PAPER - I

CODE : 0

Time : 3 Hours **Maximum Marks : 264**

READ THE INSTRUCTIONS CAREFULLY

GENERAL :

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The ORS CODE is printed on its left part as well as the right part. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet, verify that the booklet contains 32 pages and that all the 60 questions along the with the options are legible.

QUESTION PAPER FORMAT AND MARKING SCHEME :

8. The question paper has three parts : Physics, Chemistry and Mathematics. Each part has three sections.
9. Carefully read the instructions given at the beginning of each section.
10. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive)
11. Section 2 contains 10 multiple choice questions with one or more than one correct option.
    Marking Scheme : +4 for correct answer, 0 if not attempted and –2 in all other cases.
12. Section 3 contains 2 “match the following” type questions and you will have to match entries in Column I with the entries in Column II.
    Marking Scheme : for each entry in Column I, +2 for correct answer, 0 if not attempted and –1 in all other cases.

OPTICAL RESPONSE SHEET :

13. The ORS consists of an original (top sheet) and its carbon-less copy (bottom sheet).
14. Darken the appropriate bubbles on the original by applying sufficient pressure. This will leave an impression at the corresponding place on the carbon-less copy.
15. The original is machine-gradable and will be collected by the invigilator at the end of the examination.
16. You will be allowed to take away the carbon-less copy at the end of the examination.
17. Do not tamper with or mutilate the ORS.
18. Write your name, roll number and the name of the examination center and sign with pen in the space provided for this purpose on the original. Do not write any of these details anywhere else. Darken the appropriate bubble under each digit of your roll number.

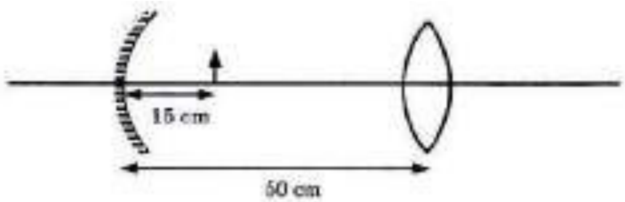
JEE-ADVANCE SOLUTIONS

PHYSICS

SECTION : 1 (Maximum Marks : 32)

- **This section contains EIGHT questions**
- **The answer to each question is a SINGLE DIGIT NUMBER ranging from 0 to 9, both inclusive.**
- **For each question, darken the bubble corresponding to the correct integer in the ORS.**
- **Marking scheme:**
  +4 If the bubble corresponding to the answer is darkened.
  0 In all other cases.

1. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept is a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



Ans. (7)

Sol: For mirror $u = -15\text{cm}\ f = -10\text{cm}$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} + \frac{1}{-15} = \frac{1}{-10}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{10}$$

$\Rightarrow v = -30$ cm

For lens

$u = -20\text{cm}\ \ f = 10\text{cm}$

$$\therefore \frac{1}{v} - \frac{1}{-20} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{20}$$

$\Rightarrow v = 20$ cm

$$|M_1| = \frac{30}{15} \times \frac{20}{20} = 2$$

In second case, for mirror

$v == -30$ cm

New focal length of lens

$$f = \frac{\frac{3}{2}-1}{\frac{3}{2}\times\frac{6}{7}-1}\times 10 = \frac{\frac{1}{2}}{\frac{9}{7}-1}\times 10 = \frac{35}{2}$$

$\therefore$ for lens

$$\frac{1}{v}-\frac{1}{-20}=\frac{2}{35}$$

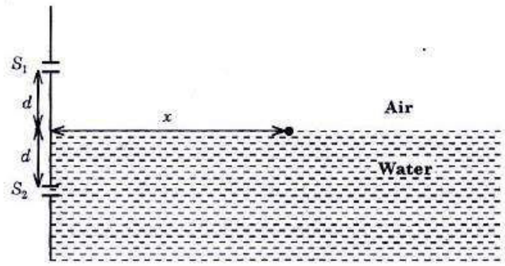
$$\therefore\ \frac{1}{v}=\frac{2}{35}-\frac{1}{20}=\frac{1}{140}$$

$\Rightarrow$ v= 140 cm

$$|M_2| = \frac{30}{15}\times\frac{140}{20} = 14$$

$$\therefore\ \left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7.$$

2. A Young's double slit interference arraignment with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is



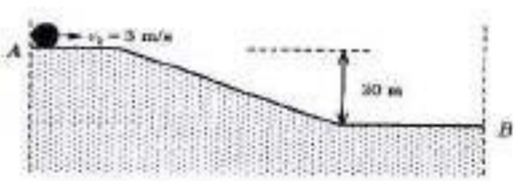
Ans. (3)

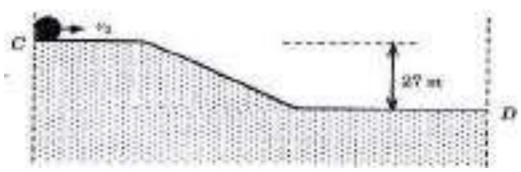
Sol: For maxima path difference = n λ

$$\therefore\ \sqrt{d^2+x^2}\left(\frac{4}{3}-1\right)=n\lambda \qquad \Rightarrow \sqrt{d^2+x^2}\left(\frac{4}{3}-1\right)=n\lambda$$

$$\therefore\ x^2 = 9n^2\lambda^2 - d^2 \qquad \Rightarrow p = 3$$

3. Two identical uniform discs roll without slipping on two different surface AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is (g = 10 m/s^2)





Ans. (7)

Sol: $\frac{3}{4}mv_1^2 + mgh_1 = \frac{3}{4}mv_2^2 + mgh_2$

$\therefore \frac{3}{4}\left(v_2^2 - v_1^2\right) = g\left(h_1 - h_2\right)$

$\Rightarrow v_2^2 - v_1^2 = 4g = 40$

$\Rightarrow v_2^2 = 49$

$\Rightarrow v_2 = 7$ m/s.

4. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity form the planet is v_{ese} = $v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

Ans. (2)

Sol: At the maximum height, acceleration due to gravity is 1/4th $\Rightarrow h_{max} = R$ (radius of planet)

Now COE $\Rightarrow \frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{2R}$

$\Rightarrow v = \sqrt{\frac{GM}{R}}$

Now $v_{esc} = \sqrt{\frac{2GM}{R}} = v\sqrt{2} \Rightarrow N = 2$

5. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

Ans. (2)

Sol: $p \propto \frac{r^2}{\lambda^4}$ (For spherical stars)

$\Rightarrow 10^4 = (400)^2 \times \left(\frac{\lambda_B}{\lambda_A}\right)^4$

$\Rightarrow \frac{\lambda_A}{\lambda_B} = 2$

6. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life *T* years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of *nT* years, then the value of *n* is

Ans. (3)

Sol: At t = 0

$P_{required} = P_{available} \times (12.5/100) = P_{available}/8$

$\therefore P_{available} = 8\, P_{required}$

$\therefore$ After first half life, $P_{available} = 4\, P_{required}$

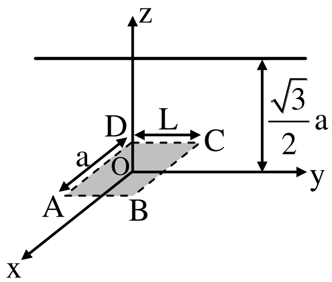
After second half life, $P_{available} = 2P_{required}$

After third half life, $P_{available} = P_{required}$

$\therefore$ Power required by the village can be met upto 3 half lives.

$\therefore$ n = 3

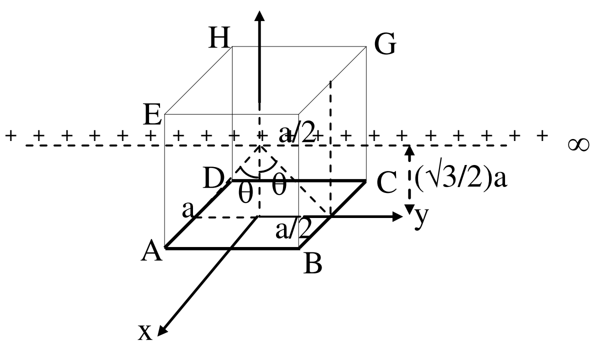
7. **An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is**



Ans. **(6)**

From figure,

$$\tan\theta = \frac{a/2}{\frac{\sqrt{3}}{2}a} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^o$$



$\therefore\ 2\theta = 60^o$

Total flux through $360^o = \frac{q_{in}}{\varepsilon_o} = \frac{\lambda L}{\varepsilon_o}$

$\therefore$ Flux through 60^o is equal to

$$\frac{\lambda L}{\varepsilon_o} \times \frac{60^o}{360^o} = \frac{\lambda L}{6\varepsilon_o} \qquad \therefore\ n = 6$$

8. **Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (*hc* = 1242 eV nm)**

Ans. **(2)**

Sol: Energy of the incident radiation = hc/λ = 1242/90 = 13.8 eV

Total energy of electron in n^{th} orbit = $\frac{13.6}{n^2}$ eV

$$\therefore\ \frac{-13.6}{n^2} + 13.8 = 10.4\,\text{eV}$$

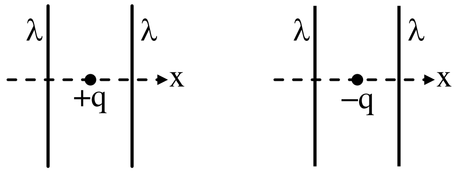
$$\frac{13.6}{n^2} = 13.8 - 10.4 = 3.4$$

$$n^2 = \frac{13.6}{3.4} = 4 \quad \Rightarrow n = 2$$

SECTION : 2 (Maximum Marks : 40)

- **This section contains TEN questions**
- **Each question has FOUR options (A), (B), (C) and (D). ONE OR ORE THAN ONE of these four option(s) is(are) correct.**
- **For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.**
- **Marking scheme:**
  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  0 If one of the bubbles is darkened. –2 In all other cases.

9. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)



(A) Both charges execute simple harmonic motion.
(B) Both charges will continue moving in the direction of their displacement.
(C) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement.
(D) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.

