

# FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Sunday 06<sup>th</sup> SEPTEMBER, 2020) TIME : 3 PM to 6 PM

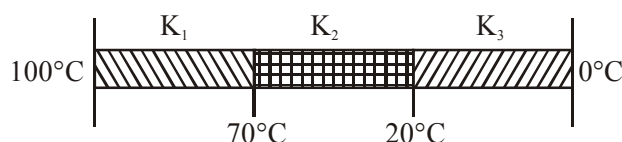
## PHYSICS

## TEST PAPER WITH ANSWER & SOLUTION

1. Two planets have masses  $M$  and  $16M$  and their radii are  $a$  and  $2a$ , respectively. The separation between the centres of the planets is  $10a$ . A body of mass  $m$  is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is :

- (1)  $\sqrt{\frac{GM^2}{ma}}$                       (2)  $\frac{3}{2}\sqrt{\frac{5GM}{a}}$   
 (3)  $4\sqrt{\frac{GM}{a}}$                       (4)  $2\sqrt{\frac{GM}{a}}$

2. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity  $K_1$ ,  $K_2$ , and  $K_3$ , respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at  $100^\circ\text{C}$  and the other at  $0^\circ\text{C}$  (see figure). If the joints of the rod are at  $70^\circ\text{C}$  and  $20^\circ\text{C}$  in steady state and there is no loss of energy from the surface of the rod, the correct relationship between  $K_1$ ,  $K_2$  and  $K_3$  is :



- (1)  $K_1 : K_3 = 2 : 3$ ;  $K_2 : K_3 = 2 : 5$   
 (2)  $K_1 < K_2 < K_3$   
 (3)  $K_1 : K_2 = 5 : 2$ ;  $K_1 : K_3 = 3 : 5$   
 (4)  $K_1 > K_2 > K_3$

3. For a plane electromagnetic wave, the magnetic field at a point  $x$  and time  $t$  is

$$\vec{B}(x, t) = [1.2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{k}] \text{ T}$$

The instantaneous electric field  $\vec{E}$  corresponding to  $\vec{B}$  is : (speed of light  $c = 3 \times 10^8 \text{ ms}^{-1}$ )

(1)  $\vec{E}(x, t) = [36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{k}] \frac{\text{V}}{\text{m}}$

(2)  $\vec{E}(x, t) = [-36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{j}] \frac{\text{V}}{\text{m}}$

(3)  $\vec{E}(x, t) = [36 \sin(1 \times 10^3 x + 0.5 \times 10^{11} t) \hat{j}] \frac{\text{V}}{\text{m}}$

(4)  $\vec{E}(x, t) = [36 \sin(1 \times 10^3 x + 1.5 \times 10^{11} t) \hat{j}] \frac{\text{V}}{\text{m}}$

4. A double convex lens has power  $P$  and same radii of curvature  $R$  of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power  $1.5 P$  is:

(1)  $\frac{R}{2}$

(2)  $2R$

(3)  $\frac{3R}{2}$

(4)  $\frac{R}{3}$

5. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit :

- (1) ammeter is always connected series and voltmeter in parallel.
- (2) Both, ammeter and voltmeter must be connected in series.
- (3) Both ammeter and voltmeter must be connected in parallel.
- (4) ammeter is always used in parallel and voltmeter is series.

6. A particle moving in the xy plane experiences a velocity dependent force  $\vec{F} = k(v_y \hat{i} + v_x \hat{j})$ , where  $v_x$  and  $v_y$  are the x and y components of its velocity  $\vec{v}$ . If  $\vec{a}$  is the acceleration of the particle, then which of the following statements is true for the particle ?

- (1) quantity  $\vec{v} \cdot \vec{a}$  is constant in time.
- (2) kinetic energy of particle is constant in time.
- (3) quantity  $\vec{v} \times \vec{a}$  is constant in time.
- (4)  $\vec{F}$  arises due to a magnetic field.

7. Consider the force  $F$  on a charge 'q' due to a uniformly charged spherical shell of radius  $R$  carrying charge  $Q$  distributed uniformly over it. Which one of the following statements is true for  $F$ , if 'q' is placed at distance  $r$  from the centre of the shell ?

