JEE Main 2020 Paper

Date of Exam: **9th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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1. A sphere of 10 cm radius has a uniform thickness of ice around it. If the ice is melting at the rate of 50 cm^3/min when thickness is 5 cm, then the rate of change of thickness is

a. $\frac{1}{12\pi}$ b. $\frac{1}{18\pi}$
c. $\frac{1}{9\pi}$ d. $\frac{1}{36\pi}$

Answer: (b)

Solution:

Let thickness of ice be x cm.

Therefore, net radius of sphere $= (10 + x)$ cm

Volume of sphere $V = \frac{4}{3}\pi(10 + x)^3$

$\Rightarrow \frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$

At $x = 5, \frac{dV}{dt} = 50 \text{ cm}^3/\text{min}$

$\Rightarrow 50 = 4\pi \times 225 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{18\pi} \text{cm/min}$

2. The number of real roots of $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is

a. 1 b. 2
c. 3 d. 4

Answer: (a)

Solution:

$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$

$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

Let $e^x + \frac{1}{e^x} = u$

Then, $u^2 + u - 6 = 0$

$\Rightarrow u = 2, -3$

$u \neq -3$ as $u > 0$ ($\because e^x > 0$)

$\Rightarrow e^x + \frac{1}{e^x} = 2 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$

Hence, only one real solution is possible.

3. If $f'(x) = \tan^{-1}(\sec x + \tan x), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f(0) = 0$, then the value of $f(1)$ is

a. $\frac{\pi-1}{4}$
b. $\frac{\pi+1}{4}$
c. $\frac{\pi+1}{2}$
d. 0

Answer: (b)

Solution:

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$$

$$f'(x) = \tan^{-1}\left(\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}\right)$$

$$f'(x) = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$$

$$f'(x) = \tan^{-1}\left[\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right]$$

$$f'(x) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi + 1}{4}$$

4. The number of solutions of $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0, 2\pi]$ is

a. 2
b. 4
c. 8
d. 6

Answer: (c)

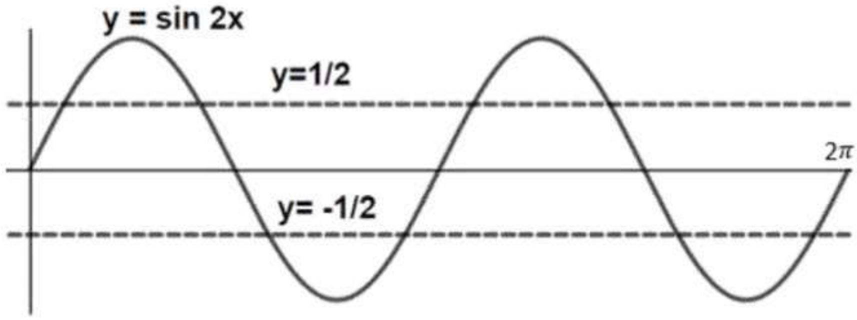
Solution:

$\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0,2\pi]$

$\Rightarrow \log_{\frac{1}{2}}|\sin x||\cos x| = 2$

$\Rightarrow |\sin x \cos x| = \frac{1}{4}$

$\therefore \sin 2x = \pm\frac{1}{2}$



$\therefore$ We have 8 solutions for $x \in [0,2\pi]$

5. If e_1 and e_2 are the eccentricities of $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and $\frac{x^2}{9} - \frac{y^2}{4} = 1$, respectively. If the points (e_1, e_2) lies on the ellipse $15x^2 + 3y^2 = k$. Then the value of k is

a. 16
b. 14
c. 15
d. 17

Answer: (a)

Solution:

$$e_1 = \sqrt{1 - \frac{4}{18}} = \frac{\sqrt{7}}{3} \ \& \ e_2 = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$$

$\because (e_1, e_2)$ lies on the ellipse $15x^2 + 3y^2 = k$

$\therefore 15e_1^2 + 3e_2^2 = k$

$\Rightarrow 15 \times \frac{7}{9} + 3 \times \frac{13}{9} = k \Rightarrow k = 16$

6. The value of $\int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$ is -

a. $7\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$

b. $7\left(\frac{x-3}{x+4}\right)^{\frac{6}{7}} + c$

c. $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$

d. $7\left(\frac{x+4}{x-3}\right)^{\frac{6}{7}} + c$

Answer: (c)

Solution:

$$I = \int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$$

$$\Rightarrow I = \int \frac{(x+4)^{\frac{6}{7}} dx}{(x-3)^{\frac{6}{7}} \times (x+4)^2} = \int \left(\frac{x-3}{x+4}\right)^{-\frac{6}{7}} \times \frac{dx}{(x+4)^2}$$

Put $\frac{x-3}{x+4} = t \Rightarrow dt = 7\left(\frac{1}{(x+4)^2}\right) dx$

$$\Rightarrow I = \frac{\int t^{-\frac{6}{7}}}{7} dt = t^{\frac{1}{7}} + c = \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$$

7. If $\left|\frac{z-i}{z+2i}\right| = 1$, $|z| = \frac{5}{2}$ then the value of $|z+3i|$ is

a. $\sqrt{10}$

b. $\sqrt{5}$

c. $\frac{7}{2}$

d. $\sqrt{3}$

Answer: (c)

Solution:

If $\left|\frac{z-i}{z+2i}\right| = 1$ & $|z| = \frac{5}{2}$

$\Rightarrow |z-i| = |z+2i|$

$\Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\Rightarrow y - 1 = \pm(y+2)$

$\Rightarrow y - 1 = -y - 2$

$\Rightarrow y = -\frac{1}{2}$

$\Rightarrow |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$

$\Rightarrow x^2 + \frac{1}{4} = \frac{25}{4}$

$\Rightarrow x = \pm\sqrt{6}$

$|z+3i| = \sqrt{x^2+(y+3)^2}$

$\Rightarrow |z+3i| = \frac{7}{2}$

8. The value of $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \ldots \infty$ is
   a. 2
   b. 1
   c. $\sqrt{2}$
   d. $2^{\frac{1}{4}}$

   Answer: (c)

   Solution:

   $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \ldots \infty = 2^{\frac{1}{4}} \times 2^{\frac{2}{16}} \times 2^{\frac{4}{48}} \ldots \infty$

   $\Rightarrow 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} \times 2^{\frac{1}{16}} \ldots \infty = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\infty}$

   $\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)} = \sqrt{2}$

9. The value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$ is -
   a. $\frac{1}{2}$
   b. $-\frac{1}{2}$
   c. $\frac{1}{\sqrt{2}}$
   d. $\frac{1}{2\sqrt{2}}$

   Answer: (d)

   Solution:

   $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8} = \cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

   $= 4\left[\cos^6\frac{\pi}{8} - \sin^6\frac{\pi}{8}\right] + 3\left[\sin^4\frac{\pi}{8} - \cos^4\frac{\pi}{8}\right]$

   $= 4\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right] - 3\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]$

   $= \left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[4\left(1 - \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right) - 3\right]$

   $= \cos\frac{\pi}{4}\left[1 - \sin^2\frac{\pi}{4}\right] = \frac{1}{2\sqrt{2}}$

10. The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x} dx$ is
    a. $4\pi^2$
    b. $2\pi^2$
    c. π^2
    d. $3\pi^2$

    Answer: (c)

Solution:

Let $I = \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (1)

$I = \int_0^{2\pi} \frac{(2\pi - x) \sin^8(2\pi - x)}{\sin^8(2\pi - x) + \cos^8(2\pi - x)} dx$

$= \int_0^{2\pi} \frac{(2\pi - x) \sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (2)

Adding (1) & (2) , we get:

$\Rightarrow 2I = 2\pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = \pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (3)

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8\left(\frac{\pi}{2} - x\right)}{\sin^8\left(\frac{\pi}{2} - x\right) + \cos^8\left(\frac{\pi}{2} - x\right)} dx = 4\pi \int_0^{\frac{\pi}{2}} \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx$ ... (4)

Adding (3) & (4) , we get -

$I = 2\pi \int_0^{\frac{\pi}{2}} 1\, dx = 2\pi \times \frac{\pi}{2} = \pi^2$

11. If $f(x) = a + bx + cx^2$ where $a, b, c \in \mathbf{R}$ then the value of $\int_0^1 f(x)dx$ is -

a. $\frac{1}{6}\left(f(1) + f(0) - 4f\left(\frac{1}{2}\right)\right)$

b. $\frac{1}{3}\left(f(1) + f(0) + 2f\left(\frac{1}{2}\right)\right)$

c. $\frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right)$

d. $\frac{1}{6}\left(f(1) - f(0) - 4f\left(\frac{1}{2}\right)\right)$

Answer: (c)

Solution:

$f(x) = a + bx + cx^2$

$f(0) = a, f(1) = a + b + c$

$f\left(\frac{1}{2}\right) = \frac{c}{4} + \frac{b}{2} + a$

$\int_0^1 f(x)dx = \int_0^1 (a + bx + cx^2)dx = a + \frac{b}{2} + \frac{c}{3}$

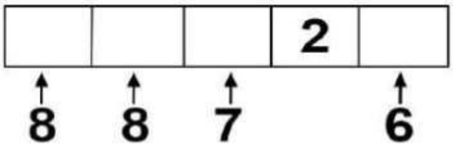
$= \frac{1}{6}(6a + 3b + 2c) = \frac{1}{6}\big(a + (a + b + c) + (4a + 2b + c)\big)$

$= \frac{1}{6}\left(f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right)$

12. If the number of ways of forming 5 digit numbers (without repeating any digit), such that the tenth place of the number must be occupied by 2 is $336k$, then the value of k is
    a. 5
    b. 6
    c. 7
    d. 8