Ans. (C)

Sol: In situation 1, let the distance between 2 line charges be 2a. Now if +q is shifted to the left by a small distance x , then

$$F_{net} = \frac{2K\lambda}{(a-x)} - \frac{2K\lambda}{(a+x)} \quad \text{(towards right)}$$

$$= 2K\lambda\frac{2x}{(a^2-x^2)} = \frac{4K\lambda}{(a^2-x^2)}x$$

$\because$ Force is opposite to direction of displacement

$$\therefore\ F_{net} = -\left[\frac{4K\lambda}{a^2-x^2}\right]x$$

Now as x << a, $\therefore\ a^2 - x^2 = a^2$

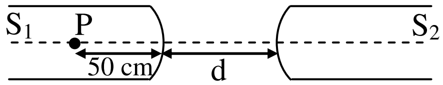
$$\Rightarrow\ F_{net} = -\left[\frac{4K\lambda}{a^2}\right]x$$

$\therefore$ +q will execute SHM

Whereas in situation 2 when –q is displaced it will continue to move in the direction of displacement.

$\therefore$ option (C) is correct.

10. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is



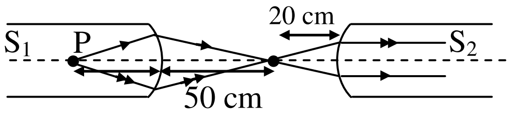
(A) 60 cm (B) 70 cm (C) 80 cm (D) 90 cm

Ans. (B)

For refraction on curved surface of S_1

$\mu_1 = 1.5,\ \mu_2 = 1,\ R = -10,\ u = -50$

using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get v = 50 cm



Now for refraction at curved surface of S_2

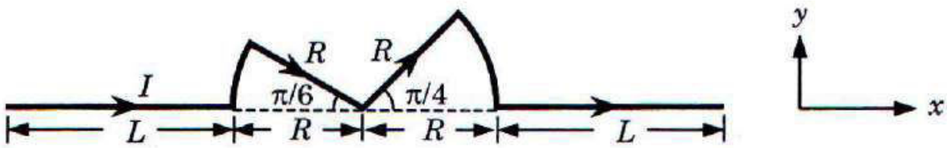
$v = \infty$ (as rays inside S_2 are parallel to the principal axis)

$\mu_1 = 1,\ \mu_2 = 1.5,\ R = +10,\ u = -x$

$\therefore \frac{1.5}{\infty} - \frac{1}{-x} = \frac{1.5-1}{10} \Rightarrow x = 20$

$\therefore$ d = 50 + 20 = 70, so option (B) is correct.

11. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$ ^. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are)



(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L+R)$

(B) If $\vec{B}$ is along $\hat{x}$, F=0

(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L+R)$

(D) If $\vec{B}$ is along $\hat{z}$, F=0

Ans. (A, B, C)

Sol: $\vec{F}_m = i\left(\vec{dl} \times \vec{B}\right)$

In (A) part $F_m \propto (L+R)$

If $\vec{B}$ is along $\hat{z}$ axis.

In (B) part $F_m = 0$

If $\vec{B}$ is along $\hat{x}$ axis.

In (C) part $F_m \propto (L+R)$

If $\vec{B}$ is along $\hat{y}$ axis.

All the result as according to above formula.

12. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are)

(A) The average energy per mole of the gas mixture is 2RT

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is ½

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$

Ans. (A, B, D)

Sol: Average energy/ mole $= \dfrac{\frac{3}{2}RT + \frac{5}{2}RT}{2} = 2RT$

(B) $$\frac{\text{Speed of sound in mixture}}{\text{Speed of sound in the Helium gas}} = \sqrt{\frac{\gamma_{mix}}{M_{mix}} \times \frac{M_{He}}{\gamma_{He}}} \qquad \left\{\because v = \sqrt{\frac{\gamma RT}{M}}\right\}$$

$$\gamma_{mix} = \frac{\Sigma n_i C_P}{\Sigma n_i C_V}$$

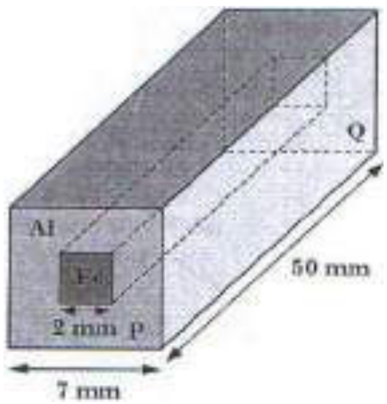
$$\text{Average molar mass of mixture} = \frac{1}{2} \times 4 + \frac{1}{2} \times 2 = 2 + 1 = 3$$

$$= \sqrt{\left(\frac{3/2}{3}\right)\left(\frac{4}{5/3}\right)} = \sqrt{\frac{6}{5}}$$

(D) $$v_{rms} \propto \frac{1}{\sqrt{M}}$$

$$\frac{(v_{rms})_{He}}{(v_{rms})_{H_2}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

13. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\,\Omega$ and $1.0 \times 10^{-7}\,\Omega\,m$, respectively. The electrical resistance between the two faces P and Q the composite bar is



(A) $\dfrac{2475}{64}\,\mu\Omega$ (B) $\dfrac{1875}{64}\,\mu\Omega$ (C) $\dfrac{1875}{49}\,\mu\Omega$ (D) $\dfrac{2475}{132}\,\mu\Omega$

Ans. (B)

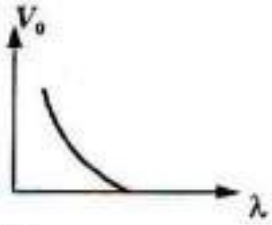
Sol: $l = 50 \times 10^{-3m}$, $\rho_{Al} = 2.7 \times 10^{-8}\,\Omega$, $\rho_{Fe} = 1.0 \times 10^{-7}\,\Omega$

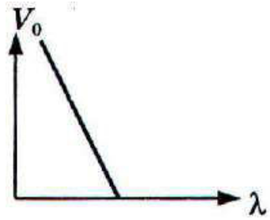
$$R_{Al} = \frac{\rho_{Al} \times l}{A} = \frac{2.7 \times 10^{-8} \times 50 \times 10^{-3}}{(7 \times 7 - 2 \times 2) \times 10^{-6}} = 3.05 \times 10^{-5}\,\Omega$$

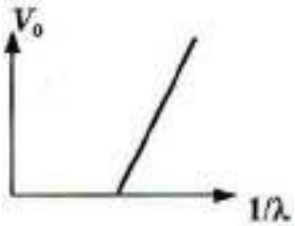
$$R_{Fe} = \frac{\rho_{Fe} \times l}{A} = \frac{1.0\times10^{-7}\times50\times10^{-3}}{(7\times7-2\times2)\times10^{-6}} = 125\times10^{-5}\,\Omega$$

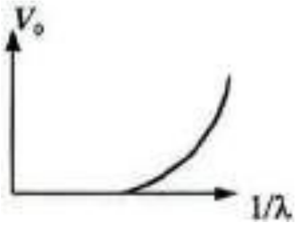
Now $R_{eqn} = \dfrac{R_{Fe}\times R_{Al}}{R_{Fe}+R_{Al}} = \left(\dfrac{1875}{64}\right)\mu\Omega$

14. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $1/\lambda$

(A) 

(B) 

(C) 

(D) 

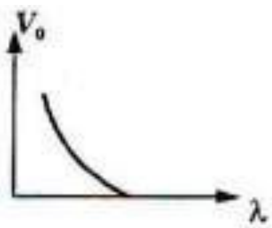
Ans. (A, C)

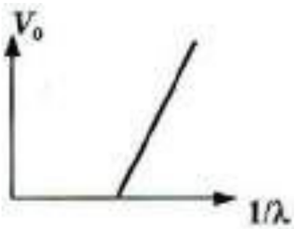
Sol: By photoelectric equation

$$hf - W_0 = eV_0$$

$$h\frac{c}{\lambda} - W_0 = eV_0$$

$$\Rightarrow \quad \frac{1}{\lambda} \propto V_0$$





15. Consider a Vernier calipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier calipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then :

(A) If the pitch of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm.

(B) If the pitch of the screw gauge is twice the least count o the Vernier calipers, the least count of the screw gauge is 0.005 mm

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm

(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.005 mm

Ans. (B, C)

Sol: L.C. of vernier caliper = 1 MSD – 1 VSD

$$=\frac{1}{8}\text{cm}-\frac{1}{2\times5}\text{cm}$$

$$=\frac{10-8}{80}=\frac{2}{80}=\frac{1}{40}\text{cm}=0.025\,\text{cm}=0.25\,\text{mm}$$

L.C. of screw gauge $=\dfrac{\text{pitch}}{\text{total no. of division on circular scale}}$

Pitch = linear distance moved on main scale in one rotation of circular scale.

16. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is (are)

(A) $M\propto\sqrt{c}$ (B) $M\propto\sqrt{G}$ (C) $L\propto\sqrt{h}$ (D) $L\propto\sqrt{G}$

Ans. (A,C, D)

Sol. $[h] = ML^2T^{-1}$ ...(i)

$[G] = M^{-1}L^3T^{-2}$.....(ii)

$[C] = LT^{-1}$ ...(iii)

From (i) and (ii)

$[hG] = L^5T^{-3} = L^2 = L^2(L^3\,T^{-3})$

$L^2\,[LT^{-1}]^3$

$$\Rightarrow L^2=\left[\frac{hG}{C^3}\right]$$

$$\Rightarrow L\propto\sqrt{G}$$

$$\Rightarrow L\propto\sqrt{h}$$

$$hc = ML^3T^{-2}$$

$$\frac{hc}{G}=\frac{ML^3T^{-2}}{M^{-1}L^3T^{-2}}$$

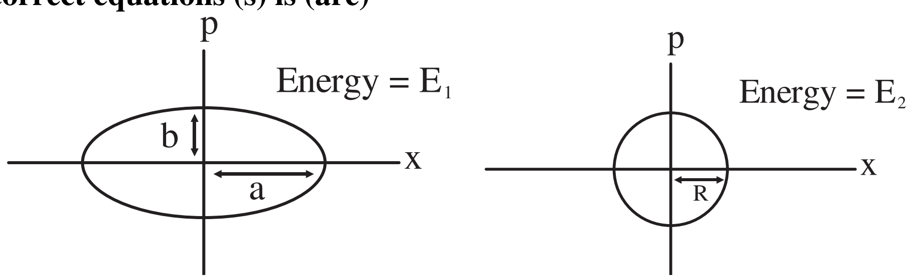
$$\frac{hc}{G}=m^2$$

$$M=\sqrt{\frac{hc}{G}}$$

$$\Rightarrow m\propto\sqrt{c}$$

Option (A), (C) and (D) are correct.

