10/04/2019
Morning

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question.

PART–A : PHYSICS

1. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$ about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is :

(1) $-\frac{I_1\omega_1^2}{12}$ (2) $\frac{3}{8}I_1\omega_1^2$

(3) $\frac{I_1\omega_1^2}{6}$ (4) $-\frac{I_1\omega_1^2}{24}$

Answer (4)

Sol. By applying conservation of angular momentum

$(I_1 + I_2)\,\omega_{common} = I_1\omega_1 + I_2\omega_2$

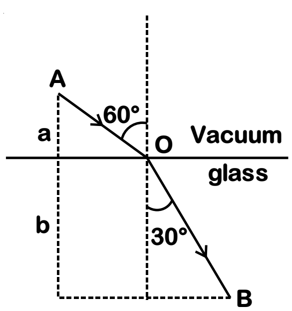
$$\omega_{common} = \frac{I_1\omega_1 + \frac{I_1\omega_1}{4}}{I_1 + \frac{I_1}{2}} = \left(\frac{5}{4}\times\frac{2}{3}\right)\omega_1$$

$$\omega_c = \frac{5\omega_1}{6}$$

$\therefore$ Loss in KE = $\left(\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2\right) - \frac{1}{2}(I_1 + I_2)\,\omega_c^2$

$\therefore$ $\Delta KE = -\frac{I_1\omega_1^2}{24}$

2. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is :

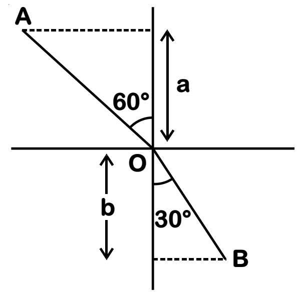


(1) $\frac{2\sqrt{3}}{a} + 2b$ (2) $2a + \frac{2b}{\sqrt{3}}$

(3) $2a + \frac{2b}{3}$ (4) 2a + 2b

Answer (4)

Sol. From the given figure



$\frac{a}{AO} = \cos 60°$

AO = 2a

$\frac{b}{BO} = \cos 30°$

$BO = \frac{2b}{\sqrt{3}}$

$\therefore$ Length of optical path = AO + BO × $\sqrt{3}$

= 2a + 2b

3. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is :

(1) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(2) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

(3) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(4) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

Answer (3)

Sol. Retardation of the particle

$a = -(g + \gamma v^2)$

$$\int_{v_0}^{0}\frac{-dv}{g+\gamma v^2} = \int_0^t dt \qquad [\text{for } H_{max}\ v = 0]$$

$$\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\frac{\sqrt{\gamma}\,v_0}{\sqrt{g}}\right) = t$$

4. A particle of mass m is moving along a trajectory given by

$x = x_0 + a \cos\omega_1 t$

$y = y_0 + b \sin\omega_2 t$

The torque, acting on the particle about the origin, at t = 0 is :

(1) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

(2) $m(-x_0 b + y_0 a)\omega_1^2 \hat{k}$

(3) $+my_0 a\omega_1^2 \hat{k}$

(4) Zero

Answer (3)

Sol. $x = x_0 + a \cos \omega_1 t$

$y = y_0 + b \sin \omega_2 t$

$\Rightarrow v_x = -a\omega_1 \sin(\omega_1 t), \quad v_y = b\omega_2 \cos(\omega_2 t)$

$a_x = -a\omega_1^2 \cos(\omega_1 t), \quad a_y = -b\omega_2^2 \sin(\omega_2 t)$

At t = 0, $\quad x = x_0 + a, \quad y = y_0$

$a_x = -a\omega_1^2, \quad a_y = 0$

$\therefore \quad \vec{\tau} = m(-a\omega_1^2) \times y_0(-\hat{k})$

$= +my_0 a\omega_1^2 \hat{k}.$

5. In a photoelectric effect experiment the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be :

Given E (in eV) = $\dfrac{1237}{\lambda(\text{in nm})}$

(1) 4.5 eV (2) 15.1 eV

(3) 3.0 eV (4) 1.5 eV

Answer (4)

Sol. Wavelength of incident wave (λ) = 260 nm

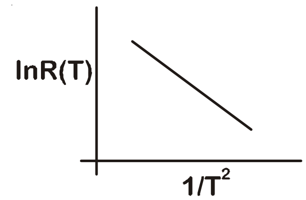
Cut off (threshold) wavelength (λ_0) = 380 nm

Then $\quad KE_{max} = \dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0}$

$= 1237\left[\dfrac{1}{260} - \dfrac{1}{380}\right]$

$= \dfrac{1237 \times 120}{380 \times 260} = 1.5 \text{ eV}$

6. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line.



One may conclude that :

(1) $R(T) = \dfrac{R_0}{T^2}$

(2) $R(T) = R_0 e^{-T_0^2/T^2}$

(3) $R(T) = R_0 e^{-T^2/T_0^2}$

(4) $R(T) = R_0 e^{T^2/T_0^2}$

Answer (2)

Sol. $\ln R(T) = a - \dfrac{a}{b} - \dfrac{1}{T^2}$

a, b are constant

$R(T) = R_0 e^{\frac{-T_0^2}{T^2}}$

7. An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is :

(1) 10^4

(2) 6×10^2

(3) 10^2

(4) 60

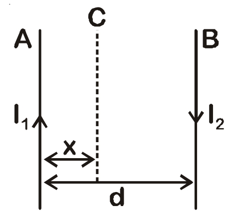
Answer (3)

Sol. $P_{gain} = \beta^2\left(\dfrac{R_{out}}{R_{in}}\right)$ & $I_{gain} = \beta$

$\therefore \quad 10^6 = \beta^2\left(\dfrac{10000}{100}\right)$

$\beta = 100$

8. Two wires A and B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are :



(1) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

(2) $x = \left(\frac{I_1}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 - I_2}\right)d$

(3) $x = \left(\frac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 - I_2}\right)d$

(4) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 + I_2}\right)d$

Answer (1)

Sol. $\Sigma\vec{F} = 0$

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(x-d)} \quad \text{since } (x > d)$$

$I_1x - I_1d = I_2x$

$$x = \frac{I_1 d}{I_1 - I_2}$$

9. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1},

(Given speed of sound = 300 m/s)

(1) 12, 16 (2) 16, 14

(3) 8, 18 (4) 12, 18

Answer (4)

Sol. Frequency of sound source (f_0) = 500 Hz

When observer is moving away from the source

Apparent frequency $f_1 = 480 = f_0\left(\frac{\nu - \nu_0'}{\nu}\right)$ ...(i)

And when observer is moving towards the source $f_2 = 530 = f_0\left(\frac{\nu + \nu_0''}{\nu}\right)$ ...(ii)

From equation (i)

$$480 = 500\left(\frac{300 - \nu_0'}{300}\right)$$

ν_0' = 12 m/s

From equation (ii)

$$530 = 500\left(1 + \frac{\nu_0''}{\nu}\right)$$

$\therefore$ ν_0'' = 18 m/s

10. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are :

(1) 440 V and 5 A (2) 220 V and 20 A

(3) 220 V and 10 A (4) 440 V and 20 A

Answer (1)

Sol. Power output (V_2I_2) = 2.2 kW

$$\therefore \quad V_2 = \frac{2.2\text{ kW}}{(10\text{A})} = 220 \text{ volts}$$

$\therefore$ Input voltage for step-down transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = 2$$

$V_{input} = 2 \times V_{output} = 2 \times 220$

= 440 V

Also $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

$$\therefore \quad I_1 = \frac{1}{2} \times 10 = 5\text{ A}$$

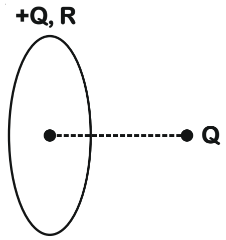
11. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at z = 4a. The minimum value of v such that it crosses the origin is :

(1) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (2) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(3) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (4) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

Answer (4)

Sol.



Potential at any point of the charged ring

$$V_P = \frac{KQ}{\sqrt{R^2+x^2}}$$

The minimum velocity (v_0) should just sufficient to reach the point charge at the center, therefore

$$\frac{1}{2}mv_0^2 = Q[V_C - V_P]$$

$$= Q\left[\frac{KQ}{3a} - \frac{KQ}{5a}\right]$$

$$v_0^2 = \frac{4KQ^2}{15\,ma} = \frac{4}{15}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{ma}$$

$$\therefore \quad v_0 = \sqrt{\frac{2}{m}}\left(\frac{2q^2}{15\times4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$$

12. n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is :

(1) $\frac{4nR}{C_V+nR}$ (2) $\frac{nR}{C_V+nR}$

(3) $\frac{4nR}{C_V-nR}$ (4) $\frac{nR}{C_V-nR}$

Answer (Bonus)

Sol. For Isobaric process

Work done (W) = $nR\Delta T$

and Heat given (Q) = $nC_P\Delta T$

$$\therefore \quad \frac{W}{Q} = \frac{R}{C_P} = \frac{R}{C_V+R}$$

13. A 25 × 10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule ?

(1) ~10^{12} (2) ~10^{10}

(3) ~10^{11} (4) ~10^{13}

Answer (2)

Sol. V = 25 × 10^{-3} m^3, N = 1 mole of O_2

T = 300 K

V_{rms} = 200 m/s

$$\because \quad \lambda = \frac{1}{\sqrt{2}N\pi r^2}$$

Average time $\frac{1}{\tau} = \frac{\langle V\rangle}{\lambda} = 200\cdot N\pi r^2\cdot\sqrt{2}$

$$= \frac{\sqrt{2}\times200\times6.023\times10^{23}}{25\times10^{-3}}\cdot\pi\times10^{-18}\times0.09$$

Average no. of collision $\approx 10^{10}$

14. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time :

(1) $\frac{1}{10\lambda}$ (2) $\frac{11}{10\lambda}$

(3) $\frac{1}{9\lambda}$ (4) $\frac{1}{11\lambda}$

Answer (3)

