

**FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020**

**(Held On Friday 04<sup>th</sup> SEPTEMBER, 2020) TIME : 9 AM to 12 PM**

| PHYSICS   | TEST PAPER WITH ANSWER & SOLUTION |           |                |         |                              |          |                                  |           |                               |          |  |
|---|-----------------------------------|-----------|----------------|---------|------------------------------|----------|----------------------------------|-----------|-------------------------------|----------|--|
| <p>1. A beam of plane polarised light of large cross sectional area and uniform intensity of <math>3.3 \text{ Wm}^{-2}</math> falls normally on a polariser (cross sectional area <math>3 \times 10^{-4} \text{ m}^2</math>) which rotates about its axis with an angular speed of <math>31.4 \text{ rad/s}</math>. The energy of light passing through the polariser per revolution, is close to :</p> <p>(1) <math>1.0 \times 10^{-5} \text{ J}</math>                      (2) <math>5.0 \times 10^{-4} \text{ J}</math><br/> (3) <math>1.0 \times 10^{-4} \text{ J}</math>                      (4) <math>1.5 \times 10^{-4} \text{ J}</math></p> |                                   |           |                |         |                              |          |                                  |           |                               |          |  |
| <p>2. Match the <math>C_p/C_v</math> ratio for ideal gases with different type of molecules :</p> <table> <tr> <th>Molecular type</th><th><math>C_p/C_v</math></th></tr> <tr> <td>(A) Monoatomic</td><td>(I) 7/5</td></tr> <tr> <td>(B) Diatomic rigid molecules</td><td>(II) 9/7</td></tr> <tr> <td>(C) Diatomic non-rigid molecules</td><td>(III) 4/3</td></tr> <tr> <td>(D) Triatomic rigid molecules</td><td>(IV) 5/3</td></tr> </table> <p>(1) A-IV, B-I, C-II, D-III<br/> (2) A-IV, B-II, C-I, D-III<br/> (3) A-III, B-IV, C-II, D-I<br/> (4) A-II, B-III, C-I, D-IV</p>  | Molecular type                    | $C_p/C_v$ | (A) Monoatomic | (I) 7/5 | (B) Diatomic rigid molecules | (II) 9/7 | (C) Diatomic non-rigid molecules | (III) 4/3 | (D) Triatomic rigid molecules | (IV) 5/3 | <p>3. Choose the correct option relating wavelengths of different parts of electromagnetic wave spectrum :</p> <p>(1) <math>\lambda_{\text{x-rays}} &lt; \lambda_{\text{micro waves}} &lt; \lambda_{\text{radio waves}} &lt; \lambda_{\text{visible}}</math><br/> (2) <math>\lambda_{\text{visible}} &gt; \lambda_{\text{x-rays}} &gt; \lambda_{\text{radio waves}} &gt; \lambda_{\text{micro waves}}</math><br/> (3) <math>\lambda_{\text{radio waves}} &gt; \lambda_{\text{micro waves}} &gt; \lambda_{\text{visible}} &gt; \lambda_{\text{x-rays}}</math><br/> (4) <math>\lambda_{\text{visible}} &lt; \lambda_{\text{micro waves}} &lt; \lambda_{\text{radio waves}} &lt; \lambda_{\text{x-rays}}</math></p> <p>4. A air bubble of radius 1 cm in water has an upward acceleration <math>9.8 \text{ cm s}^{-2}</math>. The density of water is <math>1 \text{ gm cm}^{-3}</math> and water offers negligible drag force on the bubble. The mass of the bubble is (<math>g = 980 \text{ cm/s}^2</math>)</p> <p>(1) 3.15 gm                      (2) 4.51 gm<br/> (3) 4.15 gm                      (4) 1.52 gm</p> |
| Molecular type  | $C_p/C_v$                         |           |                |         |                              |          |                                  |           |                               |          |  |
| (A) Monoatomic  | (I) 7/5                           |           |                |         |                              |          |                                  |           |                               |          |  |
| (B) Diatomic rigid molecules  | (II) 9/7                          |           |                |         |                              |          |                                  |           |                               |          |  |
| (C) Diatomic non-rigid molecules  | (III) 4/3                         |           |                |         |                              |          |                                  |           |                               |          |  |
| (D) Triatomic rigid molecules   | (IV) 5/3                          |           |                |         |                              |          |                                  |           |                               |          |  |

