Attachment for #515

OUTLINE

This attachment mainly covers the following points:

- For Reviewer#3:
  - More comprehensive description of BCR construction in §1.1.1
  - Literature review for BCR construction in §1.2
  - ToR protocol specification in §1.3
  - Gateway latency cost evaluation in §1.4
- For Reviewer#4:
  - Proof of BCR's uniqueness and completeness in §2.1

1. For Reviewer#3

1.1. Description of BCR construction

For a comprehensive description of constructing BCR, we update and present it in a more detailed and clear style in §1.1.1. During the rebuttal response period, we also find that the algorithm could achieve only 1 hash persistent storage at the best-case, $log_2 k$ at the worst-case space, and $\frac{log_2 k}{2}$ at the average. We demonstrate this in §1.1.2.

1.1.1. Algorithm1 Description. This Algorithm1 needs to achieve two objectives: 1. Update the BCR by treating the current cross-chain message as a new leaf node and using the MerkleUpdatePath. The MerkleUpdatePath is a list that stores several intermediate node values of the Merkle tree, allowing the current cross-chain message to compute the new BCR. 2. Generate the MerkleUpdatePath for the next leaf node. This is an online algorithm, meaning it needs to generate and store a new MerkleUpdatePath for computing the new BCR for the next leaf node.

The algorithm needs several key global variables stored in the cross-chain service contract on the parachain: 1) $MerkleUpdatePath[h]$ stores the latest MerkleUpdatePath for the block at height h, used to update the BCR. 2) $Cnt[h]$ stores the number of cross-chain messages in the block at height h and serves as the index of the current cross-chain message in the leaf nodes (starting from 1). 3) $Bcr[h]$ stores the BCR for the block at height h.

The algorithm needs the input of the cross-chain message cm and the block height h. By height h, we can get $MerkleUpdatePath[h]$ and $Cnt[h]$, which are the node list and the index of the current cross-chain message in the leaf nodes (starting from 1) for updating the BCR.

The algorithm iterates from the leaf node to the root node (line:6 and line:25) layer by layer, while completing the above two objectives.

To achieve the first objective, during the upward iteration process:

- If the current node's index in the current layer is odd (line:11), then it needs to calculate the hash of the parent node together with the right sibling node. Since the right node is empty, its value is assigned to the hash of the current node. And then the hash of the parent node is calculated (line:16);
- If the current node's index in the current layer is even (line:17), then it needs to calculate the hash of the parent node together with the current node's left sibling node. The left sibling node is a historical node that has been saved in $MerkleUpdatePath[h]$, so it needs to be obtained from $MerkleUpdatePath[h]$ (line:18), and then calculate the parent node's hash value (line:23). The values saved in $MerkleUpdatePath[h]$ are generated when executing the algorithm with the cross-chain message at the index $Cnt[h] - 1$;

To achieve the second objective, during the upward iteration process, it is necessary to add the necessary nodes in order to prepare for the cross-chain message at the index $Cnt[h] + 1$. To achieve this goal, we introduce a flag to record the distribution of odd and even nodes in the upward iteration process. The flag has two possible values: ALLEVEN and ANYODD. ALLEVEN means that all nodes in the upward iteration process are even, and ANYODD means that at least one node in the upward iteration process is odd. Specifically:

- If the current node's index in the current layer is odd (line:11), check the flag's state. If the flag is ALLEVEN (line:12), it means all nodes in the iteration are even, indicating the subtree rooted at the current node is a full binary tree without any new cross-chain message. The new cross-chain message will be added to the subtree rooted at the right sibling node, so the current node needs to be added to mp (line:13). If the flag is ANYODD, meaning at least one node in the iteration is odd, the subtree is not full, and new nodes will be added, so the current node does not need to be added to mp. Finally, set the flag to ANYODD (line:15).
- If the current node's index in the current layer is even (line:17), calculate the parent node's hash with the left sibling node, whose hash is obtained from MerkleUpdatePath[h] (line:18). If the flag is ANYODD (line:20), it means the subtree is not full, and the left sibling node needs to be added to mp (line:21) for the new cross-chain message. If the flag is ALLEVEN, the subtree is full, and no new leaf nodes will be added,

so the left sibling node does not need to be added to mp.

Algorithm 1 Constructing BCR: A^{u}_{BCR}.

