JEE Main 2020 Paper

Date: 9th January 2020

Time: 02.30 PM – 05:30 PM

Subject: Chemistry

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1. 5 g of Zn reacts with
   I. Excess of NaOH
   II. Dilute HCl , then the volume ratio of H_2 gas evolved in I and II is:

   a. 2:1 b. 1:2
   c. 1:1 d. 3:1

Answer: c

Solution:
$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$
$Zn + 2HCl \rightarrow ZnCl_2 + H_2$
So, the ratio of volume of H_2 released in both the cases is 1:1.

2. Given, K_{sp} for $Cr(OH)_3$ is 6×10^{-31} then determine $[OH^-]$:

   a. $(18 \times 10^{-31})^{1/4}$ M b. $(18 \times 10^{-31})^{1/2}$ M
   c. $(6 \times 10^{-31})^{1/4}$ M d. $\left(\frac{6}{27} \times 10^{-31}\right)^{1/4}$ M

Answer: a

Solution:
$Cr(OH)_{3(s)} \rightarrow Cr^{3+}_{(aq.)} + 3OH^-_{(aq.)}$
1-S S 3S

$K_{sp} = 27S^4$
$6\times10^{-31} = 27S^4$
S= $\left[\frac{6}{27}\times10^{-31}\right]^{1/4}$
$[OH^-] = 3S = 3\times\left[\frac{6}{27}\times10^{-31}\right]^{1/4} = (18\times10^{-31})^{1/4}M$

3. Select the correct statements among the following:

A. LiCl does not dissolve in pyridine
B. Li does not react ethyne to form ethynide
C. Li and Mg react slowly with water
D. Among the alkali metals, Li has highest tendency for hydration

a. B, C, D
b. A, B, C, D
c. A, B, C
d. C, D

Answer: a

Solution:
Only LiCl amongst the first group chlorides dissolve in pyridine because the solvation energy of lithium is higher than the other salts of the same group.
Lithium does not react with ethyne to form ethynilide due to its small size and high polarizability.
Lithium and Magnesium both have very small sizes and very high ionization potentials so, they react slowly with water.
Amongst all the alkali metals, Li has the smallest size hence, the hydration energy for Li is maximum.

4. Given an element having the following ionization enthalpies, $IE_1 = 496$ kJ mol^{-1} and $IE_2 = 4562$ kJ mol^{-1} . One mole of hydroxide of this element is treated separately with HCl and H_2SO_4 respectively. Moles of HCl and H_2SO_4 reacted respectively are:

a. 1, 0.5
b. 0.5, 1
c. 2, 0.5
d. 0.5, 2

Answer: a

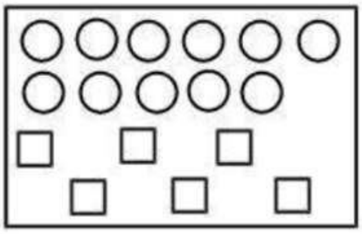
Solution:
The given data for ionization energies clearly shows that $IE_2 \gg IE_1$. So, the element belongs to the first group. Therefore, we can say that this element will be monovalent and hence forms a monoacidic base of the type MOH.

$$MOH + HCl \rightarrow MCl + H_2O$$

$$2MOH + H_2SO_4 \rightarrow M_2SO_4 + 2H_2O$$

So, from the above equation we can say that,
1 mole of metal hydroxide requires 1 mole of HCl and 0.5 mole of H_2SO_4.

5. Reactant A is represented by the squares which is in equilibrium with product B represented by circles. Then the value of equilibrium constant is:



a. 1
b. 2
c. 3
d. 4

Answer: b

Solution:
Let us assume the equation to be $A \rightleftharpoons B$,
Number of particles of A = 6
Number of particles of B = 11
$K = \frac{11}{6} \approx 2$

6. Given following complexes:
   I. $Na_4[Fe(CN)_6]$
   II. $[Cr(H_2O)_6]Cl_2$
   III. $(NEt_4)_2[CoCl_4]$
   IV. $Na_3[Fe(C_2O_4)_3]$ ($\Delta_0 > P$)

   The correct order of spin only magnetic moment for the above complexes

a. (II)>(II)>(IV)>(I)
b. (II)>(IV)>(III)>(I)
c. (I)>(III)>(III)>(II)
d. (II)>(I)>(IV)>(III)

Answer: a

Solution:

Complex (I) has the central metal ion as Fe^{2+} with strong field ligands.
Configuration of Fe^{2+}= [Ar] $3d^6$
Strong field ligands will pair up all the electrons and hence the magnetic moment will be zero.

Complex (II) has the central metal ion as Cr^{2+} with weak field ligands.
Configuration of Cr^{2+}= [Ar] $3d^4$
As weak field ligands are present, pairing does not take place. There will be 4 unpaired electrons and hence the magnetic moment = $\sqrt{24}$ B.M.

Complex (III) has the central metal ion as Co^{2+} with weak field ligands.
Configuration of Co^{2+}= [Ar] $3d^7$
As weak field ligands are present no pairing can occur. There will be 3 unpaired electrons and hence the magnetic moment = $\sqrt{15}$ B.M.

Complex (IV) has the central metal ion as Fe^{3+} with strong field ligands.
Configuration of Fe^{3+} =[Ar] $3d^5$
Strong field ligands will pair up the electrons but as we have a [Ar] $3d^5$ configuration, one electron will remain unpaired and hence the magnetic moment will be $\sqrt{3}$ B.M.

7. Select the correct option:
   a. Entropy is a function of temperature and also entropy change is a function of temperature.
   b. Entropy is a function of temperature & entropy change is not a function of temperature.
   c. Entropy is not a function of temperature & entropy change is a function of temperature.
   d. Both entropy & entropy change are not a function of temperature.

Answer: a

Solution:
Entropy is a function of temperature, at any temperature, the entropy can be given as:
$S_T = \int_0^T \frac{nCdT}{T}$

Change in entropy is also a function of temperature, at any temperature, the entropy change can be given as:
$\Delta S = \int \frac{dq}{T}$

8. A compound (A: $B_3N_3H_3Cl_3$) reacts with $LiBH_4$ to form inorganic benzene (B). (A) reacts with (C) to form $B_3N_3H_3(CH_3)_3$. (B) and (C) respectively are:
   a. Boron nitride, MeMgBr
   b. Boron nitride, MeBr
   c. Borazine, MeBr
   d. Borazine, MeMgBr

Answer: d

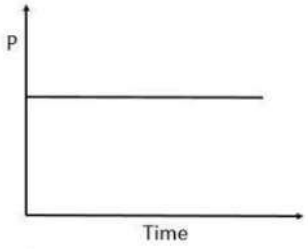
Solution:
$B_3N_3H_3Cl_3 + LiBH_4 \rightarrow B_3N_3H_6 + LiCl + BCl_3$

$B_3N_3H_3Cl_3 + 3CH_3MgBr \rightarrow B_3N_3H_3(CH_3)_3 + 3MgBrCl$

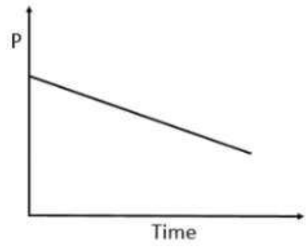
So, we can say that,
B is $B_3N_3H_6$
C is CH_3MgBr

9. In a box, a mixture containing H_2, O_2 and CO along with charcoal is present. Then, the variation of pressure with time will be:

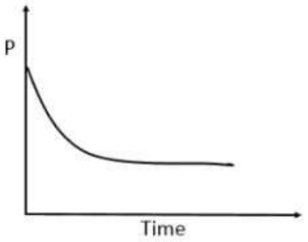
a.



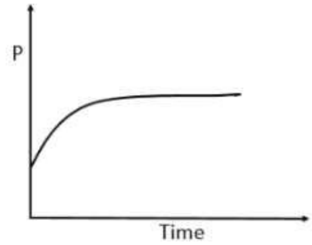
b.



c.



d.



Answer: c

Solution:
As H_2, O_2 and CO gets adsorbed on the surface of charcoal, the pressure decreases. So, option (a) and (d) can be eliminated. After some time, as almost all the surface sites are occupied, the pressure becomes constant.

10. Given the complex: $[Co(NH_3)_4Cl_2]$. If in this complex, the Cl–Co–Cl bond angle is 90°, then it is a:
   a. Cis-isomer
   b. Trans-isomers
   c. Meridional and Trans
   d. Cis and trans both

Answer: a

Solution:
In cis-isomer, similar ligands are at an angle of 90°.

11. Amongst the following, which has the least conductivity?

a. Distilled water
b. Sea water
c. Saline water used for intra venous injection
d. Well water

Answer: a

Solution:
In distilled water there are no ions present except H^+ and OH^- ions, both of which are immensely minute in concentration, that renders their collective conductivity negligible.

