y} = f(\hat{k})$$

技术进步是外生给定的

假定A以固定比例a增长:  $\dot{A}/A = a$

基本方程推导

$$k = \frac{K}{AN}$$

$$\frac{\dot{N}}{N} = n, \frac{\dot{A}}{A} = a$$

$\downarrow$  两边关于时间变量t求导

$$\begin{aligned} \dot{k} &= \frac{AN}{AN^2} \dot{K} - \frac{K}{AN^2} [A\dot{N} + \dot{A}N] \\ &= \frac{\dot{K}}{AN} - \frac{K}{AN} \cdot \frac{\dot{N}}{N} - \frac{K}{AN} \cdot \frac{\dot{A}}{A} \leftarrow n \quad a \\ &= \frac{sY - \delta K}{AN} - \hat{k} \cdot n - \hat{k} \cdot a = \frac{sY}{AN} - (\delta + n + a)\hat{k} \end{aligned}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\dot{\hat{k}} = sf(\hat{k}) - (n + \delta + a)\hat{k}$$

具有技术进步的新古典增长模型的基本方程

每单位有效劳动的资本存量的变化

=每单位有效劳动的实际投资量-投资平衡水平

必须维持一定投资水平的原因:

磨损折旧

劳动(人口)和知识的增长,有效劳动不断增加

## 5 稳态时相关内生变量的增长率(稳态增长率)

没有技术进步

内生变量	g
K/N	0
Y/N	0
K	n
Y	n

有技术进步

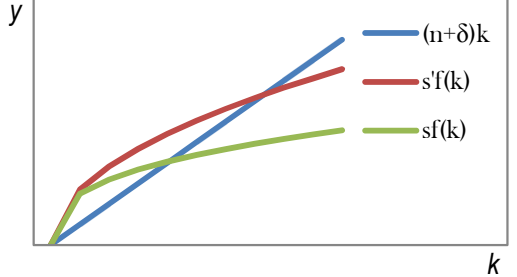
内生变量	g	内生变量	g
K/N	a	K/AN	0
Y/N	a	Y/AN	0
K	n+a		
Y	n+a		

应用新古典增长模型

对收入差异的解释

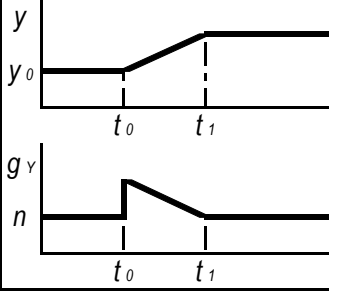
进行比较静态分析,考虑储蓄率增加和人口增长对经济稳态的影响

储蓄率增加



储蓄率↑⇒稳态的人均资本↑人均产量↑

短期:更高的储蓄率⇒总产量↑人均产量↑  
上图两个交点都是稳态,稳态中总产量增长率独立于储蓄率



新古典增长理论的结论(与现实吻合)

储蓄率的增加不能影响到稳态增长率,但能提高收入的稳态水平

只有水平效应,没有增长效应

对增长率差异的解释

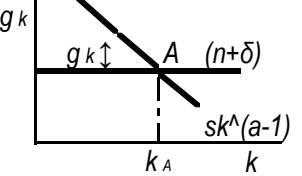
在特定生产函数 $y=k^a$ 假定下

$$\dot{k} = sk^a - (n + \delta)k, 0 < a < 1$$

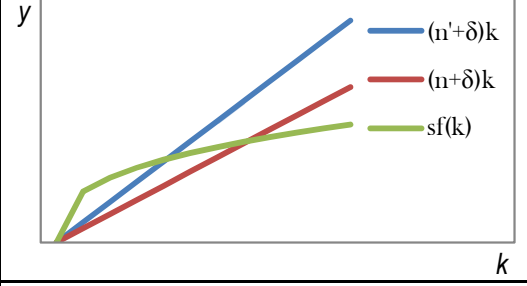
↓ 两边同除以k

$$g_k = \dot{k}/k = sk^{a-1} - (n + \delta)$$

人均资本增长率方程



人口增加



新古典增长理论的结论(与现实吻合)

人口增长率↑⇒(n+δ)k上移至(n'+δ)k  
⇒稳态的人均资本↓人均产量↓

另一个结论:

人口增长率↑⇒总产量的稳态增长率↑  
人均产量的下降<人口增长率的增加

资本的黄金律水平

如何处理积累和消费

人均消费最大化:资本的黄金律水平

$$f'(k_{gold}) = n + \delta$$

稳态人均资本量的选择应使:

资本的边际产品=劳动增长率+折旧率

对应的储蓄率:储蓄率的黄金律水平

内生增长理论

基本模型(AK模型)

生产函数: $Y=AK$ ;资本存量K,A是常量

不存在资本边际收益递减(与新古典区别)

资本积累

$$\Delta K = sY - \delta K$$

$$\Delta Y/Y = \Delta K/K = sA - \delta$$

只要 $sA > \delta$ ,没有外生技术进步也会增长

关于放弃资本边际收益递减的讨论

资本边际收益会递减

若K只包括厂房,设备存量等

资本边际收益不会递减

若知识也视为一种资本

内生增长理论:知识收益递增

如果可以被累积的生产要素有固定报酬,则  
稳态增长率将被这些要素的积累率所影响

两部门模型

制造业企业:生产物品和劳务,可用于消费和投资;

研究性大学:生产知识(生产要素),供两部门免费使用

$$Y = F[K, (1-u)EN]$$

$$\Delta E = g(u)E$$

$$\Delta K = sY - \delta K$$

u 大学的劳动力比例;(1-u)企业的劳动力比例;

E 知识存量;(1-u)EN 有效工人;g(u)知识增长取决于u

一般,假设企业的生产函数是规模收益不变

$$\lambda Y = F[\lambda K, \lambda(1-u)EN]$$

该模型除大学生产函数外,与新古典模型一致

若u不变,则E按不变比率g(u)增长(与新古典一致)

新古典增长模型三个预言

- ① 两国储蓄率(或投资率)相同,则初始人均资本较低的国家有更高的经济增长
- ② 两国初始人均资本相同,投资率较高的国家有更高的经济增长
- ③ 如果一国提高投资水平,则收入增长率也会提高

新古典增长模型分析相对经济增长率的关键在于考察未处于稳态水平的经济

∴ 稳态下,经济增长率=n

消费

消费:一国居民对本国和外国生产的最终产品和服务的支出

总支出的最大组成部分

跨期消费决策(1930,欧文·费雪)

假设

两个时期(青年,老年);  
时期1:收入 $y_1$ ,消费 $c_1$ ;  
时期2:收入 $y_2$ ,消费 $c_2$ ;  
没有通货膨胀;可以借贷或储蓄

① 跨期消费的预算约束

时期1:储蓄 $s=y_1-c_1$   
时期2:消费 $c_2=(1+r)s+y_2$   
 $r$ :实际利率;  
无时期3,∴消费者时期2不储蓄和借贷,而是花光所有积蓄

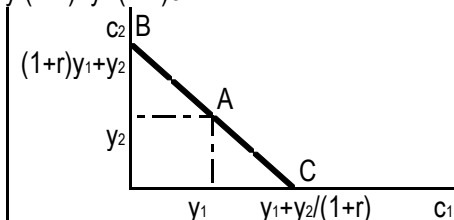
$$c_2 = (1+r)(y_1 - c_1) + y_2$$

↓

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

消费者跨期消费预算约束(标准方法)

$\Sigma$ 两期消费现值= $\Sigma$ 两期收入现值  
 $c_2 = y_1(1+r) + y_2 - (1+r)c_1$



② 跨期消费的消费者偏好

斜率表示两期消费之间的边际替代率;  
偏好具有非饱和性

跨期消费最优决策行为

必须满足条件:

必须是消费者最偏好的两期消费组合;  
必须位于给定的预算约束线上

无差异曲线与预算线的切点

边际替代率=预算约束线的斜率

$$MRS = -(1+r)$$

凯恩斯消费理论

人们在特定时期的消费与该时期的可支配收入相关

收入变动对消费的影响

$y_1 \uparrow$  or  $y_2 \uparrow$ : 预算约束线外移  
所消费的商品都是正常品

消费平稳化

无论哪期收入增长,消费者都把它分摊到两个时期的消费上去

收入的现值= $y_1 + y_2/(1+r)$

消费取决于 $\Sigma$ 两期收入现值,是以消费者预期在其一生中所得到的资源为基础

实际利率变动对消费的影响

$r \uparrow \Rightarrow$  预算约束线围绕 $(y_1, y_2)$ 顺时针旋转

收入效应: 利率上升使得消费者收入增加,从而可以向更高的无差异曲线移动,并把增加的收入分摊到两期消费中

$r \uparrow \Rightarrow$  收入  $\uparrow \Rightarrow c_1 \uparrow, c_2 \uparrow$

替代效应: 两期消费的相对价格变动造成两期消费量的变动

$r \uparrow \Rightarrow c_1$  的机会成本  $\uparrow \Rightarrow c_1 \downarrow$

两个效应都增加 $c_2$

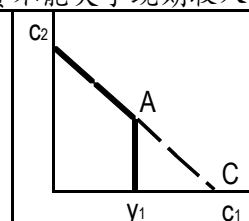
两个效应对 $c_1$ 影响相反

借贷约束/流动性约束

借贷限制

现期消费不能大于现期收入

$$c_1 \leq y_1$$



消费者希望 $c_1 < y_1$

借贷约束没有约束力

消费者希望 $c_1 > y_1$

借贷约束有约束力(角点解)

消费的随机游走假说(1978, 罗伯特·霍尔)

假设:

永久收入假说正确;  
消费者理性预期

结论

消费随时间推移而发生的变动就是不可预测的(随机游走)

相对收入消费理论(杜森贝利)

消费者会受自己过去的消费习惯和周围消费水准的影响来决定消费

长期消费函数: $c = \beta y$

长期: 消费与收入比率固定

棘轮效应: 消费者易随收入的提高增加消费, 不易随收入降低而减少消费

示范效应: 消费者的消费行为受周围人消费水准

生命周期的消费理论(莫迪利安尼)

侧重对储蓄动机的分析(财富作为变量)

人们会在更长时间范围内计划消费开支, 以达到整个生命周期内的最佳配置

$$C = aWR + cYL$$

$WR$  实际财富;  $a$  财富的边际消费倾向;

$YL$  工作收入;  $c$  工作收入的边际消费倾向

总储蓄和总消费部分地依赖于人口年龄分布

年轻人 & 老年人比例大, 消费倾向大;

中年人比例大, 消费倾向小

侧重个人如何预测自己未来收入

永久收入的消费理论(弗里德曼)

消费者的消费支出由永久收入决定:  $C = cY_P$

永久收入: 消费者可以预计到的长期收入

大致可根据观察到的年收入数值的加权平均数计得

$$Y_P = \theta Y + (1-\theta)Y_{-1}$$

距现在时间越近, 权数越大

$Y_P$  永久收入;  $\theta$  权数

$$C = cY_P = c\theta Y + (1-\theta)cY_{-1}; \text{当前收入的MPC} = c\theta < c$$

短期边际消费倾向较低的原因:

收入  $\uparrow \Rightarrow$  人们不能确信收入的增加是否持续

$\Rightarrow$  不会立刻充分调整其消费

只有收入变动最终证明是永久的, 人们才会在永久收入上充分调整其消费

政府的税收政策无法影响总需求

减税带来的收入增加, 不会立刻用于消费

单个消费者是前向预期决策者

消费不只同现期收入相关;

一次性暂时收入变化引起的消费支出变动很小;

政府增减税若只是临时的, 消费不会受到很大影响

投资

企业固定资产投资

企业固定资产投资(=Δ资本存量)

企业购买用于生产的机器设备和建筑物的活动

企业固定资产的存量(资本存量)

企业在生产和服务中使用的机器设备和建筑物

新古典投资模型

企业分类

生产企业(生产职能)

租赁资本以生产产品和提供服务

租赁企业(投资职能)

从事固定资产投资,把资本转租给生产企业

最优资本存量的决定

生产企业(使用资本和劳动)

二者可以相互替代

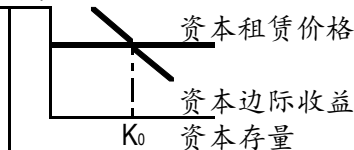
资本存量的决定

租金率=R,产品价格P

每单位资本实际成本=R/P

MR=MC,MR=MP×P,MC=R

MP=R/P



资本租赁价格由供求决定

需求:Σ生产企业资本边际收益曲线

供给:垂直线(市场上资本存量固定)

租赁企业(购买资本品并转租)

收益:出租每单位资本的实际租赁价格R/P

成本:利息成本;价格波动成本;资本折旧成本

单位资本的成本 =  $P_K(i - \pi^e + \delta) = P_K(r + \delta)$

单位资本价格PK;名义利率i;预期通胀率π

折旧率δ;实际利率r

存货投资

存货:企业持有作为储备的产品

占总支出很小,但在经济周期中变动很大,成为经济波动研究的重点

企业持有存货的原因

保证生产的平稳化

避免脱销

提高经营效率

在产品

加速模型

假设:企业存货量与产出水平成比例

$N = aY \Rightarrow I = \Delta N = a\Delta Y$

存货量N,产量Y

最优资本存量的动态调整

企业实际投资存量≠最优资本存量

⇒迅速调整成本较高⇒逐步调整

现期结束的资本存量

$K = K_{-1} + \lambda(K^* - K_{-1}), 0 < \lambda < 1$

$K_{-1}$ 上期结束时的资本存量;

$K^*$ 最优资本存量

对投资波动的理解:

$K^*$ 本身的不稳定性

K不断趋向于 $K^*$

以柯布-道格拉斯生产函数为例

柯布-道格拉斯生产函数

$Q = AK^aL^{1-a}, 0 < a < 1$

↓ Q产量;a资本在产出中所占份额

↓ K&L资本和劳动的投入量

$MP_K = aA(L/K)^{1-a}$  资本的边际产量

↓ 均衡状态下,

↓ 资本实际租赁价格R/P=资本边际产量

$R/P = aA(L/K)^{1-a}$

资本存量低,劳动投入量高,技术水平高

⇒资本实际租赁价格高

住房投资

购房目的:自住,收租金,增值

对住房投资的分析

存量均衡;流量供给

住房相对价格取决于现有住房存量的供求

供给是固定的,垂直线;

需求右下方倾斜,位置取决于:

① 人们的财富↑,住房需求↑

② 拥有住房的真实净收益↑,住房需求↑

总收益:自住的隐性收益,出租租金,

增值的资本收益

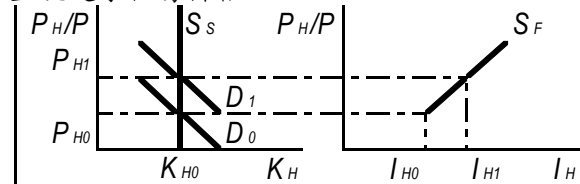
总成本:抵押贷款,物业费,物业费,折旧

③ 其他资产的真实净收益↑,住房需求↓

新住房供给的决定

成本取决于平均价格水平

收益决定于住房价格



纵轴为住房相对价格

短期中,住房存量市场的供求决定住房价格

长期中,新住房投资会将 $S_s$ 右移

两个结论

长期中,如果人口或财富以固定比率增长,

均衡状态的投资率必须能够补偿折旧率

和住房存量的增长率

因为时滞,住房供给实际上是对预期住房

建成后的价格作出反应

投资:购置物质资本的活动

① 固定资产投资:在新资本品上的支出

企业固定资产投资;住房投资

② 存货投资:公司持有存货的增加



## 货币需求

### 建立货币需求模型的思路

影响货币需求的宏观经济变量

价格水平↑, 货币需求↑

实际收入↑, 货币需求↑

利率↑, 货币需求↓

利率是非货币资产预期收益

货币需求函数

$$M^d = P \cdot L(Y, r)$$

↓ 货币需求与价格同比例变化

$$M^d/P = L(Y, r) \quad \text{货币需求函数}$$

实际货币需求(实际余额)

用货币所能购买的各种产品来测量货币需求量

影响货币需求的其他因素

预期股票收益(negative)

预期债券收益(negative)

预期通胀率(negative)

实际财富(positive)

资产组合理论

$$M^d/P = L(r_s, r_b, \pi^e, W)$$

把货币作为资产组合的一部分, 货币提供不同于其他资产的风险与收益的组合, 货币本身没有收益

强调货币价值储藏手段

### 货币需求的交易理论

强调持有货币是为了进行购买, 通过权衡持有货币的成本和收益, 来决定持有的货币量

#### 鲍莫尔-托宾模型

持有货币是有机会成本的

购买生息资产获得的利息

持有货币是为了交易方便

收益在于减少交易成本

假设

消费者一年中计划逐渐支出Y元; 物价水平不变; 支出均以现金进行

过程

① 消费者去N次银行/年;

每次提取Y/N美元

② 在一年的第1/N时期中逐渐花完

货币持有量



假定

消费者每次去银行成本为F;

推导

去银行总成本=FN; 放弃的利息=rY/(2N)

持有的平均货币量=Y/(2N);

总成本:

$$C = rY/(2N) + FN$$

两边对N求导

$$N^* = (rY/2F)^{1/2}$$

平均货币持有量

$$= \frac{Y}{2N^*} = (YF/2r)^{1/2}$$

更广义的理解

r: 货币资产与非货币资产的收益差;

F: 非货币资产转换为货币资产的成本

货币需求正向取决于支出, 反向取决于利率



新货币数量论

费雪:交易方程,1911

$$Py=MV$$

价格水平P;流通中的货币数量M;  
实际国民收入y;货币流通速度V

V,y被视为常量

V由支付习惯,交通通信等制度  
上的因素决定,短期内不会变化

y取决于资源,技术条件(充分就  
业下,不会变化)

$$P \propto M$$

强调货币交易媒介,流通手段作用

庇古:剑桥方程,1917

$$M=kY-kPy$$

名义or实际国民生产总值Y or y;  
k:经常持有的货币量

k,y被视为常量

k取决于商业习惯和制度等因素;  
y在充分就业均衡时是已知常数

$$P \propto M$$

强调货币贮藏手段,货币持有方面  
(货币需求)

相同点: $P \propto M$ ;不同点:强调点不同

凯恩斯:引入灵活偏好

$$M/P=L(y,r)=L_1(y)+L_2(r)$$

货币主义对凯恩斯的批评

只注意到利息率和收入对货币需求  
的影响,忽略人们对财富的持有量  
也是决定货币需求的重要因素

自然率假说

任何资本主义社会都存在一个自然失  
业率,大小取决于该社会的技术水平,资源数  
量和文化传统.长期中,该社会的经济总是  
趋向于自然失业率

自然失业率=摩擦性失业+自愿失业

资本主义长期中不存在非自愿失业

弗里德曼的货币需求函数

$$M = f \left( P, r_b, r_e, \frac{1}{P} \cdot \frac{dP}{dt}, w, Y, u \right)$$

M财富持有者手中保存的名义货币量;

P价格水平; $r_b$ 债券利息率; $r_e$ 股票收益率;

$\downarrow (1/P)/(dP/dt)=r_p$ 预期的物质资产的收益率;

Y名义收入;u其他因素; 价格预期变动率

w非人力财富与人力财富之比

价格及货币收入单位变化,货币需求应同比例变化

$\downarrow$  方程是P与Y的一次齐次式

$$f(\lambda P, r_b, r_e, r_p, w, \lambda Y, u) = \lambda f(P, r_b, r_e, r_p, w, Y, u)$$

$\downarrow$  令 $\lambda=1/P$

$$\frac{M}{P} = f(r_b, r_e, r_p, w, y, u)$$

对实际货币需求的关系

货币需求取决因素:

总财富

其中之一是个人挣钱能力(永久收入y)

非人力财富在总财富中的比例

人力财富转为非人力财富比较困难;

非人力财富占比越小,对货币需求越大

越缺钱,越需要钱

各种非人力财富的预期报酬率

预期报酬率越高,货币需求越小

其他因素

$$\frac{M}{Y} = f \left( r_b, r_e, r_p, w, \frac{P}{Y}, u \right)$$

货币流通速度的倒数

新货币数量论

货币流通速度不是常数,稳定的是函数,而  
不是函数值

市场出清

劳动市场的工资和产品市场的  
价格都具有充分的灵活性,可以  
根据供求迅速调整

不存在超额供给

理论观点

- ① 货币供给对名义收入变动具有决定性作用  
货币供给与影响货币需求的因素完全无关
- ② 长期中,货币数量的作用主要在于影响价  
格及其他用货币表示的量,而不能影响就  
业量和实际国民收入
- ③ 通货膨胀归根到底是一种货币现象  
短期中,货币供给量可以影响实际变量  
V在短期中可以有轻微变动
- ④ 私人经济具有内在稳定性,国家经济政策  
会破坏这一稳定性

政策主张

反对凯恩斯主义的财政政策

挤出效应,非生产性投资代替生产性投资

反对斟酌使用的货币政策

政策的滞后性

力主单一政策规则

以货币供给量作为货币政策的唯一控制  
指标,排除利率,信贷流量,准备金等因素

货币政策能够胜任的任务(弗里德曼)

- ① 防止货币成为经济混乱的主要根源
- ② 给经济提供一个稳定的环境

新古典宏观经济学的基本假设

个体利益最大化

宏观经济理论必须有微观经济理论的基础

理性预期

理性预期:在有效地利用一切信息的前  
提下,对经济变量作出的在长期中平均  
说来最为准确的,而又与所使用的经济  
理论,模型相一致的预期

- ① 作出经济决策的经济主体是有理性的
- ② 为了作出正确预期,经济主体会力图得到有关的一切信息
- ③ 经济主体在预期时不会犯系统性错误  
长期中,经济主体对某一经济变量的未  
来预期值与未来实际值会一致

自然率

## 实际经济周期理论

总产量的不稳定性

源于实际冲击而非货币冲击

作为波动源的技术冲击

技术变化

任何使生产函数发生移动,而不涉及投入要素数量变化的因素

用以解释经济增长

劳动生产率的变动

## 基本理论

假设:人口和劳动力固定

## 总量生产函数

$$y = zf(K)$$

实际收入 $y$ ,资本存量 $K$ ,技术状况 $z$

折旧率 $\delta \Rightarrow$ 未折旧资本存量 $(1-\delta)K$

经济中可利用资源 $=zf(K)+(1-\delta)K$

当期产量+未折旧资本存量

假定每个人具有相同偏好(存在代表)

代表的偏好(仅依赖于)

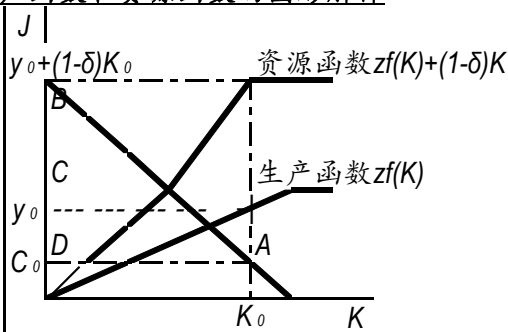
可延续未来无限期的每年的消费

消费中获得的边际效用递减

$\therefore$ 在整个生命期内均匀地消费

## 总资源函数 $=zf(K)+(1-\delta)K$

## 生产函数和资源函数的图形解释



约束线(消费和资本积累可能线)

斜率 $=-1, (K, C)$ 的组合

下期资本存量 $K_0$ ;实际收入 $y_0$ ;

消费 $C_0$ ;投资 $I_0 = y_0 - C_0$

若资本存量和生产函数不变 $\Rightarrow$ 将重复下去

图中资源函数在交点处弯折

是作图所致,本应平滑

## 一个新古典宏观经济学AD-AS模型(卢卡斯模型)

卢卡斯总供给函数

设企业 $i$ 的供给函数:

$$y_i = h(P_i - P) + y_i^*, h > 0$$

$\Downarrow$   $P_i$ 产品价格; $P$ 市场价格总水平;

$\Downarrow$   $y$ 产量; $y^*$ 企业潜在或正常产量

$\Downarrow$   $h$ 企业对两个价格偏离的反应

$\Downarrow$  纵坐标 $P_i$ ;横坐标 $y_i$ ;正斜率

$\Downarrow \Leftarrow$  企业无法知道 $P$ ,因此用预期

$$y_i = h(P_i - P^e) + y_i^*$$

$\Downarrow$  企业对价格总水平的预期(两部分组成)

$$\Downarrow \Leftarrow P^e = \bar{P} + b(P_i - \bar{P})$$

$\Downarrow$   $\bar{P}$ : 社会有关机构预测并

公布的价格预测值

$\Downarrow$  企业根据其经验对预测值调整

$\Downarrow$   $b$ 为调整系数

$$y_i = h(1-b)(P_i - \bar{P}) + y_i^*$$

$\Downarrow \Leftarrow$  加总获得总供给函数

$$y = nh(1-b)(P - \bar{P}) + y^*$$

$\Downarrow$   $y^*$ 经济的潜在产量

$\Downarrow$  经济总产量&未预测到的价格上升

$$y = y^* + \gamma(P - \bar{P}), \gamma > 0$$

含义:偏离(预期价格与实际价格)

$\Rightarrow$  偏离(实际产出与经济正常产出)

## 对宏观经济波动的解释

假定:技术进步

$\Rightarrow$ 生产函数和资源函数上移

## 资本存量不变情况下:

产量 $\uparrow \uparrow \Leftrightarrow$ 总资源 $\uparrow \uparrow \Leftrightarrow$ 下期

社会选择 $\Leftarrow$ 约束线右移 $\Leftarrow C \uparrow K \uparrow$

新组合点 $\Leftarrow$ 资本存量 $\uparrow \Leftarrow$ 产量 $\uparrow$

新稳态 $\Leftarrow$ 约束线右移(递减) $\Leftarrow$ 总资源 $\uparrow$

技术变化的影响

技术:跳跃增长,之后恒定;

投资:跳跃增长,之后增长递减;

收入:增长持续递减

## 实际经济周期理论

强调技术变化是收入和投资变动的根源

模型的基本思想

说明

对货币量和一般价格水平的不完全信

息怎样导致货币的非中性

$\Delta$ 货币量怎样导致对一般价格和相对

价格变化的短期混淆

政策含义

① 系统的货币政策无效;

② 随机的货币政策有害

## 简化的新古典宏观经济模型

设经济的总需求函数:

$$y_t^d = \alpha_t + \beta(m_t - p_t), \beta > 0$$

$\alpha$ :除货币供给以外其他引起总需求变化的因素

$m, p$ :货币供给和价格水平取对数后的值

$(m-p)$ :实际货币供给量 $(M/P)$ 的对数

$\beta$ :货币供给量对总需求的乘数

## 附加预期的总供给函数(卢卡斯供给函数):

$$y_t^s = y_n + \gamma(p_t - p_t^e), \gamma > 0$$

均衡条件

$$y_t = y_t^d = y_t^s$$

理性预期下:

对总需求和总供给函数两边取期望:

$$y_t^e = \alpha_t^e + \beta(m_t^e - p_t^e)$$

$$y_t^e = y_n$$

联立解出理性预期的均衡价格水平:

$$p_t^e = m_t^e - \frac{1}{\beta}(y_n - \alpha_t^e)$$

预期不是理性:

总需求和总供给函数对期望式做差:

$$y_t - y_n = (\alpha_t - \alpha_t^e) + \beta(m_t - m_t^e) - \beta(p_t - p_t^e)$$

$$y_t - y_n = \gamma(p_t - p_t^e) \quad \text{总需求中未预期的部分}$$

产出对自然率水平的偏离

联立解出产出和价格:

$$y_t = y_n + \frac{\gamma}{\gamma + \beta} [(\alpha_t - \alpha_t^e) + \beta(m_t - m_t^e)]$$

$$p_t = m_t^e - \frac{1}{\beta}(y_n - \alpha_t^e) + \frac{1}{\gamma + \beta} [(\alpha_t - \alpha_t^e) + \beta(m_t - m_t^e)]$$

公众预期到的总需求变动不能影响产出

预期到和未预期到的总需求变化都能影响价格

## 新古典宏观经济学和新凯恩斯主义经济学(4-3)

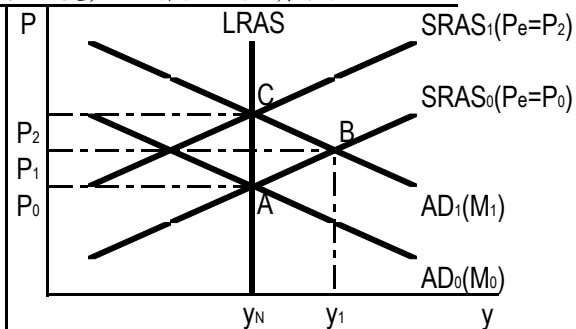
### 一个新古典宏观经济学AD-AS模型(卢卡斯模型)

#### 新古典宏观经济学模型的政策含义

波动的根源是货币冲击

货币冲击一般由央行引起

只要货币当局的政策具有系统性, 就不能改变产出增长的长期路径



当事人完全遇到货币供给:  $A \rightarrow C$

AD右移(货币增加), SRAS左移(预期调整)

当事人未能预期:  $A \rightarrow B \rightarrow C$

AD右移(货币增加)

当事人发现相对价格未变, SRAS左移(预期调整)

