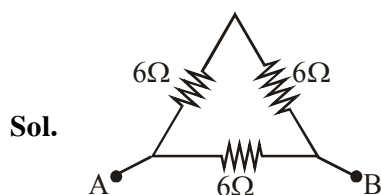# TEST PAPER OF JEE(MAIN) EXAMINATION – 2019
(Held On Thursday 10th JANUARY, 2019) TIME : 09 : 30 AM To 12 : 30 PM
PHYSICS

1. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is :
(1) 8 Ω   (2) 12 Ω   (3) 4 Ω   (4) 2Ω

Ans. (3)

Sol.

R_{eq} between any two vertex will be

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} \Rightarrow R_{eq.} = 4\Omega$$

2. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is :

(1) $\frac{3}{2}mv^2$ (2) mv^2

(3) $2mv^2$ (4) $\frac{1}{2}mv^2$

Ans. (2)

Sol. At height r from center of earth. orbital velocity

$$= \sqrt{\frac{GM}{r}}$$

∴ By energy conservation

KE of 'm' + $\left(-\frac{GMm}{r}\right) = 0 + 0$

(At infinity, PE = KE = 0)

$\Rightarrow$ KE of 'm' = $\frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$

3. A solid metal cube of edge length 2 cm is moving in a positive y direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is :
(1) 6 mV   (2) 1 mV   (3) 12 mV   (4) 2 mV

Ans. (3)

Sol. Potential difference between two faces perpendicular to x-axis will be

$$\ell.(\vec{V} \times \vec{B}) = 12 mV$$

4. A parallel plate capacitor is of area 6 cm² and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1, = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be :
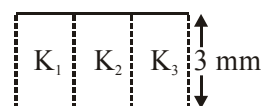
(1) 12   (2) 4   (3) 36   (4) 14

Ans. (1)

Sol. Let dielectric constant of material used be K.

$$\therefore \frac{10 \in_0 A/3}{d} + \frac{12 \in_0 A/3}{d} + \frac{14 \in_0 A/3}{d} = \frac{K \in_0 A}{d}$$

$\Rightarrow$ K = 12

5. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is :
(1) 63 mA   (2) 0.4 mA
(3) 100 mA   (4) 20 mA

Ans. (4)

Sol. P = i²R.
∴ for i_{max}, R must be minimum
from color coding R = 50×10²Ω
∴ i_{max} = 20mA

6. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\dfrac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are :

(1) 380 nm, 500 nm   (2) 625 nm, 500 nm

(3) 380 nm, 525 nm   (4) 400 nm, 500 nm

Ans. (2)

Sol. Path difference $= d \sin\theta \approx d\theta$

$= 0.1 \times \dfrac{1}{40}$ mm $= 2500$nm

or bright fringe, path difference must be integral multiple of λ.

$\therefore \ 2500 = n\lambda_1 = m\lambda_2$

$\therefore \ \lambda_1 = 625, \ \lambda_2 = 500$ (from m=5)
   (for n = 4)

7. A magnet of total magnetic moment $10^{-2}\,\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\,\hat{i}\,(\cos\omega t)$ where $B = 1$ Tesla and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at $t = 1$ second, is :

(1) 0.007 J (2) 0.014 J (3) 0.01 J (4) 0.028 J

Ans. (2)

Sol. Work done, $W = (\Delta\vec{\mu}) \cdot \vec{B}$

$= 2 \times 10^{-2} \times 1 \cos(0.125)$

$= 0.02$ J

$\therefore$ correct answer is (2)

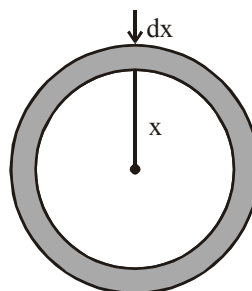**8.** To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is :

(1) $\dfrac{2}{3}\mu FR$ (2) $\mu FR/3$

(3) $\mu FR/2$ (4) $\mu FR/6$

Ans. (1)

Sol.

Consider a strip of radius x & thickness dx,

Torque due to friction on this strip.

$$\int d\tau = \int_0^R \frac{x\mu F \cdot 2\pi x dx}{\pi R^2}$$

$$\tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$

$\therefore$ correct answer is (1)

9. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At $t = 0$ it was 1600 counts per second and $t = 8$ seconds it was 100 counts per second. The count rate observed, as counts per second, at $t = 6$ seconds is close to :

(1) 150   (2) 360

(3) 200   (4) 400

Ans. (3)

Sol. at $t = 0$, $A_0 = \dfrac{dN}{dt} = 1600 \,C/s$

at $t = 8s$, $A = 100$ C/s

$\dfrac{A}{A_0} = \dfrac{1}{16}$ in 8 sec

Therefor half life is $t_{1/2} = 2$ sec

$\therefore$ Activity at $t = 6$ will be $1600\left(\dfrac{1}{2}\right)^3$

$= 200$ C/s

$\therefore$ correct answer is (3)

2
   E

10. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8/m/s)

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$$ then the

maximum electric field associated with it is :

(1) 4×10^4 N/C   (2) 4.5×10^4 N/C

(3) 6×10^4 N/C   (4) 3×10^4 N/C

Ans. (4)

Sol. $E_0 = B_0 \times C$

$= 100 \times 10^{-6} \times 3 \times 10^8$

$= 3 \times 10^4$ N/C

$\therefore$ correct answer is 3×10^4 N/C

11. A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be :

(1) $\dfrac{Q}{4\pi\varepsilon_0(a+b+c)}$

(2) $\dfrac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

(3) $\dfrac{Q}{12\pi\varepsilon_0}\dfrac{ab+bc+ca}{abc}$

(4) $\dfrac{Q}{4\pi\varepsilon_0}\dfrac{(a^2+b^2+c^2)}{(a^3+b^3+c^3)}$

Ans. (2)

Sol.
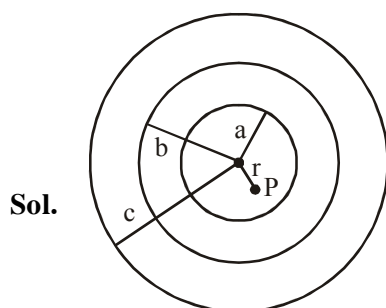
Potential at point P, $V = \dfrac{kQ_a}{a} + \dfrac{kQ_b}{b} + \dfrac{kQ_c}{c}$

$\because Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$

     {since $\sigma_a = \sigma_b = \sigma_c$}

$\therefore Q_a = \left[\dfrac{a^2}{a^2+b^2+c^2}\right]Q$

$Q_b = \left[\dfrac{b^2}{a^2+b^2+c^2}\right]Q$
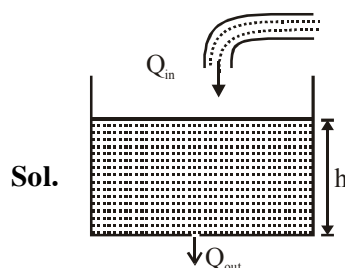
$Q_c = \left[\dfrac{c^2}{a^2+b^2+c^2}\right]Q$

$V = \dfrac{Q}{4\pi \in_0}\left[\dfrac{(a+b+c)}{a^2+b^2+c^2}\right]$

$\therefore$ correct answer is (2)

12. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^{3}s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is:

(1) 4 cm   (2) 2.9 cm   (3) 1.7 cm   (4) 5.1 cm

Ans. (4)

Sol.

Since height of water column is constant therefore, water inflow rate (Q_{in})

            = water outflow rate

$Q_{in} = 10^{-4}$ m^{3}s^{-1}

$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$10^{-4} = 10^{-4}\sqrt{20 \times h}$

$h = \dfrac{1}{20}$ m

$h = 5$cm

$\therefore$ correct answer is (4)

13. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : (g =10ms^{-2})

(1) 30 m   (2) 10 m   (3) 40 m   (4) 20 m

Ans. (3)

Sol.
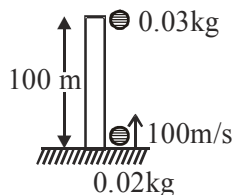
Time taken for the particles to collide,

$$t = \frac{d}{V_{rel}} = \frac{100}{100} = 1 \text{sec}$$

Speed of wood just before collision = gt = 10 m/s & speed of bullet just before collision v-gt = 100 – 10 = 90 m/s

Now, conservation of linear momentum just before and after the collision -

–(0.02) (1v) + (0.02) (9v) = (0.05)v

$\Rightarrow$ 150 = 5v

$\Rightarrow$ v = 30 m/s

Max. height reached by body h = $\frac{v^2}{2g}$

$$h = \frac{30 \times 30}{2 \times 10} = 45m$$

$\therefore$ Height above tower = 40 m

14. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is :

