JEE Main 2020 Paper

Date: 7th January 2020

Time: 02.30 PM – 05:30 PM

Subject: Chemistry

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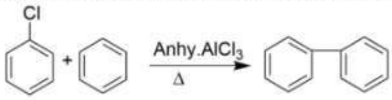
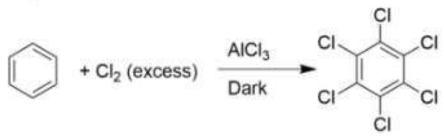
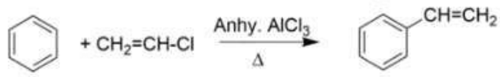
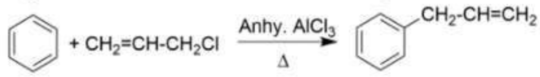
1. Which of the following reactions are possible?

A. 

B. 

C. 

D. 

a. A, B, C	b. B ,D
c. A, C, D	d. A, C

Answer: b

Solution:

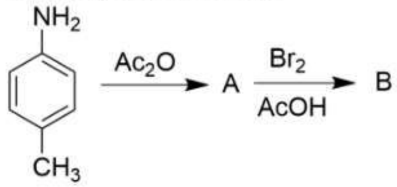
In aryl halides, due to the partial double bond character generated by chlorine, the aryl cation is not formed.

Vinyl halides do not give Friedel-Crafts reaction, because the intermediate that is generated (vinyl cation) is not stable.

$H_2C=\overset{+}{C}H$

vinyl
cation

2. B in the given reaction is?

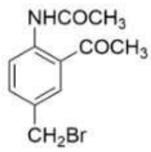


a.

b.

c.

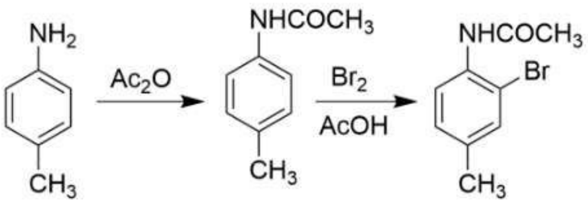
d.



Answer: a

Solution:

During trisubstitution, the acetanilide group attached to the benzene ring is more electron donating than the methyl group attached, owing to +M effect, and therefore, the incoming electrophile would prefer ortho w.r.t the acetanilide group.



3. The correct statement about gluconic acid is:

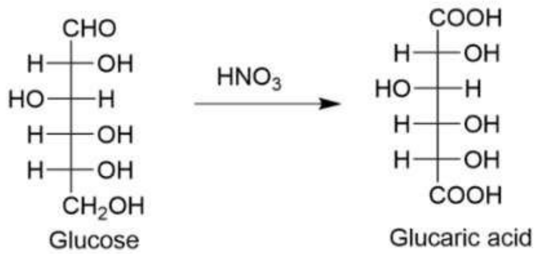
   a. It is prepared by oxidation of glucose with HNO_3
   b. It is obtained by partial oxidation of glucose
   c. It is a dicarboxylic acid
   d. It forms hemiacetal or acetal

   Answer: b

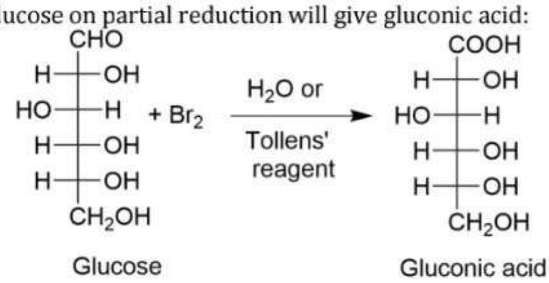
Solution:

The gluconic acid formed is a monocarboxylic acid which is formed during the partial oxidation of glucose

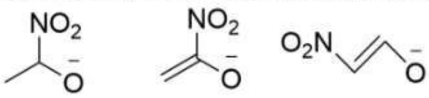
(a) Glucose on reaction with HNO_3 will give glucaric acid:



(b) Glucose on partial reduction will give gluconic acid:



4. The stability order of the following alkoxide ions are:



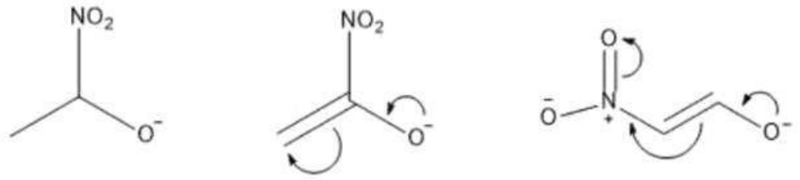
(A) (B) (C)

a. C > B > A
b. A > C > B
c. B > A > C
d. C > A > B

Answer: a

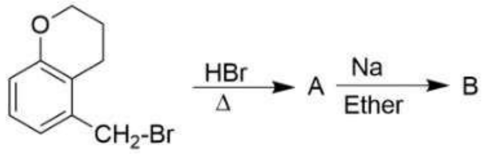
Solution:

Higher the delocalization of the negative charge, more will be the stability of the anion.



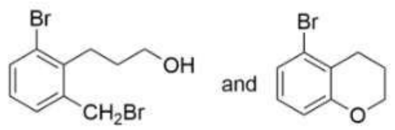
(A) The negative charge is stabilized only through –I effect exhibited by the – NO_2 group.
(B) The negative charge is stabilized by the delocalization of the double bond and the –I effect exhibited by the – NO_2 group.
(C) The negative charge is stabilized by extended conjugation.

5.

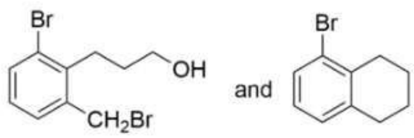


A and B are:

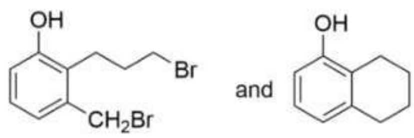
a.



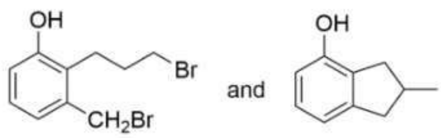
b.



c.

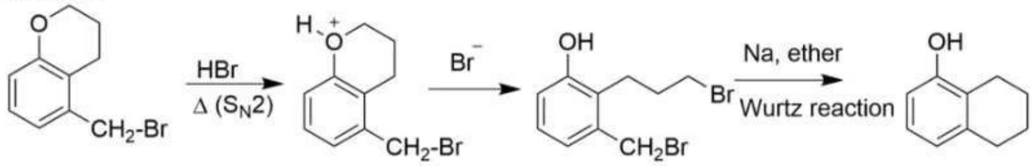


d.



Answer: c

Solution:

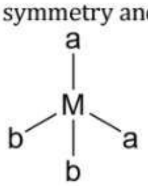


6. For the complex $[Ma_2b_2]$ if M is sp^3 or dsp^2 hybridized respectively then the total number of optical isomers respectively, are:

a. 1, 1
b. 2, 1
c. 0, 0
d. 1, 2

Answer: c

Solution:
Case 1: If M is sp^3 hybridized, the geometry will be tetrahedral. There will be a plane of symmetry and thus it does not show optical activity.



Case 2: If M is dsp^2 hybridized, the geometry will be square planar. Due to the presence of a plane of symmetry, it does not show optical activity.

7. The bond order and magnetic nature of CN^- respectively, are

a. 3, diamagnetic
b. 3, paramagnetic
c. 2.5, paramagnetic
d. 2.5, diamagnetic

Answer: a

Solution:

CN^- is a 14 electron system. The bond order and magnetism can be predicted using MOT.

The MOT electronic configuration of CN^- is:

$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\pi_{2p_x}^2\pi_{2p_y}^2\sigma_{2p_z}^2$

Bond order $= \frac{1}{2} \times (N_{bonding} - N_{antibonding}) = 3$

As CN^- does not have any unpaired electrons, and hence it is diamagnetic.

