JEE Main 2020 Paper

Date of Exam: **7th January 2020 (Shift 1)**
Time: **9:30 am- 12:30 pm**
Subject: **Physics**

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1. A polarizer-analyzer set is adjusted such that the intensity of light coming out of the analyzer is just 10 % of the original intensity. Assuming that the polarizer-analyzer set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero is

a. 45^o b. 71.6^o

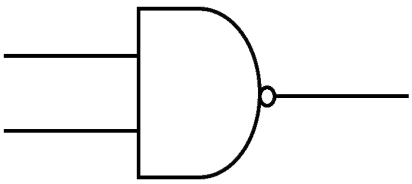
c. 90^o d. 18.4^0

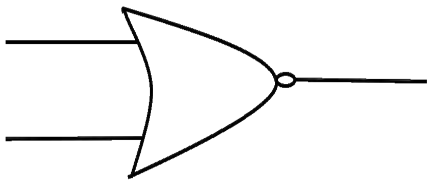
Solution:(d)

$$\text{Intensity after polarisation through polaroid} = I_o cos^2\phi$$
$$\text{So, } 0.1I_o = I_o cos^2\phi$$
$$\Rightarrow cos\phi = \sqrt{0.1}$$
$$\Rightarrow cos\phi = 0.316$$

Since, $\cos\phi < \cos 45^o$ therefore, $\phi > 45^o$ If the light is passing at 90^o from the plane of polaroid, than its intensity will be zero.
Then, $\theta = 90^o - \phi$ therefore, θ will be less than 45^o. So, the only option matching is option d which is 18.4^o

2. Which of the following gives reversible operation?


a.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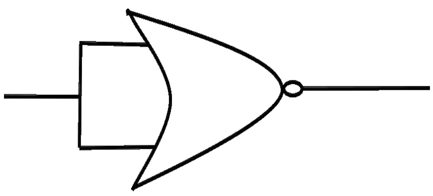
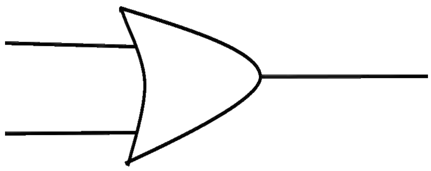
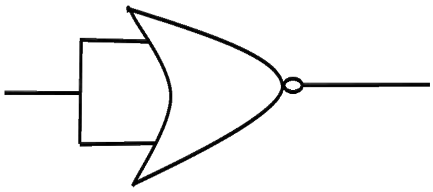

c.


d.

Solution: (c)



Since, there is only one input hence the operation is reversible.

3. A 60 HP electric motor lifts an elevator with a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Given 1 HP = 746 W, $g = 10\ m/s^2$)

a. $1.5\ m/s$
b. $2.0\ m/s$
c. $1.7\ m/s$
d. $1.9\ m/s$

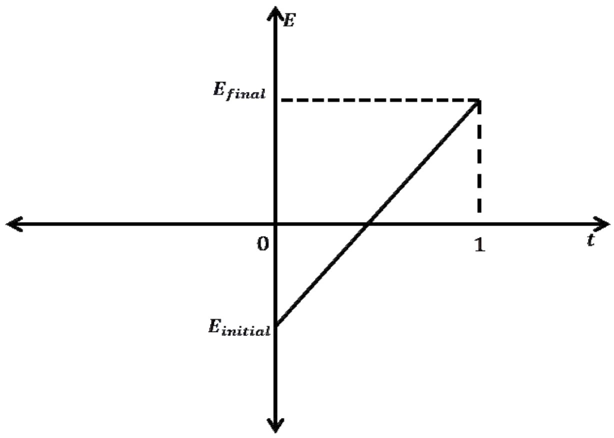
Solution:(d)
Friction will oppose the motion

$$\text{Net force} = 2000g + 4000 = 24000\ N$$
$$\text{Power of lift} = 60\ \text{HP}$$
$$\text{Power} = \text{Force} \times \text{Velocity}$$
$$v = \frac{P}{F} = \frac{60 \times 746}{24000}$$
$$v = 1.86\ m/s$$

4. A long solenoid of radius R carries a time (t) dependent current $I(t) = I_0 t(1-t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time instant $0 \leq t \leq 1$, the induced current (I_R) and the induced EMF (V_R) in the ring changes as:

a. Direction of I_R remains unchanged and V_R is maximum at $t = 0.5$
b. Direction of I_R remains unchanged and V_R is zero at $t = 0.25$
c. At $t = 0.5$ direction of I_R reverses and V_R is zero
d. At $t = 0.25$ direction of I_R reverses and V_R is maximum

Solution:(c)



Field due to solenoid near the middle $= \mu_o NI$

$$\text{Flux,} \quad \phi = BA \qquad \text{where}(A = \pi(R)^2)$$
$$= \mu_o NI_o t(1-t)\pi R^2$$
$$E = -\frac{d\phi}{dt} \qquad \text{[By Lenz's law]}$$
$$E = -\frac{d}{dt}(\mu_o NI_o t(1-t)\pi R^2)$$
$$E = -\mu_o NI_o \pi R^2 \frac{d}{dt}[t(1-t)]$$
$$E = -\pi\mu_o I_o NR^2(1-2t)$$

Current will change its direction when EMF will be zero

$$\implies (1-2t) = 0$$
$$So, \qquad t = 0.5\ sec$$

5. Two moles of an ideal gas with $\frac{C_p}{C_v} = 5/3$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_v} = 4/3$. The value of $\frac{C_p}{C_v}$ for the mixture is

a. 1.47

b. 1.42

c. 1.45

d. 1.50

Solution:(b)

$$\text{For first gas having } \gamma = \frac{C_p}{C_v} = \frac{5}{3}$$
$$\text{Using formula } C_p = \frac{R\gamma}{\gamma - 1}$$
$$C_v = \frac{R}{\gamma - 1}$$

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$$C_p = \frac{5R}{2} \quad C_v = \frac{3R}{2}$$

Similarly for 2^{nd} gas having $\gamma = \frac{C_p}{C_v} = \frac{4}{3}$

$$C_p = 4R \quad C_v = 3R$$

Now γ of mixture $= \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$

Given that $n_1 = 2$ and $n_2 = 3$

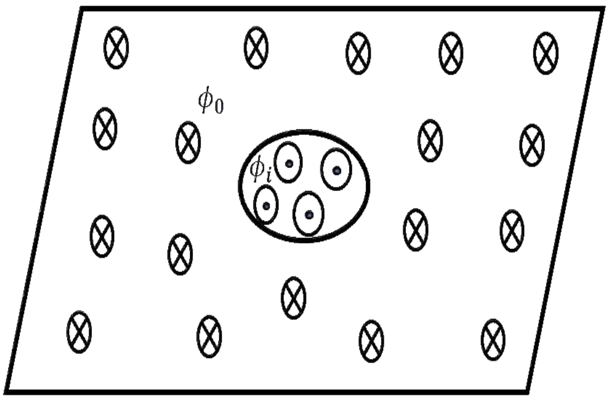
$$\gamma = \frac{2 \times \frac{5R}{2} + 3 \times 4R}{2 \times \frac{3R}{2} + 3 \times 3R} = \frac{17}{12} = 1.42$$

6. Consider a circular coil of wire carrying current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct?

   a. $\phi_i = -\phi_o$ b. $\phi_i > \phi_o$

   c. $\phi_i < \phi_o$ d. $\phi_i = \phi_o$

   Solution:(a)



As magnetic field line of ring will form close loop therefore all outgoing from circular hole passing through the infinite plate.

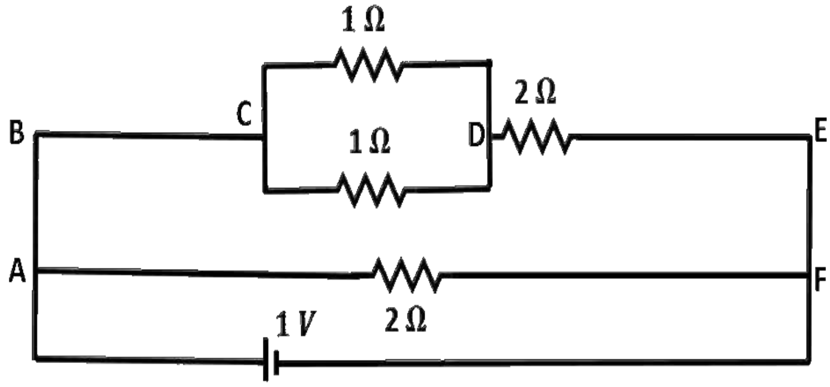
$\therefore \phi_1 = -\phi_0$ (because the magnetic field lines going inside is equal to the magnetic field lines coming out.)