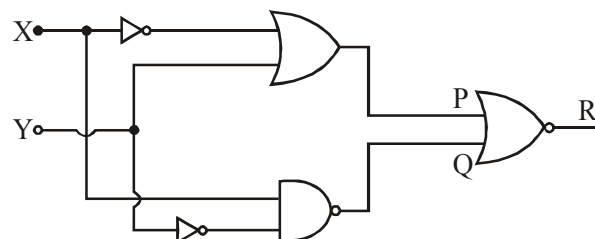(1) 410   (2) 640   (3) 16   (4) 40

Ans. (4)

Sol.

$$\frac{128kg}{m^3} = \frac{125(50g)(20)}{(25cm)^3(4)^3}$$

$$= \frac{128}{64}(20) \text{units}$$

= 40 units

15. To get output '1' at R, for the given logic gate circuit the input values must be :

(1) X = 0, Y = 1   (2) X =1 , Y = 1

(3) X = 0, Y = 0   (4) X = 1, Y = 0

Ans. (4)

Sol. $p = \bar{x} + y$

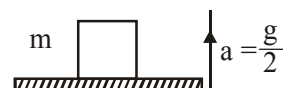$Q = \overline{\bar{y}.x} = y + \bar{x}$

$O/P = \overline{P+Q}$

To make O/P

P + Q must be 'O'

SO,   y = 0

x = 1

16. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. Work done by normal reaction on block in time t is :

(1) 0   (2) $\frac{3mg^2t^2}{8}$

(3) $-\frac{mg^2t^2}{8}$ (4) $\frac{mg^2t^2}{8}$

Ans. (2)

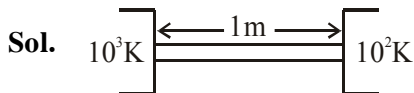**Sol.** $N - mg = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

Now, work done $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}gt^2\right)$

$\Rightarrow W = \frac{3mg^2t^2}{4}$

4

E

17. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is :

(1) 90 Wm^{-2} (2) 200 Wm^{-2}

(3) 65 Wm^{-2} (4) 120 Wm^{-2}

Ans. (1)

Sol.

$$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{\ell}$$

$$\Rightarrow \frac{1}{A}\left(\frac{dQ}{dt}\right) = \frac{(0.1)(900)}{1} = 90\,W/m^2$$

18. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS(Line of Sight) mode ? (Given : radius of earth = 6.4×10^6m).

(1) 80 km   (2) 48 km

(3) 40 km   (4) 65 km

Ans. (4)

Sol. Maximum distance upto which signal can be broadcasted is

$$d_{max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where h_T and h_R are heights of transmiter tower and height of receiver respectively.

Putting all values -

$$d_{max} = \sqrt{2 \times 6.4 \times 106}\left[\sqrt{104} + \sqrt{40}\right]$$

on solving, $d_{max} = 65$ km

19. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf $\varepsilon/2$ and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is :
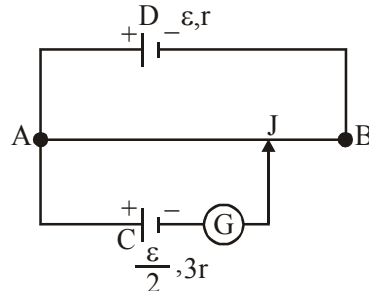
(1) $\frac{5}{12}L$ (2) $\frac{11}{24}L$ (3) $\frac{11}{12}L$ (4) $\frac{13}{24}L$

Ans. (4)

Sol. $i = \frac{\varepsilon}{13r}$

$$i\left(\frac{x}{L}12r\right) = \frac{\varepsilon}{2}$$

$$\frac{\varepsilon}{13r}\left[\frac{x}{L}.12r\right] = \frac{\varepsilon}{2} \Rightarrow \boxed{x = \frac{13L}{24}}$$

20. An insulating thin rod of length ℓ has a x linear charge density $p(x) = \rho_0\,\frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is :

(1) $\frac{\pi}{4}np\ell^3$ (2) $np\ell^3$ (3) $\pi np\ell^3$ (4) $\frac{\pi}{3}np\ell^3$

Ans. (1)

Sol. ∵ M = NIA

dq = λdx & A = πx^2

$$\int dm = \int (x)\frac{\rho_0 x}{\ell}dx.\pi x^2$$

$$M = \frac{n\rho_0\pi}{\ell}.\int_0^\ell x^3.dx = \frac{n\rho_0\pi}{\ell}.\left[\frac{L^4}{4}\right]$$

$$M = \frac{n\rho_0\pi\ell^3}{4}\ \text{ or }\ \frac{\pi}{4}np\ell^3$$

E

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21. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is :

(1) 1 : 2   (2) 1 : 4   (3) 1 : 8   (4) 1 : 16

Ans. (4)

Sol. $R = \dfrac{u^2 \sin 2\theta}{g}$

$A = \pi R^2$

$A \propto R^2$

$A \propto u^4$

$\dfrac{A_1}{A_2} = \dfrac{u_1^4}{u_2^4} = \left[\dfrac{1}{2}\right]^4 = \dfrac{1}{16}$

22. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The siring is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to :

(1) 16.6 cm   (2) 20.0 cm

(3) 10.0 cm   (4) 33.3 cm

Ans. (2)

Sol. Velocity of wave on string

$V = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{8}{5} \times 1000} = 40\,\text{m/s}$

Now, wavelength of wave $\lambda = \dfrac{v}{n} = \dfrac{40}{100}\,\text{m}$

Separation b/w successive nodes, $\dfrac{\lambda}{2} = \dfrac{20}{100}\,\text{m}$

$= 20\,\text{cm}$

23. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is :

(1) 18/17   (2) 19/18   (3) 20/19   (4) 21/20

Ans. (2)

Sol. $f_{app} = f_0 \left[\dfrac{v_2 \pm v_0}{v_2 \mp v_s}\right]$

$f_1 = f_0 \left[\dfrac{340}{340 - 34}\right]$

$f_2 = f_0 \left[\dfrac{340}{340 - 17}\right]$

$\dfrac{f_1}{f_2} = \dfrac{340 - 17}{340 - 34} = \dfrac{323}{306} \Rightarrow \dfrac{f_1}{f_2} = \dfrac{19}{18}$

24. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to :

(1) 100 keV   (2) 500 keV

(3) 25 keV   (4) 1 keV

Ans. (3)

Sol. $\lambda = \dfrac{h}{p}$ $\{\lambda = 7.5 \times 10^{-12}\}$
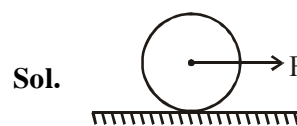
$P = \dfrac{h}{\lambda}$

$KE = \dfrac{P^2}{2m} = \dfrac{(h/\lambda)^2}{2m} = \dfrac{\left\{\dfrac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right\}}{2 \times 9.1 \times 10^{-31}}\,J$

$KE = 25\,\text{Kev}$

25. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is:

(1) $\dfrac{3F}{2m\,R}$ (2) $\dfrac{F}{3m\,R}$

(3) $\dfrac{2F}{3m\,R}$ (4) $\dfrac{F}{2m\,R}$

Ans. (3)

Sol.

$FR = \dfrac{3}{2}MR^2\alpha$

$\alpha = \dfrac{2F}{3MR}$

26. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as :

(1) $\mu_1 + \mu_2 = 3$

(2) $2\mu_1 - \mu_2 = 1$
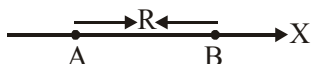
(3) $2\mu_2 - \mu_1 = 1$

(4) $3\mu_2 - 2\mu_1 = 1$

Ans. (2)

Sol. $\dfrac{1}{2f_2} = \dfrac{1}{f_1} = (\mu_1 - 1)\left(\dfrac{1}{\infty} - \dfrac{1}{-R}\right)$

$\dfrac{1}{f_2} = (\mu_2 - 1)\left(\dfrac{1}{-R} - \dfrac{1}{\infty}\right)$

$\dfrac{(\mu_1 - 1)}{R} = \dfrac{(\mu_2 - 1)}{2R}$

$2\mu_2 - \mu_2 = 1$

27. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ placed on the x-axis with a separation R, as shown in the figure

$$\underrightarrow{\quad R \quad}\!\!\underleftarrow{\qquad}$$
A   B   X

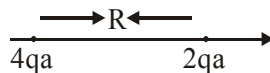The distance from A at which both of them produce the same potential is :

(1) $\dfrac{\sqrt{2}R}{\sqrt{2}+1}$

(2) $\dfrac{R}{\sqrt{2}+1}$

(3) $\dfrac{\sqrt{2}\,R}{\sqrt{2}-1}$

(4) $\dfrac{R}{\sqrt{2}-1}$

Ans. (3)

