

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021**(Held On Thursday 26th August, 2021)****TIME : 9 : 00 AM to 12 : 00 NOON****PHYSICS****TEST PAPER WITH SOLUTION****SECTION-A**

1. The fractional change in the magnetic field intensity at a distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at the centre of the same coil is :

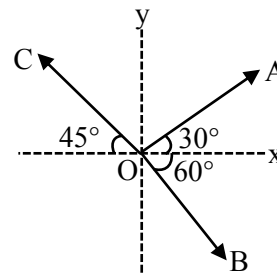
(Take $r < a$)

(1) $\frac{3}{2} \frac{a^2}{r^2}$

(2) $\frac{2}{3} \frac{a^2}{r^2}$

(3) $\frac{2}{3} \frac{r^2}{a^2}$

(4) $\frac{3}{2} \frac{r^2}{a^2}$



(1) $\tan^{-1} \frac{(1 - \sqrt{3} - \sqrt{2})}{(1 + \sqrt{3} + \sqrt{2})}$

(2) $\tan^{-1} \frac{(\sqrt{3} - 1 + \sqrt{2})}{(1 + \sqrt{3} - \sqrt{2})}$

(3) $\tan^{-1} \frac{(\sqrt{3} - 1 + \sqrt{2})}{(1 - \sqrt{3} + \sqrt{2})}$

(4) $\tan^{-1} \frac{(1 + \sqrt{3} - \sqrt{2})}{(1 - \sqrt{3} - \sqrt{2})}$

2. The magnitude of vectors \vec{OA} , \vec{OB} and \vec{OC} in the given figure are equal. The direction of $\vec{OA} + \vec{OB} - \vec{OC}$ with x-axis will be :-

3. Car B overtakes another car A at a relative speed of 40 ms^{-1} . How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A?

(1) 4 ms^{-1} (2) 0.2 ms^{-1}
 (3) 40 ms^{-1} (4) 0.1 ms^{-1}

4. Inside a uniform spherical shell :
- (a) the gravitational field is zero
 - (b) the gravitational potential is zero
 - (c) the gravitational field is same everywhere
 - (d) the gravitation potential is same everywhere
 - (e) all of the above

Choose the most appropriate answer from the options given below :

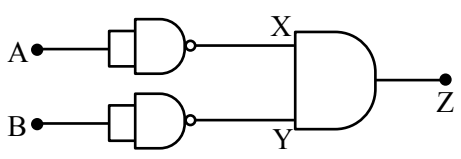
(1) (a), (c) and (d) only
 (2) (e) only
 (3) (a), (b) and (c) only
 (4) (b), (c) and (d) only

5. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube. [Take surface tension of water $T = 7.3 \times 10^{-2} \text{ Nm}^{-1}$, angle of contact = 0, $g = 10 \text{ ms}^{-2}$ and density of water = $1.0 \times 10^3 \text{ kg m}^{-3}$]

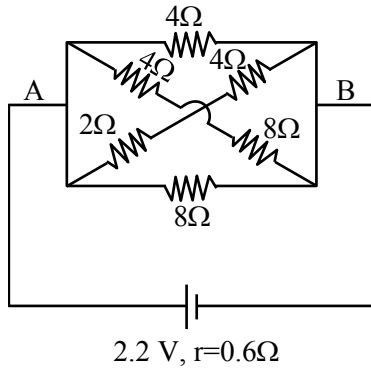
(1) 3.62 mm (2) 2.19 mm
 (3) 5.34 mm (4) 4.97 mm

6. An electric appliance supplies 6000 J/min heat to the system. If the system delivers a power of 90W. How long it would take to increase the internal energy by $2.5 \times 10^3 \text{ J}$?