5. Dimensional formula for thermal conductivity is (here K denotes the temperature)

- (1)  $\text{MLT}^{-3}\text{K}$  (2)  $\text{MLT}^{-2}\text{K}$   
(3)  $\text{MLT}^{-2}\text{K}^{-2}$  (4)  $\text{MLT}^{-3}\text{K}^{-1}$

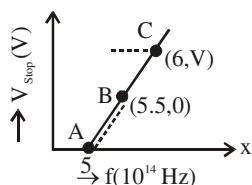
6. On the x-axis and a distance x from the origin, the gravitational field due to a mass distribution is given by  $\frac{Ax}{(x^2 + a^2)^{3/2}}$  in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is :

- (1)  $\frac{A}{(x^2 + a^2)^{1/2}}$  (2)  $\frac{A}{(x^2 + a^2)^{3/2}}$   
(3)  $A(x^2 + a^2)^{3/2}$  (4)  $A(x^2 + a^2)^{1/2}$

7. Starting from the origin at time  $t = 0$ , with initial velocity  $5\hat{j} \text{ ms}^{-1}$ , a particle moves in the x-y plane with a constant acceleration of  $(10\hat{i} + 4\hat{j})\text{ms}^{-2}$ . At time t, its coordinates are (20 m,  $y_0$  m). The values of t and  $y_0$ , are respectively :

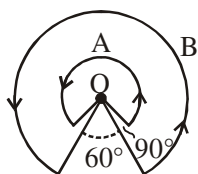
- (1) 4s and 52 m (2) 2s and 24 m  
(3) 2s and 18 m (4) 5s and 25 m

8. Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is : (Plancks constant  $h = 6.62 \times 10^{-34} \text{ J.s}$ )



- (1) 2.27 eV                      (2) 2.59 eV  
(3) 1.93 eV                      (4) 2.10 eV

9. A wire A, bent in the shape of an arc of a circle, carrying a current of 2A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is :



- (1) 4 : 6                      (2) 6 : 4  
(3) 6 : 5                      (4) 2 : 5

10. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are :

- (1) 1, 2, 3, .....                      (2)  $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots$   
(3) 1, 3, 5, .....                      (4)  $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \dots$

11. A small bar magnet placed with its axis at  $30^\circ$  with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is :

- (1)  $9.2 \times 10^{-3} \text{ J}$                       (2)  $6.4 \times 10^{-2} \text{ J}$   
(3)  $11.7 \times 10^{-3} \text{ J}$                       (4)  $7.2 \times 10^{-2} \text{ J}$

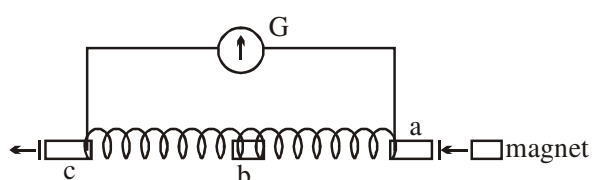
12. Particle A of mass  $m_A = \frac{m}{2}$  moving along the x-axis with velocity  $v_0$  collides elastically with another particle B at rest having mass  $m_B = \frac{m}{3}$ .

If both particles move along the x-axis after the collision, the change  $\Delta\lambda$  in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength ( $\lambda_0$ ) before collision is :

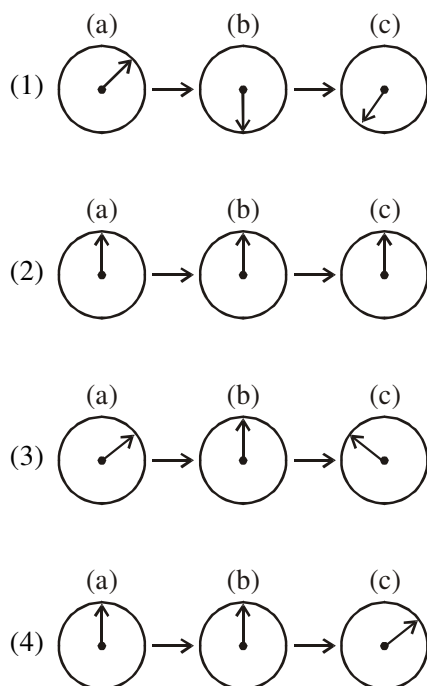
(1)  $\Delta\lambda = 4\lambda_0$                       (2)  $\Delta\lambda = \frac{5}{2}\lambda_0$

(3)  $\Delta\lambda = 2\lambda_0$                       (4)  $\Delta\lambda = \frac{3}{2}\lambda_0$

13. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil ?