#### 观点

能预期到的货币供给的变化只能改变价格水平, 对实际产量和就业量没有影响

只有未预期到的货币供给的变化才影响实际产量

宏观经济政策要想有效, 必须具有欺骗性质

#### 古典二分法

名义变量对实际变量没有实质性影响

工资, 价格由伸缩性

新凯恩斯主义: 古典二分法失效

#### 名义黏性

出现名义需求扰动时, 某种因素使得名义价格水平变动的比例不同于名义需求变动的比例

#### 实际黏性

某种因素阻止实际工资调整, 或一种工资/价格相对另一种工资/价格的黏性

### 新凯恩斯主义形成的理论背景和特征

#### 凯恩斯主义阵营(关键区别)

新古典综合派

假定一个固定的名义工资

新凯恩斯主义

为工资和价格黏性提供微观基础

#### 新凯恩斯主义形成的理论背景

客观条件

原凯恩斯主义的理论缺陷

缺乏微观基础

使用需求不足和名义工资刚性解释

失业, 但没有说明名义工资刚性

新古典宏观经济学解释现实乏力

对比吸收

新凯恩斯: 工资&价格黏性, 非市场出清

原凯恩斯: 工资&价格刚性, 非市场出清

新古典: 工资&价格伸缩性, 市场出清

将假设与宏观的产量&就业量等结合

#### 新凯恩斯主义的假设条件

非市场出清(最重要假设)

在出现需求或供给冲击后, 工资和价格

不能迅速调整到使市场出清的状态

调整过程中, 经济处于持续的非均衡

工资和价格黏性

经济当事人最大化原则

厂商利润最大化, 家庭效用最大化

理性预期

#### 新凯恩斯主义的特征

货币是非中性的(∴黏性)

经济中实际市场的不完全性对

理解经济波动十分关键

对企业定价行为的看法

新古典宏观经济学

企业是价格接受者

新凯恩斯主义

企业是制定价格的不完全竞争

市场中的企业

(市场不完全性+不对称信息)

⇒ 工资和价格具有黏性

### 名义黏性

工人: 理性的效用最大化者

厂商: 理性的利润最大化者

#### 名义工资黏性

早期: 长期劳动合同

劳动合同中的工资并非完全刚性

#### 合同影响经济的关键:

对工资变化的规定

有计划的变化;

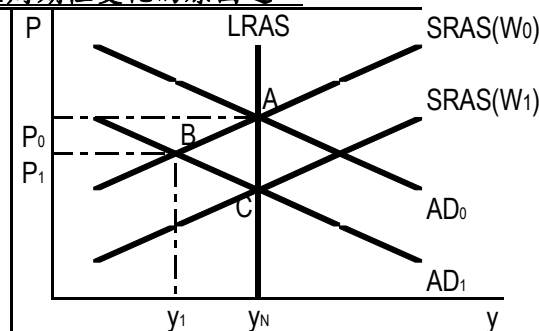
按生活费用协议的规定变化

对合同期限的规定

合同交错: 所有合同不会同时期结束

长期劳动合同是实际收入和失业

呈周期性变化的原因之一



AD左移⇒AS短期不变,  $A \rightarrow B$

⇒AS长期调整,  $B \rightarrow C$

长期劳动合同对厂商和工人均有益

① 工资谈判费时

② 谈判破裂可能性存在, 代价太大

③ 面临不利的需求冲击, 对厂商而言, 调整工资率不是最优策略

#### 名义价格黏性

菜单成本理论

垄断厂商决定价格, 但更改价格有成本

菜单成本: 厂商调整价格花费的成本

包括调整的机会成本

利润函数  $\pi(P)$ , 利润最大化价格  $P^*$

价格不调整到  $P^*$  的损失:

$$\pi(P^*) - \pi(P) = \pi'(P^*)(P^* - P) - \frac{1}{2} \pi''(P^*)(P^* - P)^2$$

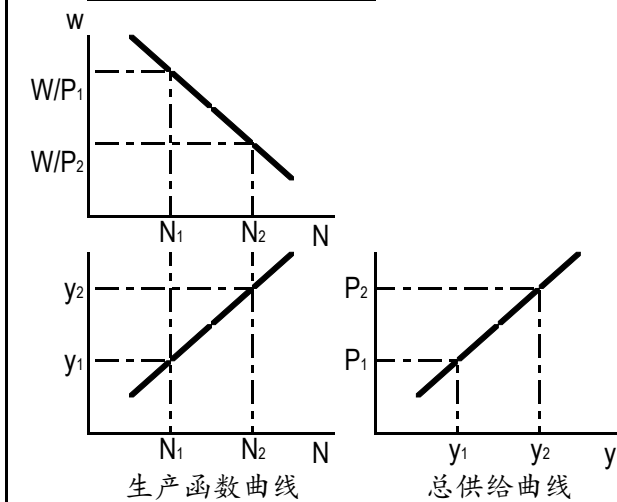
$\pi'(P^*)=0$ , 价格稍微偏离, 不调整的损失很小

### 新凯恩斯主义的短期供给曲线

#### 短期供给曲线的推导

$P \rightarrow W/P$

- ① 名义工资黏性;
- ② 给定固定名义工资,则实际工资  
W/P取决于价格水平
- ③ 劳动由需求决定  
取决于劳动需求曲线

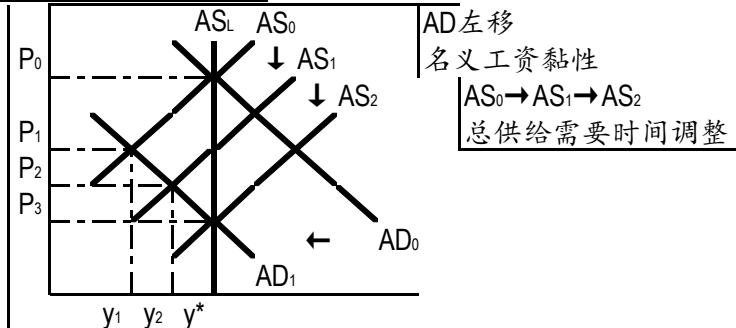


给定固定名义工资  
价格水平  $\uparrow \Rightarrow$  实际工资  $\downarrow \Rightarrow$  劳动需求  $\uparrow$   
 $\Rightarrow$  就业  $\uparrow \Rightarrow$  产出  $\uparrow$

#### 短期供给曲线的移动

- ① 名义工资  $\uparrow \Rightarrow$  实际工资  $\uparrow \Rightarrow$  AS 左移
- ② 全要素生产率  $A \downarrow \Rightarrow$  生产函数曲线  $\downarrow \Rightarrow$  AS 左移

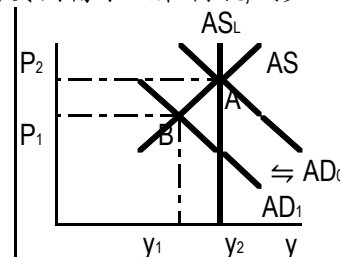
#### 对宏观经济波动的考察



### 新凯恩斯主义的稳定化政策

$\therefore$  工资和价格黏性

经济在需求冲击后,回归均衡过程缓慢  
需要用需求政策刺激,减少短期痛苦



总需求  $\downarrow$ , AD 左移:

政府不行动,经济停在B点(至少短期如此);  
政府刺激需求,AD右移,回归A点

#### 目前宏观经济学的基本共识

- ① 在长期,一国生产物品和劳务的能力决定该国居民的生活水平
- ② 制度对于长期经济增长是非常重要的  
基础性制度(产权,无腐败等)
- ③ 在长期,货币增长率决定通胀率
- ④ 在短期,总需求影响一国生产的物品与服务的数量  
在短期,价格是黏性的,  $\therefore$  总需求至关重要
- ⑤ 在短期,政策制定者面临通胀和失业之间的权衡
- ⑥ 预期是重要的



# 经济学流派整理

色诺芬,柏拉图,亚里士多德	托马斯·孟,让·博丹,科尔贝尔	配第,洛克,休谟,斯图亚特(英);布阿吉贝尔,坎蒂隆(法)	魁奈,杜尔哥(法)	亚当·斯密
公元前480-公元1566	15世纪-17世纪中叶	17世纪中叶-19世纪20年代	18世纪50-70年代	
古希腊,古罗马,中世纪经济思想	英法重商主义	古典政治经济学	重农学派	

富兰克林,汉密尔顿	李斯特(历史学派),屠能	西尼尔(英);巴师夏(法);穆勒(英)	马尔萨斯,萨伊,李嘉图体系解体	西斯蒙第(法)	李嘉图
19世纪左右	19世纪上半期	19世纪30-60年代	19世纪初		
美国经济思想	德国经济学说	英法经济学说			

凯里,乔治	门格尔(奥);杰文斯,瓦尔拉斯(法)	门格尔,庞巴维克,维塞尔	杰文斯,威克斯蒂德	瓦尔拉斯,帕累托	克拉克
19世纪末	19世纪70年代	19世纪70年代-20世纪初	19世纪70年代	19世纪末	
美国	边际效用学派创立,边际革命开始	奥地利学派	英国边际效用学派	洛桑学派	美国理论学派

米塞斯	维克塞尔,卡塞尔,俄林,缪尔达尔,林达尔	成本曲线,序数论,不完全竞争,福利	马歇尔(美)	凡勃仑	瓦格纳
新自由主义	20世纪20-30年代		20世纪20年代	19世纪末	20世纪左右
新奥地利学派	瑞典学派	新古典学派	边际革命完成	美国制度学派	德国新历史学派

坎南,罗宾斯	米塞斯,泰勒,兰格,哈耶克	奈特,西蒙斯,哈耶克	康蒙斯,米契尔,艾尔斯,贝利	熊彼特(奥)	凯恩斯
20世纪20-30年代	20世纪20年代	20世纪初	20世纪上半叶	20世纪30年代后	
伦敦学派	关于社会主义的讨论	芝加哥学派	美国制度学派	凯恩斯革命	

欧肯,艾哈德	琼·罗宾逊,斯拉法,卡尔多,卡莱斯基	汉森,托宾,希克斯,哈罗德,多马,索罗,莫迪利安尼,杜森贝利,萨缪尔森
20世纪30-40年代	20世纪50年代	20世纪30-70年代
弗莱堡学派(德)(新自由主义)	新剑桥学派(新凯恩斯主义)	新古典综合派

加尔布雷恩,鲍尔丁	林德伯克	弗里德曼	拉弗,费尔德斯坦	布坎南	科斯,诺斯,威廉姆森
20世纪50年代	20世纪60-80年代	20世纪50年代至今	20世纪70年代	20世纪60年代	20世纪下半叶
新制度学派(美)	之前瑞典学派	现代货币学派	供给学派	公共选择学派	新制度经济学学派

管理,计量,发展,比较,人口,信息,演化,实验		价格和工资黏性,效率工资,经济周期	假设:货币经济周期,实际经济周期,李嘉图等价
当代		20世纪70年代	
经济学学科发展	二者融合	新凯恩斯主义	新古典主义(理性预期学派)

correlation

scatter plots

sample covariance&correlation

$$\text{Cov}(X, Y) = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

$$r = \frac{\text{Cov}(X, Y)}{S_x S_y} \quad -1 < r < +1$$

population:  $\rho$ ; sample:  $r$

**significant test ( $H_0: \rho=0$ ) of the correlation**

$H_0: \rho=0$ ;  $H_A: \rho \neq 0$

test statistic:

$$t = \frac{r-0}{\sqrt{((1-r^2)/(n-2))}} \quad df=n-2$$

decision rule

reject  $H_0$  if  $t > t_{\text{critical}}$  or  $t < -t_{\text{critical}}$

conclusion

significantly different from zero

limitations to correlation analysis

outliers

spurious correlation

chance relationships  
divided by a third variable  
relation to a third variable

nonlinear relationships

linear regression

Y: dependent, explained, endogenous, predicted

X: independent, explanatory, exogenous, predicting

$$Y_i = b_0 + b_1 X_i + \varepsilon_i$$

interpretation of regression coefficient

$b_0$ : estimated intercept coefficient

$b_1$ : estimated slope coefficient

**6 assumptions**

linear relationship exists between X&Y;

X is not random ( $\rho_{X,\varepsilon} = 0$ );  $E(\varepsilon_i) = 0$ ;

Var( $\varepsilon$ ) is constant (homoskedastic);

Cov( $\varepsilon_i, \varepsilon_j$ ) = 0;  $\varepsilon$  is normally distributed.

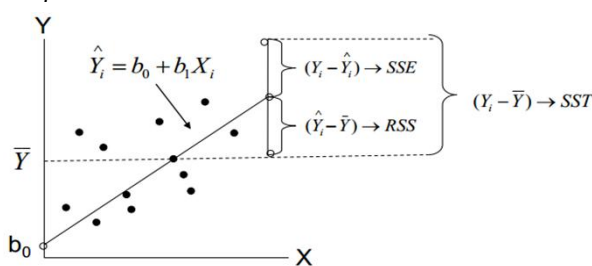
ordinary least squares (OLS)

minimize the squared residuals

$$b_1 = \frac{\text{Cov}(X, Y)}{\text{Var}(X)} \quad b_0 = Y_{\text{avg}} - b_1 X_{\text{avg}}$$

Analysis of Variance (ANOVA) table

component



**ANOVA table**

	df	SS	MSS(variance)
Regression	k=1	RSS	MSR=RSS/k
Error	n-2	SSE	MSE=SSE/(n-2)
Total	n-1	SST	-

**RSS: regression/explan**

**SSE: error/residual**

standard error of estimate (SEE)

smaller, better

$$SEE = \sqrt{SSE/(n-2)} = \sqrt{MSE}$$

coefficient determination ( $R^2$ )

$$R^2 = RSS/SST = 1 - SSE/SST$$

for simple linear regression:  $R^2 = r^2$

BUT they are different

**hypothesis test**

$$(\pm)t = \frac{b_{\text{predict}} - b_{\text{null}}}{S_{b_{\text{predict}}}}$$

**confidence interval**

$$(b_{\text{null}} =) b_{\text{predict}} \pm t S_{b_{\text{predict}}}$$

$H_0: b_1 = 0$

regression coefficient **confidence interval**

doesn't include 0  $\rightarrow$  null is rejected

**hypothesis testing** about regression coefficient (df=n-2)

$t \notin [-t_{\text{critical}}, +t_{\text{critical}}] \rightarrow$  null is rejected

p-value

$p < \alpha \rightarrow$  null is rejected

predicted value of the dependent variable

point estimate

$$\hat{Y} = \hat{b}_0 + \hat{b}_1 X'$$

confidence interval estimate

$$\hat{Y} \pm (t_c \times s_f)$$

$t_c$  = the critical t-value with df=n-2

$s_f$  = the standard error of the forecast, SEE  $\uparrow$ ,  $s_f \uparrow$

$$s_f = SEE \times \sqrt{1 + \frac{1}{n} + \frac{(X' - \bar{X})^2}{(n-1)s_x^2}} = SEE \times \sqrt{1 + \frac{1}{n} + \frac{(X' - \bar{X})^2}{\sum (X_i - \bar{X})^2}}$$

limitations of regression analysis

regression relations **changes** over time

if others are also **aware of and act on** the relationship

regression **assumptions** are violated

heteroskedastic

autocorrelation

# Quant.3-2 Multiple regression and issues in regression analysis

## Multiple regression and issues in regression analysis

$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + \dots + b_k X_{ki} + \varepsilon_i$$

interpretation of regression coefficient

slope: holding other independent variables constant

### assumptions

1. linear relationship exists between X&Y
2. X is not random ( $\rho_{x,\varepsilon} = 0$ ) **Cov(X<sub>1</sub>, X<sub>2</sub>)=0**; 3.  $E(\varepsilon_i) = 0$ ;
4. Var( $\varepsilon$ ) is constant (homoskedastic); 5. Cov( $\varepsilon_i, \varepsilon_j$ )=0;
6.  $\varepsilon$  is normally distributed

dummy variables

can only take two values, 0&1

distinguish n categories, need n-1 dummy variables

Analysis of Variance (ANOVA)

### ANOVA table

	df	SS	MSS (variance)
Regression	k	RSS	MSR = RSS/k
Error	n-k-1	SSE	MSE = SSE/(n-k-1)
Total	n-1	SST	-

standard error of estimate (SEE)

$$SEE = \sqrt{SSE/(n-k-1)} = \sqrt{MSE}$$

coefficient determination ( $R^2$ )

$$R^2 = RSS/SST = 1 - SSE/SST$$

adjusted  $R^2$

$$ad. R^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)} = 1 - \left[ \left( \frac{n-1}{n-k-1} \right) \times (1-R^2) \right]$$

$$adjusted R^2 \leq R^2$$

adjusted  $R^2$  may be less than zero

unbiased & consistent

unbiased:  $E(\text{parameter sample}) = \text{parameter population}$

consistent: sample size  $\uparrow$ , accuracy  $\uparrow$

### hypothesis testing about regression coefficient

significance test for a regression coefficient ( $b_j, j \in [1, k]$ )

$$H_0: b_j = 0$$

test statistic

$$t = \frac{b_{predict}}{S_{b_{predict}}} \quad df = n - k - 1$$

p-value (the smallest significance level)

reject  $H_0$  if p-value <  $\alpha$

fail to reject  $H_0$  if p-value >  $\alpha$

regression coefficient **confidence interval**

$$b_{predict} \pm t S_{b_{predict}}$$

estimate regression coefficient  $\pm$  (critical value) (coefficient standard error)

## assumptions $\rightarrow$ violations

### heteroskedasticity (violate 4)

unconditional heteroskedasticity

conditional heteroskedasticity

$S^2_{error}$  is related to the level of independent variables

create significant problems for statistical inference

effective of heteroskedasticity on regression analysis

not affect consistency,  $b_j$  are not affected

standard errors are usually unreliable estimates

$$S_{b_1} \uparrow \Rightarrow t = \frac{b_0}{S_{b_1}}, \downarrow \Rightarrow \text{hard to reject, type II}$$

$$S_{b_1} \downarrow \Rightarrow t = \frac{b_0}{S_{b_1}}, \uparrow \Rightarrow \text{easy to reject, type I}$$

F-test is also unreliable

**summary: not affect ( $b_0, b_j$ ); affect ( $t, F, SEE, R^2$ )**

detecting heteroskedasticity

residual scatter plots ( $\varepsilon$  & x)

**Breusch-Pager  $\chi^2$  test (BP test)**

$H_0$ : no heteroskedasticity, one-tailed test

Chi-square test:  $BP = n \times R^2_{residual}$ ,  $df = k$

$$R^2_{residual} \rightarrow \varepsilon^2 = b_0 + b_1 X + \varepsilon$$

**decision rule: BP test statistic big, reject  $H_0$**

correcting heteroskedasticity

robust standard errors (**White-corrected** standard errors)

generalized least squares

### multicollinearity (violate 2)

effect all ( $b_0, b_1, t, F, SEE, R^2$ , type II)

**detect** (fail: not reject; pass: reject)

**t-test fail, F-test pass,  $R^2$  high;**

$$|\rho(X_i, X_j)| > 0.7$$

**correct**: omit one or more  $X_i$

### serial correlation (autocorrelation, violate 5)

effective of serial correlation (like hetero.)

**positive** serial correlation (++++)

$S_{b_1} \downarrow$ , **type I**, F-test unreliable

negative serial correlation (+--+)

**type II**

detecting serial correlation

residual scatter plots

**Durbin-Watson test**

$H_0$ : no serial correlation

$$DW \approx 2 \times (1 - r), \quad r = r(\varepsilon_i, \varepsilon_j)$$

decision rule

reject $H_0$	not	reject $H_0$
++++	-	-
0	$d_1$	$4 - d_1$
	$d_u$	$4 - d_u$

$H_0$ : no positive serial correlation

$$DW \approx 2 \times (1 - r)$$

decision rule

reject $H_0$	-	fail to reject
0	$d_1$	$d_u$

correcting serial correlation

**Hansen method**

(also correct for conditional hetero.)

only hetero.: White-corrected

improve model (time-series)

model misspecification

functional form can be misspecified

important variables are omitted;

variables should be transformed (eg. ln);

data is improperly pooled.

time series misspecification

a lagged Y used as a X;

a function of Y used as a X (forecasting the past);

X is measured with  $\varepsilon$ .

effect  $b_0, b_1$

qualitative dependent variables

probit model

logit model

discriminant model (Z-score)

# Quant.3-3 Time-series analysis

## Time-series analysis

### trend models

linear:  $y_t = b_0 + b_1 t + \varepsilon_t$

limitation

log-linear:  $y_t = \exp(b_0 + b_1 t)$

inconsistent  $b_0, b_1$

$\ln(y_t) = b_0 + b_1 t + \varepsilon_t$

$\mu, \sigma^2$  of time series changes over time

## ★ autoregressive models(AR)

**AR(p):**  $X_t = b_0 + b_1 X_{t-1} + b_2 X_{t-2} + \dots + b_p X_{t-p} + \varepsilon_t$

### 3 assumptions

#### no autocorrelation

detecting autocorrelation in an AR

$H_0: r_{\varepsilon t, \varepsilon t-k} = 0$

$$t = \frac{r_{\varepsilon t, \varepsilon t-k} - 0}{S_r} = \frac{r_{\varepsilon t, \varepsilon t-k}}{1/\sqrt{n}}$$

differ significantly from 0, modify model

correction: add lagged values

seasonality

seasonal autocorrelation of  $\varepsilon$  differ from 0

$$X_t = b_0 + b_1 X_{t-1} + b_2 X_{t-4} + \varepsilon_t$$

compare forecasting power with RMSE(AR)

$RMSE = (\sum \varepsilon^2_i) / n$ , like SEE, smaller, better

regression with more than one time series

any time series contains a unit root

OLS may be invalid

use DF tests for each time series to detect unit root

none has: we can use multiple regression

some has, some not: we can't use

all have: weather the time series are cointegrated

#### cointegration

use Dickey-Fuller Engle-Granger test( **DF-EG test** )

$H_0$ : no cointegration

#### covariance-stationary series

3 conditions

constant & finite  $\mu, \sigma^2$  Cov of time series

past stationary not guarantee future stationary

all covariance-stationary time series

have a finite mean-reverting level

#### mean reversion

$$\text{mean reverting level: } x_t = \frac{b_0}{1-b_1}$$

$x_t > MRL \Rightarrow x_{t+1} < x_t$  but no less than MRL

instability of regression coefficient

financial & economic relationship are dynamic

models estimated with shorter time series

are more stable than longer

#### no conditional heteroskedasticity

test(ARCH(1))

$$\varepsilon^2_t = a_0 + a_1 \varepsilon^2_{t-1} + u_t$$

$a_1$  differ from 0, time series is ARCH(1)

$\Rightarrow$  heteroskedasticity

correction: generalized least squares

#### random walks

random walk without a drift

$$X_t = X_{t-1} + \varepsilon_t \quad b_0 = 0, b_1 = 1$$

random walk with a drift

$$X_t = b_0 + X_{t-1} + \varepsilon_t$$

features

has an undefined mean reverting level

is not covariance stationary

#### unit root test( $b_1=1$ )

have a unit root  $\Rightarrow b_1=1$

#### Dickey-Fuller test(DF test)

$$X_t = b_0 + b_1 X_{t-1} + \varepsilon_t$$

$$\Rightarrow X_t - X_{t-1} = b_0 + (b_1 - 1)X_{t-1} + \varepsilon_t$$

$$\Rightarrow X_t - X_{t-1} = b_0 + gX_{t-1} + \varepsilon_t$$

$H_0: g=0$  (have a unit root, nonstationary)

$H_A: g<0$  (don't have a unit root, stationary)

cpt conventional t-statistic, use revised t-table

#### correction

first differencing

$$\text{define } y_t = x_t - x_{t-1} = \varepsilon_t$$

$$AR(1): y_t = b_0 + b_1 y_{t-1} + \varepsilon_t, \quad b_0 = b_1 = 0$$

$y_t$  is covariance stationary

Probabilistic approaches: senario analysis, decision trees, simulations

risk type and probabilistic approaches			
discrete/ continuous	correlated/ independent	sequential/ concurrent	risk approach
discrete	correlated	sequential	decision trees
discrete	independent	concurrent	senario analysis
continuous	either	either	simulations



exchange rate

A:B=2 ⇒ 2 B/A

A:base currency; B:price currency

nominal exchange rate&real exchange rate

FX real(d/f)=FX nominal(d/f)×CPI f/CPI d

spot rate&forward rate

bid-ask spread (bank/dealer)

spread quoted by dealer(bank-client)

spread in the interbank market(benchmark);

size of transaction(larger,larger spread);

relationship between dealer&client.

1 interbank spread

currencies involved(high-volume:\$,€,£, ¥);

time of day(New York&London ⇒highest volume);

market volatility(higher,higher);

spread forward increases with maturity.

2 cross rate

相乘同边(\$/¥, €/£: bid×bid, ask×ask)

相除对角(¥/\$, €/£: bid/ask, ask/bid)

triangular arbitrage (eg.\$,€,¥)

\$→€→¥→\$

\$→¥→€→\$

1mkt1,2 1mkt3

step1.calculate cross rate €/¥ in mkt 1&2;

step2.compare with €/¥ in mkt 3;

step3.buy low sell high(mkt1,2).

乘小除大(\$ buy €: ×€/£; ÷\$/€)

3 forward discount (-)&premium (+)=F - S<sub>0</sub>

forward point=F - S<sub>0</sub>

4 market-to-market value(of a forward)

time T: V<sub>T</sub>=(FP<sub>T</sub>-FP)×contract size

FP<sub>T</sub>:spot rate at T; FP:forward price

time t: V<sub>t</sub>= (FP<sub>t</sub>-FP)×contract size

1+R×(days/360)

经济学:先轧差, 再折现

衍生品:先折现, 再轧差

5 international parity relationship

1 interest rate parity(IRP, FX&r)

covered interest rate parity

covered:bound by arbitrage

assumptions:

currencies freely traded;

forward contracts available.

interest differential ≈ forward differential

int.rate parity relationship

F&S: X/Y; r:nominal R<sub>f</sub> in X/Y

(1/S)(1+r<sub>y</sub>)F=1+r<sub>x</sub>

⇒  $\frac{F}{S} = \frac{1+r_x}{1+r_y}$

Δ%FX=(F-S)/S=(r<sub>x</sub>-r<sub>y</sub>)(1+r<sub>y</sub>) ≈ r<sub>x</sub>-r<sub>y</sub>

⇒ Δ%FX ≈ r<sub>x</sub>-r<sub>y</sub>

day count convention(days/360)

$\frac{F}{S} = \frac{1+r_x(\text{days}/360)}{1+r_y(\text{days}/360)}$

$\Delta\%FX = \frac{(r_x - r_y)(\text{days}/360)}{1+r_y(\text{days}/360)}$

arbitrage

|F/S>(1+r<sub>x</sub>)/(1+r<sub>y</sub>), borrow X

profit=(F/S)(1+r<sub>y</sub>) - (1+r<sub>x</sub>)

calculate:1.F or S; 2.forward point;

3.Δ%FX=(F-S)/S; 4.arbitrage profit.

uncovered interest rate parity

assume that investors are risk-neutral

$\frac{E(S_t)}{S} = \frac{1+r_x}{1+r_y}$

Δ%FX=(E(S<sub>t</sub>)-S)/S ≈ r<sub>x</sub>-r<sub>y</sub>

⇒ Δ%FX ≈ r<sub>x</sub>-r<sub>y</sub>

(r<sub>x</sub>-r<sub>y</sub>)>0 ⇒ Δ%FX>0, Y ↑

(r<sub>x</sub>-r<sub>y</sub>)<0 ⇒ Δ%FX<0, Y ↓

compare

covered	uncovered
no-arbitrage forward rate	expected future spot rate
by arbitrage	if uncovered holds, F is an unbiased predictor of E(S <sub>t</sub> )
all ✓	in the short run ✗; in the long run ✓

2 international Fisher relation (R&π; short run ✗; long run ✓)

int.rate differential ≈ expected inflation differential

assume that real int.rate are stable and equal( R<sub>real,x</sub>=R<sub>real,y</sub>)

1+R<sub>n,x</sub>=(1+R<sub>r,x</sub>)(1+π<sub>e,x</sub>)

|| stable&equal, ≈ Δ%GDP<sub>x</sub>(or Δ%GDP<sub>y</sub>)

1+R<sub>n,y</sub>=(1+R<sub>r,y</sub>)(1+π<sub>e,y</sub>) n:nominal; r:real; e:expected; π:inflation

exact methodology

linear approximation

$\frac{1+R_x}{1+R_y} = \frac{1+\pi_{e,x}}{1+\pi_{e,y}}, R_x - R_y \approx \pi_{e,x} - \pi_{e,y}$

3 purchasing power parity(PPP, FX&π)

absolute PPP

law of one price;

might not hold weights goods in 2 economies not be same.

relative PPP

ex-post  $\frac{S_1}{S_0} = \frac{1+\pi_x}{1+\pi_y}$

ex-ante version of PPP(short run ✗; long run ✓)

$E(S_t)/S_0 = \left( \frac{1+\pi_{e,x}}{1+\pi_{e,y}} \right)^t, \Delta\%FX \approx \pi_{e,x} - \pi_{e,y}$

compare

covered ✓ uncovered:if F=E(S<sub>1</sub>), ✓

if uncovered&Fisher ✓, ex-ante version of PPP ✓

combining Fisher&relative PPP ⇒ uncovered

if relative PPP holds at any moment, real exchange rate → constant

equilibrium real exchange rate(mean-reversion)

6 long-run fair value of an exchange rate

macroeconomic balance approach

BOT: current account balance ⇒ FV of exchange rate

external(debt) sustainability approach

external debt/GDP (20%-40%) towards its sustainable level

reduced-form econometric model approach

developed markets: relative ineffective

① preconditions for growth  
savings&investment  
financial mkt&intermediaries  
political stability  
investment in human capital  
tax and regulatory systems  
free trade&unrestricted capital flow  
stock market,sustainable growth rate  
 $P=GDP \times (E/GDP) \times (P/E)$   
 $\Rightarrow \Delta\%P = \Delta\%GDP + \Delta\%(E/GDP) + \Delta\%(P/E)$   
long-term: $\Delta\%(E/GDP)=0$ ;  $\Delta\%(P/E)=0$   
**long time horizon: $\Delta\%P=\Delta\%GDP$**