- (1)  $F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$  for  $r > R$
- (2)  $\frac{1}{4\pi\epsilon_0} \frac{qQ}{R^2} > F > 0$  for  $r < R$
- (3)  $F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$  for all  $r$
- (4)  $F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{R^2}$  for  $r < R$

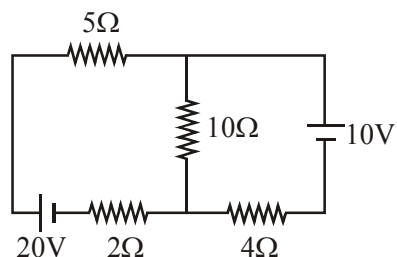
8. Given the masses of various atomic particles  $m_p = 1.0072u$ ,  $m_n = 1.0087u$ ,  $m_e = 0.000548u$ ,  $m_{\bar{\nu}} = 0$ ,  $m_d = 2.0141u$ , where  $p \equiv$  proton,  $n \equiv$  neutron,  $e \equiv$  electron,  $\bar{\nu} \equiv$  antineutrino and  $d \equiv$  deuteron. Which of the following process is allowed by momentum and energy conservation ?

- (1)  $n + p \rightarrow d + \gamma$
- (2)  $e^+ + e^- \rightarrow \gamma$
- (3)  $n + n \rightarrow$  deuterium atom  
(electron bound to the nucleus)
- (4)  $p \rightarrow n + e^+ + \bar{\nu}$

9. Two identical electric point dipoles have dipole moments  $\vec{p}_1 = p\hat{i}$  and  $\vec{p}_2 = -p\hat{i}$  and are held on the x axis at distance 'a' from each other. When released, they move along the x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm', their speed when they are infinitely far apart is:

- (1)  $\frac{p}{a} \sqrt{\frac{1}{\pi\epsilon_0 ma}}$
- (2)  $\frac{p}{a} \sqrt{\frac{3}{2\pi\epsilon_0 ma}}$
- (3)  $\frac{p}{a} \sqrt{\frac{1}{2\pi\epsilon_0 ma}}$
- (4)  $\frac{p}{a} \sqrt{\frac{2}{\pi\epsilon_0 ma}}$

10. In the figure shown, the current in the 10 V battery is close to :



- (1) 0.36 A from negative to positive terminal.
- (2) 0.71 A from positive to negative terminal.
- (3) 0.21 A from positive to negative terminal.
- (4) 0.42 A from positive to negative terminal.

- 11.** The linear mass density of a thin rod AB of length  $L$  varies from A to B as

$$\lambda(x) = \lambda_0 \left( 1 + \frac{x}{L} \right), \text{ where } x \text{ is the distance}$$

from A. If  $M$  is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is:

- (1)  $\frac{5}{12}ML^2$                       (2)  $\frac{3}{7}ML^2$   
 (3)  $\frac{2}{5}ML^2$                       (4)  $\frac{7}{18}ML^2$

- 12.** A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm; 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as :

- (1)  $(5.5375 \pm 0.0739) \text{ mm}$   
 (2)  $(5.538 \pm 0.074) \text{ mm}$   
 (3)  $(5.54 \pm 0.07) \text{ mm}$   
 (4)  $(5.5375 \pm 0.0740) \text{ mm}$

- 13.** When a particle of mass  $m$  is attached to a vertical spring of spring constant  $k$  and released, its motion is described by  $y(t) = y_0 \sin^2 \omega t$ , where ' $y$ ' is measured from the lower end of unstretched spring. Then  $\omega$  is :

- (1)  $\sqrt{\frac{g}{y_0}}$                       (2)  $\sqrt{\frac{g}{2y_0}}$   
 (3)  $\frac{1}{2}\sqrt{\frac{g}{y_0}}$                       (4)  $\sqrt{\frac{2g}{y_0}}$

14. In a dilute gas at pressure  $P$  and temperature  $T$ , the mean time between successive collisions of a molecule varies with  $T$  as :

- (1)  $\sqrt{T}$  (2)  $\frac{1}{T}$   
(3)  $\frac{1}{\sqrt{T}}$  (4)  $T$

15. A fluid is flowing through a horizontal pipe of varying cross-section, with speed  $v \text{ ms}^{-1}$  at a point where the pressure is  $P$  Pascal. At another point where pressure is  $\frac{P}{2}$  Pascal its speed is  $V \text{ ms}^{-1}$ . If the density of the fluid is  $\rho \text{ kg m}^{-3}$  and the flow is streamline, then  $V$  is equal to :