17. Two independent harmonic oscillators of equal mass are oscillation about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figure. If $\frac{a}{b}=n^2$ and $\frac{a}{R}=n$, then the correct equations (s) is (are)



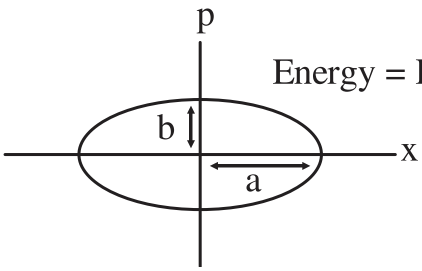
(A) $E_1\omega_1 = E_2\omega_2$ (B) $\dfrac{\omega_2}{\omega_1} = n_2$ (C) $\omega_1\omega_2 = n^2$ (D) $\dfrac{E_1}{\omega_1} = \dfrac{E_2}{\omega_2}$

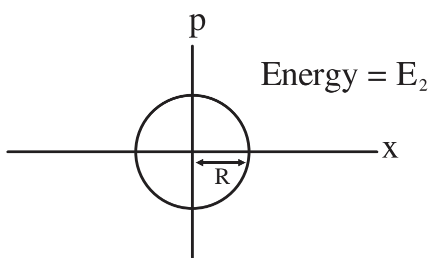
Ans. (B,D)

Sol.

Given $\dfrac{a}{n} = n^2, \dfrac{a}{R} = n$

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2$$





From figure

$$E_1 = \frac{m\omega_1^2}{2}a^2$$

$$E_2 = \frac{m}{2}\omega_2^2 R^2$$

$$\therefore \frac{E_1}{E_2} = \left(\frac{\omega_1}{\omega_2}\right)^2\left(\frac{a}{R}\right)^2 = \left(\frac{\omega_1}{\omega_2}\right)^2 .n^2 \text{.....(i)}$$

Also, $$\frac{E_1}{E_2} = \frac{\frac{p_1^2}{2m}}{\frac{p_2^2}{2m}} = \left(\frac{p_1}{p_2}\right)^2 = \left(\frac{b}{R}\right)^2$$

Or, $$\frac{E_1}{E_2} = \left(\frac{a}{n^2 \times \frac{a}{n}}\right)^2 = \frac{1}{n^2} \quad \text{.....(ii)}$$

From (i) and (ii)

$$\left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{1}{n^4} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{n^2}$$

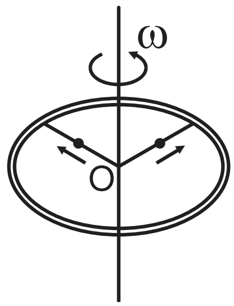
$$\Rightarrow \frac{\omega_2}{\omega_1} = n^2$$

And $$\frac{E_1}{E_2} = \frac{\omega_1}{\omega_2}$$

$$\Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Hence, (B) and (D) are correct options

18. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is

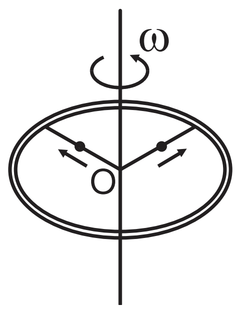


(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

Ans. (D)

Sol. Using conservation of angular momentum

We have



$$\left(MR^2\right)\omega = \left(MR^2 + \frac{M}{8}\times\frac{9}{25}R^2 + \frac{M}{8}x^2\right)\times\frac{8}{9}\omega$$

$$\Rightarrow \frac{9}{8}R^2 = R^2 = \frac{9R^2}{200} + \frac{x^2}{8}$$

$$\Rightarrow \frac{1}{8}R^2 = \frac{9}{200}R^2 + \frac{x^2}{8}$$

$$\Rightarrow x^2 = \frac{16}{25}R^2$$

$$\Rightarrow x = \frac{4}{5}R$$

SECTION : 3 (Maximum Marks : 16)

- **This section contains TWO questions.**
- **Each question contains two columns, Column I and Column II.**
- **Column I has four entries (A), (B), (C) and (D).**
- **Column II has five entries (P), (Q), (R), (S) and (T).**
- **Match the entries in column I with the entries in column II.**
- **One or more entries in column I may match with one or more entries in column II.**
- **The ORS contains a 4 × 5 matrix.**
- **For each entry in column I, darken the bubbles of all the matching entries. For example, if entry (A) in column I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).**
- **Marking scheme:**
  For each entry in column I.
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened.
  0 If one of the bubbles is darkened.
  –1 In all other cases.
  –2 In all other cases.

19. **Match the nuclear process given in column I with the appropriate option(s) in column II.**

	Column I		Column II
(A)	Nuclear fusion	(P)	Absorption of thermal neutrons by $^{235}_{92}U$
(B)	Fission in a nuclear reactor	(Q)	$^{60}_{27}Co$ nucleus
(C)	β -decay	(R)	Energy production in stars via hydrogen conversion to helium
(D)	$\gamma - \text{ray}$ emission	(S)	Heavy water
		(T)	Neutrino emission

Ans. A→R,T; B→P,S,T; C→Q,T; D→R,T

Sol. **Based on theory**

20. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement (s) in column II.

	Column I		**Column II**
(A)	$U_1(x)=\frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	**(P)**	**The force acting on the particle is zero at x = a**
(B)	$U_2(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2$	**(Q)**	**The force acting on the particle is zero at x = 0**
(C)	$U_3(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	**(R)**	**The force acting on the particle is zero at x = -a**
(D)	$U_4(x)=\frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	**(S)**	**The particle experiences an attractive force towards x = 0 in the region** $\lvert x\rvert < a$.
		(T)	**The particle with total energy** $\frac{U_0}{4}$ **can oscillate about the point x = -a**

Ans. A→P,Q,R,T; B→Q,S; C→P,Q,R,S; D→P,R,T.

Sol. $$F_1(x)=-\frac{du_1}{dx}=\frac{2U_0x}{a^2}\left(1-\frac{x^2}{a^2}\right)$$

$$F_2(x)=\frac{-U_0}{2}\times\frac{2x}{a^2}$$

$$F_3(x)=\frac{-U_0}{2}\left[\frac{2x}{a^2}e^{-\frac{x^2}{a^2}}+\frac{x^2}{a^2}\times e^{-x^2/a^2}\times\frac{-2x}{a^2}\right]$$

$$\Rightarrow F_3=\frac{2U_0}{a^2}xe^{-\frac{x^2}{a^2}}\left(1-\frac{x^2}{a^2}\right)$$

$$F_4=-\frac{U_0}{2}\left[\frac{1}{a}-\frac{1}{3}\times\frac{3x^2}{a^3}\right]$$

PART II: CHEMISTRY

SECTION : 1 (Maximum Marks : 32)

- **This section contains EIGHT questions**
- **The answer to each question is a SINGLE DIGIT NUMBER ranging from 0 to 9, both inclusive.**
- **For each question, darken the bubble corresponding to the correct integer in the ORS.**
- **Marking scheme:**
  +4 If the bubble corresponding to the answer is darkened.
  0 In all other cases.

21. If the freezing point of a 0.01 molal aqueous solution of a cobalt (III) chloride-ammonia complex (which behaves as a strong electrolyte) is –0.0558°C, the number of chloride(s) in the coordination sphere of the complex is

Key: (1)

Sol. $\Delta T_f = i\,K_f\,m$

$0.0558 = i \times 1.86 \times 0.01$

$i = \frac{0.0558}{0.0186} = 3$

$CoCl_3.x\,NH_3$

$\therefore$ If i = 3, then total ions = 3

$\therefore$ No. of Cl^- ions in the coordination sphere = 1

The complex could be $\left[Co(NH_3)_x\,Cl\right]Cl_2$

22. All the energy released from the reaction $X \rightarrow Y, \Delta G^o = -193\,kJ\,mol^{-1}$

is used for oxidizing M^+ as $M^+ \longrightarrow M^{3+} + 2e^-, E^o = -0.25\,V$.

Under standard conditions the number of moles of M^+ oxidized when one mole of X is converted to Y is [F = 96500 C mol^{-1}]

Key: (4)

Sol. Energy released when one mole of X converts into Y = 193 kJ.

Energy needed for oxidation of 1 mole of M^+ under standard condition = $-nFE^o$

$= -2 \times 96500 \times (-0.25)\,J$

$= 48.25\,kJ$

$\therefore$ No. of moles of M^+ oxidized $= \frac{193}{48.25} = 4$

23. For the octahedral complexes of Fe^{3+} in SCN^- (thicyanato-S) and in CN^- ligand environments, the difference between the spin-only magnetic moments in Bohr magnetons (when approximated to the nearest integer) is [Atomic number of Fe = 26]

Key: 4

Sol. Fe^{3+} with SCN^- (weak filed ligand)

High spin octahedral complex

$\left[Fe(SCN)_6\right]^{3-}$

$Fe^{3+} \rightarrow tg^3eg^2$

$\mu_I = \sqrt{5(5+2)} = \sqrt{35}$ BM

Fe^{3+} with CN^- (strong field ligand)

Low spin octahedral complex

$\left[Fe(CN)_6\right]^{3-}$

$Fe^{3+} \rightarrow t_2g^5$

$\mu_{II} = \sqrt{1(1+2)} = \sqrt{3}$ BM

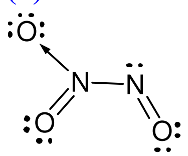
$\mu_I - \mu_{II} = \sqrt{35} - \sqrt{3}$

$= 5.91 - 1.73 = 4.18 \approx 4$ BM

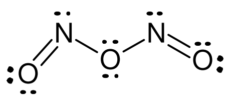
24. The total number of lone pairs of electrons in N_2O_3 is

Key: (8)

Sol.



(Asymmetric)



(symmetric)

Total number of lone pairs are 8 in N_2O_3 molecule.

25. Among the triatomic molecules/ions, BeCl₂, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^- and XeF₂, the total number of linear molecule(s)/ion(s) where the hybridization of the central atom does not have contribution from the d-orbital(s) is [Atomic number : S = 16, Cl = 17, I = 53 and Xe = 54]

Key: (4)

Sol. $BeCl_2$: sp

N_3^- : sp

N_2O : sp

NO_2^+ : sp

Other linear molecules/ions involve d orbitals.