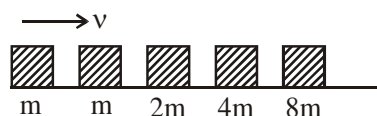
Three positions shown describe : (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.



14. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is :

- (1) 0.50 W                      (2) 0.125 W  
(3) 0.072 W                    (4) 0.10 W

15. Blocks of masses  $m$ ,  $2m$ ,  $4m$  and  $8m$  are arranged in a line on a frictionless floor. Another block of mass  $m$ , moving with speed  $v$  along the same line (see figure) collides with mass  $m$  in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass  $8m$  starts moving the total energy loss is  $p\%$  of the original energy. Value of 'p' is close to :

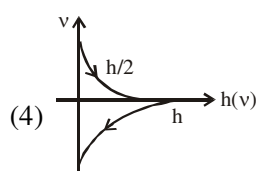
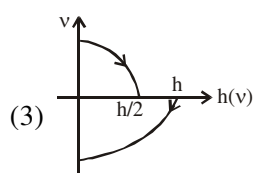
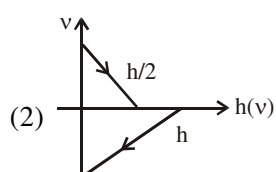
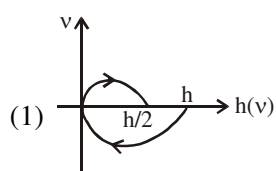


- (1) 77                                      (2) 37  
(3) 87                                      (4) 94

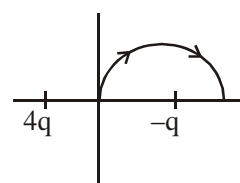
16. The specific heat of water =  $4200 \text{ J kg}^{-1} \text{ K}^{-1}$  and the latent heat of ice =  $3.4 \times 10^5 \text{ J kg}^{-1}$ . 100 grams of ice at  $0^\circ\text{C}$  is placed in 200 g of water at  $25^\circ\text{C}$ . The amount of ice that will melt as the temperature of water reaches  $0^\circ\text{C}$  is close to (in grams) :

- (1) 61.7                                      (2) 63.8  
(3) 69.3                                      (4) 64.6

17. A Tennis ball is released from a height  $h$  and after freely falling on a wooden floor it rebounds and reaches height  $\frac{h}{2}$ . The velocity versus height of the ball during its motion may be represented graphically by :  
(graph are drawn schematically and on not to scale)

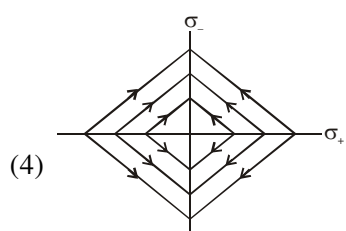
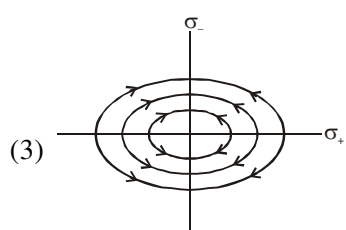
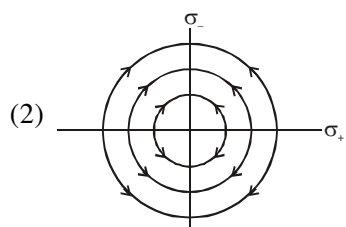
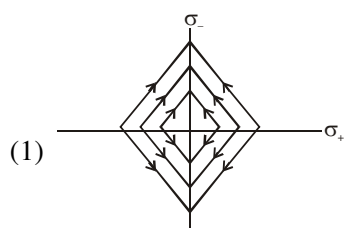


18. A two point charges  $4q$  and  $-q$  are fixed on the  $x$ -axis at  $x = -\frac{d}{2}$  and  $x = \frac{d}{2}$ , respectively. If a third point charge ' $q$ ' is taken from the origin to  $x = d$  along the semicircle as shown in the figure, the energy of the charge will :

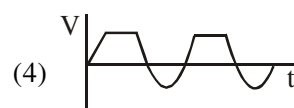
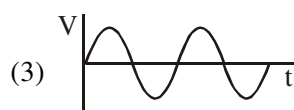
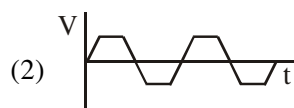
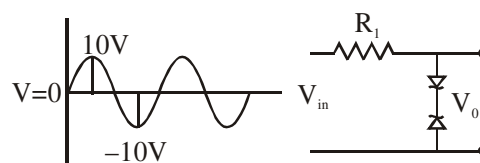