Answer: (d)

Solution:



Total numbers that can be formed are

$= 8 \times 8 \times 7 \times 6$

$= 8 \times 336$

$\therefore k = 8$

13. If D is the centroid of the ΔABC having vertices $A(3, -1), B(1, 3), C(2, 4)$ and P is the point if intersection of lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$, then which of the following point lies on the line joining D and P?
    a. $(-9, -6)$
    b. $(9, -6)$
    c. $(9, 6)$
    d. $(-9, -7)$

Answer: (a)

Solution:

Coordinates of D are $\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) = (2, 2)$

Point of intersection of two lines

$x + 3y - 1 = 0$ and $3x - y + 1 = 0$

is $P\left(\frac{-1}{5}, \frac{2}{5}\right)$

Equation of line DP is $8x - 11y + 6 = 0$

Point $(-9, -6)$ lies on DP

14. If $f(x)$ is twice differentiable and continuous function in $x \in [a, b]$. Also $f'(x) > 0$ and $f''(x) < 0$ and $c \in (a, b)$, then $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

a. 1

b. $\frac{a+b}{b-c}$

c. $\frac{b-c}{c-a}$

d. $\frac{c-a}{b-c}$

Answer: (d)

Solution:

$\therefore c \in (a, b)$ and f is twice differentiable and continuous function (a, b)

$\therefore$ LMVT is applicable

For $p \in (a, c)$, $\quad f'(p) = \frac{f(c)-f(a)}{c-a}$

For $q \in (c, b)$, $\quad f'(q) = \frac{f(b)-f(c)}{b-c}$

$\therefore f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$f'(p) > f'(q)$

$$\Rightarrow \frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c}$$

$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c}$ (as $f'(x) > 0 \Rightarrow f(x)$ is increasing)

15. If three planes
$x + 4y - 2z = 1$
$x + 7y - 5z = \beta$
$x + 5y + \alpha z = 5$
intersect in a line, then $\alpha + \beta =$

a. -10

b. 0

c. 2

d. 10

Answer: (d)

Solution:

The given planes intersect in a line

$\therefore D = D_x = D_y = D_z = 0$

$D = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$$

$\Rightarrow \alpha = -3$

$D_z = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$$

$\Rightarrow \beta = 13$

$\therefore \alpha + \beta = 10$

16. $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If mean and variance of observations $(x_1 - 3), (x_2 - 3) \ldots\ldots (x_{10} - 3)$ is λ and μ respectively, then ordered pair (λ, μ) is

a. $(1,1)$

b. $(1,3)$

c. $(3,1)$

d. $(3,3)$

Answer: (d)

Solution:

$$\sum_{i=1}^{10}(x_i - 5) = 10 \Rightarrow \sum_{i=1}^{10} x_i - 50 = 10$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 60$$

$$\lambda = \frac{\sum_{i=1}^{10}(x_i - 3)}{10} = \frac{\sum_{i=1}^{10} x_i - 30}{10} = 3$$

Variance is unchanged, if a constant is added or subtracted from each observation

$$\mu = Var(x_i - 3) = Var(x_i - 5) = \frac{\sum_{i=1}^{10}(x_i - 5)^2}{10} - \left(\frac{\sum(x_i - 5)}{10}\right)^2$$

$$= \frac{40}{10} - \left(\frac{10}{10}\right)^2 = 3$$

17. 20 cards are placed in a bag with 10 named as A and another 10 named as B. If cards are draw n one by one (with replacement), then the probability that second A comes before third B is

a. $\frac{11}{16}$

b. $\frac{7}{16}$

c. $\frac{9}{16}$

d. $\frac{13}{16}$

Answer: (a)

Solution:

Here $P(A) = P(B) = \frac{1}{2}$

Then, these following cases are possible $\rightarrow AA, BAA, ABA, ABBA, BBAA, BABA$

So, the required probability is $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

18. The negation of '$\sqrt{5}$ is an integer or 5 is an irrational number' is
    a. $\sqrt{5}$ is an integer and 5 is not an irrational number.
    b. $\sqrt{5}$ is not an integer and 5 is not an irrational number.
    c. $\sqrt{5}$ is not an integer or 5 is not an irrational number.
    d. $\sqrt{5}$ is not an integer and 5 is an irrational number.

Answer: (b)

Solution:

p: $\sqrt{5}$ is an integer

q: 5 is an irrational number

Given statement : $p \vee q$

Required negation statement: $\sim(p \vee q) = \sim p \wedge \sim q$

'$\sqrt{5}$ is not an integer and 5 is not an irrational number'

19. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj}(A)$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|}$ is
    a. 2
    b. 4
    c. 8
    d. 16

Answer: (c)

Solution:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 13 + 1 - 8 = 6$$

$$B = \text{adj}(A) \Rightarrow |\text{adj } B| = |\text{adj}(\text{adj } A)| = |A|^4 = 6^4$$

$$|C| = |3A| = 3^3|A| = 3^3 \times 6$$

$$\frac{|\text{adj } B|}{|C|} = \frac{6^4}{3^3 \times 6} = \frac{2^3 \times 3^3}{3^3} = 8$$

20. If a circle touches y-axis at (0,4) and passes through (2,0), then which of the following can be the tangent to the circle?

a. $3x + 4y - 24 = 0$

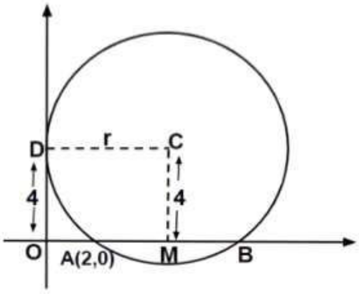
b. $4x - 3y - 17 = 0$

c. $4x + 3y - 6 = 0$

d. $3x + 4y - 6 = 0$

Answer: (d)

Solution:



$OD^2 = OA \times OB \Rightarrow 16 = 2 \times OB \Rightarrow OB = 8$

$\therefore AB = 6$

$\therefore AM = 3, CM = 4 \Rightarrow CA = 5$

$\therefore OM = 5$

Centre will be $(5,4)$ and radius is 5

Now checking option (d)

$3x + 4y - 6 = 0$

$$\frac{15 + 16 - 6}{\sqrt{3^2 + 4^2}} = 5 \ \ (p = r)$$

21. $(1 + x)\frac{dy}{dx} = [(1 + x)^2 + (y - 3)]$. If $y(2) = 0$, then the value of $y(3)$ is

Answer: (3)

Solution:

$$(1 + x)\frac{dy}{dx} = [(1 + x)^2 + (y - 3)]$$

$$\Rightarrow (1 + x)\frac{dy}{dx} - y = (1 + x)^2 - 3$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(1 + x)}y = 1 + x - \frac{3}{1 + x}$$

$$\text{I. F.} = e^{-\int \frac{1}{1+x}dx} = \frac{1}{1 + x}$$

$$y \times \frac{1}{1+x} = \int 1 - \frac{3}{(1+x)^2} dx$$

$$\frac{y}{1+x} = x + \frac{3}{1+x} + c$$

$$\Rightarrow y = x(1+x) + 3 + c(1+x)$$

At $x = 2, y = 0$, we get

$$0 = 6 + 3 + 3c$$

$$\Rightarrow c = -3$$

$\Rightarrow$ At $x = 3$,

$$y = x^2 - 2x = 9 - 6 = 3$$

$$\Rightarrow y(3) = 3$$

22. Function $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}, & x < 0 \\ b, & x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{4}{3}}}, & x > 0 \end{cases}$ is continuous at $x = 0$. The value of $a + 2b$ is

Answer: (0)

Solution:

$f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x \to 0^-} f(x) = b = \lim_{x \to 0^+} f(x)$$

$$b = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{(h+3h^2)^{\frac{1}{3}} - h^{\frac{1}{3}}}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \to 0} \frac{h^{\frac{1}{3}}\left[(1+3h)^{\frac{1}{3}} - 1\right]}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \to 0} \frac{(1+3h)^{\frac{1}{3}} - 1}{h}$$

$$\Rightarrow b = \lim_{h \to 0} \frac{1}{3}(1+3h)^{-\frac{2}{3}} \times 3$$

or, $b = 1$

$$\lim_{x\to 0^-} f(x) = 1 \Rightarrow \lim_{h\to 0} \frac{\sin(a+2)(-h)) + \sin(-h)}{-h} = 1$$

$\Rightarrow a + 3 = 1 \Rightarrow a = -2$

$\Rightarrow a + 2b = 0$

23. The coefficient of x^4 in $(1 + x + x^2)^{10}$ is

Answer: (615)

Solution:

General term of the given expression is given by $\frac{10!}{p!q!r!} x^{q+2r}$

Here, $q + 2r = 4$

For $p = 6, q = 4, r = 0$, coefficient $= \frac{10!}{6!\times 4!} = 210$

For $p = 7, q = 2, r = 1$, coefficient $= \frac{10!}{7!\times 2!\times 1!} = 360$

For $p = 8, q = 0, r = 2$, coefficient $= \frac{10!}{8!\times 2!} = 45$

Therefore, sum $= 615$

24. If $\vec{P} = (a+1)\hat{\imath} + a\hat{\jmath} + a\hat{k}$

$\vec{Q} = a\hat{\imath} + (a+1)\hat{\jmath} + a\hat{k}$

$\vec{R} = a\hat{\imath} + a\hat{\jmath} + (a+1)\hat{k}$

and $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors and $3(\vec{P}.\vec{Q})^2 - \lambda|\vec{R}\times\vec{Q}|^2 = 0$, then value of λ is

Answer: (1)

Solution:

As $\vec{P}, \vec{Q}, \vec{R}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a+1)\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$(3a+1)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$3a + 1 = 0$