Sol. Number of nuclei present at any time t

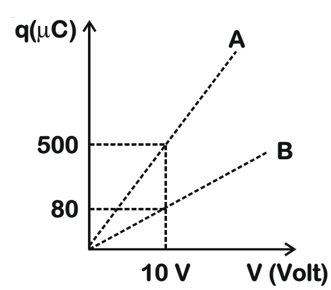
$N = N_0e^{-\lambda t}$

$$\therefore \quad \frac{N_A}{N_B} = e^{(\lambda_B-\lambda_A)t} = \frac{1}{e}$$

$(\lambda_A - \lambda_B)\cdot t = 1$

$$\therefore \quad t = \frac{1}{-\lambda+10\lambda} = \frac{1}{9\lambda}$$

15. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are :



(1) 40 μF and 10 μF (2) 20 μF and 30 μF

(3) 60 μF and 40 μF (4) 50 μF and 30 μF

Answer (1)

Sol. Equivalent capacitance for series combination

$$C' = \frac{C_1C_2}{C_1 + C_2}$$

For parallel combination $C'' = C_1 + C_2$

Also $C'' > C'$

$$C_1 + C_2 = \frac{500}{10} = 50\ \mu F$$

$$\text{and } \frac{C_1C_2}{C_1 + C_2} = \frac{80}{10} = 8\ \mu F$$

$\therefore$ $C_1C_2 = 400\ \mu F$

Solving $C_1 = 40\ \mu F$ $C_2 = 10\ \mu F$

16. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are :

(1) 4 ; 2×10^8 Hz
(2) 4 ; 1×10^8 Hz
(3) 0.25 ; 1×10^8 Hz
(4) 0.25 ; 2×10^8 Hz

Answer (4)

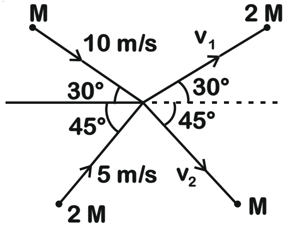
Sol. Range of frequency = $(f_c - f_m)$ to $(f_c + f_m)$

$\therefore$ Band width = $2f_m = 2 \times 100 \times 10^6$ Hz

$= 2 \times 10^8$ Hz

and Modulation index = $\frac{A_m}{A_C} = \frac{100}{400}$ = 0.25

17. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly :



(1) 3.2 m/s and 12.6 m/s
(2) 3.2 m/s and 6.3 m/s
(3) 6.5 m/s and 3.2 m/s
(4) 6.5 m/s and 6.3 m/s

Answer (4)

Sol. Apply conservation of linear momentum in X and Y direction for the system then

M(10cos30°) + 2M(5cos45°) = 2M ($v_1$cos30°) + M($v_2$cos45°)

$$5\sqrt{3} + 5\sqrt{2} = \sqrt{3}\,v_1 + \frac{v_2}{\sqrt{2}} \qquad ...(1)$$

Also

2M(5sin45°) – M(10sin30°) = 2M$v_1$sin30° – M$v_2$sin45°

$$5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \qquad ...(2)$$

Solving equation (1 and 2)

$$(\sqrt{3}+1)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5 \quad \Rightarrow \quad v_1 = 6.5\ \text{m/s}$$

v_2 = 6.3 m/s

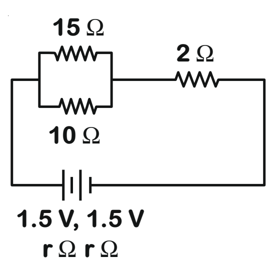
18. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0 – 5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0–10 mA is :

(1) 200 Ω
(2) 500 Ω
(3) 100 Ω
(4) 10 Ω

Answer (Bonus)

Sol. Data contradictory

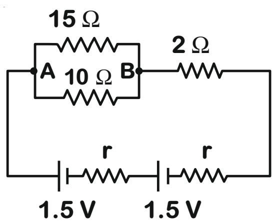
19. In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2 V. The internal resistance r, of each cell is :



(1) 0.5 Ω
(2) 0 Ω
(3) 1.5 Ω
(4) 1 Ω

Answer (1)

Sol. For the given circuit



Given that $V_{AB} = 2$ V

$$\therefore \quad I = \frac{2}{15} + \frac{2}{10} = \frac{1}{3} A$$

Also $\quad I(2r + 2) = 1.5 + 1.5 - V_{AB}$

$\Rightarrow \quad 2r + 2 = (3-2)3$

$$\Rightarrow \quad r = \frac{1}{2} \Omega$$

20. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m)

(1) 9.0×10^6 m
(2) 6.4×10^6 m
(3) 1.6×10^6 m
(4) 2.6×10^6 m

Answer (4)

Sol. Given

$$g_{height} = \frac{g_{surface}}{2} = 4.9\ m/s^2$$

$$\text{As} \quad g_h = g\left(1 + \frac{h}{R_e}\right)^{-2}$$

$$h = R_e\left(\sqrt{2} - 1\right)$$

$h = 6400 \times 0.414$

$h = 2649.6$ km

$h = 2.6 \times 10^6$ m

21. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to :

(1) 4/5
(2) 2/3
(3) 3/5
(4) 2/5

Answer (4)

Sol. Ratio of surface tension

$$\frac{S_{Hg}}{S_{Water}} = 7.5$$

$$\frac{\rho_{Hg}}{\rho_w} = 13.6 \ \& \ \frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135°}{\cos 0°} = \frac{1}{\sqrt{2}}$$

$$\frac{R_{Hg}}{R_{Water}} = \left(\frac{S_{Hg}}{S_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$= 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

22. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :

(1) $\frac{MR^2}{3}$
(2) $\frac{MR^2}{6}$
(3) $\frac{MR^2}{2}$
(4) $\frac{2MR^2}{3}$

Answer (4)

Sol. Surface mass density $(\sigma) = kr^2$

$$\text{Mass of disc } M = \int_0^R (kr^2)2\pi r dr$$

$$= 2\pi k\frac{R^4}{4} = \frac{\pi k R^4}{2}$$

$\therefore$ Moment of inertia about the axis of the disc.

$$I = \int dI = \int (dm)r^2 = \int \sigma dA r^2$$

$$= \int (Kr^2)(2\pi r dr)r^2$$

$$= \int_0^R 2\pi k\, r^5 dr = \frac{\pi k R^6}{3} = \frac{2}{3}MR^2$$

23. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J $mol^{-1}K^{-1}$]

(1) 700 J
(2) 350 J
(3) 374 J
(4) 748 J

Answer (4)

Sol. No. of moles of He at STP $= \frac{67.2}{22.4} = 3$

As the volume is constant $\rightarrow$ Isochoric proces

$$Q = nC_v\Delta T = 3\times\frac{3R}{2}\times 20 = 90R = 90 \times 8.31 \simeq 748\text{ J.}$$

24. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

(1) $r_e < r_p < r_{He}$ (2) $r_e > r_p = r_{He}$

(3) $r_e < r_p = r_{He}$ (4) $r_e > r_p > r_{He}$

Answer (3)

Sol. Radius of circular path (r) in a perpendicular uniform magnetic field $= \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

For proton, electron and α-particle,

$m_\alpha = 4m_p$ and $m_p >> m_e$

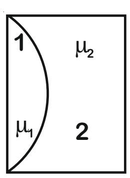
Also $q_\alpha = 2q_p$ and $q_p = q_e$

$\therefore$ As KE of all the particles is same then,

$$r \propto \frac{\sqrt{m}}{q}$$

$\therefore$ $r_\alpha = r_p > r_e$

25. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is :



(1) $\frac{R}{2(\mu_1 - \mu_2)}$

(2) $\frac{R}{2-(\mu_1 - \mu_2)}$

(3) $\frac{R}{\mu_1 - \mu_2}$

(4) $\frac{2R}{\mu_1 - \mu_2}$

Answer (3)

Sol. Focal length of plano-convex lens-

$$f_1 = \frac{R}{(\mu_1 - 1)}$$

Focal length of plano concave lens-

$$f_2 = \frac{-R}{(\mu_2 - 1)}$$

For the combination of two lens-

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R}$$

$$= \frac{\mu_1 - \mu_2}{R}$$

$$\therefore \quad f_{eq} = \frac{R}{\mu_1 - \mu_2}$$

26. The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0\hat{i}\cos(kz)\cos(\omega t)$

The corresponding magnetic field $\vec{B}$ is then given by :

(1) $\vec{B} = \frac{E_0}{C}\hat{j}\ \sin(kz)\cos(\omega t)$

(2) $\vec{B} = \frac{E_0}{C}\hat{j}\ \cos(kz)\sin(\omega t)$

(3) $\vec{B} = \frac{E_0}{C}\hat{j}\ \sin(kz)\sin(\omega t)$

(4) $\vec{B} = \frac{E_0}{C}\hat{k}\ \sin(kz)\cos(\omega t)$

Answer (3)

Sol. $\frac{E_0}{B_0} = C$

$$\therefore \quad B_0 = \frac{E_0}{C}$$

Given that $\vec{E} = E_0\cos(kz)\cos(\omega t)\hat{i}$

$$\vec{E} = \frac{E_0}{2}\left[\cos(kz - \omega t)\hat{i} - \cos(kz + \omega t)\hat{i}\right]$$

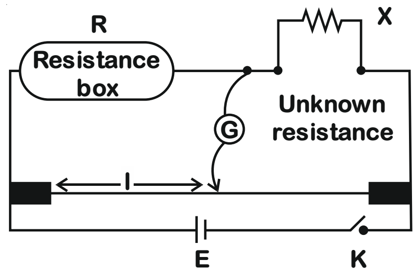
Correspondingly

$$\vec{B} = \frac{B_0}{2}\left[\cos(kz - \omega t)\hat{j} - \cos(kz + \omega t)\hat{j}\right]$$

$$\vec{B} = \frac{B_0}{2}\times 2\sin kz\ \sin\omega t$$

$$\vec{B} = \left(\frac{E_0}{C}\sin kz \sin\omega t\right)\hat{j}$$

27. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure.



Sl. No	R (Ω)	l (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent ?

