TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 2 : 30 PM To 05 : 30 PM

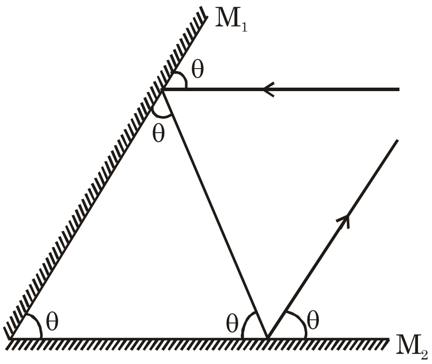
PHYSICS

1. Two plane mirrors arc inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be :

(1) 90° (2) 45° (3) 75° (4) 60°

Ans. (4)

Sol.



Assuming angles between two mirrors be θ as per geometry,

sum of anlges of Δ

$3\theta = 180°$

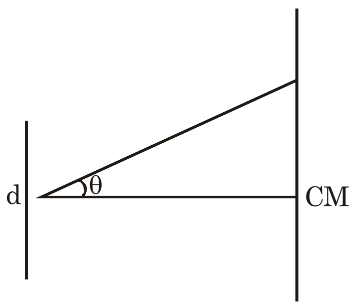
$\theta = 60°$

2. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$is:

(1) 320 (2) 641 (3) 321 (4) 640

Ans. (2)

Sol.



Pam difference

$d\sin\theta = n\lambda$

where d = seperation of slits

λ = wave length

n = no. of maximas

$0.32 \times 10^{-3} \sin 30 = n \times 500 \times 10^{-9}$

n = 320

Hence total no. of maximas observed in angular range $-30° \le \theta \le 30°$ is

maximas = 320 + 1 + 320 = 641

3. At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. [f the half-life of A is m_2, the half-life of B is :

(1) $\frac{ln2}{2}$ (2) $2ln2$ (3) $\frac{ln2}{4}$ (4) $4ln2$

Ans. (3)

Sol. Half life of A = $\ell n2$

$t_{1/2} = \frac{\ell n2}{\lambda}$

$\lambda_A = 1$

at t = 0 $R_A = R_B$

$N_A e^{-\lambda AT} = N_B e^{-\lambda BT}$

$N_A = N_B$ at t = 0

at t = t $\frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B t}}{N_0 e^{-\lambda_A t}}$

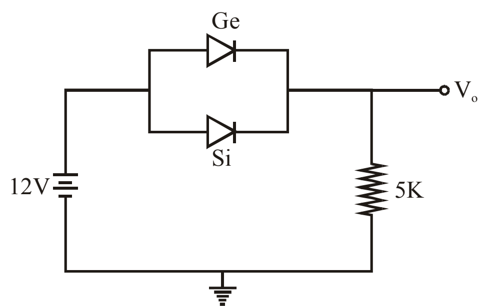
$e^{-(\lambda_B - \lambda_A)t} = e^{-t}$

$\lambda_B - \lambda_A = 3$

$\lambda_B = 3 + \lambda_A = 4$

$t_{1/2} = \frac{\ell n2}{\lambda_B} = \frac{\ell n2}{4}$

4. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by : (assume that the Ge diode has large breakdown voltage)



(1) 0.6 V (2) 0.8 V (3) 0.4 V (4) 0.2 V

Ans. (3)

Sol. Initially Ge & Si are both forward biased so current will effectivily pass through Ge diode with a drop of 0.3 V

if "Ge" is revesed then current will flow through "Si" diode hence an effective drop of (0.7 – 0.3) = 0.4 V is observed.

5. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :

(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77

Ans. (2)

Sol. Frequency of torsonal oscillations is given by

$$f = \frac{k}{\sqrt{I}}$$

$$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$$

$$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$$

$f_2 = 0.8\ f_1$

$$\frac{m}{M} = 0.375$$

6. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about :

[Take R = 8.3 J/ K mole]

(1) 10 kJ (2) 0.9 kJ (3) 6 kJ (4) 14 kJ

Ans. (1)

Sol. $Q = nC_v\Delta T$ as gas in closed vessel

$$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

Q = 10000 J = 10 kJ

7. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$ (3) $\frac{A}{\sqrt{2}}$ (4) A

Ans. (3)

Sol. Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

x = $\pm\frac{A}{\sqrt{2}}$

$\therefore$ Correct answer is (3)

8. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to :

(1) 753 Hz (2) 500 Hz

(3) 333 Hz (4) 666 Hz

Ans. (4)

Sol. Frequency of the sound produced by flute,

$$f = 2\left(\frac{v}{2\ell}\right) = \frac{2 \times 330}{2 \times 0.5} = 660\text{Hz}$$

Velocity of observer, $v_0 = 10 \times \frac{5}{18} = \frac{25}{9}\text{m/s}$

$\therefore$ frequency detected by observer, f' = $\left[\frac{v + v_0}{v}\right]f$

$$\therefore f' = \left[\frac{\frac{25}{9} + 330}{330}\right]660$$

= 335.56 × 2 = 671.12

$\therefore$ closest answer is (4)

9. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3 \times 10^8$ m/s, $h = 6.6 \times 10^{-34}$ J-s)

(1) 3.75×10^6 (2) 4.87×10^5

(3) 3.86×10^6 (4) 6.25×10^5

Ans. (4)

Sol. $f = \dfrac{3\times10^8}{8\times10^{-7}} = \dfrac{30}{8}\times10^{14}\,\text{Hz}$

$= 3.75 \times 10^{14}$ Hz

1% of f = 0.0375×10^{14} Hz

$= 3.75 \times 10^{12}$ Hz $= 3.75 \times 10^{6}$ MHz

number of channels = $\dfrac{3.75\times10^6}{6}$ = 6.25×10^5

$\therefore$ correct answer is (4)

10. Two point charges $q_1(\sqrt{10}\,\mu C)$ and $q_2(-25\,\mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is,

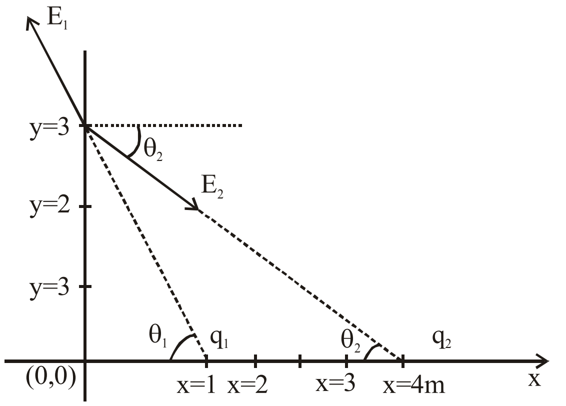
$\left[\text{take}\ \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}\right]$

(1) $(-63\hat{i}+27\hat{j})\times10^2$ (2) $(81\hat{i}-81\hat{j})\times10^2$

(3) $(63\hat{i}-27\hat{j})\times10^2$ (4) $(-81\hat{i}+81\hat{j})\times10^2$

Ans. **(3)**

Sol.



Let $\vec{E}_1$ & $\vec{E}_2$ are the vaues of electric field due to q_1 & q_2 respectively magnitude of

$$E_2 = \frac{1}{4\pi\in_0}\frac{q_2}{r^2}$$

$$E_2 = \frac{9\times10^9\times(25)\times10^{-6}}{(4^2+3^2)}\,\text{V/m}$$

$E_2 = 9 \times 10^3$ V/m

$$\therefore \vec{E}_2 = 9\times10^3\left(\cos\theta_2\hat{i}-\sin\theta_2\hat{j}\right)$$

$$\because \tan\theta_2 = \frac{3}{4}$$

$$\therefore \vec{E}_2 = 9\times10^3\left(\frac{4}{5}\hat{i}-\frac{3}{5}\hat{j}\right) = \left(72\hat{i}-54\hat{j}\right)\times10^2$$

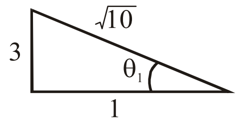
Magnitude of $E_1 = \dfrac{1}{4\pi\in_0}\dfrac{\sqrt{10}\times10^{-6}}{\left(1^2+3^2\right)}$

$= \left(9\times10^9\right)\times\sqrt{10}\times10^{-7}$

$= 9\sqrt{10}\times10^2$

$\therefore \vec{E}_1 = 9\sqrt{10}\times10^2\left[\cos\theta_1\left(-\hat{i}\right)+\sin\theta_1\hat{j}\right]$