(1) $2.5 \times 10^2 \text{ s}$ (2) $4.1 \times 10^1 \text{ s}$
 (3) $2.4 \times 10^3 \text{ s}$ (4) $2.5 \times 10^1 \text{ s}$

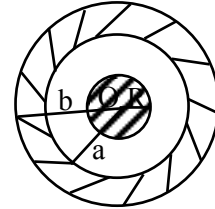
7. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds :
 (1) 0.4 (2) 0.8 (3) 0.125 (4) 0.2
8. A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance $R = 3 \text{ k}\Omega$, an inductor of inductive reactance $X_L = 250 \pi \Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be : (Take $\pi^2 = 10$)
 (1) 4 μF (2) 25 μF (3) 400 μF (4) 40 μF
9. Identify the logic operation carried out by the given circuit :-

 (1) OR (2) AND (3) NOR (4) NAND
10. A particular hydrogen like ion emits radiation of frequency $2.92 \times 10^{15} \text{ Hz}$ when it makes transition from $n = 3$ to $n = 1$. The frequency in Hz of radiation emitted in transition from $n = 2$ to $n = 1$ will be :
 (1) 0.44×10^{15} (2) 6.57×10^{15}
 (3) 4.38×10^{15} (4) 2.46×10^{15}
11. In a photoelectric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work function $\phi = 2.5 \text{ eV}$. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. ($h = 6.63 \times 10^{-34} \text{ Js}$, $c = 3 \times 10^8 \text{ ms}^{-1}$)
 (1) 1.3 V (2) 1.1 V (3) 1.9 V (4) 0.6 V

12. In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6Ω . Calculate the power dissipated in the whole circuit :



- (1) 1.32 W (2) 0.65 W
(3) 2.2 W (4) 4.4 W

13. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in figure. The approximate variation electric field \vec{E} as a function of distance r from centre O is given by



- (1)
- (2)
- (3)
- (4)

14. The rms speeds of the molecules of Hydrogen, Oxygen and Carbondioxide at the same temperature are V_H , V_O and V_C respectively then :
- (1) $V_H > V_O > V_C$ (2) $V_C > V_O > V_H$
 (3) $V_H = V_O > V_C$ (4) $V_H = V_O = V_C$

15. In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20th division of the circular scale coincides with reference line. Calculate the true reading.
- (1) 5.00 mm (2) 5.25 mm
 (3) 5.15 mm (4) 5.20 mm
16. What equal length of an iron wire and a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3Ω ?
- (Given resistivities of iron and copper-nickel alloy wire are $12 \mu\Omega \text{ cm}$ and $51 \mu\Omega \text{ cm}$ respectively)
- (1) 82 m (2) 97 m
 (3) 110 m (4) 90 m
17. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2} . The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket :[Use $g = 10 \text{ m/s}^2$]
- (1) $6.0 \times 10^2 \text{ kg s}^{-1}$ (2) 500 kg s^{-1}
 (3) 10 kg s^{-1} (4) 60 kg s^{-1}

18. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are :-

- (1) $[M^0 L^1 T^0]$ (2) $[M^{-1} L^{-1} T^2]$
 (3) $[M^1 L^1 T^{-2}]$ (4) $[M^0 L^0 T^0]$

19. The material filled between the plates of a parallel plate capacitor has resistivity $200 \Omega m$. The value of capacitance of the capacitor is 2 pF . If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is : (given the value of relative permittivity of material is 50)

- (1) $9.0 \mu A$ (2) 9.0 mA
 (3) 0.9 mA (4) $0.9 \mu A$

20. **Statement-I** : By doping silicon semiconductor with pentavalent material, the electrons density increases.

Statement-II : The n-type semiconductor has net negative charge.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Statement-I is true but Statement-II is false.
 (2) Statement-I is false but Statement-II is true.
 (3) Both Statement-I and Statement-II are true.
 (4) Both Statement-I and Statement-II are false.

SECTION-B

1. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is (Take $g = 10 \text{ m/s}^2$)

2. The electric field in a plane electromagnetic wave is given by

$$\vec{E} = 200 \cos \left[\left(\frac{0.5 \times 10^3}{\text{m}} \right) x - \left(1.5 \times 10^{11} \frac{\text{rad}}{\text{s}} \times t \right) \right] \frac{\text{V}}{\text{m}} \hat{j}$$

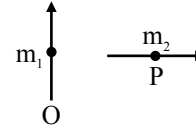
If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2 . If the radiation pressure exerted by the E.M. wave on the surface during a 10 minute exposure is $\frac{x}{10^9} \frac{\text{N}}{\text{m}^2}$. Find the value of x.

4. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of m. (Take $g = 10 \text{ m/s}^2$)

3. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be Hz.

5. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is cm.

7. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is $\times 10^{-7} \text{ Nm}$.

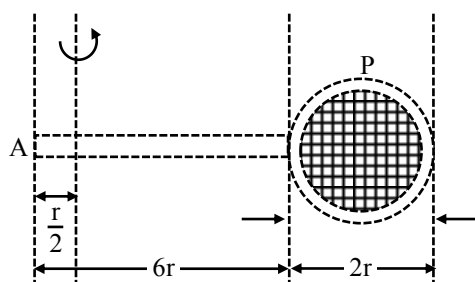


6. An amplitude modulated wave is represented by $C_m(t) = 10(1 + 0.2 \cos 12560t) \sin(111 \times 10^4 t)$ volts. The modulating frequency in kHz will be

8. Two travelling waves produces a standing wave represented by equation,
 $y = 1.0 \text{ mm} \cos(1.57 \text{ cm}^{-1})x \sin(78.5 \text{ s}^{-1})t$.
 The node closest to the origin in the region $x > 0$ will be at $x = \dots\dots\dots \text{ cm}$.

9. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and violet light is nm.

10. Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be Mr^2 .

FINAL JEE-MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021)

TIME : 9 : 00 AM to 12 : 00 NOON

CHEMISTRY

SECTION-A

1. Which one of the following complexes is violet in colour?

- (1) $[\text{Fe}(\text{CN})_6]^{4-}$ (2) $[\text{Fe}(\text{SCN})_6]^{4-}$
(3) $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3 \cdot \text{H}_2\text{O}$ (4) $[\text{Fe}(\text{CN})_5\text{NOS}]^{4-}$

2. Which one of the following is correct for the adsorption of a gas at a given temperature on a solid surface?

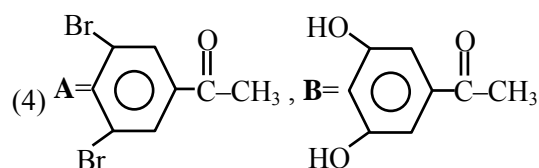
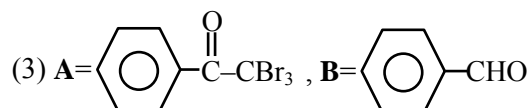
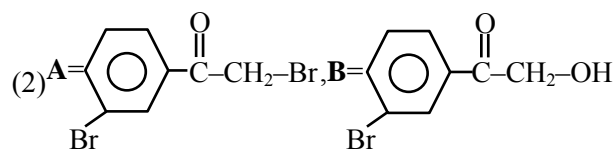
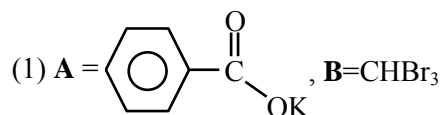
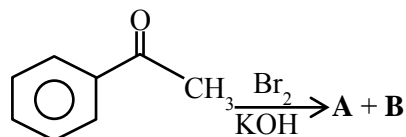
- (1) $\Delta H > 0$, $\Delta S > 0$ (2) $\Delta H > 0$, $\Delta S < 0$
(3) $\Delta H < 0$, $\Delta S < 0$ (4) $\Delta H < 0$, $\Delta S > 0$

3. Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere?

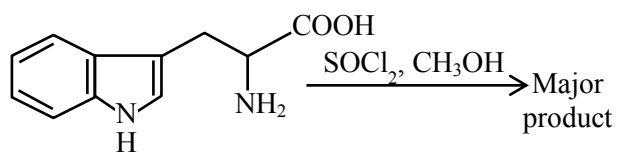
- (1) CuCl_2 (2) AgCl
(3) ZnCl_2 (4) Cu_2Cl_2

TEST PAPER WITH SOLUTION

4. The major products formed in the following reaction sequence **A** and **B** are :



5. The major product formed in the following reaction is :



- (1)
CC(C(N)C1=CC=C2C(=C1)N=C2C(=O)OC)ClCOC(=O)C1=CC=C2C(=C1)N=C2C(=O)O
- (2)
CC(C(N)C1=CC=C2C(=C1)N=C2C(=O)OC)ClCOC(=O)C1=CC=C2C(=C1)N=C2C(=O)O
- (3)
CC(C(N)C1=CC=C2C(=C1)N=C2C(=O)OC)ClCOC(=O)C1=CC=C2C(=C1)N=C2C(=O)O
- (4)
CC(C(N)C1=CC=C2C(=C1)N=C2C(=O)OC)ClCOC(=O)C1=CC=C2C(=C1)N=C2C(=O)O