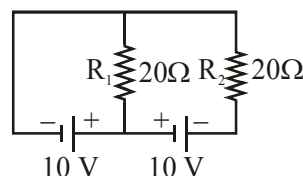**Sol.** $V = \dfrac{4qa}{(R+x)} = \dfrac{2qa}{\left(x^2\right)}$

$\sqrt{2}x = R + x$

$x = \dfrac{R}{\sqrt{2}-1}$

$\underrightarrow{\qquad}R\underleftarrow{\qquad}$
4qa   2qa

dist $= \dfrac{R}{\sqrt{2}-1} + R = \dfrac{\sqrt{2}R}{\sqrt{2}-1}$

28. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1, and R_2 respectively, are:
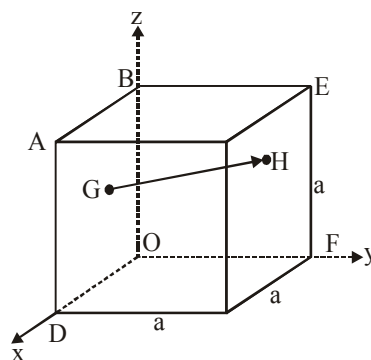

(1) 2, 2

(2) 0,1

(3) 1,2

(4) 0.5,0

Ans. (4)

Sol. $i_1 = \dfrac{10}{20} = 0.5A$

$i_2 = 0$

29. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be:

(1) $\dfrac{1}{2}a\left(\hat{i} - \hat{k}\right)$

(2) $\dfrac{1}{2}a\left(\hat{j} - \hat{i}\right)$

(3) $\dfrac{1}{2}a\left(\hat{k} - \hat{i}\right)$

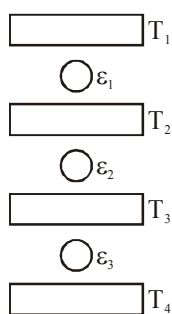(4) $\dfrac{1}{2}a\left(\hat{j} - \hat{k}\right)$

Ans. (2)

Sol. $\vec{r}_g = \dfrac{a}{2}\hat{i} + \dfrac{a}{2}\hat{k}$

$\vec{r}_H = \dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}$

$\vec{r}_H - \vec{r}_g = \dfrac{a}{2}\left(\hat{j} - \hat{i}\right)$

30. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2, and T_3, as shown, with $T_2 > T_2 > T_3 > T_4$. The three engines are equally efficient if:

(1) $T_2 = \left(T_1^2 T_4\right)^{1/3}; T_3 = \left(T_1 T_4^2\right)^{1/3}$

(2) $T_2 = \left(T_1 T_4^2\right)^{1/3}; T_3 = \left(T_1^2 T_4\right)^{1/3}$

(3) $T_2 = \left(T_1^3 T_4\right)^{1/4}; T_3 = \left(T_1 T_4^3\right)^{1/4}$

(4) $T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$

Ans. (1)

Sol. $t_1 = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{T_2}{T_2} = 1 - \dfrac{T_4}{T_3}$

$\Rightarrow \dfrac{T_2}{T_1} = \dfrac{T_3}{T_4} = \dfrac{T_4}{T_3}$

$\Rightarrow T_2 = \sqrt{T_1 T_3} = \sqrt{T_1 \sqrt{T_2 T_4}}$

$T_3 = \sqrt{T_2 T_4}$

$T_2^{3/4} = \sqrt{T_1^{1/2}}\, T_4^{1/4}$

$T_2 = T_1^{2/3} T_4^{1/3}$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019)   TIME : 9 : 30 AM To 12 : 30 PM
CHEMISTRY

1. Two pi and half sigma bonds are present in:

(1) N_2^+ (2) N_2 (3) O_2^+ (4) O_2

Ans. (1)

Sol.

$$N_2^{\oplus} \Rightarrow BO = 2.5 \Rightarrow \left[\pi - Bond = 2 \ \& \ \sigma - Bond = \frac{1}{2}\right]$$

$N_2 \Rightarrow B.O. = 3.0 \Rightarrow [\pi\text{-Bond} = 2 \ \& \ \sigma\text{-Bond} = 1]$

$O_2^{\oplus} = B.O. \Rightarrow 2.5 \Rightarrow [\pi\text{-Bond} = 1.5 \ \& \ \sigma\text{-Bond} = 1]$

$O_2 \Rightarrow B.O. \Rightarrow 2 \Rightarrow [\pi\text{-Bond} \Rightarrow 1 \ \& \ \sigma\text{-Bond} = 1]$

2. The chemical nature of hydrogen preoxide is :-

(1) Oxidising and reducing agent in acidic medium, but not in basic medium.

(2) Oxidising and reducing agent in both acidic and basic medium

(3) Reducing agent in basic medium, but not in acidic medium

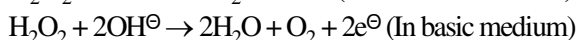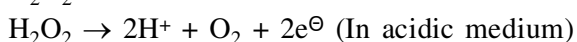(4) Oxidising agent in acidic medium, but not in basic medium.

Ans. (2)

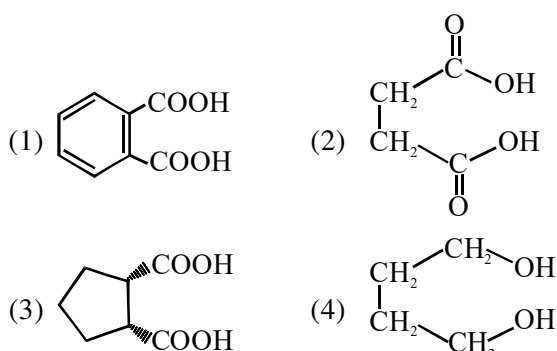**Sol.** H_2O_2 act as oxidising agent and reducing agent in acidic medium as well as basic medium.

H_2O_2 Act as oxidant :-

$H_2O_2 + 2H^{\oplus} + 2e^{\ominus} \rightarrow 2H_2O$ (In acidic medium)

$H_2O + 2e^{\ominus} \rightarrow 2OH^{\ominus}$ (In basic medium)

H_2O_2 Act as reductant :-

$H_2O_2 \rightarrow 2H^+ + O_2 + 2e^{\ominus}$ (In acidic medium)

$H_2O_2 + 2OH^{\ominus} \rightarrow 2H_2O + O_2 + 2e^{\ominus}$ (In basic medium)
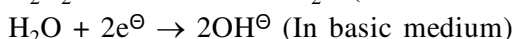
3. Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride :

(1)

(2)

(3)

(4)

Ans. (4)

Sol. Adipic acid $CO_2H–(CH_2)_4–CO_2H \xrightarrow[\text{agent}]{\text{dehydrating}}$

7 membered cyclic anhydride (Very unstable)

4. Which premitive unit cell has unequal edge lenghs (a ≠ b ≠ c) and all axial angles differetn from 90° ?

(1) Tetragonal   (2) Hexagonal

(3) Monoclinic   (4) Triclinic

Ans. (4)

Sol. In Triclinic unit cell

a ≠ b ≠ c & α ≠ β ≠ γ ≠ 90°

5. Wilkinson catalyst is :

(1) $[(Ph_3P)_3RhCl]$ (Et = C_2H_5)

(2) $[Et_3P)_3IrCl]$

(3) $[Et_3P)_3RhCl]$

(4) $[Ph_3P)_3IrCl]$
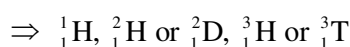
Ans. (1)

Sol. Wilkinsion catalyst is $[(ph_3P)_3RhCl]$

6. The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively, are :

(1) 2 and 0   (2) 3 and 2

(3) 3 and 1   (4) 2 and 1
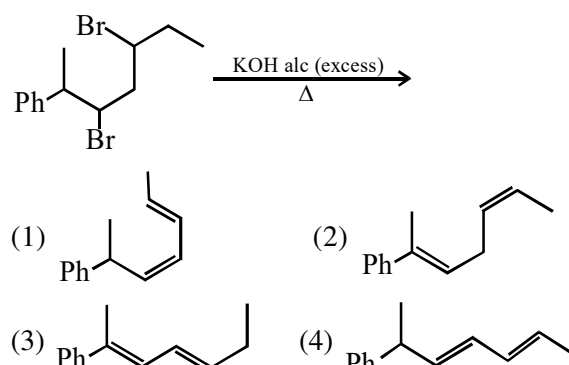
Ans. (3)

Sol. Total number of isotopes of hydrogen is 3

$\Rightarrow {}_1^1H, \ {}_1^2H \text{ or } {}_1^2D, \ {}_1^3H \text{ or } {}_1^3T$

and only ${}_1^3H \text{ or } {}_1^3T$ is an Radioactive element.