7. The current (i_1) (in A) flowing through $1\ \Omega$ resistor in the following circuit is

   a. $0.40\ A$ b. $0.20\ A$

   c. $0.25\ A$ d. $0.5\ A$

   Solution:(b)



Net resistance across CD $= \dfrac{1}{2}\,\Omega$

Net resistance across BE $= 2 + \dfrac{1}{2} = \dfrac{5}{2}\,\Omega$

Net resistance across BE $= \dfrac{\frac{5}{2}\times 2}{\frac{5}{2}+2} = \dfrac{10}{9}\,\Omega.$

Total current in circuit $= \dfrac{V}{R} = \dfrac{9}{10}\,A$

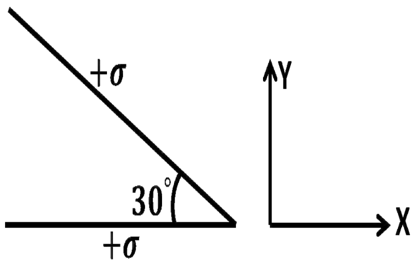
In the given circuit,voltage across BE = voltage across BF $= 1\,V$

Current across BE $= \dfrac{V_{BE}}{R} = \dfrac{2}{5}\,A$

Current across CD and DE will be same which is $\dfrac{2}{5}\,A.$

Now, current across any $1\,\Omega$ resistor will be same and given by $= I = \dfrac{1}{2}\times\dfrac{2}{5} = \dfrac{1}{5} = 0.20\,A$

8. Two infinite planes each with uniform surface charge density $+\sigma\ C/m^2$ are kept in such a way that the angle between them is 30^o. The electric field in the region shown between them is given by:



a. $\dfrac{\sigma}{2\epsilon_0}\left[(1-\dfrac{\sqrt{3}}{2})\hat{y} - \dfrac{1}{2}\hat{x}\right]$

b. $\dfrac{\sigma}{2\epsilon_0}\left[(1+\dfrac{\sqrt{3}}{2})\hat{y} - \dfrac{1}{2}\hat{x}\right]$

c. $\dfrac{\sigma}{2\epsilon_0}\left[(1-\dfrac{\sqrt{3}}{2})\hat{y} + \dfrac{1}{2}\hat{x}\right]$

d. $\frac{\sigma}{2\epsilon_0}\left[(1+\frac{\sqrt{3}}{2})\hat{y}+\frac{1}{2}\hat{x}\right]$

Solution:(a)

$$\text{Field due to single plate} = \frac{\sigma}{2\epsilon_o} = \left[\vec{E_1}\right] = \left[\vec{E_2}\right]$$
$$\text{Net electric field } \vec{E} = \vec{E_1} + \vec{E_2}$$
$$= \frac{\sigma}{2\epsilon_0}\cos 30^0(-\hat{j}) + \frac{\sigma}{2\epsilon_0}\sin 30^0(-\hat{i}) + \frac{\sigma}{2\epsilon_0}(\hat{j})$$
$$= \frac{\sigma}{2\epsilon_0}\left(1-\frac{\sqrt{3}}{2}\right)(\hat{j}) - \frac{\sigma}{4\epsilon_0}(\hat{i})$$
$$= \frac{\sigma}{2\epsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x}\right]$$

9. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{j}\ T$ then what will be expression for electric field?

a. $\vec{E} = 3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{i}\ V/m$

b. $\vec{E} = 3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{j}\ V/m$

c. $\vec{E} = 60\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{k}\ V/m$

d. $\vec{E} = 9\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{k}\ V/m$

Solution:(d)

We know that,

$$\left|\frac{E_0}{B_0}\right| = c$$
$$B_0 = 3\times10^{-8}$$
$$\implies E_0 = B_0\times c = 3\times10^{-8}\times3\times10^8$$
$$= 9\ N/C$$
$$\therefore E = E_o\sin(\omega t - kx + \phi)\hat{k} = 9\sin(\omega t - kx + \phi)\hat{k}$$
$$\vec{E} = 9\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{k}\ V/m$$

10. The time period of revolution of electron in its ground state orbit in a hydrogen atom is $1.6\times10^{-16}s$. The frequency of revolution of the electron in its first excited state (in s^{-1}) is :

a. 6.2×10^{15}

b. 1.6×10^{14}

c. 7.8×10^{14}

d. 5.6×10^{12}

Solution:(c)

Time period is proportional to $\frac{n^3}{Z^2}$.

Let T_1 be the time period in ground state and T_2 be the time period in it's first excited state.

$$T_1 = \frac{n^3}{2^2}$$

(Where, n = excitation level and 2 is atomic no.)

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$$

Given,

$$T_1 = 1.6 \times 10^{-16} s$$

So,

$$\frac{1.6 \times 10^{-16}}{T2} = \left(\frac{1}{2}\right)^3$$

$$T_2 = 12.8 \times 10^{-16}\ s$$

Frequency is given by $f = \dfrac{1}{T}$

$$f = \frac{1}{12.8} \times 10^{16}\ Hz$$

$$f = 7.8128 \times 10^{14}\ Hz$$

11. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence will be

a. $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

b. $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

c. $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

d. $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

Solution:(d)

For damped oscillator by Newton's second law

$$-kx - bv = ma$$

$$kx + bv + ma = 0$$

$$kx + b\frac{dx}{dt} + m\frac{d^2x}{dt^2} = 0$$

For LCR circuit by KVL

$$-IR - L\frac{dI}{dt} - \frac{q}{c} = 0$$

$$\implies IR + L\frac{dI}{dt} + \frac{q}{c} = 0$$

$$\implies \frac{q}{c} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} = 0$$

By comparing

$$R \implies b$$

$$c \implies \frac{1}{k}$$
$$m \implies L$$

12. Visible light of wavelength $6000 \times 10^{-8} cm$ falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minima is at 60^o from the central maxima. If the first minimum is produced at θ_1, then θ_1 is close to

a. 20^o b. 30^o
c. 45^o d. 25^o

Solution:(d)
For single slit diffraction experiment:
Angle of minima are given by

$$\sin\theta_n = \frac{n\lambda}{d} \quad (\sin\theta_n \neq \theta_n \text{ as } \theta \text{ is large })$$
$$\sin\theta_2 = \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{2\lambda}{d} = \frac{2 \times 6000 \times 10^{-10}}{d} \quad (1)$$
$$\sin\theta_1 = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{d} \quad (2)$$

Dividing (1) and (2)

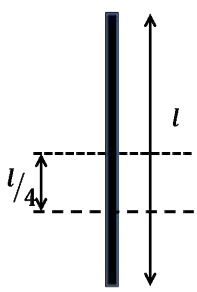
$$\implies \quad \frac{\sqrt{3}}{2sin\theta_1} = 2 \implies sin\theta_1 = \frac{\sqrt{3}}{4} = 0.43$$

As, the value is coming less than 30^o the only available option are 20^o and 25^o but by using approximation we get $\theta_1 = 25^\circ$

13. The radius of gyration of a uniform rod of length l about an axis passing through a point $l/4$ away from the center of the rod, and perpendicular to it, is

a. $\sqrt{\frac{7}{48}}l$ b. $\sqrt{\frac{3}{8}}l$
c. $\frac{1}{4}l$ d. $\frac{1}{8}l$

Solution:(a)



Moment of inertia of rod about axis perpendicular to it passing through its centre is given by

$$
\begin{aligned}
I &= \frac{Ml^2}{12} + M\left(\frac{l}{4}\right)^2 \\
&= \frac{3Ml^2 + 4Ml^2}{48} \\
&= \frac{7Ml^2}{48}
\end{aligned}
$$

Now, comparing with $I = Mk^2$ where k is the radius of gyration

$$
\begin{aligned}
k &= \sqrt{\frac{7l^2}{48}} \\
k &= l\sqrt{\frac{7}{48}}
\end{aligned}
$$

14. A satellite of mass m is launched vertically upward with an initial speed u from the surface of the earth. After it reaches height R(R = radius of earth), it ejects a rocket of mass $m/10$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G = gravitational constant; M is the mass of earth)

a. $5m\left[u^2 - \frac{119}{200}\frac{GM}{R}\right]$

b. $\frac{m}{20}\left[u - \sqrt{\frac{2GM}{3R}}\right]^2$

c. $\frac{3m}{8}\left[u + \sqrt{\frac{5GM}{6R}}\right]^2$

d. $\frac{m}{20}\left[u^2 + \frac{113}{200}\frac{GM}{R}\right]$

Solution:(a)
As we know,

$$
\begin{aligned}
&T.E_{ground} = T.E_R \\
&\frac{1}{2}mu^2 + \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv^2 + \left(\frac{-GMm}{2R}\right) \\
&\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \left(\frac{-GMm}{2R}\right) \\
&v^2 = u^2 + \left(\frac{-GM}{R}\right) \qquad (1) \\
&\implies v = \sqrt{u^2 + \left(\frac{-GM}{R}\right)}
\end{aligned}
$$