$\therefore\ \tan\theta_1 = 3$



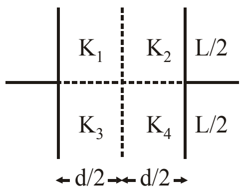
$$E_1 = 9\times\sqrt{10}\times10^2\left[\frac{1}{\sqrt{10}}\left(-\hat{i}\right)+\frac{3}{\sqrt{10}}\hat{j}\right]$$

$$E_1 = 9\times10^2\left[-\hat{i}+3\hat{j}\right] = \left[-9\hat{i}+27\hat{j}\right]10^2$$

$$\therefore \vec{E} = \vec{E}_1+\vec{E}_2 = \left(63\hat{i}-27\hat{j}\right)\times10^2\,\text{V/m}$$

$\therefore$ correct answer is (3)

11. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be :



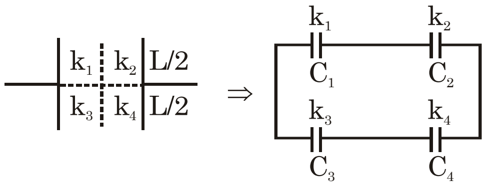
(1) $= \dfrac{(K_1+K_2)(K_3+K_4)}{{}_1+{}_2+{}_3+{}_4}$

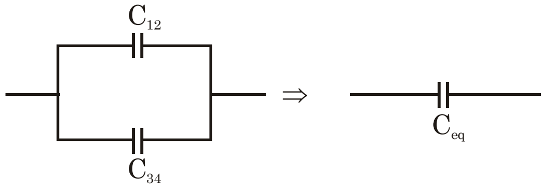
$= \dfrac{(K_1+K_2)(K_3+K_4)}{{}_1+{}_2+{}_3+{}_4}$

$= \dfrac{(K_1+K_4)(K_2+K_3)}{{}_1+{}_2+{}_3+{}_4}$

$K = \dfrac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

Ans. **(Bonus)**

Sol. 



$$C_{12} = \frac{C_1C_2}{C_1+C_2} = \frac{\dfrac{k_1 \in_0 \frac{L}{2}\times L}{d/2}\cdot\dfrac{k_2\left[\in_0 \frac{L}{2}\times L\right]}{d/2}}{(k_1+k_2)\left[\dfrac{\in_0 \cdot \frac{L}{2}\times L}{d/2}\right]}$$

$$C_{12} = \frac{k_1k_2}{k_1+k_2}\frac{\in_0 L^2}{d}$$

in the same way we get, $C_{34} = \dfrac{k_3k_4}{k_3+k_4}\dfrac{\in_0 L^2}{d}$

$$\therefore\ C_{eq} = C_{12} + C_{34} = \left[\frac{k_1k_2}{k_1+k_2}+\frac{k_3k_4}{k_3+k_4}\right]\frac{\in_0 L^2}{d} \quad ..(i)$$

Now if $k_{eq} = k$, $C_{eq} = \dfrac{k \in_0 L^2}{d}$ .....(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq} = \frac{k_1k_2(k_3+k_4)+k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any of the options so probably they have assumed the wrong combination

$$C_{13} = \frac{k_1 \in_0 L\frac{L}{2}}{d/2} + k_3 \in_0 \frac{L.\frac{L}{2}}{d/2}$$

$$= (k_1 + k_3)\frac{\in_0 L^2}{d}$$

$$C_{24} = (k_2 + k_4)\ \frac{\in_0 L^2}{d}$$

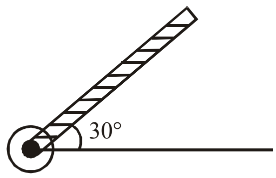
$$C_{eq} = \frac{C_{13}C_{24}}{C_{13}C_{24}} = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}\frac{\in_0 L^2}{d}$$

$$= \frac{k \in_0 L^2}{d}$$

$$k = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}$$

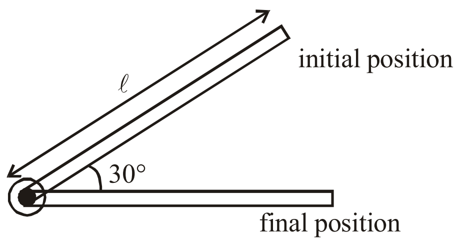
However this is one of the four options.
It must be a "Bonus" logically but of the given options probably they might go with (4)

12. A rod of length 50cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10ms^{-2}$)



(1) $\sqrt{30}$ (2) $\sqrt{\dfrac{30}{2}}$ (3) $\dfrac{\sqrt{30}}{2}$ (4) $\dfrac{\sqrt{20}}{3}$

Ans. **(2)**

Sol. 

Work done by gravity from initial to final position is,

$$W = mg\frac{\ell}{2}\sin 30°$$

$$= \frac{mg\ell}{4}$$

According to work energy theorem

$$W = \frac{1}{2}I\omega^2$$

$$\Rightarrow \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{4}$$

$$\omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3\times10}{2\times0.5}}$$

$\omega = \sqrt{30}$ rad / sec

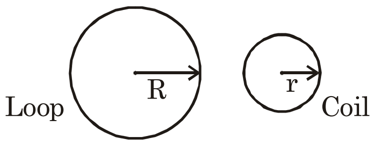
$\therefore$ correct answer is (1)

13. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. R $\frac{B_L}{B_C}$ will be :

(1) $\frac{1}{N}$ (2) N^2 (3) $\frac{1}{N^2}$ (4) N

Ans. (3)

Sol.



L = 2πR L = N $\times$ 2πr

R = Nr

$B_L = \frac{\mu_0 i}{2R}$ $B_C = \frac{\mu_0 N i}{2r}$

$B_C = \frac{\mu_0 N^2 i}{2R}$

$$\frac{B_L}{B_C} = \frac{1}{N^2}$$

14. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4 × 10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is:

(1) 1.28 × 10^4 km (2) 6.4 × 10^3 km

(3) 3.2 × 10^3 km (4) 1.6 × 10^3 km

Ans. (3)

Sol. $U_{surface} + E_1 = U_h$

KE of satelite is zero at earth surface & at height h

$$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re+h)}$$

$$E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e+h}\right)$$

$$E_1 = \frac{GM_e m}{(R_e+h)} \times \frac{h}{R_e}$$

Gravitational attraction $F_G = ma_C = \frac{mv^2}{(R_e+h)}$

$$E_2 \Rightarrow \frac{mv^2}{(R_e+h)} = \frac{GM_e m}{(R_e+h)^2}$$

$$mv^2 = \frac{GM_e m}{(R_e+h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e+h)}$$

$E_1 = E_2$

$$\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200\text{km}$$

15. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then :

(1) $U_E = \frac{U_B}{2}$ (2) $U_E < U_B$

(3) $U_E = U_B$ (4) $U_E > U_B$

Ans. (3)

Sol. Average energy density of magnetic field,

$u_B = \frac{B_0^2}{2\mu_0}$, B_0 is maximum value of magnetic field.

Average energy density of electric field,

$$u_E = \frac{\varepsilon_0 \in_0^2}{2}$$

now, $\in_0 = CB_0$, $C^2 = \frac{1}{\mu_0 \in_0}$

$$u_E = \frac{\in_0}{2} \times C^2 B_0^2$$

$= \frac{\epsilon_0}{2} \times \frac{1}{\mu_0 \epsilon_0} \times B_0^2 = \frac{B_0^2}{2\mu_0} = u_B$

$u_E = u_B$

since energy density of electric & magnetic field is same, energy associated with equal volume will be equal.