- (1) increase by  $\frac{2q^2}{3\pi\epsilon_0 d}$   
 (2) increase by  $\frac{3q^2}{4\pi\epsilon_0 d}$   
 (3) decrease by  $\frac{4q^2}{3\pi\epsilon_0 d}$   
 (4) decrease by  $\frac{q^2}{4\pi\epsilon_0 d}$

19. Two charged thin infinite plane sheets of uniform surface charge density  $\sigma_+$  and  $\sigma_-$  where  $|\sigma_+| > |\sigma_-|$  intersect at right angle. Which of the following best represents the electric field lines for this system :



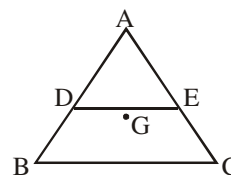
20. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is : (Graphs drawn are schematic and not to scale)





21. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is \_\_\_\_\_.

22. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is  $I_0$ . If part ADE is removed, the moment of inertia of the remaining part about the same axis is  $\frac{NI_0}{16}$  where N is an integer. Value of N is \_\_\_\_\_.



23. A circular disc of mass  $M$  and radius  $R$  is rotating about its axis with angular speed  $\omega_1$ . If another stationary disc having radius  $\frac{R}{2}$  and same mass  $M$  is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed  $\omega_2$ . The energy lost in the process is  $p\%$  of the initial energy. Value of  $p$  is \_\_\_\_\_.

24. A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be closed to \_\_\_\_\_.

25. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is  $304 \text{ \AA}$ . The corresponding difference for the Paschan series in  $\text{\AA}$  is : \_\_\_\_\_.

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**CHEMISTRY**

**TEST PAPER WITH ANSWER & SOLUTION**

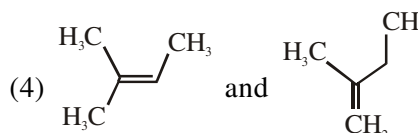
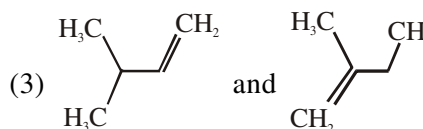
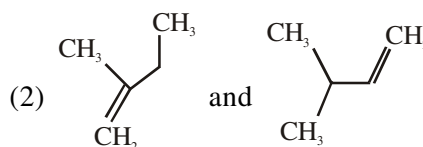
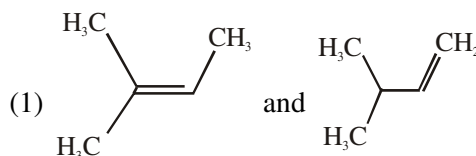
1. On heating, lead(II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is :

- (1) +5 (2) +2  
(3) +4 (4) +3

2. Which of the following will react with  $\text{CHCl}_3$  + alc. KOH ?

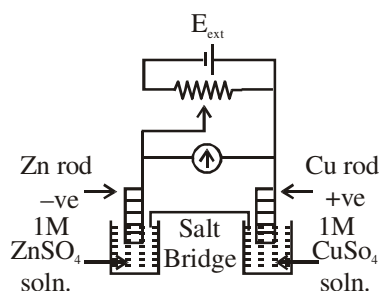
- (1) Adenine and lysine  
(2) Adenine and thymine  
(3) Adenine and proline  
(4) Thymine and proline

3. When neopentyl alcohol is heated with an acid, it slowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes ?



4. Among statements (a) - (d), the correct ones are :
- Lime stone is decomposed to CaO during the extraction of iron from its oxides.
  - In the extraction of silver, silver is extracted as an anionic complex.
  - Nickel is purified by Mond's process.
  - Zr and Ti are purified by Van Arkel method.
- (c) and (d) only
  - (a), (c) and (d) only
  - (b), (c) and (d) only
  - (a), (b), (c) and (d)

5.