26. Not considering the electronic spin, the degeneracy of the second excited state (n = 3) of H atom is 9, while the degeneracy of the second excited state of H⁻ is

Key. (3)

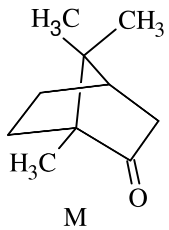
Sol. The possible excited states are

$1s^1 2p_x^1$

$1s^1 2p_y^1$

$1s^2 2p_z^1$

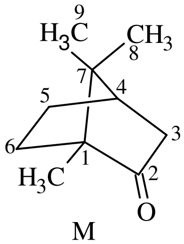
27. The total number of stereoisomers that can exist for M is



M

Key. (2)

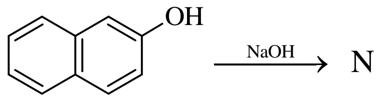
Sol.



M

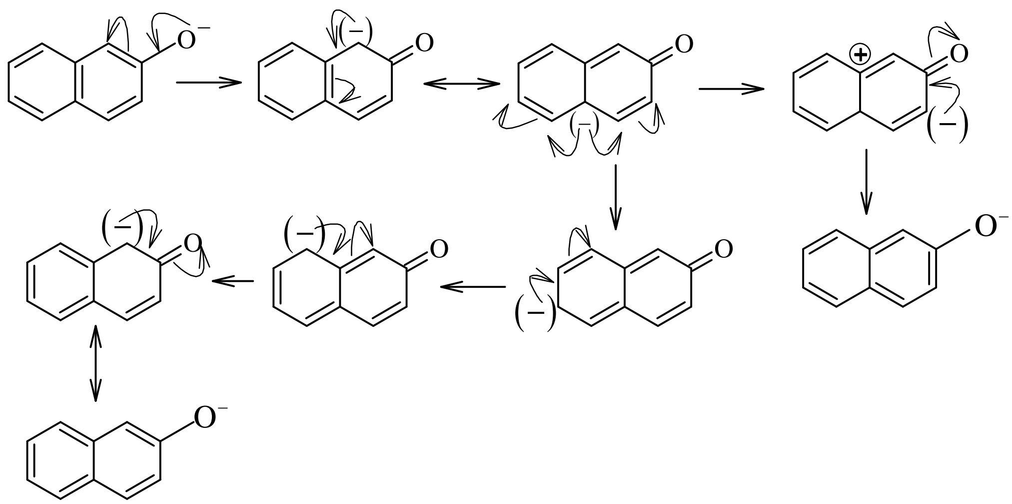
The compound M has two stereogenic carbons, hence 1R4R, 1S4S, 1R4S and 1S4R configurations are possible. Due to cage like structure it is not possible to have R configuration at C_1 and S-configuration at C_4. So total number of stereoisomers are 2.

28. The number of resonance structures for N is



Key. (9)

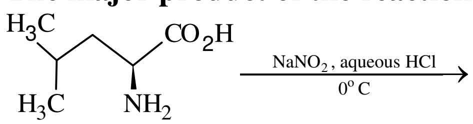
Sol.

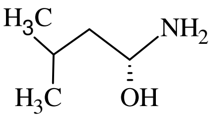


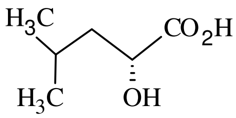
SECTION : 2 (Maximum Marks : 40)

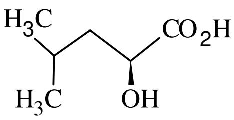
- **This section contains TEN questions**
- **Each question has FOUR options (A), (B), (C) and (D). ONE OR ORE THAN ONE of these four option(s) is(are) correct.**
- **For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.**
- **Marking scheme:**
  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  0 If one of the bubbles is darkened.
  –2 In all other cases.

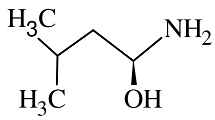
29. The major product of the reaction is



(A) 

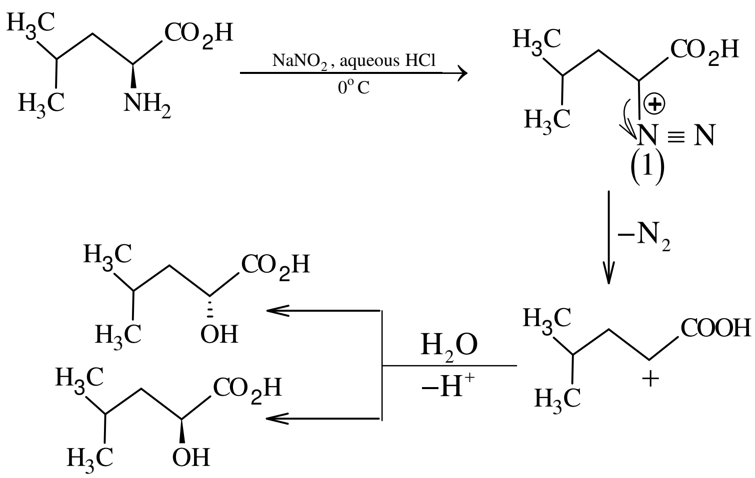
(B) 

(C) 

(D) 

Key. (B)

Sol.



30. The correct statement(s) about Cr^{2+} and Mn^{3+} is(are)
[Atomic numbers of Cr = 24 and Mn = 25]

(A) Cr^{2+} is a reducing agent
(B) Mn^{3+} is an oxidizing agent
(C) Both Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration
(D) When Cr^{2+} is used as a reducing agent, the chromium ion attains d^5 electronic configuration

Key. (A), (B), (C)

Sol.

$Cr^{2+} \rightarrow Cr^{3+}$

$t_{2g^3}e_g^1$ $\quad$ t_{2g^3} (more stable)

$$Mn^{3+} + e^- \quad \rightarrow \quad Mn^{2+}$$
$$3d^4 \qquad\qquad 3d^5 \text{ (stable electronic configuration)}$$

31. **Copper is purified by electrolytic refining of blister copper. The correct statement(s) about this process is (are)**

(A) Impure Cu strip is used as cathode
(B) Acidified aqueous $CuSO_4$ is used as electrolyte
(C) Pure Cu deposits at cathode
(D) Impurities settle as anode mud

Key. **(B, C, D)**

Sol. Anodes are of impure Cu and Cu-strip is cathode. The electrolyte is acidified $CuSO_4$ and the net process is,

Anode: $Cu \rightarrow Cu^{2+} + 2e^-$

Cathode: $Cu^{2+} + 2e^- \rightarrow Cu(s)$

Impurities from the blister Cu deposited as anode mud.

32. **Fe^{3+} is reduced to Fe^{2+} by using**

(A) H_2O_2 in presence of NaOH
(B) Na_2O_2 in water
(C) H_2O_2 in presence of H_2O_4
(D) Na_2O_2 in presence of H_2SO_4

Key. **(A, B)**

Sol. In acidic medium, $H_2O_2 + 2FeSO_4 + H_2SO_4 \rightarrow Fe_2(SO_4)_3 + 2H_2O$

In Basic medium, $H_2O_2 + 2K_3[Fe(CN)_6] + 2KOH \rightarrow 2K_4[Fe(CN)_6] + 2H_2O + O_2$

$$H_2O_2 + 2Fe + 2OH^- \rightarrow 2Fe^{2+} + 2H_2O + O_2$$

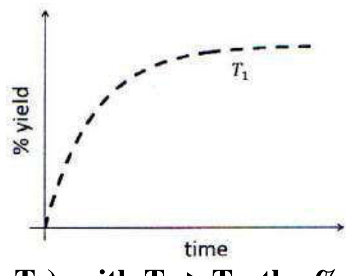
Since, $Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$

Therefore, Na_2O_2 in water also represent same reaction as that of basic H_2O_2.

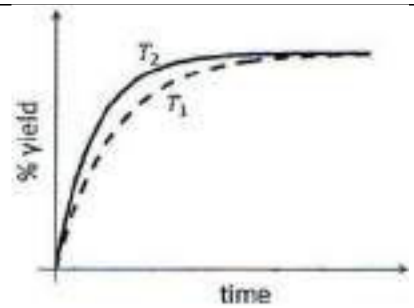
33. **The % yield of ammonia as a function of time in the reaction**

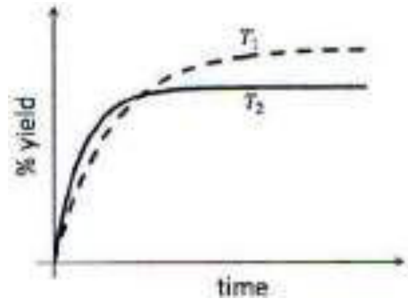
$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g),\ \Delta H < 0$$

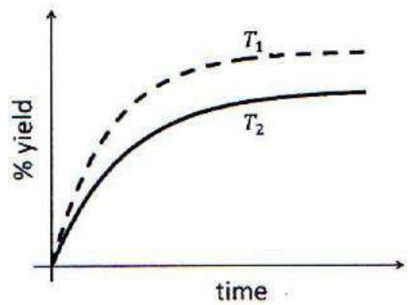
At (P, T_1) is given below

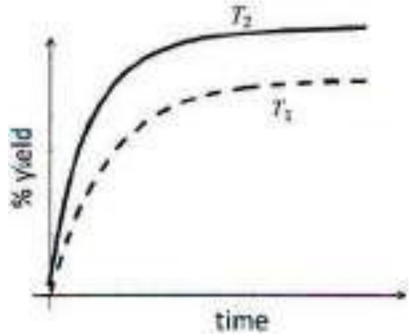


If this reaction is conducted at (P, T_2), with $T_2 > T_1$, the % yield of ammonia as a function of time is represented by

(A) 

(B) 

(C) 

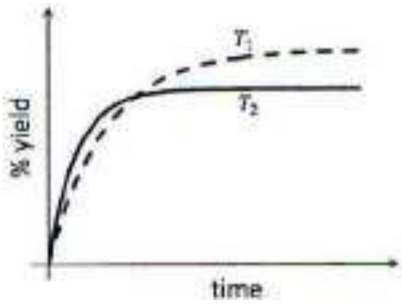
(D) 

Key. (B)

Sol. $N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g),\ \Delta H<0$

With referenced to the graph given at)P, T_1)