7. The major product of the following reaction is

(1)

(2)

(3)
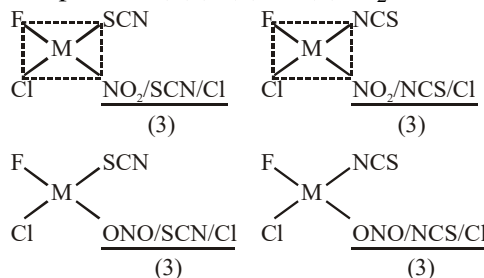
(4)

Ans. (3)

E

1

Sol. Example of E_2 elimination and conjugated diene is formed with phenyl ring in conjugation which makes it very stable.

8. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is :

(1) 12   (2) 8   (3) 16   (4) 4

Ans. (1)

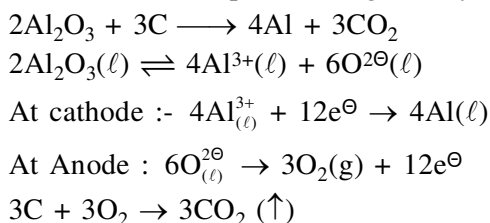**Sol.** The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is 12.

9. Hall-Heroult's process is given by "

(1) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(2) Cu^{2+} (aq.) $+ H_2(g) \rightarrow Cu(s) + 2H^+$ (aq)

(3) $ZnO + C \xrightarrow{Coke,1673K} Zn + CO$

(4) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

Ans. (4)

Sol. In Hall-Heroult's process is given by
$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$
$2Al_2O_3(\ell) \rightleftharpoons 4Al^{3+}(\ell) + 6O^{2\ominus}(\ell)$

At cathode :- $4Al^{3+}_{(\ell)} + 12e^{\ominus} \rightarrow 4Al(\ell)$

At Anode : $6O^{2\ominus}_{(\ell)} \rightarrow 3O_2(g) + 12e^{\ominus}$

$3C + 3O_2 \rightarrow 3CO_2 (\uparrow)$

10. The value of K_p/K_C for the following reactions at 300K are, respectively :
(At 300K, RT = 24.62 dm^{3}atm mol^{-1})

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
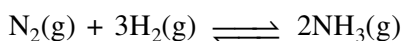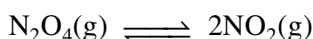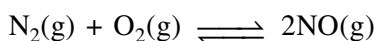
(1) 1, 24.62 dm^{3}atm mol^{-1}, 606.0 dm^{6}atm^{2}mol^{-2}

(2) 1, 4.1 × 10^{-2} dm^{-3}atm^{-1} mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}

(3) 606.0 dm^{6}atm^{2}mol^{-2}, 1.65 × 10^{-3} dm^{3}atm^{-2} mol^{-1}

(4) 1, 24.62 dm^{3}atm mol^{-1}, 1.65 × 10^{-3} dm^{-6}atm^{-2} mol^2

Ans. (4)

Sol. $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$$\frac{k_p}{k_c} = (RT)^{\Delta n_g} = (RT)^0 = 1$$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$$\frac{k_p}{k_c} = (RT)^1 = 24.62$$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
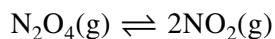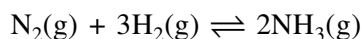
$$\frac{k_p}{k_c} = (RT)^{-2} = \frac{1}{(RT)^2} = 1.65 \times 10^{-3}$$

11. If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct ?

(1) DCM and H_2O would stay as lower and upper layer respectively in the S.F.

(2) DCM and H_2O will be miscible clearly

(3) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel (S.F.)

(4) DCM and H_2O will make trubid/colloidal mixture

Ans. (1)

12. The type of hybridisation and number of lone pair(s) of electrons of Xe in $XeOF_4$, respectively, are :

(1) sp^3d and 1

(2) sp^3d and 2

(3) sp^3d^2 and 1

(4) sp^3d^2 and 2

Ans. (3)

Sol. $sp^3d^2 \Rightarrow$ [5σ-bond +1 l.p.]
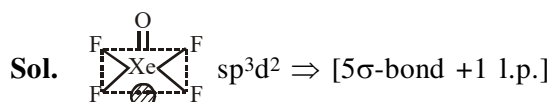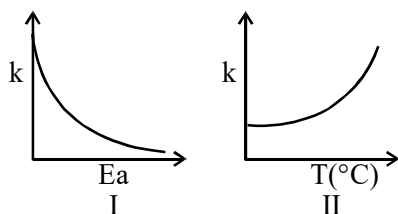
13. The metal used for making X-ray tube window is :

(1) Mg   (2) Na   (3) Ca   (4) Be

Ans. (4)

Sol. "Be" Metal is used in x-ray window is due to transparent to x-rays.

2

E

14. Consider the given plots for a reaction obeying Arrhenius equation ($0°C < T < 300°C$) : (k and E_a are rate constant and activation energy, respectively)

Choose the correct option :

(1) Both I and II are wrong

(2) I is wrong but II is right

(3) Both I and II are correct

(4) I is right but II is wrong

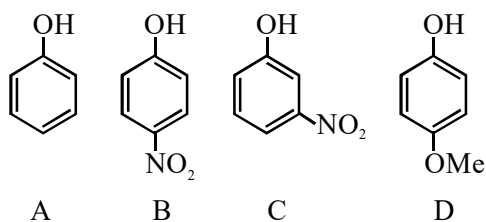**Ans. (4)**

Sol. On increasing E_a, K decreases

15. Water filled in two glasses A and B have BOD values of 10 and 20, respectively. The correct statement regarding them, is :

(1) A is more polluted than B

(2) A is suitable for drinking, whereas B is not

(3) B is more polluted than A

(4) Both A and B are suitable for drinking

Ans. (3)

Sol. Two glasses "A" and "B" have BOD values 10 and "20", respectively.

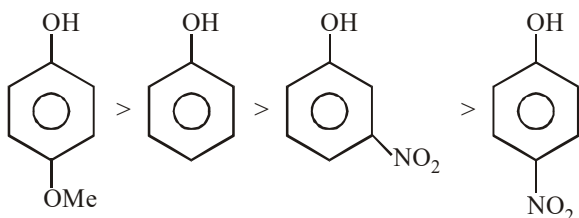Hence glasses "B" is more polluted than glasses "A".

16. The increasing order of the pKa values of the following compounds is :

(1) D < A < C < B   (2) B < C < D < A

(3) C < B < A < D   (4) B < C < A < D

Ans. (4)

Sol. Acidic strength is inversely proportional to pka.

17. Liquids A and B form an ideal solution in the entire composition range. At 350 K, the vapor pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively. The composition of the vapor in equilibrium with a solution containing 40 mole percent of A at this temperature is :

(1) $x_A = 0.37$; $x_B = 0.63$

(2) $x_A = 0.28$; $x_B = 0.72$

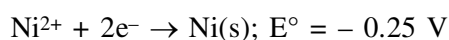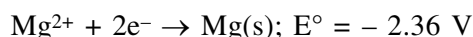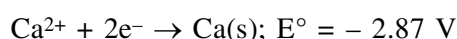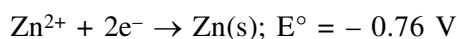(3) $x_A = 0.76$; $x_B = 0.24$

(4) $x_A = 0.4$; $x_B = 0.6$

Ans. (2)

Sol. $y_A = \dfrac{P_A}{P_{Total}} = \dfrac{P_A^o x_A}{P_A^o x_A \times p_B^o x_B}$

$= \dfrac{7 \times 10^3 \times 0.4}{7 \times 10^3 \times 0.4 + 12 \times 10^3 \times 0.6}$

$= \dfrac{2.8}{10} = 0.28$

$y_B = 0.72$

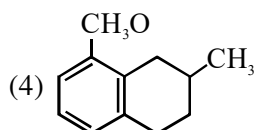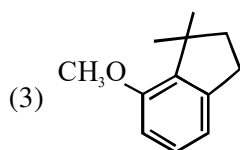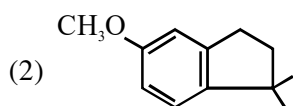**18.** Consider the following reduction processes :

$Zn^{2+} + 2e^- \rightarrow Zn(s)$; $E° = -0.76$ V

$Ca^{2+} + 2e^- \rightarrow Ca(s)$; $E° = -2.87$ V

$Mg^{2+} + 2e^- \rightarrow Mg(s)$; $E° = -2.36$ V

$Ni^{2+} + 2e^- \rightarrow Ni(s)$; $E° = -0.25$ V

The reducing power of the metals increases in the order :

(1) Ca < Zn < Mg < Ni

(2) Ni < Zn < Mg < Ca

(3) Zn < Mg < Ni < Ca
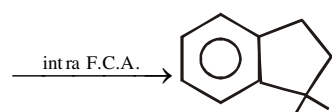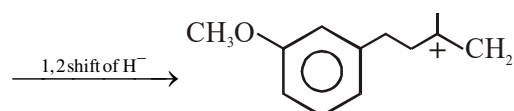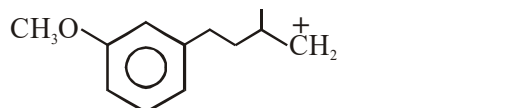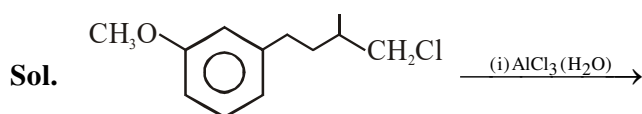
(4) Ca < Mg < Zn < Ni

Ans. (2)

Sol. Higher the oxidation potential better will be reducing power.