8. Which of the following is incorrect?

a. $\Lambda_m^0 NaCl - \Lambda_m^0 NaBr = \Lambda_m^0 KCl - \Lambda_m^0 KBr$
b. $\Lambda_m^0 H_2O = \Lambda_m^0 HCl + \Lambda_m^0 NaOH - \Lambda_m^0 NaCl$
c. $\Lambda_m^0 NaI - \Lambda_m^0 NaBr = \Lambda_m^0 NaBr - \Lambda_m^0 KBr$
d. $\Lambda_m^0 NaCl - \Lambda_m^0 KCl = \Lambda_m^0 NaBr - \Lambda_m^0 KBr$

Answer: c

Solution:

$\Lambda_m^0 NaI - \Lambda_m^0 NaBr = \Lambda_m^0 NaBr - \Lambda_m^0 KBr$

$[\lambda_m^0 Na^+ + \lambda_m^0 I^-] - [\lambda_m^0 Na^+ + \lambda_m^0 Br^-] = [\lambda_m^0 Na^+ + \lambda_m^0 Br^-] - [\lambda_m^0 K^+ + \lambda_m^0 Br^-]$

$$\lambda_m^0 I^- - \lambda_m^0 Br^- \neq \lambda_m^0 Na^+ - \lambda_m^0 K^+$$

9. $NaOH + Cl_2 \rightarrow A + \text{Other products}$
Hot & conc.
$Ca(OH)_2 + Cl_2 \rightarrow B + \text{Other products}$
Cold & dil.
A and B respectively are:
a. $NaClO_3$, $Ca(OCl)_2$
b. $NaClO_3$, $Ca(ClO_3)_2$
c. $NaCl$, $Ca(ClO_3)_2$
d. $NaClO$, $Ca(ClO_3)_2$

Answer: a

Solution:

$6NaOH + 3Cl_2 \rightarrow 5NaCl + NaClO_3 + 3H_2O$
$2Ca(OH)_2 + Cl_2 \rightarrow Ca(OCl)_2 + CaCl_2 + H_2O$

10. There are two beakers (I) having pure volatile solvent and (II) having a volatile solvent and a non-volatile solute. If both the beakers are placed together in a closed container then:
a. Volume of solvent beaker will decrease and solution beaker will increase
b. Volume of solvent beaker will increase and solution beaker will also increase
c. Volume of solvent beaker will decrease and solution beaker will also decrease
d. Volume of solvent beaker will increase and solution beaker will decrease

Answer: a

Solution:

Consider beaker I contains the solvent and beaker 2 contains the solution. Let the vapour pressure of the beaker I be P^o and the vapour pressure of beaker II be P^s. According to Raoult's law, the vapour pressure of the solvent (P^o) is greater than the vapour pressure of the solution (P^s)

($P^o > P^s$)

Due to a higher vapour pressure, the solvent flows into the solution. So volume of beaker II would increase.

In a closed beaker, both the liquids on attaining equilibrium with the vapour phase will end up having the same vapour pressure. Beaker II attains equilibrium at a lower vapour pressure and so in its case, condensation will occur so as to negate the increased vapour pressure from beaker I, which results in an increase in its volume.

On the contrary, since particles are condensing from the vapour phase in beaker II, the vapour pressure will decrease. Since beaker I at equilibrium attains a higher vapour pressure, there, evaporation will be favoured more so as to compensate for the decreased vapour pressure, as mentioned in the previous statement.

11. Metal with low melting point containing impurities of high melting point can be purified by

a. Zone refining
b. Vapor phase refining
c. Distillation
d. Liquation

Answer: d

Solution:

Liquation is the process of refining a metal with a low melting point containing impurities of high melting point

12. Which of the following statements are correct?

I. On decomposition of H_2O_2, O_2 gas is released.
II. 2-ethylanthraquinol is used in the preparation of H_2O_2
III. On heating $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$, O_2 gas is released.
IV. In the preparation of sodium peroxoborate, H_2O_2 is treated with sodium metaborate.

a. I,II, IV
b. II, III, IV
c. I, II, III, IV
d. I, II, III

Answer: c

Solution:

Decomposition of $H_2O_2 : 2H_2O_2(l) \rightarrow O_2(g) + 2H_2O(l)$

Industrially, H_2O_2 is prepared by the auto-oxidation of 2-alklylanthraquinols.

$2KClO_3 \xrightarrow{150-300^\circ C} 2KCl + 3O_2$

$2Pb(NO_3)_2 \xrightarrow{200-470^\circ C} 2PbO + 4NO_2 + O_2$

$2NaNO_3 \rightarrow 2NaNO_2 + O_2$

Synthesis of sodium perborate:

$Na_2B_4O_7 + 2NaOH + 4H_2O_2 \rightarrow 2NaBO_3 + 5H_2O$

13. Among the following, which is a redox reaction?

a. $N_2 + O_2 \xrightarrow{2000\text{ K}}$
b. Formation of O_3 from O_2
c. Reaction between NaOH and H_2SO_4
d. Reaction between $AgNO_3$ and NaCl

Answer: a

Solution:

$N_2 + O_2 \xrightarrow{2000\text{ K}} 2NO$: The oxidation state of N changes from 0 to +2, and the oxidation state of O changes from 0 to -2

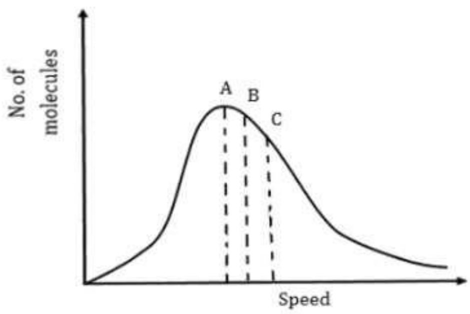
In all the remaining reactions, there is no change in oxidation states of the elements participating in the reaction.

$3O_2 \rightarrow 2O_3$

$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$ (Neutralisation reaction)

$AgNO_3 + NaCl \rightarrow NaNO_3 + AgCl$ (Double displacement)

14.



Select the correct options:

a. $A = C_{MPS}, B = C_{Average}, C = C_{RMS}$
b. $A = C_{Average}, B = C_{MPS}, C = C_{RMS}$
c. $A = C_{RMS}, B = C_{Average}, C = C_{MPS}$
d. $A = C_{Average}, B = C_{MPS}, C = C_{RMS}$

Answer: a

Solution:

$C_{RMS} = \sqrt{\frac{3RT}{M}}$

$C_{Average} = \sqrt{\frac{8RT}{\pi M}}$

$C_{MPS} = \sqrt{\frac{2RT}{M}}$

$\sqrt{3} > \sqrt{\frac{8}{\pi}} > \sqrt{2}$

$C_{RMS} > C_{Average} > C_{MPS}$

15. Which one of the following, among each pairs, will release maximum energy on gaining one electron? (A= F, Cl), (B= S, Se), (C= Li, Na)
    a. (A)= Cl, (B)= S, (C)= Li
    b. (A)= F, (B)= Se, (C)= Na
    c. (A)= F, (B)= S, (C)= Na
    d. (A)= Cl, (B)= Se, (C)= Li

Answer: a

Solution:

Element	First Electron gain enthalpy(kJ/mol)
Li	-60
Na	-53
F	-320
S	-200
Cl	-340
Se	-195

Despite F being more electronegative than Cl, due to the small size of F, Cl would have a more negative value of electron gain enthalpy because of inter-electronic repulsions.

As we go down group, the negative electron gain enthalpy decreases.

16. Which of the following statements are incorrect?
    a) Co^{3+} with strong field ligand forms a high magnetic moment complex.
    b) For Co^{3+}, if pairing energy (P)>Δ_o, then the complex formed will have t_{2g}^4, e_g^2 configuration
    c) For $[Co(en)_3]^{3+}$, $\lambda_{absorbed}$ is less than $\lambda_{absorbed}$ for $[CoF_6]^{3-}$
    d) If $\Delta_o = 18000\ cm^{-1}$ for Co^{3+}, then with same ligands for it $\Delta_t = 16000\ cm^{-1}$

    a. A, D
    b. B, C
    c. A, B
    d. A, B, C, D

Answer: a

Solution:

Co^{3+} has d^6 electronic configuration. In the presence of strong field ligand, $\Delta_o > P$. Thus the splitting occurs as: t_{2g}^6, e_g^0; so the magnetic moment is zero.