Global States:
$Mp[h]$ ▷ Merkle path for updating BCR;
$Cnt[h]$ ▷ number of the cross-chain messages;
$Bcr[h]$ ▷ BCR at block h;
Input:
h ▷ block height;
cm ▷ cross-chain message;
Output:
r ▷ the root of the Merkle tree that updated;
1: $tempNode \leftarrow \mathcal{H}(cm)$ ▷ hash of cm
2: $index \leftarrow Cnt[h] + 1$ ▷ index of cm
3: $pathPoint \leftarrow 0$ ▷ point to the position in $Mp[h]$
4: $flag \leftarrow$ ALLEVEN ▷ initialize flag
5: $mp \leftarrow EmptyList$ ▷ store the new Merkle path
6: **while** $index \geq 1$ **do**
7: **if** $index = 1$ **then**
8: mp.append($tempNode$)
9: **break**
10: **end if**
11: **if** $index \bmod 2 = 1$ **then**
12: **if** $flag$ is ALLEVEN **then**
13: mp.append($tempNode$)
14: **end if**
15: $flag \leftarrow$ ANYODD
16: $tempNode \leftarrow \mathcal{H}(tempNode \parallel tempNode)$
17: **else**
18: $node \leftarrow Mp[h][pathPoint]$
19: $pathPoint \leftarrow pathPoint + 1$
20: **if** $flag$ is ANYODD **then**
21: mp.append($node$)
22: **end if**
23: $tempNode \leftarrow \mathcal{H}(node \parallel tempNode)$
24: **end if**
25: $index \leftarrow \lceil index/2 \rceil$
26: **end while**
27: $Mp[h] \leftarrow mp$
28: $Bcr[h] \leftarrow tempNode$
29: $Cnt[h] \leftarrow Cnt[h] + 1$
30: **return** $tempNode$

1.1.2. Storage Analysis. Assume the number of cross-chain messages in a block at height h is k.

For the best-case, only 1 hash(i.e. 32 bytes) is needed to store persistently in a block. From Algorithm1, we can see that during the upward iteration process, if the index is always even(line:17), then only node in the toppest layer(line:8) is needed to be appended to the mp. Because ALLEVEN indicates that the whole tree is full, the new coming cross-chain message only serves as a new leaf node within the next adjacent tree, which merely needs the current tree's root to calculate BCR. Therefore, the minimal number of hash persistent storage is 1, with $k = 2^p$ and $p \in \mathbb{N}$. It also means that the cross-chain message at the index $k = 2^p + 1$ only needs one old hash to calculate BCR.

For the worst-case, $log_2 k$ hash persistent storage is needed in a block. From Algorithm1, we can see that during the upward iteration process, if the index of the leaf node is odd(line:11) but all the upward iterated nodes' indexes are even(line:17), then the leaf node and upward iterated nodes' left sibling nodes should be appended to the mp. The size of mp is $log_2 k$. Therefore, the minimal number of hash persistent storage is $log_2 k$, with $k = 2^p - 1$ and $p \in \mathbb{N}$. It also means that the cross-chain message at the index $k = 2^p$ needs $log_2 k$ old hashes to calculate BCR.

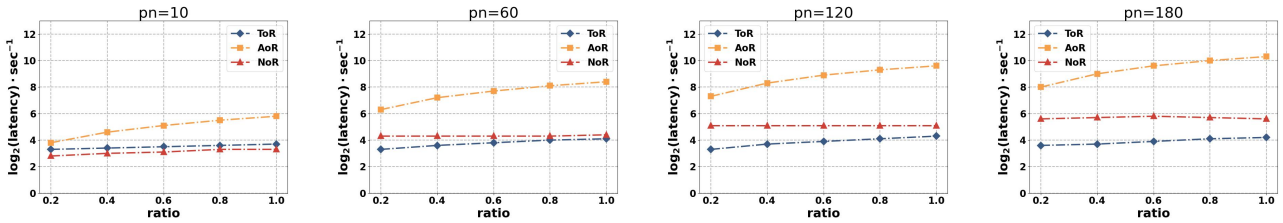
For the average storage, $\frac{log_2 k}{2}$ hash persistent storage is needed in a block. we can find that if a node's index at a layer is even, then its left sibling node is needed to calculate the parent node's hash. Therefore, for a cross-chain message at the index k, during the upward iteration process, the number of even indexes is equal to the size of mp which generated by the cross-chain message at the index $k-1$. For each iteration, the probability that the node's index is even is $\frac{1}{2}$, leading to the expected mp size $\frac{log_2 k}{2}$.