12. Number of sp^2 hybrid orbitals in Benzene is:

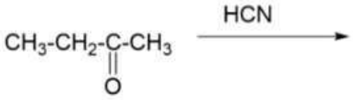
a. 18
b. 24
c. 6
d. 12

Answer: a

Solution:
Benzene (C_6H_6) has 6 sp^2 hybridized carbons. Each carbon has 3 σ-bonds and 1 π-bond. 3 σ-bonds means that there are 3 sp^2 hybrid orbitals for each carbon. Hence, the total number of sp^2 hybrid orbitals is 18.

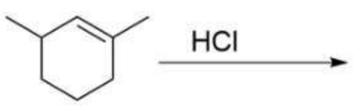
13. Which of the following reaction will not give a racemic mixture as the product?

a.



b. $(CH_3)_2\text{-}CH\text{-}CH{=}CH_2 \xrightarrow{HCl}$

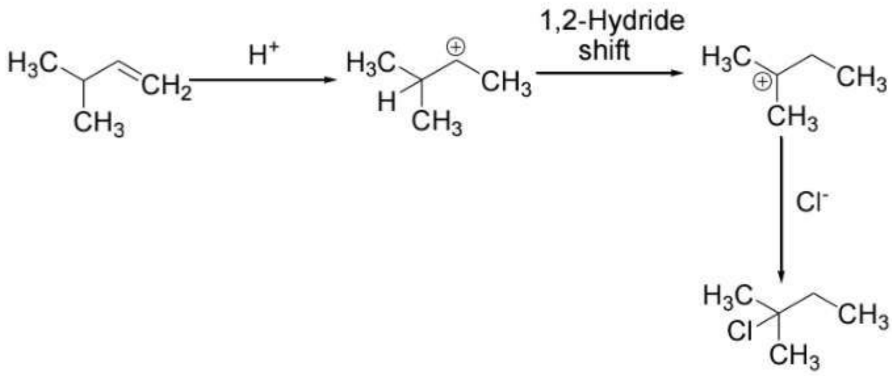
c.



d. $CH_3CH_2\text{-}CH{=}CH_2 \xrightarrow{HBr}$

Answer: b

Solution:



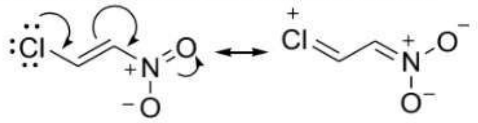
14. In which compound is the C–Cl bond length the shortest?

a. $Cl - CH = CH_2$
b. $Cl - CH = CH - CH_3$
c. $Cl - CH = CH - OCH_3$
d. $Cl - CH = CH - NO_2$

Answer: d

Solution:

There is extended conjugation present in option (d), which will reduce the length of C-Cl bond to the greatest extent which can be represented as follows:



15. Biochemical oxygen demand (BOD) is defined as .... in ppm of O_2.

a. Required to sustain life.
b. The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water.
c. The amount of oxygen required by anaerobic bacteria to break down the inorganic matter present in a certain volume of a sample of water.
d. Required photochemical reaction to degrade waste.

Answer: b

Solution:

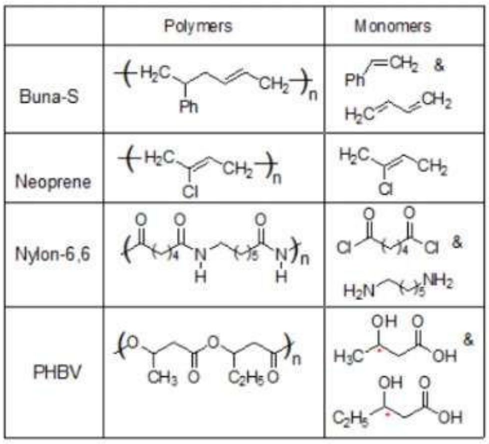
Biochemical oxygen demand (BOD) is the amount of dissolved oxygen used by microorganisms in the biological process of metabolizing organic matter in water.

16. Monomer(s) of which of the given polymer is chiral?

a. Buna-S
b. Neoprene
c. Nylon-6,6
d. PHBV

Answer: d

Solution:



17.

Lab tests			
Compound	**Molisch's test**	**Barfoed's test**	**Biuret test**
A	✓	✗	✗
B	✓	✓	✗
C	✗	✗	✓

Which of the following options is correct?

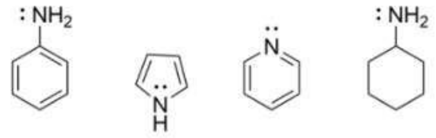
	A	B	C
a.	Lactose	Glucose	Albumin
b.	Lactose	Glucose	Alanine
c.	Lactose	Fructose	Alanine
d.	Glucose	Sucrose	Albumin

Answer: a

Solution:

Lactose, glucose and fructose gives positive Molisch's test.
Glucose gives positive Barfoed's test whereas sucrose gives a negative for Barfoed's test.
Albumin gives positive for Biuret test whereas alanine gives a negative Biuret test.

18. The order of basic character is:



(I) (II) (III) (IV)

a. I > II > III > IV

b. IV > III > I > II

c. II > I > III > IV

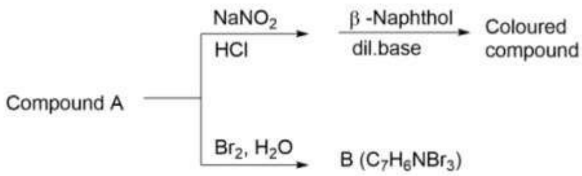
d. IV > I > II > III

Answer: b

Solution:

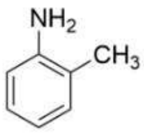
The basicity of the compound depends on the availability of the lone pairs.
In compound IV, Nitrogen is sp^3 hybridized.
In compound III, Nitrogen is sp^2 hybridized and the lone pairs are not involved in resonance.
In compound I, Nitrogen is sp^2 hybridized and the lone pairs are involved in resonance.
In compound II, Nitrogen is sp^2 hybridized and the lone pairs are involved in resonance such that, they are contributing to the aromaticity of the ring.
From the above points we can conclude that the basicity order should be IV > III > I > II.

19.

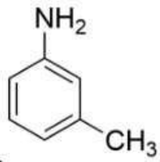


Compound A will be:

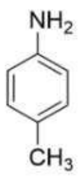
a.



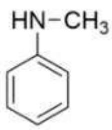
b.



c.

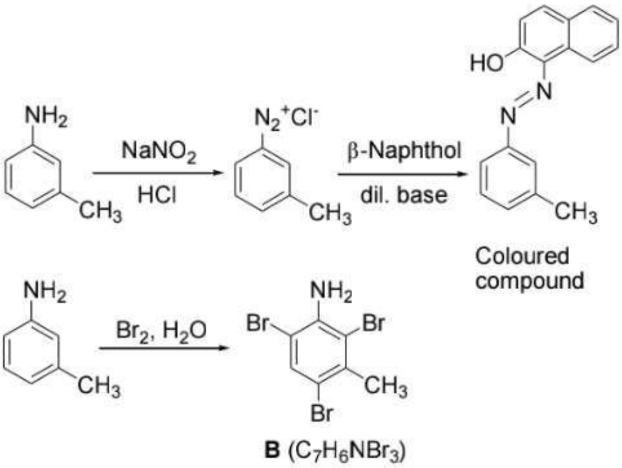


d.

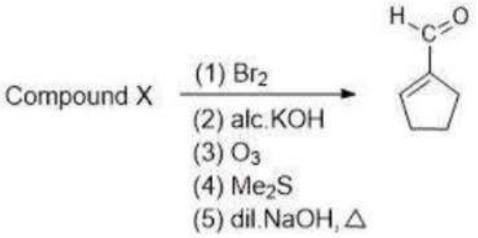


Answer: b

Solution:



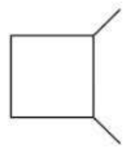
20.



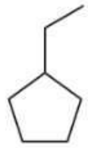
Compound X will be:

a.

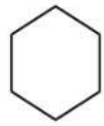
b.



c.

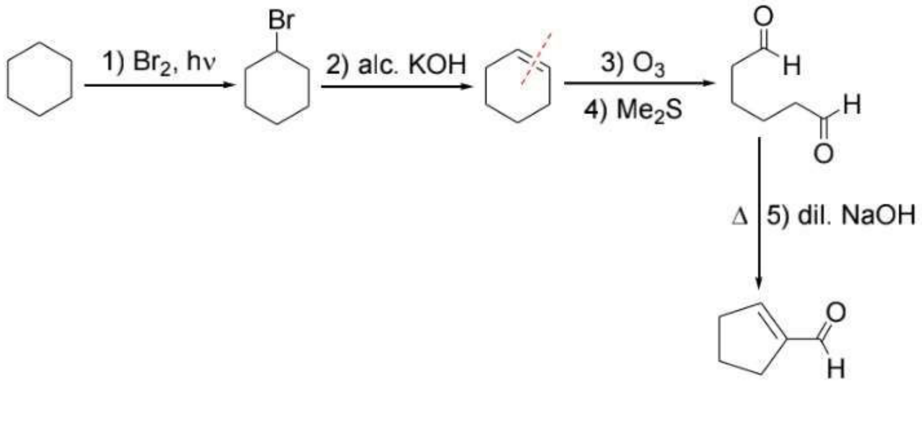


d.