$u_E = u_B$

16. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is :

(1) 2.26×10^3 J (2) 3.39×10^3 J

(3) 5.65×10^2 J (4) 5.17×10^2 J

Ans. **(4)**

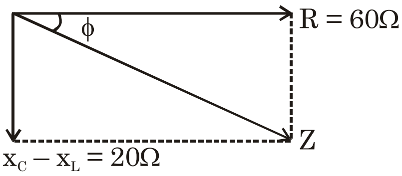
Sol. $R = 60\Omega$ $f = 50\text{Hz}, \omega = 2\pi f = 100\pi$

$x_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}}$

$x_C = 26.52\ \Omega$

$x_L = \omega L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega$

$x_C - x_L = 20.24 \approx 20$



$z = \sqrt{R^2 + (x_C - x_L)^2}$

$z = 20\sqrt{10}\Omega$

$\cos\phi = \frac{R}{z} = \frac{3}{\sqrt{10}}$

$P_{avg} = VI\cos\phi,\ I = \frac{v}{z}$

$= \frac{v^2}{z}\cos\phi$

$= 8.64$ watt

$Q = P.t = 8.64 \times 60 = 5.18 \times 10^2$

17. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to :

(1) $\sqrt{\frac{Gh}{c^3}}$ (2) $\sqrt{\frac{hc^5}{G}}$

(3) $\sqrt{\frac{c^3}{Gh}}$ (4) $\sqrt{\frac{Gh}{c^5}}$

Ans. **(4)**

Sol. $F = \frac{GM^2}{R^2} \Rightarrow G = [M^{-1}L^3T^{-2}]$

$E = h\nu \Rightarrow h = [ML^2T^{-1}]$

$C = [LT^{-1}]$

$t \propto G^x h^y C^z$

$[T] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

on comparing the powers of M, L, T

$-x + y = 0 \Rightarrow x = y$

$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ ....(i)

$-2x - y - z = 1 \Rightarrow 3x + z = -1$ ...(ii)

on solving (i) & (ii) $x = y = \frac{1}{2}, z = -\frac{5}{2}$

$t \propto \sqrt{\frac{Gh}{C^5}}$

18. The magnetic field associated with a light wave is given, at the origin, by

$B = B_0[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons ?

($c = 3 \times 10^8 ms^{-1}$, $h = 6.6 \times 10^{-34}$ J-s)

(1) 7.72 eV (2) 8.52 eV

(3) 12.5 eV (4) 6.82 eV

Ans. **(1)**

Sol. $B = B_0\sin(\pi \times 10^7C)t + B_0\sin(2\pi \times 10^7C)t$

since there are two EM waves with different frequency, to get maximum kinetic energy we take the photon with higher frequency

$B_1 = B_0\sin(\pi \times 10^7C)t$ $\nu_1 = \frac{10^7}{2} \times C$

$B_2 = B_0\sin(2\pi \times 10^7C)t$ $\nu_2 = 10^7C$

where C is speed of light $C = 3 \times 10^8$ m/s

$\nu_2 > \nu_1$

so KE of photoelectron will be maximum for photon of higher energy.

$\nu_2 = 10^7C$ Hz

$h\nu = \phi + KE_{max}$

energy of photon

$E_{ph} = h\nu = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^9$

$E_{ph} = 6.6 \times 3 \times 10^{-19}$J

$= \frac{6.6 \times 3 \times 10^{-19}}{1.6 \times 10^{-19}}eV = 12.375eV$

$KE_{max} = E_{ph} - \phi$

$= 12.375 - 4.7 = 7.675\ eV \approx 7.7\ eV$

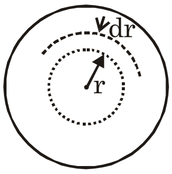
19. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :

(1) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ (2) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(3) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (4) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

Ans. **(4)**

Sol.



$Q = \int \rho dv$

$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$

$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$

$= 4\pi A\int_0^R e^{-2r/a}dr$

$= 4\pi A\left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R$

$= 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a}-1\right)$

$Q = 2\pi aA(1 - e^{-2R/a})$

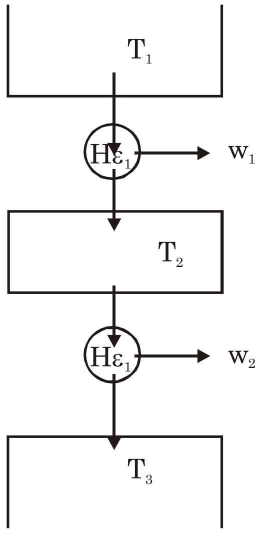
$R = \frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

20. Two Carrnot engines A and B are operated in series. The first one, A, receives heat at T_1(= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T_3(= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal :

(1) 400 K (2) 600 K (3) 500 K (4) 300 K

Ans. **(3)**

Sol.



$w_1 = w_2$

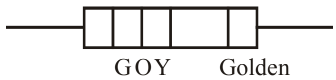
$\Delta u_1 = \Delta u_2$

$T_3 - T_2 = T_2 - T_1$

$2T_2 = T_1 + T_3$

$T_2 = 500$ K

21. A carbon resistance has a following colour code. What is the value of the resistance ?

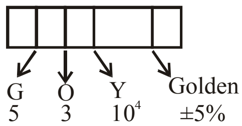


(1) 1.64 MΩ ± 5% (2) 530 kΩ ± 5%

(3) 64 kΩ ± 10% (4) 5.3 MΩ ± 5%

Ans. **(2)**

Sol.



$R = 53 \times 10^4 \pm 5\% = 530$ kΩ ± 5%

22. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?

(1) 850 J (2) 900 J

(3) 950 J (4) 875 J

Ans. **(2)**

Sol. $x = 3t^2 + 5$

$v = \frac{dx}{dt}$

$v = 6t + 0$

at $t = 0$ $v = 0$

$t = 5$ sec $v = 30$ m/s

W.D. = ΔKE

W.D. $= \frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900$J

23. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is :

(1) $a\omega$ (2) $\sqrt{3}\,a\omega$

(3) $\sqrt{2}\,a\omega$ (4) $2a\omega$

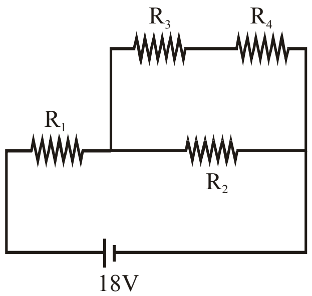
Ans. **(3)**

Sol. $v_x = -a\omega\sin\omega t \Rightarrow v_y = a\omega\cos\omega t$

$v_z = a\omega \Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$v = \sqrt{2}a\omega$

24. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5V, then the value R_2 will be :

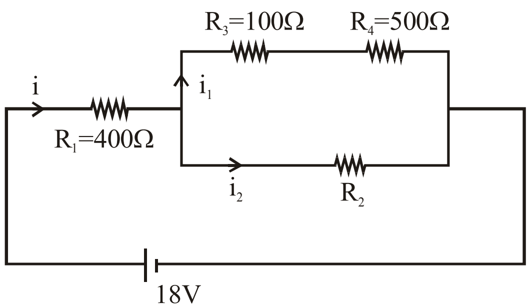


(1) 300 Ω (2) 230 Ω

(3) 450 Ω (4) 550 Ω

Ans. **(1)**

Sol.



$V_4 = 5V$

$i_1 = \frac{V_4}{R_4} = 0.01\ A$

$V_3 = i_1R_3 = 1V$

$V_3 + V_4 = 6V = V_2$

$V_1 + V_3 + V_4 = 18V$

$V_1 = 12\ V$

$i = \frac{V_1}{R_1} = 0.03 Amp.$

$i_2 = 0.02$ Amp $V_2 = 6V$

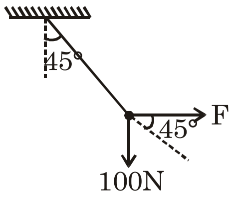
$R_2 = \frac{V_2}{i_2} = \frac{6}{0.02} = 300\Omega$

25. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$)

(1) 200 N (2) 100 N (3) 140 N (4) 70 N

Ans. **(2)**

Sol.



at equation

$\tan 45° = \frac{100}{F}$

$F = 100\ N$

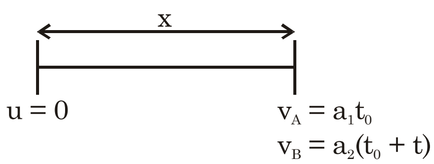
26. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to

(1) $\frac{a_1 + a_2}{2}t$ (2) $\sqrt{2a_1a_2}\,t$

(3) $\frac{2a_1a_2}{a_1 + a_2}t$ (4) $\sqrt{a_1a_2}\,t$

Ans. **(4)**

Sol. For A & B let time taken by A is t_0



from ques.