- (1)  $\sqrt{\frac{P}{2\rho} + v^2}$  (2)  $\sqrt{\frac{P}{\rho} + v^2}$   
(3)  $\sqrt{\frac{2P}{\rho} + v^2}$  (4)  $\sqrt{\frac{P}{\rho} + v}$

16. Assuming the nitrogen molecule is moving with r.m.s. velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to :  
(Given : nitrogen molecule weight :  $4.64 \times 10^{-26} \text{ kg}$ , Boltzman constant :  $1.38 \times 10^{-23} \text{ J/K}$ , Planck constant:  $6.63 \times 10^{-34} \text{ J.s}$ )
- (1)  $0.34 \text{ \AA}$  (2)  $0.24 \text{ \AA}$   
(3)  $0.20 \text{ \AA}$  (4)  $0.44 \text{ \AA}$

17. Particle A of mass  $m_1$  moving with velocity  $(\sqrt{3}\hat{i} + \hat{j})\text{ms}^{-1}$  collides with another particle B of mass  $m_2$  which is at rest initially. Let  $\vec{V}_1$  and  $\vec{V}_2$  be the velocities of particles A and B after collision respectively. If  $m_1 = 2m_2$  and after collision  $\vec{V}_1 = (\hat{i} + \sqrt{3}\hat{j})\text{ms}^{-1}$ , the angle between  $\vec{V}_1$  and  $\vec{V}_2$  is :

(1)  $60^\circ$       (2)  $15^\circ$       (3)  $-45^\circ$       (4)  $105^\circ$

18. A charged particle going around in a circle can be considered to be a current loop. A particle of mass  $m$  carrying charge  $q$  is moving in a plane with speed  $v$  under the influence of magnetic field  $\vec{B}$ . The magnetic moment of this moving particle :

(1)  $-\frac{mv^2\vec{B}}{B^2}$                       (2)  $-\frac{mv^2\vec{B}}{2\pi B^2}$   
 (3)  $\frac{mv^2\vec{B}}{2B^2}$                       (4)  $-\frac{mv^2\vec{B}}{2B^2}$

19. A square loop of side  $2a$  and carrying current  $I$  is kept in  $xz$  plane with its centre at origin. A long wire carrying the same current  $I$  is placed parallel to  $z$ -axis and passing through point  $(0, b, 0)$ , ( $b \gg a$ ). The magnitude of torque on the loop about  $z$ -axis is will be :

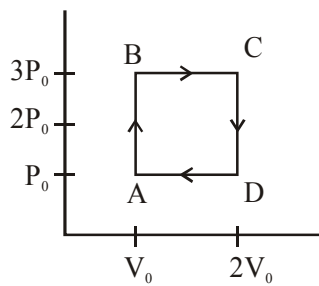
(1)  $\frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}$       (2)  $\frac{\mu_0 I^2 a^2 b}{2\pi(a^2 + b^2)}$

(3)  $\frac{\mu_0 I^2 a^2}{2\pi b}$       (4)  $\frac{2\mu_0 I^2 a^2}{\pi b}$

20. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed  $v$ , he sees that rain drops are coming at an angle  $60^\circ$  from the horizontal. On further increasing the speed of the car to  $(1 + \beta)v$ , this angle changes to  $45^\circ$ . The value of  $\beta$  is close to:
- (1) 0.41      (2) 0.50  
(3) 0.37      (4) 0.73

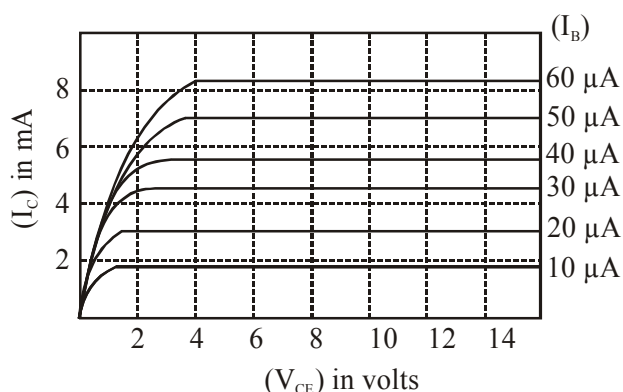


21. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to \_\_\_\_\_ .