$\Rightarrow a = -\frac{1}{3}$

$\vec{P} = \frac{1}{3}(2\hat{\imath} - \hat{\jmath} - \hat{k}),\quad \vec{Q} = \frac{1}{3}(-\hat{\imath} + 2\hat{\jmath} - \hat{k}),\quad \vec{R} = \frac{1}{3}(-\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$$\vec{R} \times \vec{Q} = \frac{1}{9}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$$

$\vec{R} \times \vec{Q} = \frac{1}{9}(-3\hat{\imath} - 3\hat{\jmath} - 3\hat{k}) = -\frac{1}{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$|\vec{R} \times \vec{Q}|^2 = \frac{1}{3}$

$\vec{P}.\vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$

$3(\vec{P}.\vec{Q})^2 - \lambda|\vec{R} \times \vec{Q}|^2 = 0$

$\Rightarrow \frac{1}{3} - \lambda \times \frac{1}{3} = 0 \Rightarrow \lambda = 1$

25. Points $A(2,4,0), B(3,1,8), C(3,1,-3), D(7,-3,4)$ are four points. The projection of line segment AB on line CD is

Answer: (8)

Solution:

$\overrightarrow{AB} = \hat{\imath} - 3\hat{\jmath} + 8\hat{k}$

$\overrightarrow{CD} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is $= \frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{CD}|} = \frac{4+12+56}{\sqrt{4^2+4^2+7^2}} = \frac{72}{9} = 8$

JEE Main 2020 Paper

Date: 9th January 2020

Time: 09.30 AM – 12:30 PM

Subject: Chemistry

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1. Determine the wavelength of electron in the 4th Bohr's orbit:

a. $4\pi a_0$ b. $2\pi a_0$

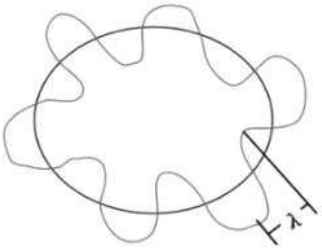
c. $8\pi a_0$ d. $6\pi a_0$

Answer: c

Solution: n=4

Z=1

Λ=?



Circumference $(2\pi r) = n\lambda$

$\frac{2\pi a_0 n^2}{z} = n\lambda$

On solving, we get $8\pi a_o$

2. Which of the following species have one unpaired electron each?

a. O_2, O_2^- b. O_2, O_2^+

c. O_2^+, O_2^- d. O_2, O_2^{2-}

Answer: c

Solution:

$O_2: \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2py^2}\ \pi^*_{2p_x^1} = \pi^*_{2p_y^1}$

$O_2^-: \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2py^2}\ \pi^*_{2p_x^2} = \pi^*_{2p_y^1}$

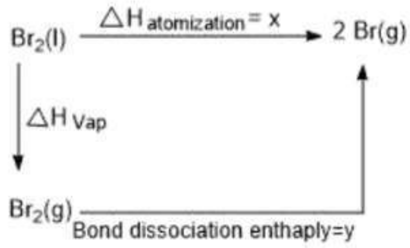
$O_2^+: \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2py^2}\ \pi^*_{2p_x^1}$

3. For $Br_2(l)$, the enthalpy of atomisation = x kJ/mol and the bond dissociation enthalpy of bromine = y kJ/mol. Then,

a. $x > y$
b. $x < y$
c. $x = y$
d. Relation does not exist.

Answer: a

Solution:



$\Delta H_{atomisation} = \Delta H_{vap} + y$

$x - y = \Delta H_{vap}$

4. Which of the following oxides are acidic, basic and amphoteric, respectively?

a. MgO, P_4O_{10}, Al_2O_3
b. N_2O_3, Li_2O, Al_2O_3
c. SO_3, Al_2O_3, Na_2O
d. P_4O_{10}, Al_2O_3, MgO

Answer: b

Solution:

Non-metallic oxides are acidic in nature, metallic oxides are basic in nature and Al_2O_3 is amphoteric in nature

5. The complex $Cr(H_2O)_6Cl_n$, shows geometrical isomerism and also reacts with $AgNO_3$ solution. Given: The spin only magnetic moment = 3.8 B. M. What is the IUPAC name of the complex?

a. Hexaaquachromium(III) chloride
b. Tetraaquadichloridochromium(III) chloride dihydrate
c. Hexaaquachromium(IV) chloride
d. Tetraaquadichloridochromium(IV) chloride dehydrate

Answer: b

Solution:

Spin only magnetic moment = 3.8 B. M. This implies, $\mu = \sqrt{(n(n+2))}$ B.M.

$(\sqrt{16} = 4$ implies that $\sqrt{15}$ should be less than four.

This means, n=3 as $\sqrt{15} = \sqrt{(3(3+2))}$

Cr (24) = $[Ar]4s^1\ 3d^5$

(g.s)

For 3 unpaired electrons, the oxidation state of Cr should be +3

Cr^{3+} can be attained if the complex has a structure that looks like: $[Cr(H_2O)_4Cl_2]Cl.\ 2H_2O$

$[Cr(H_2O)_4Cl_2]Cl.\ 2H_2O$ has the IUPAC name : Tetraaquadichloridochromium(III) chloride dihydrate

6. The electronic configuration of bivalent Europium and trivalent Cerium, respectively is:
(Atomic Number : Xe = 54, Ce = 58, Eu = 63)

a. $[Xe]4f^7, [Xe]4f^1$
b. $[Xe]4f^7 6s^2, [Xe]4f^1$
c. $[Xe]4f^7 6s^2, [Xe]4f^1 5d^1 6s^2$
d. $[Xe]4f^7, [Xe]4f^1\ 5d^1 6s^2$

Answer: a

Solution:

Ce (58): $[Xe]\ 6s^2 4f^2$

(g.s)

$Ce^{3+}: [Xe]4f^1$

$Eu(63) : [Xe]6s^2 4f^7$

(g.s)

$Eu^{2+} : [Xe]4f^7$

7. K_{sp} of $PbCl_2 = 1.6 \times 10^{-5}$. On mixing 300 mL, 0.134 M $Pb(NO_3)_2(aq.)$ and 100 mL, 0.4 M $NaCl(aq.)$:

a. $Q > K_{sp}$
b. $Q < K_{sp}$
c. $Q = K_{sp}$
d. Relation does not exist

Answer: a

Solution: Given K_{sp} of $PbCl_2 = 1.6 \times 10^{-5}$

$Pb(NO_3)_2$: mmoles= 300 mL × 0.134 M = 40.2

NaCl: mmoles = 100 mL × 0.4 M = 40

This implies, $[Pb]^{2+} = \frac{40.2}{400} \approx 0.1$ M

$$[Cl]^- = \frac{40}{400} = 0.1 \text{ M}$$

$$Q_{sp} = [Pb^{2+}][2Cl^-]^2 = 4 \times 10^{-3} > K_{sp}$$

8. Which of the following cannot act as both oxidising and reducing agent?

a. H_2SO_3
b. HNO_2
c. H_3PO_4
d. H_2O_2

Answer: c

Solution:

When the oxidation state is maximum it acts like a strong oxidising agent

When the oxidation state is minimum it acts like a strong reducing agent

When the oxidation state is between its maximum and minimum, it acts like both an oxidizing and as a reducing agent

In H_3PO_4, P has a +5 oxidation state and hence can act like a strong oxidising agent. In the rest, the oxidation state is between their maximum and minimum.

9. The first Ionisation energy of Be is higher than that of Boron. Select the correct statements regarding this:

(i) It is easier to extract electron from 2p orbital than 2s orbital
(ii) Penetration power of 2s orbital is greater than 2p orbital
(iii) Shielding of 2p electron by 2s electron
(iv) Radius of Boron atom is larger than that of Be

a. (i), (ii), (iii), (iv)
b. (i), (iii), (iv)
c. (ii), (iii), (iv)
d. (i), (ii), (iii)

Answer: d

Solution:

Be (4): $1s^2 2s^2$

B (5): $1s^2 2s^2 2p^1$

The electron in $2p^1$ can easily be extracted.

The penetrating power is of the order: s > p > d > f

The shielding power order: s > p > d > f

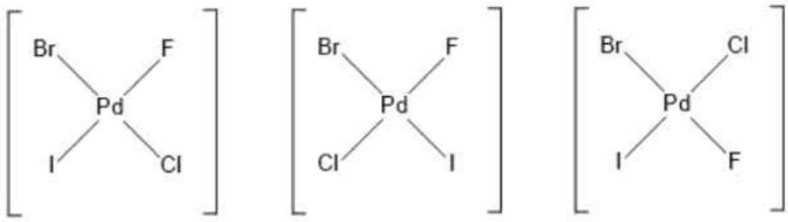
As we move along the period, the size decreases, as Z_{eff} increases. Hence the radius of B is smaller than the radius of Be.

10. For $[PdFClBrI]^{2-}$, the number of geometrical Isomers = n. Determine the spin only magnetic moment and CFSE for $[Fe(CN)_6]^{n-6}$ (Ignore pairing energy).

a. 1.73 B. M., $-2\Delta_0$
b. 2.84 B. M., $-1.6\Delta_0$
c. 0, $-1.6\Delta_0$
d. 5.92 B. M., $-2.4\Delta_0$

Answer: a

Solution:



Number of geometrical isomers (n) = 3

$[Fe(CN)_6]^{n-6} = [Fe(CN)_6]^{3-6} = [Fe(CN)_6]^{-3}$

This implies, that Iron is in its +3 oxidation state.

$Fe^{3+}(26)$: $[Ar]3d^5$

CN^- is a strong ligand in $[Fe(CN)_6]^{-3}$ and causes pairing. Hence, according to CFT, the configuration will be $t_{2g}^5\, e_g^0$.

