09/04/2019
Evening

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

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PART–A : PHYSICS

1. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r)=\frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T :

(1) T/R is a constant (2) TR is a constant

(3) T/R^2 is a constant (4) T^2/R^3 is a constant

Answer (1)

Sol. $M_{(in)}=\int_0^R 4\pi r^2 dr\cdot\frac{k}{r^2}$

$$M_{(in)}=4\pi KR$$

$$\therefore\quad G\cdot\frac{4\pi KR}{R^2}=\frac{V^2}{R}\Rightarrow v=\sqrt{4\pi GK}$$

$$\therefore\quad T=\frac{2\pi R}{v}=\frac{2\pi\cdot R}{\sqrt{4\pi GK}}$$

$$\therefore\quad \frac{T}{R}=\text{constant}$$

2. The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then :

(1) A is rigid but B has a vibrational mode.

(2) A has a vibrational mode but B has none.

(3) Both A and B have a vibrational mode each.

(4) A has one vibrational mode and B has two.

Answer (2)

Sol. For (A) C_P = 29, C_V = 22

For (B) C_P = 30 C_V = 21

$$\therefore\quad \gamma_A=\frac{C_{P_A}}{C_{V_A}}=\frac{29}{22}=1.31$$

When A has vibrational degree of freedom, then $\gamma_A = 9/7 \simeq 1.29$.

$$\gamma_B=\frac{C_{P_B}}{C_{V_B}}=\frac{30}{21}=1.42$$

$\Rightarrow$ B has no vibrational degree of freedom

3. The position vector of a particle changes with time according to the relation $\vec{r}(t)=15t^2\hat{i}+(4-20t^2)\hat{j}$. What is the magnitude of the acceleration at t = 1?

(1) 50 (2) 100

(3) 40 (4) 25

Answer (1)

Sol. $\vec{r}=15t^2\hat{i}+4\hat{j}-20t^2\hat{j}$

$$\frac{d\vec{r}}{dt}=30t\hat{i}-40t\hat{j}$$

$$\frac{d^2\vec{r}}{dt^2}=30\hat{i}-40\hat{j}$$

$$\therefore\quad \left|\frac{d^2\vec{r}}{dt^2}\right|=50\text{ m/s}^2$$

4. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to (c = 3 × 10^8 m/s) :

(1) 20 × 10^{-8} N

(2) 35 × 10^{-8} N

(3) 15 × 10^{-8} N

(4) 10 × 10^{-8} N

Answer (1)

Sol. $\because\quad P=\frac{W}{c}$ and pressure = I/c

$$\therefore\quad \frac{h}{\lambda}+\frac{h}{4\lambda}\Rightarrow \Delta P=\frac{5h}{4\lambda}\text{ for one photon}$$

$$\therefore\quad \frac{5}{4}\cdot\frac{Nh}{\lambda\Delta tA}=\text{pressure}$$

$$\text{But}\left(\frac{Nhc}{\lambda\cdot\Delta tA}\right)=50\text{ W/m}^2$$

$$\therefore\quad \frac{5}{4}\times\frac{50}{c}=\text{pressure}$$

$$\therefore\quad F_n=\frac{5\times50\times1\,m^2}{4\times c}=20\times10^{-8}\text{ N}$$

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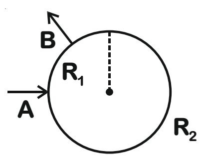
5. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be :

(1) $\frac{5}{3}\ \Omega$ (2) $\frac{5}{2}\ \Omega$

(3) $\frac{7}{2}\ \Omega$ (4) $\frac{12}{5}\ \Omega$

Answer (1)

Sol.



$R_0 = \rho\frac{l}{A} = 3\ \Omega$

Now if l = 2l

Then A = $\frac{A}{2}$

$\therefore\quad R = \frac{\rho 2l \times 2}{A} = 12\ \Omega$

$R_1 = \frac{12}{6} = 2\ \Omega$

$R_2 = 10\ \Omega$

$\therefore\quad \frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{10} = \frac{6}{10}$

$\therefore\quad R_{eq} = \frac{5}{3}\Omega$

6. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be :

(1) $\frac{M\omega_0}{M+3m}$

(2) $\frac{M\omega_0}{M+m}$

(3) $\frac{M\omega_0}{M+6m}$

(4) $\frac{M\omega_0}{M+2m}$

Answer (3)

Sol. Initial angular momentum = Final Angular Momentum

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + 2\frac{mL^2}{4}\right)\omega$$

$$\Rightarrow\quad \omega = \frac{M\omega_0}{M+6m}$$

7. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of

(1) 2.5 s (2) 2 s

(3) 5 s (4) 3 s

Answer (2)

Sol. I = 150; α = 20 rad/s^2

$\therefore\quad \omega = \alpha t$

$$E = \frac{1}{2}I\omega^2 = 1200$$

$$\frac{1}{2} \times 1.5 \times (20t)^2 = 1200\ \text{J}$$

$\Rightarrow\quad t = 2$ s

8. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A?

(1) 0.2 ohm

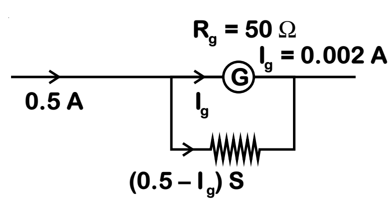
(2) 0.002 ohm

(3) 0.5 ohm

(4) 0.02 ohm

Answer (1)

Sol.



We have

$I_g R_g = (0.5 - I_g)\,S$

$\Rightarrow\quad (0.002)(50) = (0.5 - I_g)\,S$

$$\Rightarrow\quad S \simeq \frac{0.002 \times 50}{0.5} = 0.2\ \Omega$$

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9. A He^+ ion is in its first excited state. Its ionization energy is :

(1) 13.60 eV (2) 6.04 eV

(3) 48.36 eV (4) 54.40 eV

Answer (1)

Sol. $E_n: \dfrac{-E_0 z^2}{n^2} = \dfrac{-E_0 \times 4}{4} = -E_0$

To ionise it E_0 energy must be supplied.

$\therefore$ E_0 = 13.6 eV.

10. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B?

(speed of sound in air = 340 ms^{-1})

(1) 2150 Hz (2) 2300 Hz

(3) 2060 Hz (4) 2250 Hz

Answer (4)

Sol.

$$f = \frac{(v \pm u_0)}{(v \pm u_s)} \cdot f_0 = \frac{(v-20)}{(v+20)} \cdot f_0$$

$$\Rightarrow \quad 2000 = \frac{320}{360} \cdot f_0$$

$$\Rightarrow \quad \frac{2000 \times 9}{8} = f_0 = 2250\,\text{Hz}.$$

11. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is :

(1) 3 : 1 (2) 2 : 1

(3) 4 : 3 (4) 5 : 3

Answer (1)

Sol. $\therefore \quad \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\therefore$ v = (± 2u)

$\therefore \quad \dfrac{1}{2u} + \dfrac{1}{u} = \dfrac{1}{20} \quad \Rightarrow \dfrac{3}{2u} = \dfrac{1}{20}$

$\therefore$ u_1 = 30 cm

And $\quad \dfrac{1}{u} - \dfrac{1}{2u} = \dfrac{1}{20}$

$\therefore \quad \dfrac{1}{2u} = \dfrac{1}{20} \therefore u_2 = 10$

$\therefore \quad \dfrac{30}{10} = 3.$

12. The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be :

(1) $a + \dfrac{b^2}{4c}$ (2) $a + \dfrac{b^2}{3c}$

(3) $a + \dfrac{b^2}{2c}$ (4) $a + \dfrac{b^2}{c}$

Answer (2)

Sol. $x = at + bt^2 - ct^3$

$\dot{x} = a + 2bt - 3ct^2$

$\ddot{x} = 2b - 6ct$

For $\ddot{x} = 0 \quad t = +\dfrac{b}{3c}$

$$\therefore \quad v = \dot{x} = a + 2b\left(\frac{+b}{3c}\right) - 3c\left(\frac{b^2}{3c \times 3c}\right)$$

$$\Rightarrow \quad v = -\frac{b^2}{3c} + \frac{2b^2}{3c} + a = a + \frac{b^2}{3c}$$

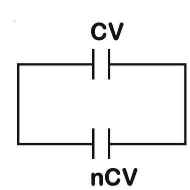
13. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :

(1) $\dfrac{(n+1)V}{(K+n)}$ (2) $\dfrac{V}{K+n}$

(3) V (4) $\dfrac{nV}{K+n}$

Answer (1)

Sol. Initially



Q = CV(1 + n)

$\therefore$ C_{eq} = (K + n)C

$$\therefore \quad V = \frac{CV(1+n)}{(K+n)C} = \frac{V(1+n)}{(K+n)}$$

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14. The physical sizes of the transmitter and receiver antenna in a communication system are:

(1) Inversely proportional to modulation frequency

(2) Inversely proportional to carrier frequency

(3) Proportional to carrier frequency

(4) Independent of both carrier and modulation frequency

Answer (2)

Sol. Size of antenna depends on wavelength of carrier wave.

15. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :

(1) $\frac{v^2}{g}$ (2) $\frac{2v^2}{5g}$

(3) $\frac{2v^2}{7g}$ (4) $\frac{v^2}{2g}$

Answer (2)

Sol. mv = (4m + m)v′

$\therefore$ Common speed $v' = \frac{v}{5}$

$$mgh + \frac{1}{2}5m\cdot\frac{v^2}{25} = \frac{1}{2}mv^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2\left(1-\frac{1}{5}\right) = \frac{1}{2}mv^2\cdot\frac{4}{5}$$

$$\therefore h = \frac{2mv^2}{5\times mg} = \frac{2v^2}{5g}$$

16. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is

(1) 0.5 (2) 0.8

(3) 0.7 (4) 0.6

Answer (4)

Sol. $V\sigma g = \frac{4}{5}v\rho_\omega g$ ...(1)

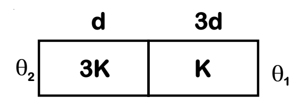
$$V\sigma g = \frac{v}{2}\rho_\omega g + \frac{v}{2}\rho_0 g$$

$$\Rightarrow \left(\frac{\rho_\omega}{2} + \frac{\rho_{oil}}{2}\right) = \frac{4}{5}\rho_\omega$$

$$\Rightarrow \frac{\rho_{oil}}{2} = \rho_\omega\left(\frac{4}{5}-\frac{1}{2}\right) = \frac{3}{10}\rho_\omega$$

$$\Rightarrow \rho_{oil} = \frac{3}{5}\rho_\omega = 0.6\rho_\omega$$

17. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is:



(1) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$ (2) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$

(3) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$ (4) $\frac{\theta_2 + \theta_1}{2}$

Answer (2)

Sol. $H = \frac{3KA}{d}(\theta_2 - \theta) = \frac{KA}{3d}(\theta - \theta_1)$

$$\Rightarrow \theta = \frac{9\theta_2}{10} + \frac{\theta_1}{10}$$

18. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is :

(1) 37.03 cm^2 (2) 37.0 cm^2

(3) 37.030 cm^2 (4) 37 cm^2

Answer (2)

Sol. 5.29 × 7 = 37.0 cm^2

Answer should be in 3 significant digits.

19. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm. coming from a distant object, the limit of resolution of the telescope is close to :

(1) 1.5 × 10^{-7} rad

(2) 3.0 × 10^{-7} rad

(3) 2.0 × 10^{-7} rad

(4) 4.5 × 10^{-7} rad

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Answer (2)

Sol. $\theta = \frac{1.22\lambda}{D}$

$\Rightarrow \quad \theta = \frac{1.22 \times 600 \times 10^{-9}}{250} \times 100 = 2.92 \times 10^{-7}$

$\Rightarrow \quad \theta = 3 \times 10^{-7}$ rad

20. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is

(1) $\frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$ (2) $\lambda_x - \lambda_y$

(3) $\lambda_x + \lambda_y$ (4) $\frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$

Answer (1)

Sol. $P_1 = \frac{h}{\lambda_x}$ $\qquad P_2 = \frac{h}{\lambda_y}$

$P = P_1 - P_2 = h\left(\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right)$

$\therefore \quad P = \frac{h}{\lambda}$

$\frac{h}{\lambda} = h\left(\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right)$

$\frac{1}{\lambda} = \frac{|\lambda_y - \lambda_x|}{\lambda_x \lambda_y}$

$\lambda = \frac{\lambda_x . \lambda_y}{|\lambda_x - \lambda_y|}$

21. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately

(1) 10^{-4} (2) 10^{-2}

(3) 10^{-1} (4) 10^{-3}

Answer (4)

Sol. NIAB = KQ

$175 \times 1 \times 10^{-3} \times 1 \times 10^{-4} \times B = \frac{10^{-6} \times \pi}{180}$

$\Rightarrow \quad B = \frac{\pi}{180} \times \frac{10}{175} \approx 9.97 \times 10^{-4}$ T.

$\Rightarrow \quad B = 10^{-3}$ T.

22. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25 fs (femto second), it's approximate resistivity is

($m_e = 9.1 \times 10^{-31}$ kg)

(1) 10^{-5} Ωm (2) 10^{-6} Ωm

(3) 10^{-7} Ωm (4) 10^{-8} Ωm

Answer (4)

Sol. $J = \eta e v_d$ $\qquad \because \quad v_d = \frac{eE\tau}{m}$

$\therefore \quad J = \frac{\eta e \cdot eE \cdot \tau}{m} = \sigma\vec{E}$

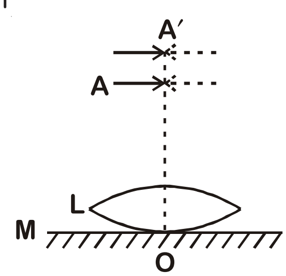
$J = \frac{\eta e^2 \tau}{m} \cdot E = \sigma\vec{E}$

$\therefore \quad \rho = \frac{m}{\eta e^2 \tau}$

$= \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-15}}\,\Omega$ m

$= \frac{9.1}{8.5 \times 2.56 \times 25} \times 10^{(-59+53)} = 1.67 \times 10^{-8}\ \Omega$ m

23. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A′, such that OA′ = 27 cm, to get its inverted real image at A′ itself. The value of μ_l will be



(1) $\sqrt{2}$ (2) $\frac{4}{3}$

(3) $\sqrt{3}$ (4) $\frac{3}{2}$

Answer (2)

Sol. f_L = 18 cm

$\frac{1}{18} = 0.5 \times \frac{2}{R} \quad \Rightarrow \quad R = 18$ cm

$\frac{1}{f_2} = (\mu_l - 1)\left(-\frac{1}{18}\right)$

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$$\frac{1}{27}=\frac{1}{18}-\frac{(\mu_l-1)}{18}=\frac{1-\mu_l+1}{18}$$

$\Rightarrow$ $2 = 3(2 - \mu_l) = 6 - 3\,\mu_l$

$\Rightarrow$ $\mu_l=\frac{4}{3}$

24. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through Q. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is

(1) 6.67×10^{-3} Wb (2) 6.67×10^{-4} Wb

(3) 3.67×10^{-3} Wb (4) 3.67×10^{-4} Wb

Answer (2)

Sol. $\phi_Q = Mi$

$\Rightarrow$ $10^{-3} = M(3)$

$\phi_P = M(2)$

$\therefore$ $\frac{10^{-3}}{3}=\frac{\phi_P}{2}$

$\phi_P=\frac{20}{3}\times10^{-4}=6.67\times10^{-4}$ Wb.

25. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K)

(1) 10^{-5} K (2) 10^{-1} K

(3) 10^{-3} K (4) 10^{-4} K

Answer (1)

Sol. $\frac{1}{2}k\Delta x^2 = E(\text{dissipated})$

$\therefore$ $\frac{1}{2}\times800\times\left(\frac{2\times2}{100\times100}\right)=\frac{16}{100}$ J

$$\frac{16}{100}=\frac{1}{2}\times400\times\Delta T+1\times4184\times\Delta T$$

$\Rightarrow$ $\frac{16}{100}=(200+4184)\Delta T=4384\ \Delta T$

$\therefore$ $\Delta T=\frac{16}{4384\times100}=3.6\times10^{-5}$ K

26. A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction.

The speed of each of the moving particle will be

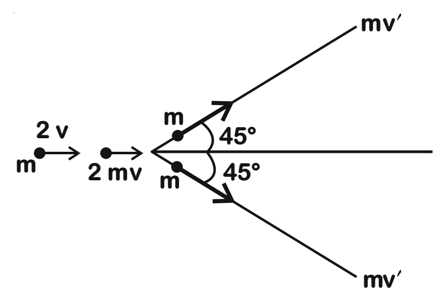
(1) $v/(2\sqrt{2})$ (2) $v/\sqrt{2}$

(3) $\sqrt{2}v$ (4) $2\sqrt{2}v$

Answer (4)

Sol. Initial momentum · $P_i = 2mv + 2mv = 4\,mv$

Let v' be the speed of I particle



$\therefore$ $2\frac{mv'}{\sqrt{2}}=4mv$

$\Rightarrow$ $v'=2\sqrt{2}\,v$

27. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is

(1) 320 m/s, 120 Hz

(2) 320 m/s, 80 Hz

(3) 180 m/s, 80 Hz

(4) 180 m/s, 120 Hz

Answer (2)

Sol. $\frac{\lambda}{2}=\frac{2}{3}$

$\Rightarrow$ $\lambda=\frac{4}{3}$m

$\therefore$ $v=\frac{4}{3}\times240$ = 320 m/s

3rd harmonic

$f_n = nf_0$

$f_0=\frac{240}{3}=80$ Hz

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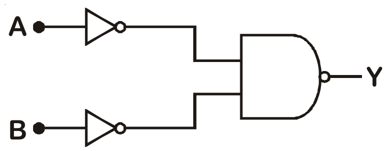
28. The logic gate equivalent to the given logic circuit is



