ToR: A Highly Scalable, General and Secure Cross-Chain Protocol

This supplment file contains three parts, including:

(1) Detailed evaluations in §1, which covers the latency §1.1, throughput §1.2 and parachain workload §1.3.
(2) Proof of workload optimization rate in §2, which covers Lemma 1, Lemma 2 and Theorem 1.

1 DETAILED EVALUATIONS

1.1 Latency

Interoperabliliy latency refers to the time taken for a cross-chain message to be initiated from the source chain, processed through the relay chain (not in NoR), and ultimately confirmed on the target chain. The experiment set $pn = [10, 60, 120, 180]$ and the $ratio$ to [0.2, 0.4, 0.6, 0.8, 1.0]. Fig.1 shows that the latency of ToR remains consistently low across different ratios, while the latency of AoR increases linearly with $ratio$. The NoR latency also remains relatively stable, but there is a significant increase in latency as pn increases.

The increase of latency with the $ratio$ rise in AoR is attributed to the elevated number of cross-chain messages. Under AoR, the relay chain is responsible for verifying and storing all cross-chain messages. Thus, an escalation in cross-chain messages volume augments the workload on the relay chain, leading to transaction congestion. This congestion amplifies the queuing latency of transactions on the relay chain. Higher $ratio$ exacerbates queuing delays, resulting in increased overall cross-chain interoperabliliy latency.

1.2 Throughput

Throughput (TPS) is defined as N_{tx}/D, where N_{tx} represents the total number of cross-chain messages, and D denotes the total time taken for cross-chain messages from issuing on the source to confirm on the target. The Fig.2 shows that ToR has a higher throughput than NoR and AoR overall. When $pn = 10$, the difference among the three protocols are negligible. However, when the number of Parachains increases ($pn = [60, 120, 180]$), the throughput of ToR significantly outperforms that of NoR and AoR. Additionally, for AoR, the throughput tends to plateau when $pn \geq 60$ as the $ratio$ increases, reaching its performance upper limit. It is worth noting that at $pn = 180$, NoR's throughput is remarkably low, far below that of AoR and ToR.

1.3 Workload on Parachains

After significantly reducing the relay chain's workload, is the parachain's workload affected? To answer this question, this experiment monitors the workload variation.

From Fig.3, it can be seen that when pn is small, the differences of parachains' workload among the three protocols are minimal and their workload gradually decrease until the end of the observation. However, when pn is large, it can be seen that the parachains' workload in NoR no longer decreases but continues to increase. It is because the parachains in NoR have to synchronize and verify the block headers from all other parachains. When pn increases, the number of block header transactions that each parachain must process increases until it exceeds the performance cap of the parachain. Therefore, even if cross-chain messages no longer exist in the system, the parachains' workload continues to increase.

The workload variation between ToR and AoR is quite similar because neither requires the parachain to process block headers from any other parachains but the relay chain. Although it is similar, the detailed figure shows that the ToR's workload on parachains is slightly higher than that of AoR. This difference is from that ToR also requires the target chain to verify the second layer proof $VP2$. Additionally, each block from the source chain only corresponds to only one $VP2$ (according to the "Reduce Verification Redundancy" strategy). Thus, the parachain's workload tends to be slightly higher in ToR.

2 PROOF OF WORKLOAD OPTIMIZATION RATE

Model. We denote the relay chain's throughput as α, meaning it can process α transactions per second. Given $n \geq 2$ parachains in the cross-chain system, fully-connected, denoted as $\{P_i \mid i \in [1, n]\}$. For the parachain P_i, it can generate $\{b_i \mid i \in [1, n]\}$ blocks per second, and the number of cross-chain messages created per block is denoted as $\{x_j \mid j \in [1, b_i]\}$. We consider the average workload during the time interval D which is divided it into a discrete time series $TS = \{t_1, t_2, ..., t_d\}$ with a 1-second interval. For convenience, we define $h(x) = \frac{|x|+x}{2}$, meaning that if $x > 0$ then $h(x) = x$, and if $x <= 0$ then $h(x) = 0$.

Lemma 1. *Based on the above model, in ToR, the average workload of the relay chain is*

$$W^{\circ}_{ToR} = \begin{cases} y_{ToR}, & \alpha \geq y_{ToR} \\ y_{ToR} + \dfrac{(d-1)}{2} * (y_{ToR} - \alpha), & \alpha < y_{ToR} \end{cases}$$

, where $y_{ToR} = \sum_{i=1}^{n} b_i$.