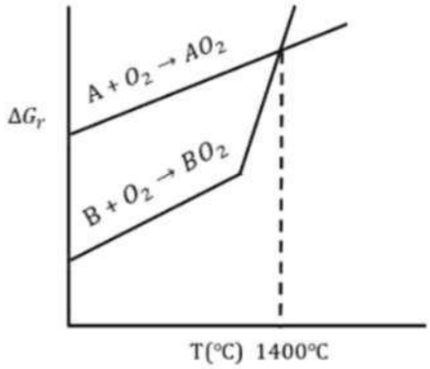
Hence, there is only 1 unpaired electron, i.e, n=1 in $\sqrt{n(n+2)} = \sqrt{3}$ = 1.73 B.M

CFSE = $(-0.4 \times n_{t2g} + 0.6 \times n_{eg})\Delta_0$

$= (-0.4 \times 5 + 0.6 \times 0)\Delta_0$

$= -2\Delta_0$

11. A can reduce BO_2 under which conditions?



a. $> 1400\ °C$

b. $< 1400\ °C$

c. $> 1200\ °C$

d. $< 1200\ °C$

Answer: a

Solution: In Ellingham's diagram, the line of the element that lies below can reduce the oxide of the element which lies above it. Therefore, for A to reduce BO_2, the temperature when the line for element A is below that of BO_2, according to the graph when $T > 1400\ °C$.

For $T > 1400\ °C$, $\Delta G_r < 0$ for $A + BO_2 \rightarrow B + AO_2$

12. $A \rightarrow B$; 700 K

$A \overset{C}{\rightarrow} B$; 500 K

Rate of the reaction in absence of catalyst at 700 K is same as in presence of catalyst at 500 K. If catalyst decreases the activation energy barrier by 30 kJ/mol, determine the activation energy in presence of catalyst. (Assume 'A' factor to be same in both cases)

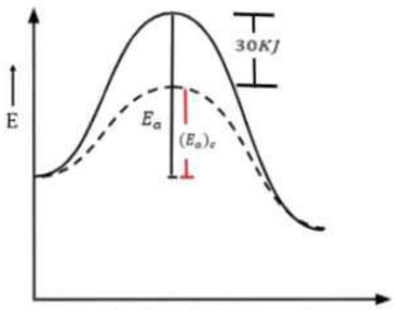
a. 75 kJ
b. 135 kJ
c. 105 kJ
d. 125 kJ

Answer: c

Solution:

$K = Ae^{\left(-\frac{E_a}{RT}\right)}$

$K_{\text{catalyst}} = K_{\text{without catalyst}}$



$$Ae^{\left(-\frac{(E_a)c}{RT_{500k}}\right)} = Ae^{\left(-\frac{(E_a)}{RT_{700k}}\right)}$$

$$e^{\left(-\frac{(E_a)c}{RT_{500k}}\right)} = e^{\left(-\frac{(E_a)}{RT_{700k}}\right)}$$

$$-\frac{(E_a)\,c}{RT_{500k}} = -\frac{(E_a)}{RT_{700k}}$$

$$(E_a)c = E_a - 30$$

$$-\frac{(E_a - 30)}{T_{500k}} = -\frac{(E_a)}{T_{700k}}$$

On Solving, $E_a = 105 \text{ kJmol}^{-1}$

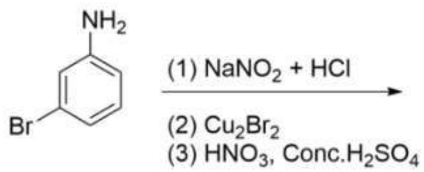
13. A substance 'X' having low melting point, does not conduct electricity in both solid and liquid state. 'X' can be :

a. Hg
b. SiC
c. ZnS
d. CCl_4

Answer: d

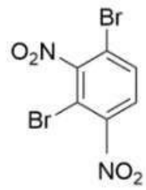
Solution: CCl_4 is non polar and does not conduct in either solid or liquid state.

14.

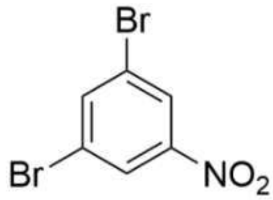


The major product for above sequence of reaction is:

a. 

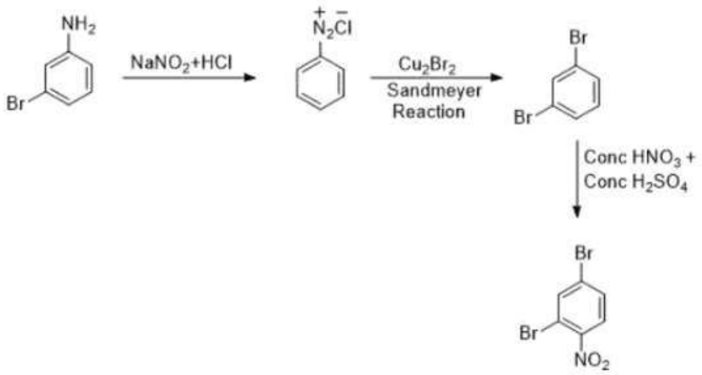
b. 

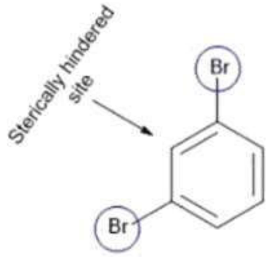
c. 

d. 

Answer: c

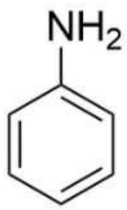
Solution:

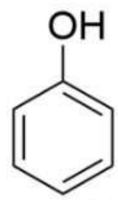


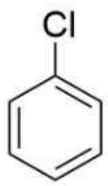


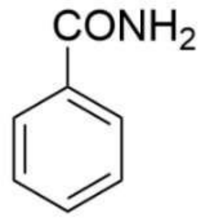
Hence, major product formed is that of option b.

15. Which of the following can give the highest yield in Friedel-Craft's reaction?

a. 

b. 

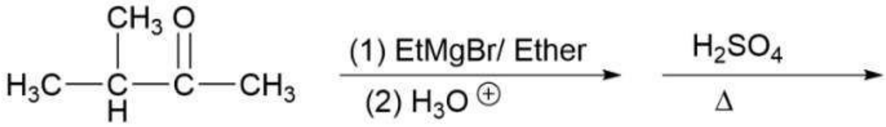
c. 

d. 

Answer: b

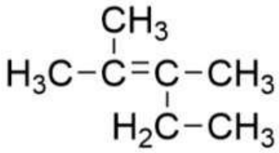
Solution: Out of the four options given, only aniline and phenol show strong +R effects, but as we know, aniline is a Lewis base and can react with a Lewis acid that is added during the reaction. Hence, Phenol gives the highest yield in Friedel-Craft's reaction.

16.

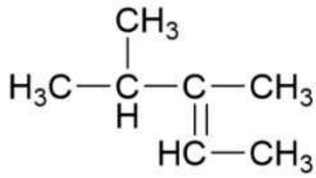


What will be the major product?

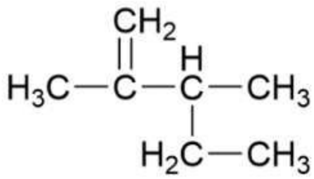
a.



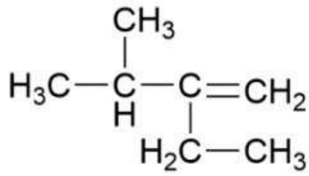
c.



b.

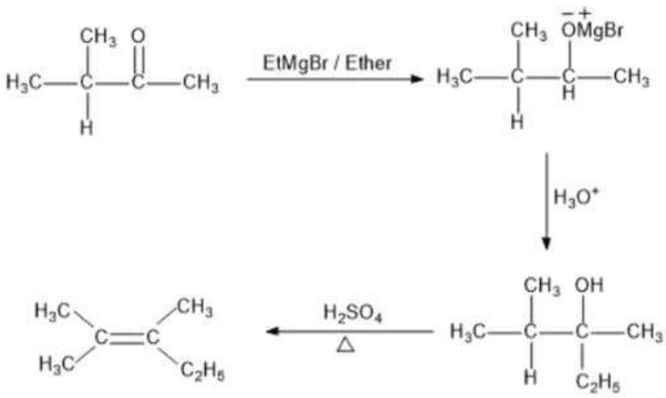


d.

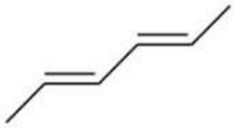
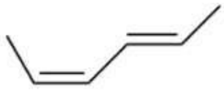


Answer: a

Solution:



17. Which of the following is the correct order for heat of combustion?

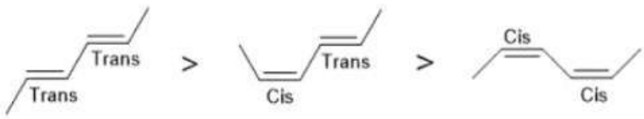
(A)  (B)  (C) 

a. C > B > A
b. B > A > C
c. A > B > C
d. C > A > B

Answer: c

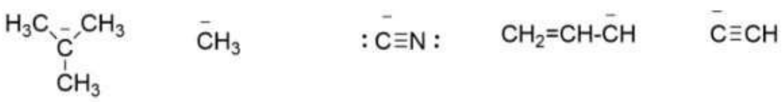
Solution: Heat of combustion $\alpha \frac{1}{\text{stability}}$

The trans-isomer is more stable than the cis-isomer. More the number of trans forms in a structure, higher the stability.