(1) OR

(2) NAND

(3) AND

(4) NOR

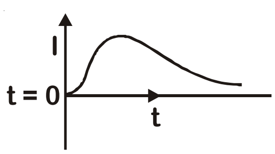
Answer (1)

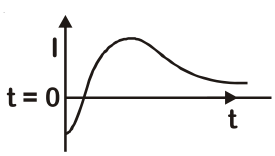
Sol. $y = \overline{\overline{A} \cdot \overline{B}} = A + B$

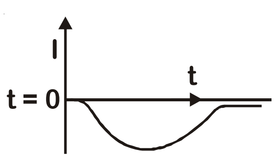
$y = A + B$

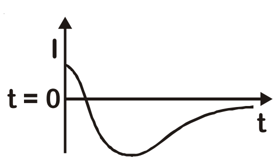
$\Rightarrow$ OR gate

29. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ ($k > 0$), as a function of time ($t \geq 0$). Counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by:

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. $i = te^{-\alpha t} \cdot k$

$\therefore \quad \phi = k_1 i = k_1 te^{-\alpha t} \qquad [k_1 = k\pi(2R)^2]$

$$\therefore \quad E = -\left(\frac{d\phi}{dt}\right) = -k_1 e^{-\alpha t} + k_1 \alpha t e^{-\alpha t}$$

$$= -k_1 e^{-\alpha t}(1 - \alpha t)$$

$\therefore$ Induced current $I = \dfrac{E}{r} = -k_1 e^{-\alpha t}(1 - \alpha t)$

30. For point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d and y =+2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D >> d, will behave as:

(1) $E \propto \dfrac{1}{D^3}$

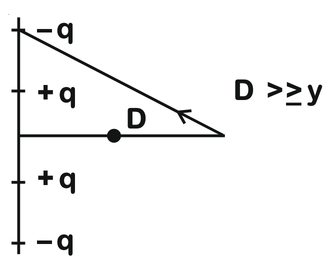
(2) $E \propto \dfrac{1}{D}$

(3) $E \propto \dfrac{1}{D^4}$

(4) $E \propto \dfrac{1}{D^2}$

Answer (3)

Sol.



$$\vec{E} = \frac{2qD}{4\pi\varepsilon_0\left(D^2 + 4y^2\right)^{\frac{3}{2}}} - \frac{2qD}{4\pi\varepsilon_0(D^2 + y^2)^{\frac{3}{2}}} \quad (-\text{ve } \hat{x})$$

$$\left|\vec{E}\right| = \frac{2qD}{4\pi\varepsilon_0 D^3}\left[\frac{1}{\left(1 + \left(\frac{2y}{D}\right)^2\right)^{\frac{3}{2}}} - \frac{1}{\left(1 + \left(\frac{y}{D}\right)^2\right)^{\frac{3}{2}}}\right]$$

$$\Rightarrow \quad \vec{E} = \frac{2q}{4\pi\varepsilon_0 D^2}\left(1 - \frac{3}{2}\cdot\frac{4y^2}{D^2} - 1 + \frac{3}{2}\frac{y^2}{D^2}\right) = \frac{9qy^2}{4\pi\varepsilon_0 D^4}$$

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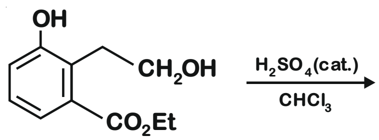
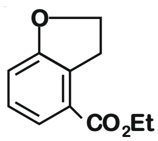
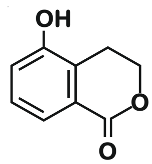
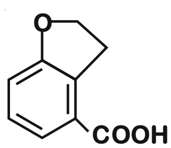
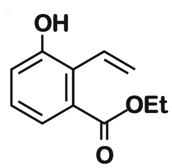
PART–B : CHEMISTRY

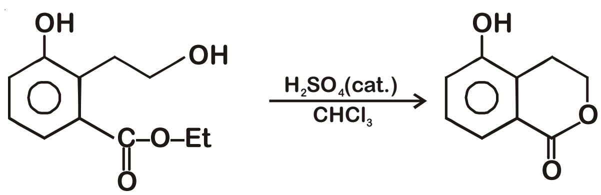
1. The major product of the following reaction is :



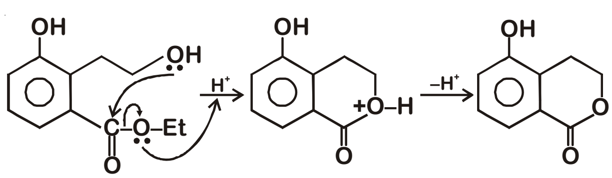
(1)  (2) 

(3)  (4) 

Answer (2)

Sol. 

Acid catalysed intramolecular esterification



2. The peptide that gives positive ceric ammonium nitrate and carbylamine tests is

(1) Ser - Lys (2) Lys - Asp

(3) Gln - Asp (4) Asp - Gln

Answer (1)

Sol. Ceric ammonium nitrate test is given by alcohol. Only serine(ser) contain –OH group.

3. Which of the following compounds is a constituent of the polymer

$$\left[HN - \overset{O}{\overset{\|}{C}} - NH - CH_2 \right]_n ?$$

(1) Formaldehyde (2) Ammonia

(3) Methylamine (4) N-Methyl urea

Answer (1)

Sol. $\left[NH - \overset{O}{\overset{\|}{C}} - NH - CH_2 \right]$ -urea-formaldehyde resin

$\therefore$ Monomers : Urea and formaldehyde

4. During compression of a spring the work done is 10 kJ and 2 kJ escaped to the surroundings as heat. The change in internal energy, ΔU (in kJ) is

(1) 12 (2) –12

(3) 8 (4) –8

Answer (3)

Sol. w = 10 kJ

q = – 2 kJ

$\Delta U = q + w = 10 - 2 = 8$ kJ

5. What would be the molality of 20% (mass/mass) aqueous solution of KI? (molar mass of KI = 166 g mol^{-1})

(1) 1.48 (2) 1.51

(3) 1.08 (4) 1.35

Answer (2)

Sol. 20% W/W KI solution

i.e. 100 g solution contains 20 g KI

$\therefore$ Mass of solvent = 100 – 20 = 80 g

$\therefore$ Molality $= \dfrac{20 \times 1000}{166 \times 80}$

$\simeq 1.51$ molar

6. A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode?

(1) 0.20

(2) 0.15

(3) 0.10

(4) 0.05

Answer (4)

Sol. 0.1 F of electricity is passed through $Ni(NO_3)_2$ solution

$\therefore$ Amount of Ni deposited = 0.1 eq

$\therefore$ Moles $= \dfrac{0.1}{2} = 0.05$

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7. HF has highest boiling point among hydrogen halides, because it has

(1) Strongest hydrogen bonding

(2) Lowest dissociation enthalpy

(3) Strongest van der Waals' interactions

(4) Lowest ionic character

Answer (1)

Sol. HF has highest boiling point among the hydrogen halides due to strong H-bonding between HF molecules.

8. The one that is not a carbonate ore is

(1) Bauxite (2) Calamine

(3) Siderite (4) Malachite

Answer (1)

Sol. Bauxite $\rightarrow$ $AlO_x(OH)_{3-2x}$

Calamine $\rightarrow$ $ZnCO_3$

Siderite $\rightarrow$ $FeCO_3$

Siderite $\rightarrow$ $FeCO_3$

Malachite $\rightarrow$ $CuCO_3 \cdot Cu(OH)_2$

9. Noradrenaline is a / an

(1) Neurotransmitter (2) Antihistamine

(3) Antacid (4) Antidepressant

Answer (1)

Sol. Noradrenaline is neurotransmitter.

10. Among the following species, the diamagnetic molecule is

(1) CO (2) NO

(3) O_2 (4) B_2

Answer (1)

Sol.

Molecule	No. of unpaired electrons
NO	1
CO	Zero
O_2	2
B_2	2

$\therefore$ Diamagnetic species is CO

11. The correct statements among I to III regarding group 13 element oxides are,

(I) Boron trioxide is acidic.

(II) Oxides of aluminium and gallium are amphoteric.

(III) Oxides of indium and thallium are basic.