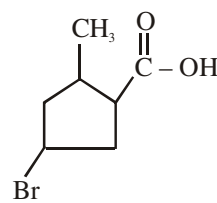
$$E^\circ_{\text{Cu}^{2+}|\text{Cu}} = +0.34\text{V}$$

$$E^\circ_{\text{Zn}^{2+}|\text{Zn}} = -0.76\text{V}$$

Identify the incorrect statement from the options below for the above cell :

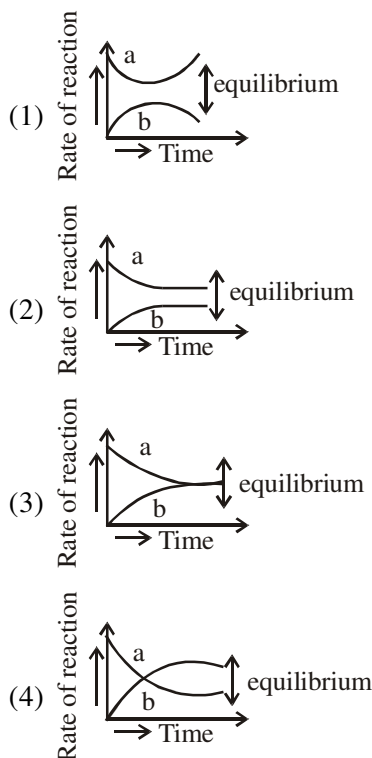
- If  $E_{\text{ext}} > 1.1\text{ V}$ , Zn dissolves at Zn electrode and Cu deposits at Cu electrode
- If  $E_{\text{ext}} > 1.1\text{ V}$ ,  $e^-$  flows from Cu to Zn
- If  $E_{\text{ext}} = 1.1\text{ V}$ , no flow of  $e^-$  or current occurs
- If  $E_{\text{ext}} < 1.1\text{ V}$ , Zn dissolves at anode and Cu deposits at cathode

6. The IUPAC name of the following compound is :

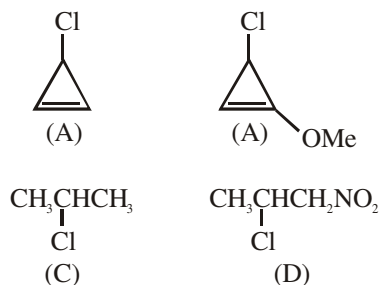


- 4-Bromo-2-methylcyclopentane carboxylic acid
- 5-Bromo-3-methylcyclopentanoic acid
- 3-Bromo-5-methylcyclopentane carboxylic acid
- 3-Bromo-5-methylcyclopentanoic acid

7. For the equilibrium  $A \rightleftharpoons B$ , the variation of the rate of the forward (a) and reverse (b) reaction with time is given by

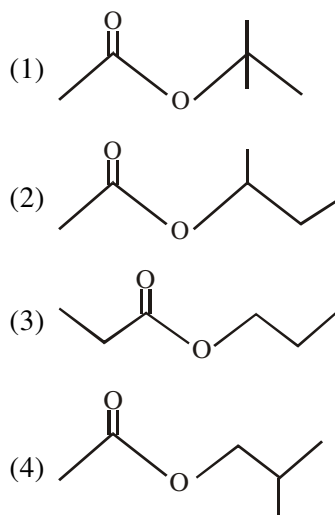


8. The decreasing order of reactivity of the following organic molecules towards  $\text{AgNO}_3$  solution is :



- (1) (A) > (B) > (D) > (C)  
 (2) (A) > (B) > (C) > (D)  
 (3) (C) > (D) > (A) > (B)  
 (4) (B) > (A) > (C) > (D)

9. An organic compound (A) (molecular formula  $\text{C}_6\text{H}_{12}\text{O}_2$ ) was hydrolysed with dil.  $\text{H}_2\text{SO}_4$  to give a carboxylic acid (B) and an alcohol (C). 'C' give white turbidity immediately when treated with anhydrous  $\text{ZnCl}_2$  and conc.  $\text{HCl}$ . The organic compound (A) is :



10. Match the following :

- |               |                |
|---------------|----------------|
| (i) Foam      | (a) smoke      |
| (ii) Gel      | (b) cell fluid |
| (iii) Aerosol | (c) jellies    |
| (iv) Emulsion | (d) rubber     |
|               | (e) froth      |
|               | (f) milk       |

(1) (i)-(b), (ii)-(c), (iii)-(e), (iv)-(d)

(2) (i)-(d), (ii)-(b), (iii)-(e), (iv)-(f)

(3) (i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)

(4) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(e)

11. The elements with atomic numbers 101 and 104 belong to, respectively :

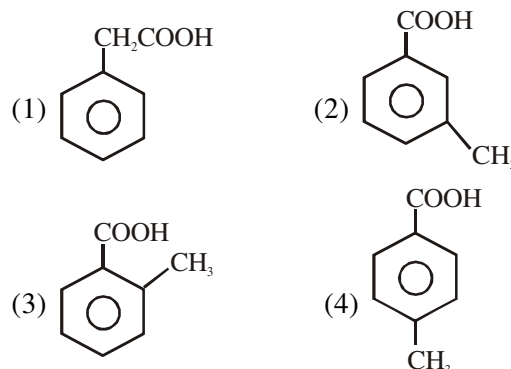
- (1) Group 11 and Group 4
- (2) Actinoids and Group 4
- (3) Actinoids and Group 6
- (4) Group 6 and Actinoids