② importance of potential GDP  
higher potential GDP growth implies  
**higher real interent rates&real asset returns;**  
expected credit risk ↓  
credit quality of all debt issues ↑  
actual GDP - potential GDP=rising price

③ factors inputs and economic growth  
**Cobb-Douglas production function**  
 $Y=TK^\alpha L^{1-\alpha}$   
**T:total factor productivity(TFP)**  
technological progress of economy  
 **$\alpha+\beta=1$ :exhibits constant returns to scale**  
 $K \times 2, L \times 2 \Rightarrow Y \times 2$

④ output per worker(Y/L,labor productivity) ( $X/L=x$ )  
 $Y/L=T(K/L)^\alpha \rightarrow y=Tk^\alpha$   
K/L:capital per worker(capital deepening)  
 **$\Delta\%y=\Delta\%T+\alpha\Delta\%k$**   
 $\Delta\%$ per-capita potential GDP( $\Delta\%$ labor productivity)  
 $=\Delta\%$ technology $+\alpha\Delta\%$ capital-to-labor ratio  
 $\alpha<1$ :add. capital  $\rightarrow$  a diminishing effect on productivity  
capital deepening  $k \uparrow$ :developing nations gain more  
in sready state,marginal product of K=marginal cost of K  
 $MP=d(TK^\alpha L^{1-\alpha})/dK=\alpha Y/K=r=MC \Rightarrow \alpha=rK/Y$   
 $\Rightarrow \alpha=rK/Y$ : amount of Y that is allocated to providers of K  
growth accounting relations  $r$ :int.rate  
**② $\Delta\%Y=\Delta\%T+\alpha\Delta\%K+(1-\alpha)\Delta\%L$**   
**③ $Y=(Y/L) \times L=y \times L \Rightarrow \Delta\%Y=\Delta\%y+\Delta\%L$**

④ factors influencing economic growth  
natural resources  
role in economic growth is complex  
Dutch disease  
global demand for a country's resources ↑  
 $\Rightarrow EX \uparrow \Rightarrow DC \uparrow \Rightarrow$ other industries  $EX \downarrow$

labor supply factors  
demographics  
population ages&individuals live>working age  
 $\Rightarrow$ labor force decline  
labor force participation= $\frac{\text{labor force}}{\text{working age population}}$   
immigration; average hours worked

human capital  
physical capital  
telecommunications(ICT)&non-ICT capital  
public infrastructure  
tech. development:developed:R&D; developing:copy

⑤ economic growth theories  
classical growth theory(exogenous)  
growth in real GDP isnot permanent  
tech.  $\uparrow \Rightarrow$ invest in new  $K \uparrow \Rightarrow y \uparrow$   
 $\Rightarrow$ real wages  $\uparrow \Rightarrow$ population  $\uparrow \Rightarrow$ real GDP  $\downarrow$   
subsistence real wage

neoclassical growth theory (exogenous)  
long-term steady state growth rate  
**tech.  $\uparrow \Rightarrow K \uparrow \Rightarrow k \uparrow \Rightarrow y \uparrow$  (marginal  $\downarrow$ )  $\Rightarrow$ stable**  
**economy equilibrium:constant  $Y/K, \Delta\%k=\Delta\%y$**   
sustainable growth of output per capita  $g^*$   
 **$g^*=\Delta\%y=\theta/(1-\alpha), \theta=\Delta\%T$**   
sustainable growth rate of output  $G^*$   
 **$G^*=\Delta\%Y=\theta/(1-\alpha)+\Delta\%L$**   
**capital deepening: $k \uparrow \Rightarrow Y \uparrow$ ;  $k \uparrow \nRightarrow \Delta\%Y$**   
 **$k \uparrow \Rightarrow y \uparrow \Rightarrow$ stable**  
economy's growth rate  $\rightarrow$  steady state  
**steady state:  $MPK=\alpha Y/K$**   
MPK:marginal product of capital, $\Delta Y/\Delta K$   
marginal productivity( $\Delta y/\Delta k$ ) diminishing  
endogenous growth theory(tech.  $\Rightarrow K \uparrow$ )

⑥ convergence hypotheses  
absolute convergence hypotheses  
conditional convergence hypotheses  
club convergence hypotheses  
countries can 'join' club by making changes  
expected impact of removing trade barriers

economics of regulation  
regulations  
statutes:laws made by legislative bodies  
administrative regulations:government  
judicial law:court

regulators  
independent regulators(SEC,etc)  
self-regulating organizations(SROs)  
outside bodies may have conflicts of interest

economic rationale for regulatory intervention  
informational frictions  
externalities

regulatory interdependencies  
**regulatory capture theory** (by industry experts)  
**regulatory competition**  
**regulatory arbitrage**

tools of regulatory intervention  
price mechanisms(taxes&subsidies)  
restricting/requiring certain activities  
provision of public goods/financial of private projects

regulating commerce%financial mkt  
commerce:laws  
financial mkt:securities mkt  
prudential supervision  $\rightarrow$  financial institutions

antitrust regulation  
cost benefit analusis of regulation  
regulatory burden&sunset clause(period of validity)  
effects of a specific regulation  
help or hinder

overview	<u>passive</u>	<u>active</u>		
	financial assets	associates	business combination	joint ventures
degree of influence	no significant	significant	control	shared control
typical % interest	< 20%	20%-50%	50% <	varies ( $\frac{1}{n}$ )
US GAAP	cost or mkt HTM, AFS;	equity method	acquisition method	equity method (proportionate consolidation)
IFRS	FV through P/L.			

financial assets

held-to-maturity(HTM)

positive intent & ability to HTM  
only for debt securities  
initial recognition(similar)  
IFRS:FV+transaction costs  
US:at cost including transaction costs  
over the holding period  
 $B:BV_{BGN} = \Sigma PVCF, \text{discount rate} = r_{mkt \text{ int.}}$   
 $A: \text{Int.} = BGN \times r_{mkt \text{ int.}} \rightarrow \text{Gain, I/S}$   
 $S: \text{coupon} \rightarrow CF$   
 $E: BV_{END} = B + A - S$

FV through P/L

debt&equity held for trading (TS)  
designated at FV

available-for-sale(AFS)

not classified as HTM, FV through P/L  
assets  $\rightarrow$  at FV  
realized G/L  $\rightarrow$  I/S  
unrealized G/L  $\rightarrow$  OCI  
FX changes  
P/L: IFRS, debt (other changes  $\rightarrow$  OCI)  
OCI: all others (all US GAAP)

reclassification

IFRS: only AFS  $\leftrightarrow$  HTM  
US: all

impairment(IFRS)

effect CF (be estimated reliably)  
loss event  
debt: default  
equity: extended decline  
unlikely recover  
reversals (in I/S)  
debt: recovery event  
equity: not permitted (I/S)

HTM (debt)

carrying amount > PVCF  
impairment loss  $\rightarrow$  I/S  
reversal  $\rightarrow$  I/S  
loss event disappear

AFS (debt&equity)

carrying amount > FV  
cumulative OCI  $\rightarrow$  I/S  
reversal  $\rightarrow$  I/S  
debt: I/S  
equity: not I/S

impairment (US GAAP)

decline no temporary  
no reversal (I/S)  
IFRS 9 (reclassification: D)  
amortized cost (D) (HTM)  
FVPL (D&E) (FV thr. P/L)  
FVOCI (E only) (AFS)

associate (A  $\Rightarrow$  invest  $\Rightarrow$  B, x%)

significant influence

representation of board;  
participation in policy making;  
material transactions;  
interchange of mgt personnel;  
technological dependency.

1 equity method (basic)

one-line consolidation

B/S: recognize cost of investment  
B/S&I/S: share NI  
B/S: decrease if div.  
B/S: carrying amount  
 $\text{invest}_{END} = \text{cost} + \text{accum}(\text{NI-div.}) \times x\%$   
I/S: gain = NI  $\times x\%$

2 equity method (complicated)

acquisition cost		
FV of net <u>identifiable</u> assets		GW
BV of net identifiable assets	FV app.	GW
<u>unidentifiable assets:GW</u>		
appreciation     ↳ <u>not list separately</u>		
PP/E → dep.		
intangible → amort.		
inventory → COGS		
<b><math>NI_{aj}=NI-(dep.+amort.+COGS)</math> FV app.</b>		
<b><math>invest_{END}=invest_{BGN}+(NI_{aj}-div.) \times x\%</math></b>		

FV option

US: allow equity method at FV;  
IFRS: only available to VC, mutual fund;  
use of FV option is irrevocable;  
changes in value  $\rightarrow$  I/S.

impairment

IFRS: loss event (future CF);  
entire carrying amount;  
impairment loss  $\rightarrow$  I/S.  
US: FV < carrying value, permanent  
impairment loss  $\rightarrow$  I/S  
both prohibit the reversal

3 equity method (transaction with associates)

upstream (A  $\Leftarrow$  sell  $\Leftarrow$  B)  
downstream (A  $\Rightarrow$  sell  $\Rightarrow$  B)  
elimination of unrealized profit  
transaction in = t in (A buy from B)  
transaction out = t out (A sell out)  
unrealized profit = pft<sub>un</sub>  
$$\text{pft}_{un} = (t_{in} - \text{cost}_{in}) \frac{t_{in} - t_{out}}{t_{in}} \times x\%$$
  
$$\text{equity income}_{aj} = \text{NI}_{aj} \times x\% - \text{pft}_{un}$$

analyst issues

equity method higher earning than passive  
 $\therefore \text{NI} > \text{div.}$   
one line consolidation (vs acquisition method)  
quality of the equity earning

joint ventures

US GAAP&IFRS  
equity method  
rare circumstances (US&IFRS)  
proportionate consolidation  
similar to a business acquisition  
report proportionate of B/S, I/S  
no minority interest

summary	HTM	AFS	FV thr. P/L
carrying V	amortized	FV	FV
I/S	int. realized G/L	int.&div. realized G/L	int.div. realized G/L
OCI		unrealized G/L	unrealized G/L

no FV, no unrealized G/L



**controlling interest investment(A ⇒invest ⇒B,x%)**

controlling interests  
parent → subsidiary

① **acquisition method(basic)**  
 step1.-investment;-investment income  
 step2.+TA,+TL;+I/S  
 step3.+MI;-P/L attribute to MI holders

minority interest  
an isolated item

transactions among entities  
are eliminated

**business combination**  
merger: A+B=A  
acquisition: A+B=(A+B)  
consolidation: A+B=C  
SPE or VIE

② **acquisition method(complicated)**  
step1.parent post-acq.,FV,A&L + target FV,A&L  
step2.-investment  
step3.+ Goodwill (separate from investment)  
step4.+ MI  
step5.=equity parent  
step1.+I/S;-investment income  
step2.-(dep.+amort.+COGS) FV app.  
step3.-P/L attribute to MI

**goodwill**  
 not amortized  
 an impairment test at least annually  
carrying value > FV,impaired

bargain acquisition  
negative GW( → gain)

**full goodwill ( IFRS & US GAAP )**

$GW = \frac{\text{consideration}}{x\%} - \text{equity}_{B,FV}$
$MI = \frac{\text{consideration}}{x\%} (1-x\%)$

**partial goodwill ( IFRS )**  
 $GW = \text{consideration} - \text{equity}_{B,FV} \times x\%$   
 $MI = \text{equity}_{B,FV} \times (1-x\%)$   
full GW × x% = partial GW

**goodwill impairment**  
 unit:associate;subsidiary  
 IFRS : cash generating unit  
carring amount unit > recoverable amount  
recoverable amount  
=max(going concern,bankruptcy liquidation)  
impairment loss  
 US : reporting unit  
carring value unit > FV unit  
cpt implied GW  
impairment loss=GW-implied GW

summary( equity,acq.,propor. . )  
 same NI  
equity: acq.(+MI) > equity=propor.  
assets,liabilities,sales,exp.  
acq. > propor. > equity

acquisition vs pooling-of-interest(has been eliminated)  
purchase method:acquisition method(before)

	acquisition	pooling
combination	FV	BV
pre-acq.earn	not recog.	recg.
post-acq.earn	(dep.,etc) app.	no
profit margin	lower	higher
ROA	lower	higher
ROE	lower	higher

**SPE&VIE**

in the past,SPEs off-balance-sheet  
 US GAAP,VIE  
 at-risk equity insufficient without add financial support;  
 equity investors **lack any one** of following  
decision making rights;  
obligation to absorb losses;  
right to receive expected residual returns.  
must be consolidated by the primary beneficiary

overview

typical

**defined-contribution pension plan(DC)**  
**defined-benefit pension plan(DB)**  
**other post retirement benefits(OPB,like DB)**

key to identify DC/DB  
who bear the investment risk  
employer's future obligation  
 OPB regarded as an extension of DB

	DC	DB	OPB
amount of benefit	not determined depend on FV of plan assets	pre-determined	depend on specifications of plan
investment risk	born by employee	born by employer	depend
employer's obligation	make periodic contributions	make pre-determined pmt to retiree	similar to PB usually unfunded

illustration of DB

**PBO (PV(pension))**  
**PBO ,projected benefit obligation(US GAAP)**  
**PVDBO ,PV of defined benefit obligation(IFRS)**

**ABO**  
accumulated benefit obligation

**VBO**  
vested benefit obligation

	definition	calculation base/assumptions
PBO	actuarial PV of all benefits(plan's benefit formula)	expected future salary increases; going concern
ABO	actuarial PV of benefits based on current&past	current compensation levels; liquidation of pension obligation
VBO	amount of ABO which employee is entitled	ABO; vesting schedule

**funded status( IFRS&US GAAP )**

**=FV of plan assets-PV of PBO**

as a net pension assets( ceiling )/lia.;  
plan asset > PBO, overfunded plan;  
plan asset < PBO, underfunded plan.

**FV of plan assets**

**B: FV<sub>BGN</sub>**

**A: employer contributions (EC)**  
**actual return**

fund operation

**S: benefit paid to employee (BP) = = =**

**E: FV<sub>END</sub> = B + A - S**

**PV of PBO**

**B: PBO<sub>BGN</sub>**

**A: current service cost (C.S.C.)**  $\rightarrow$

PV( $\Delta$ pension) certain

**interest cost (int.)** — — —  $\downarrow$

$r \times PBO_{BGN}$

**past service cost (P.S.C.)**

plan amendments during the year

**actuarial L/G (act. L/G)**

changes in actuarial assumptions

**S: benefit paid to employee (BP)**

**E: PBO<sub>END</sub> = B + A - S = PV(PVCF<sub>retire date</sub>)**

**assumptions of DB obligation**

firm discloses

**discount rate (r)**

int. rate of high quality corp. bond

duration: bond = PBO

**rate of compensation growth**

**expected vesting rate(US)**

**expected return on plan assets(IFRS)**

estimate of future compensation

final-pay plan(in CFA)

**periodic pension cost ( US GAAP )**

certain

**+ current service cost(C.S.C.)**

**+ interest cost(int.)**

**- expected return(ER)**

uncertain(amortized)

**+ actuarial L/G(general)**

act. L/G: changes in actuarial assumptions

**ER-actual R**

corridor approach

act.L/G(g) > max(PBO<sub>BGN</sub>, FV<sub>BGN</sub>) × 10%,

must amort.; amort. must consistent

amort. at least remaining service life

**+ past service cost(P.S.C.)**

amort. over remaining service life

**OCI (unamortized)**

- past service cost(P.S.C.)

- actuarial L/G(general)

**periodic pension cost( IFRS ,no amortized)**

**service cost**

+ current service cost(C.S.C.)

+ past service cost(P.S.C.)

**net interest expense/income**

= interest cost-expected return

=  $r \times (PBO_{BGN} - FV_{BGN})$

expected rate of return = discount rate

**remeasurement (actuarial L/G(general))**

actuarial L/G (act. L/G)

"ER"-actual R, r = discount rate

**OCI (not amortized)**

remeasurement

**effect of changing pension assumption**

**discount rate  $\uparrow$**

reduce PV:PBO  $\downarrow$ , funded status  $\uparrow$

service cost  $\downarrow$

int.  $\downarrow$  =  $PBO \times r$ , (usually)

unless plan is mature(duration)

**rate of compensation growth  $\downarrow$**

reduce pmt:PBO  $\downarrow$ , funded status  $\uparrow$

current service cost  $\downarrow$ , int.cost  $\downarrow$

**expected return on plan assets**

not affect PBO&funded status

**other post-employment benefits (OPB)**

assumptions

similar  $\rightarrow$  PBO

**healthcare inflation rate**

ultimate healthcare trend rate

replace compensation growth rate

firm can reduce OPB by:

near-term healthcare infl. rate ①  $\downarrow$

ultimate healthcare trend rate ②  $\downarrow$

time needed to reach ②  $\downarrow$

**share-based compensation**

**stock compensation plan(forms)**

equity settles	cash settles
stock options	stock appreciation rights
stock grants	phantom shares

equity settled

recognize exp. over svrvice period

disadvantages

mgt have limited influence over MV

risk averse/excessive risk-taking

dilute

accounting:FV&vesting schedule

cash settled

stock appreciation right(SARs,public com.)

phantom stock  $\times \Delta P$ , FV

phantom shares(private com.)

by private companies

US GAAP	B/S	I/S
- cash $-\Delta$		
② FV <sub>BGN</sub> — — ① — — — PBO <sub>BGN</sub>		
+ EC $+\Delta$	+ C.S.C. — — ③ — — — C.S.C.	
	+ int. — — ④ — — — int.	
+ actual R	+ P.S.C. — — ⑤ — — — P.S.C.,amort.	+ ER — — — — —
	+/- act.L/G — — ⑥ — — — act.L/G,amort.	
- BP — — — ⑧ — — — BP		- [ER-actual R],amort.
	OCI:	
	- P.S.C.,unamort.	
	-/+ act.L/G,,unamort.	
	- [ER-actual R],unamort — — — — — ⑦	

IFRS	B/S	I/S
- cash $-\Delta$		
② FV <sub>BGN</sub> — — ① — — — PBO <sub>BGN</sub>		
+ EC $+\Delta$	+ C.S.C. — — ③ — — — C.S.C. (service cost)	
	+ P.S.C. — — ⑤ — — — P.S.C. (service cost)	
	+ int. — — ④ — — — int. = $r \times PBO_{BGN}$ (net int. E/I)	
+ actual R	+/- act.L/G — — — — —	+ "ER" = $r \times FV_{BGN}$
- BP — — — ⑧ — — — BP		
	OCI: ⑥	
	-/+ act.L/G — — — — —	
	- "ER"-actual R — — — — — ⑦	

US GAAP&IFRS:funded status  $\rightarrow$  same

evaluating the disclosure-assumptions

**disclosure assumptions**

- discount rate
- expected compensation increases
- medical expense inflation
- expected return on plan assets

**evaluate these assumptions**

- consistent with other comparable companies
- consistent with economic environment
- consistent internally

**B/S: net pension liability or asset**

adjust the net for gross

**I/S: total periodic pension costs**

- net periodic pension cost
- total period pension cost
- economic pension cost

**cpt net periodic pension cost**

=  $\Delta$  **funded status** <sub>END-BGN</sub> - employer's contribution

funded status: assets → +  
liability → -

result: + → pension income  
- → pension expense

= actual return - 4A of PBO

**adjust US GAAP to IFRS**

- + past service costs
- amort. of past service costs
- + return of plan by discount rate rather than expected

**adjust: (pension cost → operating)**

- current service cost still operating
- int. expense → financing cost
- return of plan → other income

**cash flow**

**overcontribution**

- contribution > total periodic pension costs (TPPC)
- CFF - (contribution - TPPC) × (1 - t), repayment
- CFO + (contribution - TPPC) × (1 - t)

**undercontribution**

- contribution < total periodic pension costs (TPPC)
- CFF + (TPPC - contribution) × (1 - t), borrowing
- CFO - (TPPC - contribution) × (1 - t)

**classification of currency**

- local currency** (subsidiary)
- functional currency** (sales, finances, etc.)
- reporting currency** / presentation currency
  - parent
  - local = functional: independent
  - reporting = functional: nonindependence

**two methods of translation**

- reporting currency
  - ↑ **current rate method (CR); translation**
- functional currency
  - ↑ **temporal method (T); re-measurement**
- local currency

**comparison of 2 methods**

B/S	rate under T	rate under CR
monetary A/L	current	current
non-monetary A/L (inv., FA, u earned rev.)	historical	current
capital (g)	historical	historical
R/E	balancing	balancing
equity (as a whole)	mixed	current

capital (g) = capital + additional paid-in-capital

I/S	rate under T	rate under CR
sales & other exp.	average	average
COGS	historical	average
depreciation	historical	average
rev. & other exp	average	average
translation G/L	recognize on I/S	recognize in OCI (CTA)

**CTA: cumulative translation adjustment**

**foreign currency transactions**

- date of transactions** → foreign currencies into function currency
- balance sheet date** → re-valued monetary A/L in foreign currencies
- settlement date**
- foreign currency appreciation → export sales: gain; import purchase: loss
- recognized on the I/S (IFRS & US GAAP) → operating or non-operating

**exposure to translation** (translation G/L, risky → ending rate)

- temporal method
  - exposure = monetary assets - monetary liabilities
- current rate method
  - exposure = equity
- translation G/L = flow effect + holding G/L effect**
- flow effect (in \$) =  $\Delta$  exposure (in LC) × (ending rate - average rate)**
- holding G/L effect**
  - = exposure <sub>BGN</sub> (in LC) × (ending rate - beginning rate)

**current rate method**

2nd. B/S	1st. I/S
assets (ending rate)	sales
liabilities (ending rate)	COGS
	SG&A
	D&A
	int
	tax
TA (ending rate)	
	TL (ending rate)
	capital (historical)
	R/E <sub>END</sub> = R/E <sub>BGN</sub> + NI-div. ← NI, avg
	OCI: CTA
	TA = TL + capital + R/E <sub>END</sub> + CTA
	(ending rate = current rate)

**temporal method**

1st. B/S	2nd. I/S
money (ending)	money (ending)
non-money (historical)	non-money (historical)
	TL
	capital (historical)
TA	
	R/E = TA - TL - capital
	R/E <sub>END</sub> = R/E <sub>BGN</sub> + NI (after) - div. → NI (after translation)

FSA 5-5 Multinational operations  
translation of foreign currency FS

summary(before&after CR method)

	pure B/S& pure I/S	mix ratios
LC appreciating	same	lower(ROA,ROE,t urnover)
LC depreciating	same	higher(ROA,ROE ,turnover)

comparison of ratios under 2 methods

LC depreciating	temporal	current rate
current ratio	higher	lower
quick ratio	same	same
A/R turnover	same	same
inventory turnover	uncertain	uncertain
fixed assets turnover	lower	higher
GP%	lower	higher
NI%,ROE,ROA (translation G/L)	uncertain	uncertain
int. coverage	lower	higher
LTD-to-total capital(E+D)	lower	higher

hyperinflation economies

3 years > cumulative 100% or 26%/1 year  
US GAAP:temporal  
IFRS:restated for inflation,  
then translation using current rate

statutory tax rate&effective tax rate

statutory tax rate  
+effect of disallowed exp.  
-effect of exempt income  
**+effect of tax in foreign jurisdictions**  
-effect of recognition of prior losses  
=effective tax rate

revenue ↑ =price×volume×FX

larger proportion of accrual component in earnings,  
the faster the reversion will occur

aggregated accruals=accrual-based earnings - cash earnings

AA:aggregated accruals  
NOA:net operating assets  
AR:accrual ratio

Evaluating quality of financial reporting

quality of financial reports

2 basic points

**GAAP-compliant&decision-useful  
results are highly quality**

aggressive vs. conservative

potential problems

measurement&timing issues

	recongnition	NI	E	L	A
sales	aggressive, premature	↑	↑	-	↑
exp.	omission delayed	↑	↑	↓	↑

classification issues

classification choices

B/S,CI,CF/S

mergers and acquisitions issues

acquirer

target company

FS that diverges from economic reality

Integration of FSA techniques

framework

define the purpose&context  
collect input  
process data  
analyze/interpret data  
develop&communicate  
conclusions&recommendations  
follow up

case1 LT equity investment

Dupont analysis

case2 Off-BS leverage from operating lease

case3 changes in accounting

financial reporting quality

decomposition with BS approach

$NOA = (TA - \text{cash}) - (TL - \text{total debt})$

$AA_{B/S} = NOA_t - NOA_{t-1}$

$(AR)_{B/S} = AA_{B/S} / ((NOA_t + NOA_{t-1}) / 2)$

decomposition with CF approach

$AA_{CF} = NI_t - (CFO_t + CFI_t)$

$(AR)_{CF} = AA_{CF} / ((NOA_t + NOA_{t-1}) / 2)$

evaluating the quality of financial reports

Beneish model

M-score: +DSRI,GMI,AQI,SGI,DEPI,Accruals; -GAI,LEVI (I:index)

**DSRI(days sales receivables)** =(recievables/sales) <sub>t</sub> /(recievables/sales) <sub>t-1</sub>

**GMI(gross margin)** =(gross margin) <sub>t-1</sub> /(gross margin) <sub>t</sub>

**AQI(asset quality)** =(1-(PPE+CA)/TA) <sub>t</sub> /(1-(PPE+CA)/TA) <sub>t-1</sub>

**SGI(sales growth)** =sales <sub>t</sub> /sales <sub>t-1</sub>

**DEPI(dep.)** =(dep.rate) <sub>t-1</sub> /(dep.rate) <sub>t</sub>

dep.rate=dep./ (dep.+PPE)

**Accruals** =(income before extra-items - CFO)/TA

**SGAI(SG&A)** =(SGA/sales) <sub>t</sub> /(SGA/sales) <sub>t-1</sub>

**LEVI(leverage)** =leverage <sub>t</sub> /leverage <sub>t-1</sub> , leverage=D/A

**N(M-score)=probability of earnings manipulation**

limitation

1.accounting data; 2.game the measures used

bankruptcy prediction models(Altman model)

z<1.81:high probability of bankruptcy

z>3.00:low probability of bankruptcy

limitation

1.accounting data; 2.single-period static model

I/S quality of earnings

high-quality

sustainable&adequate(cover company's cost of capital)

model

**Earnings <sub>t+1</sub> =α+β 1 Earnings <sub>t</sub> +ε , β 1 :persistence**

**Earnings <sub>t+1</sub> =α+β 1 CF <sub>t</sub> +β 2 Accruals <sub>t</sub> +ε**

mean reversion in earnings

evaluate the earnings quality

revenue&expense recognition

CF/S indicators of cash flow quality

focus on OCF

manipulate

timing issues(sell recievables/delay paying)

classification issues

evaluate the cash flow quality

unusual items/revenue quality/strategic provisioning

B/S quality of balance sheet

high-quality: completeness,unbiased measurement,clear presentation

completeness off-BS obligations(take-or-pay contracts,operating lease)

intercorporate investments

sources of information about risk

FS,auditor's report,footnotes,MD&A,SEC from "NT",financial press



## cash flow projection

classified as:

replacement; new product/mkt;  
expansion; mandatory investment.

## principles of capital budgeting

decision based on CF (incremental)

not accounting income;  
sunk cost ✗; externalities ✓.

CF based on opportunity costs;

timing; after tax basis;

**financing cost reflected in  $R$  required rate.**

## capital budgeting process

1. idea generation; 2. analyzing project proposals;
3. create the firm-wide capital budget;
4. monitor decisions & conduct a post-audit.

## categories of capital budgeting projects

**replacement projects** (maintain & cost reduction)

**expansion projects** (existing/new product)

mandatory investment (regulatory)

other projects (pet)

## MACRS

accelerated depreciation method

**1st year: straight line**

## ★ expansion project

**initial outlay** =  $-FC_{INV} - NWC_{INV}$

**OCF** =  $(S - C - \text{dep.}) \cdot (1 - t) + D = (S - C) \cdot (1 - t) + \text{dep.} \cdot t$

**TNOCF** =  $NWC_{INV} + Sal_T - t(Sal_T - BV_T)$

TNOCF: terminal year after-tax non-operating CF

## ★ replacement capital project

assumption: same useful life of old & new assets

**initial outlay** =  $-FC_{INV} - NWC_{INV} + [Sal_o - (Sal_o - BV_o) \cdot t]$

**ΔCF** =  $(\Delta S - \Delta C - \Delta \text{dep.}) \cdot (1 - t) + \Delta D = (\Delta S - \Delta C) \cdot (1 - t) + \Delta \text{dep.} \cdot t$

VS. expansion project: **incremental x**

**TNOCF** =  $NWC_{INC} + Sal_T - t(Sal_T - BV_T)$

## cash flow projection

current after-tax salvage value of old assets

reduces the initial outlay

depreciation is Δ

accelerated depreciation method:  $ATCF \uparrow$   $NPV \uparrow$

**interest is not included in OCF**

## ★ inflation effects on capital budgeting

nominal → nominal; real → real

if inflation is higher than expected:

probability of investment < expected;

**tax shelter** ↓ (dep.), NPV underestimated;

corporate's real taxes ↑ (tax shelter ↓);

value of fixed pmt to bondholder ↓.

**inflation affects costs & sales differently.**

## mutually exclusive projects with different lives

projects with different lives → can not compare

methods **least common multiple of lives approach**

**equivalent annual annuity (EAA)**

$N, I/Y, PV, FV \rightarrow \text{cpt PMT}$

**PMT should be discounted.**

## capital rationing

**hard/soft capital ration** (budget cannot/can ↑)

less capital than profitable projects → combination

## ★ risk analysis

stand-alone methods

**sensitivity analysis** (change only 1 variable)

**scenario analysis** (all independent variable)

worst case, best case, base case

simulation analysis (Monte Carlo)

probability distributions of each input

## market risk methods

**discount rate is risk-adjusted, not WACC.**

WACC: company's risk, not project's risk.