According to the spectrochemical series, en is a stronger ligand than F and therefore promotes pairing. This implies that the Δ_o of en is more than the Δ_o of F.

$$\Delta_o = \frac{hc}{\lambda_{abs}}$$

$$\Delta_t = \frac{4}{9}\Delta_o = 8000\ cm^{-1}$$

17. 0.6 g of urea on strong heating with NaOH evolves NH_3. The liberated NH_3 will react completely with which of the following HCl solutions?

a. 100 mL of 0.2 N HCl

b. 400 mL of 0.2 N HCl

c. 100 mL of 0.1 N HCl

d. 200 mL of 0.2 N HCl

Answer: a

Solution:

Moles of urea= $\left(\frac{0.6}{60}\right)$= 0.01

$$NH_2CONH_2 + 2NaOH \rightarrow Na_2CO_3 + 2NH_3$$

$$0.01 \qquad\qquad\qquad\qquad 0.02$$

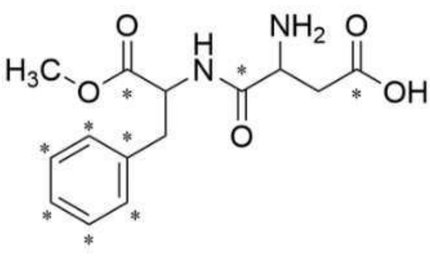
0.02 moles of NH_3 reacts with 0.02 moles of HCl.

Moles of HCl in option a= $0.2 \times \frac{100}{1000} = 0.02$

21. Number of sp^2 hybrid carbon atoms in aspartame is ____.

Answer: 9

Solution:



The marked carbons are sp^2 hybridised.

22. 3 grams of acetic acid is mixed in 250 mL of 0.1 M HCl. This mixture is now diluted to 500 mL. 20 mL of this solution is now taken in another container. $\frac{1}{2}$ mL of 5 M NaOH is added to this. Find the pH of this solution. (log 3 = 0.4771, pK_a = 4.74).

Answer: 5.22

Solution:

mmole of acetic acid in 20 mL = 2

mmole of HCl in 20 mL = 1

mmole of NaOH = 2.5

$$HCl + NaOH \rightarrow NaCl + H_2O$$

1	2.5	-	-
-	1.5	1	1

$$CH_3COOH + NaOH\ (\text{remaining}) \longrightarrow CH_3COONa + \text{water}$$

2	1.5	-	-
0.5	0	1.5	

$$pH = pK_a + \log\frac{1.5}{0.5} = 4.74 + \log 3 = 4.74 + 0.48 = 5.22$$

23. The flocculation value for As_2S_3 sol by HCl is 30 mmolL^{-1} . Calculate mass of H_2SO_4 required in grams for 250 mL sol is ____.

Answer: 0.3675 g

Solution:

For 1L sol 30 mmol of HCl is required

$\therefore$ For 1L sol 15 mmol of H_2SO_4 is required

For 250 mL of sol,

$\frac{15}{4} \times 98 \times 10^{-3}$ g of H_2SO_4= 0.3675 g

24. $NaCl \xrightarrow{K_2Cr_2O_7/Conc.H_2SO_4} (A) \xrightarrow{NaOH} (B) \xrightarrow{Dil.H_2SO_4,\ H_2O_2} (C)$

Determine the total number of atoms in per unit formula of (A), (B) & (C).

Answer: 18

Solution:

$NaCl \xrightarrow{K_2Cr_2O_7/Conc.H_2SO_4} CrO_2Cl_2 \xrightarrow{NaOH} Na_2CrO_4 + NaCl$

$Na_2CrO_4 \xrightarrow{Dil.H_2SO_4} Na_2Cr_2O_7 \xrightarrow{Dil.H_2O_2} CrO_5$

$(A) = CrO_2Cl_2$, $(B) = Na_2CrO_4$ and $(C) = CrO_5$

25. Calculate the $\Delta H_f°$ (in kJ/mol) for $C_2H_6(g)$, if $\Delta H_c° [C_{(graphite)}] = -393.5$ kJ/mol, $\Delta H_c° [H_2(g)] = -286$ kJ/mol and $\Delta H_c° [C_2H_6(g)] = -1560$ kJ/mol .

Answer: -85 kJ/mol

Solution:

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)\ \Delta H_c° = -393.5$ kJ/mol......................................(1)

$H_2(g) + 0.5O_2(g) \rightarrow H_2O(l)\quad \Delta H_c° = -286$ kJ/mol ....(2)

$C_2H_6(g) + 3.5O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)\ \Delta H_c° = -1560$ kJ/mol .... (3)

$2 \times (-393.5) + 3 \times (-286) - (-1560) = -85$ kJ/mol

By inverting (3) and multiplying (1) by 2 and (2) by 3 and adding, we get,

$2 \times (-393.5) + 3 \times (-286) - (-1560) = -85$ kJ/mol

JEE Main 2020 Paper

Date: **7th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

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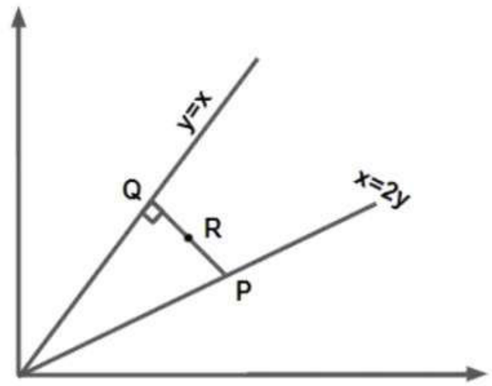
1. From any point P on the line $x = 2y$, a perpendicular is drawn on $y = x$. Let the foot of perpendicular be Q. Find the locus of mid point of PQ.

a. $5x = 7y$
b. $2x = 3y$
c. $7x = 5y$
d. $3x = 2y$

Answer: (a)

Solution:



Let R be the midpoint of PQ

PQ is perpendicular on line $y = x$

$\therefore$ Equation of the line PQ can be written as $y = -x + c$

$y = -x + c$ intersects $y = x$ at Q: $\left(\frac{c}{2}, \frac{c}{2}\right)$

$y = -x + c$ intersects $x = 2y$ at P: $\left(\frac{2c}{3}, \frac{c}{3}\right)$

$\therefore$ Midpoint R: $\left(\frac{7c}{12}, \frac{5c}{12}\right)$

Locus of R : $x = \frac{7c}{12}$

$y = \frac{5c}{12}$

$\Rightarrow 5x = 7y$

2. Let θ_1 and θ_2 (where $\theta_1 < \theta_2$) are two solutions of $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0, 2\pi)$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to

a. $\frac{\pi}{9}$

b. $\frac{2\pi}{3}$

c. $\frac{\pi}{3} + \frac{1}{6}$

d. $\frac{\pi}{3}$

Answer: (d)

Solution:

$2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0, 2\pi)$

$\Rightarrow 2\text{cosec}^2\theta - 2 - 5\text{cosec}\,\theta + 4 = 0$

$\Rightarrow 2\text{cosec}^2\theta - 4\text{cosec}\,\theta - \text{cosec}\,\theta + 2 = 0$

$\Rightarrow \text{cosec}\,\theta = 2$ or $\frac{1}{2}$ (Not possible)

As $\theta \in [0, 2\pi)$,

$\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{5\pi}{6}$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(1 + \cos 6\theta)}{2} \, d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) + \frac{\sin 6\theta}{12}\Bigg|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{\pi}{3}$$

3. Coefficient of x^7 in $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \cdots + x^{10}$ is

a. 260
b. 210
c. 420
d. 330

Answer: (d)

Solution:

Coefficient of x^7 in $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \cdots + x^{10}$

Applying sum of terms of G.P. = $\dfrac{(1+x)^{10}\left(1-\left(\frac{x}{1+x}\right)^{11}\right)}{\left(1-\frac{x}{1+x}\right)} = (1+x)^{11} - x^{11}$

Coefficient of $x^7 \Longrightarrow {}^{11}C_7 = 330$

4. Let α and β are the roots of $x^2 - x - 1 = 0$ such that $P_k = \alpha^k + \beta^k, k \geq 1$ then which one is incorrect?

a. $P_5 = P_2 \times P_3$
b. $P_1 + P_2 + P_3 + P_4 + P_5 = 26$
c. $P_5 = 11$
d. $P_3 = P_5 - P_4$

Answer: (a)

Solution:

Given α, β are the roots of $x^2 - x - 1 = 0$

$\Rightarrow \alpha + \beta = 1 \;\&\; \alpha\beta = -1$

$\Rightarrow \alpha^2 = \alpha + 1 \;\&\; \beta^2 = \beta + 1$

$P_k = \alpha^{k-2}\alpha^2 + \beta^{k-2}\beta^2$

$P_k = \alpha^{k-2}(\alpha + 1) + \beta^{k-2}(\beta + 1)$

$P_k = \alpha^{k-1} + \beta^{k-1} + \alpha^{k-2} + \beta^{k-2}$

$\Rightarrow P_k = P_{k-1} + P_{k-2}$

$\Rightarrow P_3 = P_2 + P_1 = 4$

$P_4 = P_3 + P_2 = 7$

$P_5 = P_4 + P_3 = 11$

$\therefore P_5 \neq P_2 P_3$ & $P_1 + P_2 + P_3 + P_4 + P_5 = 26$

& $P_3 = P_5 - P_4$

5. The area bounded by $4x^2 \leq y \leq 8x + 12$ is

a. $\frac{127}{3}$

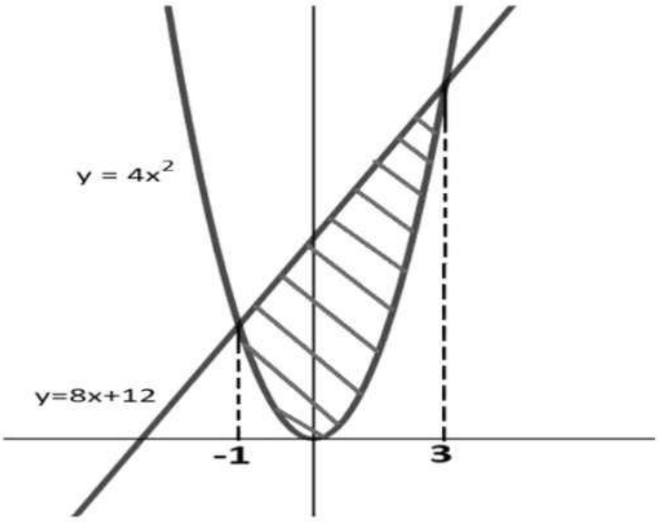
b. $\frac{125}{3}$

c. $\frac{128}{3}$

d. $\frac{124}{3}$

Answer: (c)

Solution:



For point of intersection,

$4x^2 = 8x + 12$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3, -1$

Area bounded is given by

$A = \int_{-1}^{3} (8x + 12 - 4x^2)dx$

$$A = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right]_{-1}^{3}$$

$$A = (36 + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$

$$A = 44 - \frac{4}{3} = \frac{128}{3}$$

6. Contrapositive of $A \subset B$ and $B \subset C$ then $C \subset D$

a. $C \not\subset D$ or $A \not\subset B$ or $B \not\subset C$
b. $C \subset D$ or $A \not\subset B$ and $B \not\subset C$
c. $C \subset D$ and $A \not\subset B$ or $B \not\subset C$
d. $C \subset D$ or $A \not\subset B$ or $B \not\subset C$

Answer: (d)

Solution:

Given statements: $A \subset B$ and $B \subset C$

Let $A \subset B$ be p

$B \subset C$ be q

$C \subset D$ be r

Modified statement: $(p \wedge q) \Rightarrow r$

Contrapositive: $\sim r \Rightarrow \sim (p \wedge q)$

$\therefore r \vee (\sim p \vee \sim q)$

$\Rightarrow C \subset D$ or $A \not\subset B$ or $B \not\subset C$

7. Let $3 + 4 + 8 + 9 + 13 + 14 + 18 + \ldots \ldots \ldots .40$ terms $= S$. If $S = (102)m$ then $m =$

a. 5
b. 10
c. 25
d. 20

Answer: (d)

Solution:

$S = \underline{3 + 4} + \underline{8 + 9} + 13 + 14 + \ldots \ldots$ 40 terms

$S = 7 + 17 + 27 + 37 + \cdots \ldots \ldots .20$ terms

$$S = \frac{20}{2}[14 + (19)10] = 20 \times 102$$

$\therefore m = 20$

8. $({}^{36}C_{r+1}) \times (k^2 - 3) = {}^{35}C_r \times 6$, then the number of ordered pairs (r, k), where $k \in \mathbf{I}$, are

a. 2
b. 6
c. 3
d. 4

Answer: (d)

Solution:

using ${}^{36}C_{r+1} = \frac{36}{r+1} \times {}^{35}C_r$

$\frac{36}{r+1} \times {}^{35}C_r \times (k^2 - 3) = {}^{35}C_r \times 6$

$$k^2 - 3 = \frac{r+1}{6}$$

$$k^2 = \frac{r+1}{6} + 3$$

$k \in \mathbf{I}$

$r \rightarrow$ Non-negative integer $0 \leq r \leq 35$

$r = 5 \Rightarrow k = \pm 2$

$r = 35 \Rightarrow k = \pm 3$

No. of ordered pairs $(r, k) = 4$

9. Let $f(x)$ be a five-degree polynomial which has critical points $x = \pm 1$ and $\lim_{x \to 0} \left(2 + \frac{f(x)}{x^3}\right) = 4$ then which one is incorrect.

a. $f(x)$ has minima at $x = 1$ and maxima at $x = -1$
b. $f(1) - 4f(-1) = 4$
c. $f(x)$ has maxima at $x = 1$ and minima at $x = -1$
d. $f(x)$ is odd

Answer: (a)

Solution:

Given $\lim_{x\to0}\left(2+\frac{f(x)}{x^3}\right)=4$

$\lim_{x\to0}\frac{f(x)}{x^3}=2$

$\lim_{x\to0}\frac{f(x)}{x^3}$ Limit exists and it is finite

$\therefore f(x)=ax^5+bx^4+cx^3$

$\Rightarrow \lim_{x\to0}(ax^2+bx+c)=2$

$c=2$

Also $f'(x)=5ax^4+4bx^3+6x^2$

$f'(1)=5a+4b+6=0$

$f'(-1)=5a-4b+6=0$

$b=0, \quad a=-\frac{6}{5}$

$f(x)=-\frac{6}{5}x^5+2x^3 \quad \Rightarrow f(x)$ is odd

$f'(x)=-6x^4+6x^2$

$f''(x)=-24x^3+12x \qquad (f''(1)<0, \ f''(-1)>0)$

At $x=-1$ local minima at $x=1$ local maxima

And $f(1)-4f(-1)=4$

10. If LMVT is applicable on $f(x)=x^3-4x^2+8x+11$ in $[0,1]$, the value of c is

a. $\frac{4+\sqrt{5}}{3}$

b. $\frac{4+\sqrt{7}}{3}$

c. $\frac{4-\sqrt{7}}{3}$

d. $\frac{4-\sqrt{5}}{3}$

Answer: (c)

Solution:

LMVT is applicable on $f(x)$ in $[0,1]$, therefore it is continuous and differentiable in $[0,1]$

Now, $f(0) = 11, f(1) = 16$

$f'(x) = 3x^2 - 8x + 8$

$\therefore f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1}$

$\Rightarrow 3c^2 - 8c + 8 = 5$

$\Rightarrow 3c^2 - 8c + 3 = 0$

$\Rightarrow c = \frac{8\pm2\sqrt{7}}{6} = \frac{4\pm\sqrt{7}}{3}$

As $c \in (0,1)$

We get, $c = \frac{4-\sqrt{7}}{3}$

11. Consider there are 5 machines. Probability of a machine being faulty is $\frac{1}{4}$. Probability of at most two machines being faulty is $\left(\frac{3}{4}\right)^3 k$, then the value of k is

a. $\frac{17}{4}$

b. $\frac{17}{8}$

c. $\frac{17}{2}$

d. 4

Answer: (b)

Solution:

$P(\text{machine being faulty}) = p = \frac{1}{4}$

$\therefore q = \frac{3}{4}$

$P(\text{at most two machines being faulty}) = P(\text{zero machine being faulty})$

$+P(\text{one machine being faulty}) + P(\text{two machines being faulty})$

$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3$

$= q^5 + 5pq^4 + 10p^2q^3$

$= \left(\frac{3}{4}\right)^5 + 5 \times \frac{1}{4}\left(\frac{3}{4}\right)^4 + 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

$= \left(\frac{3}{4}\right)^3 \left[\frac{9}{16} + \frac{15}{16} + \frac{10}{16}\right]$

$= \left(\frac{3}{4}\right)^3 \times \frac{34}{16} = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$

$\therefore k = \frac{17}{8}$

12. $a_1, a_2, a_3, \ldots, a_9$ are in geometric progression where $a_1 < 0$ and $a_1 + a_2 = 4, a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to

a. 171
b. -513
c. $-\frac{511}{3}$
d. -171

Answer: (d)