6. The major product formed in the following reaction is :



- (1)
CC(C)(C)Br
- (2)
CC(C)(C)Br
- (3)
CC(C)(C)Br
- (4)
CC(C)(C)Br

7. The polymer formed on heating Novolac with formaldehyde is :

- (1) Bakelite (2) Polyester
(3) Melamine (4) Nylon 6,6

8. Given below are two statements :

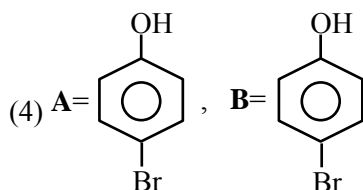
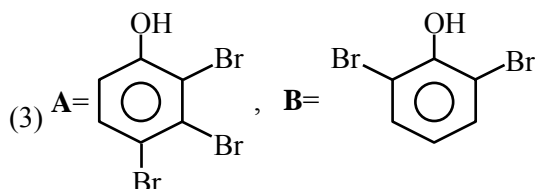
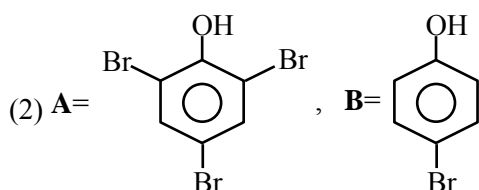
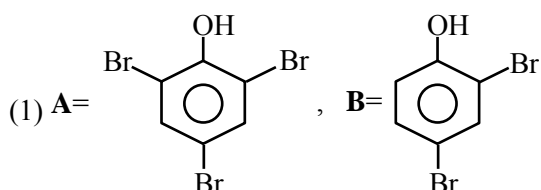
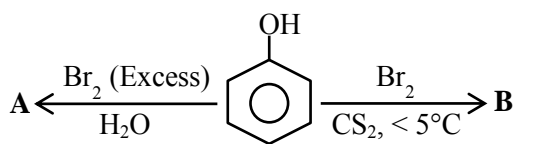
Statement I : The limiting molar conductivity of KCl (strong electrolyte) is higher compared to that of CH_3COOH (weak electrolyte).

Statement II : Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

- (1) **Statement I** is true but **Statement II** is false.
(2) **Statement I** is false but **Statement II** is true.
(3) Both **Statement I** and **Statement II** are true.
(4) Both **Statement I** and **Statement II** are false.

9. The correct options for the products **A** and **B** of the following reactions are :



Statement I: In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement II: For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

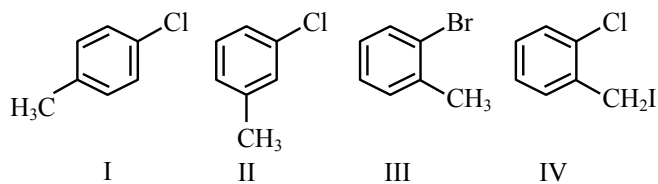
- (1) **Statement I** is false but **Statement II** is true
- (2) **Statement I** is true but **Statement II** is false
- (3) Both **Statement I** and **Statement II** are true
- (4) Both **Statement I** and **Statement II** are false

10. The conversion of hydroxyapatite occurs due to presence of F^- ions in water. The correct formula of hydroxyapatite is:

- (1) $[\text{3Ca}_3(\text{PO}_4)_2 \cdot \text{Ca}(\text{OH})_2]$
- (2) $[\text{3Ca}(\text{OH})_2 \cdot \text{CaF}_2]$
- (3) $[\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2]$
- (4) $[\text{3Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2]$

11. Given below are two statements.

12. Among the following compounds I-IV, which one forms a yellow precipitate on reacting sequentially with (i) NaOH (ii) dil. HNO_3 (iii) AgNO_3 ?