(1) 3 (2) 2

(3) 1 (4) 4

Answer (4)

Sol. $\frac{R}{X} = \frac{l}{100 - l}$

Using the above expression

$$X = \frac{R(100 - l)}{l}$$

for case (a) $x = \frac{100 \times 40}{60} = \frac{2000}{3}\ \Omega$

for case (b) $x = \frac{100 \times 87}{13} = \frac{8700}{13}\ \Omega$

for case (c) $x = \frac{10 \times 98.5}{1.5} = \frac{1970}{3}\ \Omega$

for case (d) $x = \frac{1 \times 99}{1} = 99\ \Omega$

Clearly we can see that the value of x calculate in case (d) is inconsistent than other cases.

28. A current of 5 A passes through a copper conductor (resistivity = $1.7 \times 10^{-8}\ \Omega$ m) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

(1) 1.3 m^2/Vs

(2) 1.8 m^2/Vs

(3) 1.5 m^2/Vs

(4) 1.0 m^2/Vs

Answer (4)

Sol. Mobility $(\mu) = \frac{v_d}{E}$

and resistivity $(\rho) = \frac{E}{j} = \frac{EA}{I}$

$$\therefore \quad \mu = \frac{v_d A}{i\rho}$$

$$= \frac{1.1 \times 10^{-3} \times \pi \times (5 \times 10^{-3})^2}{5 \times 1.7 \times 10^{-8}}$$

$$\mu = 1.0 \frac{m^2}{Vs}$$

29. Given below in the left column are different modes of communication using the kinds of waves given in the right column.

A. Optical Fibre Communication	P. Ultrasound
B. Radar	Q. Infrared Light
C. Sonar	R. Microwaves
D. Mobile Phones	S. Radio Waves

From the options given below, find the most appropriate match between entries in the left and the right column.

(1) A-Q, B-S, C-P, D-R

(2) A-Q, B-S, C-R, D-P

(3) A-S, B-Q, C-R, D-P

(4) A-R, B-P, C-S, D-Q

Answer (1)

Sol. Optical Fibre Communication – Infrared Light

Radar – Radio Waves

Sonar – Ultrasound

Mobile Phones – Microwaves

30. The displacement of a damped harmonic oscillator is given by
$x(t) = e^{-0.1t} \cos(10\pi t + \phi)$. Here t is in seconds.
The time taken for its amplitude of vibration to drop to half of its initial value is close to :

(1) 7 s

(2) 27 s

(3) 13 s

(4) 4 s

Answer (1)

Sol. Amplitude at (t = 0) $A_0 = e^{-0.1 \times 0} = 1$

$\therefore$ at t = t if $A = \frac{A_0}{2}$

$$\Rightarrow \frac{1}{2} = e^{-0.1t}$$

$t = 10 \ln 2 \simeq 7$ s

PART–B : CHEMISTRY

1. Consider the following statements

   (a) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3.

   (b) Ionic product of water is temperature dependent.

   (c) A monobasic acid with $K_a = 10^{-5}$ has a pH = 5. The degree of dissociation of this acid is 50%.

   (d) The Le Chatelier's principle is not applicable to common-ion effect.

   The correct statements are :

   (1) (a), (b) and (d)

   (2) (b) and (c)

   (3) (a) and (b)

   (4) (a), (b) and (c)

Answer (4)

Sol. (a) $H_2SO_4 + NaOH \rightarrow NaHSO_4 + H_2O$

	H_2SO_4	NaOH	$NaHSO_4$	H_2O
Initial moles	0.04	0.04		
	0	0	0.04	0.04

$NaHSO_4 \rightarrow Na^+ + H^+ + SO_4^{2-}$

$$[H^+] = \frac{0.04}{0.80} = 0.05\text{ M; pH} = 1.3$$

(b) Ionic product of water increases with increase of temperature because ionisation of water is endothermic.

(c) $HA \rightleftharpoons H^+ + A^-$

$C(1-\alpha) \quad C\alpha \quad C\alpha \quad pH = 5\ \&\ K_a = 10^{-5}$

$$10^{-5} = \frac{C\alpha^2}{1-\alpha}; C = 2\times10^{-5}\text{ and }\alpha = 0.5$$

2. The oxoacid of sulphur that does not contain bond between sulphur atoms is :

   (1) $H_2S_4O_6$

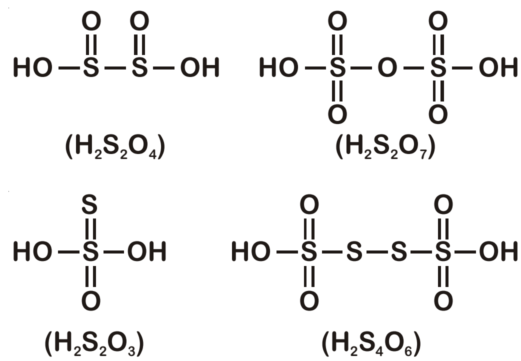
   (2) $H_2S_2O_4$

   (3) $H_2S_2O_7$

   (4) $H_2S_2O_3$

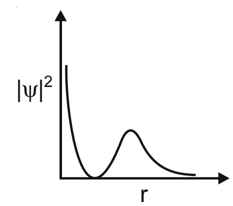
Answer (3)

Sol.



$H_2S_2O_7$ does not have S – S linkage

3. The graph between $|\psi|^2$ and r(radial distance) is shown below. This represents :

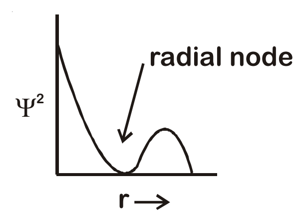


   (1) 1s orbital (2) 2s orbital

   (3) 2p orbital (4) 3s orbital

Answer (2)

Sol. The given probability density curve is for 2s orbital because it has only one radial node. Among other given orbitals, 1s and 2p do not have any radial node and 3s has two radial nodes.



4. Which of the following is a condensation polymer ?

   (1) Nylon 6, 6

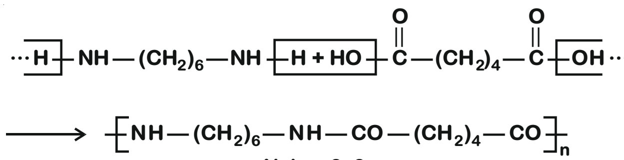
   (2) Teflon

   (3) Buna - S

   (4) Neoprene

Answer (1)

Sol. Nylon 6, 6 is obtained by condensation polymerisation of hexamethylenediamine and adipic acid



Nylon 6, 6

So, Nylon 6, 6 is a condensation polymer. Other polymers given, i.e., Buna-S, Teflon and Neoprene are addition polymers.

5. The principle of column chromatography is :

(1) Differential adsorption of the substances on the solid phase.

(2) Gravitational force.

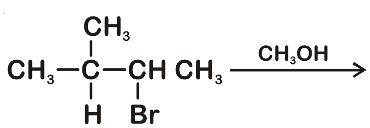
(3) Differential absorption of the substances on the solid phase.

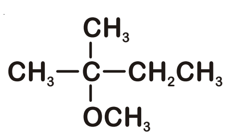
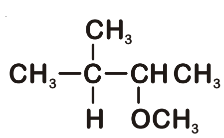
(4) Capillary action.

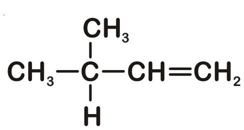
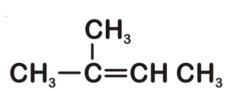
Answer (1)

Sol. In column chromatograph a solid adsorbent is packed in a column and a solution containing number of solute particles is allowed to flow down the column. The solute molecules get adsorbed on the surface of adsorbent. So it is differential adsorption of the substances on the solid phase.

6. The major product of the following reaction is :

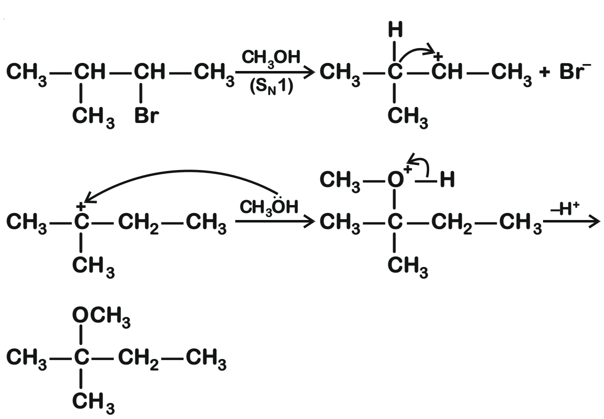


(1)  (2) 

(3)  (4) 

Answer (1)

Sol.



7. Consider the statements S1 and S2 :

S1 : Conductivity always increases with decrease in the concentration of electrolyte.

S2 : Molar conductivity always increases with decrease in the concentration of electrolyte.

The correct option among the following is :

(1) S1 is wrong and S2 is correct

(2) S1 is correct and S2 is wrong

(3) Both S1 and S2 are wrong

(4) Both S1 and S2 are correct

Answer (1)

Sol. Conductivity of an electrolyte is the conductance of 1 cm^3 of the given electrolyte. So, it increases with the increase of concentration of electrolyte. Molar conductivity (λ_m) is the conductance of a solution containing 1 mole of the electrolyte. It increases with the decrease of concentration (i) due to increase in interionic attraction for strong electrolytes and (ii) due to decrease in degree of ionisation for weak electrolytes. Therefore, (S_1) is wrong and (S_2) is correct.

8. Consider the following table :

Gas	a/(k Pa dm^6 mol^{-1})	b/(dm^3 mol^{-1})
A	642.32	0.05196
B	155.21	0.04136
C	431.91	0.05196
D	155.21	0.4382

a and b are van der Waals constants. The correct statement about the gases is :

(1) Gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D

(2) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

(3) Gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D

(4) Gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D

Answer (2)

Sol. If two gases have same value of 'b' but different values of 'a', then the gas having a larger value of 'a' will occupy lesser volume. This is because the gas having larger value of "a" will have larger force of attraction and hence lesser distance between its molecules.

If two gases have same value of 'a' but different values of 'b', then the smaller value of 'b' will occupy lesser volume and hence will be more compressible.

