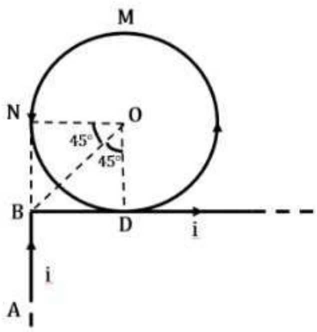
JEE Main 2020 Paper

Date of Exam: **8th January (Shift II)**
Time: **2:30 pm – 5:30 pm**
Subject: **Physics**

---

1. Find magnetic field at O. Where R is the radius of the loop



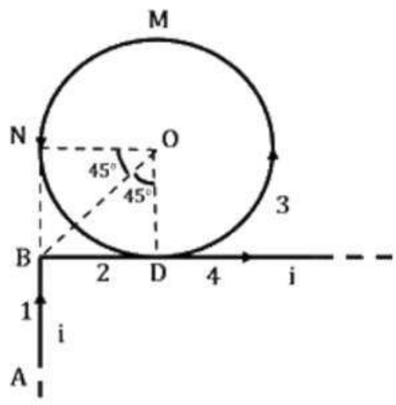
a. $\frac{\mu_0 i}{2\pi R}\left[\frac{-1}{\sqrt{2}}+\pi\right]$

b. $\frac{\mu_0 i}{2\pi R}[\pi-1]$

c. $\frac{\mu_0 i}{2R}$

d. $\frac{\mu_0 i}{2\pi R}\left[\frac{1}{\sqrt{2}}+\pi\right]$

Solution: (d)



To get magnetic field at O, we need to find magnetic field due to each current carrying part 1, 2, 3 and 4 individually.

Let's take total magnetic field as B_T, then

$$\vec{B}_T = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$$

Since 2 and 4 are parts of same wire, hence

$$\vec{B}_T = \frac{\mu_o i}{4\pi R}(\sin 90° - \sin 45°)(-\hat{k}) + \frac{\mu_o i}{2R}\,\hat{k} + \frac{\mu_o i}{4\pi R}(\sin 90° + \sin 45°)\hat{k}$$

$$= \frac{-\mu_o i}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right] + \frac{\mu_o i}{2R} + \frac{\mu_o i}{4\pi R}\left[1 + \frac{1}{\sqrt{2}}\right]\,\hat{k}$$

$$\vec{B}_T = \frac{\mu_o i}{4\pi R}\left[\sqrt{2} + 2\pi\right]\hat{k}$$

$$\vec{B}_T = \frac{\mu_o i}{2\pi R}\left[\frac{1}{\sqrt{2}} + \pi\right]\,\hat{k}$$

$\hat{k}$ denotes that direction of magnetic field is in the plane coming out of the plane of current.

2. Position of particle as a function of time is given as $\vec{r} = \cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}$, where ω is constant. Choose correct statement about $\vec{r}, \vec{v}$ *and* $\vec{a}$ where $\vec{v}$ and $\vec{a}$ are the velocity and acceleration of the particle at time t.
   a. $\vec{v}$ and $\vec{a}$ are perpendicular to $\vec{r}$
   b. $\vec{v}$ is parallel to $\vec{r}$ and $\vec{a}$ parallel to $\vec{r}$
   c. $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is away from the origin
   d. $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is towards the origin

Solution:(d)

$$\vec{r} = \cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}$$
$$\vec{v} = \frac{d\vec{r}}{dt} = \omega[-\sin\omega t\,\hat{\imath} + \cos\omega t\,\hat{\jmath}]$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2[\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}]$$
$$\vec{a} = -\omega^2\vec{r}$$

Since there is negative sign in acceleration, this means that acceleration is in opposite direction of $\vec{r}$

For velocity direction we can take dot product of $\vec{v}$ and $\vec{r}$.

$$\begin{aligned}\vec{v}.\vec{r} &= \omega(-\sin\omega t\,\hat{\imath} + \cos\omega t\,\hat{\jmath}).(\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}) \\ &= \omega[-\sin\omega t\cos\omega t + \cos\omega t\sin\omega t] = 0\end{aligned}$$

which implies that $\vec{v}$ is perpendicular to $\vec{r}$.

3. Two uniformly charged solid spheres are such that E_1 is electric field at surface of 1^{st} sphere due to itself. E_2 is electric field at surface of 2^{nd} sphere due to itself. r_1, r_2 are radius of 1^{st} and 2^{nd} sphere respectively. If $\frac{E_1}{E_2} = \frac{r_1}{r_2}$ then ratio of potential at the surface of spheres 1^{st} and 2^{nd} due to their self charges is :

a. $\frac{r_1}{r_2}$
b. $\left(\frac{r_1}{r_2}\right)^2$
c. $\frac{r_2}{r_1}$
d. $\left(\frac{r_2}{r_1}\right)^2$

Solution: (b)

$$\frac{E_1}{E_2} = \frac{r_1}{r_2}$$

$$\frac{V_1}{V_2} = \frac{E_1 r_1}{E_2 r_2} = \frac{r_1}{r_2} \times \frac{r_1}{r_2} = \left(\frac{r_1}{r_2}\right)^2$$

4. Velocity of a wave in a wire is v when tension in it is $2.06 \times 10^4\,N$. Find value of tension in wire when velocity of wave become $\frac{V}{2}$.

a. $5.15 \times 10^2\,N$
b. $9.12 \times 10^4\,N$
c. $9 \times 10^4\,N$
d. $5.15 \times 10^3 N$