**CAPM can be used:**  $R_{\text{project}} = R_f + \beta_{\text{project}} [E(R_M) - R_f]$ .

## evaluating projects with real options

similar to financial call/put; based on real assets;

offer managers flexibility that can improve NPV

**overall NPV** = project NPV + option value - option cost

4 types

**timing options** (allow company to delay investing)

**sizing options**

abandonment option (put, if do so →  $PVCF \uparrow$ )

expansion option (call)

**flexibility options**

price-setting option ( $P \uparrow$ , benefit from excess demand)

production flexibility option (profit from working overtime)

**fundamental options** (payoff contingent on an underlying)

## ★ valuation: 4 approaches

**economic income** (vs. accounting)

**EI** = (after-tax CF) + (Δproject's MV)

= CF - economic depreciation

economic depreciation =  $MV_{BGN} - MV_{END}$

calculate

$PV_0 = (PV_1 + CF_1) / (1 + r) = \Sigma PVCF$

$PV_0 \times (1 + r) = PV_1 + CF_1$

**EI** =  $CF_1 - (PV_0 - PV_1) = PV_0 \times r$

economic income (EI) vs. accounting income (AI)

EI = ATCF - eco. dep.

AI = rev. - exp.

eco. dep.: decrease in MV of investment

acct. dep.: decrease in BV based on original cost

financing cost: ignored

subtracted to arrived at NI

## economic profit approach

**economic profit (EP, or EVA)**

= NOPAT - \$WACC =  $EBIT \times (1 - T) - WACC \times \text{capital}$

reflects the income earned by all capital holders

## market value added

**MVA** =  $NPV(EP) = \Sigma [EP / (1 + WACC)^t]$

**V<sub>firm</sub>** = MVA + investment; investment: capital<sub>BGN</sub>

## residual income approach

**RI<sub>t</sub>** =  $NI_t - r_e \times B_{t-1}$

reflects the income to equity holders only

$NPV = \Sigma [RI_t / (1 + r_e)^t]$

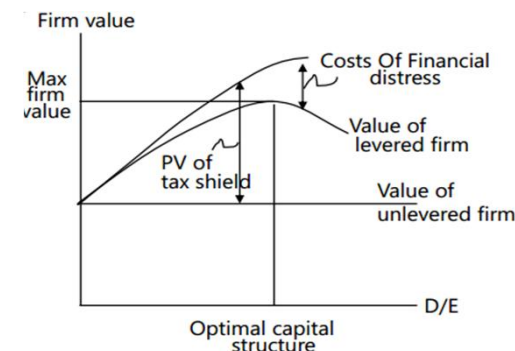
**V<sub>EQUITY</sub>** = NPV + BV<sub>EQUITY</sub>

claims valuation approach  $V_{FIRM} = ① + ②$

① FCFE discounted at  $R_{DEBT}$

② FCFE discounted at  $R_{EQUITY}$

common capital budgeting pitfalls



trade-off theory

for next page

### capital structure objective

minimize WACC → maximize  $V_{FIRM}$

$$WACC = \frac{D}{V} r_d \times (1-t) + \frac{E}{V} r_e$$

D: MV or BV;  
E: MV.

optimal capital structure **is not** the one  
that makes the maximum EPS/ROE

### capital structure theory

**MM theory** (Modigliani-Miller)

#### ★ MM proposition 1 without taxes

$V_L = V_U$  (capital structure irrelevance)

assumptions

investors agree on the expected CF;  
bonds&shares of stock are traded  
in a perfect capital market;  
investors can borrow/lend at  $R_f$ ;  
no agency costs;  
financing/investment decision: independent.

#### ★ MM proposition 2 without taxes

higher leverage raises the cost of equity

$$r_e = r_o + (r_o - r_d) \times D/E$$

$r_o$ : when debt=0,  $r_{firm} (=r_e)$

$$\beta_a = \beta_d \times (D/V) + \beta_e \times (E/V)$$

assumption  $\Rightarrow \beta_e = \beta_a + (\beta_a - \beta_d) \times (D/E)$

financial distress has no cost;  
debtholders have prior claim to A&income.

$$V = EBIT/r_o \quad r_d < r_e$$

$D/E \uparrow \Rightarrow r_e \uparrow$ , maintain constant WACC

#### ★ MM proposition 1 with taxes

$$V_L = V_U + t \times d$$

#### ★ MM proposition 2 with taxes

WACC is minimized at 100% debt.

$$V_L = EBIT(1-t)/r_o$$

$$r_e = r_o + (r_o - r_d) \times (D/E) \times (1-t)$$

do not consider: cost of financial distress/bankruptcy.

no tax: irrelevant

with tax: WACC is minimized at 100% debt

no optimal capital structure, or a structure with all debt

	without taxes	with taxes
proposition 1	$V_L = V_U$	$V_L = V_U + t \times d$
proposition 2	$r_e = r_o + (r_o - r_d) \times D/E$	$r_e = r_o + (r_o - r_d) \times (D/E) \times (1-t)$

### 3 kinds of costs

#### ★ costs of financial distress

increased costs when earing ↓  
has trouble paying fixed costs

#### two components

cost of financial distress & bankruptcy

direct(cash) & indirect(trust) costs.

probability of financial distress

operating & financial leverage;  
quality of mgt & corp. governance.

lower cost of financial distress if:

firm's assets have a ready second mkt.  
safe, tangible assets, lower cost.

#### ★ agency costs (net agency costs of equity)

costs associated with conflict of interest

managers vs. owners

smaller stake mgt. have, higher cost

#### three components

monitoring costs; bonding costs;

residual loss.

better governed, agency cost ↓.  
use of debt ↑, agency cost ↓.

#### ★ cost of asymmetric information

mgt have more information than investor

#### pecking order theory

mgt prefer way of financing:

disclose less information.

#### order

1. internally generated equity (R/E);

2. debt; 3. external equity.

capital-structure is a by-product

↓ ↓ of individual financing decisions

issuance of equity: negative signal

#### static trade-off theory

$$V_L = V_U + t \times d - PV(\text{costs of financial distress})$$

key points

with increase in financial leverage:

tax shield  $\Rightarrow$  firm value ↑

3 costs  $\Rightarrow$  firm value ↓

#### optimal capital structure

balance of tax shield & 3 costs (lowest WACC, highest  $V_{firm}$ )

### target capital structure

maximize value of firm: **target=optimal**

**in practice, actual capital structure will fluctuate:**

mgt's exploitation of mkt opportunities;  
market value fluctuate.

role of debt rating

maintaining certain minimum debt ratings.

### evaluating capital structure policy

factors should consider:

overtime;

competitors with similar business risk;  
company-specific factors (governance).

scenario analysis

international differences in leverage

include:

total debt (Japan, Europe (no UK))

longer debt maturity (north America)

emerging market differences (more total debt)

reasons:

institutional & legal factors;

financial mkt & banking system factors;

macroeconomic factor.

mergers and acquisitions (T: target; A: acquirer) (2-2)

#### ★ evaluating a merger bid

post-merger value of A

$$V_{AT} = V_A + V_T + S - C; \quad s: \text{synergies}; C: \text{cash paid}$$

gains accrued to T

$$\text{gain}_T = TP = P_T - V_T; \quad TP: \text{takeover premium}$$

gains accrued to A =  $S - TP = S - (P_T - V_T)$

who benefits from merger

short-term

stock: T 30% ↑; A 1%-3% ↓;

winner's curse; hubris of acquirers' mgt

long-term

A tend to underperform; failure to capture synergies

some merger enhance value

strong buyer; low premium;

few bidders; favorable mkt reaction

divestitures (equity carve-out; spin-off; split-off; liquidation)

## dividend policy&company value theory

### does not matter

inference from MM;  
perfect capital market assumptions;  
dividend is irrelevant to company value.  
do not mean dividend per share is irrelevant.

### bird in hand argument

pay dividends will have a lower cost of equity.

### tax aversion

investors prefers the way with lower tax;  
**dividends tax rate>capital gain's;**  
want a zero dividend payout ratio.

### clientele effect

different investors desire different dividend policy  
 **$\Delta P(1-T_{CG}) = \text{div.}(1-T_D)$** ;  $T_{CG} < T_D$ , drops < div.

### signaling

managers have more information(asymmetric);  
dividend is meaningful(∴ asymmetric);  
**unexpected dividend ↑ / ↓ :strong/negative.**

### agency issues

#### shareholders vs. managers

reduce agency cost:payout of FCF as dividends ↑

#### shareholders vs. bondholders

resolved via provisions in bond indenture

### factors affecting dividend policy

investment opportunities;  
expected volatility of future earnings;  
financial flexibility;  
tax considerations;  
flotation costs(high → less dividend);  
contractual and legal restrictions.

### repurchase vs. cash dividend

rationales for shares repurchase

**$T_D > T_{CG}$** ; share price support/signaling;  
offsetting dilution from employ stock options;  
added flexibility; increase financial leverage

global trends in payout policy(lower&trended downwards)

calculation of dividend coverage ratio

NI method: **dividend coverage ratio=NI/dividend paid**

FCF method: **FCFE coverage ration=FCFE/(dividends+ share repurchases)**

## tax consideration-taxation methods

core:effective tax rate(ETR)

### double-taxation

**effective tax rate= $t_{\text{corporate}} + (1-t_{\text{corporate}})t_{\text{individual}}$**

### split-tax rate system (t:corporate income tax)

**$NI = \text{div.} + \text{non.div.}; \text{div.} \rightarrow t_1; \text{non.div.} \rightarrow t_2$**

### tax-imputation system

**ETR=shareholder's marginal tax rate(MTR)**

**shder's MTR>company's,shders pays differences**

**shder's MTR<company's,shders receive a tax credit**

### shders preference for current income(CI) vs. capital gains(CG)

CI  **$t_{\text{dividend}} < t_{\text{capital gain}}$** ;

CG buy high-payout shares for a tax-exempt retirement account(TEA);  
tax-exempt institutions.

**$T_{CG}$  not have to be paid until being sold,  $T_D$  immediately.**

### payout policies

**stable dividend policy** (target payout adjustment model)

**expected dividend=previous dividend +**  
**(expected  $\Delta EPS$ )(target payout ratio)(adjustment factor)**  
**adjustment factor=1/years over which adjustment occur**

constant dividend payout ratio policy(seldom)

### residual dividend policy

model is **based on** firm's:

**investment opportunity schedule;**  
**target capital structure;**  
**access to and cost of external capital.**

steps to determine the target payout ratio:

- 1.identify optimal capital budget
- 2.determine financing by equity  
**for a given capital structure**
- 3.meet equity required with retained earning
- 4.pay dividend with residual earning

**dividend=earnings-(capital budget\*equity%in capital structure)**  
**or zero**

advantages

**easy; maximizes allocation of earnings to investment**

disadvantage

**div. fluctuates with investment opportunities&earnings;**  
**uncertainty cause higher expected return,lower valuation**

**long-term residual dividend(low div.&repurchase)**

## stakeholders of company

internal(stockholders,employees,mgers,board)

external

maximize shareholders' return

**⇒ maximize company's long-run profitability**

an agency relationship

potential conflicts between

### managers vs. shareholders

using funds to expand the size of firm;  
granting excessive compensation;  
investing in risky ventures;  
**not taking enough risk.**

### directors vs. shareholders

lack of independence;  
board members have personal relationship  
**have consulting/other business with firm**  
interlinked boards;  
**directors are overcompensated.**

suggestion

shape the behavior of agent(in accordance with);  
reduce information asymmetry;  
**develop mechanisms for removing bad agent.**

roots of unethical behavior

self-dealing;  
information manipulation;  
anticompetitive behavior;  
opportunistic exploitation(violate suppliers);  
substandard working conditions;  
environmental degradation;  
**corruption.**

philosophical approaches to ethics

the Friedman doctrine(stockholder theory)

**weakness:no regard for justice.**

utilitarian(greatest num. of people)

**weakness:no regard for justice.**

Kantian ethics

**weakness:no place for moral emotions**

rights theories

justice theories.

a system of principles,policies,procedures;  
clearly defined responsibilities&accountabilities  
used by stakeholders to overcome conflicts of interest.

two major objectives  
eliminate or reduce conflicts of interest;  
consistent with the best interests of investors&other stakeholders.

three major business forms  
sole proprietorships; partnerships; corporations

responsibilities of the board of directors  
**evaluate effectiveness of BOD**  
board composition and independence;  
at least 3-quarters of board members should be independent  
lack of independence  
former employment with the company;  
business relationships;personal relationships;  
interlocking directorships;  
ongoing banking or other creditor relationships.

independent chairman of the board;  
dual role of CEO and chairman of the board ✕

qualification of directors;  
annual election of directors;  
annual self-assessment;  
separate sessions of independent directors;

★ **3 committees;**  
**audit committee**  
consists only of **independent directors** ;  
has **expertise** in financial&accounting matters;  
has **full access** to and the cooperationof management;  
meets with auditors **at least** once annually.

**nominating committee**  
consists only of **independent directors**

**compensation committee**  
independent or expert legal counsel;  
corporate governance policies:should assess the following;  
codes of ethics,etc.  
disclosure and transparency;  
insider or related party transactions;  
shareholder proxy votes.

multi-term board(change 1/3):bad

mergers and acquisitions(T:target; A:acquirer)(2-1)

types of mergers  
**horizontal merger**  
**vertical merger** (forward,backward)  
**conglomerate merger**

forms of integration  
statutory merger:A+B=A  
subsidiary merger:A+B=A+B  
consolidation:A+B=C

merger motivations  
synergies  
achieving more rapid growth  
organic/external growth  
increasing market power  
gain access to unique capabilities&resources  
diversification ✕  
bootstrapping earning ✕  
EPS ↑ ✓;P/E donot change ✕  
personal benefits for managers ✕  
tax benefit ✕  
unlocking hidden value  
achieve international business goals

industry life cycle&merger motive

pioneering	conglomerate/horizontal
rapid growth	conglomerate/horizontal
mature growth	vertical/horizontal
stabilization	horizontal
decline	all

valuing a target company(equity notes)  
**takerover premium:PRM=(DP-SP)/SP**  
DP:deal price per share; SP:T'S stock price  
T's takeover price=estimated V<sub>stock</sub> × (1+PRM)

environmental,social and governance factors  
risks can be categorized:  
legislative and regulatory risk  
legal risk; reputational risk;  
operating risk; financial risk

the strength&effectiveness  
risks of an ineffective corporate system  
financial disclosure risk  
asset risk; liability risk; strategic policy risk

form of acquisition	stock purchase	asset purchase
payment	to target firm shders	to target company
approval	majority shders	no,unless substantial
tax:corporate	no	target:Tax CG
tax:shder	Tax CG	no
liabilities of T	acquirers	avoids

**method of payment(securities offering&cash offering)**  
3 main factors(considered) when deciding pmt method  
1.distribution of risk and reward between A&T  
A is confident → cash method  
2.relative valuation(A) of companies involved  
stock of A ↑ → stock offering  
3.changes in capital structure

**attitude of target mgt**  
friednly merger  
hostile merger offers  
bear hug(BOD); tender offer(shders); proxy battle

**takeover defense mechanisms**  
**pre-offer takeover defense** (shark repellents)  
poison pill  
flip-in(T's shders buy T at a discount);  
flip-over(to A's stock);  
dead hand provision(by vote of continuing directors).

poison put  
give right to bondholders of T to put the bonds to T  
states with restrictive takeover laws  
straggered board(only a part of board are due to election)  
restricted voting rights multi-term board  
supermajority voting provision for merger  
fair price ammendment; **golden parachutes**

**post-offer takeover defense**  
"just say no" defense; litigation(lawsuit)  
greenmail(allow T to repurchase back at a premium)  
share repurchase(T's shders)  
leverage recapitalization(repurchase)  
**crown jewel defense** (sell what A likes)  
pac-man defense(acquie A); **white knight/squire defense**

HHI=Σ(MS×100)<sup>2</sup>; MS i :mkt share of i

post-merger HHI	concentration	change in HHI	government
<1000	no	any amount	no action
(1000,1800)	moderately	100 or more	possible
>1800,	highly	50 or more	challenge



Equity 4-1 Valuation concept  
value&valuation

value  
intrinsic value  
 $IV_{analyst-price} = (IV_{actual-price}) + (IV_{analyst-IV_{actual}})$   
fair market value, investment value, liquidation value  
valuation on individual security ↳ particular buyer  
not to index strategy but active  
valuation process  
business-forecast-model-convert-decision  
**Porter's 5 forces**      **3 generic strategies**  
threat of new entrants; cost leadership;  
threat of substitutes; product differentiation;  
bargaining power of buyers; focus.  
bargaining power of suppliers;  
rivalry among existing competitors.  
quantitative factors(accounting&financial information)  
**qualitative factors**  
quality of mgt team;  
transparency of its performance;  
analyst's confidence in the firm's;  
industry's accounting practices.  
quality of inputs  
 $\alpha$  ex ante  $\alpha = HPR_{expected} - \text{required } R$   
ex post  $\alpha = HPR_{actual} - \text{contemporaneous required } R$   
sum-of-the-parts valuation  
 $\Sigma(\text{break-up value}) > \text{firm operates unrelated industries}$   
conglomerate discount  
internal capital inefficiency;  
endogenous factor(reverse synergy);  
research errors.

return concept

holding period return  
required return=opportunity cost=cost of capital  
=IRR(efficient market)  
**equity risk premium(ERP)**  
 $ERP = R_{required \text{ on equity index}} - r_{risk-free}$   
historical estimate  
select an index(stationary);  
time period(long).  
arithmetic mean or geometric mean;  
long term or short term as risk-free;  
survivorship bias.  
forward-looking estimate (FLE, 3 models)  
on current information&expectation;  
models&behavioral bias.  
3 models  
**GGM** (mkt datas)  
 $ERP = \frac{D_1}{P_0} + g - r_{risk-free}$   
g:earning growth  
**supply-side** estimates  
(macroeconomic model)  
 $ERP = (1+i)(1+rEg)(1+PEg) - 1 + Y - R_f$   
all the inputs are expected  
i=inflation  
rEg=real growth in GDP  
PEg=changes in the P/E ratio  
Y=yield on index  
rEg=g labor productivity + g labor supply  
**survey estimate**

required return on equity

**CAPM**

$$R_i = R_{current \text{ expected risk-free}} + \beta_i \times ERP$$

key assumptions

investors are risk aversion;  
decision by  $\mu$  &  $\sigma$  of portfolio.

**$\beta$  estimates**

**public** companies

choice of index;  
length&frequency;  
5 years of monthly data;  
2 years of weekly(fast growth mkt).

adjusted  $\beta$

$$\text{adjusted } \beta = \text{unadjusted } \beta \times 2/3 + 1.0 \times 1/3$$

$\beta$  drift:  $1.0 \times 1/3$        $\uparrow$  pure play method

**nonpublic** companies ( $\beta_{public, asset} = \beta_{non-public, asset}$ )

step1. select benchmark company(public)

step2. estimate benchmark  $\beta$

$$\text{step3. } \beta_{U, non} = \frac{\beta_{E, public}}{1 + (D/E)_{public}}$$

$$\text{step4. } \beta_{E, non} = (1 + (D/E)_{non}) \beta_{U, non}$$

$\beta_{debt} = 0$

**multifactors model**

$$R = R_{risk-free} + RP_1 + RP_2 + \dots + RP_n$$

$RP_i$  = factor sensitivity  $i \times$  factor risk premium  $i$

**FF** (Fama-French model) & **PSM** (Pastor-Stambaugh model) ( $\beta > 0 \Rightarrow$  former)

$$R = R_f + \beta_{mkt} (R_{mkt} - R_f) + \beta_{smb} (R_{s-Rb}) + \beta_{hml} (R_h - R_l) + \beta_{liq} (R_l - R_h)$$

**macroeconomic multi-factor models** (unexpected)

confidence risk( $\Delta CS$ ); time horizon risk( $Y_{T-bond} - Y_{T-bill}$ );  
inflation risk; business-cycle risk; market timing risk(all others).

**build-up method** ( $\beta = 1$ )

$$R = R_f + ERP \pm \text{one or more premium}$$

private company

$$R = R_f + ERP + \text{size premium} + \text{specific company premium}$$

bond yield plus risk premium method

$$R_{BYPRP} = YTM_{company's \text{ long-term debt}} + \text{risk premium}$$

exchange rate risk(or country risk premium, CRP)

$$ERP = ERP_{on \text{ developed mkt}} + CRP$$

country spread model(1 risk):  $R = R_f + RP_1 + RP_2 + \dots + CRP$

country risk rating model(n risks):  $R = R_f + \beta_1 RP_1$

developed  $RP_1 \rightarrow$  new country  $RP_1$

	strength	weakness		strength	weakness
HS	familiar&popular; unbias; objective quality.	precision issues; stationary assmp.; countercyclical;	GGM	popular; developed eco.; typically sample sources. (assump.)	change through time; stable growth rate.
FLE	available; non-stationarity& data bias.	survivorship bias. models bias; behavioral bias.	suppl y-side	proven models; current information.	only developed eco.
			survey	easy to obtain.	wide disparity.

$$WACC = \frac{MVD}{MVD + MVCE} r_d (1-T) + \frac{MVCE}{MVD + MVCE} r_{ce}$$

discount rate  $\Rightarrow$  CF: nominal-nominal, real-real, firm-firm, equity-equity

Equity 4-2 Valuation models  
Discounted dividend valuation

**Gordon Growth model**  $V_0 = \frac{D_1}{r-g}$

assumptions  
1. pay a dividend; 3.  $g < r$ .  
2. dividend grow at a constant rate,  $g$ ;

limitations  
sensitive to estimates of  $r$  &  $g$ ;  
difficult with non-dividend stocks;  
difficult with unpredictable growth patterns.

$V_0 = E_0 / r + PVGO$   $E$ : no-growth earnings level  
 $PVGO$ : PV of growth opportunities

**justified P/E**  
leading  $P/E = V_0 / E_1 = (1-b)/(r-g)$   
trailing  $P/E = V_0 / E_0 = (1-b)(1+g)/(r-g)$

strengths  
stable, mature, dividend paying company;  
valuing market indices;  
easily communicated & explained;  
used to determine  $g, r, PVGO$  (implied);  
supplement more complex methods.

weakness  
sensitive to estimate of  $r$  &  $g$ ;  
not applicable to non-dividend;  
not applicable to unstable growth.

valuing preferred stock  $V_P = D_P / K_P$

**multistage dividend discount models**

**two-stage DDM (1. TV; 2. PV)**

**three-stage DDM**

**H-model**

$$V_0 = \frac{D_0 \times (1 + g_L)}{r - g_L} + \frac{D_0}{r - g_L} \times \frac{t}{2} (g_S - g_L)$$

$g_S$ : short term  
 $g_L$ : long term

spreadsheet modeling  
any pattern of dividend growth;  
can project different growth rates;  
flexibility & computational accuracy.

**sustainable growth rate (SGR)**  
 $SGR = b \times ROE$ ,  $b = 1 - \text{dividend payout rate}$

**PART model**  
 $g = \text{Retention rate} \times \text{Profit\%} \times \text{Asset turnover} \times \text{Leverage}$

Free cash flow valuation

reasons  
no dividend; dividend due to board;  
for acquisition target (control perspective);  
more related to long-run profitability.

valuation approaches ( $r$ : CAPM, APT, build-up)  
 $FCFF(r \rightarrow WACC)$   
 $FCFE(r \rightarrow r_e)$

**FCFF**  
 $FCFF = EBIT \times (1-T) + NCC - WC_{INV} - FC_{INV}$   
 $FCFF = EBITDA \times (1-T) + NCC \times T - WC_{INV} - FC_{INV}$   
 $FCFF = NI + \text{int} \times (1-T) + NCC - WC_{INV} - FC_{INV}$   
 $FCFF = CFO + \text{int} \times (1-T) - FC_{INV}$   
 $NI = (EBIT - \text{int}) \times (1-T)$   
 $CFO = NI + NCC - WC_{INV}$   
 $EBIT = EBITDA - D - A = EBITDA - NCC$

**FCFE**  
 $FCFE = FCFF - \text{int} \times (1-T) + \text{net borrowing}$   
given debt ratio ( $D/A$ )  
 $\text{net borrowing} = (WC_{INV} + FC_{INV} - \text{dep}) \times DR$

**6 issues**

**#1. non-cash charges**  
1. + dep.; 2. amtz.; 3. -G/+L (long-term assets);  
4. + non-cash loss (∴ restructuring);  
5. - non-cash income (∴ restructuring);  
6. + bond discount/-premium (issuer, cash paid);  
7. deferred taxes (-DTA/+DTL) ∴

**#2.  $WC_{INV}$**  ( $NI \downarrow > CF \downarrow, +$ )  
 $WC_{INV} = WC_{END} - WC_{BGN}$ ,  $WC = CA - CL$   
excluding cash; notes, debt (with int.).

**#4. net borrowing**  
 $N.B. = \Delta(L \& S \text{ debt})$

**#5. preferred stock**  
 $NI_{common} = NI_{total} - \text{div. preferred}$

**#6. target debt ratio ( $D/A$ )**  
 $\text{net borrowing} = (WC_{INV} + FC_{INV} - \text{dep}) \times DR$

forecast FCF  
**no effect on both:** div.; share repurchases; share issues  
**leverage:** minor effect on FCFE; no effect on FCFF

free cash flow models

**single-stage model**  
 $V_{firm} = \frac{FCFF_1}{WACC - g}$ ,  $V_{equity} = \frac{FCFE_1}{r - g}$   
 $WACC$  usually calculated using target capital weights

**two & three-stage model**

sensitivity analysis  
future growth rate;  
base year for FCF growth forecast.

calculate terminal value  
single-stage;  
multiple approach  
 $TV_n = (\text{trailing } P/E) \times (\text{earnings in year } n)$   
 $TV_n = (\text{leading } P/E) \times (\text{forecast earnings in year } n+1)$

total capital =  $E + D - \text{cash}$   
 $= E + L - \text{non.int. } D - \text{cash}$   
 $= A - \text{non.int. } D - \text{cash}$   
 $= CA + \text{nonCA} - \text{non.int. } D - \text{cash}$   
 $= ((CA - \text{cash}) - \text{non.int. } D) + \text{nonCA}$   
 $= WC + \text{nonCA} = WC + pp\&e$   
 $\Delta \text{capital} = \Delta WC + \Delta FA = WC_{INV} + FA_{INV} - \text{dep.}$   $\Delta FA = \Delta(BV)$

**#3.  $FC_{INV}$**   
 **$FC_{INV} = \text{capital expenditure} - \text{proceeds}$**

no disposal  
 $FC_{INV} = \Delta(GV)$   
 $FC_{INV} = \Delta(BV) + \text{dep.}$   
 $CAPEX = \Delta(GV)$   
 $GV = BV + A.D.$   
 $\Delta(GV) = \Delta(BV) + \Delta(A.D.)$   
 $\text{dep.} = \Delta(A.D.)$

disposal  
 $FC_{INV} = \Delta(BV) + \text{dep.} - \text{gain}$   
 $CAPEX = \Delta(GV) + GV_{dis}$   
 $= BV_{END} + A.D._{END} + BV_{dis} + A.D._{dis} - BV_{BGN} - A.D._{BGN}$   
 $= BV_{END} + BV_{dis} - BV_{BGN} + \text{dep.}$   
 $\text{gain} = \text{proceeds} - BV_{dis} \rightarrow \text{proceeds} = \text{gain} + BV_{dis}$

Equity 4-3 Valuation models  
Residual income valuation

**concept**  
residual income(economic profit)  
 $RI = NI - \text{equity capital} \times \text{cost of equity}$   
EVA(economic value added)  
 $EVA = NOPAT - WACC \times \text{invested capital}$   
 $= EBIT \times (1-T) - \$WACC$   
NOPAT: net operating profit after tax  
**total capital** =  $BV_{\text{long-term debt}} + BV_{\text{equity}}$   
 $= \text{net WC} + \text{net pp\&e}$   
**market value added(MVA)** =  $MV - \text{total capital}$

**valuation models**  
single-stage RI valuation model  
 $V_0 = B_0 + \frac{(ROE - r_e)}{r_e - g} \times B_0$   
 $\Rightarrow \frac{P_0}{B_0} = 1 + \frac{(ROE - r_e)}{r_e - g}$   
justified P/B,  $ROE > r_e$ ,  $P/B > 1$   
**assume constant ROE&g**  
RI implied growth rate  
 $g = r - \frac{B_0 \times (ROE - r)}{V_0 - B_0}$ , (IV=mkt price)

**strengths**  
TV don't dominate IV;  
accounting data easy to find;  
applicable: no div.; no positive expected FCF;  
applicable: high  $\sigma_{CF}$ , (TV highly uncertain);  
economic profitability.

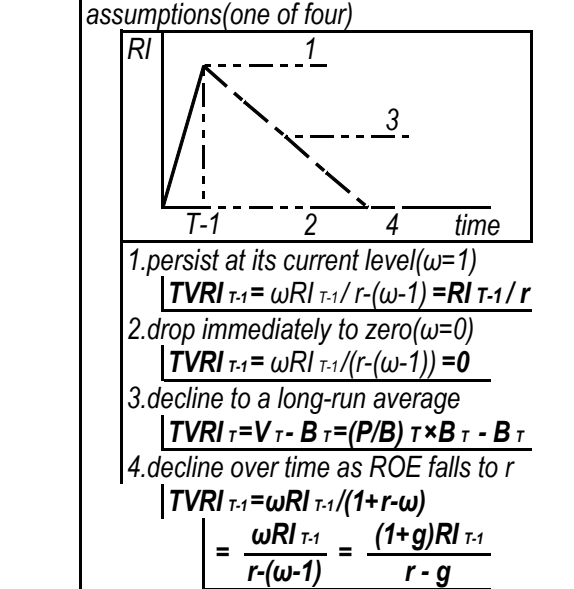
**weakness**  
accounting data can be manipulated by mgt;  
accounting data requires numerous&significant adjustment;  
clean surplus relation.

