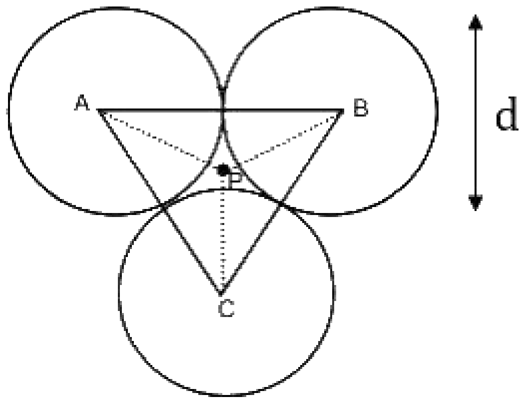
JEE Main 2020 Paper

Date of Exam: **9th January (Shift I)**
Time: **9:30 am – 12:30 pm**
Subject: **Physics**

1. Three identical solid spheres each have mass ‘m’ and diameter ‘d’ are touching each other as shown in the figure. Calculate ratio of moment of inertia about the axis perpendicular to plane of paper and passing through point P and B as shown in the figure. Given P is centroid of the triangle



a. $\frac{13}{23}$
b. $\frac{13}{15}$
c. $\frac{15}{13}$
d. $\frac{23}{13}$

Solution: (a)

Moment of Inertia of solid sphere $= \frac{2}{5} M \left(\frac{d}{2}\right)^2$

Distance of centroid ($Point\ P$) from centre of sphere $= \left(\frac{2}{3} \times \frac{\sqrt{3}d}{2}\right) = \frac{d}{\sqrt{3}}$

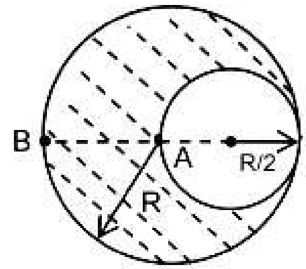
By Parallel axis theorem,

Moment of Inertia about $P = 3\left[\frac{2}{5} M \left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10} M\, d^2$

Moment of Inertia about $B = 2\left[\frac{2}{5} M \left(\frac{d}{2}\right)^2 + M(d)^2\right] + \frac{2}{5} M \left(\frac{d}{2}\right)^2 = \frac{23}{10} M\, d^2$

Now ratio $= \frac{13}{23}$

2. A solid sphere having a radius R and uniform charge density ρ. If a sphere of radius R/2 is carved out of it as shown in the figure. Find the ratio of the magnitude of electric field at point A and B



a. $\frac{17}{54}$

b. $\frac{18}{54}$

c. $\frac{18}{34}$

d. $\frac{21}{34}$

Solution: (c)

For solid sphere,

Field inside sphere, E = $\frac{\rho r}{3\epsilon_0}$ & field outside sphere, E = $\frac{\rho R^3}{3r^2\epsilon_0}$ where r is distance from centre and R is radius of sphere

Electric field at A due to sphere of radius R (sphere 1) is zero and therefore, net electric field will be because of sphere of radius $\frac{R}{2}$ (sphere 2) having charge density $(-\rho)$

$$E_A = \frac{-\rho R}{2(3\epsilon_0)}$$

$$|E_A| = \frac{\rho R}{6\varepsilon_0}$$

Similarly, Electric field at point $B = E_B = E_{1B} + E_{2B}$

E_{1B} = Electric Field due to solid sphere of radius $R = \frac{\rho R}{3\varepsilon_0}$

E_{2B} = Electric Field due to solid sphere of radius $\frac{R}{2}$ which having charge density $(-\rho)$

$$= -\frac{\rho\left(\frac{R}{2}\right)^3}{3\left(\frac{3R}{2}\right)^2 \varepsilon_0} = -\frac{\rho R}{54\varepsilon_0}$$

$$E_B = E_{1A} + E_{2A} = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17\rho R}{54\varepsilon_0}$$

$$\frac{|E_A|}{|E_B|} = \frac{9}{17} = \frac{18}{34}$$

3. Consider an infinitely long current carrying cylindrical straight wire having radius 'a'. Then the ratio of magnetic field due to wire at distance a/3 and 2a, respectively from axis of wire is

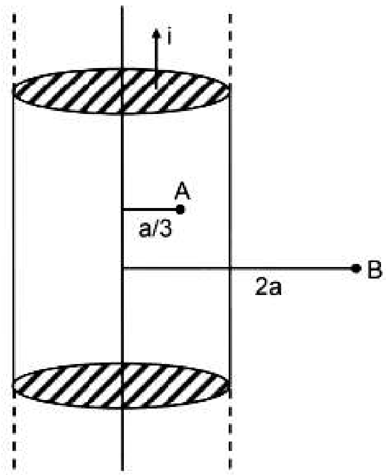
a. 3/2

b. 2/3

c. 2

d. 1/2

Solution: (b)

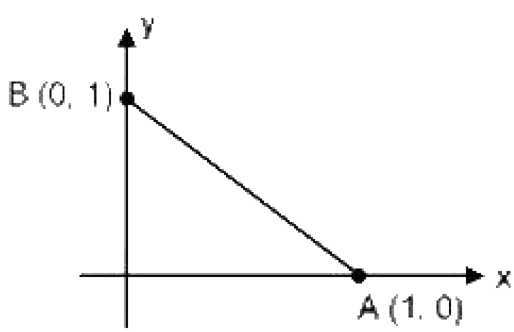


$$B_A = \frac{\mu_0 ir}{2\pi a^2} = \frac{\frac{\mu_0 ia}{3}}{2\pi a^2} = \frac{\mu_{0i}}{\pi a^2}\frac{a}{6} = \frac{\mu_{0i}}{6\pi a}$$

$$B_B = \frac{\mu_0 i(2a)}{2\pi(2a)^2} = \frac{\mu_0 i}{4\pi a}$$

$$\frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$$

4. Particle moves from point A to point B along the line shown in figure under the action of force $\vec{F} = -x\,\hat{\imath} + y\,\hat{\jmath}$. Determine the work done on the particle by $\vec{F}$ in moving the particle from point A to point B (all quantities are in SI units)



a. 1 J

b. 1/2 J

c. 2 J

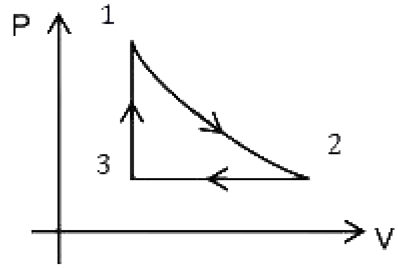
d. 3/2 J

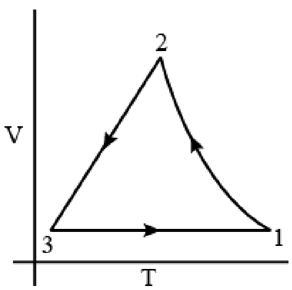
Solution: (a)

$$d\,\vec{s} = (dx\,\hat{\imath} + dy\,\hat{\jmath})$$

$$= (-x\,\hat{i} + y\,\hat{j}) . (dx\,\hat{i} + dy\,\hat{j})$$
$$= \int_1^0 -x\,dx + \int_0^1 ydy$$
$$= -\frac{x^2}{2}|_1^0 + \frac{y^2}{2}|_0^1 = \frac{1}{2} + \frac{1}{2} = 1\,J$$

5. For the given $P - V$ graph of an ideal gas, chose the correct $V - T$ graph. Process BC is adiabatic. (Graphs are schematic and not to scale)



a.


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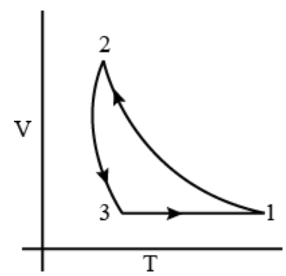
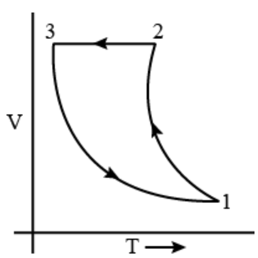
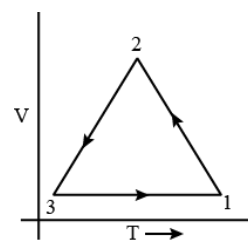


c.


d.


Solution: (a)
For process $3 - 1$; Volume is constant;
For process $1 - 2$, PV^{γ} = Constant & $PV = nRT$, therefore $TV^{\gamma-1}$ = Constant ; therefore as V increases T decreases and also relation is non linear, so curve will not be a straight line.
For process $2 - 3$; pressure is constant , therefore $V = kT$
From above, correct answer is option a.

6. An electric dipole of moment $\vec{p} = (-\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) \times 10^{-29}$ Cm is at the origin (0,0,0). The electric field due to this dipole at $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ is parallel to [Note that $\vec{r}\,.\vec{p} = 0$]

a. $\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$
b. $-\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$
c. $+\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$
d. $-\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

Solution: (c)

The electric dipole of moment $\vec{p} =$ q. a where a is distance between charge.
Electric field $(\overrightarrow{E})$ at position $\vec{r}$ is given by $\frac{2K\,\vec{p}.\vec{r}}{|r|^4}$ along radial direction and $\frac{2K\,\vec{p}\times\vec{r}}{|r|^4}$ along tangential direction , where $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k} - (0\,\hat{\imath} + 0\,\hat{\jmath} + 0\,\hat{k}) = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$
Since already in question, $\vec{p}\,.\vec{r} = 0$, this means field is along tangential direction and dipole is also perpendicular to radius vector.
Since electric field and dipole are along same line, we can write $\vec{E} = \lambda\,(\vec{p})$
where λ is an arbitrary constant
From option, on putting $\lambda =$ -1× 10^{29} , we get, $\vec{E} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

7. A body A of mass m is revolving around a planet in a circular orbit of radius R. At the instant the particle B has velocity $\vec{V}$, another particle of mass $\frac{m}{2}$ moving at velocity of $\frac{\vec{V}}{2}$ collides perfectly inelastically with the first particle. Then, the combined body

a. Fall vertically downward towards the planet.
b. Continue to move in a circular orbit
c. Escape from the Planet's Gravitational field
d. Start moving in an elliptical orbit around the planet

Solution: (d)

By conservation of linear momentum and taking velocity inline for maximum momentum transfer in single direction.
$\frac{m}{2}\frac{V}{2} + mV = (m + \frac{m}{2})V_f$
$V_f = \frac{5V}{6}$, where V is orbital velocity
Escape velocity will be $\sqrt{2}V$ and at velocity less than escape velocity but greater than orbital velocity (V), the path will be elliptical. At orbital velocity (V), path will be circular. At velocity less than orbital velocity path will remain part of ellipse and it will

either orbit in elliptical path whose length of semi major axis will be less than radius of circular orbit or start falling down and collide with the planet but it will not fall vertically down as path will remain part of ellipse. Hence the resultant mass will start moving in an elliptical orbit around the planet.