E

3

19. The major product of the following reaction is:

$$\xrightarrow[\text{(ii) H}_2\text{O}]{\text{(i) AlCl}_3 \text{(anhyd.)}}$$

(1)

(2)

(3)

(4)

Ans. (2)

Sol.

$$\xrightarrow{\text{(i) AlCl}_3 \text{(H}_2\text{O)}}$$

$$\xrightarrow{1,2 \text{ shift of H}^-}$$

$$\xrightarrow{\text{intra F.C.A.}}$$
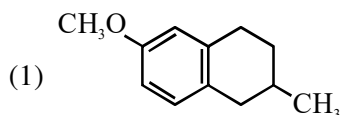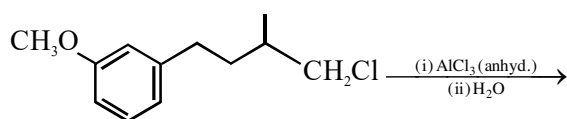
20. The electronegativity of aluminium is similar to :

(1) Boron   (2) Carbon
(3) Lithium   (4) Beryllium

Ans. (4)

Sol. E.N. of Al = (1.5) ≈ Be (1.5)

21. The decreasing order of ease of alkaline hydrolysis for the following esters is :

I

II

III

IV
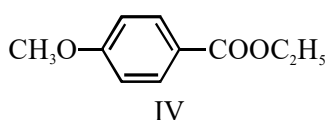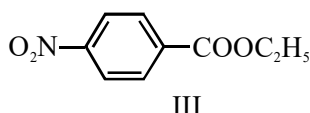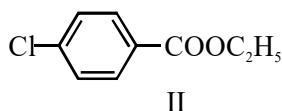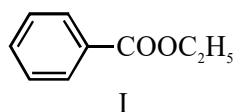
(1) IV > II > III > I

(2) III > II > I > IV

(3) III > II > IV > I

(4) II > III > I > IV

Ans. (2)

Sol. More is the electrophilic character of carbonyl group of ester faster is the alkaline hydrolysis.

22. A process has $\Delta H = 200$ Jmol^{-1} and

$\Delta S = 40$ JK^{-1}mol^{-1}. Out of the values given below, choose the minimum temperature above which the process will be spontaneous :

(1) 5 K

(2) 4 K

(3) 20 K

(4) 12 K

Ans. (1)

Sol. $\Delta G = \Delta H - T\Delta S$

$$T = \frac{\Delta H}{\Delta S} = \frac{200}{40} = 5K$$

4

E

23. Which of the graphs shown below does not represent the relationship between incident light and the electron ejected form metal surface ?

(1)

number of e⁻s vs Frequency of light

(2)

K.E. of e⁻s vs Intensity of light

(3)

K.E. of e⁻s vs Frequency of light

(4)

K.E. of e⁻s vs energy of light
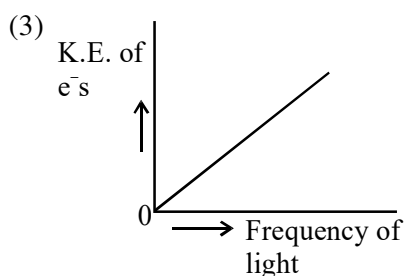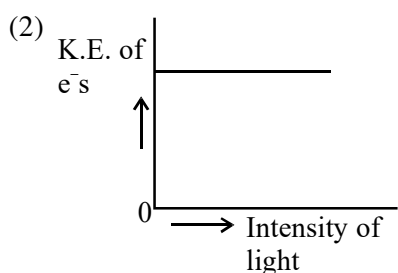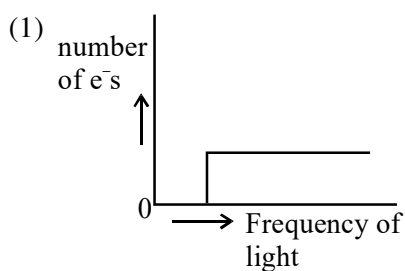
Ans. (3)

Sol. $E = W + \dfrac{1}{2} mv^2$

K.E. $= h\nu - 4\nu_0$

K.E. $= h\nu + (-h\nu_0)$

$y = mx + \underline{C}$

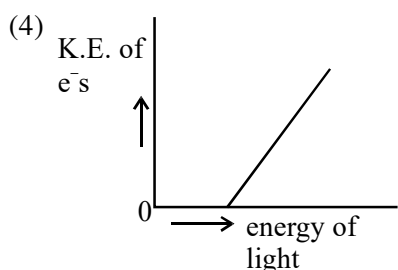**24.** Which of the following is not and example of heterogeneous catalytic reaction ?

(1) Ostwald's process

(2) Haber's process

(3) Combustion of coal

(4) Hydrogenation of vegetable oils

Ans. (3)

Sol. Then is no catalyst is required for combustion of coal.

25. The effect of lanthanoid contraction in the lanthanoid series of elements by and large means :

(1) decrease in both atomic and ionic radii

(2) increase in atomic radii and decrease in ionic radii

(3) increase in both atomic and ionic radii

(4) decrease in atomic radii and increase in ionic radii

Ans. (1)

Sol. Due to Lanthanoid contraction both atomic radii and ionic radii decreases gradually in the lanthanoid series.

26. The major product formed in the reaction given below will be :

(1)

(2)

(3)

(4)
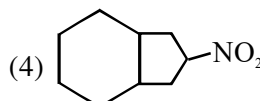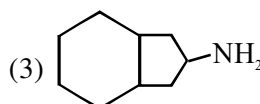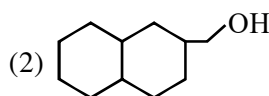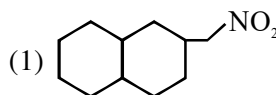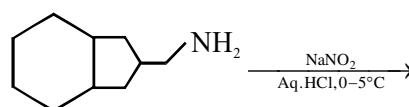
Ans. (Bonus)

Sol. Answer should be

27. The correct structure of product 'P' in the following reaction is :

Asn-Ser + $(CH_3CO)_2O \xrightarrow[\text{(excess)}]{NEt_3}$ P
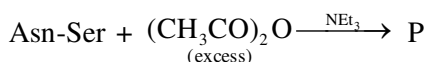
(1)

(2)

(3)

(4)

Ans. (1)

Sol. Asn–Ser is dipeptide having following structure

$$NH_2 - \underset{\underset{\underset{CONH_2}{|}}{\overset{\overset{|}{CH_2}}{|}}{C}} - \overset{O}{\overset{\|}{C}} - NH - \underset{}{\overset{\overset{CH_2OH}{|}}{CH}} - CO_2H$$

$$Asn - Ser + (CH_3CO)_2\,O \xrightarrow[\text{excess}]{NEt_3} P$$
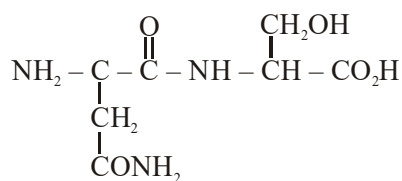
P is
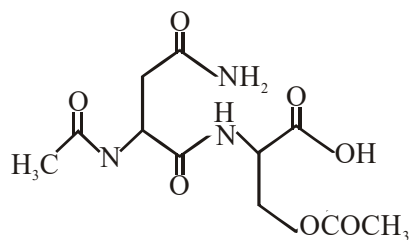
28. Which hydrogen in compound (E) is easily replaceable during bromination reaction in presence of light :

$$\underset{\delta}{CH_3} - \underset{\gamma}{CH_2} - \underset{\beta}{CH} = \underset{\alpha}{CH_2}$$
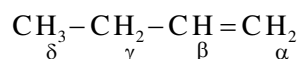
(1) β – hydrogen
(2) γ – hydrogen
(3) δ – hydrogen
(4) α – hydrogen

Ans. (2)

29. The major product 'X' formed in the following reaction is :

 $\xrightarrow[\text{MeOH}]{NaBH_4}$ X
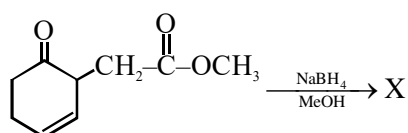
(1)