$v_A - v_B = v = (a_1 - a_2)t_0 - a_2t$ ....(i)

$x_B = x_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$

$\Rightarrow \sqrt{a_1}t_0 = \sqrt{a_2}(t_0 + t)$

$\Rightarrow (\sqrt{a_2} - \sqrt{a_2})t_0 = \sqrt{a_2}t$ ....(ii) putting t_0 in equation

$$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1}-\sqrt{a_2}} - a_2t$$

$$= \left(\sqrt{a_1}+\sqrt{a_2}\right)\sqrt{a_2}t - a_2t \Rightarrow v = \sqrt{a_1a_2}t$$

$$\Rightarrow \sqrt{a_1a_2}t + a_2t - a_2t$$

27. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V bv the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be :

(1) 25 A (2) 50 A

(3) 35 A (4) 45 A

Ans. (4)

Sol. $\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P}$

$$\Rightarrow 0.9 = \frac{23 \times I_S}{230 \times 5}$$

$$\Rightarrow I_S = 45A$$

28. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to :

(1) 9.6 m (2) 4.8 m (3) 2.9 m (4) 6.0 m

Ans. (2)

Sol. In flow volume = outflow volume

$$\Rightarrow \frac{0.74}{60} = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi}$$

$$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10}\left(\pi^2 = 10\right)$$

$$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24}$$

$$\Rightarrow h \approx 4.8m$$

29. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is :

(1) 5.755 m (2) 5.725 mm

(3) 5.740 m (4) 5.950 mm

Ans. (2)

Sol. $LC = \frac{\text{Pitch}}{\text{No. of division}}$

$LC = 0.5 \times 10^{-2}$ mm

+ve error = $3 \times 0.5 \times 10^{-2}$ mm

= 1.5×10^{-2} mm = 0.015 mm

Reading = MSR + CSR – (+ve error)

= 5.5 mm + ($48 \times 0.5 \times 10^{-2}$) – 0.015

= 5.5 + 0.24 – 0.015 = 5.725 mm

30. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron =1.6×10^{-19}C)

(1) 2.0×10^{-24} kg

(2) 1.6×10^{-19} kg

(3) 1.6×10^{-27} kg

(4) 9.1×10^{-31} kg

Ans. (1)

Sol. $\frac{mv^2}{R} = qvB$

mv = qBR ....(i)

Path is straight line

it qE = qvB

E = vB ....(ii)

From equation (i) & (ii)

$$m = \frac{qB^2R}{E}$$

$m = 2.0 \times 10^{-24}$ kg

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 2 : 30 PM To 05 : 30 PM

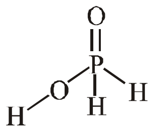
CHEMISTRY

1. lood reducing nature of H_3PO_2 ttributed to the presence of:

(1) One P-OH bond	(2) One P-H bond
(3) Two P-H bonds	(4) Two P-OH bonds

Ans. **(3)**

Sol. H_3PO_2 is good reducing agent due to presence of two P–H bonds.



2. The complex thai has highest cry splitting energy (Δ), is :

(1) $K_3[Co(CN)_6]$

(2) $[Co(NH_3)_5(H_2O)]Cl_3$

(3) $K_2[CoCl_4]$

(4) $[Co(NH_3)_5Cl]Cl_2$

Ans. **(1)**

Sol. As complex $K_3[Co(CN)_6]$ have CN^- ligand which is strongfield ligand amongst the given ligands in other complexes.

3. The metal that forms nitride by reacting directly with N_2 of air, is :

(1) K (2) Cs (3) Li (4) Rb

Ans. **(3)**

Sol. Only Li react directly with N_2 out of alkali metals

$6Li + N_2 \rightarrow 2Li_3N$

4. In which of the following processes, the bond order has increased and paramagnetic character has changed to diamagnetic ?

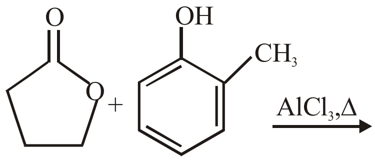
(1) $N_2 \rightarrow N_2^+$	(2) $NO \rightarrow NO^+$
(3) $O_2 \rightarrow O_2^{2-}$	(4) $O_2 \rightarrow O_2^+$

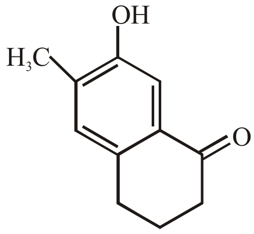
Ans. **(2)**

Sol.

Process	Change in magnetic nature	Bond Order Change
$N_2 \rightarrow N_2^+$	Dia $\rightarrow$ para	$3 \rightarrow 2.5$
$NO \rightarrow NO^+$	Para $\rightarrow$ Dia	$2.5 \rightarrow 3$
$O_2 \rightarrow O_2^{-2}$	Para $\rightarrow$ Dia	$2 \rightarrow 1$
$O_2 \rightarrow O_2^+$	Para $\rightarrow$ Para	$2 \rightarrow 2.5$

5. The major product of the following reaction is:



(1)


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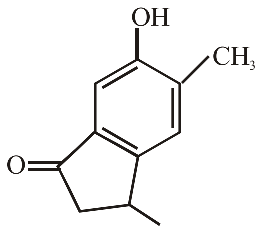

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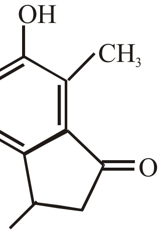

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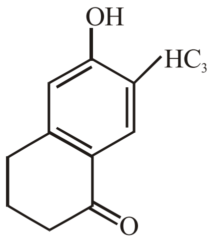
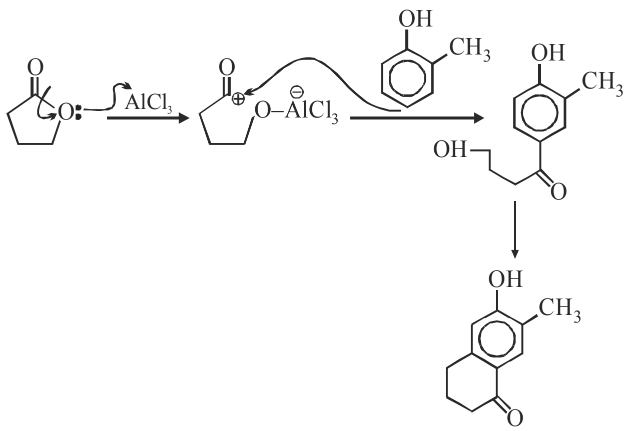


Ans. **(4)**

Sol.



6. The transition element that has lowest enthalpy of atomisation, is :

(1) Zn
(2) Cu
(3) V
(4) Fc

Ans. (2)

Sol. Since Zn is not a transition element so transition element having lowest atomisation energy out of Cu, V, Fe is Cu.

7. Which of the following combination of statements is true regarding the interpretation of the atomic orbitals ?

(a) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.

(b) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number.

(c) According to wave mechanics, the ground state angular momentum is h equal to $\frac{h}{2\pi}$.

(d) The plot of ψ Vs r for various azimuthal quantum numbers, shows peak shifting towards higher r value.

(1) (b), (c) (2) (a), (d) (3) (a), (b) (4) (a), (c)

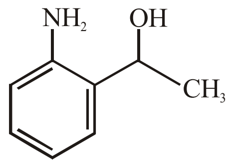
Ans. (4)

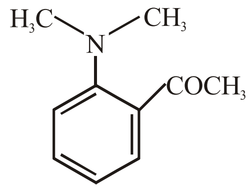
Sol. Refer Theory

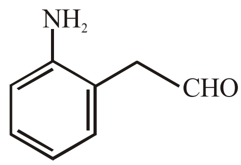
8. The tests performed on compound X and their inferences are:

Test	Inference
(a) 2,4 - DNP test	Coloured precipitate
(b) Iodoform test	Yellow precipitate
(c) Azo-dye test	No dye formation

Compound 'X' is:

(1)


(2)


(3)


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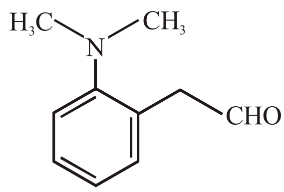
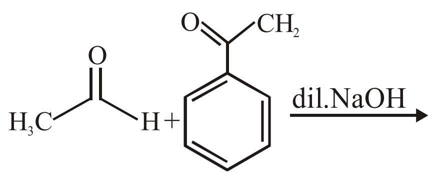


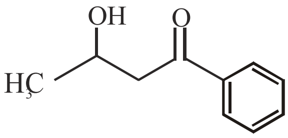
Ans. (2)

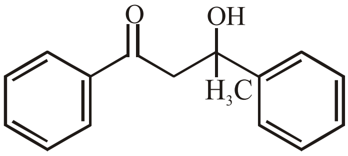
Sol. → 2,4 – DNP test is given by aldehyde on ketone

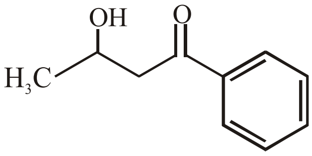
→ Iodoform test is given by compound having $CH_3-\underset{\underset{O}{\|}}{C}-$ group.