1.2. BCR Construction Literature Review

There are some Merkle tree constructing methods to calculate the root:

- **UpdateByLeaves.** Construct a new Merkle tree using the current and previous messages with $O(k)$ time complexity and $O(1)$ a space complexity (k, number of messages). For example, Bitcoin [1] uses a simple Merkle tree to construct the root committing to all transactions in a block, without any intermediate hashes storage. It is more suitable where updating is not frequent.
- **UpdateByTree.** Store the whole tree and directly update its intermediate nodes when a new leaf is appended. Existing blockchain platforms such as Ethereum [2], Solana [3], and Cosmos [4], Zilliqa [5], Near [6], and Quorum [7] use this method to construct the root committing to all states like account balances and constract storage values. This method has $O(\log(k))$ time complexity but $O(k)$ space complexity, making it more suitable where nodes update frequently and storage is cheap;

Some other methods are also proposed to improve the efficiency of Merkle tree construction, while not practical for constructing the BCR within blockchain contracts. [8] implementes an append-only Merkle tree structure with a time complexity of $O(1)$ and space complexity of $O(k)$. However, it does not store the root on-chain. BIP-0098 [9] proposes a more efficient Merkle tree structure, but it modifies the SHA-256 hash function, not compatible with existing mainstream blockchains. [10] accelerates Merkle Patricia tree construction using GPU, while not suitable for contracts. Above algorithms may not fully meet the requirements for updating BCR efficiently, economically and practically.



(a) (b) (c) (d)

Figure 1: The latency of the gateway transmitting messages from listening to the new message on the source chain to sending the message to the destination chain ($ratio$ and pn).

1.3. ToR Protocol Specification

We present the protocol description of the ToR protocol in Fig. 2, Fig. 4, and Fig. 3.

1.4. Gateway Latency Costs

We present the latency of the gateway transmitting messages from the time listening to the new message on the source chain to the time sending the message to the destination chain in Fig. 1. We issue 100,000 messages in the cross-chain system with [10, 60, 120, 180] fully connected parachains, and control the cross-chain message ratio to [0.2, 0.4, 0.6, 0.8, 1.0]. We set the y-axis to $log_2(latency) \cdot sec^{-1}$ for better visualization. Fig. 1 shows that the latency of ToR remains consistently low across different ratios, while the latency of AoR increases linearly with $ratio$. The NoR latency also remains relatively stable, but there is a significant increase in latency as pn increases. In a small-scale (pn=10), the gateway latency of ToR a bit higher than NoR, because the header synchronization overhead is relatively lower in NoR. But with the increase of pn, the latency of ToR is much lower than NoR, because the header synchronization overhead grows exponentially which makes the cross-chain message queuing delay more serious.

2. For Reviewer#4

2.1. Proof of BCR's Uniqueness and Completeness

2.1.1. Uniqueness Proof. Assume by contradiction that there exist two different valid BCRs, BCR_1 and BCR_2, for the same block B. By the BCR definition, both BCR_1 and BCR_2 must be constructed using the same algorithm MTA_0 and the same ordered list of cross-chain messages CMs.

First, the ordered list of CMs is deterministic in each block. Each cm is identified by $< block.height, msgId >$. Both $block.height$ and $msgId$ are increasing monotonically. Second, transaction execution order in each block is deterministic among different blockchain nodes. Therefore, each block generates a deterministic unique CMs. Finally,since MTA_0 is deterministic, MTA_0 must produce the same root for the same input CMs. Therefore, $BCR_1 = MTA_0(CM) = BCR_2$, which contradicts our assumption that $BCR_1 \neq BCR_2$.

2.1.2. Completeness Proof. For the BCR in $block_i$, we have:

Part1: All cross-chain messages in $block_i$ are included in the BCR. Each cross-chain message must be as input of A^u_{BCR} and update the old BCR to a new one. And each one is identified by $< block.height, msgId >$ which is unique in $block_i$, so the cross-chain message can not be overwritten by other messages. If any cross-chain message is not included in the BCR, then the contract execution is broken which contradicts with the blockchain's correctness.

Part2: Any non-cross-chain data are not included in the BCR. Only the cross-chain service contract can update the BCR by calling A^u_{BCR}. The contract's logic is deterministic and each honest node executes the same logic. Malicious nodes can run a fake contract to add non-cross-chain data to the BCR, but it won't be finalized on the chain.

Part3: Any cross-chain messages in $block_j (i \neq j)$ are not included in the BCR. If a cross-chain message is in $block_j$, then it serves as the input of A^u_{BCR} with argument $< block_j.height >$ and finally $blockMerkle[j]$ is updated. Because the message is identified by $< block.height, msgId >$ and $i \neq j$, the intersection of messages in $block_j$ and in $blockMerkle[i]$ is empty. That means any cross-chain messages in $block_j$ can not be included in the BCR in $block_i$.

From part1, part2 and part3, we can conclude that the BCR satisfies the completeness.

Cross-chain Service Contract on Parachains

Initialize(_chainId)
Deploy contract on the parachain with chainId and initialize the following global states:

- Current parachain id $srcId$ = _chainId
- Message id $msgId$ = 0
- Relay chain block headers H_{rc} = {height: header}
- Other chains' BCR B = {chainId: {height: bcr}}
- Current chain's BCR Merkle path $blockMerkle$ = {height: {root, path, index}}
- Other chains' messages ids M = {chainId: {msgId: {0,1}}}

IssueCM(dstId, payload)

- $msgId \leftarrow msgId + 1$
- $height \leftarrow$ block.height
- make message $CM \leftarrow \{srcId, dstId, msgId, height, payload\}$
- update BCR in $blockMerkle$ with $A^{u}_{BCR}(height, CM)$
- emit event $EntIssueCM(CM)$

SyncHeaderFromRC(rch)
rch is a relay chain header.