Answer: d

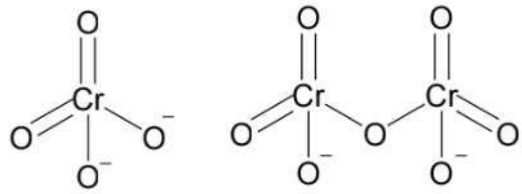
Solution:



21. Total number of Cr–O bonds in Chromate ion and Dichromate ion is:

Answer: 12

Solution:



Chromate ion Dichromate ion

22. Lacto bacillus has a generation time of 60 minutes at 300K and 40 minutes at 400K. Determine the activation energy in $\frac{\text{kJ}}{\text{mol}}$. (R = 8.3 J $K^{-1}mol^{-1}$)[$\ln\left(\frac{2}{3}\right)$ = -0.4]

Answer: 3.98

Solution:

The generation time can be utilized to get an indication of the rate ratio. Let the amount generated be (x).

$$\text{Rate} = \frac{\text{Amount generated}}{\text{Time taken}}$$

$\text{Rate}_{300\text{ K}} = \frac{(x)}{60}$ $\text{Rate}_{400\text{ K}} = \frac{(x)}{40}$

$$\frac{\text{Rate}_{300\text{K}}}{\text{Rate}_{400\text{K}}} = \frac{40}{60}$$

For the same concentration (which is applicable here), the rate ratio can also be equaled to the ratio of rate constants.

$$\ln\left[\frac{K_{\text{at }400\text{K}}}{K_{\text{at }300\text{K}}}\right] = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln\frac{60}{40} = \frac{E_a}{8.3}\left[\frac{1}{300} - \frac{1}{400}\right]$$

E_a = 0.4 x 8.3 x 1200 = 3984 J/mol = 3.98 kJ/mol

23. One litre of sea water ($d = 1.03 \frac{g}{cm^3}$) contains 10.3 mg of O_2 gas. Determine the concentration of O_2 in ppm:

Answer: 10.00

Solution:

$Ppm = \frac{w_{Solute}}{w_{Solution}} x\ 100$

Using the density of the solution and its volume ($1L = 1000\ mL = 1000\ cm^3$), the weight of the solution can be calculated.

$W_{solution} = 1.03\ x\ 1000 = 1030\ g$

Thus, $ppm = \frac{10.3\ x\ 10^{-3} g}{1030\ g} x\ 100$

24. 0.1 mole of an ideal gas has volume 1 dm^3 in a locked box with a frictionless piston. The gas is in thermal equilibrium with an excess of 0.5 m aqueous ethylene glycol at its freezing point. If the piston is released all of a sudden at 1 atm, then determine the final volume of gas in dm^3
($R = 0.08$ atm L mol^{-1} K^{-1} ; $K_f = 2.0$ K $molal^{-1}$)

Answer: 2.18

Solution:

$K_f = 2$

Molality, 'm' = 0.5

$\Delta T_f = K_f . m$

$= (0.5 \times 2) = 1$

So, the initial temperature now becomes 272 K. Further using the given value of moles and initial volume of the gas and the calculated initial temperature value, we can find out the initial pressure of the ideal gas contained inside the piston.

$P_{gas} = \frac{nRT}{V_1}$

$= (0.1)(0.08)(272) = 2.176$ atm

Now, on releasing the piston against an external pressure of 1 atm, the gas will expand until the final pressure of the gas, i.e. P_2 becomes equal to 1 atm. During this expansion, since no reaction is happening and the temperature of the gas is not changing as well, the boyle's law relation can be applied.

$P_1V_1 = P_2V_2$

$2.176\ x\ 1 = 1\ x\ V_2$

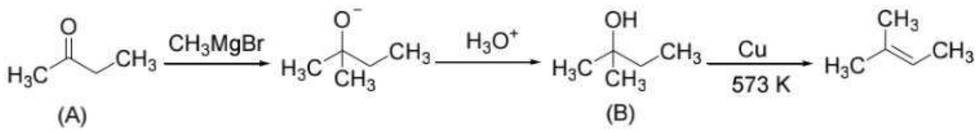
25.

$$\text{Compound A} \xrightarrow[H_3O^+]{CH_3MgBr} B \xrightarrow[573\ K]{Cu} CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH-CH_3$$

The percentage of carbon in compound A is:

Answer: 66.67

Solution:



Compound A is $CH_3(CO)CH_2CH_3$ (C_4H_8O)

The percentage of carbon in compound A by weight is $\frac{w_{Carbon}}{w_{Compound}} = \frac{12 \times 4}{72} = 66.67$

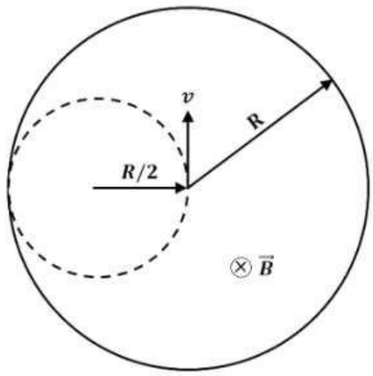
JEE Main 2020 Paper

Date of Exam: **9th January (Shift II)**
Time: **2:30 pm – 5:30 pm**
Subject: **Physics**

1. There is a long solenoid of radius R having n turns per unit length with current flowing in it. A particle having charge q and mass m is projected with speed v in the perpendicular direction of axis from a point on its axis. Find maximum value of v so that it will not collide with the solenoid.

   a. $\frac{Rq\mu_0 in}{5m}$
   b. $\frac{Rq\mu_0 in}{2m}$
   c. $\frac{3Rq\mu_0 in}{m}$
   d. $\frac{Rq\mu_0 in}{4m}$

   Solution: (b)
   Looking at the cross-section of the solenoid, R_{max} of the particle's motion has to be $\frac{R}{2}$ for it not to strike the solenoid.



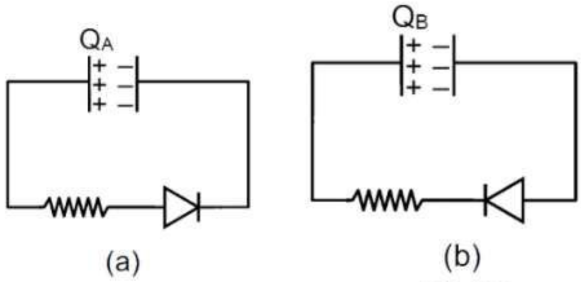
$$qvB = \frac{mv^2}{\frac{R}{2}}$$

$$R_{max} = \frac{R}{2} = \frac{mv_{max}}{q\mu_0 in}$$

$$V_{max} = \frac{Rq\mu_0 in}{2m}$$

2. A capacitor C and resistor R are connected to a battery of 5 V in series. Now the battery is disconnected and a diode is connected as shown in the figures (a) and (b) respectively.

The charge on the capacitor after time RC in (a) and (b) respectively is Q_A and Q_B. Their values are



a. $\frac{5CV}{e}, 5CV$

b. $\frac{5CV}{e}, \frac{5CV}{2e}$

c. $5CV, 5CV$

d. $5CV, \frac{5CV}{e}$

Solution:(a)

Maximum charge on capacitor = $5CV$

is forward biased and (b) is reverse biased

For case (a)

$q = q_{max}(1 - e^{\frac{-t}{RC}}) = 5CV$

$$Q_A = 5CVe^{-1}$$

For case (b)

$$Q_B = 5CV$$

3. Different values of a, b and c are given and their sum is d. Arrange the value of d in increasing order

	a	b	c
1	220.1	20.4567	40.118
2	218.2	22.3625	40.372
3	221.2	20.2435	39.432
4	221.4	18.3625	40.281

e. $d_1 = d_2 = d_3 = d_4$

f. $d_1 < d_2 < d_3 < d_4$

g. $d_1 > d_2 > d_3 > d_4$

h. $d_4 < d_1 < d_3 = d_2$

Solution:(d)

	a	b	c	$a+b+c=d$	Round off
1.	220.1	20.4567	40.118	d_1 $= 280.6747$	280.7
2.	218.2	22.3625	40.372	d_2 $= 280.9345$	280.9
3.	221.2	20.2435	39.432	d_3 $= 280.8755$	280.9
4.	221.4	18.3625	40.281	d_4 $= 280.0435$	280.0

4. A particle starts moving with a velocity $\vec{u} = 3\hat{\imath}$ from origin and an acceleration $\vec{a} = 6\hat{\imath} + 4\hat{\jmath}$. Here, if the y-coordinate of the particle is 32 m, then its x-coordinate at that instant will be

a. 60
b. 48
c. 45
d. 22

Solution:(a)

$S_y = u_y t + \frac{1}{2} a_y t^2$

$32 = 0 + \frac{1}{2} \times 4t^2 \quad \rightarrow \quad t = 4\ sec$

$S_x = u_x t + \frac{1}{2} a_x t^2$

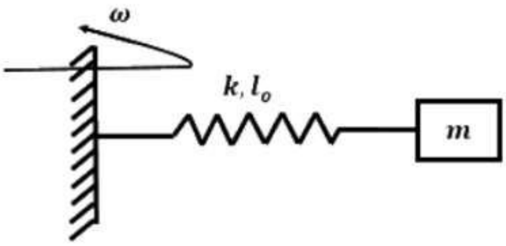
$= 3 \times 4 + \frac{1}{2} \times 6 \times 16$