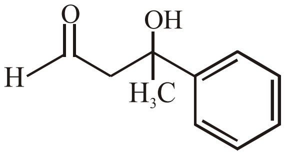
9. The major product formed in the following reaction is:



(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. Aldehyde reacts at a faster rate than keton during aldol and stericall less hindered anion will be a better nucleophile so sefl aldol at

$CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-H$ will be the major product.

10. For the reaction, $2A + B \rightarrow$ products, when the concentrations of A and B both wrere doubled, the rate of the reaction increased from 0.3 mol $L^{-1}s^{-1}$ to 2.4 mol L^{-1} s^{-1}. When the concentration of A alone is doubled, the rate increased from 0.3 mol $L^{-1}s^{-1}$ to0.6 mol $L^{-1}s^{-1}$

Which one of the following statements is correct ?

(1) Order of the reaction with respect to Bis2

(2) Order of the reaction with respect to Ais2

(3) Total order of the reaction is 4

(4) Order of the reaction with respect to B is 1

Ans. **(1)**

Sol. $r = K[A]^x[B]^y$

$\Rightarrow 8 = 2^3 = 2^{x+y}$

$\Rightarrow x + y = 3$ ...(1)

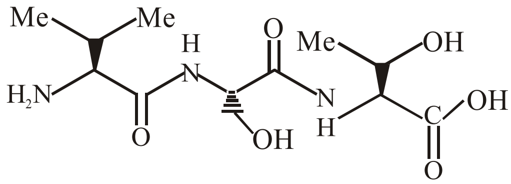
$\Rightarrow 2 = 2^x$

$\Rightarrow x = 1, y = 2$

Order w.r.t. A = 1

Order w.r.t. B = 2

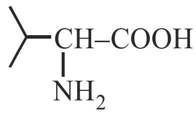
11. The correct sequence of amino acids present in the tripeptide given below is :



(1) Leu - Ser - Thr　　(2) Thr - Ser- Leu

(3) Thr - Ser - Val　　(4) Val - Ser - Thr

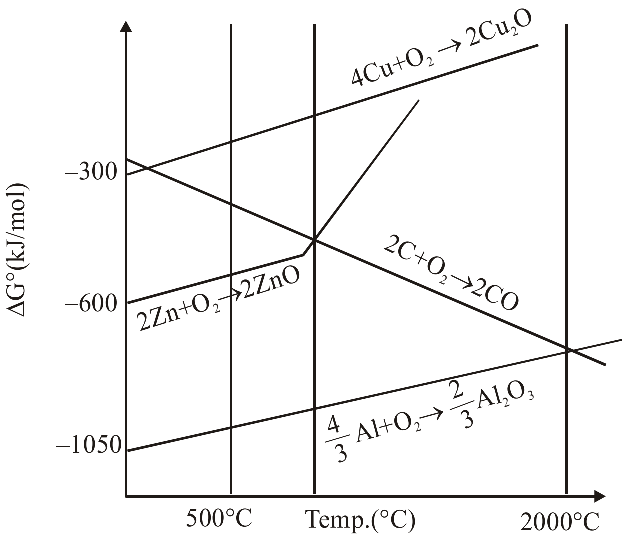
Ans. **(4)**

Sol. **Leusine** 

Serine $NO-CH_2-\underset{\underset{\displaystyle NH_2}{|}}{CH}-COOH$

Thrconine $H_3C-\underset{\underset{\displaystyle OH}{|}}{CH}-\underset{\underset{\displaystyle NH_2}{|}}{CH}-COOH$

12. The correct statement regarding the given Ellingham diagram is:



(1) At 800°C, Cu can be used for the extraction of Zn from ZnO

(2) At 500 C, coke can be used for the extraction of Zn from ZnO

(3) Coke cannot be used for the extraction of Cu from Ca_2O.

(4) At 1400°C, Al can be used for the extraction of Zn from ZnO

Ans. (4)

Sol. According to the given diagram Al can reduce ZnO.

$3ZnO + 2Al \rightarrow 3Zn + Al_2O_3$

13. For the following reaction, the mass of water produced from 445 g of $C_{57}H_{110}O_6$ is :

$2C_{57}H_{110}O_6(s) + 163O_2(g) \rightarrow 114CO_2(g) + 110\ H_2OP(1)$

(1) 495 g (2) 490 g (3) 890 g (4) 445 g

Ans. (1)

Sol. moles of $C_{57}H_{110}O_6(s) = \frac{445}{890} = 0.5$ moles

$2C_{57}H_{110}O_6(s) + 163\ O_2(g) \rightarrow 114\ CO_2(g) + 110\ H_2O(l)$

$$n_{H_2O} = \frac{110}{4} = \frac{55}{2}$$

$$m_{H_2O} = \frac{55}{2} \times 18$$

$= 495gm$

14. The correct match between Item I and Item II is :

Item I	**Item II**
(A) Benzaldehyde	(P) Mobile phase
(B) Alumina	(Q) Adsorbent
(C) Acetonitrile	(R) Adsorbate

(1) (A) → (Q);(B) → (R);(C) → (P)

(2) (A) → (P); (B) → (R); (C) → (Q)

(3) (A) → (Q); (B) → (P); (C) → (R)

(4) (A) → (R); (B) → (Q); (C) → (P)

Ans. (4)

Sol.

15. The increasing basicity order of the following compounds is :

(A) $CH_3CH_2NH_2$

(B) $CH_3CH_2NH(CH_2CH_3)$

(C) $H_3C{-}N(CH_3){-}CH_3$

(D) $Ph{-}N(CH_3){-}H$

(1) (D)<(C)<(A)<(B) (2) (A)<(B)<(D)<(C)

(3) (A)<(B)<(C)< (D) (4) (D)<(C)<(B)<(A)

Ans. (1)

Sol.

$Ph-N(CH_3)-H < CH_3-N(CH_3)-CH_3 < CH_3-CH_2-NH(CH_2-CH_3) < CH_3-CH_2-NH_2$

lone pair delocalized

more steric hinderence less solutions energy

16. For coagulation of arscnious sulphide sol, which one of the following salt solution will be most effective ?

(1) $AlCl_3$ (2) NaCl

(3) $BaCl_2$ (4) Na_3PO_4

Ans. (1)

Sol. Sulphide is –ve charged colloid so cation with maximum charge will be most effective for coagulation.

$Al^{3+} > Ba^{2+} > Na^{+}$ coagulating power.

17. At 100°C, copper (Cu) has FCC unit cell structure with cell edge length of x Å. What is the approximate density of Cu (in g cm^{-3}) at this temperature ?