Initially at high temperature yield will be more but after some interval it will be constant with type



34. If the unit cell of a mineral has cubic close packed (ccp) array of oxygen atoms with m fraction of octahedral holes occupied by aluminium ions and *n* fraction of tetrahedral holes occupied by magnesium ions, *m* and *n*, respectively, are

(A) $\frac{1}{2}, \frac{1}{8}$ (B) $1, \frac{1}{4}$ (C) $\frac{1}{2}, \frac{1}{2}$ (D) $\frac{1}{4}, \frac{1}{8}$

Key. (A)

Sol. In cubic close packing unit cell = fcc

O^{2-} form ccp lattice (Z=4)

Al^{3+} occupy m fraction of OV = 4m

Mg^{2+} occupy n fraction of TV = 8n

Charge should be neutral

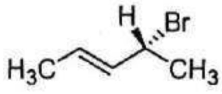
Hence, $4m \times (3) + 8n \times (2) = 4 \times (2)$

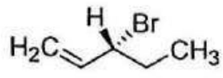
$12\ m + 16\ n = 8$

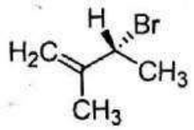
$3\ m + 4n =$

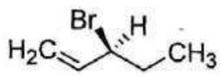
That is satisfied by when $m = \frac{1}{2},\ n = \frac{1}{8}$

35. Compound(s) that on hydrogenation produce(s) optically inactive compound (s) is (are)

(A) 

(B) 

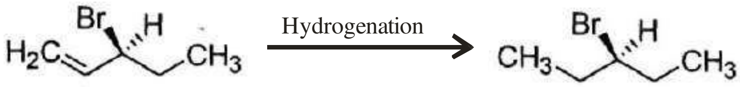
(C) 

(D) 

Key. (B, D)

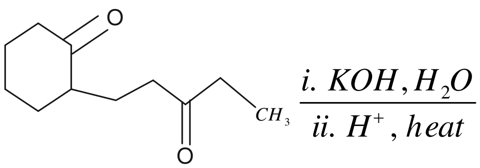
Sol. Both (B) (D) will form optically inactive product.

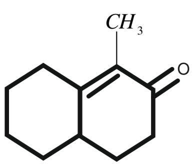
Optically inactive

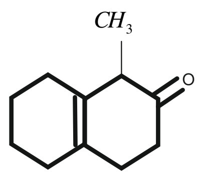


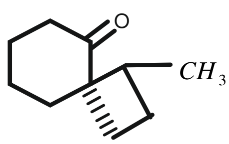
Optically inactive

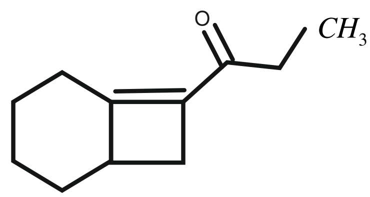
36) The major product of the following reaction is



(A) 

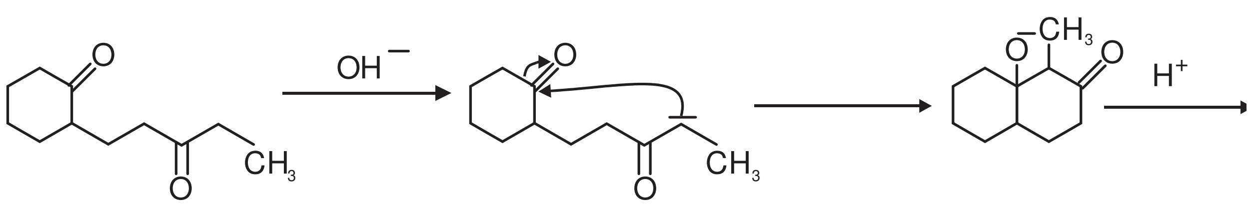
(B) 

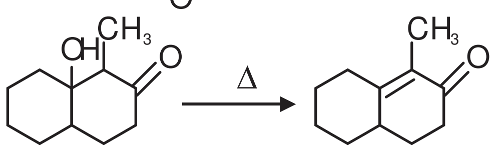
(C) 

(D) 

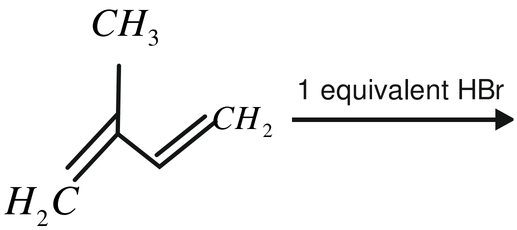
KEY: (A)

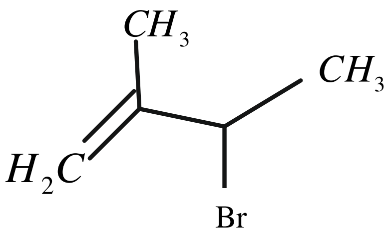
SOL:



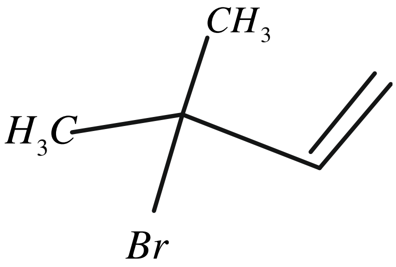


37) **In the following reaction, the major product is**

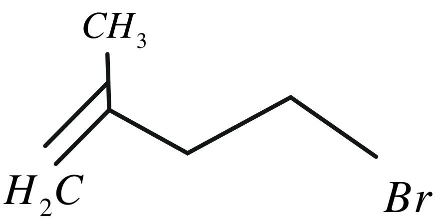




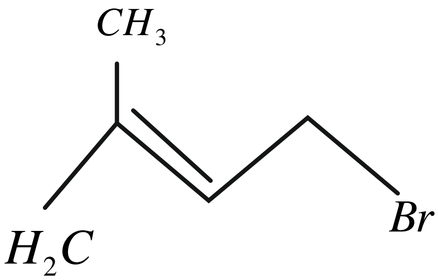
(A)



(B)



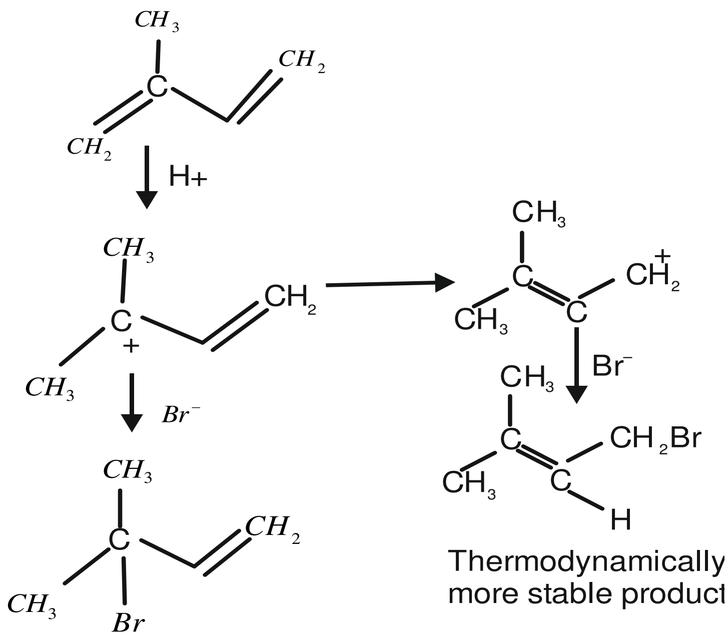
(C)



(D)

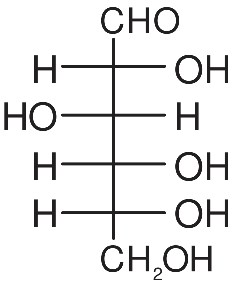
KEY: (D)

SOL:

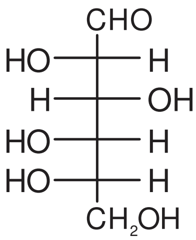
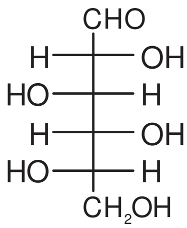
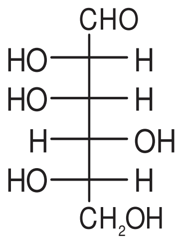
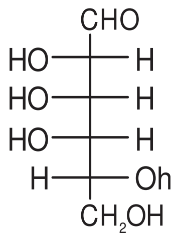


Since temperature is not given, therefore, option (B) may also be correct.

38) **The structure of D-(+) glucose is**



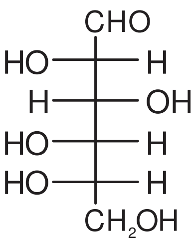
The structure of L- (-) glucose is

(A)  (B)  (C)  (D) 

KEY: (A)

SOL: L (–) glucose should be mirror image of D(+) glucose hence

The correct answer should be



Hence correct answer is (A)

SECTION : 3 (Maximum Marks : 16)

- **This section contains TWO questions.**
- **Each question contains two columns, Column I and Column II.**
- **Column I has four entries (A), (B), (C) and (D).**
- **Column II has five entries (P), (Q), (R), (S) and (T).**
- **Match the entries in column I with the entries in column II.**
- **One or more entries in column I may match with one or more entries in column II.**
- **The ORS contains a 4×5 matrix.**
- **For each entry in column I, darken the bubbles of all the matching entries. For example, if entry (A) in column I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).**
- **Marking scheme:**
  For each entry in column I.
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened.
  0 If one of the bubbles is darkened.
  –1 In all other cases.
  –2 In all other cases.