(1) (II) and (III) only (2) (I) and (II) only

(3) (I), (II) and (III) (4) (I) and (III) only

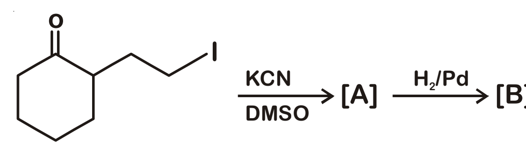
Answer (3)

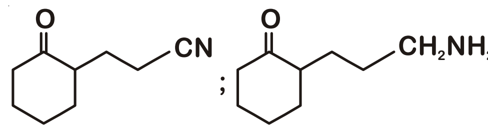
Sol. B_2O_3 is an acidic oxide

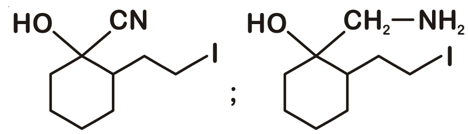
Al_2O_3 and Ga_2O_3 are amphoteric oxide

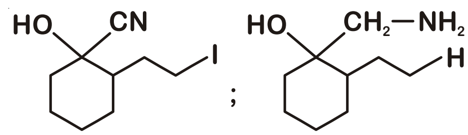
In_2O_3 and Tl_2O are basic oxide

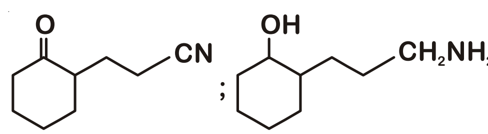
12. The major products A and B for the following reactions are, respectively



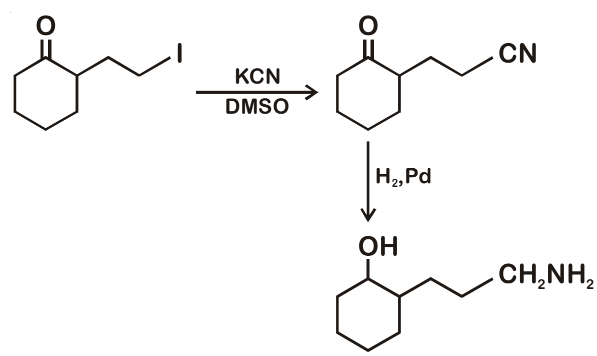
(1) 

(2) 

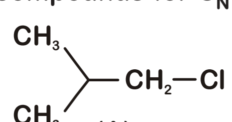
(3) 

(4) 

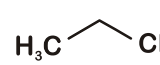
Answer (4)

Sol. 

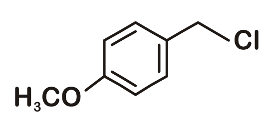
13. Increasing order of reactivity of the following compounds for S_N1 substitution is



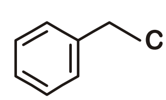
(A)



(B)



(C)



(D)

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(1) (B) < (C) < (A) < (D)

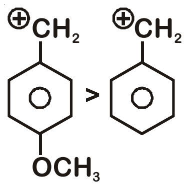
(2) (B) < (C) < (D) < (A)

(3) (B) < (A) < (D) < (C)

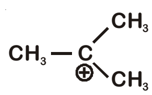
(4) (A) < (B) < (D) < (C)

Answer (3)

Sol. S_N1 reaction proceeds via formation of carbocation.



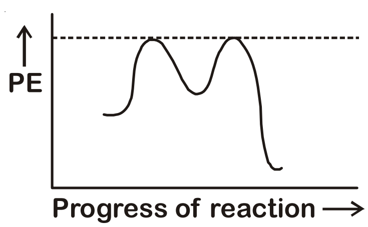
(C) (D)

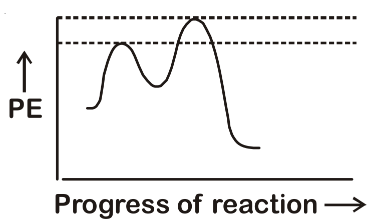
On comparing (A) and (B), in (A) there is formation of tertiary carbocation 

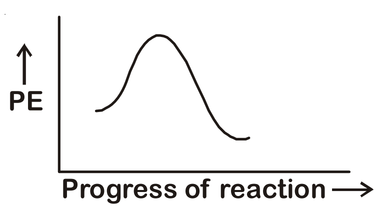
after rearrangement while (B) is primary.

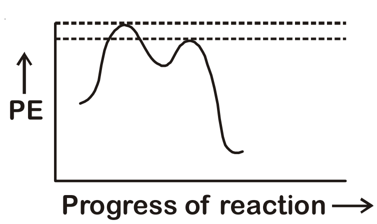
So, (C) > (D) > (A) > (B).

14. Which of the following potential energy (PE) diagrams represents the S_N1 reaction?

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. In S_N1 reaction, formation of carbocation (1^{st} step) is rate determining step (RDS)

$\therefore$ Correct graph is given in option-4.

15. Molal depression constant for a solvent is 4.0 K kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2SO_4 is

(Assume complete dissociation of the electrolyte)

(1) 0.36 K

(2) 0.18 K

(3) 0.12 K

(4) 0.24 K

Answer (1)

Sol. $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

i (Van't Hoff Factor) = 3

$\therefore \ \Delta T_f = iK_fm$

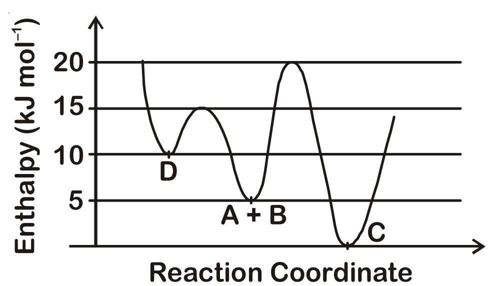
$= 3 \times 4 \times 0.03$

$= 0.36$ K

16. Consider the given plot of enthalpy of the following reaction between A and B.

$A + B \rightarrow C + D$

Identify the incorrect statement.



(1) Activation enthalpy to form C is 5 kJ mol^{-1} less than that to form D

(2) D is kinetically stable product

(3) Formation of A and B from C has highest enthalpy of activation

(4) C is the thermodynamically stable product

Answer (1)

Sol. Activation enthalpy to form C is 5 kJ more than that to form D.

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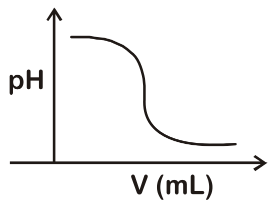
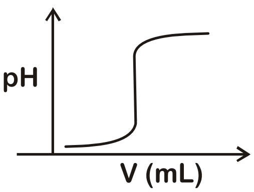
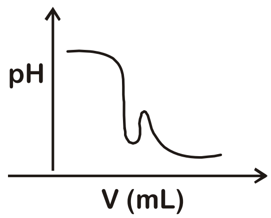
17. In an acid-base titration, 0.1 M HCl solution was added to the NaOH solution of unknown strength. Which of the following correctly shows the change of pH of the titration mixture in this experiment?



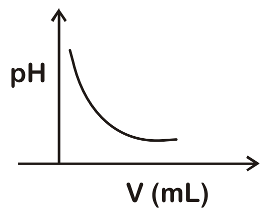
(A)



(B)



(C)



(D)

(1) (A) (2) (C)

(3) (B) (4) (D)

Answer (1)

Sol. The pH of NaOH is more than 7 and during the titration it decreases so graph (1) is correct

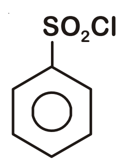
18. Hinsberg's reagent is

(1) $C_6H_5SO_2Cl$ (2) $(COCl)_2$

(3) C_6H_5COCl (4) $SOCl_2$

Answer (1)

Sol. Hinsberg's reagent is benzenesulphonyl chloride



19. The layer of atmosphere between 10 km to 50 km above the sea level is called as

(1) Stratosphere (2) Mesosphere

(3) Thermosphere (4) Troposphere

Answer (1)

Sol. Between 10-50 km above sea level lies stratosphere.

20. Assertion : For the extraction of iron, haematite ore is used.

Reason : Haematite is a carbonate ore of iron.

(1) Only the reason is correct

(2) Only the assertion is correct

(3) Both the assertion and reason are correct and the reason is the correct explanation for the assertion

(4) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion

Answer (2)

Sol. For the extraction of iron, haematite ore in used.

Haematite = Fe_2O_3

21. At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behaviour. Their equation of state is given as

$P = \frac{RT}{V-b}$ at T.

Here, b is the van der Waal's constant. Which gas will exhibit steepest increase in the plot of Z (compression factor) vs P?

(1) Kr

(2) Ar

(3) Xe

(4) Ne

Answer (3)

Sol. $P = \frac{RT}{V_m - b}$

$\Rightarrow PV_m - Pb = RT$

$\Rightarrow \frac{PV_m}{RT} = 1 + \frac{Pb}{RT}$

$\Rightarrow Z = 1 + \frac{Pb}{RT}$

Slope of Z vs P curve (straight line) = $\frac{b}{RT}$

$\therefore$ Higher the value of b, more steep will be the curve and b $\propto$ size of gas molecules

22. The amorphous form of silica is

(1) Quartz

(2) Tridymite

(3) Kieselguhr

(4) Cristobalite

Answer (3)

Sol. Quartz, tridymite and cristobalite are crystalline forms of silica.

Kieselguhr is an amorphous form of silica.

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23. The correct statements among I to III are

(I) Valence bond theory cannot explain the color exhibited by transition metal complexes.