12. On combustion Li, Na and K in excess of air, the major oxides formed, respectively, are :

- (1)  $\text{Li}_2\text{O}$ ,  $\text{Na}_2\text{O}$  and  $\text{K}_2\text{O}_2$
- (2)  $\text{Li}_2\text{O}$ ,  $\text{Na}_2\text{O}_2$  and  $\text{K}_2\text{O}$
- (3)  $\text{Li}_2\text{O}$ ,  $\text{Na}_2\text{O}_2$  and  $\text{KO}_2$
- (4)  $\text{Li}_2\text{O}_2$ ,  $\text{Na}_2\text{O}_2$  and  $\text{K}_2\text{O}_2$

13. [P] on treatment with  $\text{Br}_2/\text{FeBr}_3$  in  $\text{CCl}_4$  produced a single isomer  $\text{C}_8\text{H}_7\text{O}_2\text{Br}$  while heating [P] with sodalime gave toluene.

The compound [P] is :



14. The number of isomers possible for  $[\text{Pt}(\text{en})(\text{NO}_2)_2]$  is :

- |       |       |
|-------|-------|
| (1) 3 | (2) 2 |
| (3) 1 | (4) 4 |

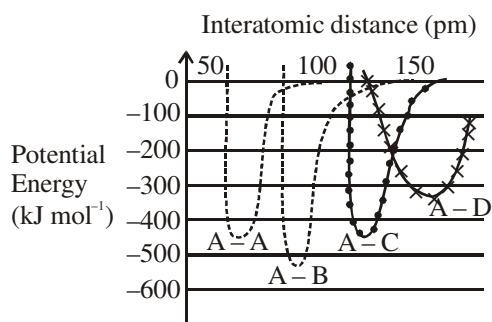
15. The ionic radii of  $O_2^-$ ,  $F^-$ ,  $Na^+$  and  $Mg^{2+}$  are in the order :

- (1)  $F^- > O_2^- > Na^+ > Mg^{2+}$
- (2)  $Mg^{2+} > Na^+ > F^- > O_2^-$
- (3)  $O_2^- > F^- > Mg^{2+} > Na^+$
- (4)  $O_2^- > F^- > Na^+ > Mg^{2+}$

16. The region in the electromagnetic spectrum where the Balmer series lines appear is

- (1) Visible
- (2) Microwave
- (3) Ultraviolet
- (4) Infrared

17. The intermolecular potential energy for the molecules A, B, C and D given below suggests that :



- (1) D is more electronegative than other atoms
- (2) A-D has the shortest bond length
- (3) A-B has the stiffest bond
- (4) A-A has the largest bond enthalpy

18. What are the functional groups present in the structure of maltose ?

- (1) One ketal and one hemiketal
- (2) One acetal and one hemiacetal
- (3) Two acetals
- (4) One acetal and one ketal

19. For one mole of an ideal gas, which of these statements must be true ?

- (a)  $U$  and  $H$  each depends only on temperature
  - (b) Compressibility factor  $z$  is not equal to 1
  - (c)  $C_{P,m} - C_{V,m} = R$
  - (d)  $dU = C_V dT$  for any process
- (1) (a), (c) and (d)      (2) (b), (c) and (d)  
 (3) (c) and (d)      (4) (a) and (c)

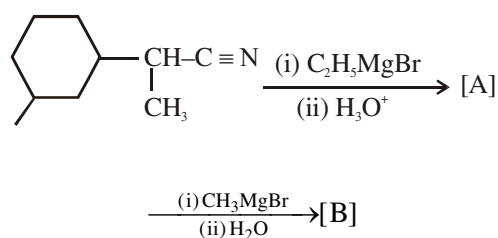


20. The pair in which both the species have the same magnetic moment (spin only) is :

- (1)  $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$  and  $[\text{Cr}(\text{H}_2\text{O})]^{2+}$
- (2)  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$  and  $[\text{CoCl}_4]^{2-}$
- (3)  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$  and  $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$
- (4)  $[\text{Co}(\text{OH})_4]^{2-}$  and  $[\text{Fe}(\text{NH}_3)_6]^{2+}$