[Atomic Mass of Cu = 63.55u]

(1) $\frac{105}{x^3}$ (2) $\frac{211}{x^3}$ (3) $\frac{205}{x^3}$ (4) $\frac{422}{x^3}$

Ans. (4)

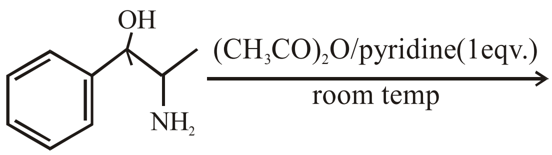
Sol. FCC unit cell Z = 4

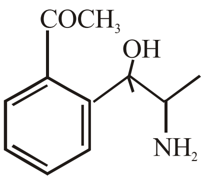
$$d = \frac{63.5\times4}{6\times10^{23}\times x^3\times10^{-24}}\ g/cm^3$$

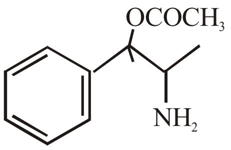
$$d = \frac{63.5\times4\times10}{6}\ g/cm^3$$

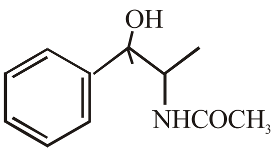
$$d = \frac{423.33}{x^3} \simeq \left(\frac{422}{x^3}\right)$$

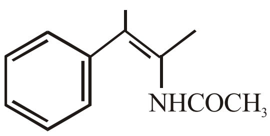
18. The major product obtained in the following reaction is :



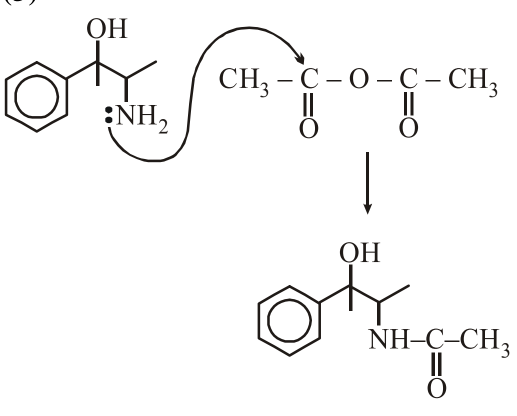
(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

Sol. 

19. Which of the following conditions in drinking water causes methemoglobinemia ?

(1) > 50ppm of load

(2) > 100 ppm of sulphate

(3) > 50 ppm of chloride

(4) > 50 ppm of nitrate

Ans. **(4)**

Sol. Concentration of nitrate >50 ppm in drinking water causes methemoglobinemia

20. Homoleptic octahedral complexes of a metal ion 'M^{3+}' with three monodentate ligands and L_1, L_2, L_3 absorb wavelengths in the region of green, blue and red respectively. The increasing order of the ligand strength is :

(1) $L_2 < L_1 < L_3$ (2) $L_3 < L_2 < L_1$

(3) $L_3 < L_1 < L_2$ (4) $L_1 < L_2 < L_3$

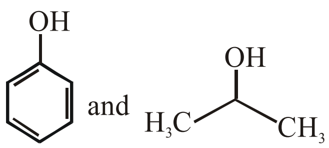
Ans. **(3)**

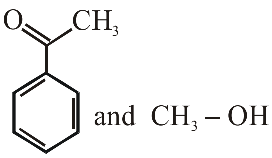
Sol. Order of λ_{abs} - $L_3 > L_1 > L_2$

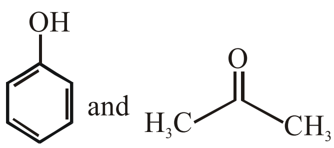
So Δ_O order will be $L_2 > L_1 > L_3$ (as $\Delta_O \propto \frac{1}{\lambda_{abs}}$)

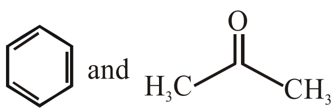
So order of ligand strength will be $L_2 > L_1 > L_3$

21. The product formed in the reaction of cumene with O_2 followed by treatment with dil. HCl are :

(1) 

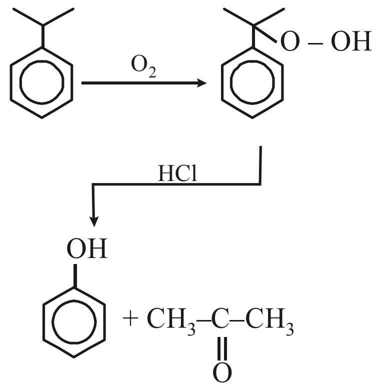
(2) 

(3) 

(4) 

Ans. **(3)**

Sol. **Cummene hydroperoxide reaction**



22. The temporary hardness of water is due to :-

(1) $Ca(HCO_3)_2$ (2) NaCl

(3) Na_2SO_4 (4) $CaCl_2$

Ans. **(1)**

Sol. $Ca(HCO_3)_2$ is reponsible for temporary hardness of water

23. The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is :

(Specific heat of water liquid and water vapour are 4.2 kJ K^{-1} kg^{-1} and 2.0 kJ K^{-1} kg^{-1} ; heat of liquid fusion and vapourisation of water are 344 kJ kg^{-1} and 2491 kJ kg^{-1}, respectively). (log 273 = 2.436, log 373 = 2.572, log 383 = 2.583)

(1) 7.90 kJ kg^{-1} K^{-1} (2) 2.64 kJ kg^{-1} K^{-1}

(3) 8.49 kJ kg^{-1} K^{-1} (4) 4.26 kJ kg^{-1} K^{-1}

Ans. **(4)**

Sol.

$$\underset{273K}{H_2O(s)} \xrightarrow[\Delta S_1]{} \underset{273K}{H_2O(\ell)} \xrightarrow[\Delta S_2]{} \underset{373K}{H_2O(\ell)}$$

$$\downarrow \Delta S_3$$

$$\underset{373K}{H_2O(g)} \xrightarrow[\Delta S_4]{} \underset{383K}{H_2O(g)}$$

$$\Delta S_1 = \frac{\Delta H_{fusion}}{273} = \frac{334}{273} = 1.22$$

$$\Delta S_2 = 4.2\ell N\left(\frac{363}{273}\right) = 1.31$$

$$\Delta S_3 = \frac{\Delta H_{vap}}{373} = \frac{2491}{373} = 6.67$$

$$\Delta S_4 = 2.0\ell n\left(\frac{383}{373}\right) = 0.05$$

$$\Delta S_{total} = 9.26 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

24. The pH of rain water, is approximately :

(1) 6.5 (2) 7.5 (3) 5.6 (4) 7.0

Ans. **(3)**

Sol. pH of rain water is approximate 5.6

25. If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction

$$Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$$

at 300 K is approximately.

(R = 8 JK^{-1} mol^{-1} , F = 96000 C mol^{-1})

(1) e^{160} (2) e^{320}

(3) e^{-160} (4) e^{-80}

Ans. **(1)**

Sol. $\Delta G° = -RT\ \ln k = -nFE^{\circ}_{cell}$

$$\ln k = \frac{n \times F \times E^{o}}{R \times T} = \frac{2 \times 96000 \times 2}{8 \times 300}$$

$\ln k = 160$

$k = e^{160}$

26. A solution containing 62 g ethylene glycol in 250 g water is cooled to –10°C. If K_f for water is 1.86 K kg mol^{-1}, the amount of water (in g) separated as ice is :

(1) 32 (2) 48 (3) 16 (4) 64

Ans. **(4)**

Sol. $\Delta T_f = K_f . m$

$$10 = 1.86 \times \frac{62/62}{W_{kg}}$$

W = 0.186 kg

$\Delta W = (250 - 186) = 64$ gm

27. When the first electron gain enthalpy ($\Delta_{eg}H$) of oxygen is –141 kJ/mol, its second electron gain enthalpy is :

(1) almost the same as that of the first

(2) negative , but less negative than the first

(3) a positive value

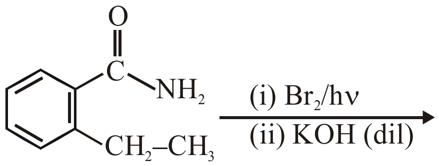
(4) a more negative value than the first

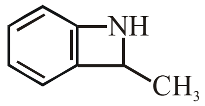
Ans. **(3)**

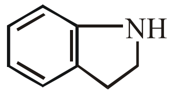
Sol. Second electron gain enthalpy is always positive for every element.

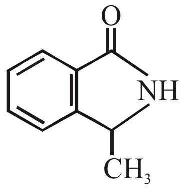
$O^{-}_{(g)} + e^{-} \rightarrow O^{-2}_{(g)}$; ΔH = positive

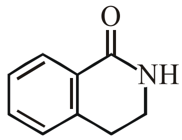
28. The major product of the following reaction is :



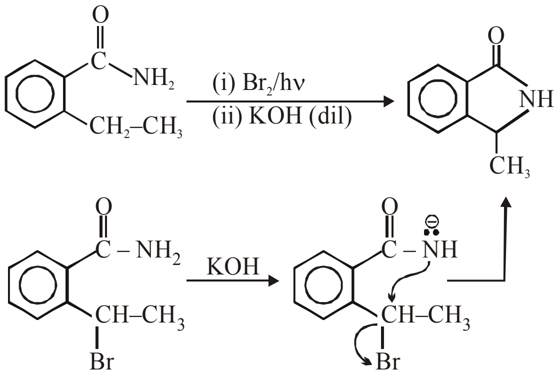
(1) 

(2) 

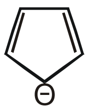
(3) 

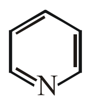
(4) 

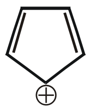
Ans. **(3)**

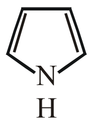
Sol. 