(II) Valence bond theory can predict quantitatively the magnetic properties of transition metal complexes.

(III) Valence bond theory cannot distinguish ligands as weak and strong field ones.

(1) (II) and (III) only (2) (I), (II) and (III)

(3) (I) and (II) only (4) (I) and (III) only

Answer (4)

Sol. Valence bond theory cannot predict quantitatively the magnetic properties of transition metal complex.

24. In the following reaction

$$\text{carbonyl compound} + \text{MeOH} \xrightleftharpoons{\text{HCl}} \text{acetal}$$

Rate of the reaction is the highest for

(1) Acetone as substrate and methanol in excess

(2) Propanal as substrate and methanol in stoichiometric amount

(3) Propanal as substrate and methanol in excess

(4) Acetone as substrate and methanol in stoichiometric amount

Answer (3)

Sol. $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$ $\quad$ CH_3-CH_2-CHO

Generally, aldehydes are more reactive than ketones in nucleophilic addition reactions.

∴ Rate of reaction with alcohol to form acetal and ketal is

$$CH_3-CH_2-CHO > CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$$

25. The structures of beryllium chloride in the solid state and vapour phase, respectively, are

(1) Chain and dimeric (2) Dimeric and dimeric

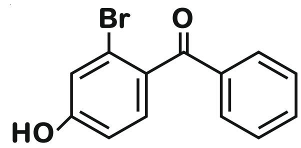
(3) Dimeric and chain (4) Chain and chain

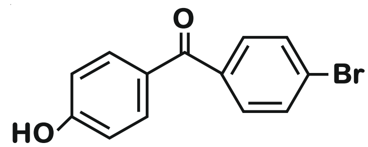
Answer (1)

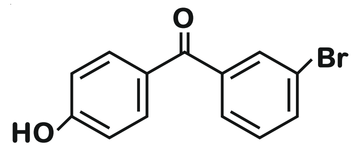
Sol. $BeCl_2$ in vapour phase exist as dimer (below 1200 K temperature)

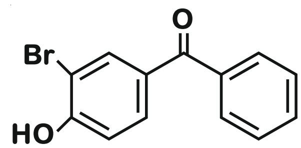
$BeCl_2$ in solid state has chain structure.

26. p-Hydroxybenzophenone upon reaction with bromine in carbon tetrachloride gives

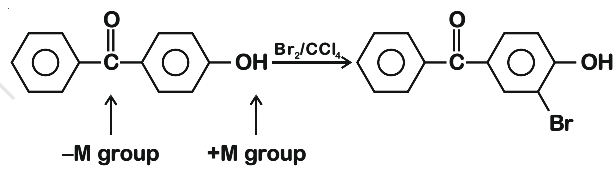
(1) 

(2) 

(3) 

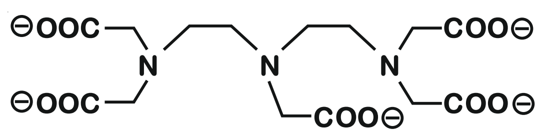
(4) 

Answer (4)

Sol. 

Product will formed as per –OH group (+M group)

27. The maximum possible denticities of a ligand given below towards a common transition and inner-transition metal ion, respectively, are



(1) 6 and 8 (2) 8 and 6

(3) 8 and 8 (4) 6 and 6

Answer (1)

Sol. The maximum possible denticities of the given ligand towards transition metal ion is 6 and towards inner transition metal ion is 8.

28. 10 mL of 1 mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as a cube, what is its edge length?

(1) 2.0 pm (2) 2.0 nm

(3) 0.1 nm (4) 1.0 pm

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Answer (1)

Sol. No. of surfactant molecule $6\times10^{23}\times\frac{10}{1000}\times10^{-3}$

$\Rightarrow$ 6 × 10^{18} molecule

Let edge length = a cm

Total surface area of surfactant = 6 × 10^{18} a^2

$\Rightarrow$ 0.24

a = 2 × 10^{-10} cm = 2 pm

29. Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect? (The Bohr radius is represented by a_0)

(1) The probability density of finding the electron is maximum at the nucleus

(2) The electron can be found at a distance $2a_0$ from the nucleus

(3) The magnitude of the potential energy is double that of its kinetic energy on an average

(4) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

Answer (4)

Sol. The total energy of the electron is minimum when it is at a distance a_0 from the nucleus for 1 s orbital.

30. The maximum number of possible oxidation states of actinoides are shown by

(1) Berkelium (Bk) and californium (Cf)

(2) Neptunium (Np) and plutonium (Pu)

(3) Actinium (Ac) and thorium (Th)

(4) Nobelium (No) and lawrencium (Lr)

Answer (2)

Sol.

Actinoids	Oxidation state shown
Th	+4
Ac	+3
Pu	+3, +4, +5, +6, +7
Np	+3, +4, +5, +6, +7
Bk	+3, +4
Cm	+3, +4
Lr	+3

$\therefore$ Maximum oxidation state is shown by (Np and Pu)

PART–C : MATHEMATICS

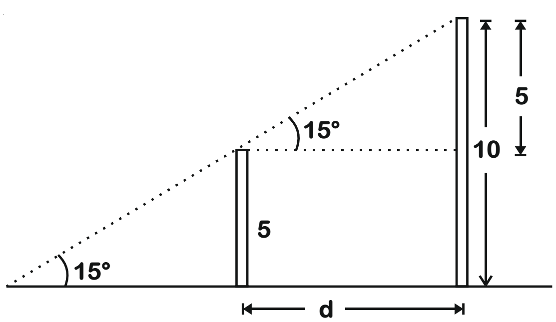
1. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is:

(1) $5\left(2+\sqrt{3}\right)$ (2) $10\left(\sqrt{3}-1\right)$

(3) $5\left(\sqrt{3}+1\right)$ (4) $\frac{5}{2}\left(2+\sqrt{3}\right)$

Answer (1)

Sol.



$$\tan 15^\circ=\frac{5}{d}\Rightarrow d=\frac{5}{\tan 15^\circ}=\frac{5\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$$

$$=\frac{5\left(4+2\sqrt{3}\right)}{2}$$

$$=5\left(2+\sqrt{3}\right)$$

2. If the system of equations 2x + 3y – z = 0, x + ky – 2z = 0 and 2x – y + z = 0 has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:

(1) $\frac{1}{2}$ (2) –4

(3) $\frac{3}{4}$ (4) $-\frac{1}{4}$

Answer (1)

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Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \boxed{k = \frac{9}{2}}$

$\therefore$ Equations are $2x + 3y - z = 0$ ...(i)

$2x - y + z = 0$ ...(ii)

$2x + 9y - 4z = 0$ ...(iii)

By (i) – (ii) $\boxed{2y = z}$ $\therefore$ $\boxed{z = -4x}$ and $\boxed{2x + y = 0}$

$$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2}$$

$$= \frac{1}{2}$$

3. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

(1) $8\sqrt{3}$ (2) $10\sqrt{5}$

(3) $4\sqrt{3}$ (4) $8\sqrt{5}$

Answer (4)

Sol. Sum of roots = $\frac{3}{m^2 + 1}$

For maximum m = 0

Hence equation becomes $x^2 - 3x + 1 = 0$

$\alpha + \beta = 3, \quad \alpha\beta = 1, \quad |\alpha - \beta| = \sqrt{5}$

$$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)| = \sqrt{5}(9 - 1) = 8\sqrt{5}$$

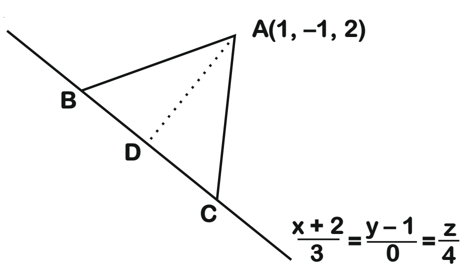
4. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is:

(1) $5\sqrt{17}$ (2) $\sqrt{34}$

(3) 6 (4) $2\sqrt{34}$

Answer (2)

Sol.



Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

Given BC = 5 so we need perpendicular distance of A from line BC.