$= 60\ m$

5. A mass m is attached to a spring of natural length l_o and spring constant k. One end of the spring is attached to the centre of a disc in the horizontal plane which is being rotated by a constant angular speed, ω. Find the extension per unit length in the spring (given $k >>> m\omega^2$)

a. $\frac{3m\omega^2}{2k}$
b. $\frac{\sqrt{2}}{3}\frac{m\omega^2}{k}$
c. $\frac{m\omega^2}{k}$
d. $\frac{3m\omega^2}{k}$

Solution: (c)



Using Newton's second law of dynamics,

$$m\omega^2(l_o + x) = kx$$

$$\left(\frac{l_o}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{l_o m\omega^2}{k - m\omega^2}$$

$$k >> m\omega^2$$

So, $\frac{x}{l_o}$ is equal to $\frac{m\omega^2}{k}$

6. A loop of radius R and mass m is placed in a uniform magnetic field B with its plane perpendicular to the field. A current i is flowing in it. Now the loop is slightly rotated about its diameter and released. Find the time period of oscillations.

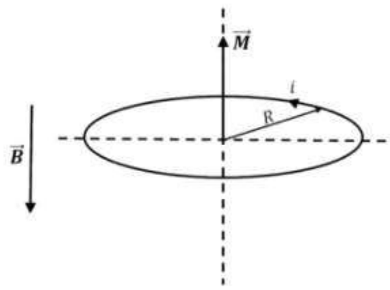
a. $2\sqrt{\frac{\pi M}{iB}}$

b. $\sqrt{\frac{2\pi M}{iB}}$

c. $2\sqrt{\frac{M}{\pi iB}}$

d. $\sqrt{\frac{M}{\pi iB}}$

Solution: (b)



Considering the torque situation on the loop,

$$\tau = MBsin\theta = -i\alpha$$

$$\pi R^2 iB\theta = -\frac{mR^2}{2}\alpha$$

The above equation is analogous to $\theta = -C\alpha$, where $C = \omega^2 = \frac{2\pi iB}{M}$

$$\omega = \sqrt{\frac{2\pi iB}{M}} = \frac{2\pi}{T}$$

$$T = \sqrt{\frac{2\pi M}{iB}}$$

7. A sphere of density ρ is half submerged in a liquid of density σ and surface tension T. The sphere remains in equilibrium. Find the radius of this sphere. (Assume the force due to surface tension acts tangentially to surface of sphere.)

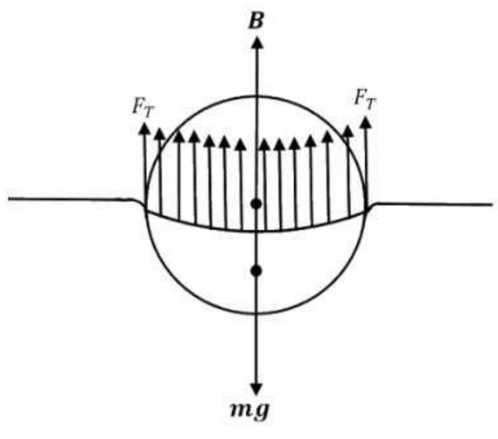
a. $\sqrt{\frac{3T}{2(\rho+\sigma)g}}$

b. $\sqrt{\frac{3T}{4(\rho-\sigma)g}}$

c. $\sqrt{\frac{3T}{2\left(\rho-\frac{\sigma}{2}\right)g}}$

d. $\sqrt{\frac{T}{(\rho+\sigma)g}}$

Solution:(c)



In equilibrium, net external force acting on the sphere is zero.

$mg = F_T + B$

$$\rho V g = \sigma\left(\frac{V}{2}\right)g + T2\pi R$$

$$\rho\frac{4}{3}\pi R^3 g = \sigma\frac{2}{3}\pi R^3 g + T2\pi R$$

$$R = \sqrt{\frac{3T}{2\left(\rho - \frac{\sigma}{2}\right)g}}$$

8. A string of mass per unit length $\mu = 6 \times 10^{-3}\ kg/m$ is fixed at both ends under the tension $540\ N$. If the string is in resonance with consecutive frequencies $420\ Hz$ and $490\ Hz$. Then what would be the length of the string?

a. 2.8 m
b. 2.1 m
c. 4.5 m
d. 4.2 m

Solution: (b)
Key Idea: The difference of two consecutive resonant frequencies is the fundamental resonant frequency.
Fundamental frequency= $490 - 420 = 70\ Hz$

$$70 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow 70 = \frac{1}{2l}\sqrt{\frac{540}{6 \times 10^{-3}}}$$

$$\Rightarrow l = \frac{1}{2 \times 70}\sqrt{90 \times 10^{-3}} = \frac{300}{140}$$

$$\Rightarrow l \approx 2.14\ m$$

9. An EM wave is travelling in $\frac{\hat{\imath}+\hat{\jmath}}{2}$ direction. Axis of polarization of EM wave is found to be $\hat{k}$. Then equation of magnetic field will be

a. $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}} cos\left(\omega t - k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
b. $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}} cos\left(\omega t + k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
c. $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}} cos\left(\omega t + k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
d. $\hat{k} cos\left(\omega t - k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$

Solution:(a)

EM wave is in direction $\frac{\hat{i}+\hat{j}}{2}$

Electric field is in direction $\hat{k}$

Direction of propagation of EM wave is given by $\vec{E} \times \vec{B}$

10. Two gases Ar (40) and Xe (131) at equal temperature have the same number density. Their diameters are 0.07 nm and 0.10 nm respectively. Find the ratio of their mean free time

a. 1.03
b. 2.04
c. 2.09
d. 2.49

Solution:(a)

Mean free time = $\frac{1}{\sqrt{2}n\pi d^2}$

$$\frac{t_{Ar}}{t_{Xe}} = \frac{d_{Xe}^2}{d_{Ar}^2} = \left(\frac{0.1}{0.07}\right)^2 = \left(\frac{10}{7}\right)^2 = 2.04$$

11. When the same mass is suspended from two steel rods, the ratio of their energy densities is 1: 4. If the lengths of both the rods are equal, then the ratio of their diameters will be

a. $\sqrt{2}:1$
b. $1:\sqrt{2}$
c. $1:2$
d. $2:1$

Solution: (a)

$$\frac{dU}{dV} = \frac{1}{2} \times stress \times \frac{stress}{Y}$$

$$= \frac{1}{2} \times \frac{F^2}{A^2Y}$$

$$\frac{dU}{dV} \propto \frac{1}{D^4}$$

$$\frac{\left(\frac{dU}{dV}\right)_1}{\left(\frac{dU}{dV}\right)_2} = \frac{{D_2}^4}{{D_1}^4} = \frac{1}{4}$$

$$\frac{D_1}{D_2} = (4)^{\frac{1}{4}}$$

$\therefore D_1 : D_2 = \sqrt{2} : 1$

12. Two planets of masses M and $\frac{M}{2}$ have radii R and $\frac{R}{2}$ respectively. If the ratio $\left(\frac{v_1}{v_2}\right)$ of the escape velocities from their surfaces is $\frac{n}{4}$, then n is

a. 8
b. 2
c. 4
d. 1

Solution: (c)

We know that the escape velocity is given by,

$$V_e = \sqrt{\frac{2GM}{R}}$$

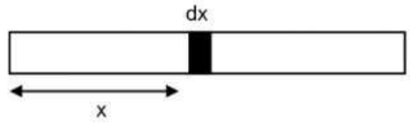
Now

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{2GM}{R}}}{\sqrt{\frac{\frac{2GM}{2}}{\frac{R}{2}}}} = 1$$

We are given that $\frac{V_1}{V_2} = \frac{n}{4}$

$$\Rightarrow \frac{n}{4} = 1$$
$$\Rightarrow n = 4$$

13. Find the centre of mass of the given rod of linear mass density $\lambda = \left(a + b\left(\frac{x}{l}\right)^2\right)$. Here, x is the distance from one of its end. (Length of the rod is l)



a. $\frac{3l}{4}\left(\frac{2a+b}{2a+b}\right)$
b. $\frac{3l}{4}\left(\frac{2a+b}{3a+b}\right)$

c. $\frac{l}{4}\left(\frac{a+b}{3a+b}\right)$

d. $l\left(\frac{a+b}{3a+b}\right)$

Solution:(b)

Here we take a small element along the length as dx at a distance x from the left end as shown.

$$x_{cm} = \frac{1}{M}\int_0^l x.\,dm$$

$$\Rightarrow dM = \lambda.\,dx = \left(a + b\left(\frac{x}{l}\right)^2\right).\,dx$$

$$x_{cm} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int_0^l x\left(a + \frac{bx^2}{l^2}\right)dx}{\int_0^l \left(a + \frac{bx^2}{l^2}\right)dx}$$

$$= \frac{a\left(\frac{x^2}{2}\right)_0^l + \frac{b}{l^2}\left(\frac{x^4}{4}\right)_0^l}{a(x)_0^l + \frac{b}{l^2}\left(\frac{x^3}{3}\right)_0^l}$$

$$= \frac{\frac{al^2}{2} + \frac{bl^2}{4}}{al + \frac{bl}{3}}$$

$$= \frac{3l}{4}\left(\frac{2a+b}{3a+b}\right)$$

14. A particle is projected from the ground with a speed u at an angle of $60°$ from horizontal. It collides with a second particle of equal mass moving with a horizontal speed u in the same direction at the highest point of its trajectory. If the collision is perfectly inelastic then find the horizontal distance travelled by them after this collision when they reached the ground.

