FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Tuesday 31st August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS | **TEST PAPER WITH SOLUTION**

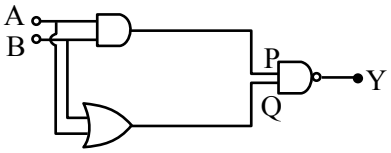
SECTION-A

1. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

(1) $\sqrt{\frac{2ghv^2+1}{h^2}}$ (2) $\sqrt{2ghv^2+h^2}$

(3) $\sqrt{\frac{2v^2h}{g}+h^2}$ (4) $\sqrt{\frac{2gh}{v^2}}+h^2$

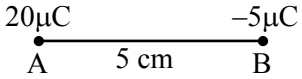
2. In the following logic circuit the sequence of the inputs A, B are (0, 0), (0,1), (1, 0) and (1, 1). The output Y for this sequence will be :



(1) 1, 0, 1, 0 (2) 0, 1, 0, 1

(3) 1, 1, 1, 0 (4) 0, 0, 1, 1

3. Two particles A and B having charges 20 μC and –5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force?



(1) At 5 cm from 20 μC on the left side of system

(2) At 5 cm from – 5 μC on the right side

(3) At 1.25 cm from – 5 μC between two charges

(4) At midpoint between two charges

4. A reversible engine has an efficiency of $\frac{1}{4}$. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink :

(1) 174°C (2) 280°C

(3) 180.4°C (4) 382°C

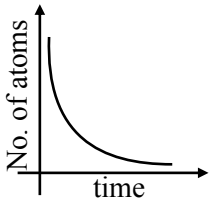
5. An object is placed at the focus of concave lens having focal length *f*. What is the magnification and distance of the image from the optical centre of the lens?

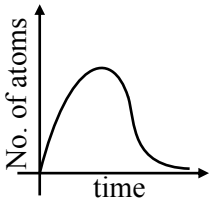
(1) 1, ∞ (2) Very high, ∞

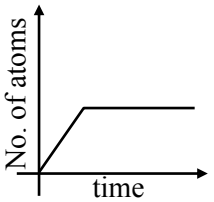
(3) $\frac{1}{2}, \frac{f}{2}$ (4) $\frac{1}{4}, \frac{f}{4}$

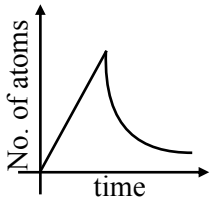
6. A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B vesus time is :

(Assume that at t = 0, there are no B atoms in the sample)

(1) 

(2) 

(3) 

(4) 

7. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil :

(1) $\frac{\mu_0 IN}{2(b-a)} \log_e\left(\frac{b}{a}\right)$ (2) $\frac{\mu_0 I}{8}\left[\frac{a+b}{a-b}\right]$

(3) $\frac{\mu_0 I}{4(a-b)}\left[\frac{1}{a}-\frac{1}{b}\right]$ (4) $\frac{\mu_0 I}{8}\left(\frac{a-b}{a+b}\right)$

8. A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is :

(1) 4 (2) 1 (3) 3 (4) 2

9. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses:

(1) $V=\frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$

(2) $V=\sqrt{\frac{4G(M_1+M_2)}{r}}$

(3) $V=\frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$

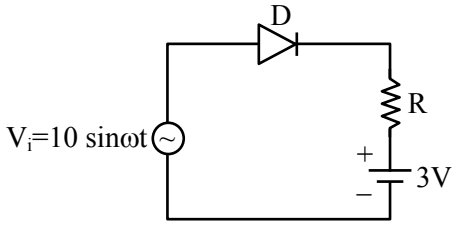
(4) $V=\frac{\sqrt{2G}(M_1+M_2)}{r}$

10. A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b >> a). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is :

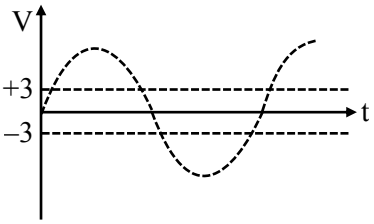
(1) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$ (2) $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{a}$

(3) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{b^2}{a}$ (4) $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{b}$

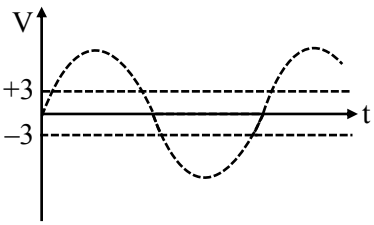
11. Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode is ideal one:



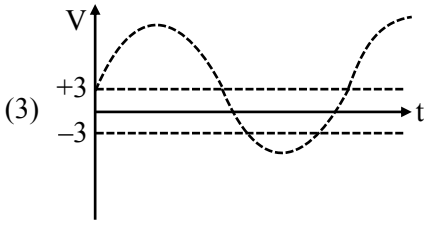
(1)



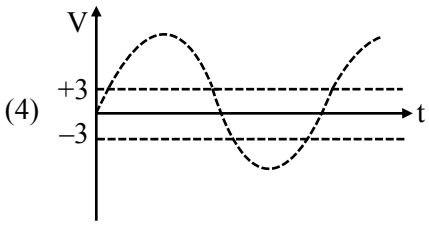
(2)



(3)



(4)

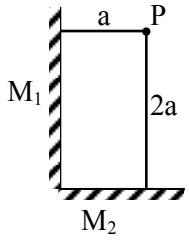


12. A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight.

(1) 2×10^{-9} m (2) 5×10^{-8} m

(3) 4×10^{-8} m (4) 5×10^{-10} m

13. Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is : (Take $\sqrt{5} = 2.3$)



(1) 3a

(2) 4.6 a

(3) 2.3 a

(4) $2\sqrt{10}$ a

14. Match **List-I** with **List-II**.

List-I	**List-II**
(a) Torque	(i) MLT^{-1}
(b) Impulse	(ii) MT^{-2}
(c) Tension	(iii) ML^2T^{-2}
(d) Surface Tension	(iv) MLT^{-2}

Choose the **most appropriate** answer from the option given below :

(1) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

(2) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

(3) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

(4) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)

15. For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If $p = p_0$ at $v = 0$ is the given boundary condition, then the maximum temperature one mole of gas can attain is :
(Here R is the gas constant)

(1) $\frac{p_0}{aeR}$

(2) $\frac{ap_0}{eR}$

(3) infinity

(4) 0ºC

16. Which of the following equations is dimensionally incorrect ?

Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume , p = pressure, W = work done, Γ = torque, $\in$= permittivity, E = electric field, J = current density, L = length.

(1) $v = \frac{\pi p a^4}{8 \eta L}$ (2) $h = \frac{2 s \cos\theta}{\rho r g}$

(3) $J = \in \frac{\partial E}{\partial t}$ (4) $W = \Gamma\theta$

17. Angular momentum of a single particle moving with constant speed along circular path :

(1) changes in magnitude but remains same in the direction

(2) remains same in magnitude and direction

(3) remains same in magnitude but changes in the direction

(4) is zero

18. In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is :

(1) $2R^2$ (2) Zero

(3) R (4) $R\sqrt{2}$

19. A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option :

(1) $K_p < K_e$ and $P_p = P_e$

(2) $K_p = K_e$ and $P_p = P_e$

(3) $K_p < K_e$ and $P_p < P_e$

(4) $K_p > K_e$ and $P_p = P_e$

20. Consider a galvanometer shunted with 5Ω resistance and 2% of current passes through it.

What is the resistance of the given galvanometer ?

(1) 300 Ω (2) 344 Ω

(3) 245 Ω (4) 226 Ω

SECTION-B

1. When a rubber ball is taken to a depth of _____m in deep sea, its volume decreases by 0.5%.

(The bulk modulus of rubber = 9.8×10^8 Nm^{-2}

Density of sea water = 10^3 kgm^{-3}

g = 9.8 m/s^2)

2. A particle of mass 1 kg is hanging from a spring of force constant 100 $Nm^{-1.}$ The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$ .The value of x is _____.

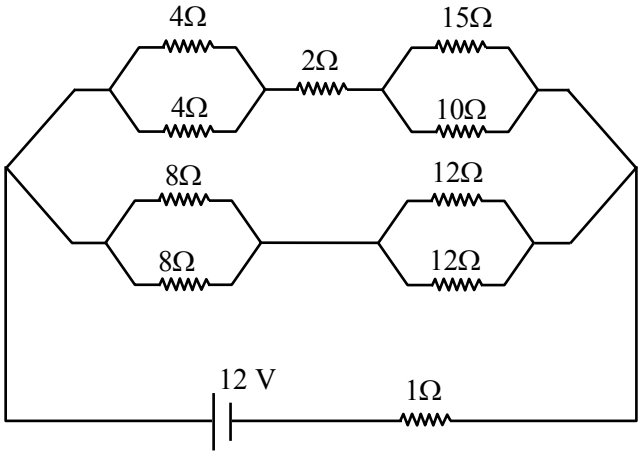
3. If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is _____km.