The rocket splits at height R. Since, sepration of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$
\begin{aligned}
\frac{m}{10}V_T &= \frac{9m}{10}\sqrt{\frac{GM}{2R}} \\
\frac{m}{10}V_r &= m\sqrt{u^2 - \frac{GM}{R}}
\end{aligned}
$$

$$\text{Kinetic energy of rocket} = \frac{1}{2} \times \frac{m}{10}(V_T^2 + V_R^2) = \frac{m}{20}\left(81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R}\right)$$

$$= \frac{m}{20}\left(100u^2 - \frac{119GM}{2R}\right)$$

$$= 5m\left(u^2 - \frac{119GM}{200R}\right)$$

15. Three point particles of mass 1 kg, 1.5 kg and 2.5 kg are placed at three corners of a right triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at the point:

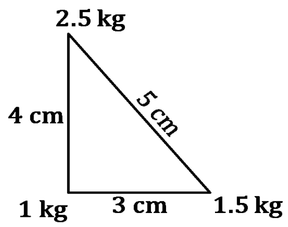
    a. 0.9 cm right and 2.0 cm above 1 kg mass

    b. 2.0 cm right and 0.9 cm above 1 kg mass

    c. 1.5 cm right and 1.2 cm above 1 kg mass

    d. 0.6 cm right and 2.0 cm above 1 kg mass

    Solution: (a)



Taking 1 kg as the origin

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$
$$x_{com} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5}$$
$$x_{com} = 0.9$$
$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$
$$y_{com} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5}$$
$$y_{com} = 2$$

Centre of mass is at $(0.9, 2)$

16. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece should be close to:

    a. 22 mm

    b. 2 mm

    c. 12 mm

    d. 33 mm

Solution:(a)

Magnification of compound microscope for least distance of distinct vision setting(strained eye)

$$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

where L is the tube length

f_0 is the focal length of objective

D is the least distance of distinct vision $= 25$ cm

$$\text{i.e.} \qquad 375 = \frac{150 \times 10^{-3}}{5 \times 10^{-3}}\left(1 + \frac{25 \times 10^{-2}}{f_e}\right)$$

$$\text{i.e.} \qquad 12.5 = 1 + \frac{25 \times 10^{-2}}{f_e}$$

$$\text{i.e.} \qquad \frac{25 \times 10^{-2}}{f_e} = 11.5$$

$$\therefore \quad f_e \approx 21.7 \times 10^{-3}\text{m} = 22 \text{ mm}$$

17. Speed of transverse wave on a straight wire (mass $6.0\ g$, length $60\ cm$ and area of cross-section $1.0\ mm^2$) is 90 m/s. If the Young's modulus of wire is $16 \times 10^{11} Nm^{-2}$, the extension of wire over its natural length is

a. $0.03\ mm$

b. $0.02\ mm$

c. $0.04\ mm$

d. $0.01\ mm$

Solution:(a)

Given, $M = 6\ grams = 6 \times 10^{-3}\ kg$

$$L = 60\ cm = 0.6\ m$$

$$A = 1\ mm^2 = 1 \times 10^{-6}\ m^2$$

$$\text{Using the relation, } v^2 = \frac{T}{\mu}$$

$$\Rightarrow T = \mu v^2 = V^2 \times \frac{M}{L}$$

$$\text{As Young's modulus, } Y = \frac{stress}{strain}$$

$$strain = \frac{stress}{Y} = \frac{T}{AY}$$

$$strain = \frac{\Delta L}{L} = \frac{V^2 \frac{M}{L}}{AY} = V^2 \frac{M}{AYL}$$

$$\Rightarrow \Delta L = \frac{V^2 M}{AY}$$

$$\Delta L = \frac{8100 \times 6 \times 10^{-3}}{1 \times 10^{-6} \times 16 \times 10^{11}}$$

$$\Delta L = 0.03\ mm$$

18. 1 liter of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.4$, the work done by air is ($3^{1.4} = 4.655$) (take air to be an ideal gas)

a. $48\ J$ b. $90.5\ J$

c. $100.8\ J$ d. $60.7\ J$

Solution:(b)
Given, $P_1 = 1\ atm, T_1 = 273\ K$ (At STP)
In adiabatic process,

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$P_2 = P_1\left[\frac{V_1}{V_2}\right]^{\gamma}$$
$$P_2 = 1 \times \left[\frac{1}{3}\right]^{1.4}$$
$$P_2 = 0.2164\ atm$$

Work done in adiabatic process is given by

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$
$$W = \frac{1 \times 1 - 3 \times .2164}{0.4} \times 101.325$$

Since, $1\ atm = 101.325\ kPa$ and $1\ Liter = 10^{-3}\ m^3$

$$W = 89.87\ J$$

19. A bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from the rest, the bob starts falling vertically. When it has covered a distance h, the angular speed of the wheel will be:

a. $r\sqrt{\frac{3}{4gh}}$ b. $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

c. $r\sqrt{\frac{3}{2gh}}$ d. $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

Solution:(b)
By energy conservation,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$\Rightarrow gh = \frac{v^2}{2} + \frac{\omega^2 r^2}{4} \tag{1}$$

Since the rope is inextensible and also it is not slipping,

$$\therefore v = r\omega \quad (2)$$

from eq. (1) and (2)

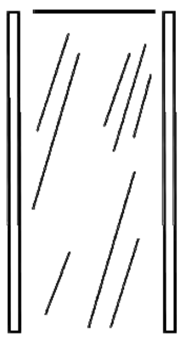
$$gh = \frac{\omega^2 r^2}{2} + \frac{\omega^2 r^2}{4}$$

$$\Rightarrow gh = \frac{3}{4} r^2 \omega^2$$

$$\Rightarrow \omega^2 = \frac{4gh}{3r^2}$$

$$\Rightarrow \omega = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

20. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant varies as $k\,(x) = k(1 + \alpha x)$, where 'x' is the distance measured from one of the plates. If $(\alpha d \ll 1)$, the total capacitance of the system is best given by the expression:



a. $\frac{A\varepsilon_0 k}{d}\left[1 + \left(\frac{\alpha d}{2}\right)^2\right]$

b. $\frac{Ak\varepsilon_0}{d}\left[1 + \left(\frac{\alpha d}{2}\right)\right]$

c. $\frac{A\varepsilon_0 k}{d}\left[1 + \left(\frac{\alpha^2 d}{2}\right)\right]$

d. $\frac{Ak\varepsilon_0}{d}\left[1 + \alpha d\right]$

Solution:(b)

Given, $k(x) = k\,(1 + \alpha x)$

dC = $\frac{A\varepsilon_0 k}{dx}$

Since all capacitance are in series, we can apply

$$\frac{1}{Ceq} = \int \frac{1}{dC} = \int_0^d \frac{dx}{k\,(1 + \alpha x)\,\epsilon_0 A}$$

$$\frac{1}{Ceq} = \left[\frac{\ln\,(1 + \alpha x)}{k\epsilon_0 A\alpha}\right]_0^d$$

On putting the limits from 0 to d

$$= \frac{\ln(1+\alpha d)}{k\epsilon_0 A\alpha}$$

Using expression $\ln(1+x) = x - \frac{x^2}{2} + ...$
And putting $x = \alpha d$ where, x approaches to 0.

$$\frac{1}{C} = \frac{d}{k\epsilon_0 A d\alpha}\left[\alpha d - \frac{(\alpha d)^2}{2}\right]$$

$$\frac{1}{C} = \frac{d}{k\epsilon_0 A}\left[1 - \frac{\alpha d}{2}\right]$$

$$C = \frac{k\epsilon_0 A}{d}\left[1 + \frac{\alpha d}{2}\right]$$

21. A non- isotropic solid metal cube has coefficient of linear expansion as $5 \times 10^{-5}/°C$ along the x-axis and $5 \times 10^{-6}/°C$ along y-axis and z-axis. If the coefficient of volumetric expansion of the solid is $C \times 10^{-6}/°C$ then the value of C is _______