39) Match the anionic species given in Column I that are present in the ore(s) given in column II

	Column I		**Column II**
(A)	**Carbonate**	**(P)**	**Siderite**
(B)	**Sulphide**	**(Q)**	**Malachite**
(C)	**Hydroxide**	**(R)**	**Bauxite**
(D)	**Oxide**	**(S)**	**Calamine**
		(T)	**Argentite**

KEY: A→P,Q,S; B→T; C→R,Q, D→R

SOL:

(P) Siderite - $FeCO_3$

(Q) Malachite - $CuCO_3.Cu(OH)_2$

(R) Bauxite - $AlO_x(OH)_{3-2x}$

(S) Calamine - $ZnCO_3$

(T) Argentite - Ag_2S

40) **Match the thermodynamics process given under Column I with the expressions given under Column II**

	Column I		**Column II**
(A)	**Freezing of water at 273 K and 1 atm**	**(P)**	**Q = 0**
(B)	**Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions**	**(Q)**	**w = 0**
(C)	**Mixing of equal volumes of two ideal gases at constant temperature and pressure in an isolated container**	**(R)**	$\Delta S_{sys} < 0$
(D)	**Reversible heating of $H_2(g)$ at 1 atm from 300 K to 600 K, followed by reversible cooling to 300 K at 1 atm**	**(S)**	$\Delta U = 0$
		(T)	$\Delta G = 0$

KEY: A→R,T; B→P,Q,S; C→P,Q,S; D→P,Q,S,T

SOL:

(A) For freezing of water at 273K and 1 atm

$\Delta S < 0, \Delta G = 0$ (it is at equilibrium)

w is not zero as there will be expansion in freezing

(B) Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions

$Q = 0, \; w = 0, \; \Delta U = 0$

(C) Mixing of equal volume of two ideal gases at constant temperature and pressure in an isolated container

$Q = 0, \; w = 0, \; \Delta U = 0$

(D) Reversible heating of $H_2(g)$ at 1 atm from 300 K to 600 K, followed by reversible cooling to 300 K at 1 atm

$Q = 0, \; w = 0, \; \Delta U = 0, \; \Delta G = 0$

MATHEMATICS

41. Let $F(x)=\int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$ for all $x\in R$ and $f:\left[0,\frac{1}{2}\right]\to[0,\infty)$ be a continuous function. For $a\in\left[0,\frac{1}{2}\right]$, if $F'(a)+2$ is the area of the region bounded by x = 0, y = 0, y = f(x) and x = a, then f(0) is

Key. (3)

Sol. $F(x)=\int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$

$F(a)=\int_{a}^{a^2+\frac{\pi}{6}} 2\cos^2 t\,dt$

$F'(a)=2\cos^2\left(a^2+\frac{\pi}{6}\right).2a-2\cos^2 a.1$

$F'(a)+2=4a\cos^2\left(a^2+\frac{\pi}{6}\right)-2\cos^2 a+2$

$=4a\cos^2\left(a^2+\frac{\pi}{6}\right)+2\sin^2 a$

$\int_0^a f(x)dx=4a\cos^2\left(a^2+\frac{\pi}{6}\right)+2\sin^2 a$

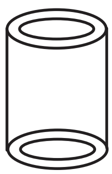
$f(a)=4\left[-a\,2\cos\left(a^2+\frac{\pi}{6}\right)\sin\left(a^2+\frac{\pi}{6}\right).2a+\cos^2\left(a^2+\frac{\pi}{6}\right).1\right]+4\sin a\cos a$

$\Rightarrow f(0)=3$

42. A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of V mm³, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.
If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is

Ans. (4)

Sol. Let inner radius = r
Height of cylinder = h
Volume of inner cylinder = v
$v=\pi r^2 h$



$h=\frac{v}{\pi r^2}$

Now, Material used

$M=\pi(r+2)^2 h-\pi r^2 h+\pi(r+2)^2 2$

$$= \pi\left((r+2)^2.\frac{v}{\pi r^2} - v + 2(r+2)^2\right)$$

$$M = \pi\left[\frac{v}{\pi}\left(1+\frac{4}{r}+\frac{4}{r^2}-v\right)+2(r+2)^2\right]$$

M is the function of r

$$f'(r) = \pi\left[\frac{v}{\pi}\left(0-\frac{4}{r^2}-\frac{8}{r^3}\right)+4(r+2)\right]=0$$

when r = 10,

$$\frac{v}{\pi}\left(\frac{-40}{1000}-\frac{8}{1000}\right)+48=0$$

$$\frac{-48v}{1000\pi}=-48$$

$$\frac{v}{250\pi}=4$$

43. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

Ans. (5)

Sol. Clearly $n = 6!5!$

Now, to calculate value of m

We make three cases

Case 1 : first four places filled by girls fifth by boy. This can be done in ${}^5P_4.5\times 5! = 5.(5!)^2$

Case 2 : Four girls are in a row having both the side a boy this can be done in

$5\times {}^5P_4 \times {}^5P_2 \times 4! = 20.(5!)^2$

Case 3 : Last four places are filled by girls and 6th place by a boy this can be done in

${}^5P_4.5.5! = 5.(5!)^2$

$m = 30.(5!)^2$

So, $\frac{m}{n} = 5$

44. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is

Ans. (8)

Sol. Let coin is tossed 'm' times, then probability of getting at least 2 heads is

$$1-\left(\frac{1}{2}\right)^m - {}^mC_1\left(\frac{1}{2}\right)^m = 1-\frac{(m+1)}{2^m}$$

Given that $1-\frac{(m+1)}{2^m} \geq 0.96$

$$\frac{m+1}{2^m} \leq 0.04 = \frac{1}{25}$$

Let $f(m)=\dfrac{m+1}{2^m}$

$f(1)=1, f(2)=\dfrac{3}{4}, f(3)=\dfrac{1}{2}, f(4)=\dfrac{5}{16}, f(5)=\dfrac{6}{32},$

$f(6)=\dfrac{7}{64}, f(7)=\dfrac{8}{128}, f(8)=\dfrac{9}{256}, f(9)=\dfrac{10}{512},....$

So minimum value of m is 8

45. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2+(y+2)^2=r^2$, then the value of r^2 is

Ans. (2)

Sol. $y^2=4x$, end points of latus rectum are $(1,\pm 2)$

Equation of normal at (1, 2) is

$y+x=3$, it touches the circle

So $\dfrac{|3+(-2)-3|}{\sqrt{1+1}}=r \Rightarrow r=\sqrt{2} \Rightarrow r^2=2$

46. Let f : R→R be a function defined by $f(x)=\begin{cases}[x], x\le 2\\ 0, x>2\end{cases}$,

Where [x] is the greatest integer less than or equal to x. If $I=\int_{-1}^{2}\dfrac{xf(x^2)}{2+f(x+1)}dx$, then the value of $(4I-1)$ is

Key. (0)

Sol. $I=\int_{-1}^{2}\dfrac{xf(x^2)}{2+f(x+1)}dx,$

$I=\int_{-1}^{0}0dx+\int_{0}^{1}0dx+\int_{1}^{\sqrt{2}}\dfrac{x}{2+0}dx+\int_{\sqrt{2}}^{\sqrt{3}}0dx+\int_{\sqrt{3}}^{\sqrt{4}}0dx$

$I=\dfrac{1}{2}.\dfrac{(x^2)_1^{\sqrt{2}}}{2}=\dfrac{1}{4}\{2-1\}=\dfrac{1}{4}$

$\Rightarrow \quad 4I-1=0.$

47. The number of distinct solutions of the equation

$$\frac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x=2$$

In the interval $[0,2\pi]$ is

Key. (8)

Sol. $\dfrac{5}{4}\cos^2 2x+1-2\sin^2 x\cos^2 x+1-3\sin^2 x\cos^2 x=2$

$\dfrac{5}{4}\cos^2 2x-\dfrac{1}{2}\sin^2(2x)-\dfrac{3}{4}\sin^2(2x)=0$

$\cos^2 2x-\sin^2 2x=0$

$\cos 4x=0$

Since, $\quad x\in[0,2\pi]$

Hence no. of distinct solution = 8.

48. **Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line y = -5, then the distance between A and B is**

Key. (4)

Sol. Let a Point $P(t^2, 2t)$ on $y^2 = 4x$

$\because$ Image of 'P' w.r.t. the line $x + y + 4 = 0$ can be given as: $\frac{x - t^2}{1} = \frac{y - 2t}{1} = -\left(t^2 + 2t + 4\right)$

$x = -2t - 4;$ $y = -t^2 - 4.$

Hence, Curve C becomes

$$\frac{(x+4)^2}{4} + 4 = -y.$$

Since, it intersect with the line y = -5

$\therefore$ $(x+4)^2 + 16 = 20$

$(x+4)^2 = 4$

$x + 4 = \pm 2$

$\Rightarrow$ $x + 4 = 2$ OR $x + 4 = -2$

$A = (-2, -5)$; $B = (-6, -5)$

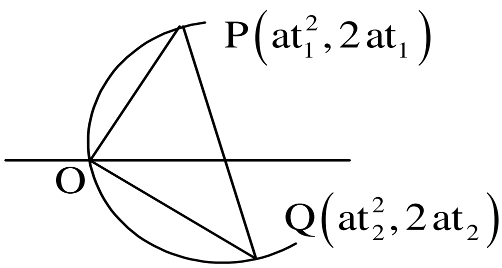
$\Rightarrow$ AB = 4.

49. **Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P ?**

(A) $\left(4, 2\sqrt{2}\right)$ (B) $\left(9, 3\sqrt{2}\right)$ (C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (D) $\left(1, \sqrt{2}\right)$

Key. (A, D)

Sol.



$t_1 t_2 = -4$ (Since, PQ is the diameter of circle)

$$\Rightarrow \quad \text{area of } \Delta OPQ = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix}$$

$$\pm 3\sqrt{2} = t_1 + \frac{4}{t_1}$$

$\Rightarrow$ $t_1 = \sqrt{2}, 2\sqrt{2}$ (Since P lies in first Quadrant)

$\Rightarrow$ $P = \left(1, \sqrt{2}\right), \left(4, 2\sqrt{2}\right)$

50. Let $y(x)$ be a solution of the differential equation $(1+e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statements is (are) true?

(A) $y(-4)=0$

(B) $y(-2)=0$

(C) $y(x)$ has a critical point in the interval $(-1,0)$

(D) $y(x)$ has no critical point in the interval $(-1,0)$

Key. (A,C)

Sol. $(1+e^x)\dfrac{dy}{dx}+y.e^x=1$

$\Rightarrow \quad \dfrac{d}{dx}\left\{y\left(e^x+1\right)\right\}=1$

$\Rightarrow \quad y\left(e^x+1\right)=x+c$

$\because \quad y(0)=2$

$\Rightarrow \quad 2(2)=c \Rightarrow c=4$

Hence, $y(e^x+1)=x+4$.