Let a point D on BC = $(3\lambda - 2, 1, 4\lambda)$

$$\overrightarrow{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$$

Also $\overrightarrow{AD}$ & $\overrightarrow{BC}$ should be perpendicular

$$\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$$

$$(3\lambda - 3)3 + 2(0) + (4\lambda - 2)4 = 0$$

$$9\lambda - 9 + 16\lambda - 8 = 0 \Rightarrow \lambda = \frac{17}{25}$$

Hence, D = $\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$$|\overrightarrow{AD}| = \sqrt{\left(\frac{1}{25} - 1\right)^2 + (2)^2 + \left(\frac{68}{25} - 2\right)^2}$$

$$= \sqrt{\left(\frac{-24}{25}\right)^2 + 4 + \left(\frac{18}{25}\right)^2}$$

$$= \sqrt{\frac{(24)^2 + 4(25)^2 + (18)^2}{25^2}}$$

$$= \sqrt{\frac{576 + 2500 + 324}{25^2}}$$

$$= \sqrt{\frac{3400}{25^2}}$$

$$= \frac{\sqrt{34} \cdot 10}{25} = \frac{2\sqrt{34}}{5}$$

Area of triangle = $\frac{1}{2} \times |\overrightarrow{BC}| \times |\overrightarrow{AD}|$

$$= \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$$

5. If $\cos x \frac{dy}{dx} - y \sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{\pi^2}{2}$ (2) $-\frac{\pi^2}{4\sqrt{3}}$

(3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2\sqrt{3}}$

Answer (4)

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Sol. $\cos x dy-(\sin x)ydx=6x\,dx$

$$\Rightarrow \int d(y\cos x)=\int 6x\,dx$$

$$\Rightarrow y\cos x=3x^2+C$$

As $y\left(\frac{\pi}{3}\right)=0 \Rightarrow (0)\times\left(\frac{1}{2}\right)=\frac{3\pi^2}{9}+C \Rightarrow C=\frac{-\pi^2}{3}$

$$\Rightarrow y\cos x=3x^2-\frac{\pi^2}{3}$$

For $y\left(\frac{\pi}{6}\right)$

$$y\frac{\sqrt{3}}{2}=\frac{3\pi^2}{36}-\frac{\pi^2}{3}$$

$$\frac{\sqrt{3}y}{2}=\frac{-3\pi^2}{12}\Rightarrow y=\frac{-\pi^2}{2\sqrt{3}}$$

6. If $\int e^{\sec x}\left(\sec x\tan x f(x)+\sec x\tan x+\sec^2 x\right)dx$

$=e^{\sec x}f(x)+C$,then a possible choice of f(x) is:

(1) $\sec x-\tan x-\frac{1}{2}$ (2) $\sec x+\tan x+\frac{1}{2}$

(3) $\sec x+x\tan x-\frac{1}{2}$ (4) $x\sec x+\tan x+\frac{1}{2}$

Answer (2)

Sol. $\int e^{\sec x}\left(\sec x\tan x f(x)+(\sec x\tan x+\sec^2 x)\right)dx$

$=e^{\sec x}f(x)+C$

$\because$ We know that

$$\int e^{g(x)}\left((g'(x)f(x))+f'(x)\right)dx=e^{g(x)}\times f(x)+C$$

$$\therefore\ f(x)=\int\left((\sec x\tan x)+\sec^2 x\right)dx$$

$$\therefore\ \boxed{f(x)=\sec x+\tan x+C}$$

7. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2:15:70, then the average of these three coefficients is:

(1) 625 (2) 964

(3) 232 (4) 227

Answer (3)

Sol. Given ${}^nC_{r-1}:{}^nC_r:{}^nC_{r+1}=2:15:70$

$$\frac{{}^nC_{r-1}}{{}^nC_r}=\frac{2}{15}\ \&\ \frac{{}^nC_r}{{}^nC_{r+1}}=\frac{15}{70}$$

$$\Rightarrow \frac{r}{n-r+1}=\frac{2}{15}\ \&\ \frac{r+1}{n-r}=\frac{3}{14}$$

$\Rightarrow$ 15r = 2n – 2r + 2 & 14r + 14 = 3n – 3r

$\Rightarrow$ 17r = 2n + 2 & 17r = 3n – 14

i.e., 2n + 2 = 3n – 14 $\Rightarrow$ n = 16 & r = 2

$$\text{Mean}=\frac{{}^{16}C_1+{}^{16}C_2+{}^{16}C_3}{3}=\frac{16+120+560}{3}$$

$$=\frac{696}{3}=232$$

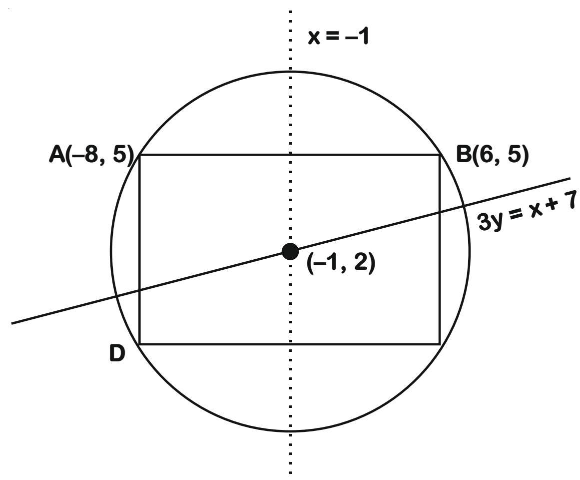
8. A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :

(1) 56 (2) 84

(3) 72 (4) 98

Answer (2)

Sol. Given situation



Perpendicular bisector of AB will pass from centre.

$\therefore$ Equation of perpendicular bisector x = –1

Hence centre (–1, 2)

Let

$$D=(\alpha,\beta)\Rightarrow \frac{\alpha+6}{2}=-1\ \&\ \frac{\beta+5}{2}=2$$

$\alpha=-8$ & $\beta=-1$ $D\equiv(-8,-1)$

|AD| = 6 & |AB| = 14

Area = 6 × 14 = 84

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9. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is :

(1) $\frac{5\pi}{12}$ (2) $\frac{2\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{5\pi}{6}$

Answer (2)

Sol. Let $\cos\alpha, \cos\beta, \cos\gamma$ be direction cosines of $\vec{a}$

Hence, by given data

$$\cos\alpha = \cos\frac{\pi}{3}, \ \cos\beta = \cos\frac{\pi}{4} \ \& \ \cos\gamma = \cos\theta$$

$$\therefore \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}, \quad \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

10. If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$ is continuous at x = 5, then the value of a – b is:

(1) $\frac{2}{\pi - 5}$ (2) $\frac{-2}{\pi + 5}$

(3) $\frac{2}{\pi + 5}$ (4) $\frac{2}{5 - \pi}$

Answer (4)

Sol. L.H.L $\lim\limits_{x\to 5} b|\pi - 5| + 3 = (5 - \pi)b + 3$

f(5) = R.H.L. $\lim\limits_{x\to 5} a|5 - \pi| + 1 = a(5 - \pi) + 1$

For continuity LHL = RHL

$(5 - \pi)b + 3 = (5 - \pi)a + 1$

$\Rightarrow 2 = (a - b)(5 - \pi)$

$\Rightarrow a - b = \frac{2}{5 - \pi}$

11. Let $z \in C$ be such that | z | < 1. If $\omega = \frac{5 + 3z}{5(1 - z)}$, then

(1) 5 Re(ω) > 4 (2) 5 Re(ω) > 1

(3) 4 Im(ω) > 5 (4) 5 Im(ω) < 1

Answer (2)

Sol. $\omega = \frac{5 + 3z}{5 - 5z} \quad \Rightarrow \quad 5w - 5wz = 5 + 3z$

$\Rightarrow 5\omega - 5 = z(3 + 5\omega)$

$\Rightarrow z = \frac{5(\omega - 1)}{3 + 5\omega}$

Given |z| < 1

$\Rightarrow 5|\omega - 1| < |3 + 5\omega|$

$\Rightarrow 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) < 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega}$ (using $|z|^2 = z\bar{z}$)

$\Rightarrow 16 < 40\omega + 40\bar{\omega}$

$\Rightarrow \omega + \bar{\omega} > \frac{2}{5}$

$\Rightarrow 2\text{Re}(\omega) > \frac{2}{5} \quad \Rightarrow \quad \text{Re}(\omega) > \frac{1}{5}$

12. Let P be the plane, which contains the line of intersection of the planes, x + y + z – 6 = 0 and 2x + 3y + z + 5 = 0 and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :

(1) $63\sqrt{5}$ (2) $\frac{17}{\sqrt{5}}$

(3) $205\sqrt{5}$ (4) $\frac{11}{\sqrt{5}}$

Answer (4)

Sol. Let the plane be

$P \equiv (2x + 3y + z + 5) + \lambda(x + y + z - 6) = 0$

As the above plane is perpendicular to xy plane

$\Rightarrow ((2 + \lambda)\hat{i} + (3 + \lambda)\hat{j} + (1 + \lambda)\hat{k}).\hat{k} = 0$

$\Rightarrow \lambda = -1$

$P \equiv x + 2y + 11 = 0$

Distance from (0, 0, 256)

$$\left|\frac{0 + 0 + 11}{\sqrt{5}}\right| = \frac{11}{\sqrt{5}}$$

13. If f(x) = [x] – $\left[\frac{x}{4}\right]$, $x \in R$, where [x] denotes the greatest integer function, then :

(1) $\lim\limits_{x\to 4+} f(x)$ exists but $\lim\limits_{x\to 4-} f(x)$ does not exist

(2) f is continuous at x = 4

(3) $\lim\limits_{x\to 4-} f(x)$ exists but $\lim\limits_{x\to 4+} f(x)$ does not exist