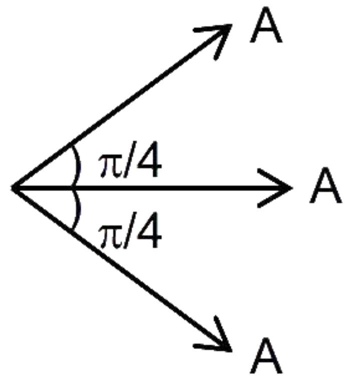
8. Two particles of equal mass m have respective initial velocities $\overrightarrow{u_1} = u\,\hat{\imath}$ and $\overrightarrow{u_2} = \frac{u}{2}\hat{\imath} + \frac{u}{2}\hat{\jmath}$. They collide completely inelastically. Find the loss in kinetic energy.
   a. $\frac{3mu^2}{4}$
   b. $\frac{\sqrt{2}mu^2}{\sqrt{3}}$
   c. $\frac{mu^2}{3}$
   d. $\frac{mu^2}{8}$

Solution: (d)

Let v_1 and v_2 be the final velocities after collision in x and y direction respectively.
Conserving linear momentum
$mu\hat{\imath} + m(\frac{u}{2}\hat{\imath} + \frac{u}{2}\hat{\jmath}) = 2m(v_1\hat{\imath} + v_2\hat{\jmath})$
By equating $\hat{\imath}$ and $\hat{\jmath}$
$v_1 = \frac{3u}{4}$ and $v_2 = \frac{u}{4}$
Initial K.E $= \frac{mv^2}{2} + \frac{m}{2} \times (\frac{u}{\sqrt{2}})^2 = \frac{3mu^2}{4}$
Final K.E $= \frac{2m}{2} \times (\frac{u\sqrt{10}}{4})^2 = \frac{5mu^2}{8}$
Change in KE =
$\frac{3mu^2}{4}$ - $\frac{5mu^2}{8} = \frac{mu^2}{8}$

9. Three harmonic waves of same frequency (v) and intensity (I_0) having initial phase angles $0, \frac{\pi}{4}, -\frac{\pi}{4}$ rad respectively. When they are superimposed, the resultant intensity is close to
   a. $5.8\,I_0$
   b. I_0
   c. $3\,I_0$
   d. $0.2\,I_0$

   Solution: (a)



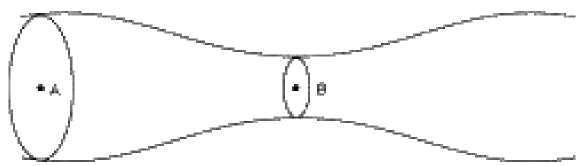
Amplitudes can be added using vector addition

$A_{resultant} = (\sqrt{2} + 1)A$

Since, $I \propto A^2$, Where I is intensity.

Therefore, $I_{res} = (\sqrt{2} + 1)^2 I_0 = 5.8\ I_0$ (approx.)

10. An ideal liquid (water) flowing through a tube of non-uniform cross-sectional area, where area at A and B are 40 cm^2 and 20 cm^2 respectively. If pressure difference between A & B is 700 N/m^2, then volume flow rate is (density of water = $1000\ kgm^{-3}$)



a. 2720 cm^3/s
b. 2420 cm^3/s
c. 1810 cm^3/s
d. 3020 cm^3/s

Solution: (a)

Using equation of continuity

$V_A \times Area_A = V_B \times Area_B$

$40V_A = 20V_B$

$2V_A = V_B$

Using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$\Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$$

$$\Delta P = 500 \times \frac{3V_B^2}{4}$$

$$V_B = \sqrt{\frac{(\Delta P)\times 4}{1500}} = \sqrt{\frac{(700)\times 4}{1500}} = \sqrt{\frac{28}{15}}\,\mathrm{m/s}$$

Volume flow rate $= \mathrm{V_B} \times \mathrm{Area_B} = (20 \times 100 \times \sqrt{\frac{28}{15}})\ \mathrm{cm}^3/\mathrm{s} = 2732.5\ \mathrm{cm}^3/\mathrm{s}$

So, answer comes nearly 2720 cm^3/s

11. A screw gauge advances by 3 mm on main scale in 6 rotations. There are 50 divisions on circular scale. Find least count of screw gauge?

a. 0.01 cm
b. 0.001 cm
c. 0.001 mm
d. 0.02 mm

Solution: (b)

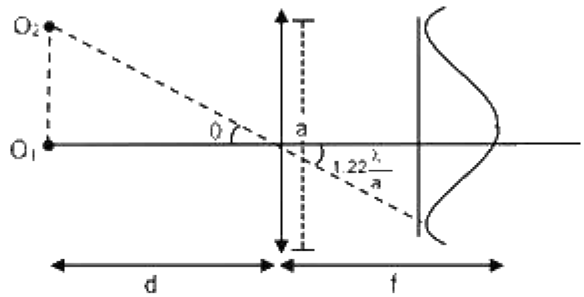
Pitch $= \frac{3}{6} = 0.5$ mm

Least count $= \frac{\text{Pitch}}{\textit{Number of divisions}} = \frac{0.5\text{mm}}{50} = \frac{1}{100}\,\text{mm} = 0.01\ \text{mm} = 0.001\ \text{cm}$

12. A telescope of aperture diameter 5 m is used to observe the moon from the earth. Distance between the moon and earth is 4×10^5 km. The minimum distance between two points on the moon's surface which can be resolved using this telescope is close to (Wavelength of light is 5500 Å)

a. 60 m
b. 20 m
c. 600 m
d. 200 m

Solution: (a)



Minimum angle for clear resolution,

$$\theta = 1.22\frac{\lambda}{a}$$

$$distance = O_1O_2 = d\theta$$

$$= 1.22\frac{\lambda}{a}d$$

$$\text{Distance} = O_1O_2 = \frac{1.22\times5500\times10^{-10}\times4\times10^{8}}{5} = 53.68\ m$$

∴ Nearest option is 60 m

13. Radiation with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of largest circular path followed by electron is 10 mm, the work function of metal is close to

a. 1.8 eV
b. 0.8 eV
c. 1.1 eV
d. 1.6 eV

Solution: (c) (Challenged question)
From photoelectric equation,

$$\frac{hc}{\lambda} = W + K.E_{max}$$

Where, hc = 12400 eV Å

$$\Rightarrow \frac{12400}{6561} = W + K.E_{max}$$

$$\Rightarrow 1.89\ eV = W + K.E_{max} - - - - - (1)$$

Radius of charged particle moving in a magnetic field is given by

$r = \frac{mv}{qB}$ and $\frac{1}{2}mv^2 = K.E_{max} = eV$

$\Rightarrow r = \frac{\sqrt{\frac{2eV}{m}} \times m}{eB} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$

$\Rightarrow 10^{-2} = \frac{1}{3 \times 10^{-4}}\sqrt{\frac{2 \times 9.1 \times 10^{-31} \times V}{1.6 \times 10^{-19}}}$

$\Rightarrow V = 0.8\ V$

So, $K.E_{max} = 0.8\ eV$

Substituting in (1),

$1.89 = W + 0.8$

i.e. $W = 1.1\ eV$ (approx)

14. Kinetic energy of the particle is E and it's de–Broglie wavelength is λ. On increasing its K.E by ΔE, it's new de–Broglie wavelength becomes $\frac{\lambda}{2}$. Then ΔE is

a. 3E
b. E
c. 2E
d. 4E

Solution: (a)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m(KE)}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\frac{\lambda}{\lambda/2} = \sqrt{\frac{KE_f}{KE_i}}$$

$$4KE_i = KE_f$$

$$\Rightarrow \Delta E = KE_f - KE_i = 4KE_i - KE_i = 3KE_i = 3E$$

15. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G is universal gravitational constant and h is the Planck's constant. Dimension of f is that of

a. area
b. energy
c. volume
d. Momentum

Solution: (b)

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow\ hc = E\,\lambda$$
$$Since, [E] = [ML^2T^{-2}]$$

Therefore,

$$[hc] = [ML^3T^{-2}]$$
$$[c] = [LT^{-1}]$$
$$[G] = [[M^{-1}L^3T^{-2}]$$
$$\left[\sqrt{\frac{hc^5}{G}}\right] = [ML^2T^{-2}]$$

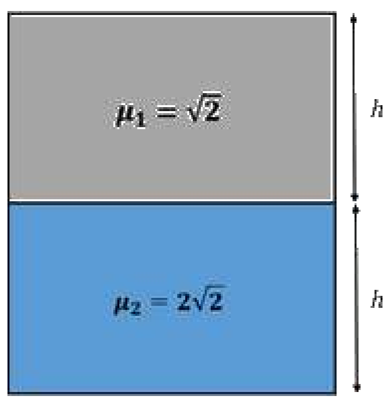
The above dimension is of energy.

16. A vessel of depth $2h$ is half filled with a liquid of refractive index $\sqrt{2}$ in upper half and with a liquid of refractive index $2\sqrt{2}$ in lower half. The liquids are immiscible. The apparent depth of inner surface of the bottom of the vessel will be

a. $\frac{3h\sqrt{2}}{4}$
b. $\frac{h}{\sqrt{2}}$
c. $\frac{h}{3\sqrt{2}}$
d. $\frac{h}{2(\sqrt{2}+1)}$

Solution: (a)

Assume, air is present outside container



Apparent height as seen from liquid 1 (having refractive index $\mu_1 = \sqrt{2}$) to liquid 2 (refractive index $\mu_2 = 2\sqrt{2}$)

D = $\frac{h\mu_1}{\mu_2} = \frac{h}{2}$

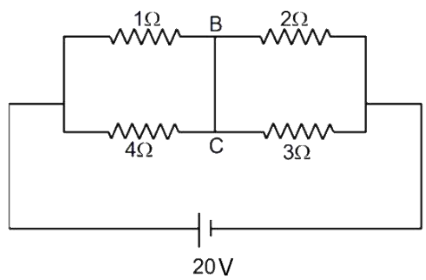
Now, actual height perceived from air, h $+ \frac{h}{2} = \frac{3h}{2}$

Therefore, apparent depth of bottom surface of the container (apparent depth as seen from air (having refractive index $\mu_0 = 1$) to liquid 1(having refractive index $\mu_1 = \sqrt{2}$)

$= \frac{3h}{2} \times \frac{\mu_0}{\mu_1}$

$= \frac{3h}{2} \times \frac{1}{\sqrt{2}} = \frac{3h}{2\sqrt{2}} = \frac{3\sqrt{2}h}{4}$

17. In the given circuit diagram, a wire is joining point B & C. Find the current in this wire



a. 0.4 A

b. 2 A

c. zero

d. 4 A

Solution: (b)

Since resistance 1 Ω and 4 Ω are in parallel

$\therefore R' = \frac{4\times1}{4+1} = \frac{4}{5}$

Similarly we can find equivalent resistance (R'') for resistances 2 Ω and 3 Ω

$\Rightarrow R'' = \frac{6}{5}$

And R' and R'' are in series

$\therefore\ R_{eff} = \frac{4}{5} + \frac{6}{5} = 2\ \Omega$

So total current flowing in the circuit 'i' can be given as

$$i = \frac{V}{R_{eff}} = \frac{20}{2} = 10\ A$$

Current will distribute in ratio opposite to resistance.