(2)

(3)

(4)

Ans. (4)

30. A mixture of 100 m mol of $Ca(OH)_2$ and 2g of sodium sulphate was dissolved in water and the volume was made up to 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively, are : (Molar mass of $Ca(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1}, respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})

(1) 1.9 g, 0.14 mol L^{-1}

(2) 13.6 g, 0.14 mol L^{-1}

(3) 1.9 g, 0.28 mol L^{-1}

(4) 13.6 g, 0.28 mol L^{-1}

Ans. (3)

Sol. $Ca(OH)_2 + Na_2SO_4 \longrightarrow CaSO_4 + 2NaOH$

100 m mol 14 m mol   —   —

—   —   14 m mol   28 m mol
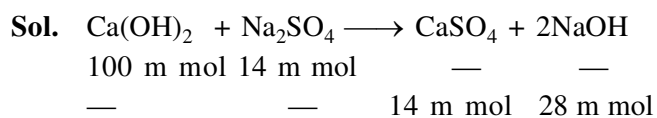
$$w_{CaSO_4} = 14 \times 10^{-3} \times 136 = 1.9gm$$

$$[OH^-] = \frac{28}{100} = 0.28M$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019
(Held On Thursday 10ᵗʰ JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM
MATHEMATICS

1. Consider a triangular plot ABC with sides AB=7m, BC=5m and CA=6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:

(1) $7\sqrt{3}$ (2) $\frac{2}{3}\sqrt{21}$ (3) $\frac{3}{2}\sqrt{21}$ (4) $2\sqrt{21}$

Ans. (2)

Sol.
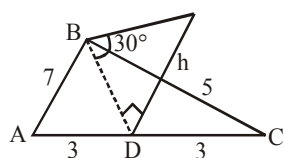
BD = hcot30° = $h\sqrt{3}$

So, $7^2 + 5^2 = 2(h\sqrt{3})^2 + 3^2)$

$\Rightarrow$ 37 = 3h² + 9.

$\Rightarrow$ 3h² = 28

$\Rightarrow$ $h = \sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}$

2. Let f : R→R be a function such that f(x) = x³+x²f'(1) + xf''(2)+f'''(3), x∈R.
Then f(2) equal :

(1) 8   (2) –2   (3) –4   (4) 30

Ans. (2)

Sol. $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2x f'(1) + f''(x)$ .....(1)

$\Rightarrow f''(x) = 6x + 2f'(1)$ .....(2)

$\Rightarrow f'''(x) = 6$ .....(3)

put x = 1 in equation (1) :

$f'(1) = 3 + 2f'(1) + f''(2)$ .....(4)

put x = 2 in equation (2) :

$f''(2) = 12 + 2f'(1)$ .....(5)

from equation (4) & (5) :

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow 3f'(1) = -15$

$\Rightarrow f'(1) = -5 \Rightarrow f''(2) = 2$ ....(2)

put x = 3 in equation (3) :

$f'''(3) = 6$

$\therefore$ $f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

3. If a circle C passing through the point (4,0) touches the circle x² + y² + 4x − 6y = 12 externally at the point (1, −1), then the radius of C is :

(1) $\sqrt{57}$ (2) 4   (3) $2\sqrt{5}$ (4) 5

Ans. (4)

Sol. x² + y² + 4x − 6y − 12 = 0

Equation of tangent at (1, −1)

x − y + 2(x + 1) − 3(y − 1) − 12 = 0

3x − 4y − 7 = 0

$\therefore$ Equation of circle is

(x² + y² + 4x − 6y − 12) + λ(3x − 4y − 7) = 0

It passes through (4, 0) :

(16 + 16 − 12) + λ(12 − 7) = 0

$\Rightarrow$ 20 + λ(5) = 0

$\Rightarrow$ λ = −4

$\therefore$ (x² + y² + 4x − 6y − 12) − 4(3x − 4y − 7) = 0

or x² + y² − 8x + 10y + 16 = 0

Radius = $\sqrt{16 + 25 - 16} = 5$

4. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and theose whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :

(1) 102   (2) 42   (3) 1   (4) 38

Ans. (4)

Sol. Let n(A) = number of students opted Mathematics = 70,

n(B) = number of students opted Physics = 46,

n(C) = number of students opted Chemistry = 28,

n(A ∩ B) = 23,

$n(B \cap C) = 9,$

$n(A \cap C) = 14,$

$n(A \cap B \cap C) = 4,$

Now $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
$- n(A \cap C) + n(A \cap B \cap C)$

$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

So number of students not opted for any course

$= \text{Total} - n(A \cup B \cup C)$

$= 140 - 102 = 38$

5. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :

(1) 1365   (2) 1256   (3) 1465   (4) 1356

Ans. (4)

Sol. $\displaystyle\sum_{r=2}^{13}(7r+2) = 7.\frac{2+13}{2}\times 6 + 2\times 12$

$= 7 \times 90 + 24 = 654$

$\displaystyle\sum_{r=1}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right)\times 13 + 5\times 13 = 702$

Total $= 654 + 702 = 1356$

6. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is :-

(1) $\left(\frac{1}{2}, 4, -2\right)$ (2) $\left(-\frac{1}{2}, 4, 0\right)$

(3) $(1, 3, 1)$ (4) $(1, 5, 1)$

Ans. (2)

Sol. $4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$

$\Rightarrow 3 - \lambda_2 = 2\lambda_1 \Rightarrow 2\lambda_1 + \lambda_2 = 3$ ....(1)

Given $\vec{a}.\vec{c} = 0$

$\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$\Rightarrow 2\lambda_1 + \lambda_3 = -1$ .....(2)

Now $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$

Now check the options, option (2) is correct

7. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is :

(1) $x - y + 9 = 0$

(2) $x - y + 7 = 0$

(3) $x - y + 1 = 0$

(4) $x - y - 3 = 0$

Ans. (3)

Sol. Hyperbola $\dfrac{x^2}{5} - \dfrac{y^2}{4} = 1$

slope of tangent = 1

equation of tangent $y = x \pm \sqrt{5-4}$

$\Rightarrow y = x \pm 1$

$\Rightarrow y = x + 1$ or $y = x - 1$

8. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k>0)$, is 1 square unit. Then k is:

(1) $\dfrac{1}{\sqrt{3}}$ (2) $\dfrac{2}{\sqrt{3}}$ (3) $\dfrac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

Ans. (1)

Sol. Area bounded by $y^2 = 4ax$ & $x^2 = 4by$, $a, b \neq 0$

is $\left|\dfrac{16ab}{3}\right|$

by using formula : $4a = \dfrac{1}{k} = 4b, k > 0$

Area $= \left|\dfrac{16.\dfrac{1}{4k}.\dfrac{1}{4k}}{3}\right| = 1$

$\Rightarrow k^2 = \dfrac{1}{3}$

$\Rightarrow k = \dfrac{1}{\sqrt{3}}$

9. Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then S:

(1) is an empty set

(2) equals $\{-2, -1, 1, 2\}$

(3) equals $\{-2, -1, 0, 1, 2\}$

(4) equals $\{-2, 2\}$

Ans. (3)

Sol. $f(x) = \begin{cases} 8 + 2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \le x \le 2 \\ 8 - 2x, & 2 < x \le 4 \end{cases}$
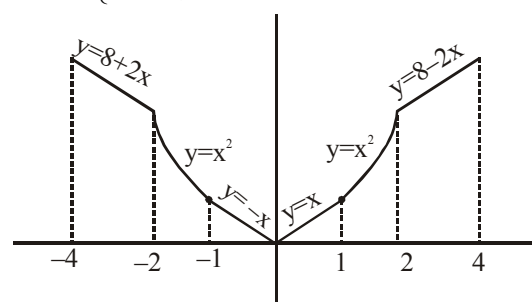
$f(x)$ is not differentiable at $x = \{-2, -1, 0, 1, 2\}$

$\Rightarrow S = \{-2, -1, 0, 1, 2\}$

2

E

10. If the parabolas $y^2=4b(x-c)$ and $y^2=8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c)

(1) $(1, 1, 0)$ (2) $\left(\dfrac{1}{2},2,3\right)$

(3) $\left(\dfrac{1}{2},2,0\right)$ (4) $(1, 1, 3)$

Ans. (1,2,3,4)

Sol. Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$,

$y = mx - 4am - 2am^3$

If they have a common normal

$(c + 2b) m + bm^3 = 4am + 2am^3$

Now $(4a - c - 2b) m = (b - 2a)m^3$

We get all options are correct for $m = 0$

(common normal x-axis)

Ans. (1), (2), (3), (4)

Remark :

If we consider question as

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal other than x-axis, then which one of the following is a valid choice for the ordered triad (a, b, c) ?