(Take radius of Earth = 6400 km)

4. A square shaped wire with resistance of each side 3Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be _____.

5. A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is _____m.

6. The voltage drop across 15Ω resistance in the given figure will be_____V.



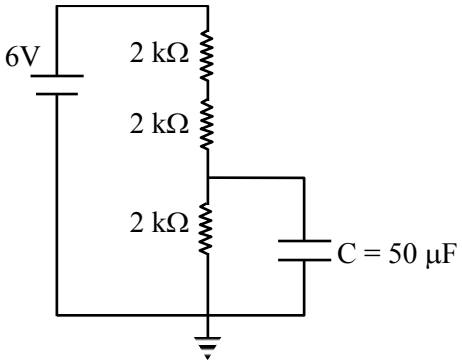
7. A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be x : 4 where x =_____.

8. The electric field in an electromagnetic wave is given by E = (50 NC^{-1}) sinω (t–x/c)

The energy contained in a cylinder of volume V is 5.5 × 10^{-12} J. The value of V is _____cm^3.

$\left(\text{given} \in_0 = 8.8 \times 10^{-12} C^2 N^{-1} m^{-2}\right)$

9. A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is_____μC.



10. A car is moving on a plane inclined at 30º to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car.The angle in degrees which the string makes with the vertical is_____. (Take g = 10 ms^{-2})

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

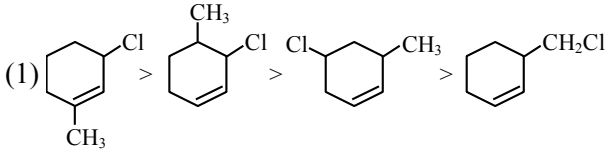
(Held On Tuesday 31st August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

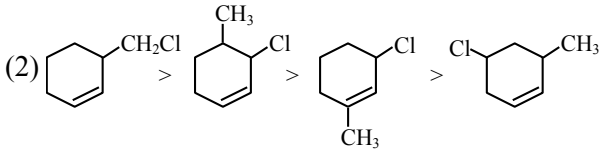
CHEMISTRY

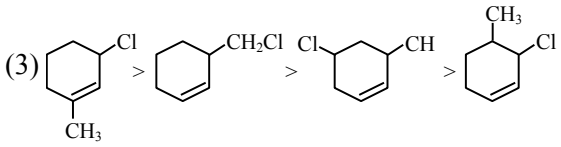
TEST PAPER WITH SOLUTION

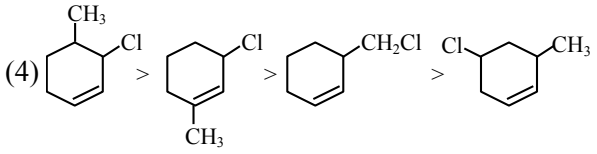
SECTION-A

1. The **correct** order of reactivity of the given chlorides with acetate in acetic acid is :

(1) 

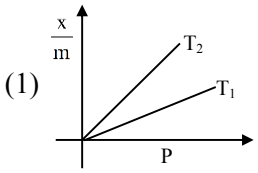
(2) 

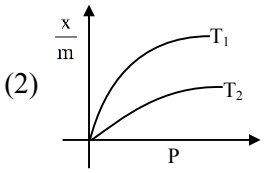
(3) 

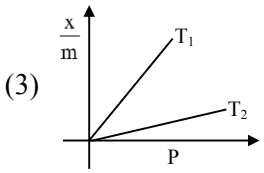
(4) 

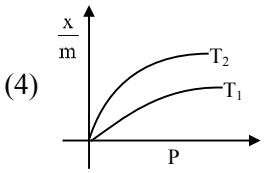
2. Select the graph that correctly describes the adsorption isotherms at two temperatures T_1 and T_2 ($T_1 > T_2$) for a gas :

(x – mass of the gas adsorbed ; m – mass of adsorbent ; P – pressure)

(1) 

(2) 

(3) 

(4) 

3. The major component/ingredient of Portland Cement is :

(1) tricalcium aluminate

(2) tricalcium silicate

(3) dicalcium aluminate

(4) dicalcium silicate

4. In the structure of the dichromate ion, there is a :

(1) linear symmetrical Cr–O–Cr bond.