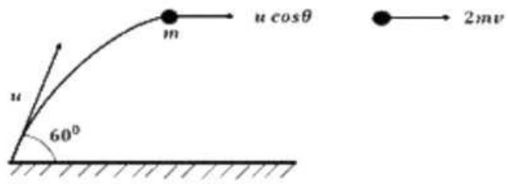
a. $\frac{3\sqrt{6}u^2}{8g}$

b. $\frac{3\sqrt{3}u^2}{8g}$

c. $\frac{u^2}{8g}$

d. $\frac{\sqrt{3}u^2}{g}$

Solution: (b)



The only external force acting on the colliding system during the collision is the gravitational force. Since gravitational force is non- impulsive, the linear momentum of the system is conserved just before and just after the collision.

$$p_i = p_f$$

$$mu + mu\,cos\theta = 2mv$$

$$\Rightarrow \quad v = \frac{u(1 + cos60^0)}{2} = \frac{3}{4}u$$

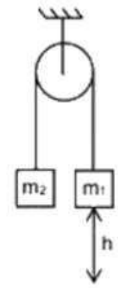
So the horizontal range after the collision $= vt$

$$= v\sqrt{\frac{2H_{\max}}{g}}$$

$$= \frac{3}{4}u\sqrt{\frac{2u^2 \sin^2 60^0}{2g^2}}$$

$$= \frac{3}{4}u^2\frac{\sqrt{\frac{3}{4}}}{g} = \frac{3\sqrt{3}u^2}{8g}$$

15. System is released from rest. Moment of inertia of pulley is I. Find angular speed of pulley when m_1 block falls by h. (Given $m_1 > m_2$ and assume no slipping between string and pulley)



a. $\frac{1}{R}\sqrt{\frac{2(m_1-m_2)gh}{m_1+m_2+\frac{I}{R^2}}}$

b. $\frac{1}{R}\sqrt{\frac{4(m_2+m_1)gh}{m_1+m_2+\frac{I}{R^4}}}$

c. $\frac{1}{R}\sqrt{\frac{(m_1-m_2)gh}{m_1+m_2+\frac{I}{R}}}$

d. $\frac{1}{R}\sqrt{\frac{2(m_2+m_1)gh}{m_1+m_2+\frac{I}{R^2}}}$

Solution: (a)

Assume initial potential energy of the blocks to be zero. Initial kinetic energy is also zero since the blocks are at rest.

When block m_1 falls by h, m_2 goes up by h (because of length constraint)

Final P.E $= m_2gh - m_1gh$

Let the final speed of the blocks be v and angular velocity of the pulley be ω

Final K.E $= \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}I\omega^2$

Total energy is conserved. Hence,

$$0 = m_2gh - m_1gh + \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}I\omega^2$$

$v = \omega r$ (due to no slip condition)

$$\Rightarrow \frac{1}{2}(m_1+m_2)\omega^2R^2 + \frac{1}{2}I\omega^2 = (m_1-m_2)gh$$

$$\Rightarrow \omega^2\left[\frac{1}{2}(m_1+m_2)R^2 + \frac{1}{2}I\right] = (m_1-m_2)gh$$

$$\Rightarrow \omega^2 = \frac{2(m_1-m_2)gh}{R^2\left[(m_1+m_2)+\frac{I}{R^2}\right]}$$

$$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{2(m_1-m_2)gh}{\left[(m_1+m_2)+\frac{I}{R^2}\right]}}$$

16. An H-like atom has its ionization energy equal to 9R. Find the wavelength of light emitted (in nm) when an electron jumps from the second excited state to the ground state. (R is Rydberg constant)

a. 12.40

b. 11.39

c. 5.80

d. 22.76

Solution:(b)

$$\frac{hc}{\lambda} = (13.6\ eV)Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$n_1 = 1$$

$$n_2 = 3$$

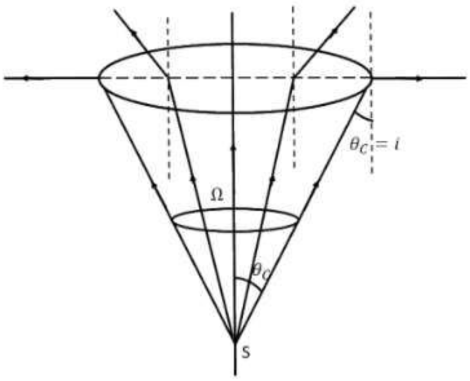
For an H-like atom, ionization energy is $(R)Z^2$.
This gives $Z = 3$

$$\frac{hc}{\lambda} = (13.6\ eV)(3^2)\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$
$$\Rightarrow \frac{hc}{\lambda} = (13.6\ eV)(9) \times \frac{8}{9}$$
$$\text{wavelength} = \frac{1240}{8 \times 13.6}\ nm$$
$$\lambda = 11.39\ nm$$

17. A point source is placed at a depth h in a liquid of refractive index is $\frac{4}{3}$. Find the percentage of energy of light that escapes from the liquid. (Assuming 100 % transmission of emerging light)

a. 17 %
b. 25 %
c. 10 %
d. 8 %

Solution:(a)



The portion of light escaping into the air from the liquid will form a cone. As long as the angle of incidence on the liquid – air interface is less than the critical angle, i.e. $i < \theta_C$, the light rays will undergo refraction and emerge into the air.
For $i > \theta_C$, the light rays will suffer TIR. So, these rays will not emerge into the air.
The portion of light rays emerging into the air from the liquid will form a cone of half angle = θ_C

$$\sin\theta_C = \frac{1}{\mu_{Liq}} = \frac{3}{4}, \quad \cos\theta_C = \frac{\sqrt{7}}{4}$$

Solid angle contained in this cone is

$\Omega = 2\pi(1-\cos\theta_C)$

Percentage of light that escapes from liquid $= \frac{\Omega}{4\pi} \times 100$

Putting values we get

Percentage $= \frac{4-\sqrt{7}}{8} \times 100 \approx 17\%$

18. An electron $(-|e|, m)$ is released in Electric field E from rest. Rate of change of de-Broglie wavelength with time will be.

a. $-\frac{h}{2|e|}$

b. $-\frac{h}{2|e|t}$

c. $-\frac{h}{|e|Et^2}$

d. $-\frac{2ht^2}{|e|E}$

Solution:(c)

$$\lambda_D = \frac{h}{mv}$$

where, $v = at$

$$v = \frac{eE}{M}t \quad \left(a = \frac{eE}{M}\right)$$

$$\lambda_D = \frac{h}{m\frac{eE}{M}t}$$

$$\lambda_D = \frac{h}{eEt}$$

$$\frac{d\lambda_D}{dt} = \frac{h}{|e|Et^2}$$

19. An AC source is connected to the LC series circuit with $V = 10\sin(314t)$. Find the current in the circuit as function of time ? $(L = 40\ mH, C = 100\ \mu F)$

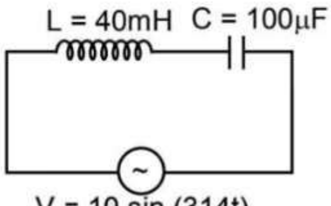
a. $10.4\sin(314t)$

b. $0.52\cos(314t)$

c. $0.52\sin(314t)$

d. $5.2\cos(314t)$

Solution:(b)



Impedance $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{(X_C - X_L)^2}$

$= X_C - X_L$

$= \frac{1}{\omega C} - \omega L$

$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$

$= 31.84 - 12.56 = 19.28\ \Omega$

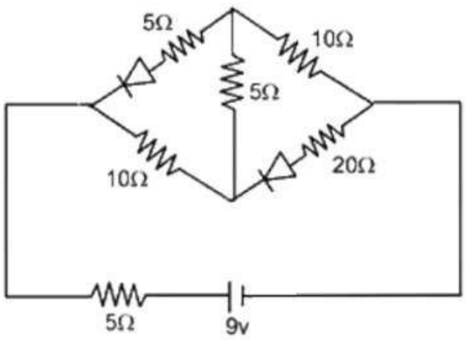
For $X_C > X_L$, current leads voltage by $\frac{\pi}{2}$

$\therefore i = \frac{V}{Z} = \frac{10\sin(314t + \frac{\pi}{2})}{19.28}$

$= 0.52 \cos\ (314t)$

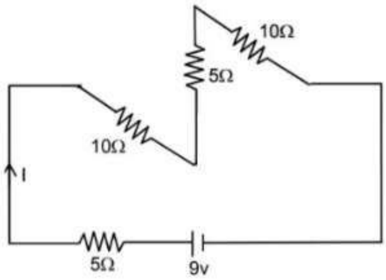
20. Find the current supplied by the battery.

a. $0.1\,A$
b. $0.3\,A$
c. $0.4\,A$
d. $0.5\,A$



Solution:(b)

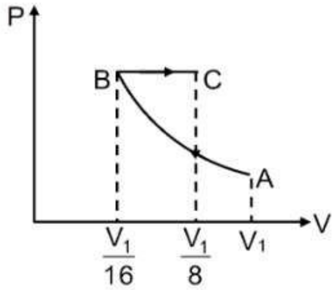
Since the diodes are reverse biased, they will not conduct.
Hence, the circuit will look like



$$R_{eff} = 5 + 10 + 5 + 10 = 30\ \Omega$$
$$I = \frac{9}{30} = 0.3\ A$$

21. An ideal gas at an initial temperature 300 K is compressed adiabatically ($\gamma = 1.4$) to its initial volume. The gas is then expanded isobarically to double its volume. Then the final temperature of the gas rounded off to nearest integer is.