So, distribution will be as

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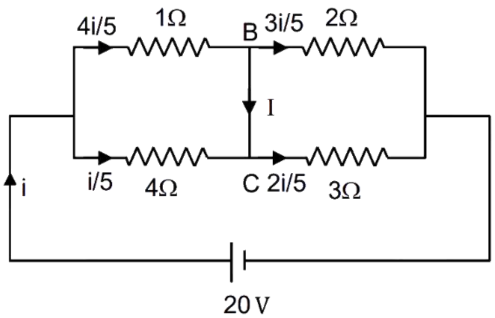


So current in the branch BC will be

$$I = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = \frac{10}{5} = 2\ A$$

18. Two plane electromagnetic waves are moving in vacuum in whose electric field vectors are given by $\vec{E}_1 = E_o\hat{\jmath}\ \cos(kx - \omega t)$ and $\vec{E}_2 = E_o\hat{k}\cos(ky - \omega t)$. At t = 0 A charge q is at origin with velocity $\vec{v} = 0.8c\ \hat{\jmath}$ (c is speed of light in vacuum). The instantaneous force on this charge (all data are in SI units)

a. $qE_o(0.4\ \hat{\imath} - 3\ \hat{\jmath} + 0.8\ \hat{k})$
b. $qE_o(0.8\ \hat{\imath} + \hat{\jmath} + 0.2\ \hat{k})$
c. $qE_o(0.8\ \hat{\imath} - \hat{\jmath} + 0.4\ \hat{k})$
d. $qE_o(-0.8\ \hat{\imath} + \hat{\jmath} + \hat{k})$

Solution: (b)

Given that the magnetic field vectors are:

$$\vec{E}_1 = E_o\hat{\jmath}\cos(kx - \omega t)$$
$$\vec{E}_2 = E_o\hat{k}\cos(ky - \omega t)$$

Since, the variation of $\vec{E}_1$ & $\vec{E}_2$ is along x and y respectively. Therefore, direction of propagation of $\vec{E}_1$ & $\vec{E}_2$ will be along x and y respectively. Since, $\vec{E} \times \vec{B}$ gives direction of propagation & $\frac{E_o}{B_0} = c$ and variation of magnetic field will be same to magnetic field.

So, the magnetic field vectors of the electromagnetic wave are given by

$$\vec{B}_1 = \frac{E_o}{c}\hat{k}\cos(kx - \omega t)$$
$$\vec{B}_2 = \frac{E_o}{c}\hat{\imath}\cos(ky - \omega t)$$

Then force is

$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

$= q(\vec{E}_1 + \vec{E}_2) + q(\vec{v} \times (\vec{B}_1 + \vec{B}_2)$

Now if we put the values of $\vec{E}_1, \vec{E}_2, \vec{B}_1 and\ \vec{B}_2$ we can get the net Lorentz force as

$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

Putting values and solving we get

$\vec{F} = qE_o[\cos(kx - \omega t)\,\hat{j} + (\cos ky - \omega t)\,\hat{k} + 0.8\cos(kx - \omega t)\,\hat{\imath} - 0.8(\cos ky - \omega t)\,\hat{k}]$

$\vec{F} = qE_o[0.8\cos(kx - \omega t)\,\hat{\imath} + \cos(kx - \omega t)\,\hat{j} + 0.2(\cos ky - \omega t)\,\hat{k}]$

Now at $t = 0$ and $x = y = 0$ we get

$\vec{F} = qE(0.8\,\hat{\imath} + \hat{j} + 0.2\,\hat{k})$

19. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibration mode and have a mass $\frac{m}{4}$. The ratio of molar specific heat at constant volume of gas A and B is

a. 7/9
b. 5/9
c. 3/5
d. 5/7

Solution: (d)

We know that,

Molar heat capacity at constant volume, $C_V = \frac{fR}{2}$ (Where f is degree of freedom)

Since, A is diatomic and rigid, degree of freedom for A is 5

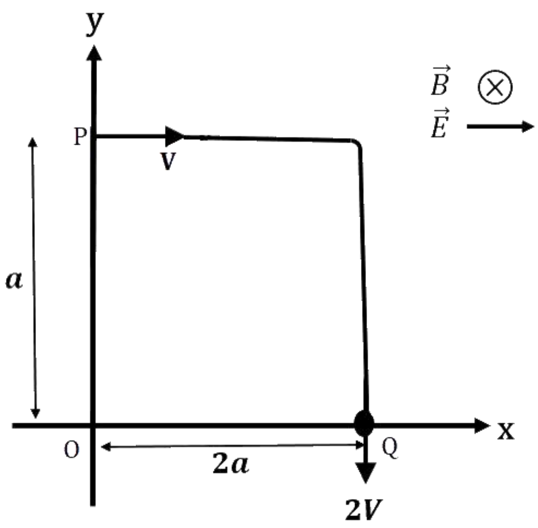
Therefore, Molar heat capacity of A at constant volume$(C_V)_A = \frac{5R}{2}$

Since, B is diatomic and has extra degree of freedom because of vibration; degree of freedom for B is $5 + 2\times 1 = 7$ (1 vibration for each atom).

Therefore, Molar heat capacity of B at constant volume$(C_V)_B = \frac{7R}{2}$

Ratio of molar specific heat of A and B= $\frac{(C_V)_A}{(C_V)_B} = \frac{5}{7}$

20. A charged particle of mass $'m'$ and charge $'q'$ is moving under the influence of uniform electric field $E\ \hat{\imath}$ and a uniform magnetic field B $\hat{k}$ follow a trajectory from P to Q as shown in figure. The velocities at P and Q are respectively $v\,\hat{\imath}$ and $-2v\,\hat{j}$. Then which of the following statements (A, B, C, D) are correct? (Trajectory shown is schematic and not to scale)



A. Magnitude of electric field $\vec{E} = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$
B. Rate of work done by electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$
C. Rate of work done by both fields at Q is zero
D. The difference between the magnitude of angular momentum of the particle at P and Q is $2mva$

a. A, C and D are correct
b. A, B and C are correct
c. A, B, C and D are correct
d. B, C and D are correct

Solution: (b)

Considering statement A
Let, Net work done by magnetic field be W_B and net work done by electric field be W_E
By Work-Energy theorem

$$W_B + W_E = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$

$$\Rightarrow \quad 0 + qE_o 2a = \frac{3}{2}mv^2$$

$$E_o = \frac{3}{4}\frac{mv^2}{qa}$$

So, statement A is correct
Now, considering statement B
Rate of work done at P = Power of electric force

$$= qE_o v$$

$$= \frac{3}{4}\frac{mv^3}{a}$$

So, statement B is correct

Now, considering statement C

At Q,

$\vec{E} \perp \vec{v}$ and already $\vec{B} \perp \vec{v}$

So, $\frac{dw}{dt} = 0$ for both forces as $\frac{dw}{dt} = q(\vec{E} + \vec{v} \times \vec{B}).\vec{v}$

So, statement C is correct

Now, considering statement D

Angular momentum should be defined about a point which is not given in question but let's find angular momentum about origin.

Change in magnitude of angular momentum of the particle at P and Q about origin

$\Delta\vec{L} = \Delta\overrightarrow{L_P} - \Delta\overrightarrow{L_q}$

$\overrightarrow{L_q} = m(2v)(2a)$

$\overrightarrow{L_p} = m(v)(a)$

Hence, $\Delta L = 3mva$

So, statement D is wrong

21. In a fluorescent lamp choke (a small transformer) 100 V of reversible voltage is produced when choke changes current in from 0.25 A to 0 A in 0.025 ms. The self-inductance of choke (in mH) is estimated to be

Solution: (10)

Fluorescent lamp choke will behave as an inductor

By using faraday law to write induced emf,

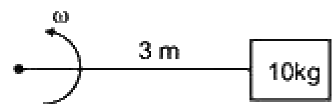
$$\in -\mathrm{L}\frac{dI}{dt} = 0$$

$$\Rightarrow 100 = \frac{L(0.25)}{0.025} \times 10^3$$

$$L = 100 \times 10^{-4}\ \mathrm{H}$$
$$= 10\ \mathrm{mH}$$

22. A wire of length l = 0.3 m and area of cross section 10^{-2} cm^2 and breaking stress 4.8×10^7 N/m^2 is attached with block of mass 10 kg. Find the maximum possible value of angular velocity (rad/s) with which block can be moved in a circle with string fixed at one end.

Solution: (4)



Breaking stress

$$\sigma = \frac{T}{A}$$

$$T = m\omega^2 l$$

$$\Rightarrow \sigma = \frac{m\omega^2 l}{A}$$

$$\Rightarrow \omega^2 = \frac{\sigma A}{ml} = \frac{4.8\times10^7\times10^{-6}}{10\times0.3} = 16$$

$$\Rightarrow \omega = 4\, rad/s$$

23. The distance x covered by a particle in one dimension motion varies as with time t as $x^2 = at^2 + 2bt + c$, where a, b, c are constants. Acceleration of particle depend on x as x^{-n}, the value of n is

Solution: (3)

Let, v be velocity, α be the acceleration then,

$$x^2 = at^2 + 2bt + c$$

$$2\,x\,v = 2\,a\,t + 2\,b$$

$$x\,v = a\,t + b \qquad \_\_\_\_\_\_(1)$$

$$\Rightarrow v = \frac{at + b}{x}$$

Now, differentiating equation (1),

$v^2 + \alpha x = a$

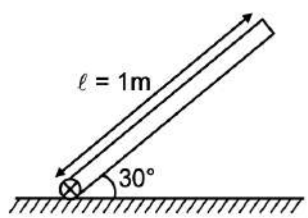
$\alpha x = a - \left(\frac{at+b}{x}\right)^2$

$\alpha = \frac{a(at^2 + 2bt + c) - (at+b)^2}{x^3}$

$\alpha = \frac{ac - b^2}{x^3}$

$$\alpha \propto x^{-3}$$

24. A rod of length 1 m pivoted at one end is released from rest when it makes 30° from the horizontal as shown in the figure below.



