Date of Exam: 7th January (Shift II)

Time: 2:30 pm - 5:30 pm

Subject: **Physics**

1. A box weighs 196 N on a spring balance at the North Pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g=10~m/s^2$ at the North Pole and radius of the Earth = 6400 km)

- a. 194.32 N
- c. 195.32 *N*

- b. 194.66 *N*
- d. 195.66 N

2. In a building, there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be approximately

- a. 10 *A*
- c. 25 A

- b. 20 A
- d. 15 *A*

- 3. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently, the mean collision time between the gas molecules changes from τ_1 to τ_2 . If $\frac{C_p}{C_n} = \gamma$ for this gas, then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by

 - c. $\left(\frac{1}{2}\right)^{\gamma}$

- b. $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ d. 2

- 4. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals (Take $g = 10 \text{ m/s}^2$ and rope to be massless)
 - a. 100 *N*

b. 90 N

c. 75 N

d. 70 N

5. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is

a.
$$2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5}\right)$$

b.
$$2\pi a^4 \left(\frac{aA}{4} + \frac{B}{5}\right)$$

c.
$$\pi a^4 \left(\frac{A}{4} + \frac{aB}{5}\right)^2$$

d.
$$2\pi a^4 \left(\frac{A}{4} + \frac{B}{5}\right)^2$$

6. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures T_1 and T_2 . The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2 . T is the temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2 if both the engines perform equal amount of work?

a.
$$T = \frac{2T_1T_2}{T_1+T_2}$$

b.
$$T = \frac{T_1 + T_2}{2}$$

c.
$$T=0$$

d.
$$T = \sqrt{T_1 T_2}$$

7. The activity of a radioactive substance falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half-life is close to

a. 66 min

b. 62 min

c. 52 min

d. 72 min

8. In a Young's double slit experiment, the separation between the slits is $0.15 \, mm$. In the experiment, a source of light of wavelength $589 \, nm$ is used and the interference pattern is observed on a screen kept $1.5 \, m$ away. The separation between the successive bright fringes on the screen is

a. 5.9 mm

b. 3.9 mm

c. 6.9 mm

d. 4.9 mm

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9. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 *cm* and 4.8 *cm*, respectively. The ratio of minimum and maximum velocities of fluid in this pipe is

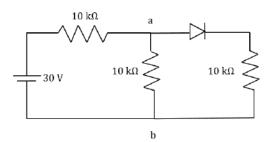
a.
$$\sqrt{\frac{3}{2}}$$

c.
$$\frac{3}{4}$$

b.
$$\frac{9}{16}$$

d.
$$\frac{3}{4}$$

10. In the figure, potential difference between a and b is



11. A particle of mass m and charge q has an initial velocity $\vec{v} = v_o \, \hat{\jmath}$. If an electric field $\vec{E} = E_0 \, \hat{\imath}$ and magnetic field $\vec{B} = B_0 \, \hat{\imath}$ act on the particle, its speed will double after a time

a.
$$\frac{\sqrt{3}mv_o}{qE_0}$$

C.
$$\frac{qE_0}{3mv_o}$$

b.
$$\frac{\sqrt{2}mv_o}{aE_0}$$

$$d. \frac{2mv_o}{2qE_0}$$

12. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one receded with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is $v_0 = 1400 \, Hz$ and the velocity of sound in air is $350 \, m/s$. The speed of each tuning fork is close to

a.
$$\frac{1}{4} m/s$$

c.
$$\frac{1}{2} m/s$$

d.
$$\frac{1}{8} m/s$$

13. An electron (of mass m) and a photon have the same energy E in the range of few eV. The ratio of the de Broglie wavelength associated with the electron and the wavelength of the photon is. (c = speed of light in vacuum)