(2) non-linear symmetrical Cr–O–Cr bond.

(3) linear unsymmetrical Cr–O–Cr bond.

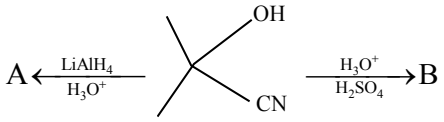
(4) non-linear unsymmetrical Cr–O–Cr bond.

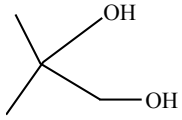
5. Which one of the following compounds contains β-C_1-C_4 glycosidic linkage ?

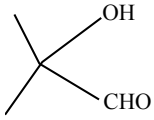
(1) Lactose (2) Sucrose

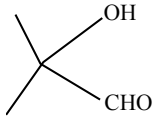
(3) Maltose (4) Amylose

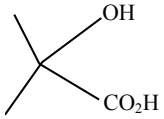
6. The major products A and B in the following set of reactions are :

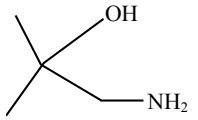


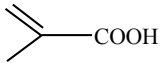
(1) A = 

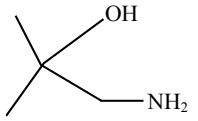
, B = 

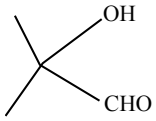
(2) A = 

, B = 

(3) A = 

, B = 

(4) A = 

, B = 

7. Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature ? (Given Z for Nd = 60, Yb = 70, La = 57, Ce =58)

(1) Nd (2) Yb (3) La (4) Ce

8. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite.

Reason (R) : The oxidation state of Al in cryolite is +3.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

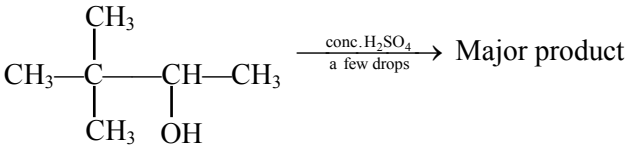
(1) **(A)** is true but **(R)** is false

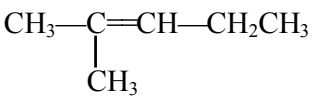
(2) **(A)** is false but **(R)** is true.

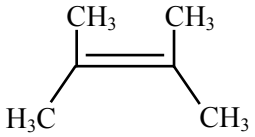
(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

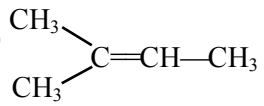
(4) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

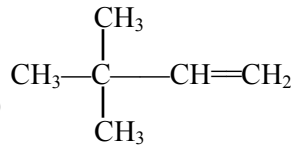
9. The major product formed in the following reaction is :



(1) 

(2) 

(3) 

(4) 

10. Monomer of Novolac is :

(1) 3-Hydroxybutanoic acid

(2) phenol and melamine

(3) o-Hydroxymethylphenol

(4) 1,3-Butadiene and styrene

11. Given below are two statements :

Statement-I : The process of producing syn-gas is called gasification of coal.

Statement-II : The composition of syn-gas is $CO + CO_2 + H_2$ (1 : 1 : 1)

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement-I** is false but **Statement-II** is true

(2) **Statement-I** is true but **Statement-II** is false

(3) Both **Statement-I** and **Statement-II** are false

(4) Both **Statement-I** and **Statement-II** are true

12. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Treatment of bromine water with propene yields 1-bromopropan-2-ol.

Reason (R) : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(4) **(A)** is true but **(R)** is false

13. The denticity of an organic ligand, biuret is :

(1) 2 (2) 4

(3) 3 (4) 6

14. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

Reason (R) : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

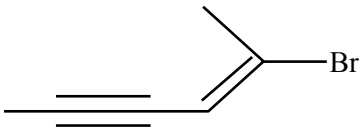
(1) **(A)** is false but **(R)** is true.

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(4) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

15. Choose the **correct** name for compound given below :



(1) (4E)-5-Bromo-hex-4-en-2-yne

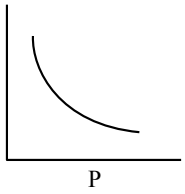
(2) (2E)-2-Bromo-hex-4-yn-2-ene

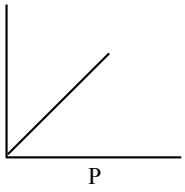
(3) (2E)-2-Bromo-hex-2-en-4-yne

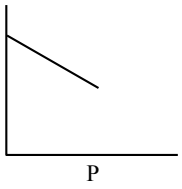
(4) (4E)-5-Bromo-hex-2-en-4-yne

16. Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas ? (P and V stand for pressure and volume of the gas respectively)

(1)

(2)


(3)


(4)


17. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)** :

Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.