$\therefore \quad y(-4)=0$

$\because \quad y=\dfrac{x+4}{e^x+1}$

$$\frac{dy}{dx}=\frac{\left(e^x+1\right)-(x+4)e^x}{\left(e^x+1\right)^2}$$

For critical point

$e^x(x+3)=1$

Let: $\quad g(x)=e^x-\dfrac{1}{x+3}$

Now, $\quad g(-1)=\dfrac{1}{e}-\dfrac{1}{2}<0$

And $\quad g(0)=1-\dfrac{1}{3}>0$

Hence, $g(x)=0$ must has one real root in (-1,0).

51. Consider the family of all circles whose centers lie on the straight line y = x. If this family of circles is represented by the differential equation $Py''+Qy'+1=0$, where P, Q are functions of x, y and y' (here $y'=\dfrac{dy}{dx}$, $y''=\dfrac{d^2y}{dx^2}$), then which of the following statements is (are) true?

(A) $P=y+x$

(B) $P=y-x$

(C) $P+Q=1-x+y+y'+(y')^2$

(D) $P-Q=x+y-y'-(y')^2$

Key. (B, C)

Sol. Equation of family of circles is

$x^2+y^2-2\alpha x-2\alpha y+\lambda=0$

$$(y-\alpha)\frac{dy}{dx}=(\alpha-x) \qquad \text{........(i)}$$

$$(y-\alpha)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-1 \qquad \text{.......(ii)}$$

$$\alpha=\frac{yy''+(y')^2+1}{y''}$$

Putting the value of α in (i)

$$y'\left[y-\frac{y''y+(y')^2+1}{y''}\right]=\frac{y''y+(y')^2+1}{y''}-x$$

or $y''(y-x)+y'\left(y'+(y')^2+1\right)+1=0$

$$P=y-x$$

$$Q=y'+(y')^2+1$$

$$P+Q=1-x+y+y'+(y')^2$$

52. **Let $g:\mathbb{R}\to\mathbb{R}$ be a differentiable function with g(0) =0, $g'(0)=0$ and $g'(1)\neq 0$. Let**

$$f(x)=\begin{cases}\dfrac{x}{|x|}g(x), & x\neq 0\\ 0, & x=0\end{cases}$$

And $h(x)=e^{|x|}$ for all $x\in\mathbb{R}$. Let $(f\circ h)(x)$ denote $f(h(x))$ and $(h\circ f)(x)$ denote $h(f(x))$. Then which of the following is (are) true?

(A) f is differentiable at x = 0
(B) h is differentiable at x = 0
(C) f o h is differentiable at x = 0
(D) h o f is differentiable at x = 0

Key. **(A, D)**

Sol. $f'(0^+)=\lim\limits_{h\to0^+}\dfrac{f(0+h)-f(0)}{h}$

$$=\lim_{h\to0^+}\frac{\frac{h}{|h|}g(h)-0}{h}$$

$$=\lim_{h\to0^+}\frac{g(h)}{h}=0$$

(As $g'(0)$ exists and $g'(0)=0$)

$$\Rightarrow \quad \lim_{h\to0}\frac{g(0+h)-g(0)}{h}=0$$

$$\Rightarrow \quad \lim_{h\to0}\frac{g(h)}{h}=0$$

$$f'(0^-)=\lim_{h\to0^-}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to0^-}\frac{\frac{h}{|h|}g(h)-0}{h}$$

$$=\lim_{h\to0^-}-\left(\frac{g(h)}{h}\right)$$

$=(-1)\times 0=0$

$\Rightarrow$ function f is differentiable at $x = 0$

$$h(x)=\begin{cases} e^{-x} & x<0 \\ e^{x} & x\ge 0 \end{cases}$$

$$h'(x)=\begin{cases} -e^{-x} & x<0 \\ e^{x} & x>.0 \end{cases}$$

$h'(0^-)=-1$; $h'(0^+)=1$

h is not differentiable at $x = 0$

$f(h(x))=f\left(e^{|x|}\right)$

$$=\frac{e^{|x|}}{\left|e^{|x|}\right|}g\left(e^{|x|}\right)=g\left(e^{|x|}\right)$$

$f(h(x))$ is not differentiable at x =0 as $g'(1)\ne 0$

$$h(f(x))=\begin{cases} h\left(\frac{x}{|x|}g(x)\right) & x\ne 0 \\ h(0) & x=0 \end{cases}$$

$$=\begin{cases} e^{\left|\frac{x}{|x|}g(x)\right|} & x\ne 0 \\ 1 & x=0 \end{cases}$$

$$=\begin{cases} e^{|g(x)|} & x\ne 0 \\ 1 & x=0 \end{cases}$$

$h(f(x))$ is continuous at $x = 0$

Also $h(f(x))$ is differentiable at $x = 0$.

53. Let $f(x)=\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x\in\mathbb{R}$ and $g(x)=\frac{\pi}{2}\sin x$ for all $x\in\mathbb{R}$. Let $(f\,o\,g)(x)$ denote $f(g(x))$ and $(g\,o\,f)(x)$ denote $g(f(x))$. Then which of the following is (are) true?

(A) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(B) Range of f o g is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(C) $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}=\frac{\pi}{6}$

(D) There is an $x\in\mathbb{R}$ such that $(g\,o\,f)(x)=1$

Key. (A, B, C)

Sol. $f(x)=Sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

$\forall\, x\in R\quad -\frac{\pi}{2}\le\frac{\pi}{2}\sin x\le\frac{\pi}{2}$

$$-1\le\sin\left(\frac{\pi}{2}\sin x\right)\le 1$$

$$-\frac{\pi}{6}\le\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\le\frac{\pi}{6}$$

$$-\frac{1}{2} \le \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right) \le \frac{1}{2}$$

Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$$f\left(g(x)\right) = f\left(\frac{\pi}{2}\sin x\right) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$$

Range of $f\left(g(x)\right)$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$$\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)} \times \frac{\pi}{6} = \frac{\pi}{6}$$

$$g\left(f(x)\right) = g\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$$

$$= \frac{\pi}{2}\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$$

$$-\frac{1}{2} \le \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right) \le \frac{1}{2}$$

$$-\frac{\pi}{4} \le \frac{\pi}{2}\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right) \le \frac{\pi}{4}$$

for no $x \in R$; $g\left(f(x)\right) = 1$

54. Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are) true?

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$ (B) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

(C) $|\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = 48\sqrt{3}$ (D) $\vec{a}.\vec{b} = -72$

Key. (A, C, D)

Sol. $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$, $\vec{c} = \overrightarrow{PQ}$

$|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$, $\vec{b}.\vec{c} = 24$

As $\vec{a} + \vec{b} + \vec{c} = 0$

$$|\vec{b} + \vec{c}| = |-\vec{a}|$$

$$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}.\vec{c} = |\vec{a}|^2$$

or $48 + |\vec{c}|^2 + 48 = 144$

$$|\vec{c}|^2 = 48$$

$$\frac{|\vec{c}|^2}{2} - |\vec{a}| = \frac{48}{2} - 12 = 12$$

$$\frac{|\vec{c}|^2}{2} + |\vec{a}| = \frac{48}{2} + 12 = 36$$

$$\vec{a}+\vec{b}=-\vec{c}$$

$$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=|\vec{c}|^2$$

$$2(\vec{a}.\vec{b})=48-144-48$$

$$(\vec{a}.\vec{b})=-72$$

$$|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=|2(\vec{a}\times\vec{b})|$$

$$=2|\vec{a}\times\vec{b}| \;=2\sqrt{|\vec{a}|^2|b|^2-(\vec{a}.\vec{b})^2}$$

$$=2\sqrt{144\times48-72\times72}$$

$$=48\sqrt{3}$$

55. Let X and Y be two arbitrary, 3×3, non zero, skew-symmetric matrices and Z be an arbitrary 3×3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(A) $Y^3Z^4-Z^4Y^3$ (B) $X^{44}+Y^{44}$ (C) $X^4Z^3-Z^3X^4$ (D) $X^{23}+Y^{23}$

Key. (C, D)

Sol. $X^T=-X$

$Y^T=-Y$

$Z^T=Z$

$$(Y^3Z^4-Z^4Y^3)^T=(Y^3Z^4)^T-(Z^4Y^3)^T$$

$$=(Z^T)^4(Y^T)^3-(Y^T)^3(Z^T)^4$$

$$=Z^4(-Y^3)-(-Y)^3(Z)^4=Y^3Z^4-Z^4Y^3$$

$$(X^{44}+Y^{44})^T=(X^T)^{44}+(Y^T)^{44}$$

$$=(-X^T)^{44}+(-Y^T)^{44}$$

$$=(-X)^{44}+(-Y)^{44}$$

$$=X^{44}+Y^{44}$$

$$(X^4Z^3-Z^3X^4)^T=(X^4Z^3)^T-(Z^3X^4)^T$$

$$=(Z^T)^3(X^T)^4-(X^T)^4(Z^T)^3$$

$$=(Z)^3(-x)^4-(-X)^4(Z)^3$$

$$=Z^3X^4-X^4Z^3=-(X^4Z^3-Z^3X^4)$$

$$(X^{23}+Y^{23})^T=(X^T)^{23}+(Y^T)^{23}$$

$$=(-X)^{23}+(-Y)^{23} \quad =-(X^{23}+Y^{23})$$

56. Which of the following values of α satisfy the equation

$$\begin{vmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+2\alpha)^2\end{vmatrix}=-648\,\alpha$$

(A) –4 (B) 9 (C) –9 (D) 4

Key (B, C)

Sol. $\begin{vmatrix} 1+\alpha^2+2\alpha & 1+4\alpha^2+4\alpha & 1+9\alpha^2+c\alpha \\ 4+\alpha^2+4\alpha & 4+4\alpha^2 8\alpha & 4+9\alpha^2+72\alpha \\ 9+\alpha^2+6\alpha & 9+4\alpha^2+12\alpha & 9+9\alpha^2+18\alpha \end{vmatrix}$

Then by simplifying

$$2\alpha^2\begin{vmatrix} 1+2\alpha^2 & 3\alpha^2+2\alpha & 1 \\ 3 & 2\alpha & 6 \\ 5 & 2\alpha & 0 \end{vmatrix}$$

$$2\alpha^2\left|6\alpha-10\alpha\right| = -4\alpha\times 2\alpha^2 \Rightarrow -8\alpha^3 = -648\alpha$$

$$\alpha^2 = 81$$

$$\alpha = \pm 9$$

57. In R^2 consider the planes P_1 : y = 0 and P_2 : x+z =1. Let P_3 be a plane different from P_1 and P_2 passes through the intersection of P_1 and P_2. If the distance of the point (0,1,0) from P_2 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relations is (are) true ?