22. The centre of mass of a solid hemisphere of radius 8 cm is X cm from the centre of the flat surface. Then value of x is \_\_\_\_\_ .

23. The output characteristics of a transistor is shown in the figure. When  $V_{CE}$  is 10 V and  $I_C = 4.0$  mA, then value of  $\beta_{ac}$  is \_\_\_\_\_ .



25. A Young's double-slit experiment is performed using monochromatic light of wavelength  $\lambda$ . The intensity of light at a point on the screen, where the path difference is  $\lambda$ , is K units. The intensity of light at a point where the path difference is  $\frac{\lambda}{6}$  is given by  $\frac{nK}{12}$ , where n is an integer. The value of n is \_\_\_\_\_ .

24. In a series LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as  $\left(\frac{n}{3\pi}\right)\mu\text{F}$ , then value of n is \_\_\_\_\_ .

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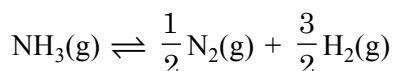
**CHEMISTRY**

**TEST PAPER WITH ANSWER & SOLUTION**

1. The value of  $K_C$  is 64 at 800 K for the reaction  

$$\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightleftharpoons 2\text{NH}_3(\text{g})$$

The value of  $K_C$  for the following reaction is :



- (1)  $\frac{1}{4}$       (2)  $\frac{1}{8}$       (3) 8      (4)  $\frac{1}{64}$

2. The element that can be refined by distillation is :

- (1) nickel                      (2) zinc  
 (3) gallium                    (4) tin

3. The correct match between **Item-I** and **Item-II** :

- | <b>Item-I</b>      | <b>Item-II</b>                      |
|--------------------|-------------------------------------|
| (a) Natural rubber | (I) 1, 3-butadiene + styrene        |
| (b) Neoprene       | (II) 1, 3-butadiene + acrylonitrile |
| (c) Buna-N         | (III) Chloroprene                   |
| (d) Buna-S         | (IV) Isoprene                       |
- (1) (a) - (III), (b) - (IV), (c) - (I), (d) - (II)  
 (2) (a) - (IV), (b) - (III), (c) - (II), (d) - (I)  
 (3) (a) - (IV), (b) - (III), (c) - (I), (d) - (II)  
 (4) (a) - (III), (b) - (IV), (c) - (II), (d) - (I)

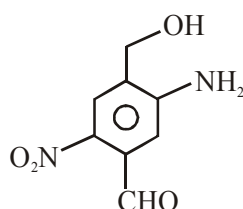
4. Mischmetal is an alloy consisting mainly of:

- (1) lanthanoid metals  
 (2) actinoid metals  
 (3) actinoid and transition metals  
 (4) lanthanoid and actinoid metals

5. Reaction of an inorganic sulphite X with dilute  $\text{H}_2\text{SO}_4$  generates compound Y. Reaction of Y with NaOH gives X. Further, the reaction of X with Y and water affords compound Z. Y and Z, respectively, are:

- (1) S and  $\text{Na}_2\text{SO}_3$   
 (2)  $\text{SO}_2$  and  $\text{NaHSO}_3$   
 (3)  $\text{SO}_3$  and  $\text{NaHSO}_3$   
 (4)  $\text{SO}_2$  and  $\text{Na}_2\text{SO}_3$

6. The IUPAC name of the following compound is :



- (1) 3-amino-4-hydroxymethyl-5-nitrobenzaldehyde
- (2) 2-nitro-4-hydroxymethyl-5-aminobenzaldehyde
- (3) 4-amino-2-formyl-5-hydroxymethylnitrobenzene
- (4) 5-amino-4-hydroxymethyl-2-nitrobenzaldehyde

7. Dihydrogen of high purity ( $> 99.95\%$ ) is obtained through:

- (1) the electrolysis of warm  $\text{Ba}(\text{OH})_2$  solution using Ni electrodes.
- (2) the reaction of Zn with dilute HCl
- (3) the electrolysis of brine solution.
- (4) the electrolysis of acidified water using Pt electrodes.