Reason (R) : Two liquids with a difference of more than 20°C in their boiling points can be separated by simple distillations.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **(A)** is false but **(R)** is true.

(2) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

18. Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression ?

(1) hydrazine (2) glucose

(3) glycine (4) $KHSO_4$

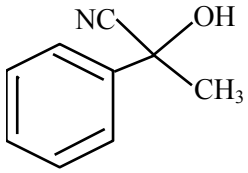
19. BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively :

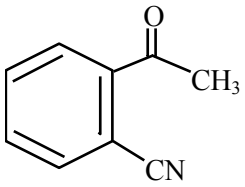
(1) A > 50, B < 27 (2) A > 25, B < 17

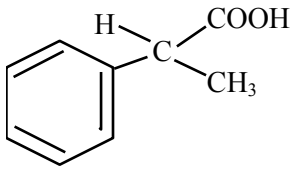
(3) A < 5, B > 17 (4) A > 15, B > 47

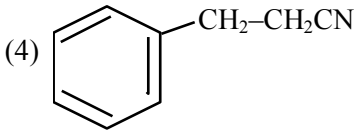
20. The structure of product C, formed by the following sequence of reactions is :

$$CH_3COOH + SOCl_2 \longrightarrow A \xrightarrow[AlCl_3]{\text{Benzene}} B \xrightarrow[-OH]{KCN} C$$

(1) 

(2) 

(3) 

(4) 

SECTION-B

1. Consider the following cell reaction :

$$Cd_{(s)} + Hg_2SO_{4(s)} + \frac{9}{5}H_2O_{(l)} \rightleftharpoons CdSO_4 \cdot \frac{9}{5}H_2O_{(s)} + 2Hg_{(l)}$$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H° = -825.2$ kJ mol^{-1}, the standard entropy change $\Delta S°$ in J K^{-1} is _____. (Nearest integer)

[Given : Faraday constant = 96487 C mol^{-1}]

2. The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ($H_2C_2O_4.2H_2O$) in 250 mL of water in mol L^{-1} is $x \times 10^{-2}$. The value of x is _____. (Nearest integer)

[Atomic mass : H : 1.0, C : 12.0, O : 16.0]

3. Consider the sulphides HgS, PbS, CuS, Sb_2S_3, As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is _____.

4. The total number of reagents from those given below, that can convert nitrobenzene into aniline is _____. (Integer answer)

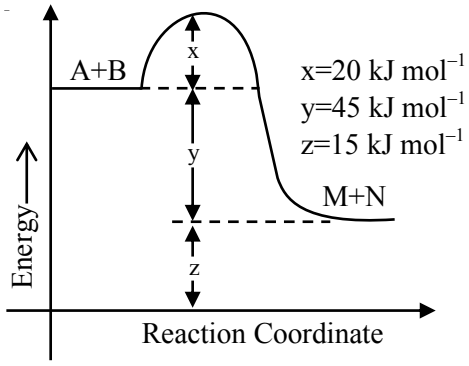
I. Sn – HCl
II. Sn – NH_4OH
III. Fe – HCl
IV. Zn – HCl
V. H_2 – Pd
VI. H_2 – Raney Nickel

5. The number of halogen/(s) forming halic (V) acid is _____.

6. For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is _____. (Integer answer)

7. The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4.5H_2O$ is _____.

8. According to the following figure, the magnitude of the enthalpy change of the reaction
$A + B \rightarrow M + N$ in kJ mol^{-1}
is equal to _____. (Integer answer)



9. Ge(Z = 32) in its ground state electronic configuration has x completely filled orbitals with $m_l = 0$. The value of x is _____.