When $m \neq 0$: $(4a - c - 2b) = (b - 2a)m^2$

$m^2 = \dfrac{c}{2a - b} - 2 > 0 \Rightarrow \dfrac{c}{2a - b} > 2$

Now according to options, option 4 is correct

11. The sum of all values of $\theta \in \left(0, \dfrac{\pi}{2}\right)$ satisfying

$\sin^2 2\theta + \cos^4 2\theta = \dfrac{3}{4}$ is :

(1) $\dfrac{\pi}{2}$ (2) π (3) $\dfrac{3\pi}{8}$ (4) $\dfrac{5\pi}{4}$

Ans. (1)

Sol. $\sin^2 2\theta + \cos^4 2\theta = \dfrac{3}{4}$, $\theta \in \left(0, \dfrac{\pi}{2}\right)$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \dfrac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta = \dfrac{1}{2} = \cos^2 \dfrac{\pi}{4}$

$\Rightarrow 2\theta = n\pi \pm \dfrac{\pi}{4}$, $n \in I$

$\Rightarrow \theta = \dfrac{n\pi}{2} \pm \dfrac{\pi}{8}$

$\Rightarrow \theta = \dfrac{\pi}{8}, \dfrac{\pi}{2} - \dfrac{\pi}{8}$

Sum of solutions $\dfrac{\pi}{2}$

12. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4 |z_2|$.

If $z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1}$ then :

(1) $|z| = \dfrac{1}{2}\sqrt{\dfrac{17}{2}}$ (2) $Re(z) = 0$

(3) $|z| = \sqrt{\dfrac{5}{2}}$ (4) $Im(z) = 0$

Ans. (Bonus)

Sol. $3|z_1| = 4|z_2|$

$\Rightarrow \dfrac{|z_1|}{|z_2|} = \dfrac{4}{3}$

$\Rightarrow \dfrac{|3z_1|}{|2z_2|} = 2$

Let $\dfrac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$

$z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1} = a + \dfrac{1}{a}$

$= \dfrac{5}{2}\cos\theta + \dfrac{3}{2}i\sin\theta$

Now all options are incorrect

Remark :

There is a misprint in the problem actual problem should be :

"Let z_1 and z_2 be any non-zero complex number such that $3|z_1| =$ **2**$|z_2|$.

If $z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1}$, then"

Given

$3|z_1| = 2|z_2|$

Now $\left|\dfrac{3z_1}{2z_2}\right| = 1$

E

3

Let $\dfrac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1}$

$= a + \dfrac{1}{a} = 2\cos\theta$

$\therefore$ Im$(z) = 0$

Now option (4) is correct.

13. If the system of equations

$x+y+z = 5$

$x+2y+3z = 9$

$x+3y+\alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals:

(1) 5   (2) 18   (3) 21   (4) 8

Ans. (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha-1 \end{vmatrix} = (\alpha-1)-4 = (\alpha-5)$

for infinite solutions $D = 0 \Rightarrow \alpha = 5$

$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta-15 & -2 & 5 \end{vmatrix} = 0$

$\Rightarrow 2+\beta-15 = 0 \Rightarrow \beta-13 = 0$

on $\beta = 13$ we get $D_y = D_z = 0$

$\alpha = 5,\ \beta = 13$

14. The shortest distance between the point $\left(\dfrac{3}{2},0\right)$ and the curve $y = \sqrt{x},(x > 0)$ is :

(1) $\dfrac{\sqrt{5}}{2}$ (2) $\dfrac{5}{4}$ (3) $\dfrac{3}{2}$ (4) $\dfrac{\sqrt{3}}{2}$

Ans. (1)

Sol. Let points $\left(\dfrac{3}{2},0\right)$, (t^2, t), $t > 0$
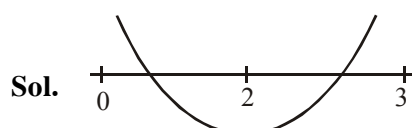
Distance $= \sqrt{t^2 + \left(t^2 - \dfrac{3}{2}\right)^2}$

$= \sqrt{t^4 - 2t^2 + \dfrac{9}{4}} = \sqrt{(t^2-1)^2 + \dfrac{5}{4}}$

So minimum distance is $\sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

15. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval $(0,2)$ and its other root lies in the interval $(2,3)$. Then the number of elements in S is :

(1) 11   (2) 18   (3) 10   (4) 12

Ans. (1)

Sol.

Let $f(x) = (c-5)x^2 - 2cx + c - 4$

$\therefore f(0)f(2) < 0$ .....(1)

& $f(2)f(3) < 0$ .....(2)

from (1) & (2)

$(c-4)(c-24) < 0$

& $(c-24)(4c-49) < 0$

$\Rightarrow \dfrac{49}{4} < c < 24$

$\therefore s = \{13, 14, 15, ... 23\}$

Number of elements in set $S = 11$

16. $\displaystyle\sum_{i=1}^{20}\left(\dfrac{^{20}C_{i-1}}{^{20}C_i + {}^{20}C_{i-1}}\right) = \dfrac{k}{21}$, then k equals :

(1) 200   (2) 50   (3) 100   (4) 400

Ans. (3)

Sol. $\displaystyle\sum_{i=1}^{20}\left(\dfrac{^{20}C_{i-1}}{^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \dfrac{k}{21}$

$\Rightarrow \displaystyle\sum_{i=1}^{20}\left(\dfrac{^{20}C_{i-1}}{^{21}C_i}\right)^3 = \dfrac{k}{21}$

$\Rightarrow \displaystyle\sum_{i=1}^{20}\left(\dfrac{i}{21}\right)^3 = \dfrac{k}{21}$

$\Rightarrow \dfrac{1}{(21)^3}\left[\dfrac{20(21)}{2}\right]^2 = \dfrac{k}{21}$

$\Rightarrow 100 = k$

4

E

17. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is :

(1) –7

(2) $2(\sqrt{2}+2)$

(3) –5

(4) $2(\sqrt{2}+1)$

Ans. (3)

Sol. $\det A = \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & -\sin\theta+2+2d \end{vmatrix}$

$(R_1 \to R_1 + R_3 - 2R_2)$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & 2+2d-\sin\theta \end{vmatrix}$

$= (2+\sin\theta)(2+2d-\sin\theta) - d(2\sin\theta - d)$

$= 4 + 4d - 2\sin\theta + 2\sin\theta + 2d\sin\theta - \sin^2\theta - 2d\sin\theta + d^2$

$= d^2 + 4d + 4 - \sin^2\theta$

$= (d+2)^2 - \sin^2\theta$

For a given d, minimum value of

$\det(A) = (d+2)^2 - 1 = 8$

$\Rightarrow d = 1$ or -5

18. If the third term in the binomial expansion of

$\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is :

(1) $2\sqrt{2}$ (2) $\dfrac{1}{8}$ (3) $4\sqrt{2}$ (4) $\dfrac{1}{4}$

Ans. (4)

Sol. $(1+x^{\log_2 x})^5$

$T_3 = {}^5C_2 \cdot (x^{\log_2 x})^2 = 2560$

$\Rightarrow 10 \cdot x^{2\log_2 x} = 2560$

$\Rightarrow x^{2\log_2 x} = 256$

$\Rightarrow 2(\log_2 x)^2 = \log_2 256$

$\Rightarrow 2(\log_2 x)^2 = 8$

$\Rightarrow (\log_2 x)^2 = 4 \quad \Rightarrow \quad \log_2 x = 2$ or -2
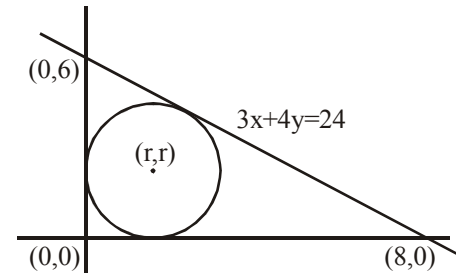
$x = 4$ or $\dfrac{1}{4}$

19. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is

(1) (3, 4)   (2) (2, 2)   (3) (4, 4)   (4) (4, 3)

Ans. (2)

Sol.