a.
$$\left(\frac{E}{2m}\right)^{\frac{1}{2}}$$

b.
$$\frac{1}{c} \left(\frac{2E}{m}\right)^{\frac{1}{2}}$$

c.
$$c(2mE)^{\frac{1}{2}}$$

d.
$$\frac{1}{c} \left(\frac{E}{2m} \right)^{\frac{1}{2}}$$

14. A planar loop of wire rotates in a uniform magnetic field. Initially at t=0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of $10\ s$ about an axis in its plane, then the magnitude of induced emf will be maximum and minimum, respectively at

a. 2.5 sec and 5 sec

b. 5 sec and 7.5 sec

c. 2.5 sec and 7.5 sec

d. 5 sec and 10 sec

15. The electric field of a plane electromagnetic wave is given by $\vec{E}(t) = E_0 \frac{\hat{(\iota+j)}}{\sqrt{2}} \cos(kz + \omega t)$. At t=0, a positively charged particle is at the point $(x,y,z)=(0,0,\pi/k)$. If its instantaneous velocity at t=0 is $v_0 \hat{k}$, the force acting on it due to the wave is

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- a. zero
- c. parallel to $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

- b. antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$
- d. parallel to \hat{k}

- 16. A thin lens made of glass (refractive index = 1.5) of focal length $f = 16 \, cm$ is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_l , then the ratio f_l/f is closest to the integer
 - a. 9
 - c. 1

- b. 17
- d. 5

- 17. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being $68 \, kg$. The mass of the elevator itself is $920 \, kg$ and it moves with a constant speed of $3 \, m/s$. The frictional force opposing the motion is $6000 \, N$. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10 \, m/s^2$) must be at least
 - a. 66000 W

b. 63360 W

c. 48000 W

d. 56300 W

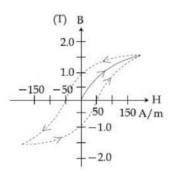
18. The figure gives experimentally measured B vs H variation in a ferromagnetic material. The retentivity, coercivity and saturation, respectively, of the material are

a. 1.5 T, 50 A/m, 1 T

b. 1 T, 50 A/m, 1.5 T

c. 1.5 T, 50 A/m, 1 T

d. 150 A/m, 1 T, 1.5 T



- 19. An emf of 20 V is applied at time t=0 to a circuit containing in series 10~mH inductor and $5~\Omega$ resistor. The ratio of the currents at time $t=\infty$ and t=40~s is close to (take $e^2=7.389$)
 - a. 1.06
 - c. 1.15

- b. 1.46
- d. 0.84

- 20. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is
 - a. $ML^{-1}T^{-2}$
 - c. MLT^{-2}

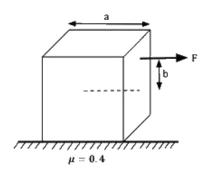
- b. ML^2T^{-2}
- d. ML^2T^{-1}

21. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) _____.

22. M grams of steam at $100^{o}C$ is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at $40^{o}C$ [heat of vaporization of water is $540 \ cal/g$ and heat of fusion of ice is $80 \ cal/g$], the value of M is _____.

23. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum value of $100 \times \frac{b}{a}$ for the box not to topple before moving is _____.

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24. The sum of two forces \vec{P} and \vec{Q} is \vec{R} such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of \vec{P} and \vec{Q} will make with \vec{Q} is _____

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25. The balancing length for a cell is $560 \ cm$ in a potentiometer experiment. When an external resistance of $10 \ \Omega$ is connected in parallel to the cell, the balancing length changes by $60 \ cm$. If the internal resistance of the cell is $\frac{N}{10} \ \Omega$, the value of N is _____

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Date: 7th January 2020

Time: 02:30 PM – 05:30 PM

Subject: Chemistry

1. Consider the following reactions:

A.

C.

+ CH₂=CH-Cl
$$\xrightarrow{\text{Anhy. AlCl}_3}$$
 CH=CH₂

+
$$CH_2$$
= CH - CH_2 CI $Anhy. AlCl_3$ CH_2 - CH = CH_2

Which of these reactions are possible?

- a. A and D
- c. B, C and D

- b. B and D
- d. A and B

2. In the following reaction sequence,

$$\begin{array}{c|c}
 & \text{NH}_2 \\
\hline
 & \text{Ac}_2\text{O} \\
\hline
 & \text{CH}_3
\end{array}$$

The major product B is:

a.

c.

b.

d.