(A) $2\alpha+\beta+2\gamma+2=0$ (B) $2\alpha-\beta+2\gamma+4=0$

(C) $2\alpha+\beta+2\gamma-10=0$ (D) $2\alpha-\beta+2\gamma-8=0$

Key (B, D)

Sol Given

P_1 = y = 0 and x + z – 1 = 0

P_2 = x + z– 1 = 0

$P_3 = P_1 + \lambda P_2 = 0$

$$\Rightarrow x+z+\lambda y-1=0$$

The perpendicular distance from (0, 1, 0) = 1

$$\Rightarrow \left|\frac{\lambda-1}{\sqrt{2+\lambda^2}}\right| = 1$$

$$\lambda = -\frac{1}{2}$$

$$P_3 = 2x-y+2z-2=0$$

The distance from α,β,γ from P_3 = 2

$$\Rightarrow \left|\frac{2\alpha-\beta+2\gamma-2}{3}\right| = 2$$

$2\alpha-\beta+2\gamma+4=0$ and $2\alpha-\beta+2\gamma-8=0$

58. In R^3, let L be a straight line passing through the origin. Suppose that all the points an L are at a constant distance from the two planes P_1 : x + 2y – z + 1 = 0: 2x – y + z –2 = 0. Let M be the locus of the feet of the perpendicular drawn from the points on L to the plane P_1. Which of the following points lie(s) of M ?

(A) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ (B) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$ (C) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ (D) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

Key (A, B)

Sol Equation of angle bisectors

$$\frac{x+2y-z+1}{\sqrt{6}}=\pm\frac{2x-y+z-1}{\sqrt{6}}$$

$\Rightarrow \quad x-3y+2z-2=0 \;\; and \;\; 3x+y=0$

Hence plane 3x+y = 0 taken, as passes through origin. Line through origin on 3x + y = 0 and parallel to

$$\begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix}=\hat{i}-3\hat{j}-5\hat{k} \text{ is } \frac{x}{1}=\frac{y}{-3}=\frac{z}{-5}=\lambda \Rightarrow \text{any point on this line is } (\lambda,-3\lambda,-5\lambda)$$

Foot of perpendicular from $(\lambda,-3\lambda,-5\lambda)$ to x + 2y –z + 1 = 0

$$\frac{x-\lambda}{1}=\frac{y+3\lambda}{2}=\frac{z+5\lambda}{-1}=\frac{-(1)}{6}$$

Point is $\left(\lambda-\frac{1}{6},-3\lambda-\frac{1}{3},-5\lambda+\frac{1}{6}\right)$, which is satisfied by A, B

59.

	Column – I	**Column – II**	
(A)	**In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i}+\beta\hat{j}$ on $\sqrt{3}\,\hat{i}+\hat{j}$ is $\sqrt{3}$ and if $\alpha=2+\sqrt{3}\,\beta$, then possible value(s) of $\lvert\alpha\rvert$ is (are)**	**(P)**	**1**
(B)	**Let a and b real numbers such that the function $f(x)=\begin{cases}-3ax^2-2, & x<1 \\ bx+a^2, & x\ge 1\end{cases}$ is differential for all $x\in R$. Then possible value (s) of a is (are)**	**(Q)**	**2**
(C)	**Let $\omega\neq 1$ be a complex cube root of unity. If $\left(3-3\omega+2\omega^2\right)^{4n+3}+\left(2+3\omega-3\omega^2\right)^{4n+3}+\left(-3+2\omega+3\omega^2\right)^{4n+3}=0$, then possible value(s) of n is (are)**	**(R)**	**3**
(D)	**Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that a, 5, q, b is an arithmetic progression, then the value(s) of $\lvert q-a\rvert$ is (are)**	**(S)**	**4**
		(T)	**5**

Key (A-P,Q, B-PQ, C-P,Q,S,T, D-Q,T)

Sol (A) $$\left|\frac{(\alpha i+\beta j).\left(\sqrt{3}\,\hat{i}+\hat{j}\right)}{2}\right|=\sqrt{3}$$

$$\left|\sqrt{3}\,\alpha+\beta\right|=2\sqrt{3}$$

$\alpha + \frac{\beta}{\sqrt{3}} = \pm 2$, compare with $\alpha = 2 + \sqrt{3}\beta$

$\beta = 0,\ \beta = -\sqrt{3}$

Then $|\alpha| = 1, 2$

(B) $f'(x) = \begin{cases} -6ax \\ b \end{cases}$

$x = 1$ for differentiable function

$-6a = b$ ....(1)

For continuous function, $-3a - 2 = -6a + a^2$ ….(2)

From equation (1) & (2) $a = 2, 1$

(C) $\left(3 - 3\omega + 2\omega^2\right)^{4n+3} + \left(2 + 3\omega - 3\omega^2\right)^{4n+3} + \left(-3 + 2\omega + 3\omega^2\right)^{4n+3}$

$= \left(3\omega^2 + 2\omega - 3\right)\left(1 + \omega^{4n+3} + \omega^{8n+6}\right) = 0$

$3\omega^2 + 2\omega - 3 \neq 0$

$\Rightarrow$ $1 + \omega^n + \omega^{2n} = 0$

Hence n is not multiple of 3

$\therefore$ n = 1, 2, 4, 5

(D) Given $\frac{2ab}{a+b} = 4,$ …(1)

$a, 5, q, b$ *are in A.P.*

$q = 10 - a$, $q = \frac{b+5}{2}$

$a = 10 - q$

$b = 2q - 5$ …(2)

From (1) and (2)

$2q^2 - 23q + 60 = 0$

$q = 4, \frac{15}{2}$ $\Rightarrow$ for q = 4, a = 6

Then q = 15/2, a = 5/2

$|q - a| = 2, 5$

60.

	Column – I	Column – II
(A)	**In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z respectively.** $2\left(a^2-b^2\right)=c^2$ **and** $\lambda=\frac{\sin\left(X-Y\right)}{\sin Z}$, **then possible values of n for which** $\cos\left(n\pi\lambda\right)=0$ **is (are)**	**(P) 1**
(B)	**In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If** $1+\cos 2X-2\cos 2Y=2\sin X\sin Y$, **then possible value(s) of** $\frac{a}{b}$ **is (are)**	**(Q) 2**
(C)	**In R^2, let** $\sqrt{3}\,\hat{i}+\hat{j},\ \hat{i}+\sqrt{3}\,\hat{j}$ **and** $\beta\hat{i}++\left(1-\beta\right)\hat{j}$ **be the position vectors of X,Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of** $\overrightarrow{OX}$ **and** $\overrightarrow{OY}$ **is** $\frac{3}{\sqrt{2}}$ **, then possible value(s) of** $\lvert\beta\rvert$ **is (are)**	**(R) 3**
(D)	**Suppose that F(x) denotes the area of the region bounded by x = 0, x = 2, y^2 = 4x and** $y=\lvert\alpha x-1\rvert+\lvert\alpha x-2\rvert+\alpha x$, **where** $\alpha\in\{0,1\}$. **Then the value(s) of** $F\left(\alpha\right)+\frac{8}{3}\sqrt{2}$, **when** $\alpha=0$ **and** $\alpha=1$, **is (are)**	**(S) 5**
		(T) 6

Key (A-P,R,S; B – P; C- P; D – S,T)

Sol (A) Given $2\left(a^2-b^2\right)=c^2$ and $\lambda=\frac{\sin\left(X-Y\right)}{\sin Z}$,

$$2\left(\sin^2 X-\sin^2 Y\right)=\sin^2 Z$$

$$2\lambda\sin\left(X+Y\right)=\sin Zz$$

$$\left(2\lambda-1\right)\sin Z=0$$

$$\lambda=\frac{1}{2}$$

$$\cos\left(\frac{n\pi}{2}\right)=0$$

$\Rightarrow$ n = odd number

(B) Given $1+\cos 2X-2\cos 2Y=2\sin X\sin Y$

$$1+1-2\sin^2 X-2+4\sin^2 Y=2\sin X\sin Y$$

$$2b^2 - a^2 = ab$$

$$\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right) - 2 = 0$$

$$\left(\frac{a}{b}\right) = 1$$

(C) Internal angular bisector is $\frac{\sqrt{3}\,\hat{i} + \hat{j} + \hat{i} + \sqrt{3}\,\hat{j}}{2}$

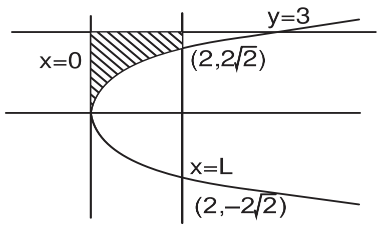
Angular bisector is $y - x = 0$

The perpendicular distance $(\beta, 1-\beta)$ is $\frac{3}{\sqrt{2}}$

$$\frac{|1-2\beta|}{\sqrt{2}} = \frac{3}{\sqrt{2}}, |\beta| = 1 \, or \, 2$$

(D) for $\alpha = 0$, $y = |\alpha x - 1| + |\alpha x - 2| + \alpha x$

y = 3

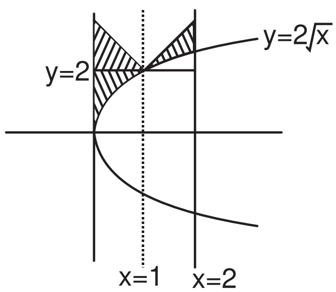


$$F(\alpha) = \left(3 - 2\sqrt{2}\right) \times 2 + \int_0^{2\sqrt{2}} \frac{y^2}{4} dy$$

$$= 6 - \frac{8\sqrt{2}}{3}$$

$$F(\alpha) + \frac{8\sqrt{2}}{3} = 6$$

For $\alpha = 1$, $y = |\alpha x - 1| + |\alpha x - 2| + \alpha x = \begin{cases} -x+3 & 0 \le x \le 1 \\ x+1 & 1 < x \le 2 \end{cases}$



$$F(\alpha) = 6 - \frac{8\sqrt{2}}{3} - 1 = 5 - \frac{8\sqrt{2}}{3}$$

$$F(\alpha) + \frac{8}{3}\sqrt{2} = 5$$