(4) Both $\lim\limits_{x\to 4-} f(x)$ and $\lim\limits_{x\to 4+} f(x)$ exist but are not equal

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Answer (2)

Sol. L.H.L $\lim\limits_{x\to 4^-}[x]-\left[\frac{x}{4}\right]=3-0=3$

$$\left(x<4\Rightarrow [x]=3\ \&\ \frac{x}{4}<1\Rightarrow\left[\frac{x}{4}\right]=0\right)$$

R.H.L $\lim\limits_{x\to 4^+}[x]-\left[\frac{x}{4}\right]=4-1=3$

$$\left(x>4\Rightarrow [x]=4\ \&\ \frac{x}{4}>1\Rightarrow\left[\frac{x}{4}\right]=1\right)$$

$f(4)=[4]-\left[\frac{4}{4}\right]=4-1=3$

LHL = f(4)= RHL

Hence f(x) is continuous at x = 4

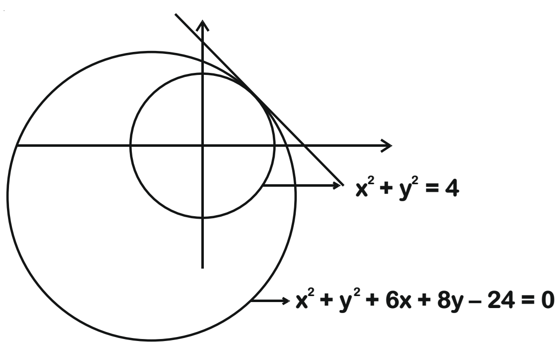
14. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :

(1) (–6, 4) (2) (–4, 6)

(3) (4, –2) (4) (6, –2)

Answer (4)

Sol. In given situation $d_{c_1c_2}=|r_1-r_2|$



Common tangent

$S_1 - S_2 = 0$

$6x + 8y - 20 = 0 \Rightarrow 3x + 4y - 10 = 0$

Hence (6, –2) lies on it

15. The domain of the definition of the function $f(x)=\frac{1}{4-x^2}+\log_{10}(x^3-x)$ is :

(1) $(-1, 0)\cup(1, 2)\cup(3, \infty)$

(2) $(-1, 0)\cup(1, 2)\cup(2, \infty)$

(3) $(1, 2)\cup(2, \infty)$

(4) $(-2, -1)\cup(-1, 0)\cup(2, \infty)$

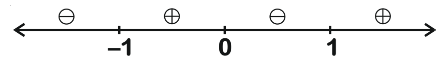
Answer (2)

Sol. For domain denominator $\neq 0$

$4-x^2\neq 0 \Rightarrow x\neq \pm 2$ ...(1)

and $x^3 - x > 0$

$\Rightarrow x(x-1)(x+1) > 0$



$x\in(-1, 0)\cup(1, \infty)$ ...(2)

Hence domain is intersection of (1) & (2) i.e.

$x\in(-1, 0)\cup(1, 2)\cup(2, \infty)$

16. The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 + ... upto 11th term is :

(1) 916 (2) 946

(3) 945 (4) 915

Answer (2)

Sol. 1 + 2.3 + 3.5 + 4.7 + ……

Lets break the sequence as shown

$$1+\underbrace{(2.3+3.5+4.7+\ldots\ldots)}_{S}$$

We find $s_{10}=\sum\limits_{n=1}^{10}(n+1)(2n+1)$

$$=\sum_{n=1}^{10}(2n^2+3n+1)$$

$$=\frac{2n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}+n(n=10)$$

$$=\frac{2.10.11.21}{6}+\frac{3.10.11}{2}+10$$

= 770 + 165 + 10 = 945

Hence required sum = 1 + 945 = 946

17. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :

(1) $4\pi\left(3+\sqrt{2}\right)$

(2) $8\pi\left(2-\sqrt{2}\right)$

(3) $8\pi\left(3-2\sqrt{2}\right)$

(4) $4\pi\left(2-\sqrt{2}\right)$

Answer (3)

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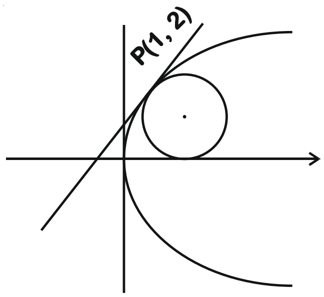

The circle and parabola will have common tangent at P(1, 2)

$\therefore$ Equation of tangent to parabola

$$\equiv y\times(2)=4\frac{(x+1)}{2}\Rightarrow 2y=2x+2$$

$$y = x + 1$$

Let equation of circle be (using family of circles)

$(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$

$\Rightarrow$ $c \equiv (x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

Also circle touches x-axis $\Rightarrow$ y-coordinate of centre = radius

$\Rightarrow$ $c \equiv x^2 + y^2 + (\lambda - 2)x + (-\lambda - 4)y + (\lambda + 5) = 0$

$$\frac{\lambda+4}{2}=\sqrt{\left(\frac{\lambda-2}{2}\right)^2+\left(\frac{-\lambda-4}{2}\right)^2-(\lambda+5)}$$

$$\Rightarrow \frac{\lambda^2-4\lambda+4}{4}=\lambda+5\Rightarrow\lambda^2-4\lambda+4=4\lambda+20$$

$\Rightarrow$ $\lambda^2 - 8\lambda - 16 = 0$

$$\Rightarrow \lambda=\frac{8\pm\sqrt{64+64}}{2}$$

$$=4\pm4\sqrt{2}$$

$\lambda=4-4\sqrt{2}$ (Other value forms bigger circle)

Hence centre of circle $(2\sqrt{2}-2, 4-2\sqrt{2})$

Radius $=4-2\sqrt{2}$

Area $=\pi(4-2\sqrt{2})^2=8\pi(3-2\sqrt{2})$

18. The total number of matrices

$$A=\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix},\ (x, y\in R, x\neq y)\text{ for}$$

Which $A^TA = 3I_3$ is :

(1) 6 (2) 3

(3) 4 (4) 2

Answer (3)

Sol. $A^TA=\begin{bmatrix}0 & 2x & 2x\\ 2y & y & -y\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{bmatrix}=3I$

$$=\begin{bmatrix}8x^2 & 0 & 0\\ 0 & 6y^2 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$$

$\Rightarrow$ $8x^2 = 3$, $6y^2 = 3$

$$x=\pm\sqrt{\frac{3}{8}},\ y=\pm\sqrt{\frac{1}{2}}$$

Total combinations of (x, y) = 2 × 2 = 4

19. If $p\Rightarrow(q\vee r)$ is false, then truth values of p, q, r are respectively :

(1) T, F, F (2) F, F, F

(3) T, T, F (4) F, T, T

Answer (1)

Sol. For $p\rightarrow q\vee r$ to be F

r must be F & $p\rightarrow q$ must be F

for $p\rightarrow q$ to be F $p\rightarrow T$ & $q\rightarrow F$

p, q, r $\equiv$ T, F, F

20. The mean and the median of the following ten numbers in increasing order

10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :

(1) $\frac{7}{3}$

(2) $\frac{8}{3}$

(3) $\frac{7}{2}$

(4) $\frac{9}{4}$

Answer (1)

Sol. Mean $=\frac{\sum x_i}{n}=\frac{x+y+300}{10}=42\Rightarrow x+y=120$

Median $=\frac{T_5+T_6}{2}=35=\frac{34+x}{2}\Rightarrow x=36\ \&\ y=84$

Hence $\frac{y}{x}=\frac{84}{36}=\frac{7}{3}$

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21. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :

(1) 157 (2) 225

(3) 262 (4) 190

Answer (4)

Sol. Balls used in equilateral triangle $= \frac{n(n+1)}{2}$

Here side of equilateral triangle has n-balls

No. of balls in each side of square is = (n – 2)

Given $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$\Rightarrow$ $n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow$ $n^2 - 9n - 190 = 0$

$\Rightarrow$ $n^2 - 19n + 10n - 190 = 0$

$\Rightarrow$ $(n - 19)(n + 10) = 0$

$\Rightarrow$ $n = 19$

Balls used to form triangle

$$= \frac{n(n+1)}{2} = \frac{19 \times 20}{2} = 190$$

22. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is :

(1) 13.9

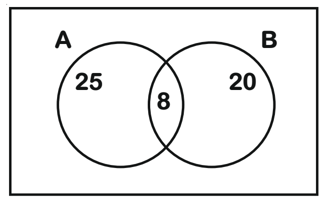
(2) 13

(3) 12.8

(4) 13.5

Answer (1)

Sol.



n(A only) = 25 – 8 = 17%

n(B only) = 20 – 8 = 12%

% of people from A only who read advertisement = 17 × 0.3 = 5.1%

% of people from B only who read advertisement = 12 × 0.4 = 4.8%

% of people from A&B both who read advertisement = 8 × 0.5 = 4%

Total % of people who read advertisement

= 5.1 + 4.8 + 4 = 13.9%

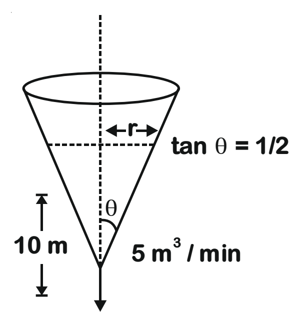
23. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is :

(1) $\frac{2}{\pi}$ (2) $\frac{1}{15\pi}$

(3) $\frac{1}{10\pi}$ (4) $\frac{1}{5\pi}$

Answer (4)

Sol.