29. Which of the following compounds is not aromatic ?

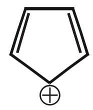
(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

Sol. 

Do not have $(4n + 2)$ π electron It has $4n$ π electrons

So it is Anti aromatic.

30. Consider the following reversible chemical reactions :

$A_2(g) + Br_2(g) \xrightleftharpoons{K_1} 2AB(g)$ .....(1)

$6AB(g) \xrightleftharpoons{K_2} 3A_2(g) + 3B_2(g)$ .....(2)

The relation between K_1 and K_2 is :

(1) $K_2 = K_1^3$ (2) $K_2 = K_1^{-3}$

(3) $K_1K_2 = 3$ (4) $K_1K_2 = \frac{1}{3}$

Ans. **(2)**

Sol. $A_2(g) + B_2(g) \xrightleftharpoons{k_1} 2AB$ ...(1)

$\Rightarrow$ eq. (1) × 3

$6\ AB(g) \rightleftharpoons 3A_2(g) + 3B_2(g)$

$\Rightarrow \left(\frac{1}{k_1}\right)^3 = k_2 \Rightarrow k_2 = (k_1)^{-3}$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let f be a differentiable function from R to R such that $|f(x)-f(y)| \le 2|x-y|^{\frac{3}{2}}$, for all x, y ε R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

Ans. (4)

Sol. $|f(x) - f(y)| \le 2|x - y|^{3/2}$

divide both sides by $|x - y|$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le 2.|x-y|^{1/2}$$

apply limit $x \to y$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0 \Rightarrow f(y) = c \Rightarrow f(x) = 1$

$$\int_0^1 1.dx = 1$$

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1 - \frac{1}{\sqrt{2}}, (k > 0)$, then the value of k is :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

Ans. (1)

Sol. $\frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\tan\theta}{\sqrt{\sec\theta}} d\theta = \frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\sin\theta}{\sqrt{\cos\theta}} d\theta$

$$= -\frac{1}{\sqrt{2k}} 2\sqrt{\cos\theta}\Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}}\left(\frac{1}{\sqrt{2}} - 1\right)$$

given it is $1 - \frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(1) 12 (2) 15 (3) 10 (4) 14

Ans. (2)

Sol. $(1 - t^6)^3 (1 - t)^{-3}$

$(1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$

$\Rightarrow$ cofficient of t^4 in $(1 - t)^{-3}$ is

${}^{3+4-1}C_4 = {}^6C_2 = 15$

4. For each xεR, let [x] be the greatest integer less than or equal to x. Then $\lim_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(1) – sin1 (2) 0 (3) 1 (4) sin1

Ans. (1)

Sol. $\lim_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$

$x \to 0^-$

$[x] = -1 \Rightarrow \lim_{x\to 0^-} \frac{x(-x-1)\sin(-1)}{-x} = -\sin 1$

$|x| = -x$

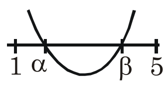
5. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval:

(1) (4,5) (2) (3,4) (3) (5,6) (4) (–5,–4)

Ans. (Bonus/1)

Sol. $x^2 - mx + 4 = 0$

$\alpha,\beta \in [1,5]$



(1) $D > 0 \Rightarrow m^2 - 16 > 0$

$\Rightarrow m \in (-\infty,-4) \cup (4,\infty)$

(2) $f(1) \ge 0 \Rightarrow 5 - m \ge 0 \Rightarrow m \in (-\infty, 5]$

(3) $f(5) \ge 0 \Rightarrow 29 - 5m \ge 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$

(4) $1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2,10)$

$\Rightarrow m \in (4,5)$

No option correct : Bonus

* If we consider $\alpha,\beta \in (1,5)$ then option (1) is correct.

6. If

$$A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

Then A is-

(1) Invertible only if $t=\frac{\pi}{2}$

(2) not invertible for any $t\varepsilon R$

(3) invertible for all $t\varepsilon R$

(4) invertible only if $t=\pi$

Ans. (3)

Sol. $$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\ t \in R$

$= 5e^{-t} \neq 0\ \forall\ t \in R$

7. The area of the region

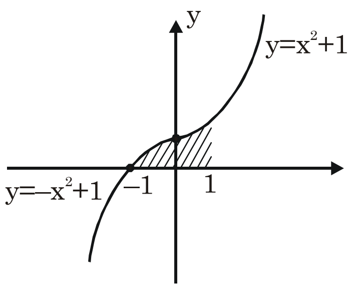
$A = \left[(x,y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\right]$

in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

Ans. (3)

Sol. The graph is a follows



$$\int_{-1}^{0}\left(-x^2+1\right)dx + \int_{0}^{1}\left(x^2+1\right)dx = 2$$

8. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) 0 (4) $\frac{\pi}{6}$

Ans. (1)

Sol. $z_0 = \omega$ or ω^2 (where ω is a non-real cube root of unity)

$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$

$z = 3 + 3i$

$\Rightarrow \arg z = \frac{\pi}{4}$

9. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

Ans. (4)

Sol. Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = |\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...(1)

and $(\vec{a}+\vec{b}) \perp \vec{c} \Rightarrow (\vec{a}+\vec{b}).\vec{c} = 0$

$\Rightarrow 5b_1 + b_2 = -10$ ...(2)

from (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$

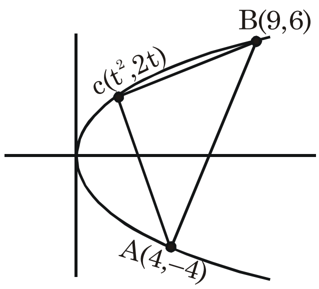
then $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + 2} = 6$

10. Let A(4,–4) and B(9,6) be points on the parabola, $y^2 + 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

Ans. (4)

Sol.



Area = $5|t^2 - t - 6| = 5\left|\left(t-\frac{1}{2}\right)^2 - \frac{25}{4}\right|$

is maximum if $t = \frac{1}{2}$

11. The logical statement

$\left[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)\right]$ is equivalent to:

(1) $(p \wedge r) \wedge \sim q$ (2) $(\sim p \wedge \sim q) \wedge r$

(3) $\sim p \vee r$ (4) $(p \wedge \sim q) \vee r$

Ans. (1)

Sol. $s\left[\sim(\sim p \vee q) \wedge (p \wedge r)\right] \cap (\sim q \wedge r)$

$\equiv \left[(p \wedge \sim q) \vee (p \wedge r)\right] \wedge (\sim q \wedge r)$

$\equiv \left[p \wedge (\sim q \vee r)\right] \wedge (\sim q \wedge r)$

$\equiv p \wedge (\sim q \wedge r)$

$\equiv (p \wedge r) \sim q$

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

Ans. (2)

Sol. E_1 : Event of drawing a Red ball and placing a green ball in the bag

E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw

$$P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$$

$$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$$

13. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which sin x-sin2x+sin3x = 0, is

(1) 2 (2) 1

(3) 3 (4) 4

Ans. (1)

Sol. sinx – sin2x + sin3x = 0

$\Rightarrow$ (sinx + sin3x) – sin2x = 0

$\Rightarrow$ 2sinx. cosx – sin2x = 0

$\Rightarrow$ sin2x(2 cosx – 1) = 0

$\Rightarrow$ sin2x = 0 or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0, \frac{\pi}{3}$

14. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is:

(1) x + 2y – 2z = 0 (2) x – 2y + z = 0

(3) 5x + 2y – 4z = 0 (4) 3x + 2y – 3z = 0

Ans. (2)

Sol. Vector along the normal to the plane containing the lines

$$\frac{x}{3}=\frac{y}{4}=\frac{z}{2} \text{ and } \frac{x}{4}=\frac{y}{2}=\frac{z}{3}$$

is $\left(8\hat{i}-\hat{j}-10\hat{k}\right)$

vector perpendicular to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $8\hat{i}-\hat{j}-10\hat{k}$ is $26\hat{i}-52\hat{j}+26\hat{k}$

so, required plane is

$26x - 52y + 26z = 0$

$x - 2y + z = 0$

15. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :

(1) $122y - 26x - 1675 = 0$

(2) $26x + 61y + 1675 = 0$

(3) $122y + 26x + 1675 = 0$

(4) $26x - 122y - 1675 = 0$

Ans. (4)

Sol. Equation of AB is

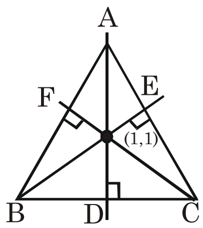
$3x - 2y + 6 = 0$

equation of AC is

$4x + 5y - 20 = 0$

Equation of BE is

$2x + 3y - 5 = 0$



Equation of CF is $5x - 4y - 1 = 0$

$\Rightarrow$ Equation of BC is $26x - 122y = 1675$

16. If $x = 3 \tan t$ and $y = 3 \sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$, is:

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$

Ans. (4)

Sol. $\frac{dx}{dt}=3\sec^2 t$

$\frac{dy}{dt}=3\sec t\tan t$

$\frac{dy}{dx}=\frac{\tan t}{\sec t}=\sin t$

$\frac{d^2y}{dx^2}=\cos t\frac{dt}{dx}$

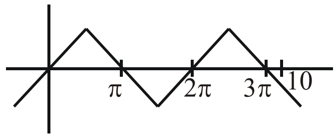
$=\frac{\cos t}{3\sec^2 t}=\frac{\cos^3 t}{3} = \frac{1}{3.2\sqrt{2}}=\frac{1}{6\sqrt{2}}$

17. If $x = \sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then $y–x$ is equal to:

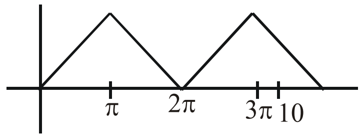
(1) π (2) 7π (3) 0 (4) 10

Ans. (1)

Sol.



$x = \sin^{-1}(\sin 10) = 3\pi - 10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$

$y - x = \pi$

18. If the lines $x = ay+b$, $z = cy + d$ and $x=a'z + b'$, $y = c'z + d'$ are perpendicular, then:

(1) $cc' + a + a' = 0$

(2) $aa' + c + c' = 0$

(3) $ab' + bc' + 1 = 0$

(4) $bb' + cc' + 1 = 0$

Ans. (2)

Sol. Line $x = ay + b$, $z = cy + d \Rightarrow \frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$

Line $x = a'z + b'$, $y = c'z + d'$

$$\Rightarrow \frac{x-b'}{a'}=\frac{y-d'}{c'}=\frac{z}{1}$$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

19. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2–11x+\alpha = 0$ are rational numbers is :

(1) 2 (2) 5 (3) 3 (4) 4

Ans. (3)

Sol. $6x^2 - 11x + \alpha = 0$

given roots are rational

$\Rightarrow$ D must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I$

$\alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I$ $\qquad \Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in I$

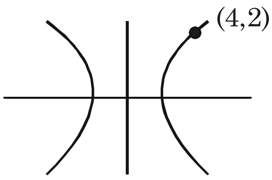
$\alpha = 5 \Rightarrow \lambda \in I$

20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{3}$ (4) 2

Ans. (1)

Sol.



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

2a = 4 a = 2

$$\frac{x^2}{4}-\frac{y^2}{b^2}=1$$

Passes through (4,2)

$$4-\frac{4}{b^2}=1 \Rightarrow b^2=\frac{4}{3} \Rightarrow e=\frac{2}{\sqrt{3}}$$

21. Let A={xεR:x is not a positive integer}

Define a function f :A→R as $f(x)=\frac{2x}{x-1}$ then f is

(1) injective but not surjective

(2) not injective

(3) surjective but not injective

(4) neither injective nor surjective

Ans. (1)

Sol. $f(x)=2\left(1+\frac{1}{x-1}\right)$

$$f'(x)=-\frac{2}{(x-1)^2}$$

⇒ f is one-one but not onto

22. If $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx,(x\geq 0)$ and $f(0)=0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (4)

Sol. $\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx$

$$=\int\frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^7}+\frac{1}{x^5}+2\right)^2}dx=\frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}}+C$$

As $f(0)=0$, $f(x)=\frac{x^7}{2x^7+x^2+1}$

$$f(1)=\frac{1}{4}$$

23. If the circles $x^2+y^2-16x-20y+164=r^2$ and $(x-4)^2+(y-7)^2=36$ intersect at two distinct points, then:

(1) $0<r<1$ (2) $1<r<11$

(3) $r>11$ (4) $r=11$

Ans. (2)

Sol. $x^2+y^2-16x-20y+164=r^2$

A(8,10), $R_1=r$

$(x-4)^2+(y-7)^2=36$

B(4,7), $R_2=6$

$|R_1-R_2|<AB<R_1+R_2$

$\Rightarrow 1<r<11$

24. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is:

(1) 9 (2) 18 (3) 32 (4) 36

Ans. (4)

Sol. Let A(α,0) and B(0,β)

be the vectors of the given triangle AOB

⇒ |αβ| = 100

⇒ Number of triangles

= 4 × (number of divisors of 100)

= 4 × 9 = 36

25. The sum of the follwing series

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+....+5^2)}{11}+....$$ up to 15 terms, is:

(1) 7820 (2) 7830 (3) 7520 (4) 7510

Ans. (1)

Sol. $T_n = \dfrac{(3+(n-1)\times 3)(1^2+2^2+....+n^2)}{(2n+1)}$

$$T_n = \frac{3.\dfrac{n(n+1)(2n+1)}{6}}{2n+1} = \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3+n^2) = \frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15\times16\times31}{6}\right]$$

$= 7820$

26. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:

(1) $\frac{1}{2}$ (2) 4

(3) 2 (4) $\frac{7}{13}$

Ans. (2)

Sol. a = A + 6d

b = A + 10d

c = A + 12d

a,b,c are in G.P.

$\Rightarrow (A + 10d)^2 = (A + 6d)(a + 12d)$

$\Rightarrow \frac{A}{d} = -14$

$$\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{6+\frac{A}{d}}{12+\frac{A}{d}} = \frac{6-14}{12-14} = 4$$

27. If the system of linear equations

x–4y+7z = g

3y – 5z = h

–2x + 5y – 9z = k

is consistent, then :

(1) g + h + k = 0

(2) 2g + h + k = 0

(3) g + h + 2k = 0

(4) g + 2h + k = 0

Ans. (2)

Sol. $P_1 \equiv x - 4y + 7z - g = 0$

$P_2 \equiv 3x - 5y - h = 0$

$P_3 \equiv -2x + 5y - 9z - k = 0$

Here $\Delta = 0$

$2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$

28. Let $f:[0,1]\to R$ be such that $f(xy) = f(x).f(y)$ for all $x,y,\varepsilon[0,1]$, and $f(0)\neq 0$. If y = y(x) satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with y(0) = 1, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(1) 4 (2) 3 (3) 5 (4) 2

Ans. (2)

Sol. $f(xy) = f(x).\ f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow f(x) = 1$

$\frac{dy}{dx} = f(x) = 1$

$\Rightarrow y = x + c$

At, x = 0, y = 1 $\Rightarrow$ c = 1

y = x + 1

$$\Rightarrow y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right) = \frac{1}{4}+1+\frac{3}{4}+1 = 3$$

29. A data consists of n observations:

$x_1, x_2, \ldots, x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is :

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

Ans. (2)

Sol. $\sum(x_i+1)^2 = 9n$ ...(1)

$\sum(x_i-1)^2 = 5n$ ...(2)

(1) + (2) $\Rightarrow \sum(x_1^2+1) = 7n$

$\Rightarrow \dfrac{\sum x_i^2}{n} = 6$

(1) - (2) $\Rightarrow 4\Sigma x_i = 4n$

$\Rightarrow \Sigma x_i = n$

$\Rightarrow \dfrac{\Sigma x_i}{n} = 1$

$\Rightarrow$ variance = 6 – 1 = 5

$\Rightarrow$ Standard diviation $= \sqrt{5}$

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to :

(1) 250 (2) 374 (3) 372 (4) 375

Ans. (2)

Sol.

a_1	a_2	a_3

Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers = 2×5^3

Required number = $5^3 + 2 \times 5^3 - 1$

= 374