**not appropriate**  
clean surplus relation is violated;  
significant uncertain  $\rightarrow BV \& ROE$ .

**clean surplus relation**  
may not hold(OCI):  
foreign currency translation gains&losses;  
pension accounting;  
changes in MV of AFS.

**calculation**  
 $RI_T = (EPS_T) - (r_e \times B_{T-1}) = (ROE - r_e) \times B_{T-1}$   
clean surplus relation:  $B_T = B_{T-1} + E_T - D_T$   
 $V = BV_{\text{equity}} + \Sigma PVRI$ , ( $NI = \$R_{\text{equity}} + RI$ )  
adjusted current BV of equity + PV of expected future RI  
relatively less sensitive to terminal value  
difference with DDM&FCFE  
assumption(clean surplus relation)  
 $RI: B + \Sigma PVRI$ ;  $GGM \& FCFE: \Sigma PVCF$

**multistage RI model (IV=BV+ΣPVRI<sub>HIGH</sub>+TVRI)**



**persistent factors** ( $\omega=1+g$ ,  $g < 0$ )  
high  $\omega$   
low dividend payouts;  
historical high RI persistence in the industry.

low  $\omega$   
high ROE;  
significant levels of nonrecurring items;  
high accounting accruals ( $NI = CF + \text{accruals}$ ).

Market-based valuation(price&enterprise value multiples)

<b>P/E</b>		
<b>justified P/E</b>		
leading $P/E_1 = \frac{1-b}{r-g}$ , trailing $P/E_0 = \frac{(1-b)(1+g)}{r-g}$		
<b>normalizing EPS</b>		
historical average EPS		
<b>good:</b> cyclical earning; <b>bad:</b> changes in business size		
average ROE		
more accurate(company's size)		
PEG not linear& differences in duration of growth(risk)		
<b>P/B (common stock)</b>		
<b>justified P/B</b>		
$P/B = \frac{ROE - g}{r - g}$		
common adjustment to BV		
excluding intangible assets(goodwill); use trailing BV.		
<b>P/S</b>		
justified P/S = $\frac{(E_0/S_0)(1-b)(1+g)}{r - g}$ , $E_0/S_0$ : profit margin		
<b>EV/EBITDA</b> (EV:enterprise value) ★★		
$EV = MV$ (common stock+preferred stock+MI+debt) - cash - (short-term investment)		
<b>dividend yield</b>		
justified D/P = $(r-g)/(1+g)$		
trailing D/P = $\frac{4 \times \text{recent Q div.}}{P}$ , leading D/P = $\frac{\text{forecast next 4Q div.}}{P}$		
<b>momentum valuation indicators</b>		
unexpected earnings(UE)=EPS-E(EPS)		
standardized unexpected earnings(SUE)=UE/ $\sigma$ (UE)		
<b>central tendency of a group of multiples( portfolio or index P/E)</b>		
weighted harmonic mean $P/E = 1 / (\sum \frac{W_i}{X_i})$ , $W = P_i / \Sigma P$		
	advantages	disadvantages
P/E	earning power; popular.	earnings can be negative; volatile earning; mgt discretion distorts.
P/B	always > 0; more stable than EPS; NAV, more fit for liquid assets.	size difference; assets off-B/S; influenced by accounting choices; $BV \neq MV$ due to inflation/tech.
P/S	for distressed firms; sales not easily manipulated; $\sigma(P/S) < \sigma(P/E)$ ; for mature, cyclical, start-up.	sales don't imply profit/CF; not capture cost structure; sales still can be distorted.
EV/EBITDA	for different financial leverage; for capital-intensive business; usually > 0.	WC $\uparrow \Rightarrow$ EBITDA overstate CFO; FCFF better.

**company-specific factors**  
stage of life cycle(less mature than public firms);  
size;  
quality&depth of mgt;  
mgt/shareholder **overlap** ;  
long-term investor;  
quality of financial and other info.(less);  
taxes(more concerned).

**stock-specific factoers**  
less liquid; restrictions on marketability;  
concentration of control.

**reasons for valuation**  
transaction-related valuation:  
VC;IPO;sale;bankruptcy;  
performance-based mgt compensation.  
compliance-related valuation  
financial reporting  
tax purposes  
litigation-related valuation  
shder suits,damage claims,etc

6 kinds of values  
fair value  
fair market value  
FV for financial reporting  
FV for litigation  
MV , IV , investment value

GPCM  
discount/premium in PE valuation  
discount of lack of control  

$$DLOC=1-\frac{1}{1+\text{control premium}}$$

discount of lack of marketability  

$$\text{total discount}=1-(1-DLOC)(1-DLOM)$$

3 issues of **financial statement adjustment**  
**normalized earnings**  
exclude nonrecurring&unusual items;  
tax-motivated expenses;  
real estate separate from firm operation.  
**strategic&nonstrategic buyers**  
synergies/not  
**estimating(choose) cash flow**  
FCFF only

3 methods for valuation  
**income approach(ΣPV)**  
FCFF method(2-stage)  
capitalized CF method(GGM)  

$$V_{\text{firm}} = \frac{FCFF_1}{WACC-g}$$

$$V_{\text{equity}} = V_{\text{firm}} - MV_D$$

$$V_{\text{equity}} = \frac{FCFE_1}{r-g}$$

**excess earnings method,EEM (RI)**  
EE=total earnings-R<sub>required</sub> on WC&FA  

$$V_{\text{intangible assets}} = EE \times (1+g)/(r-g)$$

$$V_{\text{firm}} = FV_{WC} + FV_{F.A.} + V_{I.A.}$$

**market approach**  
methods  
guideline **public company** method(GPCM)  
to estimate a control premium  
guideline **transaction** method(GTM)  
no control premium  
**prior transaction** method(PTM)  
market multiples  
**large size**: EBIT/EBITDA multiples  
**small size**: NI multiples  
**asset-based approach(ABA)**  
appropriate in followings  
little hope for better prospect;  
finance firms;  
investment companies;  
small companies/early stage companies;  
natural resource firms.



benchmark curve

**spot int.rate**

$$P(T) = \frac{1}{(1+r(T))^T}$$

**discount factor**:  $P(T)$

annualized, risk-free, no option, ZCB

**forward rate** (f(start, duration))

$$F(T', T) = \frac{1}{(1+f(T', T))^T} \quad F(T', T) = \frac{P(T'+T)}{P(T')}$$

forward **discount factor**:  $F(T', T)$

a breakeven int.rate

avg. (forward) = spot; avg. (spot) = YTM

curve marginal  $r \uparrow$ , avg.  $\uparrow$ , spot  $\uparrow$

flatten at long maturities

riding the yield curve

+bonds(mtr. longer than invest. hrz.)

**par curve**

par rate:  $P_{spot} = \text{par}$ , coupon rate = par rate

bootstrapping

par rate = coupon rate,  $P_{spot} = \text{par} = 1$

cpt: spot rate  $\Rightarrow$  par rate

**YTM**

HTM, coupon + principal made in full,

coupons reinvested at YTM

poor estimate of expected R if:

int.rate volatile

yield curve steeply sloped

risk of default

embedded options

**swap rate** (a type of par rate)

$$\sum \frac{s(T)}{(1+r(t))^t} + \frac{1}{(1+r(T))^T} = 1$$

$V_{float} = V_{fix}$ ,  $V_{float} = \text{par}$ , coupon rate

necessary mkt benchmark for int.rate

wholesale bank use swap rate

retail bank use govmt. spot curve

reflect credit spread of commercial banks

not regulated by any govmt.

yield at many mtr., T-bond only on-the-run

mtr. = maturity

hrz. = horizon

govmt. = government

spread

**swap spread** = swap rate - Treasury rate

**I-spread** = yield risky - swap rate same security

only credit & liquidity risks

**Z-spread**

**TED spread** = LIBOR - T-bill

risk of default on interbank loan

**LIBOR-OIS spread**

low  $\rightarrow$  (high market liquidity)

**term structure theories**

pure expectations theory

risk neutral,  $E(\text{future spot rate})$

local expectation theory

longer  $\rightarrow$  +risk premium

liquidity preference theory

liquidity premium

segmental market theory

each mtr. determined independent

preferred habitat theory

+premium, not related to mtr.

modern term structure models (forward)

equilibrium term structure models

**CIR model**

$$dr = \kappa(\theta - r)dt + \sigma \sqrt{r}dw$$

volatility increases with int.rate

**Vasicek model**

$$dr = \kappa(\theta - r)dt + \sigma dw$$

disadvantage

don't force int.rate  $\geq 0$

arbitrage-free model

**Ho-Lee model**

$$dr = \lambda(t)dt + \sigma dw$$

yield curve factor

change in level of int.rate (parallel), 77%

change in slope/steepness of yield curve, 17%

change in curvature of yield curve, 3%

yield curve risk

manage yield curve risk

effective duration (only for parallel)

**cpt key rate duration**

int.rate volatility

$$\sigma(t, T) = \frac{\sigma[\Delta r(t, T)/r(t, T)]}{\sqrt{\Delta t}}$$

arbitrage opportunity (law of one price)

two types

value additivity

dominance

fixed-income arbitrage

stripping

reconstitution

arbitrage-free valuation

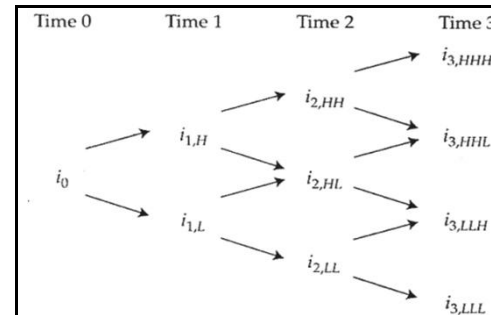
step 1. estimate the future cash flows

step 2. determine the discount rate

step 3. calculate PV of expected future CFs

interest rate trees & arbitrage-free valuation

binomial interest rate tree construction



assume equal probability (in CFA)

node, path, step ( $\Delta t$ ), period (time)

determine int.rate in int.rate trees (forward rate)

$$i_H = i_L e^{2\sigma}$$

$\sigma$ : assumed volatility of the one-year rate

estimate: historical  $\sigma$  & mkt price of int.rate derivatives

construction of a binomial int.rate tree

**valuation a option free bond**

step by step, don't forget coupons

arbitrage-free valuation

comparison: zero-coupon yield curve & binomial tree

**pathwise valuation**

binomial tree n period, will be  $2^n$  paths

path	year1	year2	year3	value(PV)
1 SUU	$i_0$	$i_U$	$i_{UU}$	
2 SUL	$i_0$	$i_U$	$i_{UL}$	
3 SLU	$i_0$	$i_L$	$i_{LU}$	
4 SLL	$i_0$	$i_L$	$i_{LL}$	
AVG				$\Sigma PV/4$

fixed-income securities with embedded option

simple options

European/American/Bermudan(lockout)

complex options

estate put(contingent put option)

sinking fund bonds(sinkers)

call provisions;delivery options

acceleration provisions

valuation and analysis:callable&putable

callable&putable bond

①calculate value of the callable/putable bond

strike time

price callable ≤ call price

price callable > call price, issuer repurchase

price putable ≥ put price

price putable < put price, investors redeem

calculate value of the call/put(not directly)

V callable = V pure - V call

V putable = V pure + V put

②calculate call on bond(directly)

call on **European** bond

only stike at a specific moment

call on **American** bond

every moment

Σprobability×PV of max (V<sub>1</sub>, V<sub>2</sub>)

rate in the up state

$R_u = R_d \times e^{2\sigma \sqrt{t}}$

①effective of volatility on V<sub>option</sub>

V callable = V pure - V call

σ ↑ ⇒ V call ↑ ⇒ V callable ↓

V putable = V pure + V put

σ ↑ ⇒ V put ↑ ⇒ V putable ↑

②effective of yield curve

call option

int.rate ↓, V call ↑

upward sloping yied curve, V call lower

put option

int.rate ↑, V put ↑

upward sloping yied curve, V call higher

OAS & Z-spread

zero-volatility spread(z-spread,one path)

z-spread = R - R<sub>free</sub>

z-spread: credit risk + liquidity risk + option risk

option adjusted spread(OAS,many paths)

OAS = credit-spread + liquidity spread

OAS: z-spread - option risk

int.rate volatility and OAS

OAS of callable bond = z - V<sub>call</sub> (= z - IV<sub>call</sub>)

call is bad, without call, risk ↓, OAS ↓

OAS of putable bond = z - V<sub>put</sub> (= z + IV<sub>put</sub>)

put is good, without put, risk ↑, OAS ↑

	Treasury benchmark	bond sector benchmark	issuer- specific
N-spread	credit	credit	liquidity
Z-spread	liquidity option	liquidity option	liquidity option
OAS	credit liquidity	credit liquidity	liquidity
OAS > 0	overvalued OAS < OAS <sub>reqd</sub> undervalued OAS > OAS <sub>reqd</sub>	overvalued OAS < OAS <sub>reqd</sub> undervalued OAS > OAS <sub>reqd</sub>	undervalued
OAS = 0	over	over	fairly priced
OAS < 0	over	over	over

OAS<sub>reqd</sub>: OAS required, OAS theory

effective duration&effective convexity

$$ED = \frac{P_{big} - P_{small}}{2P_0 \Delta y} \quad EC = \frac{P_{big} + P_{small} - 2P_0}{2P_0 \Delta y^2}$$

use binomial model to cpt effective duration&convexity

effective duration(among callable,putable,straight bonds)

ED(with an option) < ED(straight)

ED(ZCB) ≈ maturity of the bond

ED(have coupon) < maturity of the bond

ED(floater) ≈ time to next reset date

int.rate ↑ ( ↓ ), putable(callable) strike, ED ↓

float with cap(max)&floor(min)

floater: floating rate bond

capped floater protect **issuer(borrower)** against int.rate ↑

③calculate value of floater(binomial)

V capped floater = V straight - V embedded cap

V floored floater = V straight + V embedded floor

ratchet bond

ratchet down: coupon ↓

contingent put(on coupon reset date)

convertible bond(con.bond)

conversion ratio: num. of stocks for a con.bond

conversion price = bond issue price / conversion ratio

market conversion price = bond market price / conversion ratio

conversion value = market price of stock × conversion ratio

straight value(pure value)

mkt conversion premium/share

= mkt conversion price - market price(stock)

mkt conversion premium ratio

= (mkt conversion premium/share) / mkt price of stock

premium over straight value =  $\frac{\text{mkt price of con.bond}}{\text{straight value}} - 1$

effects of embedded option in con.bond

V con.bond = V straight + V call on equity

σ ↑ ⇒ V call ↑ ⇒ V con.bond ↑

stock price ↑ ↑ ⇒ V con.bond ↑

convexity

straight bonds&putable bonds, EC > 0

callable bonds

int.rate ↑, unlikely to be call, EC > 0

int.rate ↓, call is near the money, EC < 0

③one-side durations

price sensitivity of bond with option: not symmetrical

callable bond(down&up: int.rate)

one-side down-duration < one-side up duration

putable bond

one-side down-duration > one-side up duration

key rate duration

bond with low coupon rate may have negative KRD(≈ 0)

## Credit analysis models

## credit risk

$$EL = PD \times LGD \times EAD$$

**PD**, probability of default, %

**EAD**, exposure at default, \$

**LGD**, loss given default, % (RR, recovery rate)

$$\text{credit spread} = YTM_{\text{credit-risky ZCB}} - r_{\text{risk-free}}$$

## PV of expected loss

two adjustments

time value adjustment

risk-neutral probabilities

$$PVEL = EL + \text{risk premium} - \text{time value discount}$$

$$PVEL = V_{\text{risk-free}} - V_{\text{risky}}$$

## structure models

$$\text{Merton model } (V_{\text{equity}} = \max(A - K, 0), V_{\text{debt}} = \min(A, K))$$

## risk-neutral

shareholder

Equity = call

$$V_{\text{equity},t} = A_t N(d_1) - Ke^{-r(T-t)} N(d_2)$$

bondholder

$$\text{debt}_{\text{risky}} + EL = \text{debt}_{\text{risk-free}}; \quad D = V - E$$

$$V_{\text{debt},t} = A_t N(-d_1) + Ke^{-r(T-t)} N(d_2)$$

$$d_1 = \frac{\ln(A_t/K) + r(T-t) + 0.5\sigma^2(T-t)}{\sigma \sqrt{T-t}}$$

$$d_2 = d_1 - \sigma \sqrt{T-t}$$

**r: risk-free rate**

$$PD = 1 - N(d_2) = N(-d_2)$$

## traditional credit models

## credit scoring (for small businesses &amp; individuals)

ordinal rankings: don't improve with economy;  
(pressure) prioritize stability; change over time;  
different PD of same borrower.

## credit rating (least accurate)

strengths

simple to understand & reduce volatility

weakness

correlation of PD & rating ↓ (∵ stability)

don't adjust with business cycle

issuer-pays model

## real world

$$EL_t = V_{\text{risk-free}} - V_{\text{risky}}$$

$$= K - [A_t N(-e_1) + Ke^{-r(T-t)} N(e_2)] e^{r(T-t)}$$

$$= KN(-e_2) - A_t e^{r(T-t)} N(-e_1)$$

$$PVEL = EL_t \times \exp(-r(T-t))$$

$$= Ke^{-r(T-t)} N(-d_2) - A_t N(-d_1)$$

$$e_1 = \frac{\ln(A_t/K) + \mu(T-t) + 0.5\sigma^2(T-t)}{\sigma \sqrt{T-t}}$$

$$e_2 = e_1 - \sigma \sqrt{T-t}$$

**μ: actual return rate of asset**

$$PD = 1 - N(e_2) = N(-e_2)$$

	structure models	reduced form models
<b>assumption</b>	assets trading in a frictionless mkt; risk-free rate is constant; simple balance sheet structure (one debt, no coupon)	a frictionless & arbitrage-free mkt; r & state of economy are stochastic (random)
<b>strengths</b>	provide option analogy to cpt PD, debt, EL	historical simulation (input are observable); credit risk fluctuate with business cycle; do not require B/S structure
<b>weaknesses</b>	assumptions are not realistic; do not consider business cycle.	λ: hard to estimate

## reduced form models

**exponential distribution** (λ = hazard rate)

$$\text{credit spread} \approx PD \times LGD$$

$$PD = 1 - \exp(-\lambda(T-t))$$

credit analysis of ABS is different from corporate bonds

credit risk metric, waterfall (tranch)

allow input parameters vary with economy  
value of debt (risk-neutral)

$$D_t = E\left(\frac{K}{1 + r_i}\right), r_i = \text{risk-free rate for year } i$$

## Credit default swaps

## CDS

insurance contract; CDS spread; fixed coupon

single-name CDS

cheapest-to-deliver

index CDS (basket)

credit event

bankruptcy

failure to pay

restructuring

## cash settlement

par value - mkt value

## physical settlement

reference obligation (bond/loan); par value

market price of CDS

CDS spread

$$\text{CDS spread} = PD \times LGD$$

λ = hazard rate

premium leg = protection leg

upfront premium on a CDS

$$\text{upfront pmt} = PV(\text{protection leg}) - PV(\text{premium leg})$$

$$\text{upfront premium \%} \approx (\text{CDS spread} - \text{CDS coupon}) \times \text{duration}$$

$$\text{price of CDS (per \$100)} \approx \$100 - \text{upfront premium (\%)}$$

$$\text{premium} + \text{coupon} = \text{protection, premium} = \text{spread} - \text{coupon}$$

profit for protection buyer (%) ≈ change in spread (%) × duration

protection buyer is short credit risk and benefit when CS widen

naked CDS

long/short trade (long & short, different spread)

basis trade

CDO, synthetic CDO (eg. CLN)

forward contracts

long forward position

party who agree to buy **assets**

**no-arbitrage principle**

assets with same identical future CF, same price  
certain payoffs → risk-free rate of return

$$FP = S_0 \times (1 + R_f)^T$$

forward arbitrage(long low, short high)

$FP > S_0 \times (1 + R_f)^T$  , cash and carry arbitrage

$FP < S_0 \times (1 + R_f)^T$  , reverse cash and carry arbitrage

$$\text{profit} = FP - S_0 \times (1 + R_f)^T$$

**pricing** a forward contract(no-arbitrage price)

★ ★  $FP = S_0 + \text{carrying costs} - \text{carrying benefits}$

predetermine pmt to buy the asset at settlement date

**valuation** of a forward contract

★ ★ determine  $V_{\text{forward}, t}, t \in [0, T]: V = PVCF_{\text{in}} - PVCF_{\text{out}}$

**forward contract price&value** ( $\Delta t = T - t, t \in [0, T]$ )

**T-bill(zero-coupon bond) forward**

$$\text{price: } FP = S_0 \times (1 + R_f)^T$$

$$\text{value: } V_{\text{long}, t} = S_t - FP \times (1 + R_f)^{-\Delta t}$$

**forward on a dividend-paying stock**

$$\text{price: } FP = (S_0 - PVD_0) \times (1 + R_f)^T$$

$$\text{value: } V_{\text{long}, t} = S_t - PVD_t - FP \times (1 + R_f)^{-\Delta t}$$
$$= PV_{t, T}[F_t(T) - F_0(T)]$$

**forward on an equity-index**

continuously compounded

$$\text{price: } FP = S_0 \times e^{(R_f - \delta)T}$$

$$\text{value: } V_{\text{long}, t} = S_t \times e^{-\delta \Delta t} - FP \times e^{-R_f \Delta t}$$

$\delta$ : dividend yield

**forward on coupon bonds**

$$\text{price: } FP = (S_0 - PVC_0) \times (1 + R_f)^T \dots \dots \dots$$

$$\text{value: } V_{\text{long}, t} = S_t - PVC_t - FP \times (1 + R_f)^{-\Delta t}$$
$$= PV_{t, T}[F_t(T) - F_0(T)]$$

**currency forward contracts**

price(covered int. rate parity)

$$FP = S_0 \times \frac{(1 + R_{dc})^T}{(1 + R_{fc})^T}$$

value:

$$V_{\text{long}, t} = \frac{S_t}{(1 + R_{fc})^{\Delta t}} - \frac{FP}{(1 + R_{dc})^{\Delta t}}$$

$R_f$ : return rate;  $R_d$ : discount rate

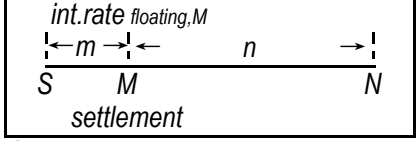
**forward rate agreement(FRAs, simple interest)**

underlying assets: LIBOR(Eurodollar)

LIBOR: London interbank offered rate(simple interest)

Euribor: Europe interbank offered rate

**M×N FRA** , forward rate =  $R_{FRA}$



**settlement**

$$\text{payoff}_L = NP \times \frac{(\text{int. rate floating, } M - R_{FRA})(n/360)}{1 + \text{int. rate floating, } M \times (n/360)}$$

$NP$ : notional principal

**FRA pricing** (no arbitrage forward rate( $R_{MN}$ ))

$$(1 + R_{SN} \times (m+n)/360) = (1 + R_{SM} \times m/360)(1 + R_{MN} \times n/360)$$

**valuation** ( $t \in [S, M]$ , long)

$$V_{FRA} = PV(CF_{\text{in}}) - PV(CF_{\text{out}})$$

$M$ : NP in;  $N$ :  $(NP + \$\text{interest}_{FRA})$  out

**T-bond futures**

underlying

hypothetical 30 year T-bond with 6% coupon rate

deliverable(deliver cheapest-to-deliver bond)

\$100,000 par value T-bond, any coupon, maturity  $\geq 15$  years

conversion factor(CF)

**pricing**

$$FP_{\text{standard}} = FP_A \times \frac{1}{CF_A}$$

**quoted futures price(QFP)**

clean price = full price - accrued interest

$$FP_{\text{clean}} = [(S_0 + AI_0) \times (1 + R_f)^T - FVC] - AI_T$$

$$QFP = FP_{\text{clean}} \times \frac{1}{CF}$$

**swaption contracts(option)**

an option to enter into a swap

2×5 swaption

after 2time, bgn a swap of 5time

maybe month/year

valuation(fixed payer)

$$V = \max(PV(NP \times \text{periodic } \Delta), 0)$$

$\Delta$  = current swap rate - swap rate

**payer swaption** (fixed payer, call on int., put on bond)

**receiver swaption** (fixed receiver, call on bond, put on int.)

swap

**interest rate swap(IRS)**

plain vanilla swap(fixed  $\rightleftharpoons$  float)

**pricing(fixed rate)**

$$V_{\text{fixed bond}} = V_{\text{float bond}}, V_{\text{float bond}} = \text{par} = 1$$

$$1 = C \times B_1 + C \times B_2 + \dots + C \times B_n + 1 \times B_n$$

$$\star C = \frac{1 - B_n}{\sum B}, B_n: \text{discount factor}$$

**swap rate** = (annualized C)/1

**valuation** (fixed out, float in)

$$V_t = PV_{\text{float}} - PV_{\text{fixed}}$$

$C_{\text{float}}$ : latest future coupon

**cross currency swap(CCS)**

A: \$  $\rightleftharpoons$  €; B,  $t \in [0, T]$

	A(need €)	B(need \$)
$t=0$	\$P_{\text{out}}; \text{€}P_{\text{in}}	\$P_{\text{in}}; \text{€}P_{\text{out}}
$t \in (0, T)$	\$C_{\text{in}}; \text{€}C_{\text{out}}	\$C_{\text{out}}; \text{€}C_{\text{in}}
$t=T$	\$P_{\text{in}}; \text{€}P_{\text{out}}	\$P_{\text{out}}; \text{€}P_{\text{in}}

exchange principle & do not netting

**pricing(fixed rate, like IRS)**

\$fixed  $\rightleftharpoons$  €float

\$floating rate → \$fixed rate

\$fixed  $\rightleftharpoons$  €fixed

\$floating rate → \$fixed rate

€floating rate → €fixed rate

\$float  $\rightleftharpoons$  €float

no pricing

**valuation** (BGN: \$out, €in)

$$V_t = PV_{\text{in}} - PV_{\text{out}} = PV\$t - PV\text{€}t$$

**discount rate**: \$ → \$; € → €

use **exchange rate at t**

**equity swap contracts**

$R_{\text{EQUITY}} \rightleftharpoons \text{fixed/float/}$   $R_{\text{EQUITY}}$

**pricing** (fixed rate, like IRS, only  $R_{\text{EQUITY}} \rightleftharpoons \text{fixed}$ )

$$C = \frac{1 - B_n}{\sum B}, B_n: \text{discount factor}$$

**valuation** (fixed out, float in)

$$V_t = PV_{\text{float}} - PV_{\text{fixed}}$$

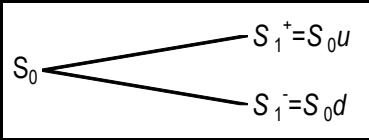


Derivative3-2 Valuation of contingent claims  
binomial model-the expectations approach

put-call parity for European options  
a fiduciary call=a protective put

★  $call + Ke^{-r\Delta t} = put + S_0$

one-period binomial model



assume:  $ud=1$

expectation approach(risk-neutral)

$\pi_u$ : risk-neutral probability of an up move

★  $\pi_u = \frac{1+R_f - d}{u - d}, \pi_u + \pi_d = 1$

valuation(X:strike price)

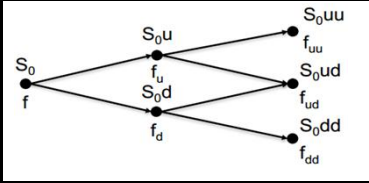
$C_u = \max(0, S_u - X), C_d = \max(0, S_d - X)$

$V_{call} = (\pi_u C_u + \pi_d C_d) \times (1+R_f)^{-T}$

hedge ratio(delta)

★  $h = (C_u - C_d) / (S_u - S_d) = \Delta C / \Delta S, \Delta C = \Delta \times \Delta S$

two-period binomial model



American vs. European

$V_{American} > V_{European}$

at t moment: if  $\max(0, S_t - X) > C_t$ , strike ( $t \in (0, T)$ )

binomial interest rate tree

int. rates at each node are 1-year forward rates

risk-neutral probability:  $\pi=0.5$

interest rate option( $M \times N$ )

interest rate call option(cap= $\Sigma$ caplet)

payoff= $\max(0, \text{reference rate} - \text{exercise rate}) \times NP$

interest rate put option(floor)

payoff= $\max(0, \text{exercise rate} - \text{reference rate}) \times NP$

Black-Scholes-Merton Model

continuous-time option pricing model

assumptions

$P \sim \text{lognormal};$

(CFA: logarithmic return is normal)

$P \rightarrow \text{continuous}; \sigma: \text{known \& constant};$

$R$ : risk-free, continuous: known & constant;

mkt: frictionless; no cash flow;

European options.

price of European option use BSM

★  $C_0 = S_0 N(d_1) - Xe^{-R_f T} N(d_2)$

★  $P_0 = Xe^{-R_f T} N(-d_2) - S_0 N(-d_1)$

$P_0 = C_0 - S_0 + Xe^{-R_f T}$

$d_{1,2} = \frac{\ln(S_0/X) + R_f T \pm 0.5\sigma^2 T}{\sigma \sqrt{T}}$

$d_2 = d_1 - \sigma \sqrt{T}$

features of BSM PV of option payoff( $C_0$ )

leveraged stock investment

$C_0$ : borrow money ( $XN(d_2)e^{-R_f T}$ )

to invest in stock ( $S_0 N(d_1)$ )