(A) (B) (C)

18. Write the correct order of basicity:



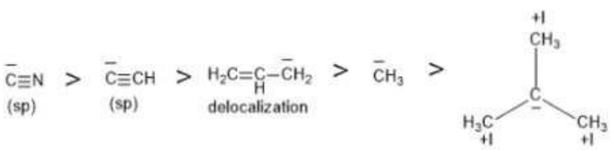
(A) (B) (C) (D) (E)

a. A > B > D > E > C
b. B > A > D > C > E
c. A > B > E > D > C
c. C > E > D > B > A

Answer: a

Solution: As we know weaker the conjugate base, stronger the acid.

The order of stability of conjugate base:



C E D B A

Hence, the order of basicity or acidic strength is:

A > B > D> E> C

19. A, B, C, and D are four artificial sweetners.

(i) A & D give positive test with ninhydrin.

(ii) C form precipitate with AgNO3 in the lassaigne extract of the sugar.

(iii) B & D give positive test with sodium nitroprusside.

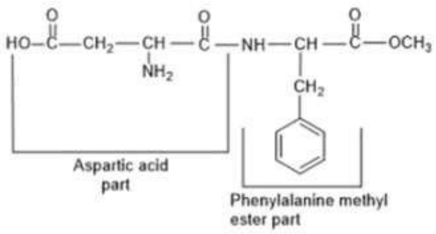
Correct option is :

a. A – Saccharine, B – Aspartame, C – Sucralose, D – Alitame
b. A – Aspartame, B – Saccharine, C – Sucralose, D – Alitame
c. A – Saccharine, B – Aspartame, C – Alitame , D – Sucralose
d. A – Aspartame, B – Sucralose, C – Saccharine, D – Alitame

Answer: b

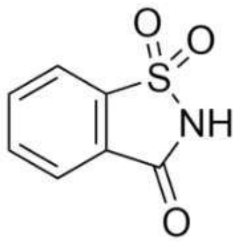
Solution:

A - Aspartame



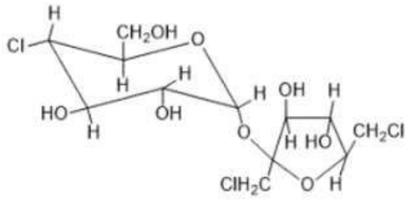
It has a free amine group and hence reacts with ninhydrin to give a purple colour known as Ruhemann's purple.

B- Saccharine



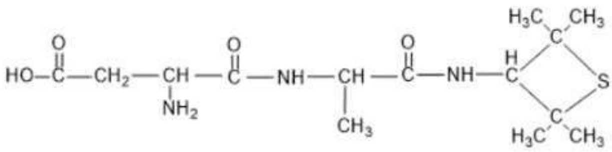
It has Sulphur, therefore, it will give a positive test with sodium nitroprusside.

C - Sucralose



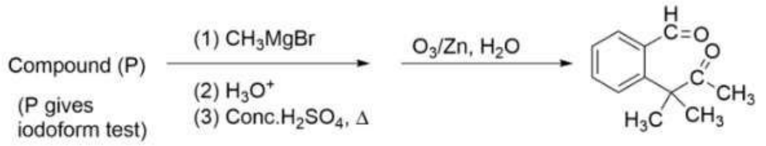
It has chlorine and hence it forms a precipitate with $AgNO_3$ in the Lassaigne's extract of the sugar.

D - Alitame

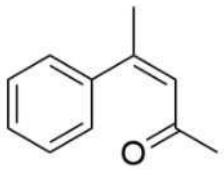


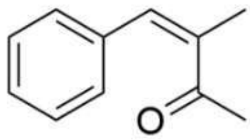
It has a free amine group and hence reacts with ninhydrin to give purple colour known as Ruhemann's purple. Also, it has Sulphur, therefore, it will give positive test with sodium nitroprusside.

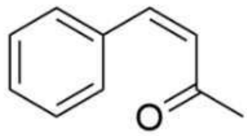
20.

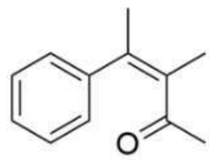


Predict the compound (P) on the basis of above sequence of the reactions, where compound (P) gives positive Iodoform test:

a. 

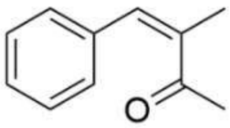
b. 

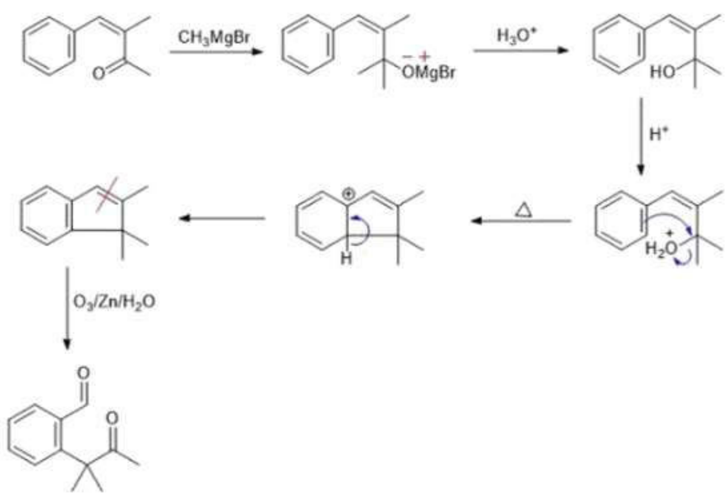
c. 

d. 

Answer: b

Solution:

 is a methyl ketone, which gives positive Iodoform test.



21. Given a solution of HNO_3 of density 1.4 g/mL and 63% $\frac{w}{w}$. Determine molarity of HNO_3 solution.

Answer: 14.00

Solution: $\%\frac{w}{w} = 63\%$

$\rho = 1.4$ g/mL

$$M = \frac{\left(\%\frac{w}{w} \times \rho \times 10\right)}{MM}$$

$$M = \frac{(63 \times 1.4 \times 10)}{63}$$

M= 14 mol/L

22. Determine the degree of hardness in terms of ppm of $CaCO_3$ of 10^{-3} molar $MgSO_4$ (aq).

Answer: 100.00

Solution:

Hardness of water is measured in ppm in terms $CaCO_3$.

$n_{CaCO_3} = n_{MgSO_4}$

ppm is the parts (in grams) present per million i.e, 10^6

1000 mL has 10^{-3} moles of $MgSO_4$.

Grams of $CaCO_3$ in 1000 mL = $10^{-3} \times 100$ grams

Grams of $CaCO_3$ in 1 mL = $\frac{10^{-3} \times 100}{1000 \text{ mL}}$ grams

Hardness = $\frac{10^{-3} \times 100}{1000 \text{ mL}} \times 10^6 = 100$

23. Determine the amount of NaCl to be dissolved in 600 g of H_2O to decrease the freezing point by 0.2°C. Given : k_f of $H_2O = 2 \text{ Km}^{-1}$

Answer: 1.76

Solution: NaCl is strong electrolyte and gives 2 ions in the solution. This implies, i=2.

Molatility= $\frac{w \times 1000}{58.5 \times 600}$

$\Delta T_f = 0.2°C$

$\Delta T_f = i \times k_f \times m$

On solving we get,

w= 1.76grams

24. On passing a particular amount of electricity in $AgNO_3$ solution, 108 g of Ag is deposited. What will be the volume of $O_2(g)$ in litres liberated at 1 bar, 273 K by the same quantity of electricity?

Answer: 5.68

Solution: On applying Faraday's 1st law,

Moles of Ag deposited= 108/108= 1 mol.

$Ag^+ + e^- \rightarrow Ag$

1Faraday is required to deposit 1 mole of Ag.

$H_2O \rightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$

$\frac{1}{2}$ moles of O_2 are deposited by 2 F of charge.
This implies, 1 F will deposit $\frac{1}{4}$ moles of O_2
Using PV=nRT
P= 1 bar
T= 273 K
R= 0.0823 Lbar $mol^{-1}K^{-1}$
On solving we get,
V = 5.68 L

25. Find percentage nitrogen by mass in Histamine?

Answer: 37.84

Solution:

Molecular mass of Histamine= 111

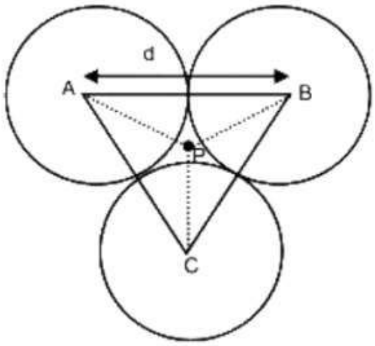
In Histamine, 3 Nitrogens are present (42g)

The percentage of Nitrogen by mass in Histamine = $\frac{42}{111} \times 100 = 37.84\%$

JEE Main 2020 Paper

Date of Exam: **9th January (Shift I)**
Time: **9:30 am – 12:30 pm**
Subject: **Physics**

1. Three identical solid spheres each have mass 'm' and diameter 'd' are touching each as shown in the figure. Calculate ratio of moment of inertia about the axis perpendicular to plane of paper and passing through point P and B as shown in the figure. Given P is centroid of the triangle



a. $\frac{13}{23}$
b. $\frac{11}{19}$
c. $\frac{7}{9}$
d. $\frac{13}{11}$

Solution: (a)

Moment of Inertia of solid sphere $= \frac{2}{5}M\left(\frac{d}{2}\right)^2$

Distance of centroid (*Point P*) from centre of sphere $= \left(\frac{2}{3} \times \frac{\sqrt{3}d}{2}\right) = \frac{d}{\sqrt{3}}$