Solution:(60)
We know that, $V = xyz$

$$\frac{\Delta v}{v} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\frac{1}{T}\frac{\Delta v}{v} = \frac{1}{T}\frac{\Delta x}{x} + \frac{1}{T}\frac{\Delta y}{y} + \frac{1}{T}\frac{\Delta z}{z}$$

$$y = \alpha_n + \alpha_y + \alpha_z$$

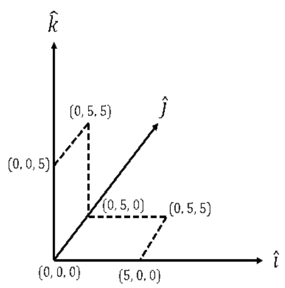
$$y = 50 \times 10^{-6}/°C + 5 \times 10^{-6}/°C + 5 \times 10^{-6}/°C$$

$$y = 60 \times 10^{-6}/°C$$

$$\therefore C = 60$$

22. A loop $ABCDEFA$ of straight edges has six corner points A(0,0,0), B(5,0,0), C(5,5,0), D(0,5,0), E(0,5,5), F(0,0,5). The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})\,T$. The quantity of flux through the loop $ABCDEFA$ (in Wb) is _______

Solution:(175)



As we know, magnetic flux $= \vec{B} \cdot \vec{A}$

$$\implies (B_x + B_z) \cdot (A_x + A_z)$$
$$\implies (3\hat{i} + 4\hat{k}) \cdot (25\hat{i} + 25\hat{k})$$
$$\implies (75 + 100)\, Wb$$
$$\implies 175\, Wb$$

23. A carnot engine operates between two reservoirs of temperature 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy in(J) delivered by the engine to the low temperature reservoir, in a cycle, is _______

Solution:(600 J)

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{2}{3}$$

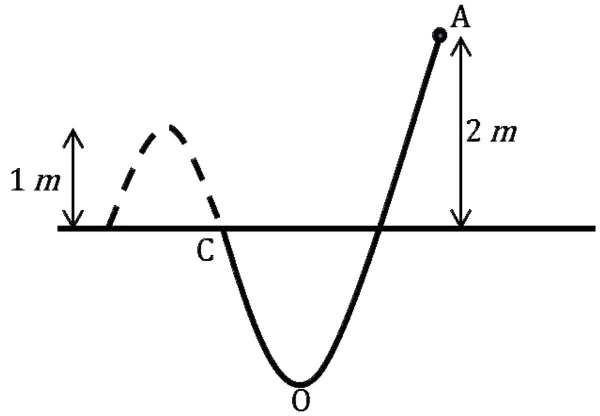
Given, $W = 1200\ J$

From conservation of energy

$$Q_1 - Q_2 = W$$
$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1} \implies Q_1 = 1800 \text{ J}$$
$$\Rightarrow Q_2 = Q_1 - W = 600 \text{ J}$$

24. A particle of mass 1 kg slides down a frictionless track (AOC) starting from rest at a point A(height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P(height 1 m) the kinetic energy of the particle (in J) is:(Figure drawn is schematic and not to scale; take $g = 10\ m/s^2$)_______

Solution:(10)



As the particle starts from rest the total energy at point $A = mgh = T.E_A$(where $h = 2\ m$)

After reaching point P

$$T.E_c = K.E. + mgh$$

By conservation of energy

$$T.E_A = T.E_p$$

$$\implies K.E. = mgh = 10\ J$$

25. A beam of electromagnetic radiation of intensity 6.4$\times\ 10^{-5}\ W/cm^2$ is comprised of wavelength, $\lambda = 310\ nm$. It falls normally on a metal (work function $\phi = 2\ eV$) of surface area $1\ cm^2$. If one in 10^3 photons ejects an electron, total number of electrons ejected in $1s$ is 10^x ($hc = 1240\ eV - nm, 1\ eV = 1.6 \times 10^{-19}\ J$), then x is -------

Solution:(11)

$$\begin{aligned} P &= \text{Intensity} \times \text{Area} \\ &= 6.4 \times 10^{-5}\text{W} - \text{cm}^{-2} \times 1\ \text{cm}^2 \\ &= 6.4 \times 10^{-5}\ \text{W} \end{aligned}$$

For photoelectric effect to take place, energy should be greater than work function

Now,

$$E = \frac{1240}{310} = 4\ eV > 2\ eV$$

Therefore, photoelectric effect takes place

Here n is the number of photons emitted.

$$n \times E = I \times A$$

$$\implies n = \frac{IA}{E} = \frac{6.4 \times 10^{-5}}{6.4 \times 10^{-19}} = 10^{14}$$

Where, n is number of incident photon

Since, 1 out of every 1000 photons are sucessfull in ejecting 1 photoelectron

Therefore, the number of photoelectrons emitted is

$$= \frac{10^{14}}{10^3}$$

$$\therefore x = 11$$

JEE Main 2020 Paper

Date: 7th January 2020

Time: 09:30 am – 12:30 pm

Subject: Chemistry

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1. The relative strength of interionic/ intermolecular forces in decreasing order is:
   a) ion-dipole > dipole-dipole > ion-ion
   b) dipole-dipole > ion-dipole > ion-ion
   c) ion-ion > ion-dipole > dipole-dipole
   d) ion-dipole > ion-ion > dipole-dipole

Answer: c

Solution:

Ion-ion interactions are stronger because they have stronger electrostatic forces of attraction whereas dipoles have partial charges and hence the electrostatic forces in their case would be relatively weak.

2. Oxidation number of potassium in K_2O, K_2O_2 and KO_2, respectively, is :
   a) $+2, +1$ and $+\frac{1}{2}$
   b) $+1, +2$ and $+4$
   c) $+1, +1$ and $+1$
   d) $+1, +4$ and $+2$

Answer: c

Solution:

Alkali metals always possess a +1 oxidation state, whereas oxygen present in K_2O (oxide) is -2, and in K_2O_2 (peroxide) is -1 and in KO_2 (superoxide) is $-\frac{1}{2}$.

3. At 35^0C, the vapour pressure of CS_2 is 512 mm Hg and that of acetone is 344 mm Hg. A solution of CS_2 in acetone has a total vapour pressure of 600 mm Hg. The false statement amongst the following is :
   a) CS_2 and acetone are less attracted to each other than to themselves
   b) heat must be absorbed in order to produce the solution at 35^0C
   c) Raoult's law is not obeyed by this system
   d) a mixture of 100 mL CS_2 and 100 mL acetone has a volume < 200 mL

Answer: d

Solution:
$P_{Total} = P_T = p_A^o X_A + p_B^o X_B$

The maximum value X_A can hold is 1, and hence the maximum value of P_T should come out to be 512 mm of Hg, which is less than the value of P_T observed (600 mm of Hg). Therefore, positive deviation from Raoult's law that is observed. This implies that A-A interactions and B-B interactions are stronger than A-B interactions. As we know, for a system not obeying Raoult's law and showing positive deviation, $\Delta V_{mix}>0$, $\Delta H_{mix}>0$.

4. The atomic radius of Ag is closest to :
 a) Ni
 b) Cu
 c) Au
 d) Hg

Answer: c

Solution:
Because of Lanthanide contraction, an increase in Z_{eff} is observed and so, the size of Au instead of being greater, as is expected, turns out be similar to that of Ag.

5. The dipole moments of CCl_4, $CHCl_3$ and CH_4 are in the order:
 a) $CH_4 < CCl_4 < CHCl_3$
 b) $CHCl_3 < CH_4 = CCl_4$
 c) $CH_4 = CCl_4 < CHCl_3$
 d) $CCl_4 < CH_4 < CHCl_3$

Answer: c

Solution:
All the three compounds possess a tetrahedral geometry. In both CCl_4 and CH_4, $\mu_{net}=0$, whereas in $CHCl_3$, $\mu_{net} > 0$.

6. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is :
 a) less efficient as it exchanges only anions
 b) more efficient as it can exchanges only cations
 c) less efficient as the resins cannot be regenerated
 d) more efficient as it can exchange both cations as well as anions

Answer: d

7. Amongst the following statements, that which was not proposed by Dalton was :
 a) matter consists of indivisible atoms
 b) when gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same T & P.
 c) Chemical reactions involve reorganisation of atoms. These are neither created nor destroyed in a chemical reaction.
 d) all the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.

Answer: b

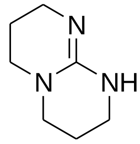
Solution:

When gases combine or react in a chemical reaction they do so in a simple ratio by volume provided all gases are maintained at the same temperature and pressure- Gay-Lussac's law.

8. The increasing order of pK_b for the following compounds will be :

$H_2N-CH=NH$

(i)



(ii)

CH_3NHCH_3

(iii)

a) ii < iii < i
b) iii < i < ii
c) i < ii < iii
d) ii < i < iii

Answer: d

Solution:

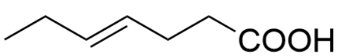
Weaker the conjugate acid, stronger the base. (ii) is the most basic as it has a guanidine like structure. It has a high tendency of accepting a proton, giving rise to a very stable conjugate acid and hence, is a very strong base.