- if rch.height in H_{rc}, then revert(EXIST)
- if not $LC.verify(rch)$, then revert(INVALID)
- $H_{rc}[rch.height] \leftarrow rch$
- emit event $EntSyncHeader(rch)$

ReceiveCM(CM, vp1, vp2, rh) $vp1$ and $vp2$ are the DLV proofs of CM. rh is height of CM's BCR finalized on the relay chain.

- $sId \leftarrow CM.srcId$;
- $mId \leftarrow CM.msgId$;
- $h \leftarrow CM.height$;
- if $M[sId][mId] = 1$, then revert(EXIST)
- $BCR' \leftarrow MTA_0.calc(CM, vp1)$
- if $B[sId][h] = \perp$, then
  - $root' \leftarrow MTA_0.calc(BCR', vp2)$
  - if $root' \neq H_{rc}[rh].root$, then revert(INVALID)
  - else $B[sId][h] \leftarrow BCR'$
- else
  - if $BCR' \neq B[sId][h]$, then revert(INVALID)
- $M[sId][mId] \leftarrow 1$
- dApp.execute(CM)
- emit event $EntReceiveCM(CM)$

Figure 2: Protocol description of Cross-chain Service Contract on Parachains

Protocol description of Gateway

Initialize(EP_s, EP_d, EP_r)
EP_s is the source chain's endpoint, EP_d is the destination chain's endpoint, EP_r is the relay chain's endpoint. The endpoint could call the corresponding contract and chain's basic functions.
Initialize the following global states:

- Other chains' headers H = {chainId $\leftarrow$ {height $\leftarrow$ header}}
- Other chains' BCR B = {chainId $\leftarrow$ {height $\leftarrow$ bcr}}

TrustRoot_S2R()
Transmit the trust root from the source chain to the relay chain.
Keep listening to the source chain's new block Blk:

- $h \leftarrow Blk.height$
- $header \leftarrow Blk.header$
- $bcr \leftarrow EP_s.getBCR(h)$
- $proof \leftarrow EP_s.getProof(h, bcr)$
- $pid \leftarrow EP_s.getChainId()$
- call $EP_r.VerifyTrustRoot(pid, header, bcr, proof)$

Header_R2D()
Transmit the header from the relay chain to the destination chain.
Keep listening to the relay chain's new block Blk:

- $header \leftarrow Blk.header$
- call $EP_d.SyncHeaderFromRC(header)$

Message_S2D()
Transmit the message from the source chain to the destination chain.
Keep listening to the source chain's event $EntIssueCM(cm)$:

- $sh \leftarrow cm.height$
- $sId \leftarrow cm.srcId$
- $vp1 \leftarrow EP_s.getProof(sh, cm)$
- wait until BCR of sh on the relay chain is finalized
- $(bcr, rcH) \leftarrow EP_r.getBCR(sId, sh)$
- if $EP_d.B[sId][sh]$ not exists, then $vp2 \leftarrow EP_d.getProof(sh, bcr)$
- else $vp2 \leftarrow \perp$
- call $EP_d.ReceiveCM(cm, vp1, vp2, rcH)$

Figure 3: Protocol description of Gateway

Cross-chain Service Contract on Relay Chain

Initialize()
Initialize the following global states:

- Other chains' headers H = {chainId $\leftarrow$ {height $\leftarrow$ header}}
- Other chains' BCR B = {chainId $\leftarrow$ {pcHeight $\leftarrow$ bcr, rcHeight}}. $pcHeight$ is the height of the BCR finalized on the parachain, $rcHeight$ is the height of BCR finalized on the relay chain.

VerifyTrustRoot(pid, header, bcr, proof)
pid is the parachain's id, $header$ is the parachain's header, bcr is the BCR in the parachain's block at $header.height$, $proof$ is bcr's state proof on the parachain.

- $h \leftarrow header.height$
- if $H[pid][h]$ exists, then revert(EXIST)
- if $LC_{pid}.verify(header)$ is false, then revert(INVALID-Header)
- if $MTA_{pid}.calc(bcr, proof) \neq header.root$, then revert(INVALID-BCR)
- $H[pid][h] \leftarrow header$
- $B[pid][h] \leftarrow bcr, block.height$
- emit event $EntVerifyTrustRoot(pid, h)$

Figure 4: Protocol description of Cross-chain Service Contract on Relay Chain

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