Solution:

$a_1 + a_2 = 4 \Rightarrow a + ar = 4 \Rightarrow a(1+r) = 4$

$a_3 + a_4 = 16 \Rightarrow ar^2 + ar^3 = 16 \Rightarrow ar^2(1+r) = 16 \Rightarrow 4r^2 = 16$

$\Rightarrow r = \pm 2$

If $r = 2, a = \frac{4}{3}$ which is not possible as $a_1 < 0$

If $r = -2, a = -4$

$\sum_{i=1}^{9} a_i = \frac{a(r^9-1)}{r-1} = \frac{(-4)[(-2)^9-1]}{-3} = \frac{4}{3}(-512-1) = 4(-171)$

$\lambda = -171$

13. If $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is

a. $\frac{2}{\sqrt{5}}$
b. $-\frac{\sqrt{5}}{2}$
c. $-\frac{\sqrt{5}}{4}$
d. $\frac{\sqrt{5}}{2}$

Answer: (b)

Solution:

$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

Differentiating w.r.t. x on both the sides, we get:

$y'\sqrt{1-x^2} + y \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -\sqrt{1-y^2} - x \times \frac{1}{2\sqrt{1-y^2}} \times (-2y)y'$

$\Rightarrow y'\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}y' = 0$

$\Rightarrow y'\left[\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right] = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$

Putting $x = \frac{1}{2}, y = -\frac{1}{4}$

$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{\frac{1}{8}}{\frac{\sqrt{15}}{4}}\right] = -\frac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{15}}\right] = -\frac{1}{4\sqrt{3}} - \frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{45}+1}{2\sqrt{15}}\right] = -\frac{1+\sqrt{45}}{4\sqrt{3}}$

$\Rightarrow y' = -\frac{\sqrt{5}}{2}$

14. Let $A = [a_{ij}], B = [b_{ij}]$ are two 3×3 matrices such that $b_{ij} = \lambda^{i+j-2}a_{ij}$ and $|B| = 81$. Find $|A|$ if $\lambda = 3$

a. $\frac{1}{81}$

b. $\frac{1}{27}$

c. $\frac{1}{9}$

d. 3

Answer: (c)

Solution:

$b_{ij} = \lambda^{i+j-2}a_{ij}\ , \lambda = 3$

$$B = \begin{bmatrix} 3^0a_{11} & 3a_{12} & 3^2a_{13} \\ 3a_{21} & 3^2a_{22} & 3^3a_{23} \\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0 a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

Taking 3^2 Common each from C_3 & R_3

$$|B| = 81 \begin{vmatrix} a_{11} & 3a_{12} & a_{13} \\ 3a_{21} & 3^2 a_{22} & 3a_{23} \\ a_{31} & 3a_{32} & a_{33} \end{vmatrix}$$

Taking 3 common each from C_2 & R_2

$$|B| = 81(9) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Given $|B| = 81$

$\Rightarrow 81 = 81(9)|A|$

$\Rightarrow |A| = \dfrac{1}{9}$

15. Pair of tangents are drawn from the origin to the circle $x^2 + y^2 - 8x - 4y + 16 = 0$, then the square of length of chord of contact is

a. $\frac{8}{5}$

b. $\frac{8}{13}$

c. $\frac{24}{5}$

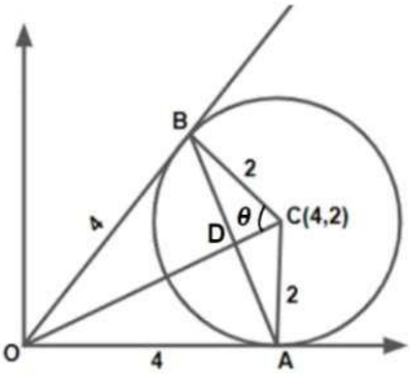
d. $\frac{64}{5}$

Answer: (d)

Solution:

$x^2 + y^2 - 8x - 4y + 16 = 0$

$(x-4)^2 + (y-2)^2 = 4 \Rightarrow$ Centre $(4,2)$, radius (2)



$OA = 4 = OB$

In ΔOBC

$\tan\theta = \frac{4}{2} = 2 \Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$

In ΔBDC

$\sin\theta = \frac{BD}{2} \Rightarrow BD = \frac{4}{\sqrt{5}}$

Length of chord of contact $(AB) = \frac{8}{\sqrt{5}}$

<u>Alternative</u>

(l) length of tangent = 4

(r) radius =2

$\Rightarrow$Length of chord of contact = $\frac{2lr}{\sqrt{(l^2+r^2)}}$

Square of length of chord of contact = $\frac{64}{5}$

16. Let $y(x)$ is the solution of differential equation $(y^2 - x)\frac{dy}{dx} = 1$ and $y(0) = 1$, then find the value of x where the curve cuts the $x - axis$.

a. $2 - e$
b. 2
c. $2 + e$
d. e

Answer: (a)

Solution:

$$(y^2 - x)\frac{dy}{dx} = 1$$

$$\frac{dx}{dy} + x = y^2$$

$$xe^y = \int y^2 e^y dy$$

$$x = y^2 - 2y + 2 + ce^{-y}$$

Given $y(0) = 1$

$\Rightarrow c = -e$

$\therefore$ Solution is $x = y^2 - 2y + 2 - e^{-y+1}$

$\therefore$ The value of x where the curve cuts the $x - axis$ will be at $x = 2 - e$

17. Let $4\alpha \int_{-1}^{2} e^{-\alpha|x|}\, dx = 5$ then $\alpha =$

a. $\ln\sqrt{2}$

b. $\ln\frac{3}{4}$

c. $\ln 2$

d. $\ln\frac{4}{3}$

Answer: (c)

Solution:

$$4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$$

$$4\alpha\left[\int_{-1}^{0} e^{-\alpha|x|} dx + \int_{0}^{2} e^{-\alpha|x|} dx\right] = 5$$

$$= 4\alpha\left[\int_{-1}^{0} e^{\alpha x} dx + \int_{0}^{2} e^{-\alpha x} dx\right] = 5$$

$$= 4\alpha\left[\left(\frac{1-e^{-\alpha}}{\alpha}\right)+\left(\frac{e^{-2\alpha}-1}{-\alpha}\right)\right] = 5$$

$$= 4[1-e^{-2\alpha}-e^{-\alpha}+1] = 5$$

Let $e^{-\alpha} = t$

$$\Rightarrow\ -4t^2-4t+3=0$$

$$\Rightarrow\ t=\frac{1}{2}=e^{-\alpha}\Rightarrow \alpha=\ln 2$$

18. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\lambda = \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ then $(\lambda, d) =$

a. $\left(\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$

b. $\left(-\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$

c. $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$

d. $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$

Answer: (c)

Solution:

Given $\vec{a}+\vec{b}+\vec{c}=\vec{0}\qquad \lambda=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

$$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right|^2 = \overrightarrow{|0|}^2$$

$$\overrightarrow{|a|}^2+\overrightarrow{|b|}^2+\overrightarrow{|c|}^2+\ 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$\lambda = -\frac{3}{2}$$

Also $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$$\Rightarrow \vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\left(-\vec{a}-\vec{b}\right)+\left(-\vec{a}-\vec{b}\right)\times\vec{a}$$

$$\Rightarrow \vec{d}=\vec{a}\times\vec{b}+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}$$

$$\Rightarrow \vec{d}=3\left(\vec{a}\times\vec{b}\right)=3\vec{a}\times\vec{b}$$

19. $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$, then the distance between foci of ellipse is

a. $2\sqrt{5}$
b. $2\sqrt{7}$
c. 4
d. $2\sqrt{3}$

Answer: (b)

Solution:

$3x + 4y = 12\sqrt{2}$ is tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $y = mx + \sqrt{a^2m^2 + 9}$

Now, $3x + 4y = 12\sqrt{2} \Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$

$\Rightarrow m = -\frac{3}{4}$ and $\sqrt{a^2m^2 + 9} = 3\sqrt{2}$

$$\Rightarrow a^2\left(-\frac{3}{4}\right)^2 + 9 = 18$$

$$\Rightarrow a^2 \times \frac{9}{16} = 9$$

$$\Rightarrow a^2 = 16 \Rightarrow a = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

Distance between foci is $2ae = 2 \times 4 \times \frac{\sqrt{7}}{4} = 2\sqrt{7}$

20. If mean and variance of 2, 3, 16, 20, 13, 7, x, y are 10 and 25 respectively then xy is equal to ______.

Answer: (124)

Solution:

Mean $= 10 \Rightarrow \frac{61 + x + y}{8} = 10$

$\Rightarrow x + y = 19$

Variance $= \frac{\sum x_i^2}{n} - (\bar{x})^2$

$\Rightarrow 25 = \frac{2^2+3^2+16^2+20^2+13^2+7^2+x^2+y^2}{8} - 100$

$\Rightarrow 1000 = 887 + x^2 + y^2$

$\Rightarrow x^2 + y^2 = 113$

$\Rightarrow (x + y)^2 - 2xy = 113$

$\Rightarrow 361 - 2xy = 113$

So, $xy = 124$

21. If $Q \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ is foot of perpendicular drawn from $P(1, 0, 3)$ onto a line L and line L is passing through $(\alpha, 7,1)$, then value of α is ________.