Solution:(1819 K)



PV^{γ} =Constant
$TV^{(\gamma-1)}$=constant

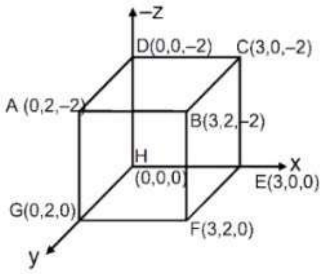
$$300(V_1)^{(1.4-1)} = T_B\left(\frac{V_1}{16}\right)^{\frac{2}{5}}$$
$$T_B = 300 \times 2^{\left(\frac{8}{5}\right)}$$

Now for BC process

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}$$

$$T_C = \frac{V_C T_B}{T_B} = 2 \times 300 \times 2^{\left(\frac{8}{5}\right)}$$
$$T_C = 1819\ K$$

22. If electric field in the space is given by $\vec{E} = 4x\hat{\imath} - (y^2 + 1)\hat{\jmath}$, and electric flux through ABCD is ϕ_1 and electric flux through BCEF is ϕ_2, then find $(\phi_1 - \phi_2)$.



Solution: (-40)
Electric flux through a surface is $\emptyset = \int \vec{E} = d\vec{A}$
For surface $ABCD$,
$d\vec{A}$ is along $(-\hat{k})$
So, at all the points of this surface,
$$\vec{E}.d\vec{A} = 0$$
Because, $\emptyset_{ABCD} = \emptyset_1 = 0$
For surface $BCEF$,
$d\vec{A}$ is along $(\hat{\imath})$
So,
$$\vec{E}.d\vec{A} = E_x dA$$
$$\emptyset_{BCEF} = \emptyset_2 = 4x(2 \times 2)$$
If $if\ x = 3$
$$\emptyset_2 = 48\ \frac{N - m^2}{C}$$
Hence, $\emptyset_1 - \emptyset_2 = -48\ \frac{N-m^2}{C}$

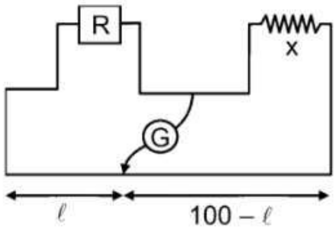
23. In a YDSE interference pattern obtained with light of wavelength $\lambda_1 = 500\ nm$, 15 fringes are obtained on a certain segment of screen. If number of fringes for light of wavelength λ_2 on same segment of screen is 10, then the value of λ_2 (in nm) is

Solution:(750)

If the length of the segment is y,
Then $y = n\,\beta$
$n =$ no. of fringes,
$\beta =$ fringe width
$15 \times 500 \times \frac{D}{d} = 10 \times \lambda_2 \times \frac{D}{d}$

$$\lambda_2 = 15 \times 50\ nm$$
$$\lambda_2 = 750\ nm$$

24. In a meter bridge experiment, the balancing length was 25 cm for the situation shown in the figure. If the length and diameter of the of wire of resistance R is halved, then the new balancing length in centimetre is

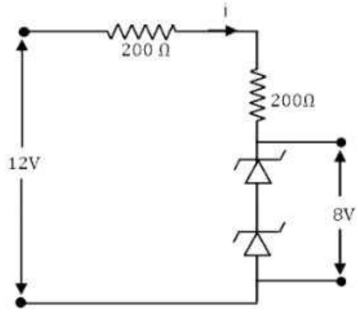


Solution:(40)

$$\frac{X}{R} = \frac{75}{25} = 3$$
$$R = \frac{\rho l}{A} = \frac{4\rho l}{\pi d^2}$$
$$R' = \frac{4\rho \frac{l}{2}}{\pi \left(\frac{d}{2}\right)^2} = 2R$$
Then $\frac{X}{R'} = \left(\frac{100-l}{l}\right)$
$$\frac{100 - l}{l} = \frac{X}{2R} = \frac{3}{2}$$

$$l = 40.00cm$$

25. Find the power loss in each diode (in mW), if potential drop across the Zener diode is $8V$.



Solution:(40)

$i = \left(\frac{12-8}{200+200}\right) A = \frac{4}{400} = 10^{-2}A$

Power loss in each diode = $(4)(10^{-2})\ W = 40\ mW$

JEE Main 2020 Paper

Date: **9th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

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1. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ and $a - 2b + c = 1$ then

   a. $f(-50) = -1$
   b. $f(50) = 1$
   c. $f(50) = -501$
   d. $f(50) = 501$

   Answer: (a)

   Solution:

   Given $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

   $a - 2b + c = 1$

   Applying $R_1 \to R_1 - 2R_2 + R_3$

   $f(x) = \begin{vmatrix} a-2b+c & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

   Using $a - 2b + c = 1$

   $\therefore f(x) = (x+3)^2 - (x+2)(x+4)$

   $\Rightarrow f(x) = 1$

   $\Rightarrow f(50) = 1$

   $\Rightarrow f(-50) = 1$

2. If $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$

   $g(x) = \left(x - \frac{1}{2}\right)^2$ then find the area bounded by $f(x)$ and $g(x)$ from $x = \frac{1}{2}$ to $x = \frac{\sqrt{3}}{2}$.

   a. $\frac{\sqrt{3}}{2} - \frac{1}{3}$
   b. $\frac{\sqrt{3}}{4} + \frac{1}{3}$
   c. $2\sqrt{3}$
   d. $3\sqrt{3}$

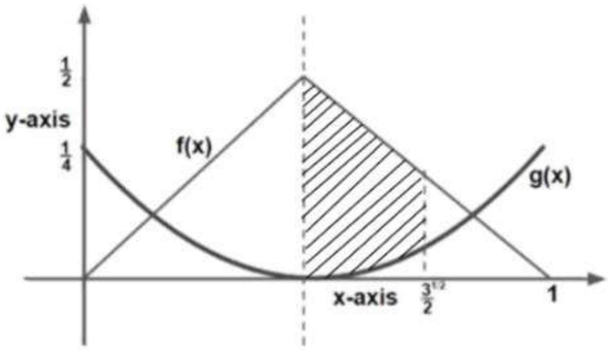
Answer: (a)

Solution:

Given $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = (x - \frac{1}{2})^2$

The area between $f(x)$ and $g(x)$ from $x = \frac{1}{2}$ to $= \frac{\sqrt{3}}{2}$:



Points of intersection of $f(x)$ and (x) :

$$1 - x = \left(x - \frac{1}{2}\right)^2$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

Required area $= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} (f(x) - g(x))dx$

$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left(1 - x - \left(x - \frac{1}{2}\right)^2\right) dx$$

$$= x - \frac{x^2}{2} - \frac{1}{3}\left(x - \frac{1}{2}\right)^3 \Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

3. If $p \rightarrow (p \wedge \sim q)$ is false. Truth value of p and q will be

a. TF

b. FT

c. TT

d. FF

Answer: (c)

Solution:

Given $p \rightarrow (p \wedge \sim q)$

Truth table:

p	q	$\sim q$	$(p \wedge \sim q)$	$p \rightarrow (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

$p \rightarrow (p \wedge \sim q)$ is false when p is true and q is true.