3. For the following reactions,

$$\begin{array}{c} & k_{S} \\ \text{CH}_{3}\text{CH}_{2}\text{CH}_{2}\text{Br} + Z^{\ominus} \\ \hline \\ & k_{e} \\ \hline & \text{Elimination} \end{array} \rightarrow \begin{array}{c} \text{CH}_{3}\text{CH}_{2}\text{CH}_{2}Z + Br^{\ominus} \\ \text{CH}_{3}\text{CH}_{2}\text{CH}_{2} + Br^{\ominus} \\ \hline \\ & \text{Elimination} \end{array}$$

Where,

$$\overset{\bigcirc}{\text{Z=CH}_{3}\text{CH}_{2}\overset{\bigcirc}{\text{O}}}\text{(A) or } \overset{\text{CH}_{3}}{\text{H}_{3}\text{C-C-O}}\text{(B)},$$

 k_s and k_e , are, respectively, the rate constants for substitution and elimination, and $\mu = \frac{k_s}{k_e}$, the correct option is _____.

- a. $\mu_A > \mu_B$ and $k_e(A) > k_e(B)$
- b. $\mu_B > \mu_A$ and $k_e(A) > k_e(B)$
- c. $\mu_A > \mu_B$ and $k_e(B) > k_e(A)$
- d. $\mu_B > \mu_A$ and $k_e(B) > k_e(A)$

- 4. Which of the following statements is correct?
 - a. Gluconic acid can form cyclic (acetal/hemiacetal) structure
 - b. Gluconic acid is a dicarboxylic acid
 - c. Gluconic acid is obtained by oxidation of glucose with HNO₃
 - d. Gluconic acid is a partial oxidation product of glucose

5. The correct order of stability for the following alkoxides is:

$$NO_2$$

$$O_2N$$

- (A)
- (B)
- (C)
- a. (C) > (A) > (B)
- b. (C) > (B) > (A)

- c. (B) > (A) > (C)
- d. (B) > (C) > (A)

6. In the following reaction sequence, structures of A and B, respectively will be:

$$\frac{\text{HBr}}{\Delta} \Rightarrow A \xrightarrow{\text{Na}} B \text{ (intramolecular product)}$$

$$CH_{2}\text{-Br}$$

a.

c.

b.

d.

- 7. A chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (A) benzanilide, (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvents, hexane: ethyl acetate (20:80), the sequence of obtained compounds is:
 - a. (B), (A) and (C)
 - b. (C), (A) and (B)
 - c. (B), (C) and (A)
 - d. (A), (B) and (C)

8. The number of possible optical isomers for the complexes $[MA_2B_2]$ with sp^3 or dsp^2 hybridized metal atom, respectively, is:

Note: A and B are unidentate neutral and unidentate monoanionic ligands, respectively.

a. 0 and1

c. 2 and 2

b. 0 and 0

d. 0 and 2

- 9. The bond order and magnetic characteristics of CN⁻ are:
 - a. 3, paramagnetic

c. 3, diamagnetic

b. 2.5, diamagnetic

d. 2.5, paramagnetic

- 10. The equation that is incorrect is:

 - a. $\Lambda_m^0 \text{NaBr} \Lambda_m^0 \text{NaI} = \Lambda_m^0 \text{KBr} \Lambda_m^0 \text{NaBr}$ b. $\Lambda_m^0 \text{NaBr} \Lambda_m^0 \text{NaCl} = \Lambda_m^0 \text{KBr} \Lambda_m^0 \text{KCl}$ c. $\Lambda_m^0 \text{KCl} \Lambda_m^0 \text{NaCl} = \Lambda_m^0 \text{KBr} \Lambda_m^0 \text{NaBr}$ d. $\Lambda_m^0 \text{H}_2 \text{O} = \Lambda_m^0 \text{HCl} + \Lambda_m^0 \text{NaOH} \Lambda_m^0 \text{NaBr}$