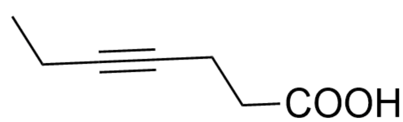
In compound (i), the N is sp^2 hybridised and its electronegativity is higher as compared to the compound (iii) which is a 2^0 amine (sp^3 hybridised). So compound (ii) is more basic compared to compound (iii).

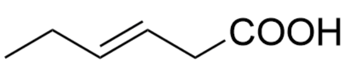
So the order of basicity is ii > i > iii and thus the order of pK_b value will be iii > i > ii

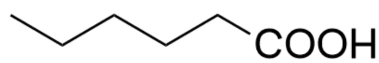
9. What is the product of the following reaction?

$$\text{Hex-3-ynal} \xrightarrow[\text{(iii) Mg/ether} \;\; \text{(iv) } CO_2/H_3O^+]{\text{(i) } NaBH_4 \;\; \text{(ii) } PBr_3}$$

a) 

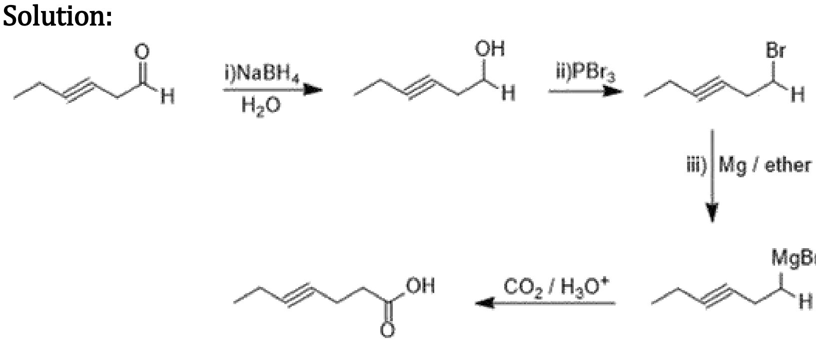
b) 

c) 

d) 

Answer: b

Solution:



10. The number of orbitals associated with quantum number n=5, m_s=+½ is :
   a) 11
   b) 15
   c) 25
   d) 50

Answer: c

Solution:
n = 5; l = (n – 1) = 4; hence the possible sub-shells for n=5 are: 5s, 5p, 5d, 5f and 5g.

The number of orbitals in each would be 1,3,5,7 and 9, respectively and summing them up gives the answer as 25.

11. The purest form of commercial iron is:
   a) cast iron
   b) wrought iron
   c) scrap iron and pig iron
   d) pig iron

Answer: b

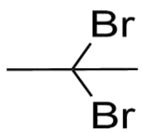
12. The theory that can completely/ properly explain the nature of bonding in [Ni(CO)4] is:
   a) Werner's theory
   b) Crystal Field Theory
   c) Molecular Orbital Theory
   d) Valence Bond Theory

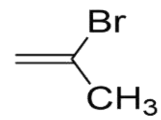
Answer: c

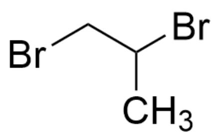
13. The IUPAC name of the complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is :
   a) Diamminechlorido(methanamine)platinum(II)chloride
   b) Bisammine(methanamine)chloridoplatinum(II) chloride
   c) Diammine(methanamine)chloridoplatinum(II)chloride
   d) Diamminechlorido(aminomethane)platinum(II)chloride

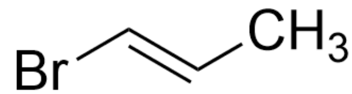
Answer: a

14. 1-methyl ethylene oxide when treated with an excess of HBr produces:

a) 

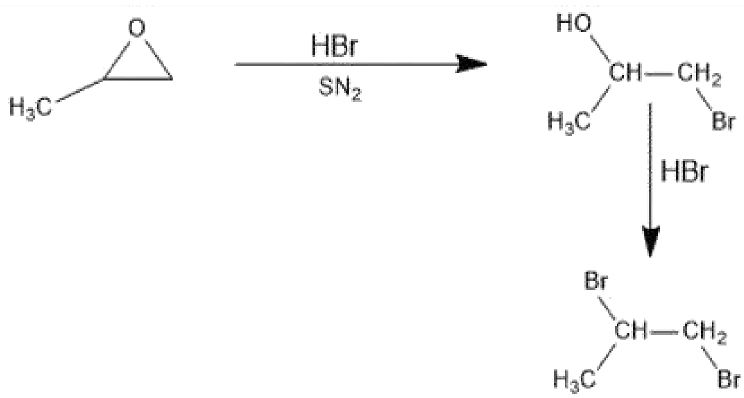
b) 

c) 

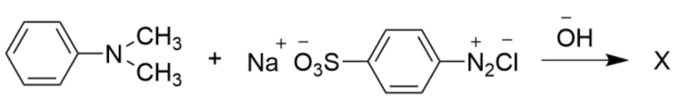
d) 

Answer: c

Solution:



15. Consider the following reaction:

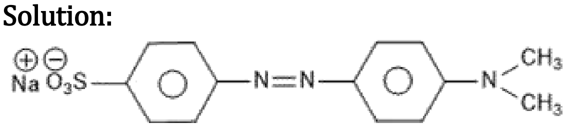


The product 'X' is used:

a) in protein estimation as an alternative to ninhydrin
b) as food grade colourant
c) in laboratory test for phenols
d) in acid-base titration as an indicator

Answer: d

Solution:



X formed is methyl orange.

16. Match the following:

List I	List II
i) Riboflavin	p) Beri beri
ii) Thiamine	q) Scurvy
iii) Ascorbic acid	r) Cheliosis
iv) Pyridoxine	s) Convulsions

	i	ii	iii	iv
a)	s	q	p	r
b)	r	p	q	s
c)	p	r	q	s
d)	s	r	q	p

Answer: b

Solution:

Vitamins	Deficiency diseases
i) Riboflavin (Vitamin B_2)	Cheilosis
ii) Thiamine (Vitamin B_1)	Beri beri
iii) Ascorbic acid (Vitamin C)	Scurvy
iv) Pyridoxine (Vitamin B_6)	Convulsions

17. Given that the standard potential; (E^o) of Cu^{2+}/Cu and Cu^{+}/Cu are 0.34 V and 0.522 V respectively, the E^o of Cu^{2+}/Cu^{+} is :

a) +0.158 V
b) -0.158 V
c) 0.182 V
d) -0.182 V

Answer: a

Solution:

$Cu^{2+} + 2e^- \rightarrow Cu \quad E^\circ = 0.340$ V

$Cu \rightarrow Cu^{+} + e^- \quad E^\circ = -0.522$ V

$Cu^{2+} + e^- \rightarrow Cu^{+} \quad E^\circ = ?$

Applying $\Delta G = -nFE^\circ$

We get,

$(-1 \times F \times E^\circ) = -2 \times F \times 0.340 + (-1 \times F \times -0.522)$

Solving, we get, $E^\circ = 0.158$ V

18. A solution of m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of $NaHCO_3$ to give fraction A. The left over organic phase was extracted with dil. NaOH solution to give fraction B. The final organic layer was labelled as fraction C. Fractions A, B and C, contain respectively:

a) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline
b) m-chlorophenol, m-chlorobenzoic acid and m-chloroaniline
c) m-chloroaniline, m-chlorobenzoic acid and m-chlorophenol
d) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol

Answer: a

Solution:
m-chlorobenzoic acid being the most acidic can be separated by a weak base like $NaHCO_3$ and hence will be labelled fraction A.

m-chlorophenol is not as acidic as m-chlorobenzoic acid, and can be separated by a stronger base like NaOH, and hence can be labelled as fraction B.

m-chloroaniline being a base, does not react with either of the bases and hence would be labelled as fraction C.