$\left| \dfrac{3r + 4r - 24}{5} \right| = r$

$7r - 24 = \pm 5r$

$2r = 24$ or $12r + 24$

$r = 14, \qquad r = 2$

then incentre is (2, 2)

20. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is :

(1) 4 : 9 (2) 6 : 7

(3) 5 : 8 (4) 10 : 3

Ans. (1)

Sol. Let two observations are x_1 & x_2

mean $= \dfrac{\sum x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13$ ....(1)

variance $(\sigma^2) = \dfrac{\sum x_i^2}{5} - 25 = 9.20$

$\Rightarrow \sum x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97$ .....(2)

by (1) & (2)

$(x_1 + x_2)^2 - 2x_1 x_2 = 97$

or $x_1 x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9$

E

21. A point P moves on the line $2x - 3y + 4 = 0$. If Q(1,4) and R(3,–2) are fixed points, then the locus of the centroid of ΔPQR is a line :

(1) parallel to x-axis 　　(2) with slope $\dfrac{2}{3}$

(3) with slope $\dfrac{3}{2}$ 　　(4) parallel to y-axis

Ans. (2)

Sol. Let the centroid of ΔPQR is (h, k) & P is (α, β), then

$$\frac{\alpha + 1 + 3}{3} = h \quad \text{and} \quad \frac{\beta + 4 - 2}{3} = k$$

$\alpha = (3h - 4)$ 　　　$\beta = (3k - 4)$

Point P(α, β) lies on line $2x - 3y + 4 = 0$

$\therefore \ 2(3h - 4) - 3(3k - 2) + 4 = 0$

$\Rightarrow$ locus is $6x - 9y + 2 = 0$

22. If $\dfrac{dy}{dx} + \dfrac{3}{\cos^2 x} y = \dfrac{1}{\cos^2 x}, x \in \left(\dfrac{-\pi}{3}, \dfrac{\pi}{3}\right)$, and

$y\left(\dfrac{\pi}{4}\right) = \dfrac{4}{3}$, then $y\left(-\dfrac{\pi}{4}\right)$ equals :

(1) $\dfrac{1}{3} + e^6$ 　　　　(2) $\dfrac{1}{3}$

(3) $-\dfrac{4}{3}$ 　　　　(4) $\dfrac{1}{3} + e^3$

Ans. (1)

Sol. $\dfrac{dy}{dx} + 3\sec^2 x.y = \sec^2 x$

I.F. $= e^{3\int \sec^2 x dx} = e^{3\tan x}$

or $y.e^{3\tan x} = \int \sec^2 x.e^{3\tan x} dx$

or $y.e^{3\tan x} = \dfrac{1}{3}e^{3\tan x} + C$ 　　　　....(1)

Given

$y\left(\dfrac{\pi}{4}\right) = \dfrac{4}{3}$

$\therefore \ \dfrac{4}{3}.e^3 = \dfrac{1}{3}e^3 + C$

$\therefore \ C = e^3$

Now put $x = -\dfrac{\pi}{4}$ in equation (1)

$\therefore \ y.e^{-3} = \dfrac{1}{3}e^{-3} + e^3$

$\therefore \ y = \dfrac{1}{3} + e^6$

$\therefore \ y\left(-\dfrac{\pi}{4}\right) = \dfrac{1}{3} + e^6$

23. The plane passing through the point (4, –1, 2) and parallel to the lines $\dfrac{x+2}{3} = \dfrac{y-2}{-1} = \dfrac{z+1}{2}$

and $\dfrac{x-2}{1} = \dfrac{y-3}{2} = \dfrac{z-4}{3}$ also passes through the point :

(1) (–1, –1, –1) 　　　　(2) (–1, –1, 1)

(3) (1, 1, –1) 　　　　(4) (1, 1, 1)

Ans. (4)

Sol. Let $\vec{n}$ be the normal vector to the plane passing through (4, –1, 2) and parallel to the lines L_1 & L_2

then $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore \ \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$\therefore$ Equation of plane is

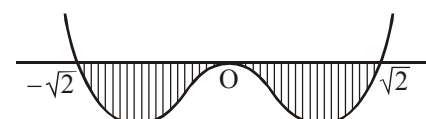$-1(x - 4) - 1(y + 1) + 1(z - 2) = 0$

$\therefore \ x + y - z - 1 = 0$

Now check options

24. Let $I = \int_a^b (x^4 - 2x^2)dx$. If I is minimum then the ordered pair (a, b) is :

(1) $\left(-\sqrt{2}, 0\right)$ 　　　　(2) $\left(-\sqrt{2}, \sqrt{2}\right)$

(3) $\left(0, \sqrt{2}\right)$ 　　　　(4) $\left(\sqrt{2}, -\sqrt{2}\right)$

Ans. (2)

Sol. Let $f(x) = x^2(x^2 - 2)$

As long as $f(x)$ lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is $(-\sqrt{2}, \sqrt{2})$ for minimum of I

25. If 5, 5r, 5r² are the lengths of the sides of a triangle, then r cannot be equal to :

(1) $\dfrac{3}{2}$ (2) $\dfrac{3}{4}$ (3) $\dfrac{5}{4}$ (4) $\dfrac{7}{4}$

Ans. (4)

Sol. r = 1 is obviously true.

Let 0 < r < 1

$\Rightarrow$ r + r² > 1

$\Rightarrow$ r² + r − 1 > 0

$\left(r-\dfrac{-1-\sqrt5}{2}\right)\left(r-\left(\dfrac{-1+\sqrt5}{2}\right)\right)$

$\Rightarrow$ $r-\dfrac{-1-\sqrt5}{2}$ or $r>\dfrac{-1+\sqrt5}{2}$

$r\in\left(\dfrac{\sqrt5-1}{2},1\right)$

$\dfrac{\sqrt5-1}{2}<r<1$

When r > 1

$\Rightarrow$ $\dfrac{\sqrt5+1}{2}>\dfrac{1}{r}>1$

$\Rightarrow$ $r\in\left(\dfrac{\sqrt5-1}{2},\dfrac{\sqrt5+1}{2}\right)$

Now check options

26. Consider the statement : "P(n): n² − n + 41 is prime." Then which one of the following is true?

(1) P(5) is false but P(3) is true

(2) Both P(3) and P(5) are false

(3) P(3) is false but P(5) is true

(4) Both P(3) and P(5) are true

Ans. (4)

Sol. P(n) : n² − n + 41 is prime

P(5) = 61 which is prime

P(3) = 47 which is also prime

27. Let A be a point on the line

$\vec{r}=(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overline{AB}$ is parallel to the plane x − 4y + 3z = 1 is :

(1) $\dfrac{1}{2}$ (2) $-\dfrac{1}{4}$ (3) $\dfrac{1}{4}$ (4) $\dfrac{1}{8}$

Ans. (3)

Sol. Let point A is

$(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$

and point B is (3, 2, 6)

then $\vec{AB}=(2+3\mu)\hat{i}+(3-\mu)\hat{j}+(4-5\mu)\hat{k}$

which is parallel to the plane x − 4y + 3z = 1

∴ 2 + 3μ − 12 + 4μ + 12 − 15μ = 0

8μ = 2

$\mu=\dfrac{1}{4}$

28. For each t∈R, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1+}\dfrac{(1-|x|+\sin|1-x|)\sin\left(\dfrac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(1) equals −1   (2) equals 1

(3) does not exist   (4) equals 0

Ans. (4)

Sol. $$\lim_{x\to 1^+}\dfrac{(1-|x|+\sin|1-x|)\sin\left(\dfrac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$=\lim_{x\to 1^+}\dfrac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\dfrac{\pi}{2}(-1)\right)$$

$$=\lim_{x\to 1^+}\left(1-\dfrac{\sin(x-1)}{(x-1)}\right)(-1)=(1-1)(-1)=0$$

29. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

(1) $\dfrac{13}{36}$　　(2) $\dfrac{19}{36}$　　(3) $\dfrac{19}{72}$　　(4) $\dfrac{15}{72}$

Ans. (3)

Sol.

$$P(A) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$$

30. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$.

Then $\displaystyle\int \frac{(\sin^n \theta - \sin\theta)^{\frac{1}{n}} \cos\theta}{\sin^{n+1}\theta} d\theta$ is equal to :

(Where C is a constant of integration)

(1) $\dfrac{n}{n^2-1}\left(1 - \dfrac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}} + C$

(2) $\dfrac{n}{n^2+1}\left(1 - \dfrac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(3) $\dfrac{n}{n^2-1}\left(1 - \dfrac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(4) $\dfrac{n}{n^2-1}\left(1 + \dfrac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

Ans. (3)

Sol. $\displaystyle\int \frac{(\sin^n \theta - \sin\theta)^{1/n} \cos\theta}{\sin^{n+1}\theta} d\theta$

$$= \int \frac{\sin\theta\left(1 - \dfrac{1}{\sin^{n-1}\theta}\right)^{1/n} \cos\theta}{\sin^{n+1}\theta} d\theta$$

Put $1 - \dfrac{1}{\sin^{n-1}\theta} = t$

So $\dfrac{(n-1)}{\sin^n\theta}\cos\theta d\theta = dt$

Now $\dfrac{1}{n-1}\int (t)^{1/n} dt$

$$= \frac{1}{(n-1)} \frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{1}{(n-1)}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{1}{n}+1} + C$$