If ω of rod is $\sqrt{n}$ at the moment it hits the ground, then find n

Solution: (15)

By using conservation of energy,

$mg\frac{l}{2}\sin 30^\circ = \frac{1}{2}\frac{ml^2}{3}\omega^2$

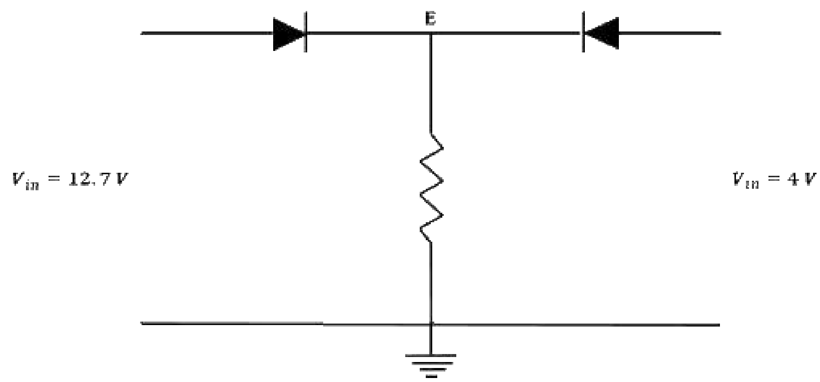
On solving

$\omega^2 = 15$

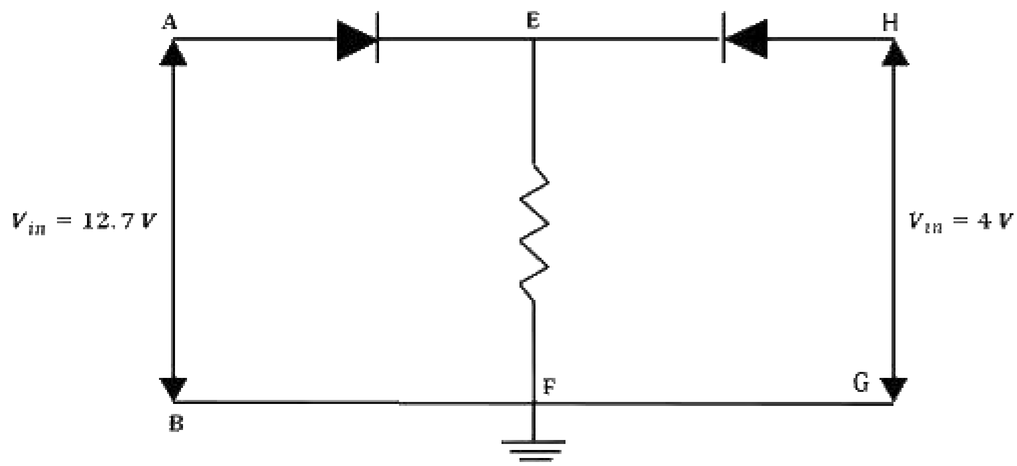
$\omega = \sqrt{15}$

Therefore, $n = 15$

25. In the given circuit both diodes are ideal having zero forward resistance and built-in potential of 0.7 V. Find the potential of point E in volts



Solution: (12)



We have to apply nodal analysis on both left and right side and check what can be voltage at E. For nodal analysis, voltage at B, F and G will be 0 volts and voltage at A will be 12.7 volt and voltage at H will be 4 volts.
If, we apply Nodal from right side, voltage at E will be 12 volt (diode between A and E will be forward biased). Now voltage at E is 12 volt and voltage at H is 4 volt and since, diode between E and H is reversed biased and any difference of voltage is possible across reverse biased. So, this is possible.
If, we apply Nodal from left side, voltage at E will be 3.3 volt (diode between E and H will be forward biased). Now voltage at E is 3.3 volt and voltage at A is 12 volt and since, diode between E and A is forward biased and in forward biased difference of voltage of 0.7 volt is allowable. So, this case is not possible. Therefore current will also not flow through GH.
Hence, $V_E = 12\ V$

JEE Main 2020 Paper

Date: 9th January 2020

Time: 09:30 AM – 12:30 PM

Subject: Chemistry

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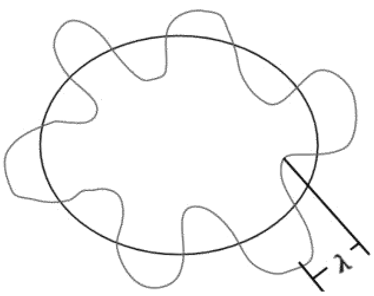
1. The de Broglie wavelength of an electron in the 4th Bohr orbit is:
   a. $4\pi a_0$
   b. $2\pi a_0$
   c. $8\pi a_0$
   d. $6\pi a_0$

Answer: c

Solution: n=4
Z=1
Λ=?



Circumference ($2\pi r$)= $n\lambda$

$\frac{2\pi a_o n^2}{z}=n\lambda$

On solving, we get $8\pi a_o$

2. If the magnetic moment of a dioxygen species is 1.73 B.M, it may be:
   a. $O_2, O_2^- \ or\ O_2^+$
   b. $O_2^- \ or\ O_2^+$
   c. $O_2 \ or\ O_2^-$
   d. $O_2 \ or\ O_2^+$

Answer: b

Solution:

$O_2: \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2py^2}\ \ \pi^*_{2p_x^1} = \pi^*_{2p_y^1}$

$O_2^-: \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2py^2}\ \ \pi^*_{2p_x^2} = \pi^*_{2p_y^1}$

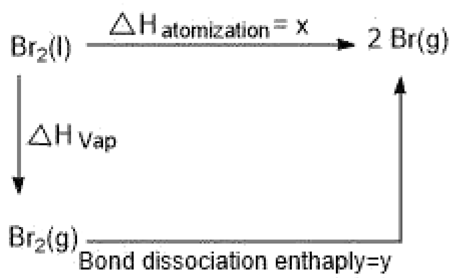
$O_2^+: \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2py^2}\ \ \pi^*_{2p_x^1}$

3. If enthalpy of atomisation for $Br_2(l)$ is x kJ/mol and bond enthalpy for Br_2 is y kJ/mol, the relation between them:

a. is $x > y$
b. is $x < y$
c. is $x = y$
d. does not exist

Answer: a

Solution:



$\Delta H_{atomisation} = \Delta H_{vap} + y$

$x - y = \Delta H_{vap}$

4. Which of the following oxides are acidic, basic and amphoteric, respectively?

a. MgO, Cl_2O, Al_2O_3
b. N_2O_3, Li_2O, Al_2O_3
c. SO_3, Al_2O_3, Na_2O
d. P_4O_{10}, Cl_2O, CaO

Answer: b

Solution:

Non-metallic oxides are acidic in nature, metallic oxides are basic in nature and Al_2O_3 is amphoteric in nature

5. Complex X of composition$Cr(H_2O)_6Cl_n$, has a spin only magnetic moment of 3.83 BM. It reacts with $AgNO_3$ and shows geometrical isomerism. The IUPAC nomenclature of X is :

a. Hexaaqua chromium(III) chloride
b. Tetraaquadichlorido chromium(III) chloride dihydrate
c. Hexaaquachromium(IV) chloride
d. Tetraaquadichlorido chromium(IV) chloride dihydrate

Answer: b

Solution:

Spin only magnetic moment = 3.8 B. M. This implies, $\mu = \sqrt{(n(n+2))}$ B.M.

($\sqrt{16} = 4$ implies that $\sqrt{15}$ should be less than four.
This means, n=3 as $\sqrt{15} = \sqrt{(3(3+2))}$
Cr (24) = $[Ar]4s^1\ 3d^5$ (g.s)

For 3 unpaired electrons, the oxidation state of Cr should be +3
Cr^{3+} can be attained if the complex has a structure that looks like: $[Cr(H_2O)_4Cl_2]Cl.2H_2O$
$[Cr(H_2O)_4Cl_2]Cl.2H_2O$ has the IUPAC name : Tetraaquadichloridochromium(III) chloride dihydrate

6. The electronic configuration of bivalent europium and trivalent cerium, are:
(Atomic Number : Xe = 54, Ce = 58, Eu = 63)
a. $[Xe]4f^7, [Xe]4f^1$
b. $[Xe]4f^7 6s^2$, $[Xe]4f^2 6s^2$
c. $[Xe]4f^2, [Xe]4f^7$
d. $[Xe]4f^4, [Xe]4f^9$

Answer: a

Solution:

Ce (58): [Xe] $6s^2 4f^2$ (g.s)

Ce^{3+}: $[Xe]4f^1$
Eu(63) : $[Xe]6s^2 4f^7$ (g.s)
Eu^{2+} : $[Xe]4f^7$

7. The K_{sp} for the following dissociation is $= 1.6 \times 10^{-5}$.

$$PbCl_2(s) \rightleftharpoons Pb^{2+}(aq) + 2Cl^-(aq)$$

Which of the following choices is correct for a mixture of 300 mL 0.134 M $Pb(NO_3)_2$ and 100mL 0.4 M NaCl ?
a. $Q > K_{sp}$
b. $Q < K_{sp}$
c. $Q = K_{sp}$
d. Not enough data provided

Answer: a

Solution: Given K_{sp} of $PbCl_2$ $= 1.6 \times 10^{-5}$

$Pb(NO_3)_2$: mmoles= 300 mL $\times$ 0.134 M = 40.2

NaCl: mmoles = 100 mL $\times$ 0.4 M = 40

This implies, $[Pb]^{2+} = \frac{40.2}{400} \approx 0.1$ M

$[Cl]^- = \frac{40}{400} = 0.1$ M

$Q_{sp} = [Pb^{2+}][2Cl^-]^2 = 4 \times 10^{-3} > K_{sp}$

8. The compound that cannot act both as oxidising and reducing agent is :

a. H_2SO_3
b. HNO_2
c. H_3PO_4
d. H_2O_2

Answer: c

Solution:

When the oxidation state is maximum it acts like a strong oxidising agent

When the oxidation state is minimum it acts like a strong reducing agent

When the oxidation state is between its maximum and minimum, it acts like both an oxidizing and as a reducing agent

In H_3PO_4, P has a +5 oxidation state and hence can act like a strong oxidising agent. In the rest, the oxidation state is between their maximum and minimum.

9. B has a smaller first ionization enthalpy than Be. Consider the following statements:

(i) It is easier to remove 2p electron than 2s electron
(ii) 2p electron of B is more shielded from the nucleus by the inner core of electrons than the 2s electron of Be
(iii) 2s electron has more penetration power than 2p electron
(iv) Atomic radius of B is more than Be
(Atomic number B=5, Be=4)
The correct statements are:

a. (i), (ii), and (iii)
b. (i), (iii) and (iv)
c. (ii), (iii) and (iv)
d. (i), (ii), (iv)

Answer: a

Solution:

Be (4): $1s^2 2s^2$

B (5): $1s^2 2s^2 2p^1$

The electron in $2p^1$ can easily be extracted.