19. The electron gain enthalpy (in kJ/mol) of fluorine, chlorine, bromine, and iodine, respectively, are:

a) -333, -325, -349 and -296
b) -333, -349, -325 and -296
c) -296, -325, -333 and -349
d) -349, -333, -325 and -296

Answer: b

Solution:
Cl > F > Br > I

20. Consider the following reactions:

a) $(CH_3)_3CCH(OH)CH_3 \xrightarrow{\text{Conc.}H_2SO_4}$

b) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{\text{Alc.KOH}}$

c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{(CH_3)_3\overset{-}{O}\overset{+}{K}}$

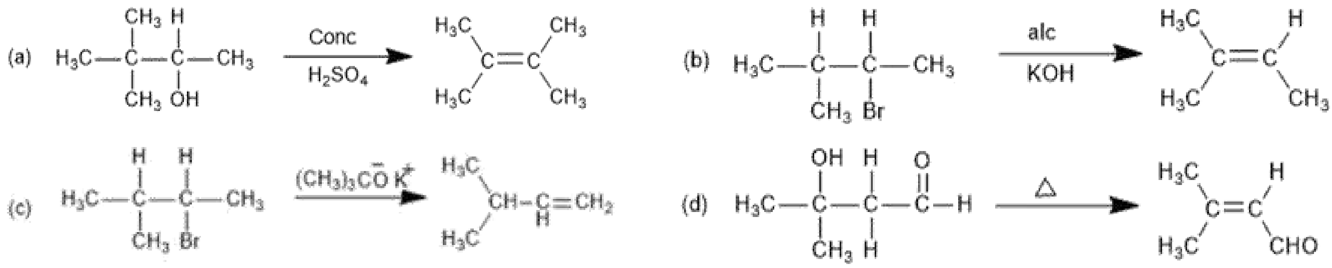
d) $(CH_3)_2\underset{\displaystyle OH}{\underset{|}{C}}-CH_2CHO \xrightarrow{\Delta}$

Which of these reaction(s) will not produce Saytzeff product?

a. b and d
b. d only
c. a, c and d
d. c only

Answer: d

Solution:



21. Two solutions A and B each of 100 L was made by dissolving 4 g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resulting solution obtained from mixing 40 L of Solution A and 10 L of Solution B is:

Answer: 10.6

Solution:
Molarity of NaOH (4 g in 100 L) = 10^{-3} M

Molarity of H_2SO_4 (9.8 g in 100 L) = 10^{-3} M

Equivalents of NaOH= $M \times V \times n_f = 10^{-3} \times 40 \times 1 = 0.04$

Equivalents of H_2SO_4= $M \times V \times n_f = 10^{-3} \times 10 \times 2 = 0.02$

$$M_{NaOH}.V_{NaOH}.(n_f)_{NaOH} - M_{H_2SO_4}.V_{H_2SO_4}.(n_f)_{H_2SO_4} = M.V_{total}$$

$10^{-3} \times 40 \times 1 - 10^{-3} \times 10 \times 2 = M.\,50$

$M = 4 \times 10^{-4}$

pOH= -log M

=4 – 2log2

=3.4

pH= 14 -3.4 = 10.6

22. During the nuclear explosion, one of the products is ^{90}Sr with half of 6.93 years. If 1 µg of ^{90}Sr was absorbed in the bones of a newly born baby in place of Ca, how much time, in years, is required to reduce it by 90% if it is not lost metabolically

Answer: 23.03

Solution:
All nuclear processes follow first order kinetics, and hence

$t_{1/2} = \frac{0.693}{\lambda}$

$\lambda = 0.1\ (\text{year})^{-1}$

$t = \frac{2.303}{\lambda}\left(\frac{\log(a_0)}{a_t}\right)$

$t = \frac{2.303}{0.1}\left(\frac{\log(a_0)}{0.1a_0}\right)$

On solving, t= 23.03 years

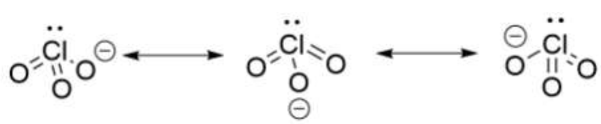
23. Chlorine reacts with hot and concentrated NaOH and produces compounds (X) and (Y). Compound (X) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (Y) is

Answer: 1.67

Solution:

$$Cl_2 + \underset{\text{(hot)}}{NaOH} \longrightarrow \underset{\text{(X)}}{NaCl} + NaClO_3 + H_2O$$

$$\text{(X)} \xrightarrow{AgNO_3} \underset{\text{(White)}}{AgCl}$$

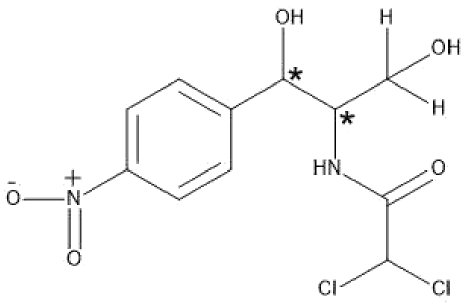


Bond order $= \frac{\text{Total no of bonds}}{\text{Total resonating structures}} = \frac{5}{3} = 1.67$

24. The number of chiral carbons in chloramphenicol is:

Answer: 2

Solution:



25. For the reaction $A_{(l)} \rightarrow 2B_{(g)}$

$\Delta U = 2.1$ kcal, $\Delta S = 20$ calK^{-1} at 300 K, Hence ΔG in kcal is

Answer: -2.7

Solution:
$\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = 2100 + (2 \times 2 \times 300)$ (R=2 cal$K^{-1}mol^{-1}$)

$= 3300$ cal

$\Delta G = \Delta H - T\Delta S$

$\Delta G = 3300 - (300 \times 20) = -2.7$ kcal

JEE Main 2020 Paper

Date of Exam: **7th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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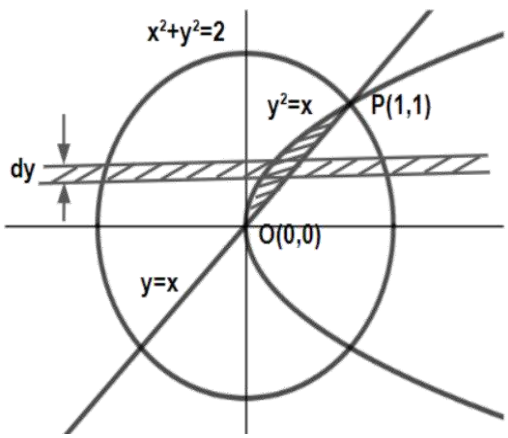
1. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is

   a. $\frac{1}{3}(12\pi - 1)$
   b. $\frac{1}{6}(12\pi - 1)$
   c. $\frac{1}{3}(6\pi - 1)$
   d. $\frac{1}{6}(24\pi - 1)$

   Answer: (b)

   Solution:

   Required area = area of the circle – area bounded by given line and parabola



   Required area = $\pi r^2 - \int_0^1 (y - y^2)\, dy$

   Area $= 2\pi - \left(\frac{y^2}{2} - \frac{y^3}{3}\right)_0^1 = 2\pi - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$ sq. units

2. Total number of six-digit numbers in which only and all the five digits 1,3,5,7 and 9 appear, is

   a. 5^6
   b. $\frac{1}{2}(6!)$
   c. $6!$
   d. $\frac{5}{2}(6!)$

   Answer: (d)

   Solution:

   Selecting all 5 digits $= {}^5C_5 = 1$ way

   Now, we need to select one more digit to make it a 6 digit number $= {}^5C_1 = 5$ ways

Total number of permutations $= \frac{6!}{2!}$

Total numbers $= {}^5C_5 \times {}^5C_1 \times \frac{6!}{2!} = \frac{5}{2}(6!)$

3. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. The expected value of X, is

a. $\frac{1}{8}$

b. $\frac{3}{16}$

c. $-\frac{1}{8}$

d. $-\frac{3}{16}$

Answer: (a)

Solution:

$k =$ no. of consecutive heads

$P(k=3) = \frac{5}{32}$ (HHHTH, HHHTT, THHHT, HTHHH, TTHHH)

$P(k=4) = \frac{2}{32}$ (HHHHT, HHHHT)

$P(k=5) = \frac{1}{32}$ (HHHHH)

$P(\bar{3} \cap \bar{4} \cap \bar{5}) = 1 - \left(\frac{5}{32} + \frac{2}{32} + \frac{1}{32}\right) = \frac{24}{32}$

$\sum XP(X) = \left(-1 \times \frac{24}{32}\right) + \left(3 \times \frac{5}{32}\right) + \left(4 \times \frac{2}{32}\right) + \left(5 \times \frac{1}{32}\right) = \frac{1}{8}$

4. If $\mathrm{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a

a. circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.

b. straight line whose slope is $\frac{3}{2}$.

c. circle whose diameter is $\frac{\sqrt{5}}{2}$.

d. straight line whose slope is $-\frac{2}{3}$.