21. The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is \_\_\_\_\_.

22. The number of chiral centres present in [B] is \_\_\_\_\_.



23. A 20.0 mL solution containing 0.2 g impure  $\text{H}_2\text{O}_2$  reacts completely with 0.316 g of  $\text{KMnO}_4$  in acid solution. The purity of  $\text{H}_2\text{O}_2$  (in %) is \_\_\_\_\_ (mol. wt. of  $\text{H}_2\text{O}_2 = 34$ ; mol. wt. of  $\text{KMnO}_4 = 158$ )

24. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) \_\_\_\_\_.
- (Take :  $\log 2 = 0.30$ ;  $\log 2.5 = 0.40$ )
25. At 300 K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state \_\_\_\_\_ ?

**FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020**

**(Held On Friday 04<sup>th</sup> SEPTEMBER, 2020) TIME : 9 AM to 12 PM**

| MATHEMATICS  | TEST PAPER WITH SOLUTION   |
|--|--|
| <p>1. If <math>A = \begin{bmatrix} \cos \theta &amp; i \sin \theta \\ i \sin \theta &amp; \cos \theta \end{bmatrix}</math>, <math>\left(\theta = \frac{\pi}{24}\right)</math> and</p> <p><math>A^5 = \begin{bmatrix} a &amp; b \\ c &amp; d \end{bmatrix}</math>, where <math>i = \sqrt{-1}</math>, then which one of the following is not true?</p> <p>(1) <math>0 \leq a^2 + b^2 \leq 1</math>      (2) <math>a^2 - d^2 = 0</math></p> <p>(3) <math>a^2 - b^2 = \frac{1}{2}</math>      (4) <math>a^2 - c^2 = 1</math></p> <p>2. Let <math>[t]</math> denote the greatest integer <math>\leq t</math>. Then the equation in <math>x</math>, <math>[x]^2 + 2[x + 2] - 7 = 0</math> has :</p> <p>(1) no integral solution</p> <p>(2) exactly four integral solutions</p> <p>(3) exactly two solutions</p> <p>(4) infinitely many solutions</p> | <p>3. Let <math>\alpha</math> and <math>\beta</math> be the roots of <math>x^2 - 3x + p = 0</math> and <math>\gamma</math> and <math>\delta</math> be the roots of <math>x^2 - 6x + q = 0</math>. If <math>\alpha, \beta, \gamma, \delta</math> form a geometric progression. Then ratio <math>(2q + p) : (2q - p)</math> is :</p> <p>(1) 3 : 1      (2) 33 : 31</p> <p>(3) 9 : 7      (4) 5 : 3</p> <p>4. Let <math>\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1</math> (<math>a &gt; b</math>) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, <math>\phi(t) = \frac{5}{12} + t - t^2</math>, then <math>a^2 + b^2</math> is equal to :</p> <p>(1) 126      (2) 135</p> <p>(3) 145      (4) 116</p> |

5. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If  $\angle BAC = 90^\circ$ , and  $\text{ar}(\triangle ABC) = 5\sqrt{5}$  sq. units, then the abscissa of the vertex C is :

- (1)  $2 + \sqrt{5}$  (2)  $1 + \sqrt{5}$   
(3)  $1 + 2\sqrt{5}$  (4)  $2\sqrt{5} - 1$

6. Let  $f(x) = |x - 2|$  and  $g(x) = f(f(x))$ ,  $x \in [0, 4]$ .

Then  $\int_0^3 (g(x) - f(x)) dx$  is equal to :

- (1)  $\frac{3}{2}$  (2) 0  
(3)  $\frac{1}{2}$  (4) 1

7. Given the following two statements :

(S<sub>1</sub>) :  $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$  is a tautology.

(S<sub>2</sub>) :  $\sim q \wedge (\sim p \leftrightarrow q)$  is a fallacy.

Then :

- (1) only (S<sub>1</sub>) is correct.  
(2) both (S<sub>1</sub>) and (S<sub>2</sub>) are correct.  
(3) both (S<sub>1</sub>) and (S<sub>2</sub>) are not correct.  
(4) only (S<sub>2</sub>) is correct.

