

Collaboration may not be tantamount to success and human progress, but all forms of it have certainly had some collaboration. I trust that you do not need me to give you examples. Collaboration is something every person, every mathematician, has done within their life. This is especially true in the connected, digital world we live in. Collaboration within mathematics specifically is an interesting and difficult question, because of the rich history of the subject and its various nuances that come with it. Collaboration within the field has been talked about extensively from developments such as the PolyMath project, spearheaded by famous mathematicians such as Terrence Tao, to discussions about the integration of AI in the research level mathematics. Whether or not as a discipline we have a lack of it, is an interesting question in and of itself. My goal is to comment on this question and other things which surround it. The words I am going to say are a shadow of my ideas. I am purposefully casting them from multiple angles, but they are mostly from the perspective of someone who has mostly been “doing some mathematics”, but has thought about lots of other things very broadly. I believe both the concepts and the words themselves can be applied to different disciplines, and more broadly, different areas of life. It’s just that my motivation for framing things may be different from yours.

However before I go any further, I would like to introduce myself.

My name is Tora Ozawa. I’m 21 years old. I am not a professional mathematician. I am an undergraduate student studying mathematics and computer science. I’ve done a little research and technical writing in both areas, but have no official journal publications. I may be “wrong” on some of these things, and my viewpoint is naive and skewed. But regardless I’d like to develop and share my perspective on the landscape of mathematics as a practice, as a discipline, and most importantly, as a community and how I think we could improve. Simply put, I am trying to paint a picture. Certainly an incomplete one. Whether or not it has value is not for me to decide.

What is mathematics to you? How do you feel about it? If I were to ask these questions to many people I’m sure I would get many different answers. Some would say it’s a tool, many would say they hate it, and some would say it’s beautiful. The reason for this is inherent to life as a whole: the various experiences people will give them different insights and opinions. One cannot expect a unified answer, on a broad question which touches everyone’s lives. That being said, I believe the disparity is in at least small part due to how we present mathematics. That is not to say we should eliminate such dissonance and gaps, but rather to understand it and see how we can shape and leverage it. But what does this have to do with collaboration? It’s almost needless to say that how we operate and present ourselves, both society as a whole and the community of mathematics, has consequences. It’s almost needless to say that several factors and influences are intertwined. But I believe it is not emphasized enough explicitly. Moreover, I believe that the very same forces which have exacerbated the countless viewpoints of what mathematics is, are in large part accountable for the lack of collaboration within mathematics.

Lots of mathematics has been revolutionized by individuals. People like Euclid, Cauchy, Gauss, Galois, Riemann, Hilbert, Ramanujan, Turing, Erdos, Grothendieck, and many more come to mind. They usually have a few core ideas or techniques, apply them to some interesting problems, and try to generalize some existing phenomena. Their work is then discovered, valued and developed further by the individual in question, some larger group of contemporaries, or by mathematicians of the future. The history of mathematics can be said to be the interweaving of these individual narrative threads, working towards mathematical goals.

Students of mathematics then learn about these people in their classes. They hear about their theorems, their theories, and bit by bit get some insight into the history of their chosen discipline. More broadly the

general public has likely heard many of these names in passing as well. This ingrains into us that mathematics is done by these sorts of individuals, these geniuses if you will. We, as a society, revere these sorts of people, and romanticize their tales and work. We try to approach this ideal by immersing ourselves in the same way we believe the greats of the past and present have.

Likewise, we also often give credit to names. We name conjectures, we name, and we cite who proves things. And while this credit is in many cases certainly due, there are also lots of cases where what is due is not assigned. Especially for results dating the 20th century, it is often the case that they were discovered by earlier civilizations or people, but we give disproportionate credit to some cultures. Moreover, now speaking about the old and the new, we usually give credit to one or a couple of names. We name who posed a certain conjecture, we usually name an individual who solved it, giving credit to the person who does "the last step". Of course, this is in many senses inevitable. It would be silly to reference every single person who contributed to any single step of a mathematical result every time we invoke it. But we cannot deny the lasting effects it has: the mathematical community is more cutthroat and individualistic because of this phenomena. We feel so strongly about those of the past and present, because we feel that it gives our chosen pursuit meaning. We do this by telling their stories in grandiose ways.

We compete for being faster, for better grades, for graduate school, for better results, for better positions, and so on. Collaboration is in some sense antithetical to this. In one's own world, it would be ideal to be able to prove every statement one comes across. We would be able to instantly understand everything and anything. Within this fantastical setting, communicating with others is purely a demerit as it wastes time. The reality is that we strike a balance. When we encounter a concept we have trouble grappling with, or a problem we can't solve, we (sometimes) look towards what others have to say. The problem is many are not equipped with the best means to communicate and connect with other people and new ideas. Moreover most collaborations are limited to one's university affiliation. This is perhaps inevitable, but I think many could do a better job of advocating for attending conferences and also potentially pursuing online collaboration.

Mathematics can be lonely. Not everyone understands our strong attachment to the pursuit, and because of their strange impression of the subject, this can lead to ostracization. And not just for the mathematicians. It ultimately deters meaningful engagement with the subject. For example, graduate school is all about finding one's specialization. Once a student finds it, one usually tries to learn everything about a few particular things, however this usually results in less and less people to be able to work with. This can be extremely isolating. One grows distant not only from the rest of the world, but also the mathematical world. Social needs are especially important for those around my age, and some in mathematics, there is an everlasting deficiency. Moreover, because of the structure of PhD programs, where one has to work towards a dissertation, graduate students are further isolated by the need to get results which are entirely their own.

I think everyone should be made more aware of this: that lone mathematicians are not necessarily the way they are by choice. As for what to do, there is no silver bullet. We can't unravel our societal norms, and we certainly can't socioeconomic structures in a day. All we can do is give people more choices. We can urge people to work with their peers, at all levels, to reach out. Let them know, and understand that they're not alone. We can get people more open to collaboration, communication, and socialization through various venues, including some online platforms. For those who insist on an individualistic paradigm for mathematics, for simply continuing how things are now I have many questions. Shouldn't we at least have the option for a more collaborative mathematical setting? Or at the very least make the level of collaboration we have more efficient? Sure incentives are important but do the ones we have in place

now really produce the best results? Why is it not more normative to learn about the technologies and tools for collaboration? Why is it not normal for mathematicians to post about fledgling ideas, and only put polished results online?

If some more options did exist, then I think mathematics as a discipline would be much more inviting. Whether or not we take it, is up to us, individually.