9. Amylopectin is composed of

(1) β-D-glucose, $C_1 — C_4$ and $C_2 — C_6$ linkages

(2) α-D-glucose, $C_1 — C_4$ and $C_2 — C_6$ linkages

(3) β-D-glucose, $C_1 — C_4$ and $C_1 — C_6$ linkages

(4) α-D-glucose, $C_1 — C_4$ and $C_1 — C_6$ linkages

Answer (4)

Sol. Starch is a polymer of α-D-glucose. It has two components, namely

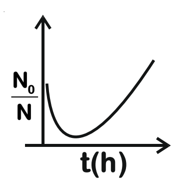
(i) Amylose and

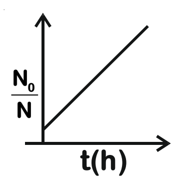
(ii) Amylopectin

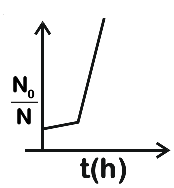
Amylose has only α-1,4-glycosidic linkage and is a linear polymer

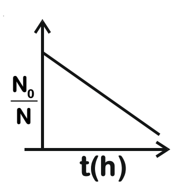
Amylopectin has α-1, 6-glycosidic linkage in addition to α-1,4-glycosidic linkage and is a cross-linked polymer.

10. A bacterial infection in an internal wound grows as $N'(t) = N_0 \exp(t)$, where the time t is in hours. A dose of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$. What will be the plot of $\frac{N_0}{N}$ vs. t after 1 hour ?

(1) 

(2) 

(3) 

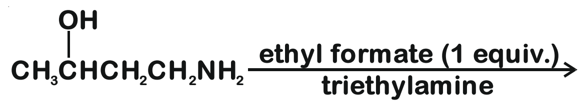
(4) 

Answer (2)

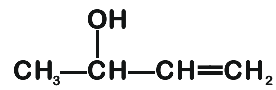
Sol. When drug is administered bacterial growth is given by $\frac{dN}{dt} = -5N^2$

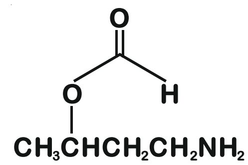
$\Rightarrow \frac{N_0}{N_t} = 1 + 5t\,N_0$. Thus $\frac{N_0}{N_t}$ increases linearly with t.

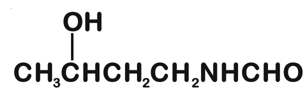
11. The major product of the following reaction is :



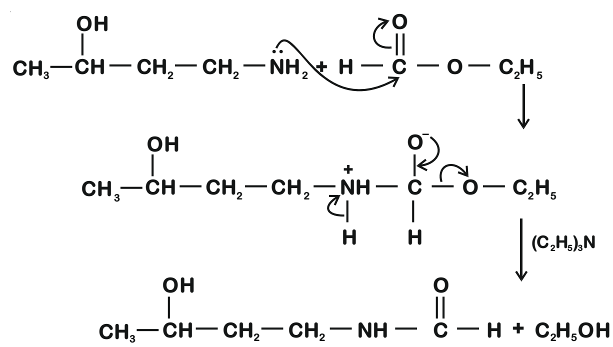
(1) $CH_3CH{=}CH—CH_2NH_2$

(2) 

(3) 

(4) 

Answer (4)

Sol. 

The acylation of NH_2 group takes place and not of OH group due to lower electronegativity of N-atom.

12. Consider the hydrated ions of Ti^{2+}, V^{2+}, Ti^{3+}, and Sc^{3+}. The correct order of their spin-only magnetic moments is :

(1) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

(2) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+}$

(3) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$

(4) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$

Answer (1)

Sol. Electronic configuration of the given transition metal ions are

Sc^{3+} (Z = 21) $1s^2 2s^2 2p^6 3s^2 3p^6$

Ti^{2+} (Z = 22) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^2$

Ti^{3+} (Z = 22) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$

V^{2+} (Z = 23) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^3$

Magnetic moment is directly proportional to the number of unpaired electrons. So the correct increasing order of magnetic moment is

$Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

0 1 2 3 unpaired electrons

13. The isoelectronic set of ions is :

(1) N^{3-}, Li^{+}, Mg^{2+} and O^{2-}

(2) Li^{+}, Na^{+}, O^{2-} and F^{-}

(3) N^{3-}, O^{2-}, F^{-} and Na^{+}

(4) F^{-}, Li^{+}, Na^{+} and Mg^{2+}

Answer (3)

Sol. Atomic numbers of N, O, F and Na are 7, 8, 9 and 11 respectively. Therefore, total number of electrons in each of N^{3-}, O^{2-}, F^{-} and Na^{+} is 10 and hence they are isoelectronic.

14. The alloy used in the construction of aircrafts is :

(1) Mg - Zn (2) Mg - Sn

(3) Mg - Mn (4) Mg - Al

Answer (4)

Sol. An alloy of Mg and Al called magnalium is used in manufacturing of aircraft due to its light weight and high strength.

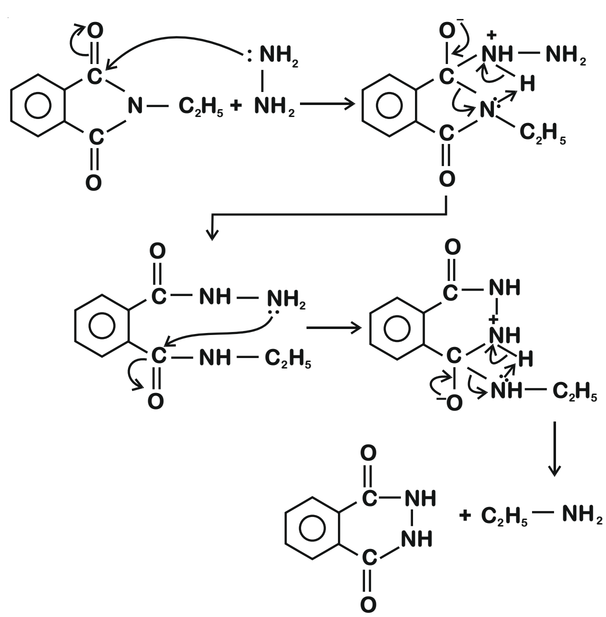
15. Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphthalimide on treatment with :

(1) NH_2NH_2 (2) $NaBH_4$

(3) H_2O (4) CaH_2

Answer (1)

Sol. N-ethyl phthalimide on treatment with $NH_2—NH_2$ gives ethylamine.



Note: In place of NH_2NH_2, H_2O can also be used in presence of H^+ or OH^- as a catalyst.

16. A process will be spontaneous at all temperatures if :

(1) $\Delta H < 0$ and $\Delta S > 0$

(2) $\Delta H > 0$ and $\Delta S < 0$

(3) $\Delta H > 0$ and $\Delta S > 0$

(4) $\Delta H < 0$ and $\Delta S < 0$

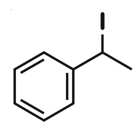
Answer (1)

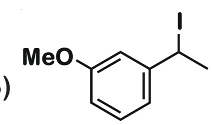
Sol. A reaction is spontaneous if ΔG_{sys} is negative.

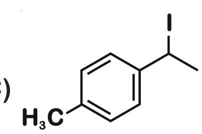
$\Delta G_{sys} = \Delta H_{sys} - T\Delta S_{sys}$

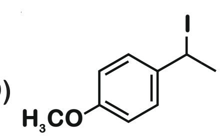
A reaction will be spontaneous at all temperatures if ΔH_{sys} is negative and ΔS_{sys} = +ve

17. Increasing rate of S_N1 reaction in the following compounds is :

(A) 

(B) 

(C) 

(D) 

(1) (B) < (A) < (C) < (D)

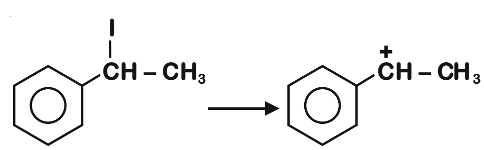
(2) (A) < (B) < (D) < (C)

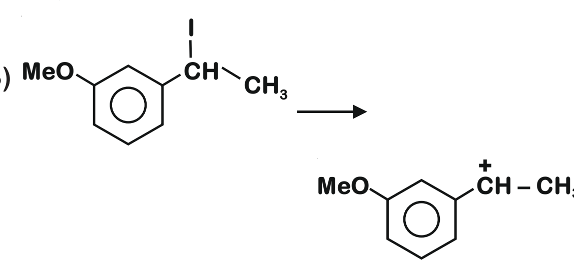
(3) (B) < (A) < (D) < (C)

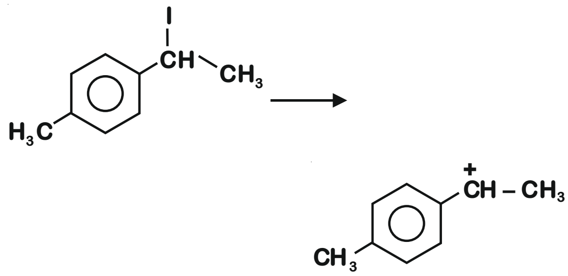
(4) (A) < (B) < (C) < (D)

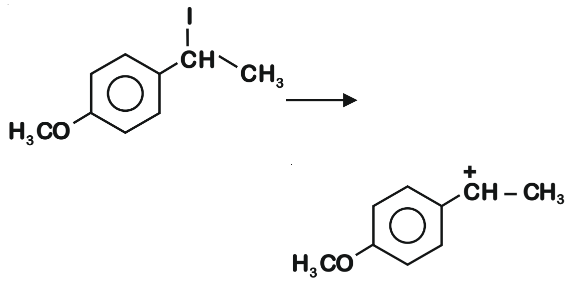
Answer (1)

Sol. The rate of S_N1 is decided by the stability of carbocation formed in the rate determining step.