10. A_3B_2 is a sparingly soluble salt of molar mass M (g mol^{-1}) and solubility x g L^{-1}. The solubility product satisfies $K_{sp} = a\left(\frac{x}{M}\right)^5$. The value of *a* is _____. (Integer answer)

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Tuesday 31st August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

(1) $* = \vee, \square = \vee$ (2) $* = \wedge, \square = \wedge$

(3) $* = \wedge, \square = \vee$ (4) $* = \vee, \square = \wedge$

2. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^{x} - 6 = 0$ is :

(1) 2 (2) 4

(3) 1 (4) 0

3. The sum of 10 terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots.$$ is :

(1) 1 (2) $\frac{120}{121}$

(3) $\frac{99}{100}$ (4) $\frac{143}{144}$

4. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :

(1) –23 (2) – 15

(3) 23 (4) 15

5. Let f be a non–negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-\left(f'(t)\right)^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$ and f(0) = 0, then $\lim\limits_{x\to 0} \frac{1}{x^2}\int_0^x f(t)\,dt$:

(1) equals 0 (2) equals 1

(3) does not exist (4) equals $\frac{1}{2}$

6. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}| = |3\vec{a}+\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 4 (2) 6

(3) 5 (4) 8

7. The function $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2-12x+4|}$ is not differentiable at exactly :

(1) four points (2) three points

(3) two points (4) one point

8. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to :

(1) $7 - 7\sqrt{3}$ (2) $7 + \sqrt{3}$

(3) $7 - \sqrt{3}$ (4) $7 + 3\sqrt{3}$

9. Which of the following is **not** correct for relation R on the set of real numbers ?

(1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.

(2) $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.

(3) $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.

(4) $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric.

10. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}} + C$ (2) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}} + C$

(3) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$ (4) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}} + C$

11. If p and q are the lengths of the perpendiculars from the origin on the lines,

$x \operatorname{cosec} \alpha - y \sec \alpha = k\cot 2\alpha$ and

$x \sin \alpha + y \cos\alpha = k \sin 2\alpha$

respectively, then k^2 is equal to :

(1) $4p^2 + q^2$ (2) $2p^2 + q^2$

(3) $p^2 + 2q^2$ (4) $p^2 + 4q^2$

12. $\operatorname{cosec}18°$ is a root of the equation :

(1) $x^2 + 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$

(3) $x^2 - 2x + 4 = 0$ (4) $x^2 - 2x - 4 = 0$

13. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a = -\frac{1}{3}, b \neq \frac{7}{3}$ (2) $a \neq \frac{1}{3}, b = \frac{7}{3}$

(3) $a \neq -\frac{1}{3}, b = \frac{7}{3}$ (4) $a = \frac{1}{3}, b \neq \frac{7}{3}$

14. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x–axis at a distance R and S (>R) respectively from the origin, is :

(1) 4(S + R) (2) 2(S – R)

(3) 4(S – R) (4) 2(S + R)

15. If the function $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , \quad x<0 \\ k & , \quad x=0 \\ \frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , \quad x>0\end{cases}$

is continuous at x = 0, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

(1) –5 (2) 5

(3) – 4 (4) 4

16. If $\frac{dy}{dx}=\frac{2^{x+y}-2^x}{2^y}$, y(0) = 1, then y(1) is equal to :

(1) $\log_2(2+e)$ (2) $\log_2(1+e)$

(3) $\log_2(2e)$ (4) $\log_2(1+e^2)$

17. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to :

(1) π^2 (2) $2\pi^2$

(3) $4\pi^2$ (4) 4π

18. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

(1) $12\sqrt{15}$ (2) $12\sqrt{10}$

(3) $8\sqrt{10}$ (4) $6\sqrt{10}$

19. If $a_r = \cos\dfrac{2r\pi}{9} + i\sin\dfrac{2r\pi}{9}$, r = 1, 2, 3,..., $i = \sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

(1) $a_2a_6 - a_4a_8$ (2) a_9

(3) $a_1a_9 - a_3a_7$ (4) a_5

20. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves?

(1) $x^2 + y^2 = 169$

(2) $144x^2 + 25y^2 = 3600$

(3) $25x^2 + 12y^2 = 3600$

(4) $x^2 + y^2 = 60$

SECTION-B

1. Let [t] denote the greatest integer $\le$ t. Then the value of $8\cdot\int\limits_{-\frac{1}{2}}^{1}\left([2x] + |x|\right)dx$ is _____.

2. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$, then the minimum value of $\left|z-9\sqrt{2}-2i\right|^2$ is equal to _____.

– =

3. The square of the distance of the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point $(-1, -1, 2)$ is _____.

4. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is _____.

5. The mean of 10 numbers

$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \ldots.$ is _____.

6. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is _____.

7. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _____.

8. If $x\,\phi(x) = \int_5^x \left(3t^2 - 2\phi'(t)\right) dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is _____.

9. If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to _____.

10. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to _____.