$P_0$ : short stock to buy the bond

probability of strike= $N(d_2)=1-N(-d_2)$

other options

options on dividend paying stock

$C_0 = S_0 N(d_1) e^{-\delta T} - XN(d_2) e^{-R_f T}$

$\delta$ : dividend yield

$d_{1,2} = \frac{\ln(S_0/X) + (R_f - \delta)T \pm 0.5\sigma^2 T}{\sigma \sqrt{T}}$

$d_2 = d_1 - \sigma \sqrt{T}$

options on currencies

$C_0 = S_0 N(d_1) \exp(-R_{foreign} T) - XN(d_2) \exp(-R_{domestic} T)$

$R_{foreign}$ : foreign  $R$  risk-free, as a return

$R_{domestic}$ : domestic  $R$  risk-free, as a discount rate

rho( $\rho$ )

$R_f$   
impact < vega

implied volatility  
use BSM

vega =  $\partial C / \partial \sigma$

vega(call)=vega(put)  
long: vega > 0

theta(time decay)

theta(call)=theta(put)  
long:  $\theta < 0$

gamma =  $\partial \Delta / \partial S$

normal distribution  
 $\Gamma(\text{call}) = \Gamma(\text{put})$   
long:  $\Gamma > 0$   
short:  $\Gamma < 0$

gamma largest: ATM

rebalance  $\Delta$ -neutral portfolio more frequently

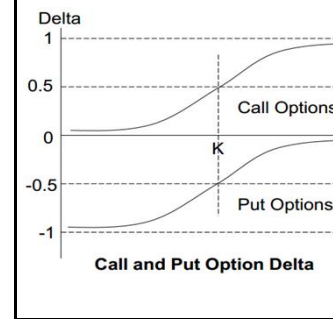
Option Greeks&Implied volatility

5 inputs of BSM model and sensitivity factors \*European put  $\theta$  may be negative

sensitivity	input	Calls	Puts
$\Delta$ : delta	$S_0$ : underlying asset price	$+, \Delta > 0$	$-, \Delta < 0$
$\Lambda$ : vega	$\sigma$ : volatility	$+, \Lambda > 0$	$+, \Lambda > 0$
$\rho$ : rho	$R_f$ : risk-free rate	$+, \rho > 0$	$-, \rho < 0$
$\theta$ : theta	$T$ : time to expiration	$\theta < 0$	$\theta < 0^*$ (time decay)
	$X$ : strike price	-	+

★ delta( $\Delta = \Delta \text{option} / \Delta \text{underlying}$ )

delta call =  $\Delta C / \Delta S$ ; delta put =  $\Delta P / \Delta S = \text{delta call} - 1$



call:  $\Delta \in (0, 1)$ ; put:  $\Delta \in (-1, 0)$

long C, short P:  $\Delta > 0$

long P, short C:  $\Delta < 0$

call:  $\Delta = N(d_1)$ ; put:  $\Delta = N(d_1) - 1$

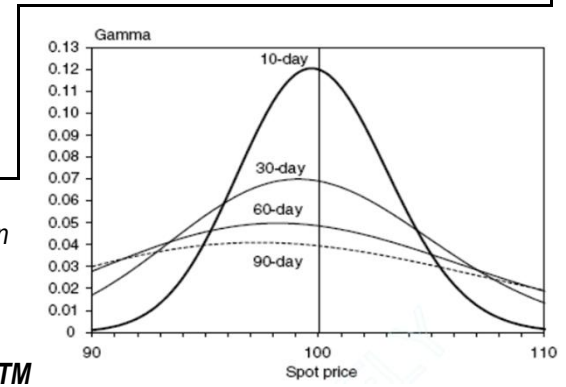
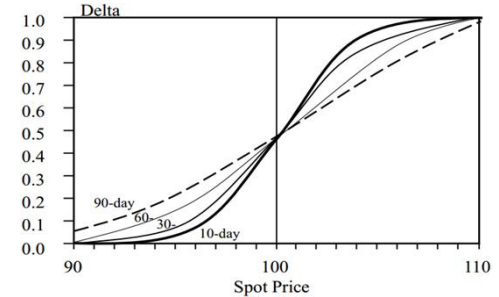
	OTM	ATM	ITM
call	$\Delta = 0$	$\Delta = 0.5$	$\Delta = 1$
put	$\Delta = 0$	$\Delta = -0.5$	$\Delta = -1$

dynamic hedging( $t \rightarrow T \Rightarrow \text{delta changes} \Rightarrow \text{change NC}$ )

$NS \times \Delta S - NC \times \Delta C = 0 \Rightarrow NS / NC = \Delta C / \Delta S = \text{delta} \Rightarrow NS = \Delta \times NC$

$NS$ : num. of shares;  
 $NC$ : num. of calls

$t \rightarrow T$ , delta is unstable



# Derivative3-3 Valuation of contingent claims binomial model-the no-arbitrage approach

hedge ration:  $h = \Delta C / \Delta S$

**$\Delta C = h \times \Delta S$ : 1 C required h S to hedge**

no-arbitrage approach

construct a portfolio with no delta risk

call option

$$C_0 - hS_0 = PV[(C_-) - (hS_-)] = PV[(C_+) - (hS_+)]$$

+: stock price up; -: stock price down

$$C_0 = hS_0 + PV[C_- - hS_-] = hS_0 + PV[C_+ - hS_+]$$

$$h = N(d_1)$$

↑↓ compare  $C_0 = +$  stock - bond (or borrow money)

$$C_0 = S_0 N(d_1) - XN(d_2) \exp(-R_f T)$$

put option

$$P_0 = hS_0 + PV[(P_-) - (hS_-)] = hS_0 + PV[(P_+) - (hS_+)]$$

↑↓ compare:  $h = -N(-d_1)$ ,  $P_0 = +$  bond - stock

$$P_0 = XN(-d_2) \exp(-R_f T) - S_0 N(-d_1)$$

## Black option valuation model

option on futures

underlying: futures

$$C_0 = \exp(-R_f T) [FP \times N(d_1) - XN(d_2)]$$

$$P_0 = \exp(-R_f T) [XN(-d_2) - FP \times N(-d_1)]$$

$$d_{1,2} = \frac{\ln(FP/X) \pm 0.5\sigma^2 T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

$$C_0 = \max(0, PV(FP - X))$$

call: long futures financed by bond

interest rate option

underlying: forward rate or FRA rate

strike int. rate are fixed in advance

$$C_0 = NP \times AP \times \exp(-rN \times 30/360) [FP_{M \times N}(d_1) - XN(d_2)]$$

$$AP = \text{accrual period} = (N - M) \times 30/360$$

option on swaptions

right to enter into an IRS

with a pre-specified exercise swap rate

underlying: forward swap rate

payer swaption

$$V_{\text{payer}} = NP \times AP \times PVA[R_{\text{fixed}} N(d_1) - R \times N(d_2)]$$

receiver swaption

$$V_{\text{payer}} = NP \times AP \times PVA[R \times N(d_1) - R_{\text{fixed}} N(d_2)]$$

$R_{\text{fixed}}$ : fixed swap rate starting when swaption expires

$R$ : exercise rate;  $PVA = \sum PV_{0,t_i}(1), (\text{annuity})$

## Derivatives strategies

derivatives application

interest rate derivatives

futures

swap(D: duration)

D payer swap

$$= D \text{ floating rate note} - D \text{ fixed bond}$$

D receiver swap

$$= D \text{ fixed bond} - D \text{ floating rate note}$$

equity derivatives

futures & swap

currency derivatives

futures & swap

index derivatives

futures

synthetic stock index fund & synthetic cash

synthetic long/short asset

call - put = asset

put - call = -asset

synthetic call/put

call = asset + put

put = -asset + call

synthetic stock

= risk-free asset + stock futures

synthetic cash (earn risk-free rate)

= stock - stock futures

## ★ combination strategies

collar = stock + p - c = protective put - c = covered call + p

$$\text{payoff} = S_T - S_0 + \max(0, (X_L - S_T)) - P - \max(0, (S_T - X_H)) + C$$

$$\text{profit max} = X_H - S_0 - P + C$$

$$\text{loss max} = S_0 - X_L + P - C$$

$$\text{breakeven: } S_T = S_0 + P - C$$

limit the downside risk

as a cost, giving up the upside return

straddle = c + p,  $k = k(k \neq k, \text{strangle})$

$$\text{payoff} = \max(0, (S_T - X)) - C - \max(0, (X - S_T)) + P$$

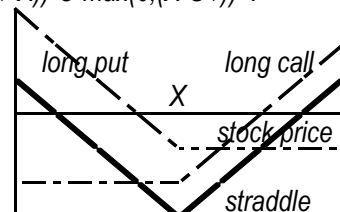
$$\text{profit max} \rightarrow +\infty$$

$$\text{loss max} = P + C$$

breakeven

$$S_T = X + (P + C)$$

$$S_T = X - (P + C)$$



option strategies

## ★ simple strategies

covered call = S - C

$$\text{payoff} = (S_T - S_0) - \max(0, (S_T - X)) + C$$

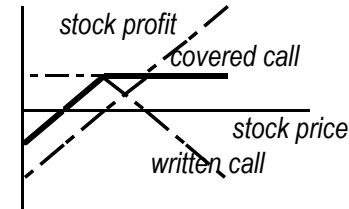
$$S_T > X, \text{ payoff max} = X - S_0 + C$$

$$S_T = 0, \text{ payoff min} = C - S_0$$

$$\text{breakeven: } S_T = S_0 - C$$

caps the upside potential at X

$$\text{delta covered call} = \text{delta stock} - \text{delta call}$$



protective put = S + P

$$\text{payoff} = (S_T - S_0) + \max(0, (X - S_T)) - P$$

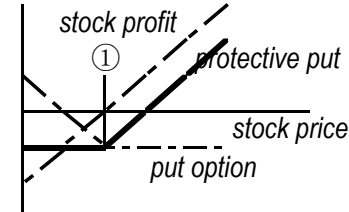
$$\text{payoff max} \rightarrow +\infty$$

$$S_T = 0, \text{ payoff min} = X - S_0 - P$$

$$\text{breakeven: } S_T = S_0 + P$$

$$\text{① deductible} = S_0 - X$$

$$\text{delta protective put} = \text{delta stock} + \text{delta put}$$



cash secured put (-put, secured with depositing the amount (= exercise price))

## ★ spread strategies (all call/all put)

bull call/put spread: long low, short high

$$\text{payoff} = \max(0, (S_T - X_L)) - \max(0, (S_T - X_H)) - C_L + C_H$$

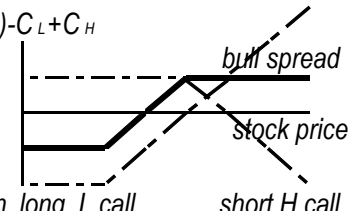
$$S_T > X_H, \text{ payoff max} = X_H - X_L + C_H - C_L$$

$$S_T < X_L, \text{ payoff min} = C_H - C_L$$

$$\text{breakeven: } S_T = X_L + C_L - C_H$$

limit the downside risk

as a cost, giving up the upside return



bear call/put spread: long high, short low

$$\text{payoff} = \max(0, (S_T - X_H)) - \max(0, (S_T - X_L)) + C_L - C_H$$

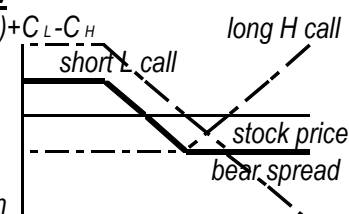
$$S_T < X_L, \text{ payoff max} = C_L - C_H$$

$$S_T > X_H, \text{ payoff min} = X_L - X_H + C_L - C_H$$

$$\text{breakeven: } S_T = X_L + C_L - C_H$$

limit the downside risk

as a cost, giving up the upside return



calendar spread

short a near-dated call, long a longer-dated one

time decay is more pronounced for a short-term option

breakeven price analytics

calculate the breakeven price for the strategy

calculate breakeven price deviation

$$\% \Delta P = \frac{|BP - CP|}{CP}$$

calculate annual breakeven volatility

$$\sigma_{\text{annual}} = \% \Delta P \times \left( \frac{252}{T_{\text{DUM}}} \right)^{0.5}$$

T<sub>DUM</sub>: trading days until maturity

# Alternative3-1 Private real estate investment

## private real estate investment

### forms of real estate

#### first dimension:private/public

##### private investments

solely owned; partnerships;  
commingled real estate funds(CREF)

##### public market investments

real estate investment trust( **REIT** )  
real estate operating company( **REOC** )  
mortgage-backed securities

#### second dimension:debt/equity

### forms of real estate

#### real estate characteristics

heterogeneity; high unit value;  
active management; lack of liquidity;  
high transaction(brokers,lawyers);  
depreciation&desirability;  
cost&availability of debt capital;  
difficulty in determining price

#### REITs' advantages

actively traded,more likely to reflect MV;  
diversified real estate portfolio;  
don't need property management

### property classifications

#### commercial real estate property

**multi-family property;** **office;**  
industrial/warehouse; **retail;**  
hospitality; parking,restaurants,etc.

#### non-commercial real estate property

##### unique categories

farmland; timberland

### valuations

#### types of value

**market value; investment value;**  
value in use; assessed value;  
mortgage lending value

#### highest and best

#### reasons to invest

current income; capital appreciation;  
inflation hedge; diversification;  
tax benefits

#### risks to invest

business conditions; cost&availability of capital;  
new property lead time(futures);  
unexpected inflation; demographic factors;  
lack of liquidity; environmental issues;  
availability of information;  
management expertise; leverage

#### indices

quarterly data → lag actual transactions;smooth  
repeat-sale index&hedonic index(factor)

### ★ income approach(vs DCF)

#### single-stage (direct capitalization method)

##### net operating income (cash)(gross lease)

= rental income from fully occupied + other income =potential gross income  
- vacancy and collection loss =effective gross income  
- operating expense

capitalization rate (cap rate)=discount rate - growth;  $V=NOI_1/cap\ rate$

stabilized NOI:remove temporary issues(renovation)

gross income multiplier=sales price/gross income

#### two-stage(DCF method)

##### ★ direct income approach(TV approach)( 2nd stage:reversion or resale )

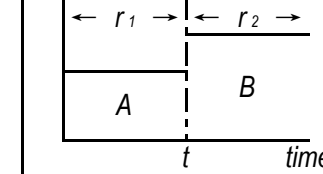
$$MV=\sum \frac{NOI_i}{(1+r)^i} + \frac{MV_n}{(1+r)^n}, \quad MV_n=\frac{NOI_{n+1}}{r-g} \quad r-g:\text{terminal cap rate}$$

limitations: difficult to selecting the appropriate cap rate

not applicable for owner-occupied propertys that provide other benefits

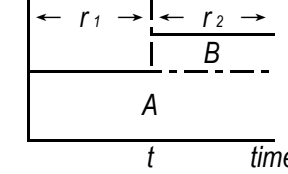
#### 2nd stage:not resale

##### ★ term and reversion approach



assume  $g=0$ , cap rate= $r$

##### ★ layer method



$$V=PV(A) \text{ by } r_1 + PV[PV(B) \text{ t, by } r_2] \text{ by } r_2$$

### ★ cost approach(vs asset-based)

$V= \text{replacement cost} - \text{dep.} - \text{obsolescence} - \text{curable} (\text{cost} + PV(\text{rent})) + MV \text{ of land}$   
(= replacement cost - weakness of target house (new house doesn't have)+ land)  
obsolescence:functional/locational/economic

### ★ sales comparison approach (C:comparable; T:target; $n \geq 3$ ) (price multiple)

$$V=\text{size}_T \times \frac{\sum [(V_c + \text{bad of C(T not have)} - \text{good of C(T not have)})/\text{size}_c]}{n}$$

reconciliation of value(average of above)

land:highest and best

### financial ratios used to analysis--debt

#### debt service coverage ratio(DSCR) (like interest coverage)

$$DSCR=\frac{\text{first-year NOI}}{\text{debt service}}, \quad \text{debt service:interest\&principal}$$

#### loan-to-value(LTV) ratio(like D/A)

$$LTV=\frac{\text{loan amount}}{\text{appraisal value}}$$

### office

#### demand

employment growth  
space used per employee  
average amount of space per employee

#### operating expenses

gross lease :operating exp. paid by owner

net lease :operating exp. paid by tenant

expenses reimbursement

### multi-family

#### demand

population growth  
population demographics  
propensity to rent in culture  
cost of rent  
interest rate  
operating expenses

### retail

#### demand

trends in consumer spending  
health of economy  
job growth,population growth,saving rates

#### unique aspect

pay fixed+floating (sales reach a certain level)

Alternative3-2 Publicly traded real estate securities  
publicly traded real estate securities

advantages

superior liquidity;  
lower minimum investment;  
limited liability;  
access to premium properties;  
active professional management;  
protection(by SEC);  
greater potential for diversification  
exemption from taxation  
predictable earning  
high yield

REITs

economic value determinants of REITs

REITs		first	second
shopping	national GDP growth	retail sales growth	
office		job creation	
healthcare		population growth	
industrial		retail sales growth	
hotel		job creation	
storage		population growth	

NAVPS in REIT valuation

net asset value per share  
**NAV=operating real estate +(cash+A/R) - debt**  
operating real estate=NOI<sub>1</sub>/cap rate  
NOI<sub>1</sub>=NOI<sub>0</sub> - **non-cash rents** + **full-year adjustment**

FFO&AFFO in REIT valuation

funds from operations(FFO)  
FFO= accounting net earnings + dep. + deferred tax exp.  
± losses/gains from sales and debt restructuring  
adjusted funds from operations(AFFO)  
also known as  
cash/funds available distribution(CAD/FAD)  
**AFFO=FFO - non-cash(straight-line)rent adjustment**  
**- recurring maintenance-type CAPEX&lease**  
P/FFO or P/AFFO multiples

distribution waterfall

deal-by-deal method  
total return method

**1st:only after entire committed capital returned to LPs**  
**2nd:excesss some minimum amount**

Private equity valuation  
private equity valuation

PE investment transaction

PE investor  
private equity firm  
portfolio company

classification of PE

venture capital(VC)  
buyout  
special situations

control mechanisms in PE transactions

compensation  
tag-along,drag-along clauses  
buy one,must to all SHDers

board representation  
noncompete clauses(buyout)  
priority in claims  
required approvals  
earn-outs

PE fund structure

limited partnership

economic terms of a PE fund

**mgt fees**:basis as % of paid-in capital  
transaction fees  
**carried interest:usually 20%**  
ratchet:allocation of equity  
between PE&mgt  
hurdle rate:above it,GP get carried interest  
target fund size  
vintage:year the fund start  
term of the fund

corporate governance terms of a PE fund

key man clause  
performance disclosure&confidentiality  
clawback  
tag-along,drag-along clauses  
no-fault divorce(LP's can fire GP no reason)  
removal for cause  
investment restrictions  
co-investment

PE  
PE(1) PE(2) PE(3)

exit

**IPO; secondary market sale; MBO; liquidation**

valuation (PE ⇒ portfolio company)

VC valuation issues (pre- and post-money valuation)

for a single financing round (price per share=invest/shares PE=pre/share pre)

**post=FV/(1+r)<sup>n</sup>** → → → → → **pre=post - invest**  
↓ **f=invest/post**  
**price per share=invest/shares PE** ← **shares PE = [shares pre/(1-f)]×f**

for multiple financing rounds

**post 2=FV/(1+r)<sup>n</sup>** → → → → → **pre 2=post 2 - invest 2**  
↓  
**pre 1=post 1 - invest 1** ← ← ← ← ← **post 1=pre 2/(1+r)<sup>n</sup>**  
↓ **f 1=invest 1/post 1**  
**shares PE,1 = [shares pre/(1-f 1)]×f 1** → **price per share 1=invest 1/shares PE,1**  
↓ **f 2=invest 2/post 2**  
**shares PE,2 = [(shares pre+shares PE,1)/(1-f 2)]×f 2**  
↓  
**price per share 2=invest 2/shares PE,2**

accounting for risk when valuing VC

adjusting r  
**r\*=[(1+r)/(1-q)]-1**  
**q:probability of failure in a year**  
adjusting TV  
scenario analysis

buyout valuation method

payoff multiple=total payoff/total investment

	c	capital called down	known
Σc	p	<b>paid-in capital</b>	
p×%	m	<b>mgt fees</b>	
	o	operating results	known
Na t-1 +c-m+o	Nb	<b>NAV before distributions</b>	
(Nb-Nb x)×%	i	<b>carried interest</b>	
	d	distributions(to LPs)	known
Nb-i-d	Na	<b>NAV after distributions</b>	

**Nb x=max(committed c,Nb t-1)**

**PIC**=paid-in capital/committed capital

**DPI**=distributed/paid-in capital **realized**

**RVPI**=residual value/paid-in capital **unrealized**

**TVPI**=total value/paid-in capital **TVPI=DPI+RVPI**



**sectors**

- energy
- grains
- industrial(base) metals
- livestock
- precious metals
- softs(cash crops)

**backwardation**

futures price<spot price

**contango**

futures price>spot price

3 theories

- insurance theory(backwardation)
  - producers as hedgers,short
- hedging pressure hypothesis
  - producers and consumers
  - producers(backwardation)
  - consumers(contango)
- theory of storage(contango)

**total return=(spot return+roll return)+ collateral return+rebalancing return**

roll return>0,backwardation	↓	
<u>roll return&lt;0,contango</u>	margin interest	↓ portfolio

portfolio perspective

Harry Markowitz: Modern Portfolio Theory (MPT)  
some pricing model (pricing return rate): CAPM, APT, ICAPM

portfolio management (steps)

① the planning step (CFA II)

① identify investor's objective & constraints

**risk objective**

specific factors that affect **ability**  
required spending needs; long-term wealth target;  
financial strengths/liabilities; health/age.  
specific factors that affect **willingness**  
return objective; habit;  
historical trading; character

**return objective**

measurement (absolute R, relative R, nominal R, etc.)  
desired return (wish) & required return (must be)

**investment constraints**

liquidity; time horizon; tax concerns;  
legal & regulatory factors; unique circumstances.

② creating IPS

**main roles:** be implemented by investment advisers  
promote long-term discipline for port. decisions  
help protect short-term shift in strategy

**elements:** a client description; objectives & constraints;  
purpose of IPS (policies, objective, goals, limitations);  
identification of duties & responsibilities;  
calendar schedule; asset allocation ranges;  
guidelines for adjustment & rebalancing

3 approaches of investment strategy

passive/active/semiactive investment

③ forming capital mkt expectations (CME)

long run forecast of risk & return for various asset classes

④ creating **strategic** asset allocation (just weighting)

② the execution step

**tactical** asset allocation

security selection/composition

③ the feedback step

monitoring & rebalance

performance evaluation

management investment portfolio

update IPS at least annual & when great event

an introduction to multifactor models (= regression models)

★ **arbitrage pricing theory** (equilibrium-pricing model, no  $\epsilon$ )

**assumptions**

a **factor (RP)** model describes asset returns;  
investor can form **well-diversified portfolio** (WDP);  
**no arbitrage opportunities** among WDP.

exactly formula (**asset not exist**)

$$E(R_p) = R_f + \beta_{p,1}(\lambda_1) + \beta_{p,2}(\lambda_2) + \dots + \beta_{p,k}(\lambda_k)$$

pure factor portfolio for factor j:  $E(r) = r_{free} + \lambda_j$   
parameters:  $R_f$ , factor risk-premiums  $\lambda_j$

assumptions for arbitrage opportunities

**extreme** long/short are permitted  
all arbitrage opportunities would be exploited

★ **multifactor model** (regression models,  $\epsilon$ )

**macroeconomic factor models**

$$R_i = E(R_i) + b_{i1}F_{GDP} + b_{i2}F_{QS} + \epsilon_i$$

$E(R_i)$ : expected return for asset i

$F_{GDP}$ : surprise in GDP rate

$F_{QS}$ : surprise in credit quality spread

$\epsilon_i$ : firm-specific surprise (out of model)

**surprise = actual value - expected value**

**slope coefficients: factor sensitivity**

**fundamental factor models**

$$R_i = a_i + b_{i1}F_{P/E} + b_{i2}F_{SIZE} + \epsilon_i$$

$$b_{ij} = \frac{\Delta \text{attribute value (asset } i - \text{average)}}{\sigma(\text{attribute value})}$$

$a_1$ : no economic interpretation (or average level)

**slope coefficients: standardized  $\beta$**

exception: dummy variables  $\rightarrow b \rightarrow b=0$  or  $1$

statistical factor models

factor analysis models & principal components models

**advantage**: minimal assumptions

**disadvantage**: no economic interpretation

mixed factor models

application

**return attribution** ( $\alpha$ )

**active return** ( $\alpha$ ) =  $R_{\text{portfolio}} - R_{\text{benchmark}}$

first component

**factor tilts** (asset allocation)

$$\text{factor return} = \sum (\beta_{pk} - \beta_{bk}) \times \lambda_k$$

$\beta_{pk}$ : for the kth factor in the active portfolio

$\beta_{bk}$ : for the kth factor in the benchmark

$\lambda_k$ : factor premium for factor k

second component

**security selection** (asset selection)

security selection return =  $\alpha$  - factor return

**risk attribution** (tracking error/volatility)

the standard deviation of  $\alpha$  ( $\sigma_\alpha$ , not  $\sigma_\alpha$ )

**active risk** =  $s(R_p - R_b)$

**information ratio** =  $\alpha / \sigma_\alpha$

**active risk squared**

= active factor risk + active specific risk

portfolio construction

passive management

can use multifactor models to match an index

active management

use  $\beta$  to evaluate portfolio manager

**Carhart four-factor model** (four factor model)

$$R = R_f + \beta_1 R_{MRF} + \beta_2 R_{SMB} + \beta_3 R_{HML} + \beta_4 R_{WML}$$

	APT	Multifactor models	CAPM
characteristics	cross-selection	time-series	
assumptions	no arbitrage opportunities, far more flexible than CAPM	regression	a lot
intercept	risk-free rate	expected return	
conclusions	multiple risk factor		only mkt premium

value at risk, VaR(\$ or %)

a time period, x%, min loss → VaR

estimating VaR

**analytical method (parameter method)**

or variance-covariance method

$$VaR(X\%) = |E(R) - z(X\%) \cdot \sigma|$$

$$VaR(x\%)_{J \text{ days}} = VaR(x\%)_{1 \text{ day}} \times J^{1/2}$$

advantages

easy to calculate & understand;  
allow modeling the correlation of risk;  
applied to shorter/longer time period.

disadvantages

assumes **normal distribution** of return;  
many assets leptokurtosis (fat tails);  
hard to estimate  $\sigma$  in very large portfolio.

**historical method**

advantages

easy to calculate & understand;  
do not assume a return distribution;  
can be applied to different time period.

disadvantages

history → future

**Monte Carlo method**

advantages & disadvantages

any assumptions;  
output only as good as input assumption.

**advantages of VaR**

simple concept; easily communicated;  
provides a basis for risk comparison;  
facilitates capital allocation decisions;  
used for performance evaluation;  
reliability can be verified;  
widely accepted by regulators.