4. $\int \frac{d\theta}{\cos^2\theta\ (\sec 2\theta + \tan 2\theta)} = \lambda \tan\theta + 2\log f(x) + c$ then ordered pair $(\lambda,\ f(x))$ is

a. $(1, 1 + \tan\theta)$

b. $(1, 1 - \tan\theta)$

c. $(-1, 1 + \tan\theta)$

d. $(-1, 1 - \tan\theta)$

Answer: (c)

Solution:

Let $I = \int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)}$

$I = \int \frac{\sec^2\theta d\theta}{\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right) + \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}$

$I = \int \frac{(1-\tan^2\theta)(\sec^2\theta)d\theta}{(1+\tan\theta)^2}$

Let $\tan\theta = k \Rightarrow \sec^2\theta\ d\theta = \mathrm{d}k$

$I = \int \frac{(1-k^2)}{(1+k)^2} dk = \int \frac{(1-k)}{(1+k)} dk$

$I = \left(\frac{2}{1+k} - 1\right) dk$

$I = 2\ln|1+k| - k + c$

$I = 2\ln|1+\tan\theta| - \tan\theta + c$

Given $I = \lambda \tan\theta + 2 \log f(x) + c$

$\therefore \lambda = -1, f(x) = |1 + tan\theta|$

5. Let a_n is a positive term of GP and $\sum_{n=1}^{100} a_{2n+1} = 200$, $\sum_{n=1}^{100} a_{2n} = 100$, then the value of $\sum_{n=1}^{200} a_n$ is

a. 150 b. 225

c. 300 d. 175

Answer: (a)

Solution:

a_n is a positive term of GP.

Let GP be $a, ar, ar^2, \ldots..$

$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots\ldots + a_{201}$

$200 = ar^2 + ar^4 + \ldots\ldots + ar^{201}$

$200 = \frac{ar^2(r^{200}-1)}{r^2-1} \quad \ldots(1)$

Also, $\sum_{n=1}^{100} a_{2n} = 100$

$100 = a_2 + a_4 + \ldots \ldots. + a_{200}$

$100 = ar + ar^3 + \ldots\ldots. + ar^{199}$

$100 = \frac{ar(r^{200}-1)}{r^2-1} \quad \ldots(2)$

From (1) and (2), r = 2

And $\sum_{n=1}^{100} a_{2n+1} + \sum_{n=1}^{100} a_{2n} = 300$

$\Rightarrow a_2 + a_3 + a_4 \ldots \ldots. + a_{200} + a_{201} = 300$

$\Rightarrow ar + ar^2 + ar^3 + \ldots\ldots + ar^{200} = 300$

$\Rightarrow r(a + ar + ar^2 + \ldots. + ar^{199}) = 300$

$\Rightarrow 2(a_1 + a_2 + a_3 + \ldots\ldots + a_{200}) = 300$

$\sum_{n=1}^{200} a_n = 150$

6. z is a complex number such that $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be equal to

a. $\sqrt{8}$ b. $\sqrt{7}$

c. $\sqrt{\frac{17}{2}}$ d. $\sqrt{10}$

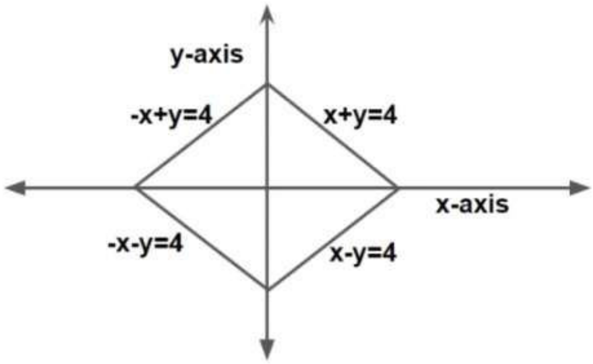
Answer: (b)

Solution:

$|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$

Let $z = x + iy$

$\Rightarrow |x| + |y| = 4$



$\therefore z$ lies on the rhombus.

Maximum value of $|z| = 4$ when $z = 4, -4, 4i, -4i$

Minimum value of $|z| = 2\sqrt{2}$ when $z = 2 \pm 2i,\ \pm 2 + 2i$

$|z| \in [2\sqrt{2}, 4]$

$|z| \in [\sqrt{8}, \sqrt{16}]$

$|z| \neq \sqrt{7}$

7. $f(x) : [0,5] \to R, F(x) = \int_0^x x^2\, g(x)dx\,, f(1) = 3, g(x) = \int_1^x f(t)dt$ then correct choice is

a. $F(x)$ has no critical point
b. $F(x)$ has local minimum at $x = 1$
c. $F(x)$ has local maximum at $x = 1$
d. $F(x)$ has point of inflection at $x = 1$

Answer: (b)

Solution:

$F(x) = x^2 g(x)$

Put $x = 1$

$\Rightarrow F(1) = g(1) = 0$ ... (1)

Now $F''(x) = 2xg(x) + g'(x)x^2$

$F''(1) = 2g(1) + g'(1)$ $\{\because g'(x) = f(x)\}$

$F''(1) = f(1) = 3$ ... (2)

From (1) and (2), $F(x)$ has local minimum at $x = 1$

8. Let $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, then the value of $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is

a. $\frac{3}{8}$

b. $\frac{5}{8}$

c. $\frac{7}{8}$

d. $\frac{3}{2}$

Answer: (a)

Solution:

$$\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\frac{dy}{dx} = \frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}$$

$$\frac{dy}{dx} = \cot\frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{2}\ cosec^2\frac{3\theta}{2}\ \frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{3}{2}\ cosec^2\frac{3\theta}{2}\right)\frac{1}{(2\cos\theta - 2\cos 2\theta)}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi} = \frac{3}{8}$$

9. If $f(x)$ and $g(x)$ are continuous functions, fog is identity function, $g'(b) = 5$ and $g(b) = a$, then $f'(a)$ is

a. $\frac{3}{5}$

b. 5

c. $\frac{2}{5}$

d. $\frac{1}{5}$

Answer: (d)

Solution:

$f(g(x)) = x$

$f'(g(x))g'(x) = 1$

Put $x = b$

$f'(g(b))g'(b) = 1$

$f'(a) \times 5 = 1$

$f'(a) = \frac{1}{5}$

10. Let $x + 6y = 8$ is tangent to standard ellipse where minor axis is $\frac{4}{\sqrt{3}}$, then eccentricity of ellipse is

a. $\frac{1}{4}\sqrt{\frac{11}{12}}$

b. $\frac{1}{4}\sqrt{\frac{11}{3}}$

c. $\sqrt{\frac{5}{6}}$

d. $\sqrt{\frac{11}{12}}$

Answer: (d)

Solution:

If $2b = \frac{4}{\sqrt{3}}$

$b = \frac{2}{\sqrt{3}}$

Comparing $y = -\frac{x}{6} + \frac{8}{6}$ with $y = mx \pm \sqrt{a^2m^2 + b^2}$

$m = -\frac{1}{6}$ and $a^2m^2 + b^2 = \frac{16}{9}$

$\frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3}$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{11}{12}}$

11. If one end of focal chord of parabola $y^2 = 8x$ is $\left(\frac{1}{2}, -2\right)$, then the equation of tangent at the other end of this focal chord is

a. $x + 2y + 8 = 0$
b. $x - 2y = 8$
c. $x - 2y + 8 = 0$
d. $x + 2y = 8$

Answer: (c)

Solution:

Let PQ be the focal chord of the parabola $y^2 = 8x$

$\Rightarrow P(t_1) = (2t_1^2, 4t_1)$ & $Q(t_2) = (2t_2^2, 4t_2)$

$\Rightarrow t_1 t_2 = -1$

$\because \left(\frac{1}{2}, -2\right)$ is one of the ends of the focal chord of the parabola

Let $\left(\frac{1}{2}, -2\right) = (2t_2^2, 4t_2)$

$\Rightarrow t_2 = -\frac{1}{2}$

$\Rightarrow$ Other end of focal chord will have parameter $t_1 = 2$

$\Rightarrow$ The co-ordinate of the other end of the focal chord will be $(8, 8)$

$\therefore$ The equation of the tangent will be given as $\rightarrow$ $8y = 4(x + 8)$

$\Rightarrow 2y - x = 8$

12. If $7x + 6y - 2z = 0$, $3x + 4y + 2z = 0$ & $x - 2y - 6z = 0$, then the system of equations has

a. No solution
b. Infinite non-trivial solution for $(x = 2z)$
c. Infinite non-trivial solution for $(y = 2z)$
d. Only trivial solution

Answer: (b)

Solution:

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$

As the system of equations are Homogeneous $\Rightarrow$ the system is consistent.

$$\Rightarrow \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0$$

$\Rightarrow$ Infinite solutions exist (both trivial and non-trivial solutions)

When $y = 2z$

Let's take $y = 2, z = 1$

When $(x, 2,1)$is substituted in the system of equations

$\Rightarrow 7x + 10 = 0$

$3x + 10 = 0$

$x - 10 = 0$ (which is not possible)

$\therefore y = 2z \Rightarrow$ Infinite non-trivial solutions does not exist.

For $x = 2z$, lets take $x = 2, z = 1, y = y$

Substitute $(2, y, 1)$in system of equations

$\Rightarrow y = -2$

$\therefore$For each pair of (x, z), we get a value of y.

Therefore, for $x = 2z$ infinite non-trivial solution exists.