The penetrating power is of the order: s > p > d > f

The shielding power order: s > p > d > f

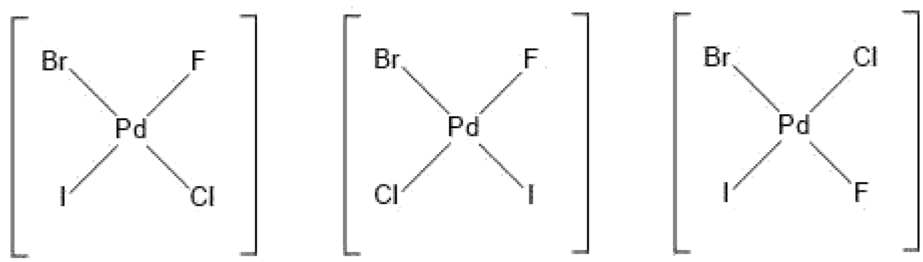
As we move along the period, the size decreases, as Z_{eff} increases. Hence the radius of B is smaller than the radius of Be.

10. $[Pd(F)(Cl)(Br)(I)]^{2-}$,has n number of geometrical Isomers. Then, the spin-only magnetic moment and crystal field stabilisation energy [CFSE] of $[Fe(CN)_6]^{n-6}$, respectively,
[Note: Ignore pairing energy].

a. 1.73 BM and $-2\ \Delta_0$
b. 2.84 BM and $-1.6\ \Delta_0$
c. 0 BM and $-2.4\ \Delta_0$
d. 5.92 BM and 0

Answer: a

Solution:



Number of geometrical isomers (n) = 3

$[Fe(CN)_6]^{n-6} = [Fe(CN)_6]^{3-6} = [Fe(CN)_6]^{-3}$

This implies, that Iron is in its +3 oxidation state.

$Fe^{3+}(26)$: $[Ar]3d^5$

CN^- is a strong ligand in $[Fe(CN)_6]^{-3}$ and causes pairing. Hence, according to CFT, the configuration will be $t_{2g}^5\ e_g^0$.

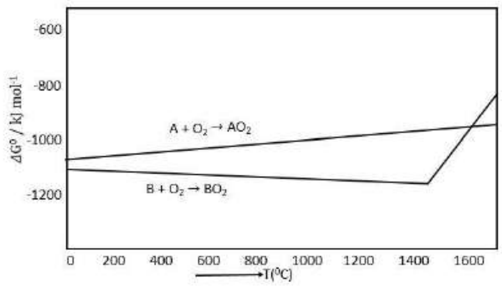
Hence, there is only 1 unpaired electron, i.e, n=1 in $\sqrt{n(n+2)} = \sqrt{3}$ = 1.73 B.M

CFSE = $\left(-0.4 \times n_{t2g} + 0.6 \times n_{eg}\right)\Delta_0$

= $(-0.4 \times 5 + 0.6 \times 0)\Delta_0$

= $-2\Delta_0$

11. According to the following diagram, A reduces BO_2 when the temperature is:



a. > 1400 °C
b. < 1400 °C
c. $>$ 1200 °C but <1400 °C
d. $<$ 1200 °C

Answer: a

Solution: In Ellingham's diagram, the line of the element that lies below can reduce the oxide of the element which lies above it. Therefore, for A to reduce BO_2, the temperature when the line for element A is below that of BO_2, according to the graph when T $>$ 1400 °C.

For T > 1400 °C , $\Delta G_r < 0$ for A + $BO_2 \rightarrow$ B + AO_2

12. For following reactions

$A \xrightarrow{700K}$ Product;

$A \xrightarrow{\text{Catalyst,500K}}$ Product;

It was found that the E_a is decreased by 30 kJ/mol in the presence of catalyst. If the rate remains unchanged , the activation energy for catalysed reaction is (Assume pre exponenetial factor is same)

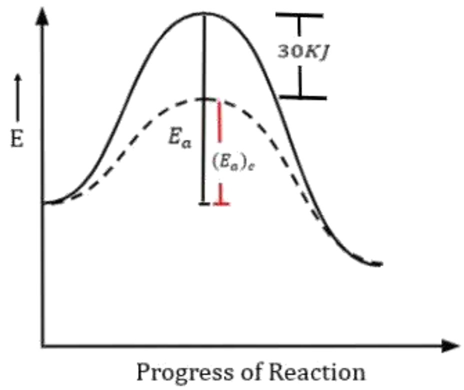
a. 75 kJ/mol
b. 135 kJ/mol
c. 105 kJ/mol
d. 198 kJ/mol

Answer: c

Solution:

$K = Ae^{\left(-\frac{E_a}{RT}\right)}$

$K_{catalyst} = K_{without\ catalyst}$



$Ae^{\left(-\frac{(E_a)c}{RT_{500k}}\right)} = Ae^{\left(-\frac{(E_a)}{RT_{700k}}\right)}$

$e^{\left(-\frac{(E_a)c}{RT_{500k}}\right)} = e^{\left(-\frac{(E_a)}{RT_{700k}}\right)}$

$-\frac{(E_a)\,c}{RT_{500k}} = -\frac{(E_a)}{RT_{700k}}$

$(E_a)c = E_a - 30$

$-\frac{(E_a-30)}{T_{500k}} = -\frac{(E_a)}{T_{700k}}$

On Solving, $E_a = 105\ kJmol^{-1}$

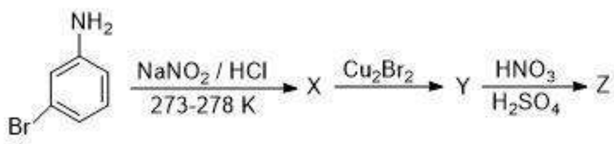
13. 'X' melts at low temperature and is a bad conductor of electricity in both liquid and solid state. X is :

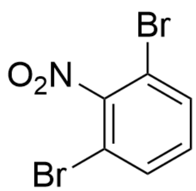
a. mercury
b. silicon carbide
c. zinc sulphide
d. carbon tetrachloride

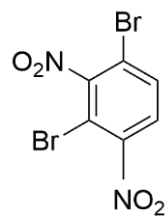
Answer: d

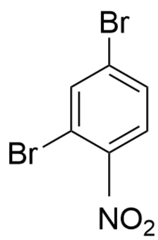
Solution: CCl_4 is non polar and does not conduct in either solid or liquid state.

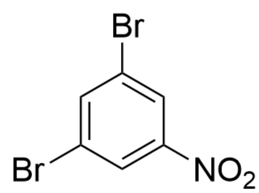
14. The major product Z obtained in the following reaction scheme is:



a. 

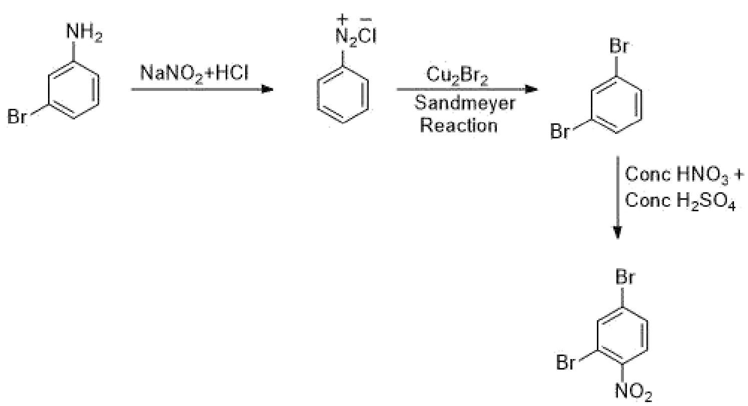
b. 

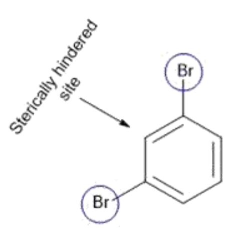
c. 

d. 

Answer: c

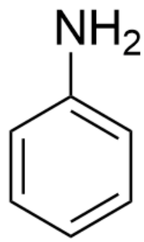
Solution:

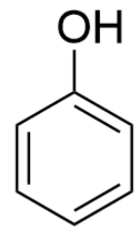


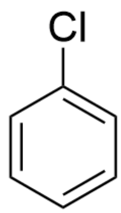


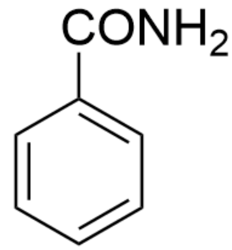
Hence, major product formed is that of option c.

15. Which of these will produce the highest yield in Friedel-Craft's reaction?

a. 

b. 

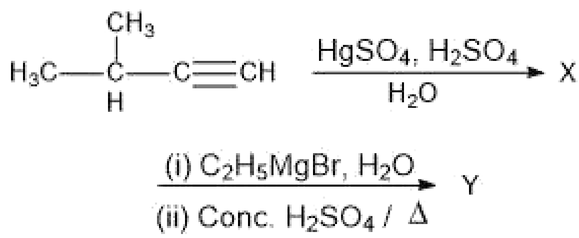
c. 

d. 

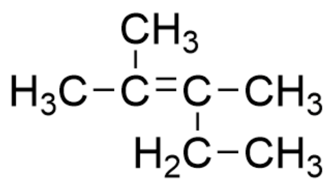
Answer: b

Solution: Out of the four options given, only aniline and phenol show strong +R effects, but as we know, aniline is a Lewis base and can react with a Lewis acid that is added during the reaction. Hence, Phenol gives the highest yield in Friedel-Craft's reaction.

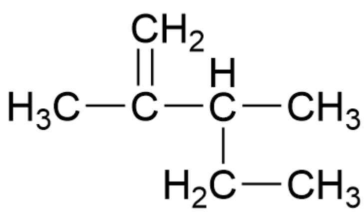
16. The major product (Y) in the following reactions is :



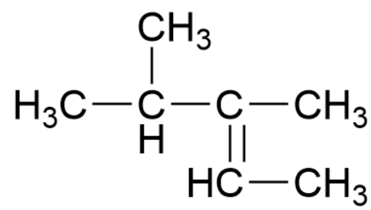
a.



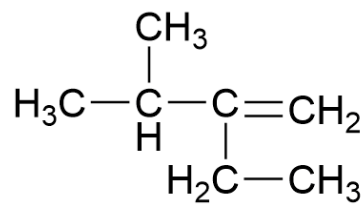
b.



c.