11. In the following reactions, product (A) and (B), respectively, are:

$$NaOH + Cl_2 \rightarrow (A) + side products$$

(hot & conc.)

$$Ca(OH)_2 + Cl_2 \rightarrow (B) + side products$$

- a. NaClO₃ and Ca(ClO₃)₂
- b. NaOCl and Ca(ClO₃)₂
- c. NaOCl and Ca(OCl)₂
- d. NaClO₃ and Ca(OCl)₂

- 12. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non-volatile solute are together sealed in a container. Over time:
 - a. the volume of the solution and the solvent does not change
 - b. the volume of the solution increases and the volume of the solvent decreases
 - c. the volume of the solution decreases and the volume of the solvent increases
 - d. the volume of the solution does not change and the volume of the solvent decreases

13.	f 3. The refining method used when the metal and the impurities have $f k$	ow and	high	melting
	temperatures, respectively, is:			

a. vapour phase refining

b. distillation

c. liquation

d. zone refining

- 14. Among statements I-IV, the correct ones are:
 - I. Decomposition of hydrogen peroxide gives dioxygen
 - II. Like hydrogen peroxide, compounds, such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$ when heated liberate dioxygen.
 - $III. \ \ 2-Ethylanthraquinone\ is\ useful\ for\ the\ industrial\ preparation\ of\ hydrogen\ peroxide.$
 - IV. Hydrogen peroxide is used for the manufacture of sodium perborate

a. I,II, III and IV

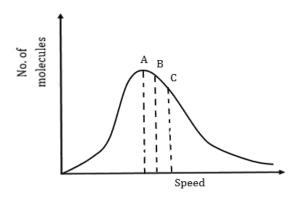
b. I, II and III only

c. I, III and IV only

d. I and III only

- 15. The redox reaction among the following is:
 - a. formation of ozone from atmospheric oxygen in the presence of sunlight
 - b. reaction of H₂SO₄ with NaOH
 - c. combination of dinitrogen with dioxygen at 2000 K
 - d. Reaction of $[Co(H_2O)_6]Cl_3$ with $AgNO_3$

16. Identify the correct labels of A, B and C in the following graph from the options given below:



Root mean square speed (V_{rms}) ; most probable speed (V_{mp}) ; average speed (V_{av})

a.
$$A = V_{mp}$$
, $B = V_{av}$, $C = V_{rms}$

b.
$$A = V_{mp}$$
, $B = V_{rms}$, $C = V_{av}$

c.
$$A = V_{av}$$
, $B = V_{rms}$, $C = V_{mp}$

d.
$$A = V_{rms}$$
, $B = V_{mp}$, $C = V_{av}$

17. For the reaction,

$$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$$

The observed rate expression is, rate = $k_f[NO]^2[H_2]$. The rate expression for the reverse reaction is:

- a. $k_b[N_2][H_20]^2$
- b. $k_b[N_2][H_20]$
- c. $k_b[N_2][H_20]^2/[H_2]$
- d. $k_b[N_2][H_20]^2/[N0]$

c

18. Within each pair of elements F & Cl, S & Se and Li & Na, respectively, the elements that release more energy upon an electron gain are:

a. Cl, Se and Na

b. Cl, S and Li

c. F, S and Li

d. F, Se and Na

- 19. Among the following statements A-D, the incorrect ones are:
 - A. Octahedral Co(III) complexes with strong field ligands have high magnetic moments
 - B. When Δ_0 < P, the d- electron configuration of Co(III) in an octahedral complex is t_{2g}^4 , e_g^2 .
 - C. Wavelength of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$.
 - D. If the Δ_o for an octahedral complex of Co(III) is 18000 cm⁻¹, the Δ_t for its tetrahedral complex with the same ligand will be16000 cm⁻¹.