Answer: (c)

Solution:

$z = x + iy$

$$\frac{x + iy - 1}{2x + 2iy + i} = \frac{(x-1) + iy}{2x + i(2y+1)}\left(\frac{2x - i(2y+1)}{2x - i(2y+1)}\right)$$

$$\frac{2x(x-1) + y(2y+1)}{4x^2 + (2y+1)^2} = 1$$

$$2x^2 + 2y^2 - 2x + y = 4x^2 + 4y^2 + 4y + 1$$

$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$

Circle's centre will be $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

Radius $= \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \frac{\sqrt{5}}{4}$

Diameter$= \frac{\sqrt{5}}{2}$

5. If $f(a + b + 1 - x) = f(x)\ \forall\ x$, where a and b are fixed positive real numbers, then $\frac{1}{(a+b)} \int_a^b x(f(x) + f(x + 1))\ dx$ is equal to

a. $\int_{a-1}^{b-1} f(x)\ dx$

b. $\int_{a+1}^{b+1} f(x + 1)\ dx$

c. $\int_{a-1}^{b-1} f(x + 1)\ dx$

d. $\int_{a+1}^{b+1} f(x)\ dx$

Answer: (c)

Solution:

$f(a + b + 1 - x) = f(x) \qquad (1)$

$x \to x + 1$

$f(a + b - x) = f(x + 1) \qquad (2)$

$I = \frac{1}{a+b} \int_a^b x(f(x) + f(x + 1))dx \qquad (3)$

From (1) and (2)

$I = \frac{1}{a+b} \int_a^b (a + b - x)(f(x + 1) + f(x))dx \qquad (4)$

Adding (3) and (4)

$$2I = \int_a^b (f(x) + f(x + 1))dx$$

$$2I = \int_a^b f(x + 1)dx + \int_a^b f(x)dx$$

$$2I = \int_a^b f(a + b - x + 1)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$$I = \int_a^b f(x)dx \qquad ; \quad x = t + 1, dx = dt$$

$$I = \int_{a-1}^{b-1} f(t + 1)dt$$

$$I = \int_{a-1}^{b-1} f(x + 1)dx$$

6. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

   a. $2\sqrt{3}$ b. $\sqrt{3}$

   c. $\frac{3}{\sqrt{2}}$ d. $3\sqrt{2}$

   Answer: (d)

   Solution:

   Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$

   Now $2ae=6$ & $\frac{2a}{e}=12$

   $\Rightarrow ae=3$ & $\frac{a}{e}=6$

   $\Rightarrow a^2=18$

   $\Rightarrow a^2e^2=c^2=a^2-b^2=9$

   $\Rightarrow b^2=9$

   Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{\sqrt{18}}=3\sqrt{2}$

7. The logical statement $(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$ is equivalent to

   a. $\sim p$ b. p

   c. q d. $\sim q$

   Answer: (a)

   Solution:

p	q	$p\Rightarrow q$	$\sim p$	$q\Rightarrow\sim p$	$(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

   Clearly $(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$ is equivalent to $\sim p$

8. The greatest positive integer k, for which 49^k+1 is a factor of the sum $49^{125}+49^{124}+\cdots+49^2+49+1$, is

   a. 32 b. 60

   c. 65 d. 63

   Answer: (d)

   Solution:

$1 + 49 + 49^2 + \cdots + 49^{125} = \frac{49^{126} - 1}{49 - 1}$

$= \frac{(49^{63}+1)(49^{63}-1)}{48}$

$= \frac{(49^{63}+1)((1+48)^{63}-1)}{48}$

$= \frac{(49^{63}+1)(1+48I-1)}{48}$; where I is an integer

= $(49^{63}+1)I$

Greatest positive integer is $k = 63$

9. A vector $\vec{a} = \alpha\hat{\imath} + 2\hat{\jmath} + \beta\hat{k}$ $(\alpha, \beta \in \boldsymbol{R})$ lies in the plane of the vectors, $\vec{b} = \hat{\imath} + \hat{\jmath}$ and $\vec{c} = \hat{\imath} - \hat{\jmath} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then

a. $\vec{a}.\hat{\imath} + 3 = 0$

b. $\vec{a}.\hat{k} + 4 = 0$

c. $\vec{a}.\hat{\imath} + 1 = 0$

d. $\vec{a}.\hat{k} + 2 = 0$

Answer: (BONUS)

Solution:

The angle bisector of vectors $\vec{b}$ and $\vec{c}$ is given by:

$$\vec{a} = \lambda\left(\hat{b} + \hat{c}\right) = \lambda\left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}} + \frac{\hat{\imath} - \hat{\jmath} + 4\hat{k}}{3\sqrt{2}}\right) = \lambda\left(\frac{4\hat{\imath} + 2\hat{\jmath} + 4\hat{k}}{3\sqrt{2}}\right)$$

Comparing with $\vec{a} = \alpha\hat{\imath} + 2\hat{\jmath} + \beta\hat{k}$, we get

$$\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$$

$\therefore \vec{a} = 4\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

None of the options satisfy.

10. If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}$ where $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is

a. $-\frac{1}{4}$

b. $\frac{4}{3}$

c. 4

d. -4

Answer: (c)

Solution:

$$y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}$$

$$y(\alpha) = \sqrt{2\frac{1}{\sin\alpha\cos\alpha\times\frac{1}{\cos^2\alpha}} + \frac{1}{\sin^2\alpha}}$$

$$y(\alpha) = \sqrt{2\cot\alpha + \text{cosec}^2\alpha}$$

$$y(\alpha) = \sqrt{(1+\cot\alpha)^2}$$

$$y(\alpha) = -1 - \cot\alpha$$

$$\frac{dy}{d\alpha} = 0 + \text{cosec}^2\alpha\Big|_{\alpha=\frac{5\pi}{6}}$$

$$\frac{dy}{d\alpha} = \text{cosec}^2\frac{5\pi}{6}$$

$$\frac{dy}{d\alpha} = 4$$

11. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to

a. -64
b. 128
c. -128
d. -32

Answer: (c)

Solution:

Any tangent to the parabola $y^2 = 4x$ is $y = mx + \frac{a}{m}$

Comparing it with $y = mx + 4$, we get $\frac{1}{m} = 4 \Rightarrow m = \frac{1}{4}$

Equation of tangent becomes $y = \frac{x}{4} + 4$

$y = \frac{x}{4} + 4$ is a tangent to $x^2 = 2by$

$\Rightarrow x^2 = 2b\left(\frac{x}{4} + 4\right)$

Or $2x^2 - bx - 16b = 0,$

$D = 0$

$b^2 + 128b = 0,$

$\Rightarrow b = 0$ (not possible),

$\Rightarrow b = -128$

12. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to

a. A
b. A^2
c. A^3
d. I_3

Answer: (c)

Solution:

The roots of equation $x^2 + x + 1 = 0$ are complex cube roots of unity.

$\therefore \alpha = \omega$ or ω^2

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\1 & \alpha & \alpha^2\\1 & \alpha^2 & \alpha^4\end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\1 & \omega & \omega^2\\1 & \omega^2 & \omega\end{bmatrix}$$

$$A^2 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & \omega & \omega^2\\1 & \omega^2 & \omega\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\1 & \omega & \omega^2\\1 & \omega^2 & \omega\end{bmatrix}$$

$$A^2 = \frac{1}{3}\begin{bmatrix}3 & 0 & 0\\0 & 0 & 3\\0 & 3 & 0\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I$$

$A^4 = I$

$A^{28} = I$

Therefore, we get

$A^{31} = A^{28}A^3$

$A^{31} = IA^3$

$A^{31} = A^3$

13. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

a. $-\frac{3}{2}$

b. $-\frac{1}{2}$

c. $\frac{1}{2}$

d. $\frac{3}{2}$

Answer: (b)

Solution:

$g(x) = x^2 + x - 1$

$gof(x) = 4x^2 - 10x + 5$

$g\big(f(x)\big) = 4x^2 - 10x + 5$

$f^2(x) + f(x) - 1 = 4x^2 - 10x + 5$

Putting $x = \frac{5}{4}$ & $f\left(\frac{5}{4}\right) = t$

$$t^2 + t + \frac{1}{4} = 0$$

$t = -\frac{1}{2}$ or $f\left(\frac{5}{4}\right) = -\frac{1}{2}$

14. Let α and β are two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\,\lambda\tan x = 1-k$, where $(k \neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then value of λ is

a. $5\sqrt{2}$
b. $10\sqrt{2}$
c. 10
d. 5

Answer: (c)

Solution:

$(k+1)\tan^2 x - \sqrt{2}\lambda\tan x = 1-k$
$\tan^2(\alpha+\beta) = 50$
$\because$ $\tan\alpha$ and $\tan\beta$ are the roots of the given equation.
Now,

$$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}, \qquad \tan\alpha\tan\beta = \frac{k-1}{k+1}$$

$$\Rightarrow \left(\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}}\right)^2 = 50$$

$$\Rightarrow \frac{2\lambda^2}{4} = 50$$

$\Rightarrow \lambda^2 = 100$
$\Rightarrow \lambda = \pm 10$

15. Let P be a plane passing through the points $(2,1,0)$, $(4,1,1)$ and $(5,0,1)$ and R be any point $(2,1,6)$. Then the image of R in the plane P is:

a. $(6,5,2)$
b. $(6,5,-2)$
c. $(4,3,2)$
d. $(3,4,-2)$

Answer: (b)