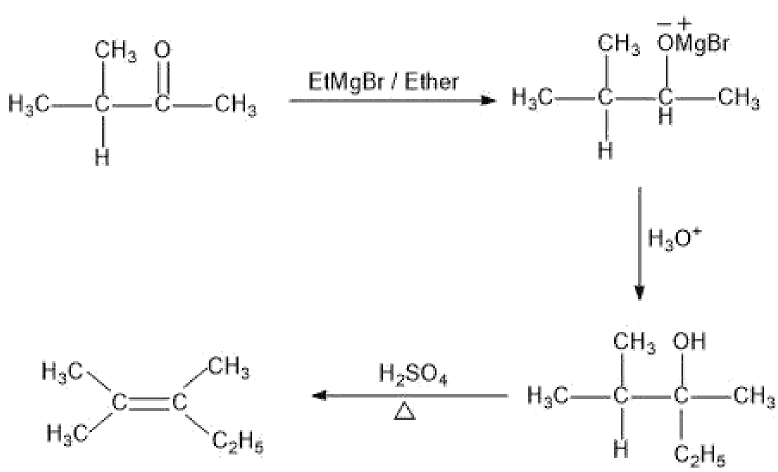


d.

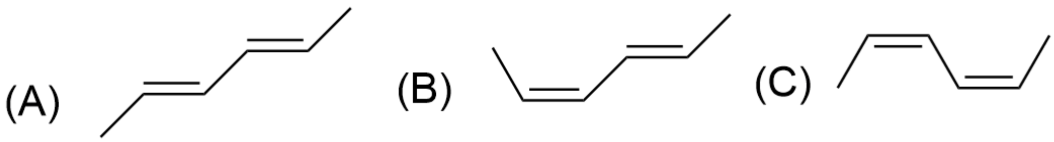


Answer: a

Solution:



17. The correct order of heat of combustion for following alkadienes is:

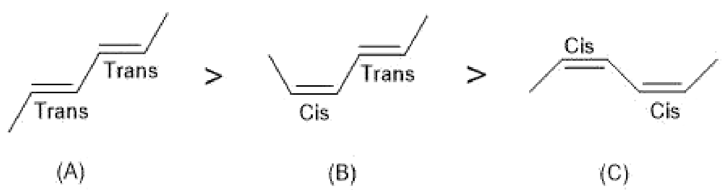


a. C > B > A
b. B > A > C
c. A > B > C
d. C > A > B

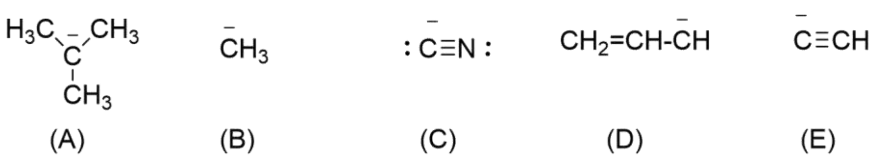
Answer: c

Solution: Heat of combustion $\alpha \frac{1}{\text{stability}}$

The trans-isomer is more stable than the cis-isomer. More the number of trans forms in a structure, higher the stability.



18. The increasing order of basicity for the following intermediates is (from weak to strong)

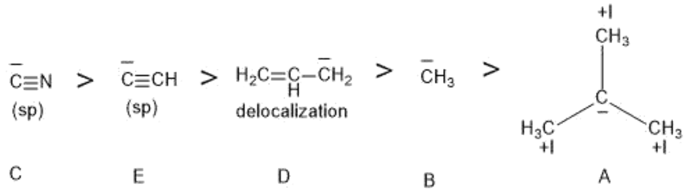


a. A > B > D > E > C
b. B > A > D > C > E
c. A > B > E > D > C
c. C > E > D > B > A

Answer: a

Solution: As we know weaker the conjugate base, stronger the acid.

The order of stability of conjugate base:



Hence, the order of basicity or acidic strength is:

A > B > D> E> C

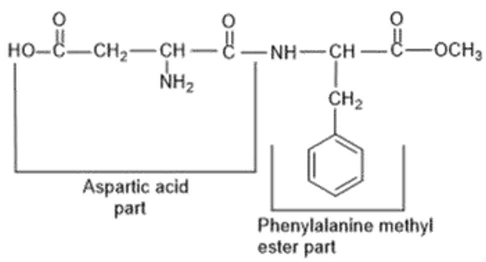
19. A chemist has 4 samples of artificial sweetener A, B, C and D. To identify these samples, he performed certain experiments and noted the following observations:
    (i) A and D both form blue-violet colour with ninhydrin.
    (ii) Lassaigne extract of C gives positive $AgNO_3$ test and negative $Fe_4[Fe(CN)_6]_3$ test.
    (iii) Lassaigne extract of B and D gives positive sodium nitroprusside test.

Based on these observations which option is correct?

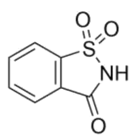
a. A – Alitame, B – Saccharin, C – Aspartame, D – Sucralose
b. A –Saccharin, B – Alimate, C – Sucralose, D – Aspartame
c. A – Aspartame, B – Alitame, C – Saccharin , D – Sucralose
d. A – Aspartame, B – Saccharin, C – Sucralose, D – Alitame

Answer: d

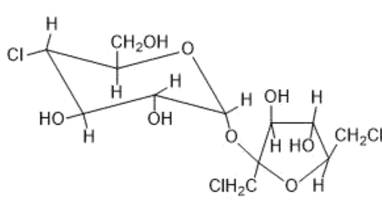
Solution:

A - Aspartame 

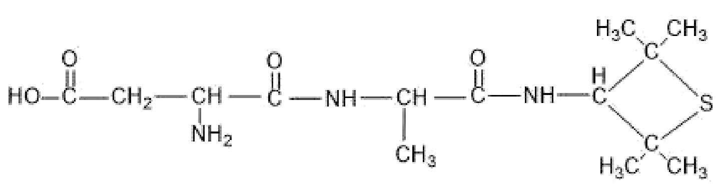
It has a free amine group and hence reacts with ninhydrin to give a purple colour known as Ruhemann's purple.

B- Saccharine 

It has Sulphur, therefore, it will give a positive test with sodium nitroprusside.

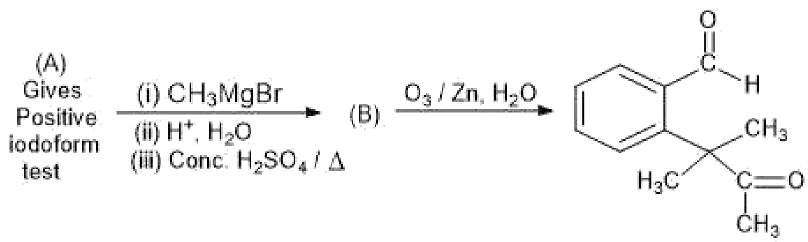
C - Sucralose 

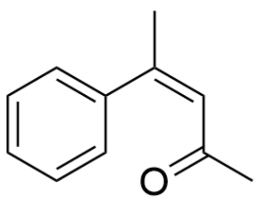
It has chlorine and hence it forms a precipitate with $AgNO_3$ in the Lassaigne's extract of the sugar.

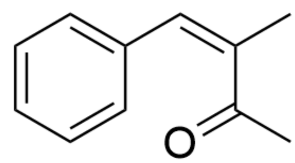
D - Alitame 

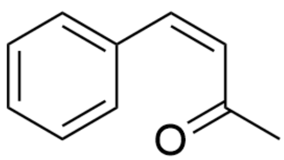
It has a free amine group and hence reacts with ninhydrin to give purple colour known as Ruhemann's purple. Also, it has Sulphur, therefore, it will give positive test with sodium nitroprusside.

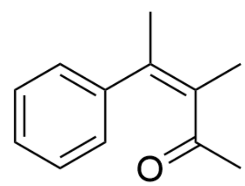
20. Identify (A) in the following reaction sequence:



a.


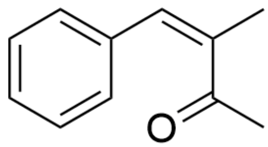
b.


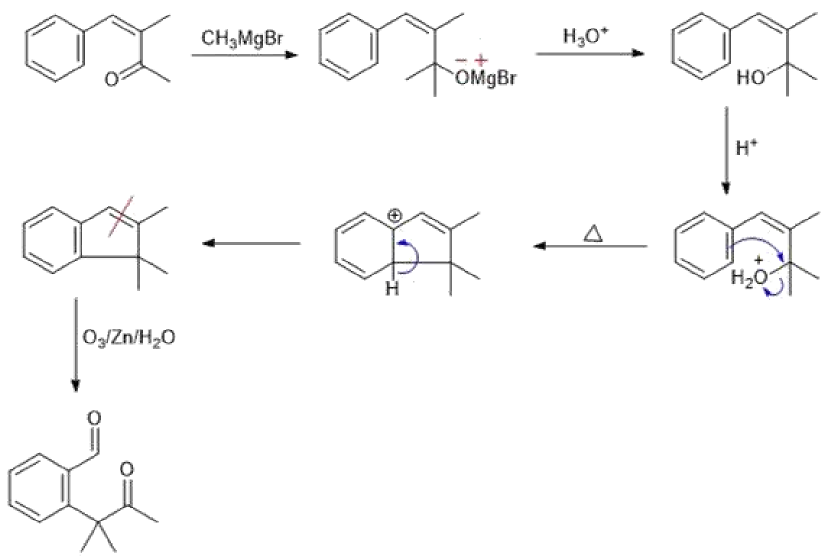
c.


d.


Answer: b

Solution:


is a methyl ketone, which gives positive Iodoform test.



21. The molarity of HNO_3 in a sample which has density 1.4 g/mL and mass percentage of 63% is ----------- (Molecular weight of HNO_3= 63).

Answer: 14.00

Solution: $\%\frac{w}{w}= 63\%$

$\rho= 1.4$ g/mL

$$M = \frac{\left(\%\frac{w}{w} \times \rho \times 10\right)}{MM}$$

$$M = \frac{(63\times 1.4\times 10)}{63}$$

M= 14 mol/L

22. The hardness of a water sample containing 10^{-3} M $MgSO_4$ expressed as $CaCO_3$ equivalents (in ppm)is ------------- (molar mass of $MgSO_4$ is 120.37 g/mol)

Answer: 100.00

Solution:

Hardness of water is measured in ppm in terms $CaCO_3$.

$n_{CaCO_3} = n_{MgSO_4}$

ppm is the parts (in grams) present per million i.e, 10^6

1000 mL has 10^{-3} moles of $MgSO_4$.

Grams of $CaCO_3$ in 1000 mL = $10^{-3} \times 100$ grams

Grams of $CaCO_3$ in 1 mL = $\frac{10^{-3} \times 100}{1000 \text{ mL}}$ grams

Hardness = $\frac{10^{-3} \times 100}{1000 \text{ mL}} \times 10^6 = 100$

23. How much amount of NaCl should be added to 600 g of water (ρ=1.00 g/mL) to decrease the freezing point of water to -0.2°C?
(The freezing point depression constant for water = 2 K Kg mol^{-1})

Answer: 1.76

Solution: NaCl is strong electrolyte and gives 2 ions in the solution. This implies, i=2.