- (1) II
(3) I

- (2) IV
(4) III

13. Which one of the following methods is most suitable for preparing deionized water?

- (1) Synthetic resin method
(2) Clark's method
(3) Calgon's method
(4) Permutit method

14. Given below are two statements.

Statement I: The choice of reducing agents for metals extraction can be made by using Ellingham diagram, a plot of ΔG vs temperature.

Statement II: The value of ΔS increases from left to right in Ellingham diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Both **Statement I** and **Statement II** are true
(2) **Statement I** is false but **Statement II** is true
(3) Both **Statement I** and **Statement II** are false
(4) **Statement I** is true but **Statement II** is false

15. What are the products formed in sequence when excess of CO_2 is passed in slaked lime?

- (1) $\text{Ca}(\text{HCO}_3)_2$, CaCO_3
(2) CaCO_3 , $\text{Ca}(\text{HCO}_3)_2$
(3) CaO , $\text{Ca}(\text{HCO}_3)_2$
(4) CaO , CaCO_3

16. Given below are two statements.

Statement I: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement II: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Both **Statement I** and **Statement II** are false
(2) Both **Statement I** and **Statement II** are true
(3) **Statement I** is false but **Statement II** is true
(4) **Statement I** is true but **Statement II** is false

17. The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is:

- (1) $\text{Br}_2/\text{AlBr}_3$, $\text{HNO}_3/\text{H}_2\text{SO}_4$, Mg/ether , CO_2 , H_3O^+
- (2) $\text{Br}_2/\text{AlBr}_3$, NaCN , H_3O^+ , $\text{HNO}_3/\text{H}_2\text{SO}_4$
- (3) $\text{Br}_2/\text{AlBr}_3$, $\text{HNO}_3/\text{H}_2\text{SO}_4$, NaCN , H_3O^+
- (4) $\text{HNO}_3/\text{H}_2\text{SO}_4$, $\text{Br}_2/\text{AlBr}_3$, Mg/ether , CO_2 , H_3O^+

18. Given below are two statements.

Statement I: Frenkel defects are vacancy as well as interstitial defects.

Statement II: Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the **most appropriate** answer for the statements from the options given below:

- (1) **Statement I** is false but **Statement II** is true
- (2) Both **Statement I** and **Statement II** are true
- (3) **Statement I** is true but **Statement II** is false
- (4) Both **Statement I** and **Statement II** are false

19. The **incorrect** statement is:

- (1) Cl_2 is more reactive than ClF .
- (2) F_2 is more reactive than ClF .
- (3) On hydrolysis ClF forms HOCl and HF .
- (4) F_2 is a stronger oxidizing agent than Cl_2 in aqueous solution

20. Excess of isobutane on reaction with Br_2 in presence of light at 125°C gives which one of the following, as the major product?

- (1) $\text{CH}_3 - \overset{\text{Br}}{\underset{\text{CH}_3}{\text{C}}} - \text{CH}_2 - \text{Br}$
- (2) $\text{CH}_3 - \underset{\text{CH}_2\text{Br}}{\text{CH}} - \text{CH}_2\text{Br}$
- (3) $\text{CH}_3 - \underset{\text{CH}_3}{\text{CH}} - \text{CH}_2\text{Br}$
- (4) $\text{CH}_3 - \overset{\text{CH}_3}{\underset{\text{CH}_3}{\text{C}}} - \text{Br}$

SECTION-B

1. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _____. (Integer answer)

2. These are physical properties of an element

- (A) Sublimation enthalpy
- (B) Ionisation enthalpy
- (C) Hydration enthalpy
- (D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is _____ (Integer answer)

3. Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of 0.10 M $\text{C}_2\text{H}_5\text{OH}$ is _____

(Integer answer)

(i) 0.10 M $\text{Ba}_3(\text{PO}_4)_2$

(ii) 0.10 M Na_2SO_4

(iii) 0.10 M KCl

(iv) 0.10 M Li_3PO_4

4. The OH^- concentration in a mixture of 5.0 mL of 0.0504 M NH_4Cl and 2 mL of 0.0210 M NH_3 solution is $x \times 10^{-6}$ M. The value of x is _____.

(Nearest integer)

[Given $K_w = 1 \times 10^{-14}$ and $K_b = 1.8 \times 10^{-5}$]

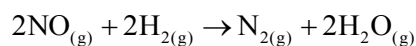
5. The number of 4f electrons in the ground state electronic configuration of Gd^{2+} is _____.