Solution:

Points $A(2,1,0)$, $B(4,1,1)$ $C(5,0,1)$
$\overrightarrow{AB} = (2,0,1)$
$\overrightarrow{AC} = (3,-1,1)$
$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = (1,1,-2)$
Equation of the plane is $x + y - 2z = 3 \ldots (1)$
Let the image of point $(2,1,6)$ is (l,m,n)

$$\frac{l-2}{1} = \frac{m-1}{1} = \frac{n-6}{-2} = \frac{-2(-12)}{6} = 4$$

$\Rightarrow l = 6, m = 5, n = -2$
Hence the image of R in the plane P is $(6,5,-2)$

16. Let $x^k + y^k = a^k$, $(a, k > 0)$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is

a. $\frac{1}{3}$

b. $\frac{3}{2}$

c. $\frac{2}{3}$

d. $\frac{4}{3}$

Answer: (c)

Solution:

$x^k + y^k = a^k$

$kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$

$\Rightarrow \frac{dy}{dx} + \left(\frac{y}{x}\right)^{1-k} = 0$

$\Rightarrow 1 - k = \frac{1}{3}$

$\Rightarrow k = \frac{2}{3}$

17. Let the function, $f: [-7, 0] \to \mathbf{R}$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval:

a. $[-6, 20]$

b. $(-\infty, 20]$

c. $(-\infty, 11]$

d. $[-3, 11]$

Answer: (b)

Solution:

$f(-7) = -3$ and $f'(x) \leq 2$

Applying LMVT in $[-7,0]$, we get

$\frac{f(-7) - f(0)}{-7} = f'(c) \leq 2$

$\frac{-3 - f(0)}{-7} \leq 2$

$f(0) + 3 \leq 14$

$f(0) \leq 11$

Applying LMVT in $[-7, -1]$, we get

$\frac{f(-7) - f(-1)}{-7 + 1} = f'(c) \leq 2$

$\frac{-3 - f(-1)}{-6} \leq 2$

$f(-1) + 3 \leq 12$

$f(-1) \leq 9$

Therefore, $f(-1) + f(0) \leq 20$

18. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to

a. $\log_e 2$
b. $2e$
c. $2 + \log_e 2$
d. $1 + \log_e 2$

Answer: (d)

Solution:

$e^y(y' - 1) = e^x$

$\Rightarrow \frac{dy}{dx} = e^{x-y} + 1$

Let $x - y = t$

$1 - \frac{dy}{dx} = \frac{dt}{dx}$

So, we can write

$\Rightarrow 1 - \frac{dt}{dx} = e^t + 1$

$\Rightarrow -e^{-t}\, dt = dx$

$\Rightarrow e^{-t} = x + c$

$\Rightarrow e^{y-x} = x + c$

$1 = 0 + c$

$\Rightarrow e^{y-x} = x + 1$

at $x = 1$

$\Rightarrow e^{y-1} = 2$

$\Rightarrow y = 1 + \log_2 2$

19. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is

a. 16
b. 27
c. 7
d. $\frac{21}{2}$

Answer: (a)

Solution:

Let 5 numbers be $a - 2d, a - d, a, a + d, a + 2d$

$5a = 25$

$a = 5$

$(a - 2d)(a - d)a(a + d)(a + 2d) = 2520$

$(25 - 4d^2)(25 - d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$4d^4 - 4d^2 - 121d^2 + 121 = 0$

$d^2 = 1 \; or \; d^2 = \frac{121}{4}$

$d = \pm\frac{11}{2}$

For $d = \frac{11}{2}, a + 2d$ is the greatest term, $a + 2d = 5 + 11 = 16$

20. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0,$

where $a, b, c \in \mathbf{R}$ are non-zero and distinct; has non-zero solution, then

a. $a + b + c = 0$

b. a, b, c are in A.P.

c. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

d. a, b, c are in G.P.

Answer: (c)

Solution:

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 2 & 2a & a \\ 0 & 3b - 2a & b - a \\ 0 & 4c - 2a & c - a \end{vmatrix} = 0$$

$\Rightarrow (3b - 2a)(c - a) - (4c - 2a)(b - a) = 0$

$\Rightarrow 3bc - 2ac - 3ab + 2a^2 - [4bc - 4ac - 2ab + 2a^2] = 0$

$\Rightarrow -bc + 2ac - ab = 0$

$\Rightarrow ab + bc = 2ac$

$\Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

21. $\lim_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{\frac{-x}{2}} - 3^{1-x}}$ is equal to ____________

Answer: (36)

Solution:

$$\lim_{x \to 2} \frac{3^x + \frac{27}{3^x} - 12}{\frac{1}{3^{\frac{x}{2}}} - \frac{3}{3^x}}$$

Put $3^{\frac{x}{2}} = t$

$$\lim_{t\to 3}\frac{t^2+\frac{27}{t^2}-12}{-\frac{3}{t^2}+\frac{1}{t}} = \lim_{t\to 3}\frac{(t^2-9)(t^2-3)}{(t-3)} = \lim_{t\to 3}(t^2-3)(t+3) = 36$$

22. If variance of first n natural numbers is 10 and variance of first m even natural numbers is 16, $m + n$ is equal to________.

Answer: (18)

Solution:

For n natural number variance is given by

$$\sigma^2 = \frac{\sum x_i{}^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$\frac{\sum x_i{}^2}{n} = \frac{1^2+2^2+3^2+\ldots . n\ term}{n} = \frac{n(n+1)(2n+1)}{6n}$$

$$\frac{\sum x_i}{n} = \frac{1+2+3+\ldots . n\ terms}{n} = \frac{n(n+1)}{2n}$$

$\sigma^2 = \frac{n^2-1}{12} = 10$ (given)

$\Rightarrow n = 11$

Variance of $(2, 4, 6 \ldots) = 4 \times$ variance of $(1, 2, 3, 4 \ldots) = 4\times \frac{m^2-1}{12} = \frac{m^2-1}{3} = 16$ (given)

$\Rightarrow m = 7$

Therefore, $n + m = 11 + 7 = 18$

23. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + x^3 \ldots . +x^{2n})(1 - x + x^2 - x^3 \ldots . + x^{2n})$ is 61, then n is equal to ________

Answer: (30)

Solution:

Let $(1 + x + x^2 + \cdots + x^{2n})(1 - x + x^2 - \cdots + x^{2n}) = a_o + a_1x + a_2x^2 + \cdots$

Put $x = 1$

$2n + 1 = a_o + a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots \qquad (1)$

Put $x = -1$

$2n + 1 = a_o - a_1 + a_2 - a_3 + \ldots\ldots\ldots\ldots \qquad (2)$

Add (1) and (2)

$2(2n + 1) = 2(a_o + a_2 + a_4 + \ldots\ldots\ldots\ldots$

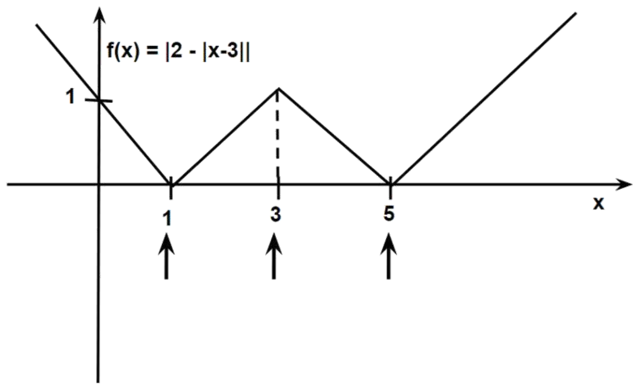
$2n + 1 = 61$

$n = 30$

24. Let S be the set of points where the function, $f(x) = \left|2 - |x - 3|\right|, x \in \mathbf{R}$, is not differentiable. Then, the value of $\sum_{x \in S} f(f(x))$ is equal to _________.

Answer: (3)

Solution:



There will be three points $x = 1, 3, 5$ at which $f(x)$ is non-differentiable.

So $f\big(f(1)\big) + f\big(f(3)\big) + f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1$

$= 3$

25. Let $A(1, 0),\ B(6, 2),\ C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line the segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is ____________

Answer: (5)

Solution:

P is the centroid which is $\equiv \left(\frac{1+6+\frac{3}{2}}{3}, \frac{1+5+2}{3}\right)$

$P = \left(\frac{17}{6}, \frac{8}{3}\right)$

$Q = \left(-\frac{7}{6}, -\frac{1}{3}\right)$

$PQ = \sqrt{(4)^2 + (3)^2} = 5$