a. B and C only

c. A and D only

b. A and B only

d. C and D only

- 20. The ammonia (NH₃) released on quantitative reaction of 0.6 g urea (NH₂CONH₂) with sodium hydroxide (NaOH) can be neutralized by:
 - a. 200 mL of 0.2 N HCl

c. 100 mL of 0.1 N HCl

b. 200 mL of 0.4 N HCl

d. 100 mL of 0.2 N HCl

21. Number of sp² hybrid carbon atoms present in aspartame is ____.

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22. 3 grams of acetic acid is added to 250 mL of 0.1 M HCl and the solution is made up to 500 mL. To 20 mL of this solution $\frac{1}{2}$ mL of 5 M NaOH is added. The pH of this solution is ____. (Given: log 3 = 0.4771, pK_a of acetic acid = 4.74, molar mass of acetic acid = 60 g/mole).

23. The flocculation value of HCl for As_2S_3 sol is 30 mmolL⁻¹. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams, of H_2SO_4 in 250 mL required for the above purpose is

24. Consider the following reactions :

 $NaCl + K_2Cr_2O_7 + H_2SO_4 \rightarrow (A) + side products$

- $(A) + NaOH \rightarrow (B) + side products$
- (B) + H_2SO_4 (dil.) + $H_2O_2 \rightarrow$ (C) + side products

The sum of the total number of atoms in one molecule of (A), (B) & (C) is $___$.

25. The standard heat of formation ($\Delta_f H_{298}^{\circ}$) of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are -1560, -393.5 and -286 kJ/mol, respectively, is_____.

7th January 2020 (Shift- 2), Chemistry

Date of Exam: 7th January 2020 (Shift 2)

Time: 2:30 P.M. to 5:30 P.M.

Subject: Mathematics

1. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$, for some $a \in R$ then the distance between the foci of the ellipse is :

a.
$$2\sqrt{5}$$

c.
$$2\sqrt{2}$$

b.
$$2\sqrt{7}$$

- 2. Let A, B, C and D be four non-empty sets. The Contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$ then $A \subseteq C$ " is :
 - a. If $A \subseteq C$, then $B \subseteq A$ or $D \subseteq B$
 - b. If $A \nsubseteq C$, then $A \subseteq B$ and $B \subseteq D$
 - c. If $A \nsubseteq C$, then $A \nsubseteq B$ and $B \subseteq D$
 - d. If $A \nsubseteq C$, then $A \nsubseteq B$ or $B \nsubseteq D$

- 3. The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+\cdots+x^{10}$ is :
 - a. 420

b. 330

c. 210

d. 120

- 4. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3$ k, then k is equal to :
 - a. $\frac{17}{2}$

b. 4

c. $\frac{17}{4}$

d. $\frac{17}{8}$

5. The locus of mid points of the perpendiculars drawn from points on the line x = 2y to the line x = y is :

a.
$$2x - 3y = 0$$

c.
$$5x - 7y = 0$$

b.
$$3x - 2y = 0$$

d.
$$7x - 5y = 0$$

6. The value of α for which $\,4\alpha\int_{-1}^{2}e^{-\alpha|x|}\,dx=$ 5, is :

a.
$$\log_e 2$$

b.
$$\log_e \sqrt{2}$$

c.
$$\log_e\left(\frac{4}{3}\right)$$

d.
$$\log_e\left(\frac{3}{2}\right)$$

- 7. If the sum of the first 40 terms of the series, $3+4+8+9+13+14+18+19+\ldots$ is (102)m, then m is equal to:
 - a. 10
 - c. 5

- b. 25
- d. 20

- 8. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0,2\pi]$, is a real number, then the argument of $\sin\theta+i\cos\theta$ is :
 - a. $\pi \tan^{-1}\left(\frac{4}{3}\right)$

b. $-\tan^{-1}\left(\frac{3}{4}\right)$

c. $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

d. $tan^{-1}\left(\frac{4}{3}\right)$

- 9. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is :
 - a. $\frac{1}{9}$

b. $\frac{1}{81}$

c. $\frac{1}{3}$

d. 3

- 10. Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x \to 0} \left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?
 - a. f(1) 4f(-1) = 4
 - b. x = 1 is a point of maxima and x = -1 is a point of minimum of f.
 - c. f is an odd function.
 - d. x = 1 is a point of minima and x = -1 is a point of maxima of f.