[Atomic number of Gd = 64]

6. The ratio of number of water molecules in Mohr's salt and potash alum is _____ $\times 10^{-1}$.

(Integer answer)

7. The following data was obtained for chemical reaction given below at 975 K.



[NO] mol L ⁻¹	[H ₂] mol L ⁻¹	Rate mol L ⁻¹ s ⁻¹
(A) 8×10^{-5}	8×10^{-5}	7×10^{-9}
(B) 24×10^{-5}	8×10^{-5}	2.1×10^{-8}
(C) 24×10^{-5}	32×10^{-5}	8.4×10^{-8}

The order of the reaction with respect to NO is _____. [Integer answer]

8. The Born-Haber cycle for KCl is evaluated with the following data :

$\Delta_f H$ for KCl = $-436.7 \text{ kJ mol}^{-1}$;

$\Delta_{\text{sub}} H$ for K = 89.2 kJ mol^{-1} ;

$\Delta_{\text{ionization}} H^\ominus$ for K = $419.0 \text{ kJ mol}^{-1}$; $\Delta_{\text{electron gain}} H^\ominus$ for $\text{Cl}_{(g)}$ = $-348.6 \text{ kJ mol}^{-1}$; $\Delta_{\text{bond}} H^\ominus$ for Cl_2 = $243.0 \text{ kJ mol}^{-1}$

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is _____ (Nearest integer)

9. The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be _____. (Integer answer)

10. An aqueous KCl solution of density 1.20 g mL^{-1} has a molality of 3.30 mol kg^{-1} . The molarity of the solution in mol L^{-1} is _____. (Nearest integer)
[Molar mass of KCl = 74.5]

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021**(Held On Thursday 26th August, 2021)****TIME : 9 : 00 AM to 12 : 00 NOON****MATHEMATICS****TEST PAPER WITH SOLUTION****SECTION-A**

1. The sum of solutions of the equation

$$\frac{\cos x}{1 + \sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\} \text{ is :}$$

(1) $-\frac{11\pi}{30}$

(2) $\frac{\pi}{10}$

(3) $-\frac{7\pi}{30}$

(4) $-\frac{\pi}{15}$

2. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :

(1) (11, 26)

(2) (10.5, 25)

(3) (11, 25)

(4) (10.5, 26)

3. On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2) \cdot A$ is :

(1) 6

(2) 12

(3) 14

(4) 24

4. Let $y = y(x)$ be a solution curve of the differential equation $(y + 1) \tan^2 x \, dx + \tan x \, dy + y \, dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim_{x \rightarrow 0^+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :

- (1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{4} - 1$
 (3) $\frac{\pi}{4} + 1$ (4) $\frac{\pi}{4}$

6. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$$(1 + \cos^2 \theta)x + \sin^2 \theta y + 4 \sin 3 \theta z = 0$$

$$\cos^2 \theta x + (1 + \sin^2 \theta) y + 4 \sin 3 \theta z = 0$$

$$\cos^2 \theta x + \sin^2 \theta y + (1 + 4 \sin 3 \theta) z = 0$$

has a non-trivial solution, then the value of θ is :

- (1) $\frac{4\pi}{9}$ (2) $\frac{7\pi}{18}$ (3) $\frac{\pi}{18}$ (4) $\frac{5\pi}{18}$

5. Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$. The largest value of p, for which $P(\text{exactly one of A, B occurs}) = \frac{5}{9}$, is :

- (1) $\frac{1}{3}$ (2) $\frac{2}{9}$
 (3) $\frac{4}{9}$ (4) $\frac{5}{12}$

7. Let $f(x) = \cos\left(2 \tan^{-1} \sin\left(\cot^{-1} \sqrt{\frac{1-x}{x}}\right)\right)$,

$0 < x < 1$. Then :

(1) $(1-x)^2 f'(x) - 2(f(x))^2 = 0$

(2) $(1+x)^2 f'(x) + 2(f(x))^2 = 0$

(3) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$

(4) $(1+x)^2 f'(x) - 2(f(x))^2 = 0$

8. The sum of the series

$$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \dots + \frac{2^{100}}{x^{2^{100}}+1} \text{ when } x = 2$$

is :