Answer: (4)

Solution:

Direction ratios of line L: $\left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right)$

$= \left(\frac{3\alpha - 5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$

Direction ratios of PQ: $\left(-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3}\right)$

As line L is perpendicular to PQ

So, $\left(\frac{3\alpha-5}{3}\right)\left(-\frac{2}{3}\right) + \left(\frac{14}{3}\right)(-\frac{7}{3}) + \left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right) = 0$

$\Rightarrow -6\alpha + 10 - 98 + 112 = 0$

$\Rightarrow 6\alpha = 24 \Rightarrow \alpha = 4$

22. If system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $3x + 2y + \lambda z = \mu$ has more than 2 solutions, then $(\mu - \lambda^2)$ is _______.

Answer: (13)

Solution:

The system of equations has more than 2 solutions

$\therefore\ D = D_3 = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow 2\lambda - 6 - \lambda + 9 + 2 - 6 = 0$$

$\Rightarrow \lambda = \mathbf{1}$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow 2\mu - 20 - \mu + 30 - 24 = 0$$

$\Rightarrow \mu = \mathbf{14}$

So, $\mu - \lambda^2 = 13$

23. If $f(x)$ is defined in $x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$ &

$$f(x) = \begin{cases} \left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases}$$

The value of k such that $f(x)$ is continuous is ______.

Answer: (5)

Solution:

As $f(x)$ is continuous

$\Rightarrow \lim_{x\to 0} f(x) = f(0) = k$

$\Rightarrow \lim_{x\to 0}\left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right) = k$

$\Rightarrow \lim_{x\to 0}\frac{3\log(1+3x)}{3x} - \lim_{x\to 0}\frac{(-2)\log(1-2x)}{(-2x)} = k$

$\Rightarrow 3 + 2 = k \Rightarrow k = 5$

24. Let $X = \{x: 1 \le x \le 50, x \in \mathbf{N}\}$, $A = \{x: x \text{ is a multiple of } 2\}$, $B = \{x: x \text{ is a multiple of } 7\}$. Then the number of elements in the smallest subset of X which contain elements of both A and B is ______.

Answer: (29)

Solution:

$A = \{x: x \text{ is multiple of } 2\} = \{2,4,6,8, \dots\}$

$B = \{x: x \text{ is multiple of } 7\} = \{7,14,21, \dots\}$

$X = \{x: 1 \le x \le 50, x \in \mathbf{N}\}$

Smallest subset of X which contains elements of both A and B is a set with multiples of 2 or 7 less than 50.

$P = \{x: x \text{ is a multiple of 2 less than or equal to } 50\}$

$Q = \{x: x \text{ is a multiple of 7 less than or equal to } 50\}$

$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$= 25 + 7 - 3$

$= 29$

JEE Main 2020 Paper

Date of Exam: **7th January (Shift II)**
Time: **2:30 pm – 5:30 pm**
Subject: **Physics**

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1. If the weight of an object at the pole is 196 N, then the weight of the object at the equator is? ($g = 10\ m/s^2$; $the\ radius\ of\ earth = 6400\ \ km$)

   a. 194.32 N b. 194.66 N
   c. 195.32 N d. 195.66 N

   Solution: (c)
   Weight of the object at the pole, $W = mg = 196\ N$
   Mass of the object, $m = \frac{W}{g} = \frac{196}{10} = 19.6\ kg$
   Weight of object at the equator(W') = Weight at pole − Centrifugal acceleration
   $$W' = mg - m\omega^2 R$$
   $$196 - (19.6)\left(\frac{2\pi}{24 \times 3600}\right)^2 \times 6400 \times 10^3 = 195.33\ N$$

2. In a house 15 bulbs of 45 W, 15 bulbs of 100 W, 15 bulbs of 10 W and two heaters of 1 kW each is connected to 220 V mains supply. The minimum fuse current will be

   a. 5 A b. 20 A
   c. 25 A d. 15 A

   Solution: (b)
   Total power consumption of the house(P) = Number of appliances × Power rating of each appliance
   $$P = (15 \times 45) + (15 \times 100) + (15 \times 10) + (2 \times 1000) = 4325\ W$$
   So, minimum fuse current $I = \frac{Total\ power\ consumption}{Voltage\ supply} = \frac{4325}{220} A = 19.66\ A$

3. In an adiabatic process, the volume is doubled. Find the ratio of final average relaxation time and initial relaxation time. Given $\frac{C_p}{C_v} = \gamma$

a. $\frac{1}{2}$

b. $(2)^{\frac{1+\gamma}{2}}$

c. $\left(\frac{1}{2}\right)^{\gamma}$

d. $\left(\frac{1}{2}\right)^{\frac{\gamma}{2}+1}$

Solution:(b)

Relaxation time (τ) dependence on volume and temperature can be given by $(\tau) \propto \frac{V}{\sqrt{T}}$

Also, for an adiabatic process,

$$T \propto \frac{1}{V^{\gamma-1}}$$

$$\Rightarrow \tau \propto V^{\frac{1+\gamma}{2}}$$

Thus,

$$\frac{\tau_f}{\tau_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}}$$

$$\frac{\tau_f}{\tau_i} = (2)^{\frac{1+\gamma}{2}}$$

4. A block of mass 10 kg is suspended from a string of length 4 m. When pulled by a force F along horizontal from the midpoint. The upper half of the string makes 45^o with the vertical, value of F is

a. 100 N

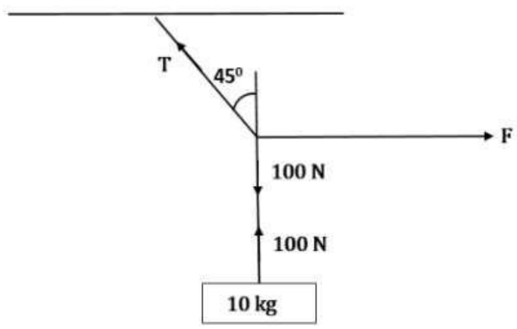
b. 90 N

c. 75 N

d. 70 N

Solution: (a)

Equating the vertical and horizontal components of the forces acting at point



$$\frac{T}{\sqrt{2}} = 100$$

$$\frac{T}{\sqrt{2}} = F$$
$$F = 100\ N$$

5. The surface mass density of a disc varies with radial distance as $\sigma = A + Br$, where A and B are positive constants. The moment of inertia of the disc about an axis passing through its centre and perpendicular to the plane is

a. $2\pi a^4\left(\frac{A}{4}+\frac{Ba}{5}\right)$
b. $2\pi a^4\left(\frac{4A}{4}+\frac{B}{5}\right)$
c. $\pi a^4\left(\frac{A}{4}+\frac{Ba}{5}\right)$
d. $2\pi a^4\left(\frac{A}{5}+\frac{Ba}{4}\right)$

Solution: (a)

$$\sigma = A + Br$$
$$\int dm = \int (A+Br)2\pi r dr$$
$$I = \int dm\, r^2$$
$$=\int_0^a (A+Br)2\pi r^3\, dr$$
$$=2\pi\left(A\frac{a^4}{4}+B\frac{a^5}{5}\right)$$
$$= 2\pi a^4\left(\frac{A}{4}+\frac{Ba}{5}\right)$$

6. Cascaded Carnot engine is an arrangement in which heat sink of one engine is source for other. If high temperature for one engine is T_1, low temperature for other engine is T_2 (Assume work done by both engines is same). Calculate lower temperature of first engine.