8. Match the following :

Test/Method	Reagent
(i) Lucas Test	(a) $\text{C}_6\text{H}_5\text{SO}_2\text{Cl/aq. KOH}$
(ii) Dumas method	(b) $\text{HNO}_3/\text{AgNO}_3$
(iii) Kjeldahl's method	(c) $\text{CuO}/\text{CO}_2$
(iv) Hinsberg Test	(d) Conc. HCl and $\text{ZnCl}_2$
	(e) $\text{H}_2\text{SO}_4$
(1) (i)-(d), (ii)-(c), (iii)-(e), (iv)-(a)	
(2) (i)-(b), (ii)-(d), (iii)-(e), (iv)-(a)	
(3) (i)-(d), (ii)-(c), (iii)-(b), (iv)-(e)	
(4) (i)-(b), (ii)-(a), (iii)-(c), (iv)-(d)	

9. The reaction of NO with  $\text{N}_2\text{O}_4$  at 250 K gives :

- (1)  $\text{N}_2\text{O}_5$
- (2)  $\text{NO}_2$
- (3)  $\text{N}_2\text{O}$
- (4)  $\text{N}_2\text{O}_3$

10. For the given cell ;

$\text{Cu(s)}|\text{Cu}^{2+}(\text{C}_1\text{M})||\text{Cu}^{2+}(\text{C}_2\text{M})|\text{Cu(s)}$  change in Gibbs energy ( $\Delta G$ ) is negative, if :

- (1)  $\text{C}_1 = 2\text{C}_2$
- (2)  $\text{C}_2 = \frac{\text{C}_1}{\sqrt{2}}$
- (3)  $\text{C}_1 = \text{C}_2$
- (4)  $\text{C}_2 = \sqrt{2}\text{C}_1$

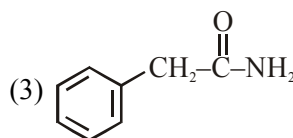
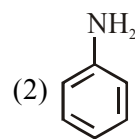
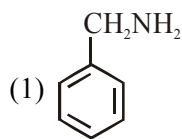
11. A crystal is made up of metal ions ' $M_1$ ' and ' $M_2$ ' and oxide ions. Oxide ions form a ccp lattice structure. The cation ' $M_1$ ' occupies 50% of octahedral voids and the cation ' $M_2$ ' occupies 12.5% of tetrahedral voids of oxide lattice. The oxidation numbers of ' $M_1$ ' and ' $M_2$ ' are, respectively :

- (1) +2, +4                      (2) +3, +1  
(3) +1, +3                      (4) +4, +2

12. For a  $d^4$  metal ion in an octahedral field, the correct electronic configuration is :

- (1)  $t_{2g}^4 e_g^0$  when  $\Delta_o < P$   
(2)  $e_g^2 t_{2g}^2$  when  $\Delta_o < P$   
(3)  $t_{2g}^3 e_g^1$  when  $\Delta_o < P$   
(4)  $t_{2g}^3 e_g^1$  when  $\Delta_o > P$

13. Which of the following compounds can be prepared in good yield by Gabriel phthalimide synthesis?



14. The correct match between **Item-I** (starting material) and **Item-II** (reagent) for the preparation of benzaldehyde is :

Item-I	Item-II
(I) Benzene	(P) HCl and $SnCl_2$ , $H_3O^+$
(II) Benzonitrile	(Q) $H_2$ , Pd-BaSO <sub>4</sub> , S and quinoline
(III) Benzoyl Chloride	(R) CO, HCl and $AlCl_3$
(1) (I)-(Q), (II)-(R) and (III)-(P)	
(2) (I)-(R), (II)-(Q) and (III)-(P)	
(3) (I)-(R), (II)-(P) and (III)-(Q)	
(4) (I)-(P), (II)-(Q) and (III)-(R)	

15. The average molar mass of chlorine is  $35.5 \text{ g mol}^{-1}$ . The ratio of  $^{35}\text{Cl}$  to  $^{37}\text{Cl}$  in naturally occurring chlorine is close to :

(1) 4 : 1  
(2) 1 : 1  
(3) 2 : 1  
(4) 3 : 1

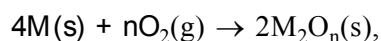
16. Which one of the following statements not true ?