**limitation of VaR**

subjective; failure to take into account liquidity;  
underestimate the frequency of extreme events;  
sensitivity to correlation;  
vulnerability to trending or volatility regimes;  
misunderstanding the meaning of VaR;  
oversimplification;  
disregard of right-tail events.

extensions of VaR

**conditional VaR (CVaR, or ES)**

**incremental VaR (IVaR, Δ)**

**marginal VaR (MVaR)**

$$IVaR \approx MVaR_i \times W_i$$

relative VaR

ex ante tracking error

other key risk measures

sensitivity

equity:  $\beta$ ; option:  $\Delta, \Gamma$ , vega  
fixed: duration, convexity

advantage

address some shortcomings  
of position size measures

limitations

do not often distinguish assets  
by  $\sigma$ , less comparable.

scenario risk measures

**historical scenarios**

**hypothetical scenarios (HY)**

advantage

do not need to rely on history  
no assumption of  $N(\mu, \sigma^2)$   
can be tailored  
allow liquidity into consideration

limitations

history → future; difficult to establish  
HY may incorrectly specify  
HY can be difficult to create

applications of risk measures

**banks**: liquidity gap (working capital);

VaR; economic capital

**asset managers**: position limit;

redemption risk; tracking error;  
HF: gross exposure; leverage

**pension fund**

surplus at risk (SAR), glide path

**insurers**

economic capital  
asset & liability matching

**execution algorithms** (how to trade)

VWAP; implementation shortfall; market participation

**high-frequency trading algorithms** (how/when/what to trade)

data

market data; quote event;  
trade event; news event

HFT-statistical arbitrage

pairs trading spread trading  
index arbitrage mean reversion  
basket trading delta neutral strategies

latency

stimulus and response

application areas

liquidity aggregation & smart order routing  
real-time pricing of instrument (millisecond)  
trading on news  
genetic tuning

risk management

turned off pre-trade risk management to latency → risk ↑

regulatory

insider trading; wash trading;  
front running orders; trade collusion;  
painting the tape;  
fictitious order  
quote stuffing, layering, spoofing

impact on securities markets

positive

liquidity & effectiveness ↑

negative

fear of unfair advantage  
market movement ↑  
gaming the market  
increased risk profile  
algorithms gone wild

confidence intervals

	CI	VaR
$\pm 1.00\sigma$	68%	16.0%
$\pm 1.65\sigma$	90%	5.0%
$\pm 1.96\sigma$	95%	2.5%
$\pm 2.33\sigma$	98%	1.0%
$\pm 2.58\sigma$	99%	0.5%

using constraints in mkt management

risk budgeting  
position limits  
liquidity limits  
stop-loss limits  
risk measures and capital allocation

framework for the analysis of financial markets

present value model

$$P_{i,t} = \sum \frac{E_t[CF_{i,t+s}]}{(1+I_{t,s} + \theta_{t,s} + \rho_{i,t,s})^s}$$

$P$ : value of  $I$  at time  $t$ ;  $CF_{i,t+s}$ : uncertain, nominal CF paid  $s$  period;  
 $E_t[CF_{i,t+s}]$ : expectation based on today's information;  
 $I$ : YTM on a real default-free investment today;  
 $\theta$ : expected inflation rate between  $t$  and  $t+s$ ;  
 $\rho$ : RP of CF of asset  $i$ ,  $s$  period in the future

present value model (complete formula)

$$P_{i,t} = \sum \frac{E_t[CF_{i,t+s}]}{(1+I_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma_{i,t,s} + \kappa_{i,t,s} + \varphi_{i,t,s})^s}$$

$I$ : Rf;  $\theta$ : inflation;  $\pi$ : uncertain inflation;  
 $\gamma$ : credit spread;  $\kappa$ : equity spread;  $\varphi$ : liquidity spread

the discount rate on real default-free bonds (TIPS)

$I$  marginal utility of consumption of investors diminishes

**inter-temporal rate of substitution**,  $m = u_t/u_0$ , always  $m < 1$

$m = u_t/u_0$ , always  $m < 1$ ,  $m$  is a random variable (∵  $u_t$  is random)

good state of the economy → lower  $m$  (∵ future, richer)

price of bond =  $m$ , return on bond = one-period risk-free rate =  $I = 1/E(m) - 1$

$m \uparrow \Rightarrow u_t \uparrow \Rightarrow u_0 \downarrow \Rightarrow \text{invest}_0 \uparrow \Rightarrow P \uparrow \Rightarrow I \downarrow$

uncertainty & risk premium

$P = E(P)/(1+I) + \text{cov}(P', m)$ ; in general,  $\text{cov}(P', m) < 0$

economy goes bad,  $\text{cov}(P', m) > 0$  (fight to quality)

economy  $\uparrow \Rightarrow I \uparrow$ ;  $\sigma_{GDP} \uparrow \Rightarrow I \uparrow$

short-term nominal interest rate (T-bills)

$\theta$  influenced by:

inflation environment; economic activity;

central bank's policy rate (Taylor rule: policy rate =  $I + 0.5(I - I^*) + 0.5(Y - Y^*)$ )

$I^*$ : target rate of inflation;  $Y^*$ : potential real GDP (log)

yield curve and business cycle (non-short-term treasury bond)

$\pi$  break-even inflation rates (BEI),  $\pi \propto GDP, \sigma_{GDP}$

$BEI = \theta + \pi = R$  zero-coupon default-free bond (nominal-real)

yield curve

level, slope, curvature

shape of yield curve

upward sloping; downward sloping; hump; flat

an inverted yield curve → a predictor of recession

applications of fundamental law

global equity strategy (TC)

selection & constraints

fixed-income strategy (IC, BR)

increasing in BR → decreasing IC

practical limitations

independence of investment decisions

overestimate own IC;

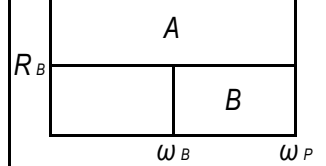
different sector, different IC

value added (active return)

$$R_A = R_P - R_B = \Sigma \Delta \omega_i R_{Ai}$$

$$R_A = A + B = \Sigma \omega_P (R_P - R_B) + \Sigma (\omega_P - \omega_B) R_B$$

$R_P$



Sharpe ratio

$$SR_P = \frac{R_P - R_F}{\sigma(R_P)}$$

SR is unaffected by  $\pm$  cash

information ratio

$$IR_P = \frac{R_P - R_B}{\sigma(R_P - R_B)} = \frac{R_A}{\sigma(R_A)}$$

unaffected by

aggressiveness of active weights;  
 $\pm$  position in the benchmark portfolio

★ constructing optimal portfolios ( $SR_{max}$ )

$$SR_P^2 = SR_B^2 + IR^2 \quad SR_P^2 = SR_B^2 + TC^2 (IR^*)^2$$

$$\sigma_{RA} = \frac{IR}{SR_B} \sigma_{RB} \quad (\sigma_{RA} = \frac{TC \times IR^*}{SR_B} \sigma_{RB})$$

$$\sigma^2(R_P) = \sigma^2(R_B) + \sigma^2(R_A); \quad \omega = \sigma^* A / \sigma_A$$

IC & TC

$$IR = TC \times IC \times \sqrt{BR}$$

IC: information coefficient

$IC = \text{correlation}(RA/\sigma, \mu/\sigma)$ ; (=cor(real, guess))

TC: transfer coefficient

$TC = \text{correlation}(\mu/\sigma, \Delta \omega \sigma)$ ; (=cor(guess, do))

size active weights (optimal active)

$$\mu_i = IC \times \sigma_i \times S_i$$

IC: expected IC

$\sigma_i$ : separate for individual security

$S_i$ : correct magnitude (+ or -)

$$\Delta \omega^* i = \frac{\mu_i}{\sigma_i^2} \frac{\sigma_A}{\sqrt{\sum \frac{\mu_i^2}{\sigma_i^2}}} = \frac{\mu_i}{\sigma_i^2} \frac{\sigma_A}{IC \sqrt{BR}}$$

basic fundamental law

expected active portfolio return

$$E(R_A) = \Sigma (\Delta \omega_i \mu_i) = IC \sqrt{BR} \times \sigma_A, \text{ optimal}$$

$$IR^* = IC \times \sqrt{BR}$$

full fundamental law

constraints to TC ( $TC < 1$ ): regulation, cost, etc.

$$IR = TC \times IC \times \sqrt{BR}$$

$$E(R_A) = TC \times IC \times \sqrt{BR} \times \sigma_A$$

ex post performance measurement

realized information coefficient ( $IC_R$ )

$$E(R_A | IC_R) = TC \times IC_R \times \sqrt{BR} \times \sigma_A$$

$$R_A = E(R_A | IC_R) + \text{Noise}$$

$$\text{cor}(R_A, E(R_A | IC_R)) = TC, \quad r^2 = R^2 = TC^2$$

$$BR = \frac{N}{1 + (N-1)\rho}$$



**Products&risk models**

*fixed-income product*

*bond basics*

*treasury bonds*

*corporate bonds*

*MBS*

*valuation of bonds*

*risk metrics*

*rating agencies*

*derivative*

*forward mkt and futures mkt*

*forward and futures prices*

*interest rate futures*

*hedging strategies using futures*

*swap mkt*

*properties of stock options*

*trading strategies involving options*

*exotic options*

*option valuation*

*risk metrics-Greek letters*

*central counterparties*

*introduction*

*exchanges,etc der.,DPCs and SPVs*

*basic principles of central clearing*

*risks caused by CCPs:risks faced by CCPs*

*risk measurement and management*

*measures of financial risk*

*putting VaR to work*

*quantifying volatility in VaR models*

*expected and unexpected loss*

*country risk*

*operational risk*

*stress test*

## fixed-income product

### bond basics

- risk-free rate
  - treasury rates
  - libor
  - repo rates
- compounding frequencies
  - simple interest
  - compounding interest
- bond
  - coupon rate**
  - face value**
  - maturity**
  - yield to maturity**
- bond pricing

### risk metrics

- one-factor risk metrics
  - duration**
    - Macaulay duration**
    - modified duration(MD)**
    - dollar duration(DD)**
    - DV01**
    - hedge ratio**
    - effective duration**
  - convexity**
    - effective convexity**
    - portfolio duration & convexity**
- multi-factor risk metrics
  - key-rate shifts
  - key rate 01s**
  - key rate duration**

$$HR = \frac{DV01(\text{per } \$100 \text{ of initial position})}{DV01(\text{per } \$100 \text{ of hedging instrument})}$$

$$DD(0) + N * DD(H) = 0$$

### corporate bonds

- role of corporate trustee
- different coupon payment
  - zero, straight, floating
- different types of corporate bonds
  - mortgage bonds
  - collateral trust bonds
  - equipment trust certificates
  - debenture bonds
  - guaranteed bonds
- zero-coupon corporate bonds
  - original issue discount(OID)**
- corporate bond retirements
  - call provision
  - sinking-find provisions
  - maintenance and replacement funds
  - tender offers
- corporate bond credit risk
  - credit default risk
  - credit-spread risk
  - issuer default rate & dollar default rate
  - recovery rate
- high-yield bond(issuers)
  - original issuers
  - fallen angels
  - restructurings and leverage buyouts
- paymentr features
  - deferred-interest bonds
  - step-up bonds
  - payment-in-kind(PIK)bonds
- spread of a bond**
- callable and puttable bond

### MBS

- mortgage loan
  - agency or conforming loans
  - non-agency or non-conforming loans
- fixed rate mortgage payments
  - interest and principal**
- prepayment option
  - valuable when rates have fallen
  - single monthly mortality rate(SMM)**
  - constant prepayment rate**
  - conditional prepayment rate (CPR)**
- mortgage-back-securities mkt
  - mortgage pass-through
  - mortgage servicers
  - mortgage guarantors
  - weighted-average coupon or WAC
  - weighted-average maturity or WAM
- agency mortgage pools trade
  - specified pools mkt
  - to be announced(TBA)
- dollar rolls
- prepayment components
- valuing MBS
  - Monte Carlo methodology
  - zero-volatility spread(Z-spread)
  - option-adjusted spread(OAS)

### treasury bonds

- treasury mkt(quoted price)
  - T-bills & T-bonds**
- quotation of treasury bonds
  - accrued interests**
  - day count conventions
  - clean price**
  - dirty price**
- treasury STRIPS

### valuation of bonds

- spot rate, forward rates, discount factor**
- par rate**
- bond replication**
  - same CFs, same price
- pricing bond using:**
  - discount factors**
  - spot rates**
  - forward rates**
- pricing of annuity and perpetuity**
  - perpetuity: c/y**
- yield to maturity**
  - yield curve(Forward > Spot > YTM)
- reinvestment risk
- interest rate risk
- decomposition of P&L**
  - carry-roll-down**
  - rate change**
  - spread change**

### rating agencies

- S&P vs. Moody
- sovereign rating
  - political stability
  - social & eco. Coherence
  - integration into global
- opinion about creditworthness
- transition matrices
- through-the- cycle
- at-the-point approach
- ratings change impact bond prices
- external and internal ratings

## derivative 2-1

### forward mkt and futures mkt

forward(otc)

futures

standardized

daily settlement

market participants

hedgers

speculators

arbitrageurs

market maker

commodity forward contract

storage costs

lease rate

convenience yields

commodity spread

financial forward contract

**forward interest agreement(FRA)**

trading manner of a futures

close out

physical delivery

cash settlement

exchange for physicals

crush spread & crack spread(soybean)

specification of futures

assets

contract size

margin requirement

initial margin

maintenance margin

variation margin

trading order

mkt,limit

stop,stop-loss

stop-limit

clearing house

### forward and futures prices

**cost of carry model(TVM)**

$$F_0 = S_0 e^{(r+u-q-y)T}$$

**interest rate parity**

$$\text{Forward} = \text{Spot} \left( \frac{1 + r_{DC}}{1 + r_{FC}} \right)^T$$

$$\text{Forward} = \text{Spot} \times e^{(r_{DC} - r_{FC})T}$$

foreign exchange risk

net exposure position

positive:  $A > L$

on-balance-sheet hedge

hedge off-balance-sheet

normal & inverted futures mkt

contango & backwardation(F,S)

**valuing the forward contract**

$$f = (F_0 - K) e^{-rT}$$

### interest rate futures

**T-bond futures (long-term)**

cheapest-to-deliver bond

**cost=quoted bond price-(QFP\*CF)**

**Eurodollar futures (short-term)**

three-month,\$1m

$$P = 1m * (1 - 0.25(100 - FQ)\%)$$

$$P = 1m * (1 - 0.25 \text{libor})$$

long:  $FQ$  1bp  $\uparrow$ , **EuroD \$25  $\uparrow$**

difference between forward&futures price

$\rho(S,r) > 0$ , futures more attractive

**Eurodollar futures vs. FRA**

$$\text{Forward Rate} = \text{Futures rate} - \frac{1}{2} \sigma^2 T_1 T_2$$

### hedging strategies using futures

short hedge&long hedge

**strip** hedge&**stack(&roll)** hedge

**minimum variance hedge ratio**

$$HR = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$$

effectiveness of the hedge

coefficient of determination:  $R^2 = \rho^2$

**using stock index futures to change a stock**

**portfolio's beta**

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

$$\text{number of contracts} = (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

$$\beta(S) * V(S) + N * \beta(H) * V(H) = \beta(T) * V(S)$$

$$N * \beta(H) * V(H) = (\beta(T) - \beta(S)) * V(S)$$

tailing the hedge

$$HR * (P(S)/P(F))$$

**basis risk : Spot-Futures**

different assets

different maturity

cross hedging

long the basis(short Futures)

short the basis(long Futures)

### swap mkt

swaps

interest rate swap

currency swap

**comparative advantage** argument

valuing interest rate swap with bonds

$$V(\text{swap}) = V(\text{fixed}) - V(\text{float})$$

$$D(\text{swap}) = D(\text{fixed}) - D(\text{float})$$

valuing currency swap with bonds

### properties of stock options

basics

call&put,European&American

moneyness

ITM,ATM,OTM

intrinsic value&time value

$$IV: C = \max(S - X, 0)$$

$$IV: P = \max(X - S, 0)$$

six factors that affect on option's price

$(S, X, T, \sigma, r, D)$  (Euro C&P, Ame C&P)

**upper and lower bounds for option prices**

Option	Proxy	Min Value	Max Value
European call	c	$\max(0, S_0 - Xe^{-rT})$	$S_0$
American call	C	$\max(0, S_0 - Xe^{-rT})$	$S_0$
European put	p	$\max(0, Xe^{-rT} - S_0)$	$Xe^{-rT}$
American put	P	$\max(0, X - S_0)$	X

**put-call parity**

$$C + Ke^{-rT} = p + S_0$$

$$S - X \leq C - P \leq S - Xe^{-rT}$$

$$p + S_0 = c + D + Xe^{-rT}$$

$$S_0 - X - D \leq C - P \leq S_0 - Xe^{-rT}$$

### trading strategies involving options

simple strategies

$$\text{covered call} = S - C$$

$$\text{protective put} = S + P$$

**spread strategies (call/put)**

**bull** call/put **spread**: long low, short high

**bear** call/put **spread**: long high, short low

**butterfly spread**: 4call/put

calendar spread

diagonal strategies

**combination strategies**

$$\text{collar} = \text{stock} + \text{put} - c$$

$$\text{straddle} = c + p, k = k(k \neq k, \text{strangle})$$

$$\text{strip} = 2p + c, k = k$$

$$\text{strap} = p + 2c, k = k$$

derivative 2-2  
exotic options

standard and nonstandard Ame. options

Bermudan option(certain dates)

compound options

options on options

forward start options

chooser option

**barrier options**

up in+up out=option

down in+down out=option

binary options

cash-or-nothing:  $QN(d_2)e^{(-rT)}$

asset-or-nothing:  $S(0)N(d_1)e^{(-rT)}$

**lookback options**

floating lookback call:  $S(T)-S(\min)$

floating lookback put:  $S(\max)-S(T)$

fixed lookback call:  $S(\max)-K$

fixed lookback put:  $K-S(\min)$

shout options

Asian options:  $S=S(\text{ave})$

volatility and variance swaps

static options replication

option valuation

**Binomial trees**

risk-neutral valuation

**assumption**

Stock:geometric Brownian motion

risk neutral

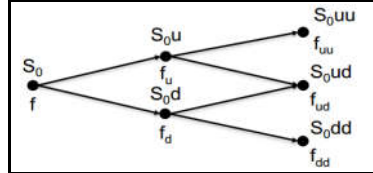
$ud=1$

**one-step Binomial trees**

$$u = e^{\sigma\sqrt{\Delta t}}; d = e^{-\sigma\sqrt{\Delta t}}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \text{ or } p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

**two-step Binomial trees**



American options(put)

**Black-Scholes-Merton model**

**assumption**

$P \sim \text{lognormal}$

$P \rightarrow \text{continuous}$

$R(f)$ :known&constant

$\sigma$ :known&constant

mkt:frictionless

no cash flow

European options

**price of European call option**

$$c = SN(d_1) - Ke^{-rT}N(d_2)$$

$$c = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

**price of European put option**

$$p = Ke^{-rT}N(-d_2) - SN(-d_1)$$

$$p = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

the early exercise of American options

risk metrics-Greek letters

**delta**  $\Delta$ , **gamma**  $\Gamma$ , **vega**  $\Lambda$ , **theta**  $\theta$ , **rho**  $\rho$

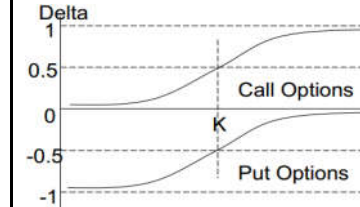
$\text{delta} = (C_1 - C_0)/(S_1 - S_0)$ ; stock :  $\Delta = 1$

**slope** of option pricing function

call:  $\Delta \in (0, 1)$ ; put:  $\Delta \in (-1, 0)$

**long C, short P:  $\Delta > 0$**

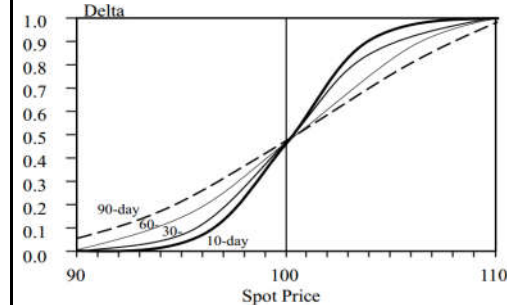
**long P, short C:  $\Delta < 0$**



**Call and Put Option Delta**

call:  $\Delta = N(d_1)$ ; put:  $\Delta = N(d_1) - 1$

CDF of a normal distribution



$t \rightarrow T$ , delta is unstable

call: ATM,  $\Delta = 0.5$ ; OTM,  $\Delta = 0$ ; ITM,  $\Delta = 1$

**forward  $\Delta = 1$  or  $e^{(-qT)}$**

**futures  $\Delta = e^{(rT)}$  or  $e^{((r-q)T)}$**

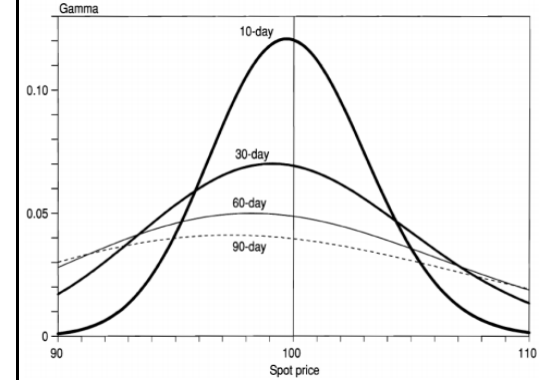
$\Delta = 0$ : delta neutral position

$\text{gamma} = \partial \Delta / \partial S$

normal distribution

**gamma(call)=gamma(put)**

**long:  $\Gamma > 0$ ; short:  $\Gamma < 0$**



**gamma neutral position**

**option  $\rightarrow \Gamma$ , stock  $\rightarrow \Delta$**

$$\Delta C = \text{delta} \Delta S + 0.5 \text{gamma} (\Delta S)^2$$

$\text{vega} = \partial C / \partial \sigma$

**vega(call)=vega(put)**

**vega(LT) > vega(ST)**

theta(time decay)

**theta(call)=theta(put)**

**long:  $\theta < 0$**

rho(sensitivity to the interest rate)

**max: ITM > ATM > OTM**



## central counterparties(CCP,after the crisis)

### introduction

OTC(risk)

CCP(central clearing of all standardized OTC der.)

- reducing counterparty risk**
- improving mkt transparency**
- reducing mkt interconnectedness**

### advantages

- setting standards for clearing members
- multilateral **netting**
- collecting and maintaining margin
- mutualization** of losses among clearing members
- enhanced trade **liquidity** & reduced overall mkt disruptions

### drawbacks

- the potential failure of a CCP
- moral hazard** problem
- increased costs arising from tying up funds as initial margin
- adverse selection**

## risks caused by CCPs:risks faced by CCPs

### major risks

- default risk**
- model risk
- liquidity risk
- operational risk
- legal risk

default correlation  $\geq$  OTC

### lesson learned

- operational risk
- variation margin(often&quickly)
- automated payments

## exchanges,otc der.,DPCs and SPVs

derivatives:bilaterally(OTC) or exchanges

### exchanges

3 primary functions

- product standardization
- trading venue
- reporting services

counterparty risk mitigants

- clearing**
- margining**
- netting**

clearing(3 forms)

- direct clearing**
- clearing ring**
- CCP**

### OTC

- advantage:customization
- disadvantage: **counterparty risk**
- OTC derivatives
  - inert rate**, foreign exchange, equity, commodity, credit

controlling counterparty risk

- special purpose vehicles( **SPV** )
  - bankruptcy remote
- derivatives product companies( **DPC** )
  - AAA
  - monolines insurance
- credit derivatives product companies( **CDPC** )

## basic principles of central clearing

### functions

- novation**
- netting
- margining
- managing **auction** process,
- loss **mutualization**

conditions needed for CCP

- product standardization
- lower complexity
- high liquidity

participants

- members
- number of CCPs

types of CCPs

- utility-driven CCP
- profit-driven CCP

failure of a CCP

advantages of CCPs

- transparency**
- offsetting**
- loss **mutualization**
- legal and operational efficiency**
- liquidity
- default management

disadvantages

- moral hazard
- adverse selection
- separation of cleared&non-cleared products
- procyclicality of margin requirement**

## risk measurement and management 2-1

### measures of financial risk

#### coherent risk measure

**monotonicity** :  $R1 \geq R2, \rho(R1) \leq \rho(R2)$

**subadditivity** :  $\rho(R1+R2) \leq \rho(R1)+\rho(R2)$

**positive homogeneity** :  $\beta > 0, \rho(\beta R) = \beta \rho(R)$

**translation invariance** :  $\rho(R+c) = \rho(R)-c$

#### mean-variance framework

#### Value at Risk

max loss, target horizon, confidence level

disadvantages

worst conditions, tail loss

not sub-additive

$$VaR(X\%) = |E(R) - z(X\%)*\sigma|$$

stock:  $VaR(X\%) = V * z(X\%)*\sigma$ , 1.65, 95% ; 2.33, 99%

square root rule

$$VaR(X\%)_{j\text{-days}} = VaR(X\%)_{1\text{-days}} \times \sqrt{j}$$

**trend** → positive correlation ( $\rho > 0$ ) →  $VaR \uparrow$

**mean reversion** → negative ( $\rho < 0$ ) →  $VaR \downarrow$

conditional  $VaR$  (expected shortfall, ES)

ave(worst 100\*(1- $\alpha$ )% of losses)

### putting VaR to work

#### linear derivatives (F, f, stock)

tiny changes, delta → constant

$VaR$  of linear derivatives ( $\Delta$  normal approximation)

$$VaR_{\text{Linear Derivative}} = \Delta \times VaR_{\text{Underlying Risk Factor}}$$

eg.  $\Delta C = \Delta * \Delta S$ ,  $VaR(\Delta C) = VaR(\Delta * \Delta S) = \Delta * VaR(\Delta S)$

#### delta-normal approach

linear approximation, underlying factor is normal

$$\Delta P = -D^*P^*\Delta y, \quad VaR(dP) = |-D^*P^*| \times VaR(dy)$$

$$\Delta C = \text{delta} * \Delta S, \quad VaR(df) = |\Delta| \times VaR(dS)$$

#### nonlinear derivatives (bond, option)

#### delta-gamma approximation

$$\Delta P = -D^*P^*\Delta y + 0.5 * C^*P^*(\Delta S)^2$$

$$\Delta C = \text{delta} * \Delta S + 0.5 * \text{gamma} * (\Delta S)^2$$

$$VaR(dP) = |-D^*P^*| \times VaR(dy) + \frac{1}{2} (C^*P^*) \times VaR(dy)^2$$

$$VaR(df) = |\Delta| \times VaR(dS) + \frac{1}{2} \Gamma \times VaR(dS)^2$$

#### full revaluation method (accurate but computationally)

historical simulation

bootstrap simulation

Monte Carlo simulation

scenario analysis

## quantifying volatility in VaR models

### reasons for fat tails

$\sigma, \mu$  of the distribution changes over time

volatility is time-varying

regime-switching volatility model

constant mean, different volatility

historical-based approach

#### parametric model

#### nonparametric approach

historical simulation method

#### hybrid approach

$$\lambda^{(i-1)*((1-\lambda)/(1-\lambda^k))}$$

#### implied volatility-based approach

uses derivative pricing model

advantages

forward-looking

reacts immediately to changing mkt

disadvantages

model dependent

multivariate density estimation (MDE)

estimating volatility

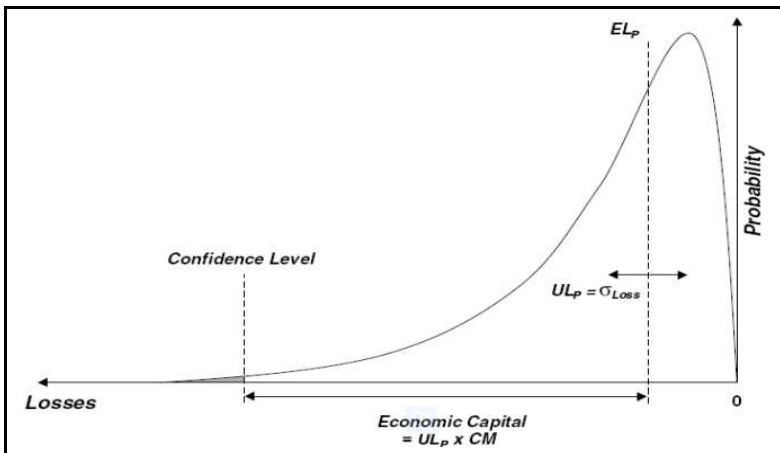
historical standard deviation approach

exponential smoothing methods

$$EWMA \quad \sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) u_{n-1}^2$$

$$GARCH \quad \sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

persistence =  $\alpha + \beta$



### expected and unexpected loss

credit risk

3 drivers

**PD**, probability of default: %

**EA**, exposure amount: \$

**LR**, loss rate: %

expected losses, **EL**

$$EL = PD * EA * LR$$

unexpected loss, **UL**

$$UL = EA \cdot \sqrt{PD \cdot \sigma_{LR}^2 + LR^2 \cdot \sigma_{PD}^2}$$

$$\sigma^* \sigma(PD) = PD(1-PD)$$

## risk measurement and management 2-1

### country risk

#### sources of country risk

##### **economic growth life cycle**

early stages of economic growth, more risk

##### **political risk**

chaos of democracy: more continuous risk

authoritarian: discontinuous

corrupt, as a cost

physical conflicts

##### **legal risk**

##### **economic structure**

#### risk services' limitations

scores are not standardized

ranking than measuring

#### sovereign default risk

#### local currency defaults

#### consequences of default

reputation loss

capital mkt turmoil

real output

political instability

#### factors determining sovereign default risk

degree of indebtedness

pensions/social service commitments

revenues/inflows to government

stability of revenues

autocracies more than democracies

implicit backing from other entities

#### sovereign ratings

not change quickly enough

rating process

**notch-up** approach: **foreign** currency rating

**notch-down** approach: **local** currency rating

criticized

upward biased

herd behavior

take too long to change rating

vicious cycle

reasons for rating agencies fail

information problem

cannot afford to hire too many analysts

revenue bias

conflict of interest

#### sovereign default spread

advantages

more granular & dynamic

weaknesses

more volatile

affected by liquidity risk

#### credit default swaps (CDS)

physical settlement

cash settlement

narrowness of the mkt

### operational risk

internal processes, people, system

7 categories of operational risk

**without** reputation & strategy

approaches to measuring

	Top-Down Approaches	Bottom-Up Approaches
Sophistication	Simple	Complex
Data Requirement	Non-intensive	Intensive
HFLS vs. LFHS	Undifferentiated	Differentiated
Diagnostic Ability	No	Yes
Perspective	Backward-looking	Forward-Looking

loss **frequency** distribution

number of losses, **Poisson distribution**

loss **severity** distribution

loss severity distribution, lognormal

data issues

#### operational risk regulatory capital

**basic indicator approach (BIA), 3 years,  $\alpha=15\%$**

$$K_{\text{operational, BIA}} = \frac{\sum_{i=\text{last three years}} (G_i \times \alpha)}{3}$$

**standardized approach**

$$K_{\text{operational, SA}} = \frac{\left\{ \sum_{i=\text{last three years}} \max \left[ \sum G_{i, \text{line } 1-8} \times \beta_{i, \text{line } 1-8}, 0 \right] \right\}}{3}$$

**AMA approach, 1 year, 99.9% confidence level**

### stress test

#### 3 steps

create a set of extreme mkt scenarios

determine the price changes to instruments,

determine change in portfolio value

summarize the result

#### advantages

complements VaR

#### disadvantages

highly subjective

unidimensional & multidimensional scenarios analysis

prospective & historical scenarios

event-driven & portfolio-driven scenarios

market risk(3-1) VaR

## estimating mkt risk measures (VaR>0) non-parametric approaches

data

profit/loss data

$$P/L_t = P_t + D_t - P_{t-1}$$

$$PV(P/L_t) = \frac{(P_t + D_t)}{(1+d)} - P_{t-1}$$

$$FV(P/L_t) = P_t + D_t - (1+d)P_{t-1}$$

loss/profit data(VaR/ES)

$$L/P_t = -P/L_t$$

arithmetic return

$$r_t = (P_t + D_t) / P_{t-1} - 1$$

geometric return( $\sim N(\mu, \sigma^2)$ )

$$R_t = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right)$$

parametric VaR

normal VaR

$$VaR = (\mu - z_{\alpha}\sigma)P_{t-1}$$

1.65, 95%; 2.33, 99%

lognormal VaR

$$VaR = (1 - \exp(\mu - z_{\alpha}\sigma))P_{t-1}$$

conditional VaR, CVaR

(expected shortfall, ES)

$$ES = E(\text{loss}_i | \text{loss}_i > VaR_{\alpha})$$

more i, bigger ES

coherent risk measures

$$M_{\phi} = \int_0^1 \phi(p) q_p dp$$

$\phi(p)$ =weighting function  
specified by user

eg.  $ES = \phi(p) = \begin{cases} 0 & p < \alpha \\ 1/(1-\alpha) & p \geq \alpha \end{cases}$