a. $\frac{2T_1T_2}{T_1+T_2}$
b. $\frac{T_1+T_2}{2}$
c. 0
d. $\sqrt{T_1T_2}$

Solution:
(b)
Heat input to 1st engine= Q_H
Heat rejected from 1st engine= Q_L
Heat rejected from 2nd engine= Q_L
Work done by 1st engine = Work done by 2nd engine

$$Q_H - Q_L = Q_L - Q_L$$
$$2\,Q_L = Q_H + Q_L$$

$$2 = \frac{T_1}{T} + \frac{T_2}{T}$$
$$T = \frac{T_1 + T_2}{2}$$

7. Activity of a substance changes from 700 s^{-1} to 900 s^{-1} in 30 minutes. Find its half-life in minutes.

a. 66
b. 62
c. 56
d. 50

Solution:
(b)
Using the half-life equation,
$\ln\frac{A_0}{A_t} = \lambda t$
At half-life, $t = t_{\frac{1}{2}}$ and $A_t = \frac{A_0}{2}$
$\Rightarrow \ln 2 = \lambda\, t_{\frac{1}{2}}$ ------------- (1)
Also given
$\ln\frac{500}{700} = \lambda\,(30)$ ------------- (2)
Dividing the equations,
$$\frac{\ln 2}{\ln\left(\frac{7}{5}\right)} = \frac{t_{\frac{1}{2}}}{30}$$
$\Rightarrow t_{\frac{1}{2}} = 61.8$ minutes

8. In YDSE, separation between slits is 0.15 mm, distance between slits and screen is 1.5 m and wavelength of light is 589 nm. Then, fringe width is

a. 5.9 mm
b. 3.9 mm
c. 1.9 mm
d. 2.3 mm

Solution:
(a)
Given,
Maximum diameter of pipe = 6.4 cm
Minimum diameter of pipe = 4.8 cm

$$\beta = \lambda\frac{D}{d} = \frac{589 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$$
$$= 5.9\, mm$$

9. An ideal fluid is flowing in a pipe in streamline flow. Pipe has maximum and minimum diameter of 6.4 cm and 4.8 cm respectively. Find out the ratio of minimum to maximum velocity.

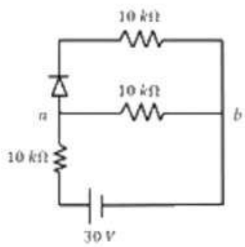
a. $\frac{81}{256}$
b. $\frac{9}{16}$
c. $\frac{3}{4}$
d. $\frac{3}{16}$

Solution:
(b)

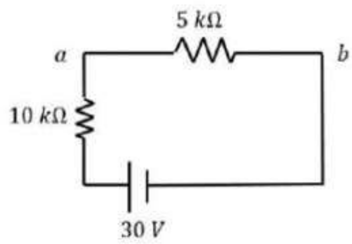
Using equation of continuity
$A_1V_1 = A_2V_2$
$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

10. There is an electric circuit as shown in the figure. Find potential difference between points a and b

a. 0 V
b. 15 V
c. 10 V
d. 5 V



Solution:
(c)
Diode is in forward bias, so it will behave as simple wire. So, the circuit effectively becomes



$V_{ab} = \frac{30}{5+10} \times 5 = 10\ V$

11. A particle of mass m and positive charge q is projected with a speed of V_o in y–direction in the presence of electric and magnetic field and both of them are in x–direction. Find the instant of time at which the speed of particle becomes double the initial speed.

a. $t = \frac{mV_o\sqrt{3}}{qE}$

b. $t = \frac{mV_o\sqrt{2}}{qE}$

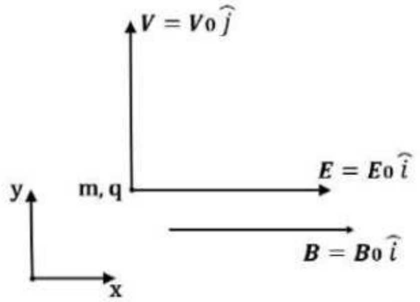
c. $t = \frac{mV_o}{qE}$

d. $t = \frac{mV_o}{2qE}$

Solution:
(a)

As $\vec{V} = V_o\hat{j}$ (magnitude of velocity does not change in y–z plane)

$$(2V_o)^2 = V_o^2 + V_x^2$$
$$V_x = \sqrt{3}V_o$$



$$\therefore\ \sqrt{3}V_x = 0 + \frac{qE}{m}t \Rightarrow t = \frac{mv_o\sqrt{3}}{qE}$$

12. Two sources of sound moving with the same speed V and emitting frequencies of 1400 Hz are moving such that one source s_1 is moving towards the observer and s_2 is moving away from the observer. If observer hears a beat frequency of 2 Hz, then find the speed of the source (Given $V_{sound} \gg V_{source}$ and $V_{sound} = 350\ m/s$.)

a. $\frac{1}{4}$

b. 4

c. 2

d. $\frac{1}{2}$

Solution:

(a)

$$f_0\left(\frac{C}{C-V}\right)-f_0\left(\frac{C}{C+V}\right)=2$$
$$V=\frac{1}{4}\,m/s$$

13. An electron and a photon have same energy E. Find the de Broglie wavelength of electron to wavelength of photon. (Given mass of electron is m and speed of light is c)

a. $\frac{2}{C}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

b. $\frac{1}{C}\left(\frac{E}{2m}\right)^{\frac{1}{3}}$

c. $\frac{1}{C}\left(\frac{E}{m}\right)^{\frac{1}{2}}$

d. $\frac{1}{C}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

Solution:

(d)

λ_d for electron $=\frac{h}{\sqrt{2mE}}$

λ for photon $=\frac{hC}{E}$

Ratio $=\frac{h}{\sqrt{2mE}}\frac{E}{hC}=\frac{1}{C}\sqrt{\frac{E}{2m}}$

14. A ring is rotated about diametric axis in a uniform magnetic field perpendicular to the plane of the ring. If initially the plane of the ring is perpendicular to the magnetic field. Find the instant of time at which EMF will be maximum and minimum respectively.

a. 2.5 sec, 5 sec

b. 5 sec, 7.5 sec

c. 2.5 sec, 7.5 sec

d. 10 sec, 5 sec

Solution:

(a)

$$\omega=\frac{2\pi}{T}=\frac{\pi}{5}$$

When $\omega t=\frac{\pi}{2}$

Then φ_{flux} will be minimum

$\therefore$ e will be maximum

$t = \frac{\frac{\pi}{2}}{\frac{\pi}{5}} = 2.5\ sec$

When $\omega t = \pi$

Then φ_{flux} will be maximum

$\therefore e$ will be minimum

$t = \frac{\pi}{\frac{\pi}{5}} = 5\ sec$

15. Electric field in space is given by $\vec{E}(t) = \frac{E_0\,(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}\cos(\omega t + kz)$. A positively charged particle at $(0, 0, \pi/k)$ is given velocity $v_0\hat{k}$ at $t = 0$. Direction of force acting on particle is

a. $f = 0$

b. Antiparallel to $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

c. Parallel to $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

d. $\hat{k}$

Solution:

(b)

Force due to electric field is in direction $-\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

Because at $t = 0, E = -\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}E_0$

Force due to magnetic field is in direction $q(\vec{v} \times \vec{B})$ and $\vec{v} \parallel \hat{k}$

$\therefore$ It is parallel to $\vec{E}$

$\therefore$ Net force is antiparallel to $\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$.