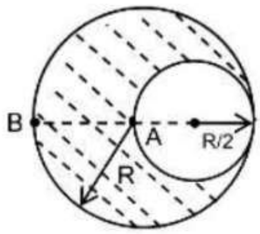
By Parallel axis theorem,

Moment of Inertia about $P = 3\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}M\,d^2$

Moment of Inertia about $B = 2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M(d)^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2 = \frac{23}{10}M\,d^2$

Now ratio $= \frac{13}{23}$

2. A sold sphere having a radius R and uniform charge density ρ has a radius R/2 as shown in the figure. Find the ratio of the magnitude of electric field at point A and B



a. $\frac{18}{19}$
b. $\frac{11}{17}$
c. $\frac{9}{17}$
d. $\frac{9}{91}$

Solution: (c)

For solid sphere,

Field inside sphere, E = $\frac{\rho r}{3\epsilon_0}$ & Field outside sphere, E = $\frac{\rho R^3}{3r^2\epsilon_0}$ where, r is distance from centre and R is radius of sphere

Electric field at A due to sphere of radius R (sphere 1) is zero and therefore, net electric field will be because of sphere of radius $\frac{R}{2}$ (sphere 2) having charge density $(-\rho)$

$$E_A = \frac{-\rho R}{2(3\epsilon_0)}$$

$$|E_A| = \frac{\rho R}{6\varepsilon_0}$$

Similarly, Electric field at point $B = E_B = E_{1B} + E_{2B}$

E_{1B} = Electric Field Due to solid sphere of radius $R = \frac{\rho R}{3\varepsilon_0}$

E_{2B} = Electric Field Due to solid sphere of radius $\frac{R}{2}$ which having charge density $(-\rho)$

$$= -\frac{\rho\left(\frac{R}{2}\right)^3}{3\left(\frac{3R}{2}\right)^2\varepsilon_0} = -\frac{\rho R}{54\varepsilon_0}$$

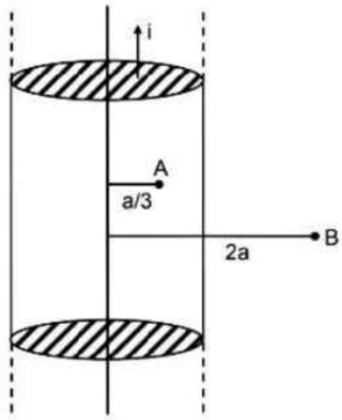
$$E_B = E_{1A} + E_{2A} = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17\rho R}{54\varepsilon_0}$$

$$\frac{|E_A|}{|E_B|} = \frac{9}{17}$$

3. Consider an infinitely long current carrying cylindrical straight wire having radius 'a'. Then the ratio of magnetic field at distance a/3 and 2a from axis of wire is.

a. 3/5
b. 2/3
c. 1/2
d. 4/3

Solution: (b)

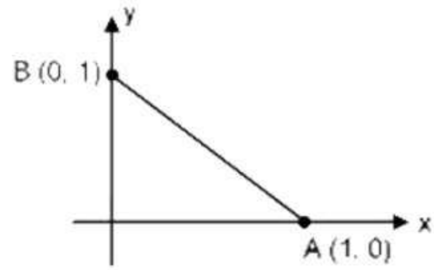


$$B_A = \frac{\mu_0 i r}{2\pi a^2} = \frac{\frac{\mu_0 i a}{3}}{2\pi a^2} = \frac{\mu_0 i}{\pi a^2}\frac{a}{6} = \frac{\mu_0 i}{6\pi a}$$

$$B_B = \frac{\mu_0 i (2a)^2}{2\pi(2a)} = \frac{\mu_0 i}{4\pi a}$$

$$\frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$$

4. Particle moves from point A to point B along the line shown in figure under the action of force. $\bar{F} = -x\hat{\imath} + y\hat{\jmath}$. Determine the work done on the particle by $\bar{F}$ in moving the particle from point A to point B



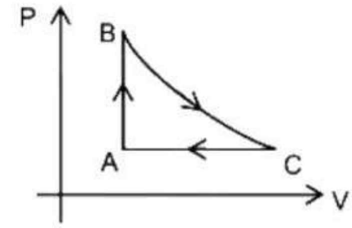
a. 1 J
b. 1/2 J
c. 2 J
d. 3 J

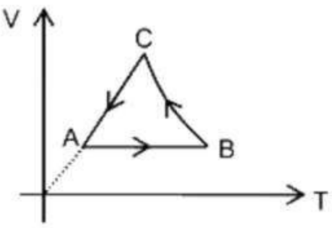
Solution: (a)

$$d\vec{s} = (d x\,\hat{\imath} + d y\,\hat{\jmath})$$
$$= (-x\hat{\imath} + y\hat{\jmath}).(d x\,\hat{\imath} + d y\,\hat{\jmath})$$
$$= \int_1^0 -x\,dx + \int_0^1 y dy$$

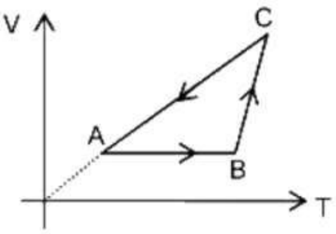
$$= -\frac{x^2}{2}\Big|_1^0 + \frac{y^2}{2}\Big|_0^1 = \frac{1}{2} + \frac{1}{2} = 1\,J$$

5. For the given P -V graph for an ideal gas, chose the correct V -T graph. Process $B\ C$ is adiabatic.

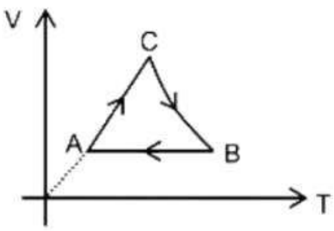




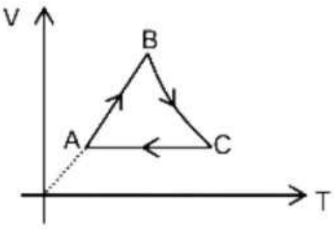
a.



b.



c.



d.

Solution: (a)
For process A−B; Volume is constant;
For process $B - C$, PV^{γ} = Constant & $PV = nRT$, Therefore $TV^{\gamma-1}$ = Constant ;
Therefore as V increases T increases.
For process $C - A$; pressure is constant, Therefore $V = k\,T$
From above, correct answer is option 1.

6. Given, Electric field at point P, $\vec{p} = -\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and $\hat{r} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$. Find vector parallel to electric field at position $\vec{r}$. [Note that $\vec{p}\,.\vec{r} = 0$]

a. $\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$
b. $3\hat{\imath} + \hat{\jmath} + 2\hat{k}$
c. $-3\hat{\imath} - \hat{\jmath} - 2\hat{k}$
d. $-\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

Solution: (a)

Since, we have to find vector parallel to electric field at position $\vec{r}$

We have to find $\vec{p}\,.\vec{r} = 0$

Since already in question, $\vec{p}\,.\vec{r} = 0$ is given we need to find E such that

$\vec{E} = \lambda\,(\vec{p})$

where λ is a arbitrary positive constant

On putting , λ = -1 , we get, $\vec{E} = \hat{\imath} + 3\,\hat{\jmath} - 2\,\hat{k}$

7. A particle of mass m is revolving around a planet in a circular orbit of radius R. At the instant the particle has velocity $\vec{V}$, another particle of mass $\frac{m}{2}$ moving at velocity of $\frac{\vec{v}}{2}$ in same direction collides perfectly in-elastically with the first particle. The new path of the combined body will take is

a. Elliptical

b. Circular

c. Straight Line

d. Spiral

Solution: (a)

By conservation of linear momentum

$\frac{m}{2}\frac{V}{2} + mV = (m + \frac{m}{2})V_f$

$V_f = \frac{5V}{6}$

Escape velocity will be at $\sqrt{2}V$ and at velocity less than escape velocity path will be elleptical or part of ellipse except for velocity V where path will be circular.

Hence the resultant mass will go on to an elliptical path

8. Two particles of same mass m moving with velocities $\vec{v_1} = v\,\hat{\imath}$ and and $\vec{v_1} = \frac{v}{2}\hat{\imath} + \frac{v}{2}\hat{\jmath}$ collide in – elastically. Find the loss in kinetic energy.

a. $\frac{mv^2}{8}$

b. $\frac{1mv^2}{8}$

c. $\frac{9mv^2}{8}$

d. $\frac{3mv^2}{8}$

Solution: (a)

Conserving linear momentum

$mv\,\hat{\imath} + m(\frac{v}{2}\hat{\imath} + \frac{v}{2}\hat{\jmath}) = 2m(v_1\,\hat{\imath} + v_2\,\hat{\jmath})$

By equating $\hat{\imath}$ and $\hat{\jmath}$

$v_1 = \frac{3v}{4}$ and $v_2 = \frac{v}{4}$

Initial K.E $= \frac{mv^2}{2} + \frac{m}{2} \times (\frac{v}{\sqrt{2}})^2 = \frac{3mv^2}{4}$

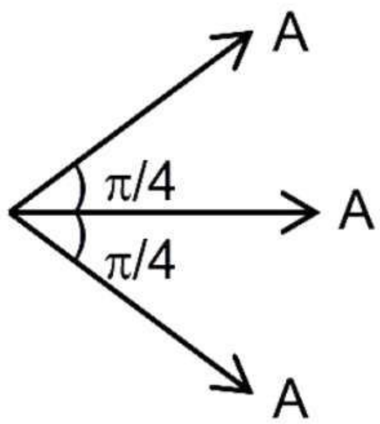
Final K.E $= \frac{2m}{2} \times (\frac{v\sqrt{10}}{4})^2 = \frac{mv^2}{8}$

Change in KE =
$\frac{3mv^2}{4} - \frac{5mv^2}{8} = \frac{mv^2}{8}$

9. Three waves of same intensity (I_0) having initial phases $0, \frac{\pi}{4}, -\frac{\pi}{4}\ rad$ respectively interfere at a point. Find the resultant intensity.