## quantile-quantile plots

estimation of HS VaR&ES

basic historical simulation

bootstrapped HS

discrete confidence intervals

solu1: linear interpolation

solu2: kernel method

curves&surfaces

VaR curve: unsteady

ES curve: smoother

## blames for traditional HS

scare of data (solu: bootstrapped)

holding period  $\uparrow \rightarrow$  # of observations  $\downarrow$

VaR: 1day, 1week or 1year

window  $\uparrow \rightarrow$  little relevance for current

past  $\rightarrow$  future (solu: stress test)

slices of data

long-run data

irrelevance&ghost effect

## ghost effect

insensitive

too insensitive

sudden to fall

## weighted historical simulation

### BRW (age-weighted HS)

$$\omega_1 + \omega_2 \lambda + \dots + \omega_n \lambda^{n-1} = 1$$

$$\omega_i = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n} \quad i=1, \text{yesterday}$$

$\lambda$  close to 1 (small  $\lambda$ , decay fast)

### attractions

a nice generation of traditional HS

(HS:  $\lambda=1$ )

a large loss will receive a higher weight

reduce ghost effects

allow us to let sample period grow

reduce the effective sample size (30 enough)

### Hull&White (volatility-weighted HS)

$$\frac{r_{t,i}^*}{\sigma_{t,i}} = \frac{r_{t,i}}{\sigma_{t,i}}, \left( \left( \frac{r}{\sigma} \right)_T = \left( \frac{r}{\sigma} \right)_t \right) \quad (\text{base: historic } \sigma, r, \text{today } \sigma)$$

correlation-weighted historical simulation

filtered historical simulation (GARCH)

neural network approaches

## advantages

intuitive&simple

no parametric assumptions

accommodate any type of position

no need for Cov. Matrices

no curses of dimensionality

data are (often) readily available

combine them with parametric

## disadvantages

depend on historical data

have difficulty handling shifts

## problems from long window

aged data

longer: ghost effects to disappear

news will be drowned out by olds

collection

## VaR mapping

### fixed income

principal mapping

duration mapping

cash flow mapping

### linear derivatives

long forward (foreign currency)

= long foreign currency spot

+ long foreign currency bill

+ short U.S. dollar bill

### options (use BSM)

long option = long  $\Delta$  asset

+ short  $(\Delta S - c)$  bill

## backtesting VaR (VaR\_95%, p=5%)

## exceptions (loss > VaR)

too many: model underestimates risk

too few: excess allocation of capital across units

## model backtesting with exceptions

### exceptions follow binomial

eg. VaR\_95%(daily)

$f(x) = C_T^x p^x (1-p)^{T-x}$	mean(u)	$252 \times (1-95\%)$
	stdev(s)	$\sqrt{5\% \times 95\% \times 252}$
$z = \frac{x - pT}{\sqrt{p(1-p)T}} \approx N(0,1)$	95%_CI	$u \pm 1.96s$
	99%_CI	$u \pm 2.58s$

### type1&type2 error

## Kupiec VaR backtest

$H_0$ : accurate model

$H_a$ : inaccurate model

test statistic:

$$LR_{uc} = -2 \ln[(1-p)^{T-N} p^N]$$

p: probability of exception,  $p=1-c$

N: num. of exceptions; T: num. of samples

$LR > 3.84$ , reject  $H_0$

interval  $\downarrow$  as confidence level  $\uparrow$

high confidence level  $\rightarrow$  exceptions are very rare

## Basel rules for backtest

mkt VaR: confidence level\_99%; past year(252 days)

zone	green	yellow					red
num. of exceptions	0-4	5	6	7	8	9	10+
increase in K(%)	0	40	50	65	75	85	100

capital =  $VaR \times (3+K) \times 8\%$

## causes for exceptions

basic integrity of the model

model accuracy

intraday trading

bad luck

## Christoffersen test

test: bonch or wrong VaR

## conclusions

horizon should be short (daily VaR, observations  $\uparrow$ )

confidence level (VaR, not backtest) not too high

high power (1-type2)



market risk(3-2) VaR  
risk measurement for the trading book

for bank:

trading book

for trading\_mkt risk\_FV,book to mkt

banking book

HTM\_credit risk\_accrued/historical cost

### VaR implementation

time-varying  $\sigma \Rightarrow \text{VaR}(t) \Rightarrow \text{horizon} \uparrow, \text{accuracy} \downarrow$

$\sigma$  generated by stochastic jumps

recognize  $\sigma(t)$  in VaR measures

or will underestimate risk

time-varying correlations( $\rho(t)$ )

num. of exceptions is small, but great loss

severity if losses in the tail  $\rightarrow$

not subadditive  $\rightarrow$  spectral risk( $\sum w_i \text{loss}_i$ )

### integrating liquidity into VaR models

exogenous liquidity

liquidity-adjusted VaR(LVaR)

endogenous liquidity

depend on trade sizes

integrated risk measurement

unified approach

consider all risk categories simultaneously

compartmentalized approach

calculate for individual risk types(Basel)

risk aggregation(top-down,bottom-up)

B/S management

eco.boom  $\Rightarrow$  relax VaR  $\Rightarrow$  expand  $\Rightarrow$  more risk

eco.bust  $\Rightarrow$  VaR constraint  $\uparrow \Rightarrow$  leverage&liquidity  $\downarrow$

※ risk measures

$$\text{VaR}_\alpha(L) = \inf\{l: F_L(l) \geq \alpha\}$$

$$\text{ES}_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 \text{VaR}_\alpha(L) du = E(L|L \geq \text{VaR}_\alpha)$$

spectral risk measures( $\sum w_i \text{loss}_i$ )

$$\text{SRM} = \int_0^1 w(u) \text{VaR}_u(L) du$$

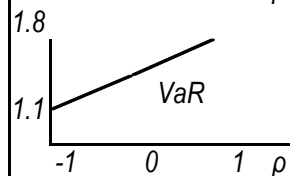
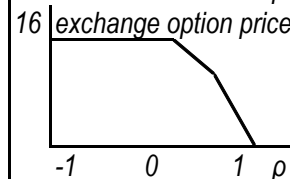
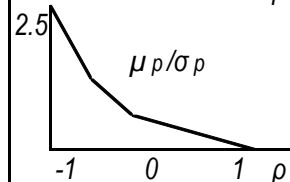
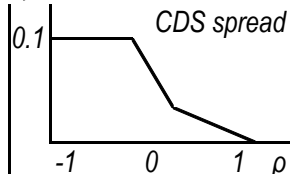
upper partial moments

$$\text{UPM}(1, \text{VaR}_\alpha) = (1-\alpha)(\text{ES}_\alpha - \text{VaR}_\alpha)$$

correlation

### some correlation basics

$\rho \uparrow, \text{value} \downarrow$



### empirical properties of correlation

equity correlation behaviors

in recessions,  $\rho \uparrow, \sigma_\rho \uparrow$

**positive relationship:  $\rho$  &  $\sigma_\rho$**

$\rho$ : a strong mean reversion of 77.51%

$$\rho_{t-1} - \rho_{t-2} \quad \frac{\partial(S_t - S_{t-1})}{\partial S_{t-1}} < 0$$

$\rho$ : autocorrelation of 22.49%

**before recession:  $\Delta \sigma_\rho \downarrow$**

bond correlations & PD correlations

similar to equity

Johnson SB distribution

$\rho_{\text{bond}}$  distribution: more normal shape

generalized extreme value distribution

|(GEV)

wrong way risk( $\rho(\text{bad thing 1, bad thing 2}) \uparrow$ )

**quanto option** (fixed ex.rate)

Ame. investor  $\rightarrow$  \$  $\rightarrow$  Jap.  $\rightarrow$  r stock

$\rho(r_{\text{stock Jap.}}, \text{yen})$  small, quanto high

eg. r stock  $\uparrow$ , yen  $\downarrow$

### correlation swap

#### 1. correlation swap

fixed rate payer  $\rightarrow$  fixed  $\rho$   $\rightarrow$  fixed rate receiver  $\leftarrow$  realized  $\rho$

#### 2. buying correlation( $\rho \propto \sigma$ )

$P/L = V_{\text{call on index}} - V_{\text{call on stock}}$

buy call index  $\rightarrow$  investor  $\rightarrow$  sell option stoc

index:  $\rho \uparrow \uparrow \rightarrow \sigma \uparrow \uparrow \rightarrow V_{\text{call}} \uparrow \uparrow$

stock:  $\rho \uparrow \rightarrow \sigma \uparrow \rightarrow V_{\text{call}} \uparrow$

#### 3. two swaps( $\rho_{\text{index}} > \rho_{\text{component}}$ )

receive var++  $\rightarrow$  variance swap on an index  $\rightarrow$  pay fix

receive fix  $\rightarrow$  variance swap on components  $\rightarrow$  pay var+

### statistical correlation models

limitation of **Pearson correlation** ( $\rho, \text{Cov.}$ )

linear dependencies

zero correlation  $\neq$  independence

linear correlation measures only if

joint distribution of x is elliptical

normal, t(fixed  $\mu, \sigma^2$ , skew)

variances of sets X&Y have to be finite

$\sigma \neq 0, \infty$

not invariant to transformations

### financial correlation modeling(copulas)

$$C_{GD}[Q_i(t), \dots, Q_n(t)] = M_n[N^{-1}(Q_1(t)), \dots, N^{-1}(Q_n(t)); \rho_M]$$

default time CDP<sub>B</sub> CDP<sub>C</sub>  
Q<sub>B</sub>(t) Q<sub>C</sub>(t)

$\downarrow$  map  $\downarrow$

NORM.S.INV(probability)

C<sub>GD</sub>: Gaussian default time copula

Q<sub>i</sub>(t): cumulative default prob. of asset i at time t

M<sub>n</sub>: joint, n-variate cumulative standard normal distribution

$\rho_M$ : n\*n symmetric, positive-definite correlation matrix of M<sub>n</sub>

### loss of hedge fund

2005

strategy

-equity tranche(insurance)

receive high spread

+mezz tranche(insurance)

pay low spread

in fact:  $\rho \downarrow$  (paper loss)

$V_{\text{equity}} \downarrow \Rightarrow$  spread of equity  $\uparrow$

$V_{\text{mezz}} \uparrow \Rightarrow$  spread of mezz  $\downarrow$

2007-2009

strategy

+equity tranche

$\rho \uparrow, V_{\text{equity}} \uparrow$

PD  $\uparrow$  &  $\rho \uparrow \Rightarrow V_{\text{equity}} \downarrow$

$\rho$  within sectors is higher than between sectors

time horizon  $\uparrow \Rightarrow$  PD  $\uparrow$

mkt  $\uparrow \Rightarrow \rho \uparrow$ ; mkt  $\downarrow \Rightarrow \rho \uparrow \uparrow$

### Spearman's rank correlation

order X; Y respect to X

$d_i = X_{\text{order}} - Y_{\text{order}}$

$$\rho_s = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 - 1)}$$

### Kendall's $\tau$

order X; Y respect to X

concordant(rank x<sub>i</sub>): x<sub>i</sub>  $\geq$  y<sub>i</sub>, x<sub>j</sub>  $\geq$  y<sub>j</sub>

neither concordant nor discordant

rank: x<sub>i</sub> = y<sub>i</sub>, x<sub>j</sub> = y<sub>j</sub>

discordant: other

$$\tau = \frac{n_c - n_d}{n(n-1)/2}$$

empirical approaches to risk metrics&hedges

DV01-neutral hedge  
 $\Delta P = -D \times P \times \Delta y + 0.5 \times C \times P \times \Delta y^2 \approx -DD \times \Delta y$   
hedge:  $DD_0 \Delta y + DD_H \Delta y = 0$   
drawbacks:  
assume  $\Delta y_H = \Delta y_0 = \Delta y$   
one-to-one relationship not always exist  
regression hedge ( $\Delta y_H \neq \Delta y_0$ )  
 $x_H \times DD_H \times \Delta y_H = x_0 \times DD_0 \times \Delta y_0$   
assume  $\Delta y_H = 1\text{bps}, \Delta y_0 = \Delta \text{bps}$   
 $x_H \times DD_H = x_0 \times DD_0 \times \Delta$   
hedger = underlying  
principal components analysis (PCA)

science of term structure models

binominal int. rate tree  
  
value an option-free bond (← discount)  
  
BSM not apply to fixed income  
BSM assumes:  
price underlying  $\in [0, +\infty)$   
risk-free rate is constant  
volatility is constant  
bonds with embedded options  
 $V_{callable} = V_{pure} - V_{call}$   
 $V_{puttable} = V_{pure} + V_{put}$

valuing a European call option  
  
 $V_{call} = \max(P_{bond} - \text{strike price}, 0)$   
constant maturity treasury swap (CMT swap)  
fixed  $\rightleftharpoons$  float, fixed payer  
  
 $P_{swap} = \text{swap}_{spot} + \Sigma PV_{swap}$

evolution of short rates, shape of term structure

Jensen's inequality  
Jensen's inequality  
 $E\left[\frac{1}{1+r}\right] > \frac{1}{E(1+r)}$   
certainty > uncertainty  
convexity effect  
can be measured by J inequality  
maturity & volatility  $\uparrow \Rightarrow$  convexity  $\uparrow$   
risk premium (RP)  
calculate price & return with RP  
  
$$P = \frac{\frac{1}{1.1 + RP} + \frac{1}{1.06 + RP}}{1.08} \times 0.5$$
  
$$R = \frac{\left[\frac{1}{1.1} + \frac{1}{1.06}\right] \times 0.5 - P}{P}$$

art of term structure models: drift, volatility and distribution

model 1 (volatility)  
 $dr = \sigma dw, r_1 = r_0 \pm \sigma \sqrt{dt}, dw = \epsilon \sqrt{dt}, dw \sim N(0, dt)$   
  
solutions for negative interest rates  
use distributions (non-negative)  
force negative interest rates = 0  
model 2 (trend + volatility)  
 $dr = \lambda dt + \sigma dw, r_1 = r_0 + \lambda dt \pm \sigma \sqrt{dt}, \lambda = c$   
  
Ho-lee model with time-dependent drift  
arbitrage model  
  
model 3  
 $dr = \lambda(t)dt + \sigma(t)dw$   
 $dr = \lambda(t)dt + \sigma \exp(-\alpha t)dw$   
Cox-Ingersoll-Ross model  
 $dr = \kappa(\theta - r)dt + \sigma \sqrt{rdw}$   
lognormal model (model 4)  
 $dr = \alpha dt + \sigma rdw$   
model 4 with deterministic drift  
 $d[\ln(r)] = a(t)dt + \sigma dw$   
model 4 with mean reversion  
 $d[\ln(r)] = \kappa(t)[\ln(\theta(t)) - \ln(r)]dt + \sigma(t)dw$

volatility smile  
implied volatility: use BSM, cpt  $\sigma$   
 $x$ : strike price,  $y$ :  $\sigma$   
foreign currency option  
  
At the Money Options  
reasons  
  
equity options  
  
At the Money Options

OIS discounting  
treasury rates lower than  $R_{free}$   
demand  $\uparrow$ , capital low, tax-free  
overnight indexed swap  
  
risk: default; overnight  
3 month LIBOR-OIS spread  
normal conditions: 10 bps  
in Oct. 2008: 364 bps  
 $V = NDV - CVA + DVA \pm FVA$   
NDV: no default value  
CVA: PV of counterparty default risk

# Credit risk 3-1

## credit risk measurement 3-1(PD)

### basic approaches

experts-based(rating)  
statistical-based(Merton,LDA,logistic)  
heuristic&numerical  
structural(eco.&fina.assump.)  
reduced-form(statistically)

### rating system

role in CR mgt.  
experts-based approaches  
foundation of CR mgt.  
ordinal measure  
supports credit pricing&capital provisions  
support decision

### features of good rating system

measurability(PD)&verifiability(backtest)  
objective&homogeneity  
specificity(only default)

### rating agencies(not directly comparable)

Moody's:issues rating  
S&P:issuer FITCH:issuer  
rating to PD & transition matrix

### measurement of PD

CDP= $\sum_{all} CDP$   
CDP:cumulative DP  
MDP:marginal DP MDP= $\Delta CDP$   
FP:forward prob. (old MDP)  
SR:survival rate SR=1-CDP  
ADR:annualized default rate

$$1-CDP=(1-ADR)^t;=exp(-ADR \times t)$$

### internal rating

### measurement from mkt price(bond)

#### credit risk(only)

accurate(1year,zero-coupon,all data)

$$P = \frac{1}{1+YTM} = \frac{PD \times RR + (1-PD) \times 1}{1+R_f}$$

$$PD = \frac{1}{1-RR} \times \frac{YTM-R_f}{1+YTM}$$

$$= \frac{1}{LGD} \times \frac{CS}{1+YTM}$$

approximate(non-zero coupon)

$$CS=PD \times LGD$$

#### credit risk&liquidity risk

accurate

$$PD = \frac{1}{1-RR} \times \left(1 - \frac{1+R_f+LS}{1+YTM}\right)$$

approximate

$$YTM=R_f+PD \times LGD+LS$$

### measurement from mkt price(equity)

#### Merton model (capital structure)

shareholder

$$E=call$$

$$E=Call=VN(d_1)-De^{-rt}N(d_2)$$

bondholder

$$D+put=debt_{risk-free}; D=V-E$$

$$D=De^{-rt}N(d_2)+VN(-d_1)$$

distance to default: $d_2$

$$d_2 = \frac{\ln(V/Ke^{(-rt)})}{\sigma \sqrt{t}} - 0.5 \sigma \sqrt{t}$$

$$PD=1-N(d_2)=N(-d_2)$$

risk-neutral: $R_{free}$ ;real-world:asset drift

#### Moody's KMV model

$$distance\ of\ default: DD = \frac{A-K}{\sigma_A}$$

$$\sigma_A: \sigma(\%) \times A; PD=N(-DD)$$

$$LT/ST < 1.5: K=ST+0.5LT$$

$$otherwise: K=0.7ST+0.7LT$$

### exponential distribution ( $\lambda$ =hazard rate)

$$CPD(t^* < t) = 1 - \exp(-\lambda t); SR = \exp(-\lambda t)$$

$$\lambda \times LGD \approx CS$$

$$P(t^* < t+\tau | t < t^*) = P(t^* < \tau)$$

### single factor model (asset return)

$$ROA = \alpha, \text{ if } \alpha < \frac{-E}{V}, \text{ default}$$

#### unconditional default: $\alpha \sim N(0,1)$

$$\alpha = \beta m + \sqrt{1-\beta^2} \epsilon$$

$$E(\alpha) = 0; Var(\alpha) = \beta^2 + 1 - \beta^2 = 1$$

$$R_{mkt}: m \sim N(0,1)$$

$$R_{stock}: \epsilon \sim N(0,1); Cov(m, \epsilon) = 0$$

$$PD = P(\alpha < R)$$

#### conditional(particular value m)

$$\alpha_i = \beta_i m + \sqrt{1-\beta_i^2} \epsilon_i \quad \alpha \sim N(\beta m, 1-\beta^2)$$

$$PD = p = \Phi\left(\frac{k_i - \beta_i m}{\sqrt{1-\beta_i^2}}\right) \quad i = 1, 2, \dots$$

### other model

#### linear discriminant analysis

$$Altman's\ Z\text{-score}(Z=2.675)$$

#### logistic regression model

$Y \sim Bernoulli$

$$g(\pi_i) = \log \frac{\pi_i}{1-\pi_i} = \beta_0 + \sum_{j=1}^p \beta_j \cdot x_{ij}$$

#### cluster analysis&principal component

#### cash flow simulation model

#### heuristic methods&numerical methods

### concentration risk(x default)

$$C_N^x p^x (1-p)^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

## credit risk identification

### classifications&drivers

default&recovery&exposure

### key indicators

expected loss  $EL=PD \times LGD \times EAD$

unexpected loss(credit VaR)

$$UL=WCL-EL(C\&O:99.9\%)$$

concentration risk

### decision&analyst

consumer:credit score

nonfinancial firm:earning&profit&CF

financial firm:capital/asset,qualitative

sovereigns:more subjective

## Credit risk 3-2

### credit risk measurement 3-2( **exposure** )

#### exposure metrics

##### current

current exposure

##### future positive

**expected exposure**

**expected positive exposure(EPE)**

##### counterparty's point of view

negative exposure

negative expected exposure(NEE)

expected negative exposure(ENE)

##### high-pencentile(VaR)

potential future exposure( **PFE** )/peak exposure

maximum PFE

##### roll-over

effective EE & effective EPE

#### credit value adjustment(CVA)

##### basic measurement

standard equation for CVA

**CVA=ΣPVEL**

$$= (1 - \text{Rec}) \sum_{i=1}^m \text{DF}(t_i) \text{EE}(t_i) \text{PD}(t_{i-1}, t_i)$$

CVA as a spread(EE → constant)

**CVA=credit spread × EPE**

incremental CVA(new trade)

marginal CVA(component)

#### WWR&RWR

##### wrong-way risk(WWR)

PD ↑ EAD ↑ → CVA ↑

##### right-way risk(RWR)

PD ↑ EAD ↓ → CVA ↓

#### stress testing counterparty exposure

##### stress test on a loan portfolio( **PD** )

stressed EL

$$EL_s = \Sigma PD_s \times EAD \times LGD$$

stress loss=EL<sub>s</sub>-EL

##### stress test on derivative portfolio( **PD&EAD** )

stressed EL

$$EL_s = \Sigma PD_s \times \alpha \times EAD_s \times LGD$$

stress loss=EL<sub>s</sub>-EL

##### stressed **CVA** & stress loss on CVA

$$CVA_s = \Sigma (LGD \times \Sigma (EE_s \times PD_s))$$

stress loss=CVA<sub>s</sub>-CVA

##### **DVA(BCVA)** & stressed DVA

$$DVA = LGD \times EE \times PD \times S_1 - LGD \times NEE \times PD \times S_2$$

$$BCVA = \sum_{n=1}^N LGD_n \cdot \sum_{i=1}^m EE_n(t_i) \cdot PD_n(t_{i-1}, t_i) \cdot S_i(t_{i-1}) - \sum_{n=1}^N LGD_n \cdot \sum_{i=1}^m NEE_n(t_i) \cdot PD_n(t_{i-1}, t_i) \cdot S_n(t_{i-1})$$

##### common pitfalls in stress testing CCR

stressing current exposure

how to aggregated for portfolio

exposure is highly nonlinear

#### credit risk measurement 3-3( **portfolio credit VaR** )

##### default correlation(**PD=π**)

$$E(X_i) = \pi_i \quad i = 1, 2$$

$$E(X_1 X_2) = \pi_{12}$$

$$V(X_i) = \pi_i (1 - \pi_i) \quad i = 1, 2$$

$$\text{Cov}(X_1, X_2) = \pi_{12} - \pi_1 \pi_2$$

$$\rho = \frac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1 (1 - \pi_1)} \sqrt{\pi_2 (1 - \pi_2)}}$$

##### portfolio credit VaR

default correlation impact

**ρ only impact to UL**

$$\rho=1, EL=EAD \times PD \times LGD$$

$$\rho=0, EL=N \times \pi \times LGD$$

marginal contribution to portfolio UL

#### credit risk management 4-1( **mitigating approaches** )

##### netting

##### types of netting

close-out netting: default → netting

novation netting: a replacement contract(new, netting payment)

influence factor	positive	negative
correlation	×	✓
initial MtM	×	✓

$$\text{netting factor} = \frac{EPE(\text{netting})}{EPE(\text{without netting})} = \frac{\sqrt{(n+n(n-1)\rho)}}{n}$$

##### collateralization

##### provision

haircut

threshold

only incremental amount need coll.

minimum transfer

independent amount(initial margin)

##### central counterparties(CCP)

##### clearing members

general clearing member(GCM)

eg. securities agency

individual clearing member(ICM)

non-clearing member(NCM)

##### initial margin

determined by

**volatility of the trade**

**tail risk(mkt variables)**

**dependency(ρ)**

VaR approaches(WCL)

##### loss waterfall

initial margin of defaulter

reserve fund of defaulter

first loss equity of CCP

remaining CCP reserve fund

addition capital from survivor

capital of CCP

liquidity support or CCP fails

##### other mitigation clause

close-out(immediate closing all)

walkaway feature

acceleration clause(accelerate given an event)

termination

break clause/liquidity put(long term contract)

additional termination event

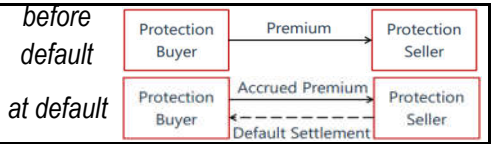


Credit risk 3-3

credit risk management 4-2( **credit derivatives** )

**credit default swap(CDS)**

characteristic



settlement

cash

liquidity:par-MV

ill-liquidity:fixed

physical

cash of par,assets transfer

basket CDS

Nth to default CDS

**first-to-default**

**second-to-default**

**subordinated basket(below)**

**senior basket(above)**

correlation impact in Nth-to-default swap

$\rho \uparrow$ , first-to-default  $\downarrow$ , N-to-default  $\uparrow$

$\rho \downarrow$ , first-to-default  $\uparrow$ , N-to-default  $\downarrow$

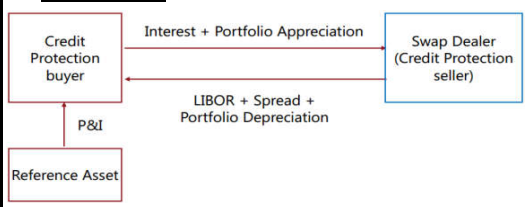
$\rho=1$ , first-to-default=N-to-default

**total return swap(TRS)**

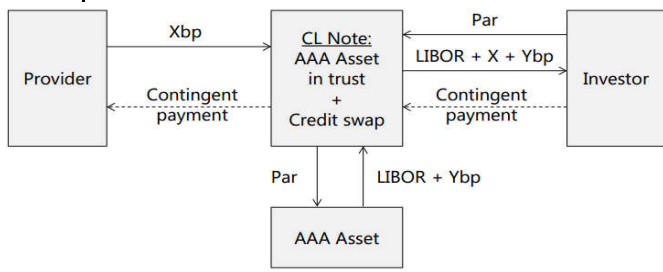
hedge against

1.default 2.credit deterioration

3.maket risk



**credit linked note(CLN)**



credit risk management 4-3( **securitization** )

securitization&structured product

structured finance

**ring-fencing(insulate)**

**securitization(true sale)**

securitization

SPV

amortizing structure

revolving structures(credit card)

master trust(defferent product)

tranching(for investor)

**waterfall(cashflow)**

structured product

covered bond(remain on BS)

CDO(loan,bond)

**cash CDO**

**synthetic CDO(like CLN)**

**singel-tranche CDOs**

performance analysis

CPR&PSA

**constant prepayment rate(CPR)**

$CPR = 1 - (1 - SMM)^{12}$

**public securities association(PSA)**

100%-PSA

CPR 0% increase by 0.2%

reach 6% in month 30

**credit scenario analysis**

mezz:high PD-equity;low PD-senior

$\rho \equiv PD \uparrow$

$PD \equiv \rho \uparrow$

V VaR

V VaR

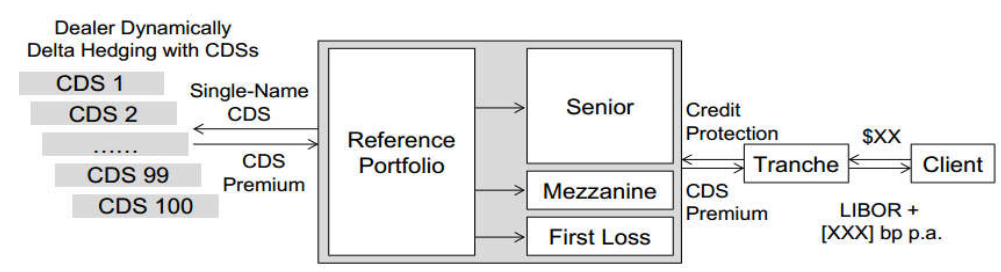
senior  $\downarrow \uparrow$

senior  $\downarrow \uparrow$

equity  $\downarrow \downarrow$

equity  $\uparrow \uparrow$

singel-tranche CDOs



credit risk management 4-4( **retail credit risk** )

retail credit risk

home mortgage

home equity loans...

nature of retail credit risk

single customer,well-diversified port.

EL rate can be built into price

corp.&comm. credit portfolio

concentrations,grographical

**dark side of retail credit risk**

lack of historical data(innovation)

economic environment

social and legal system

operational issue

**key variables in mortgage credit assess**

**document type**

full doc

stated income(employ,Y;income,N)

no income/no asset(source of income,Y)

no ratio

no doc

**FICO(above 660)**

**DTI:debt to income**

**LTV:loan to value(collateral)**

**payment type**

credit scoring model

credit bureau scores(FICO)

pooled models(by outside vendors)

custom models(in house)

cutoff scores

scorecard performance

cumulative accuracy profile,CAP

accuracy ratio,AR