Proof. The relay chain can receive up to $y_{ToR} = \sum_{i=1}^{n} b_i$ transactions (i.e., block header synchronization) per second. Given that the relay chain's throughput is α, we can know W_{ToR} as follows:

$$\begin{aligned} W_{ToR} = \{W_{t_1} &= y_{ToR}, \\ W_{t_2} &= h(W_{t_1} - \alpha) + y_{ToR}, ..., \\ W_{t_d} &= h(W_{t_{d-1}} - \alpha) + y_{ToR}\}. \end{aligned}$$

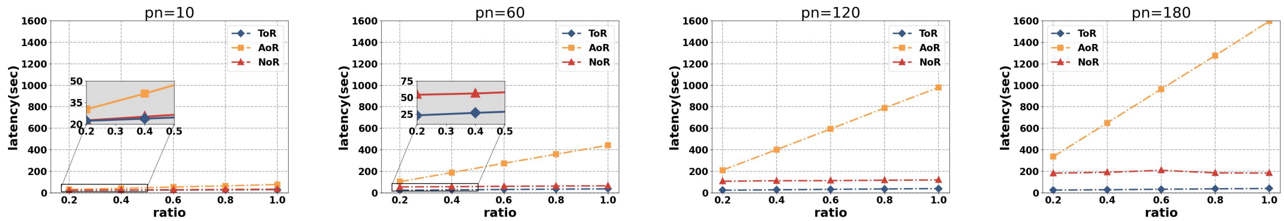
In terms of varying values of α,

i. if $\alpha \geq y_{ToR}$ then

$$W^{\circ}_{ToR} = W_{t_k} = W_{t_1} = y_{ToR},\ where\ k \in [1, d]$$

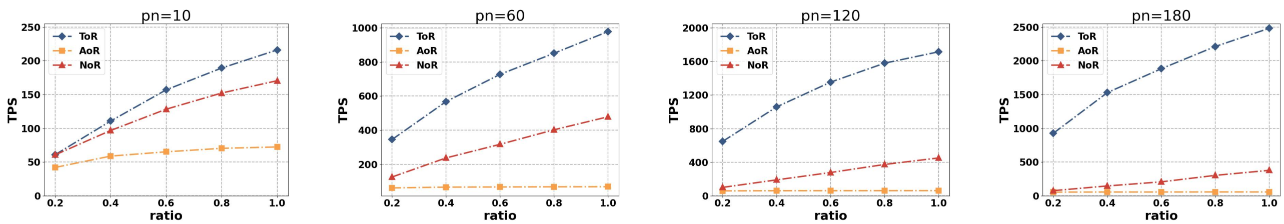
ii. if $\alpha < y_{ToR}$ then

$$W_{t_{k+1}} = W_{t_k} - \alpha + y_{ToR},\ where\ k \in [1, d-1]$$



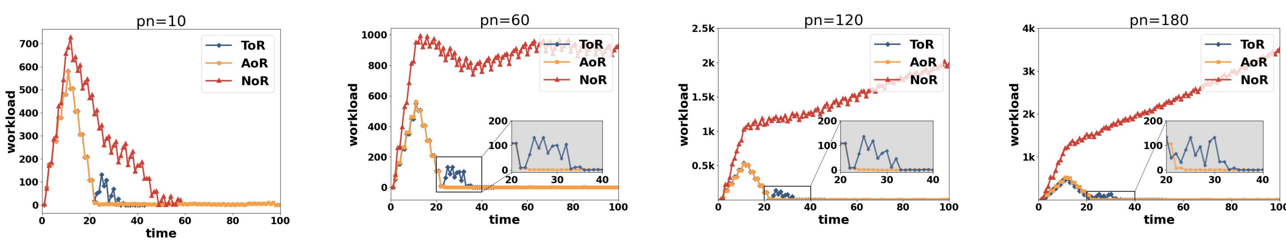
(a) (b) (c) (d)

Figure 1: In cross-chain systems with different parachain number, as increaing the ratio of cross-chain messages, it shows the latency from creating CM on the source to confirming it on the target.



(a) (b) (c) (d)

Figure 2: In cross-chain systems with different parachain number, it shows the TPS of this overall system as increaing the ratio of cross-chain messages.



(a) (b) (c) (d)

Figure 3: Parachain workload trends.

Thus, W_{ToR} forms an arithmetic sequence with a common difference of $\Delta = y_{ToR} - \alpha$. Therefore, it can be derived that:

$$W^{\circ}_{ToR} = y_{ToR} + \frac{(d-1)}{2} * (y_{ToR} - \alpha)$$

□

Lemma 2. *Based on the above model, in AoR, the average workload of the relay chain is*

$$W^{\circ}_{AoR} = \begin{cases} y_{AoR}, & \alpha \geq y_{AoR} \\ y_{AoR} + \frac{(d-1)}{2} * (y_{AoR} - \alpha), & \alpha < y_{AoR} \end{cases}$$

, where $y_{AoR} = \sum_{i=1}^{n} b_i + \sum_{i=1}^{n} \sum_{j=1}^{b_i} x_{i,j}$

Proof. the relay chain can receive up to $y_{AoR} = \sum_{i=1}^{n} b_i + \sum_{i=1}^{n} \sum_{j=1}^{b_i} x_{i,j}$ transactions (i.e., block header synchronization and cross-chain messages) per second. We can know W_{AoR} as follows:

$$\begin{aligned} W_{AoR} = \{W'_{t_1} &= y_{AoR}, \\ W'_{t_2} &= h(W'_{t_1} - \alpha) + y_{AoR}, \ldots, \\ W'_{t_d} &= h(W'_{t_{d-1}} - \alpha) + y_{AoR}\}. \end{aligned}$$