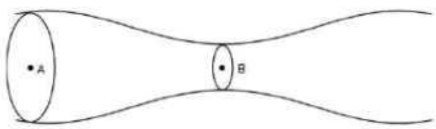
a. $5.8\, I_0$
b. I_0
c. $0.4\, I_0$
d. $0.3\, I_0$

Solution: (a)



Amplitudes can be vectorially added
$A_{resultant} = (\sqrt{2} + 1)A$
Since, $I \propto A^2$
Therefore, $I_{res} = (\sqrt{2} + 1)^2 I_0 = 5.8\, I_0$

10. An ideal liquid (water) flowing through a tube of non-uniform cross section area at A and B are 40 cm^2 and 20 cm^2 respectively. If pressure difference between A & B is 700 N/m^2 then volume flow rate is



a. 2732 cm^3/s
b. 2142 cm^3/s
c. 1832 cm^3/s
d. 3218 cm^3/s

Solution: (a)

Using equation of continuity

$V_A \times Area_A = V_B \times Area_B$

$40V_A = 20V_B$

$2V_A = V_B$

Using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$\Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$$

$$\Delta P = 500 \times \frac{3V_B^2}{4}$$

$$V_B = \sqrt{\frac{(\Delta P) \times 4}{1500}} = \sqrt{\frac{(700) \times 4}{1500}} = \sqrt{\frac{28}{15}}\,\text{m/s}$$

Volume flow rate $= V_B \times Area_B = 20 \times 100 \times \sqrt{\frac{28}{15}}\,\text{cm}^3/\text{s} = 2732\,\text{cm}^3/\text{s}$

11. A screw gauge advances by 3mm in 6 rotations. There are 50 divisions on circular scale. Find least count of screw gauge?

a. 0.002 cm
b. 0.001 cm
c. 0.01 cm
d. 0.02 cm

Solution: (b)

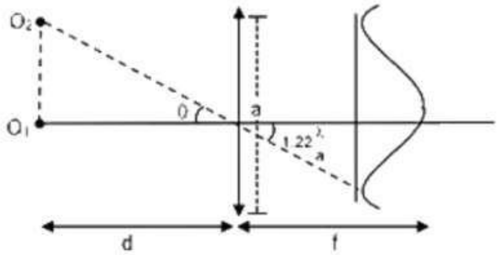
Pitch $= \frac{3}{6} = 0.5$ mm

Least count $= \frac{\text{Pitch}}{\textit{Number of division}} = \frac{0.5\text{mm}}{50} = \frac{1}{100}\text{mm} = 0.01\text{ mm} = 0.001\text{ cm}$

12. A telescope of aperture diameter 5m is used to observe the moon from the earth. Distance between the moon and earth is 4×10^5 km. Determine the minimum distance between two points on the moon's surface which can be resolved using this telescope. (Wave length of light is 5893 Å)

a. 60 m
b. 20 m
c. 600 m
d. 200 m

Solution: (a)



Minimum angle for clear resolution,

$$\theta = 1.22\frac{\lambda}{a}$$

$$distance = O_1O_2 = d\theta$$

$$= 1.22\frac{\lambda}{a}d$$

$$distance = O_1O_2 = \frac{1.22 \times 5893 \times 10^{-10} \times 4 \times 10^{8}}{5} \approx 57.52 \text{ m}$$

$\therefore$ *Nearest option is* 60 m

13. Photons of wavelength 6556 Å falls on a metal surface. If ejected electrons with maximum K.E moves in magnetic field of 3×10^{-4} T in circular orbit of radius 10^{-2}m, then work function of metal surface is

a. 1.8 eV
b. 0.8 eV
c. 1.1 eV
d. 1.4 eV

Solution: (c)

From photoelectric equation,

$$\frac{hc}{\lambda} = W + K.E_{max}$$

Where, hc = 12400 eV Å

$$\Rightarrow \frac{12400}{6556} = W + K.E_{max}$$

$$\Rightarrow 1.9 \text{ eV} = W + K.E_{max} - - - - - (1)$$

Radius of charged particle moving in a magnetic field is given by

$r = \frac{mv}{qB}$ where, $\frac{1}{2}mv^2 = K.E_{max} = eV$

$$\Rightarrow r = \frac{\sqrt{\frac{2eV}{m}} \times m}{eB} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

$$\Rightarrow 10^{-2} = \frac{1}{3 \times 10^{-4}}\sqrt{\frac{2 \times 9.1 \times 10^{-31} \times V}{1.6 \times 10^{-19}}}$$

$\Rightarrow V = 0.8\ V$

So, $K.E_{max} = 0.8\ eV$

Substituting in (1),

$1.9 = W + 0.8$

i.e. $W = 1.1\ eV$

14. Kinetic energy of the particle is E and it's De–Broglie wavelength is λ. On increasing it's K.E by ΔE, it's new De–Broglie wavelength becomes $\frac{\lambda}{2}$. Then ΔE is

a. 3E
b. E
c. 2E
d. 4E

Solution: (a)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m(KE)}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\frac{\lambda}{\lambda/2} = \sqrt{\frac{KE_f}{KE_i}}$$

$$4KE_i = KE_f$$

$$\Rightarrow \Delta E = KE_f - KE_i = 4KE_i - KE_i = 3KE_i = 3E$$

15. The dimensional formula of $\sqrt{\frac{hc^5}{G}}$ is

a. $[ML^2T^{-3}]$
b. $[ML^2T^{-2}]$
c. $[ML^{-2}T^2]$
d. $[MLT^{-2}]$

Solution: (b)

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow hc = E\lambda$$

$$Since, [E] = [ML^2T^{-2}]$$

Therefore,

$$[hc] = [ML^3T^{-2}]$$
$$[c] = [LT^{-1}]$$
$$[G] = [[M^{-1}L^3T^{-2}]$$
$$\left[\sqrt{\frac{hc^5}{G}}\right] = [ML^2T^{-2}]$$

16. Two immiscible liquids of refractive index $\sqrt{2}$ and $2\sqrt{2}$ are filled with equal height h in a vessel. Then apparent depth of bottom surface of the container given that outside medium is air

a. $\frac{3\sqrt{2}h}{4}$

b. $\frac{3h}{4}$

c. $\frac{3h}{2}$

d. $\frac{3h}{4\sqrt{2}}$

Solution: (a)

Apparent height as seen from liquid 1 (having refractive index $\mu_1 = \sqrt{2}$) to liquid 2 (refractive index $\mu_2 = 2\sqrt{2}$)

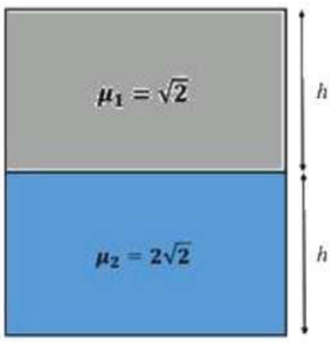
$D = \frac{h\mu_1}{\mu_2} = \frac{h}{2}$

Now, Actual height perceived from air, $h + \frac{h}{2} = \frac{3h}{2}$

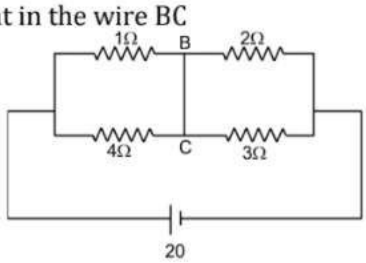
Therefore, apparent depth of bottom surface of the container (apparent depth as seen from air (having refractive index $\mu_0 = 1$) to liquid 1(having refractive index $\mu_1 = \sqrt{2}$)

$= \frac{3h}{2} \times \frac{\mu_0}{\mu_1}$

$= \frac{3h}{2} \times \frac{1}{\sqrt{2}} = \frac{3h}{2\sqrt{2}} = \frac{3\sqrt{2}h}{4}$



17. Find the current in the wire BC



a. 1.6 A

b. 2 A

c. 2.4 A

d. 3 A

Solution: (b)

Since resistance 1 Ω and 4 Ω are in parallel

$\therefore R' = \frac{4\times1}{4+1} = \frac{4}{5}$

Similarly we can find equivalent resistance (R'') for resistances 2 Ω and 3 Ω

$\Rightarrow R'' = \frac{6}{5}$

And R' and R'' are in series

$\therefore\ R_{eff} = \frac{4}{5} + \frac{6}{5} = 2\ \Omega$

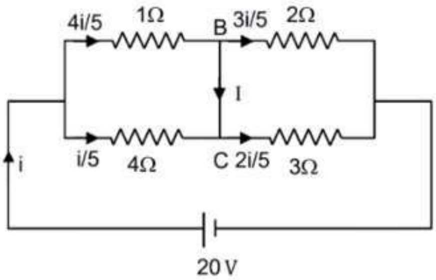
So total current flowing in the circuit 'i' can be given as

$$i = \frac{V}{R_{eff}} = \frac{20}{2} = 10\ A$$

Current will distribute in ratio opposite to resistance.