(1) Lactose contains  $\alpha$ -glycosidic linkage between  $\text{C}_1$  of galactose and  $\text{C}_4$  of glucose.  
(2) Lactose ( $\text{C}_{11}\text{H}_{22}\text{O}_{11}$ ) is a disaccharide and it contains 8 hydroxyl groups.  
(3) On acid hydrolysis, lactose gives one molecule of D(+)-glucose and one molecule of D(+)-galactose.  
(4) Lactose is a reducing sugar and it gives Fehling's test.

17. A set of solutions is prepared using 180 g of water as a solvent and 10 g of different non-volatile solutes A, B and C. The relative lowering of vapour pressure in the presence of these solutes are in the order [Given, molar mass of A =  $100 \text{ g mol}^{-1}$ ; B =  $200 \text{ g mol}^{-1}$ ; C =  $10,000 \text{ g mol}^{-1}$ ]

(1)  $A > B > C$                       (2)  $A > C > B$   
(3)  $C > B > A$                       (4)  $B > C > A$

18. For a reaction,



the free energy change is plotted as a function of temperature. The temperature below which the oxide is stable could be inferred from the plot as the point at which :

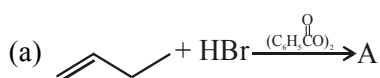
(1) the slope changes from positive to zero  
(2) the free energy change shows a change from negative to positive value  
(3) the slope changes from negative to positive  
(4) the slope changes from positive to negative

19. Match the following compounds (Column-I) with their uses (Column-II) :

S.No.	Column – I	S.No.	Column – II
(I)	$\text{Ca(OH)}_2$	(A)	casts of statues
(II)	$\text{NaCl}$	(B)	white wash
(III)	$\text{CaSO}_4 \cdot \frac{1}{2} \text{H}_2\text{O}$	(C)	antacid
(IV)	$\text{CaCO}_3$	(D)	washing soda preparation

- (1) (I)-(D), (II)-(A), (III)-(C), (IV)-(B)  
 (2) (I)-(B), (II)-(C), (III)-(D), (IV)-(A)  
 (3) (I)-(C), (II)-(D), (III)-(B), (IV)-(A)  
 (4) (I)-(B), (II)-(D), (III)-(A), (IV)-(C)

20. The increasing order of the boiling points of the major products A, B and C of the following reactions will be :



- (1)  $C < A < B$                       (2)  $B < C < A$   
 (3)  $A < B < C$                       (4)  $A < C < B$

21. For Freundlich adsorption isotherm, a plot of  $\log (x/m)$  (y-axis) and  $\log p$  (x-axis) gives a straight line. The intercept and slope for the line is 0.4771 and 2, respectively. The mass of gas, adsorbed per gram of adsorbent if the initial pressure is 0.04 atm, is  $\_\_\_\_\_\_ \times 10^{-4}\text{g}$ .

( $\log 3 = 0.4771$ )

22. A solution of phenol in chloroform when treated with aqueous NaOH gives compound P as a major product. The mass percentage of carbon in P is \_\_\_\_\_. (to the nearest integer)  
(Atomic mass : C = 12; H = 1; O = 16)
23. If the solubility product of  $AB_2$  is  $3.20 \times 10^{-11} M^3$ , then the solubility of  $AB_2$  in pure water is \_\_\_\_\_  $\times 10^{-4} \text{ mol L}^{-1}$ . [Assuming that neither kind of ion reacts with water]
24. The rate of a reaction decreased by 3.555 times when the temperature was changed from  $40^\circ\text{C}$  to  $30^\circ\text{C}$ . The activation energy (in  $\text{kJ mol}^{-1}$ ) of the reaction is \_\_\_\_\_.  
Take;  $R=8.314 \text{ J mol}^{-1} \text{ K}^{-1}$   $\ln 3.555 = 1.268$
25. The atomic number of Unnilunium is \_\_\_\_\_.



**FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020**

**(Held On Wednesday 06<sup>th</sup> SEPTEMBER, 2020) TIME : 3 PM to 6 PM**

**MATHEMATICS**

**TEST PAPER WITH SOLUTION**

1. The set of all real values of  $\lambda$  for which the function  $f(x) = (1 - \cos^2 x)(\lambda + \sin x)$ ,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , has exactly one maxima and

exactly one minima, is :

(1)  $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$       (2)  $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(3)  $\left(-\frac{3}{2}, \frac{3}{2}\right)$       (4)  $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

2. For all twice differentiable functions  $f : \mathbb{R} \rightarrow \mathbb{R}$ , with  $f(0) = f(1) = f'(0) = 0$

(1)  $f''(x) = 0$ , for some  $x \in (0, 1)$

(2)  $f''(0) = 0$

(3)  $f''(x) \neq 0$  at every point  $x \in (0, 1)$

(4)  $f''(x) = 0$  at every point  $x \in (0, 1)$

3. If the tangent to the curve,  $y = f(x) = x \log_e x$ , ( $x > 0$ ) at a point  $(c, f(c))$  is parallel to the line - segment joining the points  $(1, 0)$  and  $(e, e)$ , then  $c$  is equal to :

(1)  $\frac{1}{e-1}$

(2)  $e^{\left(\frac{1}{1-e}\right)}$

(3)  $e^{\left(\frac{1}{e-1}\right)}$

(4)  $\frac{e-1}{e}$

4. Consider the statement : "For an integer  $n$ , if  $n^3 - 1$  is even, then  $n$  is odd." The contrapositive statement of this statement is :

- (1) For an integer  $n$ , if  $n^3 - 1$  is not even, then  $n$  is not odd.  
(2) For an integer  $n$ , if  $n$  is even, then  $n^3 - 1$  is odd.  
(3) For an integer  $n$ , if  $n$  is odd, then  $n^3 - 1$  is even.  
(4) For an integer  $n$ , if  $n$  is even, then  $n^3 - 1$  is even.

5. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity  $e$  of the ellipse satisfies :

- (1)  $e^2 + 2e - 1 = 0$       (2)  $e^2 + e - 1 = 0$   
(3)  $e^4 + 2e^2 - 1 = 0$       (4)  $e^4 + e^2 - 1 = 0$

6. A plane  $P$  meets the coordinate axes at  $A$ ,  $B$  and  $C$  respectively. The centroid of  $\triangle ABC$  is given to be  $(1, 1, 2)$ . Then the equation of the line through this centroid and perpendicular to the plane  $P$  is :

(1)  $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

(2)  $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

(3)  $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

(4)  $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

7. If  $\alpha$  and  $\beta$  are the roots of the equation  $2x(2x + 1) = 1$ , then  $\beta$  is equal to :

(1)  $2\alpha^2$       (2)  $2\alpha(\alpha + 1)$

(3)  $-2\alpha(\alpha + 1)$       (4)  $2\alpha(\alpha - 1)$

8. Let  $z = x + iy$  be a non-zero complex number such that  $z^2 = iz|z|^2$ , where  $i = \sqrt{-1}$ , then  $z$  lies on the :

(1) imaginary axis      (2) real axis  
(3) line,  $y = x$       (4) line,  $y = -x$

9. The common difference of the A.P.  $b_1, b_2, \dots, b_m$  is 2 more than the common difference of A.P.  $a_1, a_2, \dots, a_n$ . If  $a_{40} = -159$ ,  $a_{100} = -399$  and  $b_{100} = a_{70}$ , then  $b_1$  is equal to :

(1) -127      (2) -81  
(3) 81      (4) 127

10. The angle of elevation of the summit of a mountain from a point on the ground is  $45^\circ$ . After climbing up one km towards the summit at an inclination of  $30^\circ$  from the ground, the angle of elevation of the summit is found to be  $60^\circ$ . Then the height (in km) of the summit from the ground is :

(1)  $\frac{1}{\sqrt{3}-1}$       (2)  $\frac{1}{\sqrt{3}+1}$   
(3)  $\frac{\sqrt{3}-1}{\sqrt{3}+1}$       (4)  $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

11. Let  $\theta = \frac{\pi}{5}$  and  $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ . If  $B = A$

+  $A^4$ , then  $\det(B)$  :

- (1) is one (2) lies in (1, 2)  
(3) is zero (4) lies in (2, 3)

12. For a suitably chosen real constant  $a$ , let a function,  $f : \mathbb{R} - \{-a\} \rightarrow \mathbb{R}$  be defined by

$$f(x) = \frac{a-x}{a+x}. \text{ Further suppose that for any real}$$

number  $x \neq -a$  and  $f(x) \neq -a$ ,  $(f \circ f)(x) = x$ . Then

$f\left(-\frac{1}{2}\right)$  is equal to :