(A) 

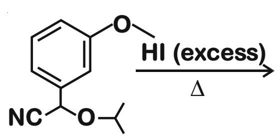
(B) 

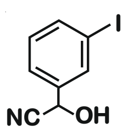
(C) 

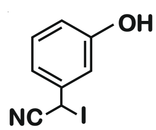
(D) 

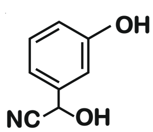
Carbocation (D) is most stable due to +R effect of CH_3O group, (C) is stabilised by +I and +H effect of CH_3 group; (B) is least stable due to –I effect of MeO group. So increasing order of rate of S_N1 is (B) < (A) < (C) < (D)

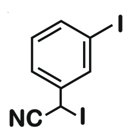
18. The major product of the following reaction is :



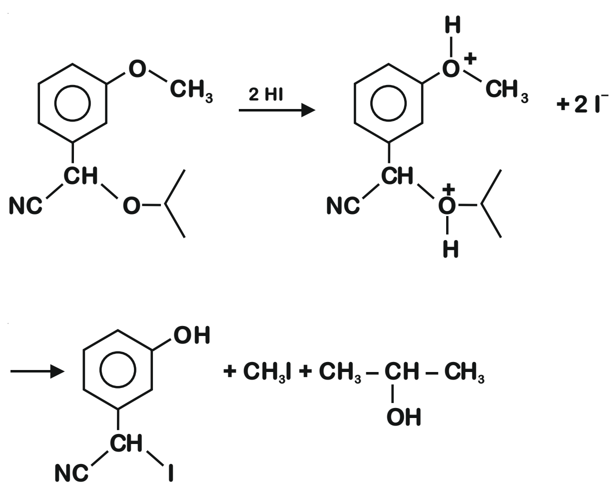
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

19. The species that can have a trans-isomer is :

(en = ethane-1, 2-diamine, ox = oxalate)

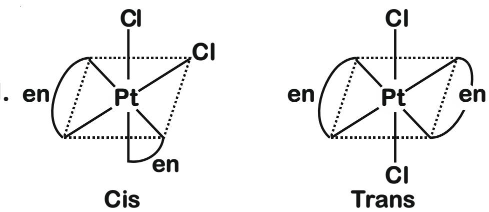
(1) $[Zn(en)Cl_2]$

(2) $[Pt(en)Cl_2]$

(3) $[Cr(en)_2(ox)]^+$

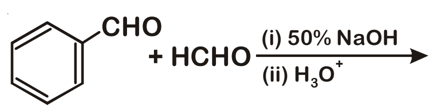
(4) $[Pt(en)_2Cl_2]^{2+}$

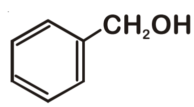
Answer (4)

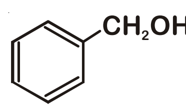
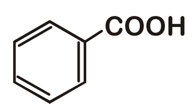
Sol. 

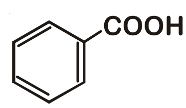
Cis-trans isomerism is possible with $[Pt(en)_2Cl_2]^{2+}$. $[Cr(en)_2Ox]^+$ shows optical isomerism but not geometrical isomerism. The other two complexes, i.e. $[Pt(en)Cl_2]$ and $[Zn(en)Cl_2]$ do not show stereoisomerism.

20. Major products of the following reaction are :



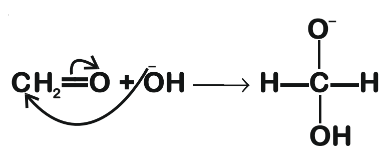
(1) HCOOH and 

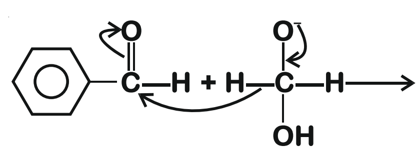
(2)  and 

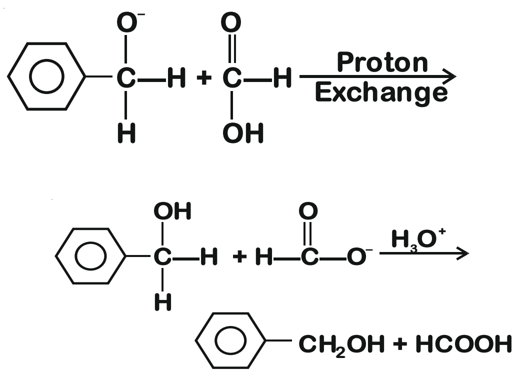
(3) CH_3OH and 

(4) CH_3OH and HCO_2H

Answer (1)

Sol. 





21. The correct order of catenation is :

(1) C > Si > Ge ≈ Sn

(2) C > Sn > Si ≈ Ge

(3) Si > Sn > C > Ge

(4) Ge > Sn > Si > C

Answer (1)

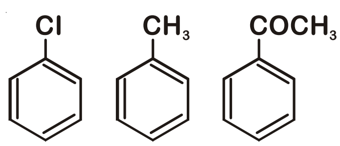
Sol. The order of catenation property amongst 14th group elements is based on bond enthalpy values of identical atoms of the same element. The decreasing order of bond enthalpy values is

Bond enthalpy $\underset{348\ \text{kJ/mol}}{\text{C–C}} > \underset{297}{\text{Si–Si}} > \underset{260}{\text{Ge–Ge}} \approx \underset{240\ \text{kJ/mol}}{\text{Sn–Sn}}$

∴ Decreasing order of catenation is

C > Si > Ge ≈ Sn

22. The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is :



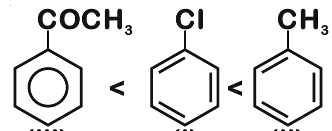
(I) (II) (III)

(1) I < III < II (2) III < II < I

(3) II < I < III (4) III < I < II

Answer (4)

Sol. CH_3 group when bonded to benzene increases the electron density of benzene by +I and hyper conjugation effects and hence makes the compound more reactive towards EAS. Cl group decreases the electron density of benzene by –I effect, and CH_3CO group strongly decreases the electron density of benzene by –I and –R effects. Therefore, correct increasing order the given compounds towards EAS is



(III) (I) (II)

23. During the change of O_2 to O_2^-, the incoming electron goes to the orbital :

(1) $\pi 2p_x$ (2) $\pi^* 2p_x$

(3) $\pi 2p_y$ (4) $\sigma^* 2p_z$

Answer (2)

Sol. Electronic configuration of O_2 is

$$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p_z}^2\pi_{2p_x}^2=\pi_{2p_y}^2\pi_{2p_x}^{*1}=\pi_{2p_y}^{*1}$$

When O_2 gains an electron to form O_2^-, the incoming electron goes to $\pi_{2p_x}^*$ or $\pi_{2p_y}^*$

24. The regions of the atmosphere, where clouds form and where we live, respectively, are :

(1) Troposphere and Troposphere

(2) Stratosphere and Troposphere

(3) Troposphere and Stratosphere

(4) Stratosphere and Stratosphere

Answer (1)

Sol. The lowest region of atmosphere in which human beings live is troposphere. It extends up to a height of 10 km from sea level. Clouds are also formed in this layer.

25. At room temperature, a dilute solution of urea is prepared by dissolving 0.60 g of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mmHg, lowering of vapour pressure will be : (molar mass of urea = 60 g mol^{-1})

(1) 0.031 mmHg (2) 0.017 mmHg

(3) 0.028 mmHg (4) 0.027 mmHg

Answer (2)

Sol. Relative lowering of VP is given by

$$\frac{P_B^\circ - P_B}{P_B^\circ} = x_A = \frac{n_A}{n_A + n_B} \simeq \frac{n_A}{n_B}$$

$$\frac{P_B^\circ - P_B}{35} = \frac{0.6 \times 18}{60 \times 360} = \frac{1}{2000}$$

On solving, $\Delta P_B = P_B^\circ - P_B = 0.017$

26. A gas undergoes physical adsorption on a surface and follows the given Freundlich adsorption isotherm equation

$$\frac{x}{m} = kp^{0.5}$$

Adsorption of the gas increases with :

(1) Increase in p and decrease in T

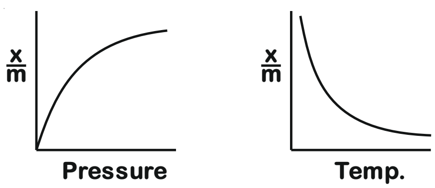
(2) Decrease in p and decrease in T

(3) Increase in p and increase in T

(4) Decrease in p and increase in T

Answer (1)

Sol. Freundlich adsorption is applicable for physical adsorption. The variation of extent of adsorption with (i) Pressure and (ii) Temp is given by the following curves.



So, extent of adsorption increases with increase of pressure and decrease of temperature.

27. The synonym for water gas when used in the production of methanol is :

(1) fuel gas

(2) syn gas

(3) laughing gas

(4) natural gas

Answer (2)

Sol. When steam is passed over red hot coke, an equimolar mixture of CO and H_2 is obtained

$$\underset{\text{Steam}}{H_2O(g)} + \underset{\text{Red hot}}{C} \longrightarrow CO + H_2$$

The gaseous mixture thus obtained is called water gas or syn. gas.

28. At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion, and 40 mL of CO_2 is formed. The formula of the hydrocarbon is :

(1) C_4H_{10} (2) C_4H_8

(3) C_4H_6 (4) C_4H_7Cl

Answer (3)

Sol. $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

10 ml 55 ml

0 $55 - 10\left(x+\frac{y}{4}\right)$ 10 x

Vol. of CO_2, 10x = 40 ; x = 4

$55 - 10\left(x+\frac{y}{4}\right) = 0$; y = 6

$\therefore$ Hydrocarbon is C_4H_6

29. Three complexes, $[CoCl(NH_3)_5]^{2+}$(I), $[Co(NH_3)_5H_2O]^{3+}$(II) and $[Co(NH_3)_6]^{3+}$(III) absorb light in the visible region. The correct order of the wavelength of light absorbed by them is :

(1) (I) > (II) > (III) (2) (II) > (I) > (III)

(3) (III) > (I) > (II) (4) (III) > (II) > (I)

Answer (1)

Sol. In a co-ordination compound, the strong field ligand causes higher splitting of the d-orbitals. Wavelength of the energy absorbed by the co-ordination compound is inversely proportional to ligand field strength of the given co-ordination compound. The decreasing order of ligand field strength is $NH_3 > H_2O > Cl$. Therefore decreasing order of wavelength absorbed is (I) > (II) > (III).