In terms of varying values of α,

i. if $\alpha < y_{AoR}$ then

$$W'_{t_{k+1}} = W'_{t_k} - \alpha + y_{AoR}, \text{ where } k \in [1, d-1].$$

Thus, W_{AoR} forms an arithmetic sequence with a common difference of $\Delta = y_{AoR} - \alpha$. Consequently, it follows that:

$$W^{\circ}_{AoR} = y_{AoR} + \frac{(d-1)}{2} * (y_{AoR} - \alpha).$$

ii. if $\alpha \geq y_{AoR}$ then

$$W^{\circ}_{AoR} = W^{'}_{t_k} = W^{'}_{t_1} = y_{AoR},\ where\ k \in [1, d-1]$$

Hence,

$$W^{\circ}_{AoR} = \begin{cases} y_{AoR}, & \alpha \geq y_{AoR} \\ y_{AoR} + \frac{(d-1)}{2} * (y_{AoR} - \alpha), & \alpha < y_{AoR} \end{cases}$$

□

Thereom 1. *Based on the above model, we have the average workload optimization ratio*

$$\tau \geq 1 - \frac{1}{\varsigma}$$

, where $\varsigma = \overline{x} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{b_i} x_{i,j}}{\sum_{i=1}^{n} b_i}$*, which refers to the average number of cross-chain messages in a parachain block.*

Proof. From Lemma 1 and Lemma 2, we can know τ as follows:

$$\tau = 1 - \frac{W^{\circ}_{ToR}}{W^{\circ}_{AoR}}$$

$$= 1 - \begin{cases} \dfrac{y_{ToR} + \frac{(d-1)}{2} * (y_{ToR} - \alpha)}{y_{AoR} + \frac{(d-1)}{2} * (y_{AoR} - \alpha)}, & \alpha < y_{ToR} \\ \dfrac{y_{ToR}}{y_{AoR} + \frac{(d-1)}{2} * (y_{AoR} - \alpha)}, & y_{ToR} \leq \alpha < y_{AoR} \\ \dfrac{y_{ToR}}{y_{AoR}}, & y_{AoR} \leq \alpha \end{cases}$$

Discuss by different conditions.

i. if $\alpha < y_{ToR}$ then

$$\tau = 1 - \frac{W^{\circ}_{ToR}}{W^{\circ}_{AoR}} = 1 - \frac{y_{ToR} - \frac{d-1}{d+1} * \alpha}{y_{AoR} - \frac{d-1}{d+1} * \alpha}$$

As the time series length d approaches sufficiently large, given $\lim_{d \to +\infty} \frac{d-1}{d+1} = 1$, we can consequently deduce that

$$\tau = 1 - \frac{y_{ToR} - \alpha}{y_{AoR} - \alpha}$$

Let $g = \frac{y_{ToR} - \alpha}{y_{AoR} - \alpha}$, then

$$g = \frac{1 - \frac{\alpha}{n\overline{b}}}{1 + \frac{\sum_{i=1}^{n}\sum_{j=1}^{b_i} x_{i,j}}{n\overline{b}} - \frac{\alpha}{n\overline{b}}} = \frac{1 - \frac{\alpha}{n\overline{b}}}{1 + \overline{x} - \frac{\alpha}{n\overline{b}}}$$

Let $v = 1 - \frac{\alpha}{n\overline{b}}$, then $g = \frac{1}{1 + \frac{\overline{x}}{v}}$.

It can be easily deduced that for $v \in (0, 1)$, $g(v)$ is monotonically increasing, and τ is monotonically decreasing. Given $\alpha \to 0$, τ approaches its minimum value $\frac{1}{1+\overline{x}}$.

Therefore, if $\alpha < y_{ToR}$, then $\tau > 1 - \frac{1}{\overline{x}}$.

ii. if $y_{ToR} \leq \alpha < y_{AoR}$, then

$$\tau = 1 - \frac{W^{\circ}_{ToR}}{W^{\circ}_{AoR}} = 1 - \frac{2 * y_{ToR}}{(d+1) * y_{AoR} - (d-1) * \alpha}$$

Let

$$g(a) = \frac{2 * y_{ToR}}{(d+1) * y_{AoR} - (d-1) * \alpha}$$

, we can know $g(a)$ is monotonically increasing, andτ is monotonically decreasing. Given $\alpha \to y_{AoR}$, τ approaches its minimum value $\frac{y_{ToR}}{y_{AoR}}$.

Let $\gamma = \frac{y_{ToR}}{y_{AoR}}$, then

$$\gamma = \frac{\sum_{i=1}^{n} b_i}{\sum_{i=1}^{n} b_i + \sum_{i=1}^{n}\sum_{j=1}^{b_i} x_{i,j}} = \frac{1}{1+\overline{x}}$$

Therefore, if $y_{ToR} \leq \alpha < y_{AoR}$, then $\tau > 1 - \frac{1}{\overline{x}}$.

iii. if $y_{AoR} \leq \alpha$ then $\tau = 1 - \frac{1}{\overline{x}}$.

In summary, from (i.,ii.,iii.), we have $\tau \geq 1 - \frac{1}{\overline{x}}$. □