- 11. The number of ordered pairs (r, k) for which $6\cdot{}^{35}C_r=(k^2-3)\cdot{}^{36}C_{r+1}$, where k is an integer, is :
 - a. 4

b. 6

c. 2

d. 3

- 12. Let $a_1, a_2, a_3, ...$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^9 a_i = 4\lambda$, then λ is equal to :
 - a. 171

b. $\frac{511}{3}$

c. -171

d. -513

13. Let \vec{a} , \vec{b} and \vec{c} be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a}$. $\vec{b} + \vec{b}$. $\vec{c} + \vec{c}$. \vec{a} and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair (λ, \vec{d}) is equal to :

a.
$$\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$$

b.
$$\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$$

c.
$$\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$$

d.
$$\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$$

14. Let y = y(x) be the solution curve of the differential equation, $(y^2 - x) \frac{dy}{dx} = 1$, satisfying y(0) = 1This curve intersects the x – axis at a point whose abscissa is :

a.
$$2 + e$$

c.
$$2 - e$$

15. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0,2\pi)-\{\pi\}$ which satisfy the equation, $2\cot^2\theta-\frac{5}{\sin\theta}+4=0$, then $\int_{\theta_1}^{\theta_2}\cos^23\theta\ d\theta$ is equal to :

a.
$$\frac{2\pi}{3}$$

b.
$$\frac{\pi}{3}$$

c.
$$\frac{\pi}{3} + \frac{1}{6}$$

d.
$$\frac{\pi}{9}$$

16. Let α and β are the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \ge 1$ then which one of the following statements is not true?

a.
$$(p_1 + p_2 + p_3 + p_4 + p_5) = 26$$

b.
$$p_5 = 11$$

c.
$$p_5 = p_2 \cdot p_3$$

d.
$$p_3 = p_5 - p_4$$

17. The area (in sq. units) of the region $\{(x,y)\in R | 4x^2 \le y \le 8x+12\}$ is :

a.
$$\frac{125}{3}$$

b.
$$\frac{128}{3}$$

c.
$$\frac{124}{3}$$

d.
$$\frac{127}{3}$$

18. The value of c in Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, where $x \in [0,1]$ is :

a.
$$\frac{4-\sqrt{7}}{3}$$

c.
$$\frac{\sqrt{7}-2}{3}$$

b.
$$\frac{2}{3}$$

d.
$$\frac{4-\sqrt{5}}{3}$$

19. Let y=y(x) be a function of x satisfying $y\sqrt{1-x^2}=k-x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right)=-\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x=\frac{1}{2}$, is equal to :

a.
$$-\frac{\sqrt{5}}{2}$$

c.
$$-\frac{\sqrt{5}}{4}$$

b.
$$\frac{\sqrt{5}}{2}$$

d.
$$\frac{2}{\sqrt{5}}$$

20. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points *A* and *B*. The $(AB)^2$ is equal to :

a.
$$\frac{32}{5}$$

c.
$$\frac{52}{5}$$

b.
$$\frac{64}{5}$$

d.
$$\frac{56}{5}$$

21. If system of linear equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$3x + 2y + \lambda z = \mu$$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____.

22. If the foot of perpendicular drawn from the point (1,0,3) on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to _____.

23. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by $f(x) = \begin{cases} \left(\frac{1}{x}\right) \log_e\left(\frac{1+3x}{1-2x}\right) & when & x \neq 0 \\ k & , & when & x = 0 \end{cases}$ is continuous, the k is equal to ______.

24. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively then xy is equal to _____.

25. Let $X = \{n \in \mathbb{N}: 1 \le n \le 50\}$. If $A = \{n \in X: n \text{ is a multiple of 2}\}$ and $B = \{n \in X: n \text{ is a multiple of 7}\}$, then the number of elements in the smallest subset of X containing both A and B is _____.

7th January 2020 (Shift 1), Mathematics