8. Let P(3, 3) be a point on the hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \text{ If the normal to it at P intersects}$$

the x-axis at (9, 0) and e is its eccentricity, then the ordered pair  $(a^2, e^2)$  is equal to :

- (1)  $\left(\frac{9}{2}, 3\right)$  (2)  $\left(\frac{9}{2}, 2\right)$   
(3)  $\left(\frac{3}{2}, 2\right)$  (4) (9, 3)

9. Let  $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$  ( $x \geq 0$ ). Then  $f(3) - f(1)$

is equal to :

(1)  $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$  (2)  $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(3)  $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$  (4)  $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If  $x\%$  of the people read both the newspapers, then a possible value of  $x$  can be:

- (1) 65 (2) 37  
(3) 29 (4) 55

11. Let  $u = \frac{2z+i}{z-ki}$ ,  $z = x + iy$  and  $k > 0$ . If the curve

represented by  $\text{Re}(u) + \text{Im}(u) = 1$  intersects the  $y$ -axis at the points P and Q where  $PQ = 5$ , then the value of  $k$  is :

- (1)  $3/2$  (2) 4  
(3) 2 (4)  $1/2$

12. Let  $x_0$  be the point of local maxima of  $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$ , where  $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$  and  $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$ . Then the value of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  at  $x = x_0$  is :
- (1) -30 (2) 14  
(3) -4 (4) -22
13. Two vertical poles  $AB = 15$  m and  $CD = 10$  m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :
- (1) 20/3 (2) 5  
(3) 10/3 (4) 6
14. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :
- (1) 7 (2) 3  
(3) 5 (4) 9
15. The integral  $\int \left( \frac{x}{x \sin x + \cos x} \right)^2 dx$  is equal to : (where C is a constant of integration)
- (1)  $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$   
(2)  $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$   
(3)  $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$   
(4)  $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$

16. If

$1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+.....+(1-20^2.19)$   
 $= \alpha - 220\beta$ , then an ordered pair  $(\alpha, \beta)$  is equal  
 to :

- (1) (10, 97)                      (2) (11, 103)  
 (3) (10, 103)                    (4) (11, 97)

17. Let  $y = y(x)$  be the solution of the differential  
 equation,  $xy' - y = x^2(x \cos x + \sin x)$ ,  $x > 0$ .

If  $y(\pi) = \pi$ , then  $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$  is equal to :

- (1)  $2 + \frac{\pi}{2}$                       (2)  $1 + \frac{\pi}{2}$   
 (3)  $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$                     (4)  $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

18. The value of  $\sum_{r=0}^{20} {}^{50-r}C_6$  is equal to :

- (1)  ${}^{51}C_7 + {}^{30}C_7$                       (2)  ${}^{51}C_7 - {}^{30}C_7$   
 (3)  ${}^{50}C_7 - {}^{30}C_7$                     (4)  ${}^{50}C_6 - {}^{30}C_6$

19. Let  $f$  be a twice differentiable function on  
 $(1, 6)$ . If  $f(2) = 8$ ,  $f'(2) = 5$ ,  $f'(x) \geq 1$  and  
 $f''(x) \geq 4$ , for all  $x \in (1, 6)$ , then :

- (1)  $f(5) \leq 10$                       (2)  $f'(5) + f''(5) \leq 20$   
 (3)  $f(5) + f'(5) \geq 28$                     (4)  $f(5) + f'(5) \leq 26$

20. If  $(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$ ,

where  $a > b > 0$ , then  $\frac{dx}{dy}$  at  $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$  is :

(1)  $\frac{a-b}{a+b}$

(2)  $\frac{a+b}{a-b}$

(3)  $\frac{2a+b}{2a-b}$

(4)  $\frac{a-2b}{a+2b}$

21. If the system of equations

$$x - 2y + 3z = 9$$

$$2x + y + z = b$$

$$x - 7y + az = 24,$$

has infinitely many solutions, then  $a - b$  is equal to \_\_\_\_\_.

22. The probability of a man hitting a target is  $\frac{1}{10}$ .

The least number of shots required, so that the probability of his hitting the target at least once

is greater than  $\frac{1}{4}$ , is \_\_\_\_\_.

23. Suppose a differentiable function  $f(x)$  satisfies the identity  $f(x + y) = f(x) + f(y) + xy^2 + x^2y$ ,

for all real  $x$  and  $y$ . If  $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$ , then  $f'(3)$

is equal to \_\_\_\_\_.



24. Let  $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ . Then  $\frac{a_7}{a_{13}}$  is equal to \_\_\_\_\_.

25. If the equation of a plane P, passing through the intersection of the planes,  $x + 4y - z + 7 = 0$  and  $3x + y + 5z = 8$  is  $ax + by + 6z = 15$  for some  $a, b \in \mathbb{R}$ , then the distance of the point  $(3, 2, -1)$  from the plane P is \_\_\_\_\_.