13. If both the roots of the equation $ax^2 - 2bx + 5 = 0$ are α and of the equation $x^2 - 2bx - 10 = 0$ are α and β. Then the value of $\alpha^2 + \beta^2$

a. 15
b. 20
c. 25
d. 30

Answer: (c)

Solution:

$ax^2 - 2bx + 5 = 0$ has both roots as α

$\Rightarrow 2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$

And $\alpha^2 = \dfrac{5}{a}$

$\Rightarrow b^2 = 5a (a \neq 0)$ ... (1)

$\Rightarrow \alpha + \beta = 2b \,\&\, \alpha\beta = -10$

$\alpha = \frac{b}{a}$ is also a root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

$\because b^2 = 5a \Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow a = \frac{1}{4} \Rightarrow b^2 = \frac{5}{4}$

$\Rightarrow \alpha^2 = 20, \beta^2 = 5$

$\Rightarrow \alpha^2 + \beta^2 = 25$

14. If $A = \{x: |x| < 2$ and $B = \{x: |x - 2| \geq 3\}$ then

a. $A \cap B = [-2, -1]$
b. $B - A = \mathbf{R} - (-2,5)$
c. $A - B = [-1,2)$
d. $A \cup B = \mathbf{R} - (2,5)$

Answer: (b)

Solution:

$A = \{x: x \in (-2,2)\}$

$B = \{x: x \in (-\infty, -1] \cup [5, \infty)\}$

$A \cap B = \{x: x \in (-2, -1]\}$

$B - A = \{x: x \in (-\infty, -2] \cup [5, \infty)\}$

$A - B = \{x: x \in (-1,2)\}$

$A \cup B = \{x: x \in (-\infty, 2) \cup [5, \infty)\}$

15. The value of $P(x_i > 2)$ for the given probability distribution is

x_i	1	2	3	4	5
P_i	k^2	$2k$	k	$2k$	$5k^2$

a. $\frac{1}{36}$
b. $\frac{23}{36}$
c. $\frac{1}{6}$
d. $\frac{7}{12}$

Answer: (b)

Solution:

We know that $\sum_{x_i=1}^{5} P_i = 1$

$\Rightarrow k^2 + 2k + k + 2k + 5k^2 = 1$

$\Rightarrow k = -1, \frac{1}{6} \therefore k = \frac{1}{6}$

$P(x_i > 2) = P(x_i = 3) + P(x_i = 4) + P(x_i = 5)$

$= k + 2k + 5k^2 = \frac{23}{36}$

16. Let the distance between the plane passing through lines $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{8}$ and $\frac{x+3}{2} = \frac{y+2}{1} = \frac{z-1}{\lambda}$ and the plane $23x - 10y - 2z + 48 = 0$ is $\frac{k}{\sqrt{633}}$, then the value of k is

a. 4
b. 3
c. 2
d. 1

Answer: (b)

Solution:

We find the point of intersection of the two lines, and the distance of given plane from the two lines is the distance of plane from the point of intersection.

$\therefore (2p - 1, 2p + 3, 8p - 1) = (2q - 3, q - 2, \lambda q + 1)$

$p = -2$ and $q = -1$

$\lambda = 18$

Point of intersection is $(-5, -3, -17)$

$\therefore \frac{k}{\sqrt{633}} = \left|\frac{-115 + 30 + 34 + 48}{\sqrt{633}}\right| \Rightarrow k = 3$

17. Let $x = \sum_{n=0}^{\infty} (-1)^n (\tan\theta)^{2n}$ and $y = \sum_{n=0}^{\infty} (\cos\theta)^{2n}$, where $\theta \in \left(0, \frac{\pi}{4}\right)$, then

a. $y(x-1) = 1$
b. $y(1-x) = 1$
c. $x(y+1) = 1$
d. $y(1+x) = 1$

Answer: (b)

Solution:

$y = 1 + \cos^2\theta + \cos^4\theta + \cdots$

$\Rightarrow y = \dfrac{1}{1-\cos^2\theta} \Rightarrow \dfrac{1}{y} = \sin^2\theta$

$x = 1 - \tan^2\theta + \tan^4\theta - \cdots$

$\Rightarrow x = \dfrac{1}{1-(-\tan^2\theta)} = \cos^2\theta$

$\therefore x + \dfrac{1}{y} = 1 \Rightarrow y(1-x) = 1$

18. If $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$, then the value of x at which $f(x) = [x^2]\sin \pi x$ is discontinuous at (where [.] denotes greatest integer function)

a. $\sqrt{A+5}$
b. $\sqrt{A+1}$
c. $\sqrt{A+21}$
d. $\sqrt{A}$

Answer: (b)

Solution:

$f(x) = [x^2]\sin \pi x$

It is continuous $\forall x \in \mathbf{Z}$ as $\sin \pi x \to 0$ as $x \to \mathbf{Z}$.

$f(x)$ is discontinuous at points where $[x^2]$ is discontinuous i.e. $x^2 \in \mathbf{Z}$ with an exception that $f(x)$ is continuous as x is an integer.

$\therefore$ Points of discontinuity for $f(x)$ would be at

$x = \pm\sqrt{2}, \pm\sqrt{3}, \pm\sqrt{5}, \ldots\ldots$

Also, it is given that $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$ (indeterminate form $(0 \times \infty)$)

$\Rightarrow \lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A$

$\Rightarrow 4 - \lim_{x\to 0}\left\{\frac{4}{x}\right\} = A$

$\Rightarrow A = 4$

$\sqrt{A+5} = 3$

$\sqrt{A+1} = \sqrt{5}$

$\sqrt{A+21} = 5$

$\sqrt{A} = 2$

$\therefore$ Points of discontinuity for $f(x)$ is $x = \sqrt{5}$

19. Circles $(x-0)^2 + (y-4)^2 = k$ and $(x-3)^2 + (y-0)^2 = 1^2$ touch each other. The maximum value of k is ________.

Answer: (36)

Solution:

Two circles touch each other if $C_1C_2 = |r_1 \pm r_2|$

$\sqrt{k} + 1 = 5$ or $|\sqrt{k} - 1| = 5$

$\Rightarrow k = 16$ or 36

Maximum value of k is 36

20. If ${}^{25}C_0 + 5\,{}^{25}C_1 + 9\,{}^{25}C_2 + \cdots + 101\,{}^{25}C_{25} = 2^{25}k$, then the value of k is ________.

Answer: (51)

Solution:

$S = {}^{25}C_0 + 5\,{}^{25}C_1 + 9\,{}^{25}C_2 + \cdots + 97\,{}^{25}C_{24} + 101\,{}^{25}C_{25} = 2^{25}k \qquad (1)$

Reverse and apply property ${}^{n}C_r = {}^{n}C_{n-r}$ in all coefficients

$S = 101\,{}^{25}C_0 + 97\,{}^{25}C_1 + \cdots + 5\,{}^{25}C_{24} + {}^{25}C_{25} \qquad (2)$

Adding (1) and (2), we get

$2S = 102[{}^{25}C_0 + {}^{25}C_1 + \cdots + {}^{25}C_{25}]$

$S = 51 \times 2^{25}$

$\Rightarrow k = 51$

21. Number of common terms in both the sequences $3, 7, 11, \ldots 407$ and $2, 9, 16, \ldots 905$ is _______.

Answer: (14)

Solution:

First common term is 23

Common difference $= \text{LCM}(7, 4) = 28$

$23 + (n-1)28 \le 407$

$n - 1 \le 13.71$

$n = 14$

22. Let $|\vec{a}| = \sqrt{3}, |\vec{b}| = 5, \vec{b}.\vec{c} = 10$ and angle between $\vec{b}$ and $\vec{c}$ is equal to $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to $\vec{b} \times \vec{c}$, then the value of $|\vec{a} \times (\vec{b} \times \vec{c})|$ is

Answer: (30)

Solution:

$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{b} \times \vec{c}| \sin\theta$ where θ is the angle between $\vec{a}$ and $\vec{b} \times \vec{c}$

$\theta = \frac{\pi}{2}$ given

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3}|\vec{b} \times \vec{c}| = \sqrt{3}|\vec{b}||\vec{c}| \sin\frac{\pi}{3}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} \times 5 \times |\vec{c}| \times \frac{\sqrt{3}}{2}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \frac{15}{2}|\vec{c}|$

Now, $|\vec{b}||\vec{c}| \cos\theta = 10$

$5|\vec{c}|\frac{1}{2} = 10$

$|\vec{c}| = 4$

23. If minimum value of term free from x for $\left(\frac{x}{\sin\theta} + \frac{1}{x\cos\theta}\right)^{16}$ is L_1 in $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ and L_2 in $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, the value of $\frac{L_2}{L_1}$ is

Answer: (16)

Solution:

$$T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r} \left(\frac{1}{x\cos\theta}\right)^{r}$$

For term independent of x,

$$16 - 2r = 0 \Rightarrow r = 8$$

$$T_9 = {}^{16}C_8 \left(\frac{1}{\sin\theta\cos\theta}\right)^8 = {}^{16}C_8 2^8 \left(\frac{1}{\sin 2\theta}\right)^8$$

$$L_1 = {}^{16}C_8 2^8 \quad \text{at } \theta = \frac{\pi}{4}$$

$$L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 2^{12} \quad \text{at } \theta = \frac{\pi}{8}$$

$$\frac{L_2}{L_1} = 16$$