Molarity= $\frac{w \times 1000}{58.5 \times 600}$

$\Delta T_f = 0.2°C$

$\Delta T_f = i \times k_f \times m$

On solving we get,

w= 1.76 grams

24. 108 g silver (molar mass 108 g mol^{-1}) is deposited at cathode from $AgNO_3$(aq) solution by a certain quantity of electricity. The volume (in L) of oxygen gas produced at 273K and 1 bar pressure from water by the same quantity of electricity is ----------------

Answer: 5.68

Solution: On applying Faraday's 1st law,

Moles of Ag deposited= 108/108= 1 mol.

$Ag^{+} + e^{-} \rightarrow Ag$

1Faraday is required to deposit 1 mole of Ag.

$H_2O \rightarrow 2H^{+} + \frac{1}{2}O_2 + 2e^{-}$

$\frac{1}{2}$ moles of O_2 are deposited by 2 F of charge.
This implies, 1 F will deposit $\frac{1}{4}$moles of O_2
Using PV=nRT
P= 1 bar
T= 273 K
R= 0.0823 Lbar $mol^{-1}K^{-1}$
On solving we get,
V = 5.68 L

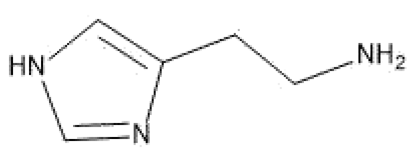
25. The mass percentage of nitrogen in histamine is -----------
Answer: 37.84

Solution:

Molecular mass of Histamine= 111

In Histamine, 3 nitrogen atoms are present (42g)

The percentage of nitrogen by mass in Histamine = $\frac{42}{111} \times 100 = 37.84\%$



JEE Main 2020 Paper

Date of Exam: **9th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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1. If C be the centroid of the triangle having vertices $(3, -1)$, $(1, 3)$ and $(2, 4)$. Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$, then the line passing through the points C and P also passes through the point:

   a. $(-9, -7)$
   b. $(-9, -6)$
   c. $(7, 6)$
   d. $(9, 7)$

   Answer: (b)

   Solution:

   Coordinates of C are $\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) = (2, 2)$

   Point of intersection of two lines

   $x + 3y - 1 = 0$ and $3x - y + 1 = 0$

   is $P\left(\frac{-1}{5}, \frac{2}{5}\right)$

   Equation of line CP is $8x - 11y + 6 = 0$

   Point $(-9, -6)$ lies on CP

2. The product $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \times 16^{\frac{1}{128}} \ldots$ to ∞ is equal to:

   a. $2^{\frac{1}{4}}$
   b. 2
   c. $2^{\frac{1}{2}}$
   d. 1

   Answer: (c)

   Solution:

   $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \ldots \infty = 2^{\frac{1}{4}} \times 2^{\frac{2}{16}} \times 2^{\frac{4}{48}} \ldots \infty$

   $\Rightarrow 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} \times 2^{\frac{1}{16}} \ldots \infty = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\infty}$

   $\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)} = \sqrt{2}$

3. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at the rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which the thickness of ice decreases, is:

a. $\frac{5}{6\pi}$

b. $\frac{1}{54\pi}$

c. $\frac{1}{36\pi}$

d. $\frac{1}{18\pi}$

Answer: (d)

Solution:

Let thickness of ice be x cm.

Therefore, net radius of sphere $= (10 + x)$ cm

Volume of sphere $V = \frac{4}{3}\pi(10 + x)^3$

$\Rightarrow \frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$

At $x = 5, \frac{dV}{dt} = 50\ \text{cm}^3/\text{min}$

$\Rightarrow 50 = 4\pi \times 225 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{18\pi} \text{cm/min}$

4. Let f be any function continuous on $[a, b]$ and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than:

a. $\frac{b-c}{c-a}$

b. 1

c. $\frac{c-a}{b-c}$

d. $\frac{b+a}{b-a}$

Answer: (c)

Solution:

$\therefore c \in (a, b)$ and f is twice differentiable and continuous function on (a, b)

$\therefore$ LMVT is applicable

For $p \in (a, c)$, $\quad f'(p) = \frac{f(c)-f(a)}{c-a}$

For $q \in (c, b)$, $\quad f'(q) = \frac{f(b)-f(c)}{b-c}$

$\therefore f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$f'(p) > f'(q)$

$$\Rightarrow \frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c}$ (as $f'(x) > 0 \Rightarrow f(x)$ is increasing)

5. The value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$ is :

a. $\frac{1}{4}$

b. $\frac{1}{2\sqrt{2}}$

c. $\frac{1}{2}$

d. $\frac{1}{\sqrt{2}}$

Answer: (b)

Solution:

$\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8} = \cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\left[\cos^6\frac{\pi}{8} - \sin^6\frac{\pi}{8}\right] + 3\left[\sin^4\frac{\pi}{8} - \cos^4\frac{\pi}{8}\right]$

$= 4\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right] - 3\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]$

$= \left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[4\left(1 - \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right) - 3\right]$

$= \cos\frac{\pi}{4}\left[1 - \sin^2\frac{\pi}{4}\right] = \frac{1}{2\sqrt{2}}$

6. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is:

a. 3

b. 4

c. 1

d. 2

Answer: (c)

Solution:

$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

$$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

Let $e^x + \frac{1}{e^x} = u$

Then, $u^2 + u - 6 = 0$

$\Rightarrow u = 2, -3$

$u \neq -3$ as $u > 0$ ($\because e^x > 0$)

$\Rightarrow e^x + \dfrac{1}{e^x} = 2 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$

Hence, only one real solution is possible.

7. The value of $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to:

a. 2π
b. 4π
c. $2\pi^2$
d. π^2

Answer: (d)

Solution:

Let $I = \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (1)

$I = \int_0^{2\pi} \frac{(2\pi - x)\sin^8(2\pi - x)}{\sin^8(2\pi - x) + \cos^8(2\pi - x)} dx$

$= \displaystyle\int_0^{2\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (2)

Adding (1) & (2) , we get:

$\Rightarrow 2I = 2\pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = \pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (3)

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8\left(\frac{\pi}{2} - x\right)}{\sin^8\left(\frac{\pi}{2} - x\right) + \cos^8\left(\frac{\pi}{2} - x\right)} dx = 4\pi \int_0^{\frac{\pi}{2}} \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx$ ... (4)

Adding (3) & (4) , we get:

$I = 2\pi \displaystyle\int_0^{\frac{\pi}{2}} 1\, dx = 2\pi \times \frac{\pi}{2} = \pi^2$

8. If for some α and β in R, the intersection of the following three planes

$x + 4y - 2z = 1$
$x + 7y - 5z = \beta$
$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to:

a. 0
b. 10
c. -10
d. 2

Answer: (b)

Solution:

The given planes intersect in a line

$\therefore D = D_x = D_y = D_z = 0$

$D = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$$

$\Rightarrow \alpha = -3$

$D_z = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$$

$\Rightarrow \beta = 13$

$\therefore \alpha + \beta = 10$

9. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$. Then k is equal to:

a. 14
b. 15
c. 17
d. 16

Answer: (d)

Solution:

$$e_1 = \sqrt{1 - \frac{4}{18}} = \frac{\sqrt{7}}{3} \ \& \ e_2 = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$$

$\because (e_1, e_2)$ lies on the ellipse $15x^2 + 3y^2 = k$

$\therefore 15e_1^2 + 3e_2^2 = k$

$\Rightarrow 15 \times \frac{7}{9} + 3 \times \frac{13}{9} = k \Rightarrow k = 16$

10. If $f(x) = \begin{cases} \frac{\sin(a+2)x+\sin x}{x}, & x<0 \\ b, & x=0 \\ \frac{(x+3x^2)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}}, & x>0 \end{cases}$ is continuous at $x=0$ then $a+2b$ is equal to:

a. -2
b. 1
c. 0
d. -1

Answer: (c)

Solution:

$f(x)$ is continuous at $x=0$

$\therefore \lim_{x\to 0^-} f(x) = b = \lim_{x\to 0^+} f(x)$

$$b = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{(h+3h^2)^{\frac{1}{3}} - h^{\frac{1}{3}}}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{h^{\frac{1}{3}}\left[(1+3h)^{\frac{1}{3}}-1\right]}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{(1+3h)^{\frac{1}{3}}-1}{h}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{1}{3}(1+3h)^{-\frac{2}{3}} \times 3$$

or, $b=1$

$$\lim_{x\to 0^-} f(x) = 1 \Rightarrow \lim_{h\to 0} \frac{\sin((a+2)(-h)) + \sin(-h)}{-h} = 1$$

$\Rightarrow a+3=1 \Rightarrow a=-2$

$\Rightarrow a+2b=0$

11. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj}\, A$ and $C = 3A$, then $\frac{|\text{adj}\, B|}{|C|}$ is equal to:

a. 16
b. 2
c. 8
d. 72

Answer: (c)

Solution:

$$A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix} \Rightarrow |A|=\begin{vmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{vmatrix}=13+1-8=6$$

$$B=\operatorname{adj}(A) \Rightarrow |\operatorname{adj} B|=|\operatorname{adj}(\operatorname{adj} A)|=|A|^4=6^4$$

$$|C|=|3A|=3^3|A|=3^3\times 6$$

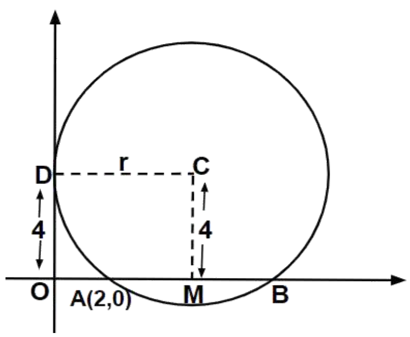
$$\frac{|\operatorname{adj} B|}{|C|}=\frac{6^4}{3^3\times 6}=\frac{2^3\times 3^3}{3^3}=8$$

12. A circle touches the y-axis at the point (0,4) and passes through the point (2,0). Which of the following lines is not a tangent to the circle?

a. $4x-3y+17=0$
b. $3x+4y-6=0$
c. $4x+3y-8=0$
d. $3x-4y-24=0$

Answer: (c)

Solution:



$$OD^2=OA\times OB \Rightarrow 16=2\times OB \Rightarrow OB=8$$

$$\therefore AB=6$$

$$\therefore AM=3, CM=4 \Rightarrow CA=5$$

$$\therefore OM=5$$

Centre will be (5,4) and radius is 5

Now checking all the options

Option (c) is not a tangent.

$$4x+3y-8=0$$

$$\frac{20+12-8}{\sqrt{3^2+4^2}}=\frac{24}{5}\quad (p\neq r)$$

13. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z+3i|$ is:

a. $\sqrt{10}$

b. $\frac{7}{2}$

c. $\frac{15}{4}$

d. $2\sqrt{3}$

Answer: (b)

Solution:

If $\left|\frac{z-i}{z+2i}\right| = 1$ & $|z| = \frac{5}{2}$

$\Rightarrow |z-i| = |z+2i|$

$\Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\Rightarrow y-1 = \pm(y+2)$

$\Rightarrow y-1 = -y-2$

$\Rightarrow y = -\frac{1}{2}$

$\Rightarrow |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$

$\Rightarrow x^2 + \frac{1}{4} = \frac{25}{4}$

$\Rightarrow x = \pm\sqrt{6}$

$|z+3i| = \sqrt{x^2 + (y+3)^2}$

$\Rightarrow |z+3i| = \frac{7}{2}$

14. If $f'(x) = \tan^{-1}(\sec x + \tan x), -\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to:

a. $\frac{\pi+1}{4}$

b. $\frac{\pi+2}{4}$

c. $\frac{1}{4}$

d. $\frac{\pi-1}{4}$

Answer: (a)

Solution:

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$$

$$f'(x) = \tan^{-1}\left(\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}\right)$$

$$f'(x) = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$$

$$f'(x) = \tan^{-1}\left[\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right]$$

$$f'(x) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi + 1}{4}$$

15. Negation of the statement: $'\sqrt{5}$ is an integer or 5 is irrational$'$ is:
    a. $\sqrt{5}$ is irrational or 5 is an integer.
    b. $\sqrt{5}$ is not an integer or 5 is not irrational.
    c. $\sqrt{5}$ is an integer and 5 is irrational.
    d. $\sqrt{5}$ is not an integer and 5 is not irrational.