(1) $1 + \frac{2^{101}}{4^{101}-1}$ (2) $1 + \frac{2^{100}}{4^{101}-1}$

(3) $1 - \frac{2^{100}}{4^{100}-1}$ (4) $1 - \frac{2^{101}}{4^{101}-1}$

9. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of

$(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2 {}^{20}C_r$ is equal to :

(1) 420×2^{19}

(2) 380×2^{19}

(3) 380×2^{18}

(4) 420×2^{18}

10. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :

(1) {80, 83, 86, 89}

(2) {84, 86, 88, 90}

(3) {79, 81, 83, 85}

(4) {84, 87, 90, 93}

11. The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with:

(1) centre at $(0, -1)$ and radius $\sqrt{2}$
 (2) centre at $(0, 1)$ and radius $\sqrt{2}$
 (3) centre at $(0,0)$ and radius $\sqrt{2}$
 (4) centre at $(0,1)$ and radius 2

12. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$. If \vec{c} is a vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to :
- (1) -2 (2) -6 (3) 6 (4) 2

13. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is :

(1) 5 (2) 7 (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

14. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right)^{1/2} dx$ is:
- (1) $\log_e 4$ (2) $\log_e 16$
 (3) $2\log_e 16$ (4) $4\log_e (3 + 2\sqrt{2})$

15. A plane P contains the line
 $x + 2y + 3z + 1 = 0 = x - y - z - 6$,
 and is perpendicular to the plane $-2x + y + z + 8 = 0$.
 Then which of the following points lies on P ?

- (1) $(-1, 1, 2)$ (2) $(0, 1, 1)$
 (3) $(1, 0, 1)$ (4) $(2, -1, 1)$

16. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -2 & 1 \\ \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, and

$Q = A^T B A$, then the inverse of the matrix
 $A Q^{2021} A^T$ is equal to :

- (1) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$
 (3) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

17. If the sum of an infinite GP a, ar, ar^2, ar^3, \dots is 15
 and the sum of the squares of its each term is 150,
 then the sum of ar^2, ar^4, ar^6, \dots is :

- (1) $\frac{5}{2}$ (2) $\frac{1}{2}$
 (3) $\frac{25}{2}$ (4) $\frac{9}{2}$

18. The value of $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{n^2}{n^2 + 4r^2}$ is:

(1) $\frac{1}{2} \tan^{-1}(2)$ (2) $\frac{1}{2} \tan^{-1}(4)$

(3) $\tan^{-1}(4)$ (4) $\frac{1}{4} \tan^{-1}(4)$

19. Let ABC be a triangle with A(-3, 1) and $\angle ACB = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan \theta$ is equal to:

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) 2

20. If the truth value of the Boolean expression $((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)) \rightarrow (p \wedge q)$ is false, then the truth values of the statements p, q, r respectively can be:

(1) T F T (2) F F T

(3) T F F (4) F T F

SECTION-B

1. Let $z = \frac{1-i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of

$21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \dots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$
is _____.

3. Let the line L be the projection of the line

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$$

in the plane $x - 2y - z = 3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to _____.

2. The sum of all integral values of k ($k \neq 0$) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is _____.

4. If ${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \dots + 15 \cdot {}^{15}P_{15} = {}^qP_r - s$, $0 \leq s \leq 1$, then ${}^{q+s}C_{r-s}$ is equal to _____.

5. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi} + 1\right)k$ is equal to _____.

6. The area of the region
 $S = \{(x, y) : 3x^2 \leq 4y \leq 6x + 24\}$ is _____.

7. The locus of a point, which moves such that the sum of squares of its distances from the points $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$ is 18 units, is a circle of diameter d . Then d^2 is equal to _____.

8. If $y = y(x)$ is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to _____.

10. Let $a, b \in \mathbf{R}$, $b \neq 0$, Define a function

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x-1), & \text{for } x \leq 0 \\ \frac{\tan 2x - \sin 2x}{bx^3}, & \text{for } x > 0. \end{cases}$$

If f is continuous at $x = 0$, then $10 - ab$ is equal to

_____.

9. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is _____.