(1)  $\frac{1}{3}$  (2) 3

(3) -3 (4)  $-\frac{1}{3}$

=

=

13. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $f(x) = \max\{x, x^2\}$ . Let  $S$  denote the set of all points in  $\mathbb{R}$ , where  $f$  is not differentiable. Then :
- (1)  $\{0, 1\}$  (2)  $\{0\}$   
(3)  $\emptyset$  (an empty set) (4)  $\{1\}$
14. The area (in sq. units) of the region enclosed by the curves  $y = x^2 - 1$  and  $y = 1 - x^2$  is equal to :
- (1)  $\frac{4}{3}$  (2)  $\frac{8}{3}$   
(3)  $\frac{16}{3}$  (4)  $\frac{7}{2}$
15. The probabilities of three events  $A$ ,  $B$  and  $C$  are given by  $P(A) = 0.6$ ,  $P(B) = 0.4$  and  $P(C) = 0.5$ . If  $P(A \cup B) = 0.8$ ,  $P(A \cap C) = 0.3$ ,  $P(A \cap B \cap C) = 0.2$ ,  $P(B \cap C) = \beta$  and  $P(A \cup B \cup C) = \alpha$ , where  $0.85 \leq \alpha \leq 0.95$ , then  $\beta$  lies in the interval:
- (1)  $[0.36, 0.40]$  (2)  $[0.35, 0.36]$   
(3)  $[0.25, 0.35]$  (4)  $[0.20, 0.25]$
16. if the constant term in the binomial expansion of  $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$  is 405, then  $|k|$  equals :
- (1) 2 (2) 1  
(3) 3 (4) 9

17. The integral  $\int_1^2 e^x \cdot x^x (2 + \log_e x) dx$  equal :

- (1)  $e(4e + 1)$                       (2)  $e(2e - 1)$   
(3)  $4e^2 - 1$                       (4)  $e(4e - 1)$

18. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point  $(-1, -4)$  in this line is :

- (1)  $\left(\frac{8}{5}, \frac{29}{5}\right)$                       (2)  $\left(\frac{29}{5}, \frac{11}{5}\right)$   
(3)  $\left(\frac{11}{5}, \frac{28}{5}\right)$                       (4)  $\left(\frac{29}{5}, \frac{8}{5}\right)$

19. If  $y = \left(\frac{2}{\pi}x - 1\right) \operatorname{cosec} x$  is the solution of the

differential equation,

$\frac{dy}{dx} + p(x)y = \frac{2}{\pi} \operatorname{cosec} x$ ,  $0 < x < \frac{\pi}{2}$ , then the function  $p(x)$  is equal to

- (1)  $\cot x$                                       (2)  $\tan x$   
(3)  $\operatorname{cosec} x$                                   (4)  $\sec x$

20. The centre of the circle passing through the point  $(0, 1)$  and touching the parabola  $y = x^2$  at the point  $(2, 4)$  is :

- (1)  $\left(\frac{3}{10}, \frac{16}{5}\right)$                       (2)  $\left(\frac{-16}{5}, \frac{53}{10}\right)$   
(3)  $\left(\frac{6}{5}, \frac{53}{10}\right)$                       (4)  $\left(\frac{-53}{10}, \frac{16}{5}\right)$

21. The sum of distinct values of  $\lambda$  for which the system of equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$$

has non-zero solutions, is \_\_\_\_\_.

22. Suppose that a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfies  $f(x + y) = f(x)f(y)$  for all  $x, y \in \mathbb{R}$  and

$$f(1) = 3. \text{ If } \sum_{i=1}^n f(i) = 363, \text{ then } n \text{ is equal to}$$

\_\_\_\_\_.

23. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is \_\_\_\_\_.

24. Consider the data on  $x$  taking the values  $0, 2, 4, 8, \dots, 2^n$  with frequencies  ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$  respectively. If the mean of this data is  $\frac{728}{2^n}$ , then  $n$  is equal to \_\_\_\_\_.
25. If  $\vec{x}$  and  $\vec{y}$  be two non-zero vectors such that  $|\vec{x} + \vec{y}| = |\vec{x}|$  and  $2\vec{x} + \lambda\vec{y}$  is perpendicular to  $\vec{y}$ , then the value of  $\lambda$  is \_\_\_\_\_.