Answer: (d)

Solution:

p: $\sqrt{5}$ is an integer

q: 5 is an irrational number

Given statement : $p \vee q$

Required negation statement: $\sim(p \vee q) = \sim p \wedge \sim q$

$'\sqrt{5}$ is not an integer and 5 is not irrational'

16. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x)dx$ is equal to:

a. $2\left(3f(1)+2f\left(\frac{1}{2}\right)\right)$

b. $\frac{1}{3}\left(f(0)+f\left(\frac{1}{2}\right)\right)$

c. $\frac{1}{2}\left(f(1)+3f\left(\frac{1}{2}\right)\right)$

d. $\frac{1}{6}\left(f(0)+f(1)+4f\left(\frac{1}{2}\right)\right)$

Answer: (d)

Solution:

$f(x)=a+bx+cx^2$

$f(0)=a, f(1)=a+b+c$

$f\left(\frac{1}{2}\right)=\frac{c}{4}+\frac{b}{2}+a$

$\int_0^1 f(x)dx=\int_0^1(a+bx+cx^2)dx\ =a+\frac{b}{2}+\frac{c}{3}$

$=\frac{1}{6}(6a+3b+2c)=\frac{1}{6}\big(a+(a+b+c)+(4a+2b+c)\big)$

$=\frac{1}{6}\left(f(0)+f(1)+4f\left(\frac{1}{2}\right)\right)$

17. If the number of five digit numbers with distinct digits and 2 at the $10th$ place is $336k$, then k is equal to:

a. 8

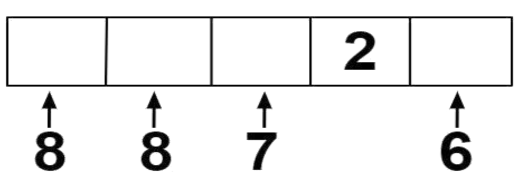
b. 7

c. 4

d. 6

Answer: (a)

Solution:



Total numbers that can be formed are

$=8\times 8\times 7\times 6$

$=8\times 336$

$\therefore k=8$

18. Let the observations $x_i(1\le i\le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i-5)=10$ and $\sum_{i=1}^{10}(x_i-5)^2=40$. If μ and λ are the mean and the variance of observations, $(x_1-3),(x_2-3)\ldots..(x_{10}-3)$, then the ordered pair (μ,λ) is equal to:

a. $(6,3)$ b. $(3,6)$
c. $(3,3)$ d. $(6,6)$

Answer: (c)

Solution:

$$\sum_{i=1}^{10}(x_i-5)=10 \Rightarrow \sum_{i=1}^{10}x_i-50=10$$

$$\Rightarrow \sum_{i=1}^{10}x_i=60$$

$$\mu=\frac{\sum_{i=1}^{10}(x_i-3)}{10}=\frac{\sum_{i=1}^{10}x_i-30}{10}=3$$

Variance is unchanged, if a constant is added or subtracted from each observation

$$\lambda=Var(x_i-3)=Var(x_i-5)=\frac{\sum_{i=1}^{10}(x_i-5)^2}{10}-\left(\frac{\sum_{i=1}^{10}(x_i-5)}{10}\right)^2$$

$$=\frac{40}{10}-\left(\frac{10}{10}\right)^2=3$$

19. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to: (where C is a constant of integration)

a. $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}}+C$ b. $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}}+C$
c. $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+C$ d. $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-\frac{13}{7}}+C$

Answer: (c)

Solution:

$$I=\int\frac{dx}{(x-3)^{\frac{6}{7}}\times(x+4)^{\frac{8}{7}}}$$

$$\Rightarrow I=\int\frac{(x+4)^{\frac{6}{7}}dx}{(x-3)^{\frac{6}{7}}\times(x+4)^2}=\int\left(\frac{x-3}{x+4}\right)^{-\frac{6}{7}}\times\frac{dx}{(x+4)^2}$$

Put $\frac{x-3}{x+4}=t \Rightarrow dt=7\left(\frac{1}{(x+4)^2}\right)dx$

$$\Rightarrow I=\frac{\int t^{-\frac{6}{7}}}{7}dt=t^{\frac{1}{7}}+C=\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+C$$

20. In a box, there are 20 cards out of which 10 are labelled as A and remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is:

a. $\frac{15}{16}$ b. $\frac{9}{16}$

c. $\frac{13}{16}$ d. $\frac{11}{16}$

Answer: (d)

Solution:

Here $P(A) = P(B) = \frac{1}{2}$

Then, these following cases are possible $\rightarrow AA, BAA, ABA, ABBA, BBAA, BABA$

So, the required probability is $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

21. If the vectors $\vec{p} = (a+1)\hat{\imath} + a\hat{\jmath} + a\hat{k}, \vec{q} = a\hat{\imath} + (a+1)\hat{\jmath} + a\hat{k}$ and $\vec{r} = a\hat{\imath} + a\hat{\jmath} + (a+1)\hat{k}$ $(a \in R)$ are coplanar and $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$, then value of λ is ____ .

Answer: (1)

Solution:

As $\vec{p}, \vec{q}, \vec{r}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a+1)\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$

$$(3a+1)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$3a + 1 = 0$

$$\Rightarrow a = -\frac{1}{3}$$

$\vec{p} = \frac{1}{3}(2\hat{\imath} - \hat{\jmath} - \hat{k}), \quad \vec{q} = \frac{1}{3}(-\hat{\imath} + 2\hat{\jmath} - \hat{k}), \quad \vec{r} = \frac{1}{3}(-\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\vec{r} \times \vec{q} = \frac{1}{9}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$

$\vec{r} \times \vec{q} = \frac{1}{9}(-3\hat{\imath} - 3\hat{\jmath} - 3\hat{k}) = -\frac{1}{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$|\vec{r} \times \vec{q}|^2 = \frac{1}{3}$

$\vec{p}.\vec{q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$

$3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$

$\Rightarrow \frac{1}{3} - \lambda \times \frac{1}{3} = 0 \Rightarrow \lambda = 1$

22. The projection of the line segment joining the points $(1, -1, 3)$ and $(2, -4, 11)$ on the line joining the points $(-1, 2, 3)$ and $(3, -2, 10)$ is ______ .

Answer: (8)

Solution:

$\overrightarrow{AB} = \hat{\imath} - 3\hat{\jmath} + 8\hat{k}$

$\overrightarrow{CD} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is $= \frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{CD}|} = \frac{4+12+56}{\sqrt{4^2+4^2+7^2}} = \frac{72}{9} = 8$

23. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0,2\pi]$, is ___________.

Answer: (8)

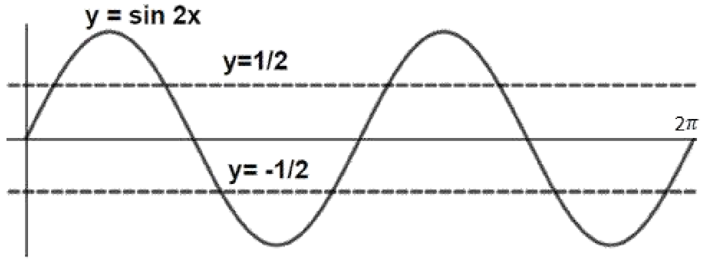
Solution:

$\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0,2\pi]$

$\Rightarrow \log_{\frac{1}{2}}|\sin x||\cos x| = 2$

$\Rightarrow |\sin x \cos x| = \frac{1}{4}$

$\therefore \sin 2x = \pm\frac{1}{2}$



$\therefore$ We have 8 solutions for $x \in [0,2\pi]$

24. If for $x \geq 0, y = y(x)$ is the solution of the differential equation $(1+x)dy = [(1+x)^2 + y - 3]dx,\ y(2) = 0$, then $y(3)$ is equal to ______ .

Answer: (3)

Solution:

$$(1+x)\frac{dy}{dx} = [(1+x)^2 + (y-3)]$$

$$\Rightarrow (1+x)\frac{dy}{dx} - y = (1+x)^2 - 3$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(1+x)}y = 1 + x - \frac{3}{1+x}$$

$$\text{I.F.} = e^{-\int \frac{1}{1+x}dx} = \frac{1}{1+x}$$

$$y \times \frac{1}{1+x} = \int 1 - \frac{3}{(1+x)^2}\,dx$$

$$\frac{y}{1+x} = x + \frac{3}{1+x} + c$$

$$\Rightarrow y = x(1+x) + 3 + c(1+x)$$

At $x = 2, y = 0$, we get

$$0 = 6 + 3 + 3c$$

$$\Rightarrow c = -3$$

$\Rightarrow$ At $x = 3$,

$$y = x^2 - 2x = 9 - 6 = 3$$

$\Rightarrow y(3) = 3$

25. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is

Answer: (615)

Solution:

General term of the given expression is given by $\frac{10!}{p!q!r!} x^{q+2r}$

Here, $q + 2r = 4$

For $p = 6, q = 4, r = 0$, coefficient $= \frac{10!}{6! \times 4!} = 210$

For $p = 7, q = 2, r = 1$, coefficient $= \frac{10!}{7! \times 2! \times 1!} = 360$

For $p = 8, q = 0, r = 2$, coefficient $= \frac{10!}{8! \times 2!} = 45$

Therefore, sum $= 615$