30. Match the refining methods (Column I) with metals (Column II).

Column I (Refining methods)	Column II (Metals)
(I) Liquation	(a) Zr
(II) Zone Refining	(b) Ni
(III) Mond Process	(c) Sn
(IV) Van Arkel Method	(d) Ga

(1) (I)-(c); (II)-(a); (III)-(b); (IV)-(d)

(2) (I)-(b); (II)-(d); (III)-(a); (IV)-(c)

(3) (I)-(c); (II)-(d); (III)-(b); (IV)-(a)

(4) (I)-(b); (II)-(c); (III)-(d); (IV)-(a)

Answer (3)

Sol. Mond's process is used for refining of Ni, Van Arkel method is used for Zr, Liquation is used for Sn and zone refining is used for Ga.

So, correct match is

(I)-(c); (II)-(d); (III)-(b); (IV)-(a)

PART–C : MATHEMATICS

1. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot\frac{1}{4^{\sin^2 y}}\le 1$ also satisfy the equation :

(1) $\sin x = |\sin y|$

(2) $\sin x = 2\sin y$

(3) $2\sin x = \sin y$

(4) $2|\sin x| = 3\sin y$

Answer (1)

Sol. $2^{\sqrt{\sin^2 x-2\sin x+5}}\le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x-2\sin x+5}\le 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x-1)^2+4}\le 2\sin^2 y$

it is true when $\sin x = 1$

$|\sin y| = 1$

so $\sin x = |\sin y|$

2. If $a > 0$ and $z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{1}{5}+\frac{3}{5}i$

(2) $-\frac{3}{5}-\frac{1}{5}i$

(3) $\frac{1}{5}-\frac{3}{5}i$

(4) $-\frac{1}{5}-\frac{3}{5}i$

Answer (4)

Sol. $z=\frac{(1+i)^2}{a-i}\times\frac{a+i}{a+i}$

$$z=\frac{(1-1+2i)(a+i)}{a^2+1}=\frac{2ai-2}{a^2+1} \quad \text{...(i)}$$

$$|z|=\sqrt{\left(\frac{-2}{a^2+1}\right)^2+\left(\frac{2a}{a^2+1}\right)^2}=\sqrt{\frac{4+4a^2}{(a^2+1)^2}}$$

$$=\sqrt{\frac{4(1+a^2)}{(1+a^2)^2}}=\frac{2}{\sqrt{1+a^2}}$$

given $|z|=\sqrt{\frac{2}{5}}$

so $\sqrt{\frac{2}{5}}=\frac{2}{\sqrt{1+a^2}}$ from equation (i)

(square both side)

$\Rightarrow \frac{2}{5}=\frac{4}{1+a^2}$

$\Rightarrow 1 + a^2 = 10$

$a^2 = 9$

$\Rightarrow a \pm 3 \quad \because (a > 0) \therefore a = 3$

Hence $z=\frac{1+i^2+2i}{3-i}=\frac{2i}{3-i}=\frac{2i(3+i)}{10}=\frac{-1+3i}{5}$

$\bar{z}=\frac{-1}{5}-\frac{3}{5}i$

3. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :

(1) 72

(2) 48

(3) 60

(4) 36

Answer (3)

Sol.

a_1	a_2	a_3	a_4	a_5	a_6

digit 0, 1, 2, 5, 7, 9

$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11K$

so (1, 2, 9) (0, 5, 7)

Now number of ways to arranging them

= 3! × 3! + 3! × 2 × 2

= 6 × 6 + 6 × 4

= 6 × 10

= 60

4. The value of $\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx$, where [t] denotes the greatest integer function, is :

(1) π

(2) $-\pi$

(3) -2π

(4) 2π

Answer (2)

Sol. $I=\int_0^{2\pi}\left[\sin 2x\left(1+\cos 3x\right)\right]dx$ ...(i)

$\because \int_0^a f(x)=\int_0^a f(a-x)\,dx$

$\therefore \quad I=\int_0^{2\pi}\left[-\sin 2x\left(1+\cos 3x\right)\right]dx$ ...(ii)

By (i) + (ii)

$2I=\int_0^{2\pi}(-1)dx$

$2I=-(x)_0^{2\pi}$

$\Rightarrow \quad I=-\pi$

5. If $\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2=\begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x\neq 0$; then

for all $\theta\in\left(0,\frac{\pi}{2}\right)$:

(1) $\Delta_1+\Delta_2=-2x^3$

(2) $\Delta_1-\Delta_2=-2x^3$

(3) $\Delta_1+\Delta_2=-2(x^3+x-1)$

(4) $\Delta_1-\Delta_2=x(\cos 2\theta-\cos 4\theta)$

Answer (1)

Sol. $\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$=x(-x^2-1)-\sin\theta(-x\sin\theta-\cos\theta)$

$+\cos\theta(-\sin\theta+x\cos\theta)$

$=-x^3-x+x\sin^2\theta+\sin\theta\cos\theta-\cos\theta\sin\theta$

$+x\cos^2\theta$

$=-x^3-x+x=-x^3$

Similarly $\Delta_1=-x^3$

$\Delta_1+\Delta_2=-2x^3$

6. If $f(x)=\begin{cases} \dfrac{\sin(p+1)x+\sin x}{x}, & x<0 \\ q, & x=0 \\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0 \end{cases}$

is continuous at x = 0, then the ordered pair (p, q) is equal to :

(1) $\left(\frac{5}{2},\frac{1}{2}\right)$ (2) $\left(-\frac{3}{2},\frac{1}{2}\right)$

(3) $\left(-\frac{3}{2},-\frac{1}{2}\right)$ (4) $\left(-\frac{1}{2},\frac{3}{2}\right)$

Answer (2)

Sol. $f(x)=\begin{cases} \dfrac{\sin(p+1)x+\sin x}{x} & x<0 \\ q & x=0 \\ \dfrac{\sqrt{x^2+x}-\sqrt{x}}{x^{\frac{3}{2}}} & x>0 \end{cases}$

is continuous at x = 0

So $f(0^-)=f(0)=f(0^+)$ ...(1)

$f(0^-)=\underset{h\to 0}{\text{lt}}\, f(0-h)$

$=\underset{h\to 0}{\text{lt}}\dfrac{\sin(p+1)(-h)+\sin(-h)}{-h}$

$=\underset{h\to 0}{\text{lt}}\left[\dfrac{-\sin(p+1)h}{-h}+\dfrac{\sin h}{h}\right]$

$=\lim_{h\to 0}\dfrac{\sin(p+1)h}{h(p+1)}\times(p+1)+\lim_{h\to 0}\dfrac{\sin h}{h}$

$=(p+1)+1=p+2$ ...(2)

Now $f(0^+)=\lim_{h\to 0}f(0+h)=\lim_{h\to 0}\dfrac{\sqrt{h^2+h}-\sqrt{h}}{h^{3/2}}$

$=\lim_{h\to 0}\dfrac{(h)^{\frac{1}{2}}\left[\sqrt{h+1}-1\right]}{h\left(h^{\frac{1}{2}}\right)}$

$=\lim_{h\to 0}\dfrac{\sqrt{h+1}-1}{h}\times\dfrac{\sqrt{h+1}+1}{\sqrt{h+1}+1}$

$=\lim_{h\to 0}\dfrac{h+1-1}{h\left(\sqrt{h+1}+1\right)}$

$=\lim_{h\to 0}\dfrac{1}{\sqrt{h+1}+1}=\dfrac{1}{1+1}=\dfrac{1}{2}$ ...(3)

Now, from equation (1)
$f(0^-) = f(0) = f(0^+)$

$p + 2 = q = \frac{1}{2}$

So, $q = \frac{1}{2}$ and $p = \frac{1}{2} - 2 = \frac{-3}{2}$

$(p, q) \equiv \left(-\frac{3}{2}, \frac{1}{2}\right)$

7. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) -1 (2) -2

(3) 1 (4) 2

Answer (1)

Sol. $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1} = p$ $\qquad P(\beta, 0, \beta)$

any point on line $A = (p, 1, -p-1)$

Now, DR of AP $\equiv \langle p - \beta, 1 - 0, -p - 1 - \beta \rangle$

Which is perpendicular to line so

$(p - \beta).1 + 0.1 - 1(-p - 1 - \beta) = 0$

$\Rightarrow p - \beta + p + 1 + \beta = 0$

$p = \frac{-1}{2}$

Point $A\left(\frac{-1}{2}, 1 - \frac{1}{2}\right)$

Now, distance AP = $\sqrt{\frac{3}{2}}$

$\Rightarrow AP^2 = \frac{3}{2}$

$\Rightarrow \left(\beta + \frac{1}{2}\right)^2 + 1 + \left(\beta + \frac{1}{2}\right)^2 = \frac{3}{2}$

$2\left(\beta + \frac{1}{2}\right)^2 = \frac{1}{2}$

$\Rightarrow \left(\beta + \frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow \beta = 0, \ -1, (\beta \neq 0)$

$\therefore \boxed{\beta = -1}$

8. $\lim_{n \to \infty} \left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \ldots\ldots + \frac{(2n)^{1/3}}{n^{4/3}} \right)$ is equal to :

(1) $\frac{4}{3}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$

(3) $\frac{4}{3}(2)^{3/4}$ (4) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$

Answer (2)

Sol. $\lim_{n \to \infty} \frac{(n+1)^{\frac{1}{3}} + (n+2)^{\frac{1}{3}} + \ldots + (n+n)^{\frac{1}{3}}}{n(n)^{\frac{1}{3}}}$

$= \lim_{n \to \infty} \sum_{r=1}^{n} \frac{(n+r)^{\frac{1}{3}}}{n.n^{\frac{1}{3}}}$ $\qquad \frac{r}{n} \to x$ and $\frac{1}{n} \to dx$

$= \int_0^1 (1+x)^{\frac{1}{3}} dx$

$= \left[\frac{3}{4}(1+x)^{\frac{4}{3}}\right]_0^1 = \frac{3}{4}(2)^{\frac{4}{3}} - \frac{3}{4}$

9. The line $x = y$ touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

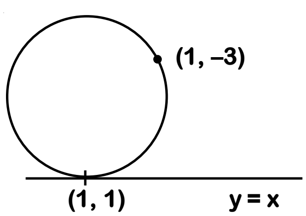
(1) $3\sqrt{2}$ (2) 2

(3) $2\sqrt{2}$ (4) 3

Answer (3)

Sol. Equation of circle = $(x - 1)^2 + (y - 1)^2 + \lambda(y - x) = 0$

Which passes through (1, –3)