16. Focal length of convex lens in air is $16\ cm$ $\left(\mu_{glass} = 1.5\right)$. Now the lens is submerged in liquid of refractive index 1.42. Find the ratio of focal length in medium to focal length in air.

a. 9

b. 17

c. 1

d. 5

Solution:

(a)

$$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_g} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_m} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{f_a}{f_m} = \frac{\left(\frac{\mu_g}{\mu_m} - 1\right)}{\left(\frac{\mu_g}{\mu_a} - 1\right)} = \frac{\left(\frac{1.50}{1.42} - 1\right)}{\left(\frac{1.50}{1} - 1\right)} = \frac{0.08}{(1.92)(0.5)}$$

$$\frac{f_m}{f_a} = \frac{(1.42)(0.5)}{0.08} = 8.875 = 9$$

17. A lift of mass 920 kg has a capacity of 10 persons. If average mass of person is 68 kg. Friction force, between lift and lift shaft is 6000 N. The minimum power of motor required to move the lift upward with constant velocity 3 m/s is $[g = 10\ m/s^2]$

a. 66000 W
b. 63248 W
c. 48000 W
d. 56320 W

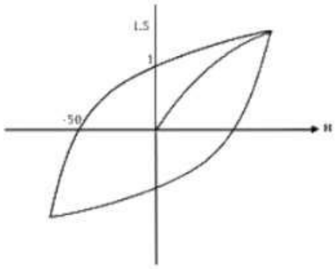
Solution:
(a)
Net force on motor will be

$$F_m = [920 + 68(10)]g + 6000$$
$$F_m = 22000\ N$$

So, required power for motor

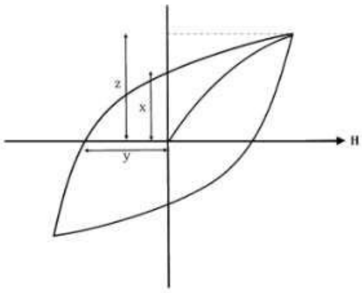
$$P_m = \overrightarrow{F_m} \cdot \vec{v}$$
$$= 22000 \times 3$$
$$= 66000\ W$$

18. The hysteresis curve for a material is shown in the figure. Then for the material retentivity, coercivity and saturation magnetization respectively will be



a. 50 A/m, 1 T, 1.5 T
b. 1.5 T, 50 A/m, 1T
c. 1 T, 50 A/m, 1.5 T
d. 50 A/m, 1.5 T, 1 T

Solution:
(c)



x = retentivity
y = coercivity
z = saturation magnetization

19. An inductor of inductance 10 mH and a resistance of 5Ω is connected to a battery of 20 V at $t = 0$. Find the ratio of current in the circuit at $t = \infty$ to current at $t = 40\ sec$.

a. 1.06
b. 1.48
c. 1.15
d. 0.84

Solution:
(a)

$$i = i_o\left(1 - e^{\frac{-t}{L/R}}\right)$$
$$= \frac{20}{5}\left(1 - e^{\frac{-t}{0.01/5}}\right)$$
$$= 4(1 - e^{-500t})$$
$$i_\infty = 4$$
$$i_{40} = 4(1 - e^{-500\times40}) = 4\left(1 - \frac{1}{(e^2)^{10000}}\right) = 4\left(1 - \frac{1}{7.29^{10000}}\right)$$

$\frac{i_\infty}{i_{40}} \approx 1$ (Slightly greater than one)

20. Find the dimensions of $\frac{B^2}{2\mu_o}$

a. $ML^{-1}T^{-2}$
b. ML^2T^{-2}
c. $ML^{-1}T^2$
d. $ML^{-2}T^{-1}$

Solution:

(a)

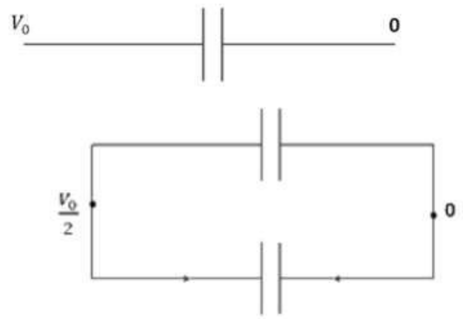
Energy density in magnetic field $= \frac{B^2}{2\mu_o}$

$$= \frac{\text{Force} \times \text{displacement}}{(\text{displacement})^3} = \frac{MLT^{-2}.L}{(L)^3} = ML^{-1}T^{-2}$$

21. A capacitor of 60 pF charged to 20 V. Now, the battery is removed, and this capacitor is connected to another identical uncharged capacitor. Find heat loss in nJ.

Solution:

(6)



$V_0 = 20\ V$

Initial potential energy $U_i = \frac{1}{2}CV_0^2$

After connecting identical capacitor in parallel, voltage across each capacitor will be $\frac{V_0}{2}$. Then, final potential energy $U_f = 2\left[\frac{1}{2}C\left(\frac{V_0}{2}\right)^2\right]$

Heat loss $= U_i - U_f$

$$= \frac{CV_0^2}{2} - \frac{CV_0^2}{4} = \frac{CV_0^2}{4} = \frac{60\times10^{-12}\times20^2}{4} = 6 \times 10^{-9} = 6 \text{ nJ}$$

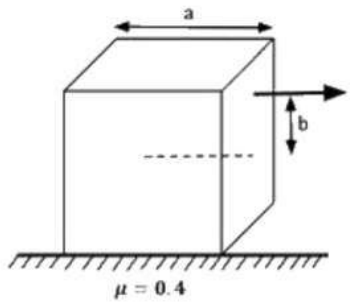
22. When m grams of steam at 100^oC is mixed with 200 grams of ice at 0^oC, it results in water at 40^oC. Find the value of m in grams

(Given, Latent heat of fusion $(L_f) = 80$ cal/g, Latent heat of vaporization $(L_v) = 540$ cal/g, specific heat of water $(C_w) = 1\ cal/g/^oC$)

Solution:

(40)
Here, heat absorbed by ice = $m_{ice}\, L_f \; + \; m_{ice} C_w (40 \; - \; 0)$
Heat released by steam = $m_{steam}\, L_v \; + \; m_{steam} C_w (100 \; - \; 40)$
Heat absorbed = heat released
$m_{ice}\, L_f \; + \; m_{ice} C_w (40 \; - \; 0) = m_{steam}\, L_v \; + \; m_{steam} C_w (100 \; - \; 40)$
$\Rightarrow \; 200 \times \; 80\, cal/g \; + \; 200 \; \times \; 1\, cal/g/°C \; \times \; (40 \; - \; 0)$
$= \; m \; \times \; 540\, cal/g \; + \; 540 \; \times \; 1\, cal/g/°C \; \times \; (100 \; - \; 40)$
$\Rightarrow \; 200\, [80 \; + \; (40)1] \; = \; m[540 \; + \; (60)1]$
$m \; = \; 40\, g$

23. A solid cube of side 'a' is shown in the figure. Find the maximum value of c $\frac{100b}{a}$ for which the block does not topple before sliding.



Solution:
(50)
F balances kinetic friction so that the block can move
So, $F \; = \mu m g$

For no toppling, the net torque about bottom right edge should be zero

i.e $$F\left(\frac{a}{2} + \; b\right) \leq \; mg\frac{a}{2}$$

$$\mu m g\left(\frac{a}{2} + \; b\right) \leq \; mg\frac{a}{2}$$

$$F\,\mu\frac{a}{2} + \; \mu b \; \leq \frac{a}{2}$$

$$0.2a \; + \; 0.4b \; \leq \; 0.5a$$

$$0.4b \le 0.3a$$

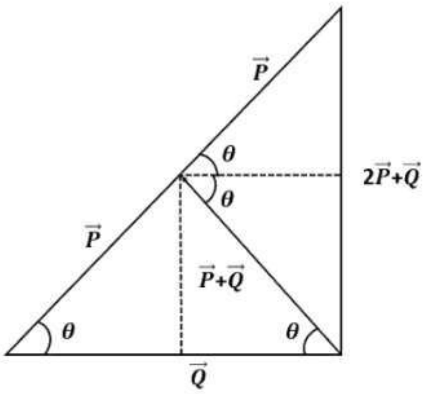
$$b \le \frac{3}{4}a$$

But, maximum value of b can only be $0.5a$

$\therefore$ Maximum value of $100\frac{b}{a}$ is 50.

24. Magnitude of resultant of two vectors $\vec{P}$ and $\vec{Q}$ is equal to magnitude of $\vec{P}$. Find the angle between $\vec{Q}$ and resultant of $2\vec{P}$ and $\vec{Q}$.

Solution:
(90°)



25. A battery of unknown emf connected to a potentiometer has balancing length 560 cm. If a resistor of resistance 10 Ω is connected in parallel with the cell the balancing length change by 60 cm. If the internal resistance of the cell is $\frac{n}{10}$ Ω, the value of $'n'$ is

Solution:

(12)

Let the emf of cell is ε internal resistance is $'r'$ and potential gradient is x.

$$\varepsilon = 560\, x \qquad (1)$$

After connecting the resistor

$$\frac{\varepsilon \times 10}{10 + r} = 500x \qquad (2)$$

From (1) and (2)

$$\frac{560 \times 10}{10 + r} = 500\, s$$

$$56 = 540 + 5r$$

$$r = \frac{6}{5} = 1.2\, \Omega$$

n = 12