So, distribution will be as

So current in the branch BC will be



$$I = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = \frac{10}{5} = 2\ A$$

$$I=\frac{4i}{5}-\frac{3i}{5}=\frac{i}{5}=\frac{10}{5}=2\ A$$

18. Two electromagnetic waves are moving in free space in x and y direction respectively whose electric field vectors are given by $\vec{E}_1=E_o\hat{j}\cos(kx-\omega t)$ and $\vec{E}_2=E_o\hat{k}\cos(ky-\omega t)$. A charge q is moving with velocity $\vec{v}=0.8c\,\hat{j}$. Find the net Lorentz force on this charge at t = 0 and when it is at origin.

a. $qE_o(0.4\,\hat{\imath}+0.2\,\hat{\jmath}+0.2\,\hat{k})$
b. $qE_o(0.8\,\hat{\imath}+\hat{\jmath}+0.2\,\hat{k})$
c. $qE_o(0.6\,\hat{\imath}+\hat{\jmath}+0.2\,\hat{k})$
d. $qE_o(0.8\,\hat{\imath}+\hat{\jmath}+\hat{k})$

Solution: (b)

Given that the magnetic field vectors are:

$$\vec{E}_1=E_o\hat{\jmath}\cos(kx-\omega t)$$
$$\vec{E}_2=E_o\hat{k}\cos(ky-\omega t)$$

So the magnetic field vectors of the electromagnetic wave are given by

$$\vec{B}_1=\frac{E_o}{c}\hat{k}\cos(kx-\omega t)$$
$$\vec{B}_2=\frac{E_o}{c}\hat{\imath}\cos(ky-\omega t)$$

Then force is

$\vec{F}=q\vec{E}+q(\vec{v}\times\vec{B})$

$\quad=q(\vec{E}_1+\vec{E}_2)+q(\vec{v}\times(\vec{B}_1+\vec{B}_2)$

Now if we put the values of $\vec{E}_1,\vec{E}_2,\vec{B}_1 and\ \vec{B}_2$ we can get the net Lorentz force as

$\vec{F}=q\vec{E}+q(\vec{v}\times\vec{B})$

Putting values and solving we get

$\vec{F}=qE_o[\cos(kx-\omega t)\,\hat{\jmath}+\cos(ky-\omega t)\,\hat{k}+0.8\cos(kx-\omega t)\,\hat{\imath}-0.8(\cos ky-\omega t)\,\hat{k}]$

$\vec{F}=qE_o[0.8\cos(kx-\omega t)\,\hat{\imath}+\cos(kx-\omega t)\,\hat{\jmath}+0.2(\cos ky-\omega t)\,\hat{k}]$

Now at $t=0$ and $x=y=0$ we get

$\vec{F}=(0.8\,\hat{\imath}+\hat{\jmath}+0.2\,\hat{k})$

19. Two ideal di-atomic gases A and B. A is rigid, B has an extra degree of freedom due to vibration. Mass of A is m and mass of B is $\frac{m}{4}$. The ratio of molar specific heat of A to B at constant volume is

a. 7/9
b. 5/9
c. 5/11
d. 5/7

Solution: (d)

We know that,

Molar heat capacity at constant volume, $C_V = \frac{fR}{2}$ (Where f is degree of freedom)

Since, A is diatomic and rigid, degree of freedom for A is 5

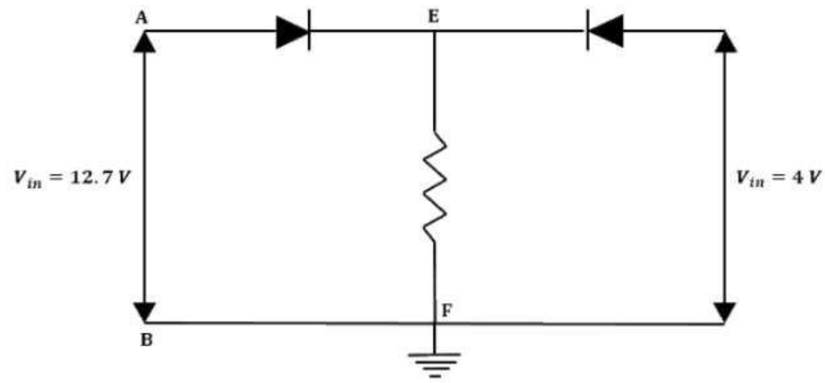
Therefore, Molar heat capacity of A at constant volume$(C_V)_A = \frac{5R}{2}$

Since, B is diatomic and have extra degree of freedom because of vibration, degree of freedom for B is $5 + 2\times 1 = 7$ (1 vibration for each atom).

Therefore, Molar heat capacity of B at constant volume$(C_V)_B = \frac{7R}{2}$

Ratio of molar specific heat of A and B= $\frac{(C_V)_A}{(C_V)_B} = \frac{5}{7}$

20. In the given circuit both diodes are ideal having zero forward resistance and built-in potential of 0.7 V. Find the potential of point E in volts

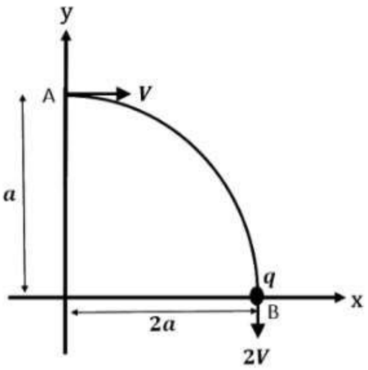


Solution: (12)

By applying Kirchhoff's Voltage Law in the loop ACBFA

$$12.7 - 0.7 - V_{EF} = 0$$
$$\Rightarrow V_{EF} = 12\ V$$
$$\Rightarrow V_E = 12\ V$$

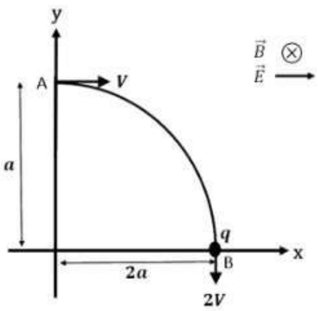
21. A particle having mass m and charge q is moving in a region as shown in figure. This region contains a uniform magnetic field directed into the plane of the figure, and a uniform electric field directed along positive x − axis. Which of the following statements are correct for moving charge as shown in figure?



A. Magnitude of electric field $\vec{E} = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$
B. Rate of change of work done at a point A is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$
C. Rate of change of work done by both fields at point B is zero
D. Change in angular momentum about the origin is $2mva$

a. A, B and C are correct
b. A, B, C and D are correct
c. A and B are correct
d. B, C and D are correct

Solution: (a)



Considering statement A
By Work-Energy theorem

$$W_{mag} + W_{ele} = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$
$$\Rightarrow \quad 0 + qE_o 2a = \frac{3}{2}mv^2$$
$$E_o = \frac{3}{4}\frac{mv^2}{qa}$$

So statement A is correct
Now considering statement B
Rate of change of work done at A = Power of electric force

$$= 9E_o v$$
$$= \frac{3}{4}\frac{mv^3}{a}$$

So statement B is correct
Coming to statement C
At B,
$\vec{E} \perp \vec{v}$
So, $\frac{dw}{dt} = 0$ for both forces
Coming to statement D.
Change in angular momentum about the origin is
$\Delta\vec{L} = \Delta\overrightarrow{L_B} - \Delta\overrightarrow{L_A}$
$\overrightarrow{L_B} = m(2v)(2a)$
$\overrightarrow{L_A} = m(v)(a)$
$Hence, \Delta L = 3mva$

22. If reversible voltage of 200 V is applied across an inductor, current in it reduces from 0.25A to 0A in 0.025ms. Find inductance of inductor (in mH).
Solution: (20)

By using KVL,

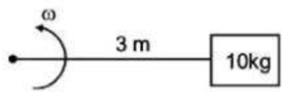
$$V - L\frac{dI}{dt} = 0$$

$$\Rightarrow 200 = \frac{L(0.25)}{0.025} \times 10^3$$

$$L = 200 \times 10^{-4}\text{ H}$$
$$= 20\text{ mH}$$

23. A wire of length l = 3 m and area of cross section 10^{-2}cm^2 and breaking stress 4.8×10^8 N/m^2 is attached with block of mass 10 kg. Find the maximum possible value of angular velocity (rad/s) with which block can be moved in circle with string fixed at one end.

Solution: (4)



Breaking stress

$$\sigma = \frac{T}{A}$$

$$T = m\omega^2 l$$

$$\Rightarrow \sigma = \frac{m\omega^2 l}{A}$$

$$\Rightarrow \omega^2 = \frac{\sigma A}{ml} = \frac{4.8\times10^8\times10^{-6}}{10\times3} = 16$$

$$\Rightarrow \omega = 4\, rad/s$$

24. Position of a particle as a function of time is given as $x^2 = at^2 + 2bt + c$, where a, b, c are constants. Acceleration of particle varies with x^{-n} then value of n is

Solution: (3)

Let, v be velocity, α be the acceleration then,

$$x^2 = at^2 + 2bt + c$$

$$2\,x\,v = 2\,a\,t + 2\,b$$

$$x\,v = a\,t + b \qquad \_\_\_\_(1)$$

$$\Rightarrow v = \frac{at+b}{x}$$

Now, differentiating equation (1),

$$v^2 + \alpha x = a$$

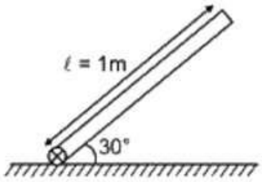
$$\alpha x = a - \left(\frac{at+b}{x}\right)^2$$

$$\alpha = \frac{a(at^2 + 2bt + c) - (at + b)^2}{x^3}$$

$$\alpha = \frac{ac - b^2}{x^3}$$

$$\alpha \propto x^{-3}$$

25. A rod of length 1 m is released from rest as shown in the figure below.



If ω of rod is $\sqrt{n}$ at the moment it hits the ground, then find n

Solution: (15)

: By using conservation of energy,

$$mg\frac{l}{2}\sin 30^\circ = \frac{1}{2}\frac{ml^2}{3}\omega^2$$

On solving

$\omega^2 = 15$

$\omega = \sqrt{15}$

Therefore, n = 15