So, $0 + 16 + \lambda(-3 - 1) = 0$

$16 + \lambda(-4) = 0$

$\boxed{\lambda = 4}$

Now equation of circle

$(x - 1)^2 + (y - 1)^2 + 4y - 4x = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

radius = $\sqrt{9 + 1 - 2} = 2\sqrt{2}$

10. Which one of the following Boolean expressions is a tautology ?

(1) $(p \vee q) \vee (p \vee \sim q)$

(2) $(p \wedge q) \vee (p \wedge \sim q)$

(3) $(p \vee q) \wedge (p \vee \sim q)$

(4) $(p \vee q) \wedge (\sim p \vee \sim q)$

Answer (1)

Sol. $(p \vee q) \vee (p \vee \sim q)$

$= p \vee (q \vee p) \vee \sim q$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$ so first statement is tautology

11. If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1} = \lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$, then k is :

(1) $\dfrac{4}{3}$

(2) $\dfrac{3}{2}$

(3) $\dfrac{8}{3}$

(4) $\dfrac{3}{8}$

Answer (3)

Sol. If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1} = \lim\limits_{x\to K}\left(\dfrac{x^3-k^3}{x^2-k^2}\right)$

L·H·S·

$\underset{x\to 1}{\text{Lt}}\dfrac{x^4-1}{x-1} = \left(\dfrac{0}{0}\text{ form}\right)$

$\underset{x\to 1}{\text{Lt}}\dfrac{4x^3}{1} = 4$

Now, $\lim\limits_{x\to K}\dfrac{x^3-k^3}{x^2-k^2} = 4$

$\Rightarrow \lim\limits_{x\to K}\dfrac{3x^2}{2x} = 4$

$\Rightarrow \dfrac{3}{2}k = 4$

$k = \dfrac{8}{3}$

12. If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

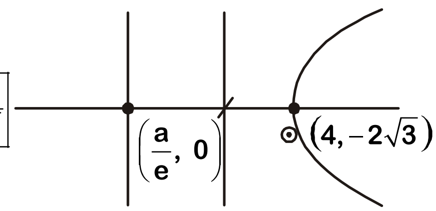
(1) $4e^4 + 8e^2 - 35 = 0$ (2) $4e^4 - 24e^2 + 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$ (4) $4e^4 - 24e^2 + 27 = 0$

Answer (2)

Sol. $x = \dfrac{4}{\sqrt{5}}$

$\because$ $\boxed{\dfrac{a}{e} = \dfrac{4}{\sqrt{5}}}$



Equation of hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ it passes through $(4, -2\sqrt{3})$

$\because$ $e^2 = 1 + \dfrac{b^2}{a^2}$

$\Rightarrow$ $\boxed{a^2e^2 - a^2 = b^2}$

$\Rightarrow$ $\dfrac{16}{a^2} - \dfrac{12}{a^2e^2 - a^2} = 1$

$\Rightarrow$ $\dfrac{4}{a^2}\left[\dfrac{4}{1} - \dfrac{3}{e^2-1}\right] = 1$

$\Rightarrow$ $4e^2 - 4 - 3 = (e^2 - 1)\left(\dfrac{a^2}{4}\right)$

$\Rightarrow$ $4(4e^3 - 7) = (e^2 - 1)\left(\dfrac{4e}{\sqrt{5}}\right)^2$

$\Rightarrow$ $4e^4 - 24e^2 + 35 = 0$

13. If the line x – 2y = 12 is tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the point $\left(3, \dfrac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

(1) 5 (2) $8\sqrt{3}$

(3) $12\sqrt{2}$ (4) 9

Answer (4)

Sol. Equation of tangent at $\left(3, -\frac{9}{2}\right)$ to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$\frac{3x}{a^2} - \frac{y9}{2b^2} = 1$ which is equivalent to x – 2y = 12

$\frac{3}{a^2} = \frac{-9}{2b^2 \cdot (-2)} = \frac{1}{12}$ (On comparing)

$a^2 = 3 \times 12$ and $b^2 = \frac{9 \times 12}{4}$

$\boxed{a = 6} \Rightarrow b = 3\sqrt{3}$

So latus rectum = $\frac{2b^2}{a} = \frac{2 \times 27}{6} = 9$

14. Let A(3, 0 –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the mid point of AC. If G divides BM in the ratio, 2 : 1 then cos(∠ GOA) (O being the origin) is equal to :

(1) $\frac{1}{6\sqrt{10}}$

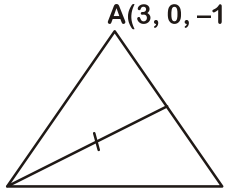
(2) $\frac{1}{\sqrt{30}}$

(3) $\frac{1}{2\sqrt{15}}$

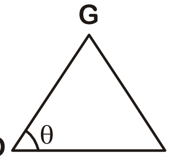
(4) $\frac{1}{\sqrt{15}}$

Answer (4)

Sol. G is the centroid of ΔABC



G (2, 4, 2)



$OG = \sqrt{4+16+4}, \quad OA = \sqrt{9+1}$

$AG = \sqrt{1+16+9}$

$\cos\theta = \frac{24+10-26}{2\sqrt{24}\sqrt{10}}$

$= \frac{8}{2\sqrt{8 \times 3 \times 2 \times 5}}$

$= \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$

15. If the system of linear equations

x + y + z = 5

x + 2y + 2z = 6

x + 3y + λz = μ, (λ, μ ∈ R), has infinitely many solutions, then the value of λ + μ is :

(1) 10 (2) 12

(3) 7 (4) 9

Answer (1)

Sol. x + y + z = 5

x + 2y + 2z = 6

x + 3y + λz = μ have infinite solution

Δ = 0, Δx = Δy = Δz = 0

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

⇒ 1(2λ –6) – 1(λ – 2) + 1 (3 – 2) = 0

⇒ 2λ – 6 – λ + 2 + 1 = 0

$\boxed{\lambda = 3}$

Now, $\Delta x = \begin{vmatrix} 5 & 1 & 1 \\ 6 & 2 & 2 \\ \mu & 3 & 3 \end{vmatrix} = 0, \ \Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu-5 & 2 \end{vmatrix} = 0$$

⇒ 1(2 – μ + 5) = 0

$\boxed{\mu = 7}$

$$\Delta z = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix} = \begin{vmatrix} 1 & 1 & 5 \\ 0 & -1 & -1 \\ 0 & 2 & \mu-5 \end{vmatrix}$$

⇒ 1 (5 – μ + 2) = 0

⇒ μ = 7

So, λ + μ = 10

16. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{11}$ (2) $\frac{1}{12}$

(3) $\frac{1}{10}$ (4) $\frac{1}{17}$

Answer (1)

Sol. A = At least two girls

B = All girls

$$P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$$

$$=\frac{P(B)}{P(A)}=\frac{\left(\frac{1}{4}\right)^4}{1-{}^4C_0\left(\frac{1}{2}\right)^4-{}^4C_1\left(\frac{1}{2}\right)^4}$$

$$=\frac{1}{16-1-4}=\frac{1}{11}$$

17. If for some $x\in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

Then the mean of the marks is :

(1) 3.2 (2) 3.0

(3) 2.5 (4) 2.8

Answer (4)

Sol. Number of students

$\Rightarrow$ $(x + 1)^2 + (2x - 5) + (x^2 - 3x) + x = 20$

$\Rightarrow$ $2x^2 + 2x - 4 = 20$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = 3$

So,

Marks	2	3	5	7
No. of students	16	1	0	3

Average marks = $\frac{32+3+21}{20}=\frac{56}{20}=2.8$

18. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for :

(1) Exactly one value of K

(2) Infinitely many values of K

(3) Exactly two values of K

(4) No value of K

Answer (4)

Sol. $S_1 \equiv x^2 + y^2 + 5Kx + 2y + K = 0$

$$S_2 \equiv x^2+y^2+Kx+\frac{3}{2}y-\frac{1}{2}=0$$

Equation of common chord is

$S_1 - S_2 = 0$

$\Rightarrow$ $4Kx+\frac{y}{2}+K+\frac{1}{2}=0$ ...(1)

$4x + 5y - K = 0$ ...(2) (given)

On comparing (1) and (2)

$$\frac{4K}{4}=\frac{1}{10}=\frac{2K+1}{-2K}$$

$\Rightarrow$ $\boxed{K=\frac{1}{10}}$ and $-2K=20K+10$

$\Rightarrow$ $22K = -10$

$\boxed{K=\frac{-5}{11}}$

$\therefore$ No value of K exists

19. The region represented by $|x-y|\le 2$ and $|x+y|\le 2$ is bounded by a :

(1) Square of side length $2\sqrt{2}$ units

(2) Square of area 16 sq. units

(3) Rhombus of side length 2 units

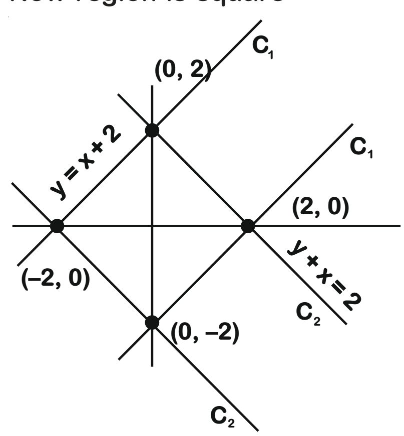
(4) Rhombus of area $8\sqrt{2}$ sq. units

Answer (1)

Sol. $C_1 : |y - x| \le 2$

$C_2 : |y + x| \le 2$

Now region is square



Length of side = $\sqrt{2^2+2^2}=2\sqrt{2}$

20. If $a_1, a_2, a_3, ..., a_n$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 98 (2) 38

(3) 64 (4) 76

Answer (4)

Sol. $3(a_1 + a_{16}) = 114$

$\boxed{a_1 + a_{16} = 38}$

Now $a_1 + a_6 + a_{11} + a_{16} = 2(a_1 + a_{16})$

$= 2 \times 38 = 76$

21. If Q(0, –1, –3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$

(3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

Answer (4)

Sol. Image of Q in plane

$$\frac{(x-0)}{3} = \frac{(y+1)}{-1} = \frac{z+3}{+4} = \frac{-2(1-12-2)}{9+1+16} = 1$$

$x = 3, y = -2, z = 1$

P(3, –2, 1), Q(0, –1, –3), R(3, –1, –2)

Now area of ΔPQR is

$$\frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{QR}| = \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 4 \\ 3 & 0 & 1 \end{matrix}\right\|$$

$$= \frac{1}{2}\left|\left\{\hat{i}(-1) - \hat{j}(3-12) + \hat{k}(3)\right\}\right|$$

$$= \frac{1}{2}\sqrt{(1+81+9)}$$

$$= \frac{\sqrt{91}}{2}$$

22. Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ?