Given $\frac{dv}{dt} = 5m^3/min$

$V = \frac{1}{3}\pi r^2 h$ ...(i) (where r is radius and h is height at any time)

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Also $\tan\theta = \frac{r}{h} = \frac{1}{2} \Rightarrow h = 2r \Rightarrow \frac{dh}{dt} = \frac{2dr}{dt}$ ...(ii)

Differentiate eq^n. (i), we get

$$\frac{dV}{dt} = \frac{1}{3}\left(\pi 2r\frac{dr}{dt}h + \pi r^2\frac{dh}{dt}\right) = \frac{1}{3}\left(100\pi\frac{1}{2} + 25\pi\right)\frac{dh}{dt}$$

at h = 10, r = 5

$$5 = \frac{75\pi}{3}\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{5\pi} m/mim$$

24. The value of the integral

$\int_0^1 x\cot^{-1}(1-x^2+x^4)\,dx$ is

(1) $\frac{\pi}{4} - \log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$

(3) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$

Answer (4)

Sol. $\int_0^1 x\cot^{-1}\left(1-x^2+x^4\right)dx = \int_0^1 x\tan^{-1}\left(\frac{1}{1+x^4-x^2}\right)$

$$\Rightarrow \int_0^1 x\tan^{-1}\left(\frac{x^2-\left(x^2-1\right)}{1+x^2\left(x^2-1\right)}\right)dx$$

$$\Rightarrow \int_0^1 x\tan^{-1}x^2dx - \int_0^1 x\tan^{-1}\left(x^2-1\right)dx$$

Put $x^2 = t \Rightarrow 2xdx = dt$, (For 1st integration)

put $x^2 - 1 = k \Rightarrow 2xdx = dk$ (For 2nd integration)

$$\Rightarrow \frac{1}{2}\int_0^1 1\tan^{-1}tdt - \frac{1}{2}\int_{-1}^0 1\tan^{-1}kdk$$

$$\Rightarrow \frac{1}{2}\left(\int_0^1 t\tan^{-1}t\Big|_0^1 - \int_0^1\frac{t}{1+t^2}dt\right) - \frac{1}{2}$$

$$\left(k\tan^{-1}k\Big|_{-1}^0 - \int_{-1}^0\frac{k}{1+k^2}dk\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+t^2\right)\right)\Big|_0^1\right) - \frac{1}{2}$$

$$\left(0 - \frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+k^2\right)\right)\Big|_{-1}^0\right)$$

$$\Rightarrow \left(\frac{\pi}{8} - \frac{1}{4}\ln 2\right) - \left(\frac{-\pi}{8} - \frac{1}{4}10 - \ln 2\right)$$

$$\Rightarrow \frac{\pi}{4} - \frac{1}{2}\ln 2$$

25. The area (in sq. units) of the region

$A = \left\{(x, y) : \frac{y^2}{2} \le x \le y + 4\right\}$ is

(1) 18

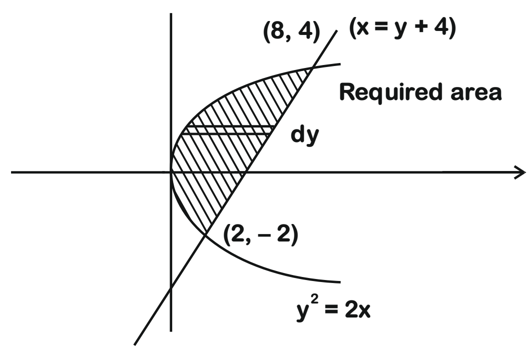
(2) 16

(3) $\frac{53}{3}$

(4) 30

Answer (1)

Sol.



Hence area $= \int_{-2}^4 xdy$

$$= \int_{-2}^4\left(y+4-\frac{y^2}{2}\right)dy$$

$$= \frac{y^2}{2} + 4y - \frac{y^3}{6}\Big|_{-2}^4 = \left(8+16-\frac{64}{6}\right) - \left(2-8+\frac{8}{6}\right)$$

$$= \left(24-\frac{32}{3}\right) - \left(-6+\frac{4}{3}\right) = \frac{40}{3} + \frac{14}{3} = \frac{54}{3} = 18$$

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26. If the two lines $x + (a-1)y = 1$ and $2x + a^2y = 1$ $(a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is

(1) $\sqrt{\frac{2}{5}}$ (2) $\frac{\sqrt{2}}{5}$

(3) $\frac{2}{5}$ (4) $\frac{2}{\sqrt{5}}$

Answer (1)

Sol. For perpendicular $m_1m_2 = -1$

$$\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$$

$\Rightarrow$ $2 = a^2(1-a)$

$\Rightarrow$ $a^3 - a^2 + 2 = 0$

$\Rightarrow$ $(a+1)(a^2 + 2a + 2) = 0$

$$\boxed{a = -1}$$

Hence lines are $x - 2y = 1$ and $2x + y = 1$

$\therefore$ Intersection point $\left(\frac{3}{5}, \frac{-1}{5}\right)$

Distance from origin $= \sqrt{\frac{9}{25} + \frac{1}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$

27. The value of $\sin 10° \sin 30° \sin 50° \sin 70°$ is

(1) $\frac{1}{18}$ (2) $\frac{1}{32}$

(3) $\frac{1}{16}$ (4) $\frac{1}{36}$

Answer (3)

Sol. $\sin(60° + A).\sin(60° - A)\sin A = \frac{1}{4}\sin 3A$

Hence, $\sin 10° \sin 50° \sin 70°$

$= \sin 10° \sin(60° - 10°) \sin(60° + 10°) = \frac{1}{4}\sin 30°$

Hence, $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{4}\sin^2 30°$

$= \frac{1}{16}$

28. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to

(1) $\sqrt{2} - 1$

(2) $\sqrt{2} + 1$

(3) $2\sqrt{2} + 1$

(4) $2\sqrt{2} - 1$

Answer (2)

Sol. Let tangent in terms of m to parabola and ellipse

i.e $y = mx + \frac{1}{4m}$ for parabola at point $\left(\frac{1}{4m^2}, \frac{-1}{2m}\right)$

and $y = mx \pm \sqrt{m^2 + \frac{1}{2}}$ for ellipse on comparing

$$\Rightarrow \frac{1}{4m} = \pm\sqrt{m^2 + \frac{1}{2}} \Rightarrow \frac{1}{16m^2} = m^2 + \frac{1}{2}$$

$\Rightarrow$ $16m^4 + 8m^2 - 1 = 0$

$$m^2 = \frac{-8 \pm \sqrt{64 + 64}}{2(16)} = \frac{-8 \pm 8\sqrt{2}}{2(16)} = \frac{\sqrt{2} - 1}{4}$$

$$\alpha = \frac{1}{4m^2} = \frac{1}{4\,\frac{\sqrt{2}-1}{4}} = \sqrt{2} + 1$$

29. If $f : R \to R$ is a differentiable function and $f(2) = 6$, then $\lim\limits_{x \to 2} \int_6^{f(x)} \frac{2t\,dt}{(x-2)}$ is

(1) 0 (2) $2f'(2)$

(3) $12f'(2)$ (4) $24f'(2)$

Answer (3)

Sol. Using L' Hospital rule and Leibnitz theorem

$$\lim_{x \to 2} \frac{\int_6^{f(x)} 2t\,dt}{(x-2)}$$

$$\lim_{x \to 2} \frac{2f(x)f'(x) - 0}{1}$$

$2f(2)f'(2) = 12f'(2)$

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30. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11th term is

(1) –36 (2) 25

(3) –25 (4) –35

Answer (3)

Sol. Let terms be a – d, a, a + d

$\Rightarrow$ 3a = 33 $\Rightarrow$ 11

Product of terms

$(a - d)\, a\, (a + d) = 11\,(121 - d^2) = 1155$

$\Rightarrow 121 - d^2 = 105 \Rightarrow d = \pm 4$

if d = 4

$$\left.\begin{matrix} T_1 = 7 \\ T_2 = 11 \\ T_3 = 15 \end{matrix}\right\} \Rightarrow T_{11} = T_1 + 10d = 7 + 10(4) = 47$$

if d = – 4

$$\left.\begin{matrix} T_1 = 15 \\ T_2 = 11 \\ T_3 = 7 \end{matrix}\right\} \Rightarrow T_{11} = T_1 + 10d = 15 + 10(-4) = -25$$

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