(1) $f(g(S)) = S$ (2) $g(f(S)) = g(S)$

(3) $g(f(S)) \neq S$ (4) $f(g(S)) \neq f(S)$

Answer (2)

Sol. $f(x) = x^2 \quad x \in R$

$g(A) = \{x \in R : f(x) \in A\} \quad S \equiv [0, 4]$

$g(S) = \{x \in R : f(x) \in S\}$

$= \{x \in R : 0 \le x^2 \le 4\}$

$= \{x \in R : -2 \le x \le 2\}$

$\therefore$ $g(S) \neq S$

$\therefore$ $f(g(S)) \neq f(S)$

$g(f(S)) = \{x \in R : f(x) \in f(S)\}$

$= \{x \in R : x^2 \in S^2\}$

$= \{x \in R : 0 \le x^2 \le 16\}$

$= \{x \in R : -4 \le x \le 4\}$

$\therefore$ $g(f(S)) \neq g(S)$

$\therefore$ $g(f(S)) = g(S)$ is incorrect

23. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(1) (– 54, 315)

(2) (28, 861)

(3) (–21, 714)

(4) 28, 315

Answer (4)

Sol. $(1 + ax + bx^2)(1 - 3x)^{15}$

Co-eff. of $x^2 = 1.{}^{15}C_2(-3)^2 + a.{}^{15}C_1(-3) + b.{}^{15}C_0$

$$= \frac{15 \times 14}{2} \times 9 - 15 \times 3a + b = 0 \text{ (Given)}$$

$\Rightarrow$ $945 - 45a + b = 0$ ...(i)

Now co-eff. of $x^3 = 0$

$\Rightarrow$ ${}^{15}C_3(-3)^3 + a.{}^{15}C_2(-3)^2 + b.{}^{15}C_1(-3) = 0$

$$\Rightarrow \frac{15 \times 14 \times 13}{3 \times 2} \times (-3 \times 3 \times 3) + a \times \frac{15 \times 14 \times 9}{2} - b \times 3 \times 15 = 0$$

$\Rightarrow$ $15 \times 3[-3 \times 7 \times 13 + a \times 7 \times 3 - b] = 0$

$\Rightarrow$ $21a - b = 273$ ...(ii)

From (i) and (ii)

$a = +28, b = 315 \equiv (a, b) \equiv (28, 315)$

24. The sum

$$\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+....$$

upto 10^{th} term, is :

(1) 620 (2) 600

(3) 680 (4) 660

Answer (4)

Sol. $T_r=\frac{(2r+1)(1^3+2^3+3^3+...+r^3)}{1^2+2^2+3^2+...+r^2}$

$$T_r=(2r+1)\left(\frac{r(r+1)}{2}\right)^2\times\frac{6}{r(r+1)(2r+1)}$$

$$T_r=\frac{3r(r+1)}{2}$$

Now,

$$S=\sum_{r=1}^{10}T_r=\frac{3}{2}\sum_{r=1}^{10}(r^2+r)$$

$$=\frac{3}{2}\left\{\frac{10\times(10+1)(2\times10+1)}{6}+\frac{10\times11}{2}\right\}$$

$$=\frac{3}{2}\left\{\frac{10\times11\times21}{6}+5\times11\right\}$$

$$=\frac{3}{2}\times5\times11\times8=660$$

25. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\ x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is :

(1) $\left[-1,\frac{-1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$ (2) $[0, \infty)$

(3) $\left[0,\frac{1}{2}\right]\cup[1,\infty)$ (4) $\left[\frac{-1}{2},0\right]\cup[1,\infty)$

Answer (3)

Sol. $f(x) = e^x - x$, $g(x) = x^2 - x$

$f(g(x)) = e^{(x^2-x)}-(x^2-x)$

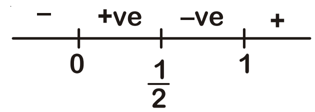
If $f(g(x))$ is increasing function

$(f(g(x)))' = e^{(x^2-x)}\times(2x-1)-2x+1$

$= (2x-1)e^{(x^2-x)}+1-2x$

$$= \underbrace{(2x-1)}_{A}\underbrace{[e^{(x^2-x)}-1]}_{B}$$

A & B are either both positive or negative



for $(f(g(x)))' \geq 0$,

$$x\in\left[0,\frac{1}{2}\right]\cup[1,\infty)$$

26. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta\in\left(0,\frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ is equal to :

(1) $\frac{2^{12}}{(\sin\theta-8)^6}$ (2) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

(3) $\frac{2^6}{(\sin\theta+8)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

Answer (4)

Sol. Given $\alpha + \beta = -\sin\theta$ and $\alpha\beta = -2\sin\theta$

$$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\sin^2\theta+8\sin\theta}$$

Hence required quantity

$$\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$$

27. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx}=(\tan x-y)\sec^2 x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{1}{e}-2$

(2) $\frac{1}{2}-e$

(3) $e - 2$

(4) $2+\frac{1}{e}$

Answer (3)

Sol. $\dfrac{dy}{dx} + y\sec^2 x = \sec^2 x \tan x$ $\rightarrow$ This is linear differential equation

$$IF = e^{\int \sec^2 x dx} = e^{\tan x}$$

Now solution is

$$y \cdot e^{\tan x} = \int e^{\tan x} \sec^2 x \tan x dx$$

$\because$ Let tanx = t

$\sec^2 x dx = dt$

$$ye^{\tan x} = \int \underset{II}{e^t}\, \underset{I}{t}\, dt$$

$ye^{\tan x} = te^t - e^t + c$

$ye^{\tan x} = (\tan x - 1)e^{\tan x} + c$

$y = (\tan x - 1) + c \cdot e^{-\tan x}$

Given y(0) = 0

$\Rightarrow$ 0 = –1 + c

$\Rightarrow$ c =1

$$y\left(-\frac{\pi}{4}\right) = -1 - 1 + e = -2 + e$$

28. Let f : R $\rightarrow$ R be differentiable at c $\in$ R and f(c) = 0. If g(x) = |f(x)|, then at x = c, g is :

(1) not differentiable if $f'(c) = 0$

(2) differentiable if $f'(c) = 0$

(3) not differentiable

(4) differentiable if $f'(c) \neq 0$

Answer (2)

Sol. $\because$ $g'(c) = \lim\limits_{x \to c} \dfrac{g(x) - g(c)}{x - c}$

$\Rightarrow$ $g'(c) = \lim\limits_{x \to c} \dfrac{|f(x)| - |f(c)|}{x - c}$

$\because$ f(c) = 0

$\Rightarrow$ $g'(c) = \lim\limits_{x \to c} \dfrac{|f(x)|}{x - c}$

$\Rightarrow$ $g'(c) = \lim\limits_{x \to c} \dfrac{f(x)}{x - c}$ if f(x) > 0

and $g'(c) = \lim\limits_{x \to c} \dfrac{-f(x)}{x - c}$ if f(x) < 0

$\Rightarrow$ $g'(c) = f'(c) = -f'(c)$

$\Rightarrow$ $2f'(c) = 0$

$\Rightarrow$ $f'(c) = 0$

29. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

(1) 20

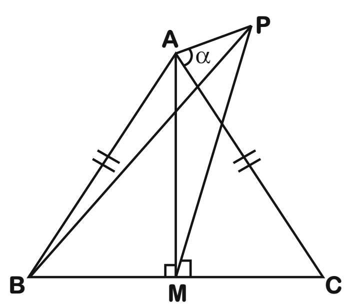
(2) $10\sqrt{5}$

(3) 25

(4) $\dfrac{100}{3\sqrt{3}}$

Answer (1)

Sol.



ΔAPM

$$\frac{h}{AM} = \frac{1}{3\sqrt{2}}$$

ΔBPM

$$\frac{h}{BM} = \frac{1}{\sqrt{7}}$$

ΔABM

$\therefore$ $AM^2 + MB^2 = (100)^2$

$\Rightarrow$ $18h^2 + 7h^2 = 100 \times 100$

$\Rightarrow$ $h^2 = 4 \times 100$

$\Rightarrow$ h = 20

30. If $\int \frac{dx}{(x^2-2x+10)^2}$

$$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$$

where C is a constant of integration, then :

(1) $A=\frac{1}{81}$ and $f(x)=3(x-1)$

(2) $A=\frac{1}{54}$ and $f(x)=3(x-1)$

(3) $A=\frac{1}{27}$ and $f(x)=9(x-1)$

(4) $A=\frac{1}{54}$ and $f(x)=9(x-1)^2$

Answer (2)

Sol. $\int \frac{dx}{(x^2-2x+10)^2}=\int \frac{dx}{((x-1)^2+9)^2}$

Let $(x-1)^2 = 9\tan^2\theta$ ...(i)

$\Rightarrow \tan\theta = \frac{x-1}{3}$

On Differentiating ...(i)

$2(x-1)dx = 18\tan\theta \sec^2\theta \, d\theta$

$$\therefore I = \int \frac{18\tan\theta \sec^2\theta \, d\theta}{2\times 3\tan\theta \times 81\sec^4\theta}$$

$$I = \frac{1}{27}\int \cos^2\theta \, d\theta = \frac{1}{27}\times\frac{1}{2}\int (1+\cos 2\theta)d\theta$$

$$I = \frac{1}{54}\left\{\theta+\frac{\sin 2\theta}{2}\right\}+c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{1}{2}\times\frac{2\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right]+c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3(x-1)}{x^2-2x+10}\right]+c$$

So $A = \frac{1}{54}$

$f(x) = 3(x-1)$

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