Solution:(d)

$$V \propto \sqrt{T}$$

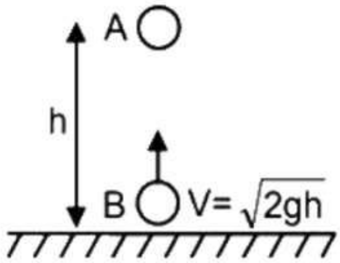
$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{2V}{V} = \sqrt{\frac{2.06 \times 10^4}{T}}$$

$$\Rightarrow T = \frac{2.06 \times 10^4}{4}\,N$$

$$= 5.15 \times 10^3\,N$$

5. There are two identical particles A and B. One is projected vertically upward with speed $\sqrt{2gh}$ from ground and other is dropped from height h along the same vertical line. Colision between them is perfectly inelastic. Find time taken by them to reach the ground after collision in terms of $\sqrt{\frac{h}{g}}$ is.



a. $\sqrt{\frac{3}{2}}$

b. $\sqrt{\frac{1}{2}}$

c. $\sqrt{3}$

d. $\sqrt{\frac{1}{5}}$

Solution:(a)

Time taken for the collision $t_1 = \frac{h}{\sqrt{2gh}}$

After t_1

$$V_A = 0 - gt_1 = -\frac{\sqrt{gh}}{2}$$

And $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right]$

At the time of collision

$$\vec{P_i} = \vec{P_f}$$

$\Rightarrow m\vec{V_A} + m\vec{V_B} = 2\,m\,\vec{V_f}$

$$\Rightarrow -\sqrt{\frac{gh}{2}} + \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right] = 2\,\vec{V_f}$$

$$V_f = 0$$

And height from the ground $= h - \frac{1}{2} g t_1^2 = h - \frac{h}{4} = \frac{3h}{4}$.

So, time taken to reach ground after collision $= \sqrt{2 \times \frac{\left(\frac{3h}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$

6. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is
   a. 99 J
   b. 90 J
   c. 1 J
   d. 100 J

Solution:(b)

For Carnot engine using as refrigerator

Work done on engine is given by

$W = Q_1 - Q_2$ ... (1)

where Q_1 is heat rejected to the reservoir at higher temperature and Q_2 is the heat absorbed from the reservoir at lower temperature.

It is given $\eta = 1/10$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$\Rightarrow \frac{Q_2}{Q_1} = \frac{9}{10}$ ..... (2)

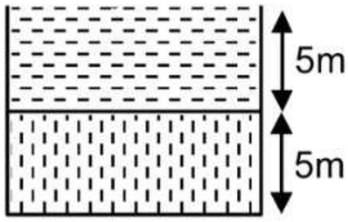
We are given, $W = 10$ J

Therefore, from equations (1) and (2),

$$Q_2 = \frac{10}{\frac{10}{9} - 1}$$

$\Rightarrow Q_2 = 90$ J

7. Two liquid columns of same height 5 m and densities ρ and 2ρ are filled in a container of uniform cross -sectional area. Then ratio of force exerted by the liquid on upper half of the wall to lower half of the wall is



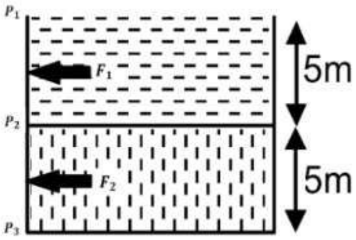
a. $\frac{2}{3}$

b. $\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{2}{5}$

Solution:(c)



The net force exerted on the wall by one type of liquid will be average value of pressure due to that liquid multiplied by the area of the wall.

Here, since the pressure due a liquid of uniform density varies linearly with depth, its average will be just the mean value of pressure at the top and pressure at the bottom.

So,

$$P_1 = 0$$

$$P_2 = \rho g \times 5$$

$$P_3 = 5\rho g + 2\rho g \times 5$$

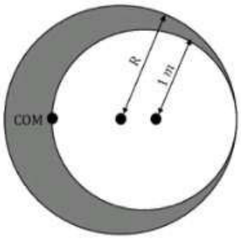
$$F_1 = \left(\frac{P_1 + P_2}{2}\right) A$$

$$F_2 = \left(\frac{P_2 + P_3}{2}\right) A$$

So,

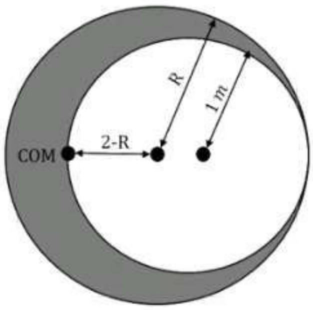
$$\frac{F_1}{F_2} = \frac{1}{4}$$

8. A uniform solid sphere of radius R has a cavity of radius $1\ m$ cut from it. If the center of mass of the system lies at the periphery of the cavity then



a. $(R^2 + R + 1)(2 - R) = 1$
b. $(R^2 - R - 1)(2 - R) = 1$
c. $(R^2 - R + 1)(2 - R) = 1$
d. $(R^2 + R - 1)(2 - R) = 1$

Solution (a)



Let M be the mass of the sphere and M' be the mass of the cavity.

Mass of the remaining part of the sphere = $M - M'$

Mass moments of the cavity and the remaining part of sphere about the original CoM should add up to zero.

$$(M - M')(2 - R) - M'(R - 1) = 0$$

(Mass of the cavity to be taken negative)

$$\Rightarrow \frac{4}{3}\pi(R^3 - 1^3)\rho g\,(2 - R) = \frac{4}{3}\pi(1)^3\rho g(R - 1)$$

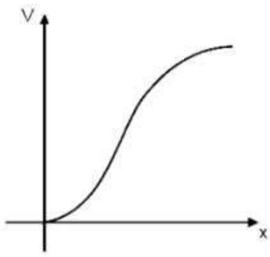
$$\Rightarrow (R^3 - 1^3)(2 - R) = (1^3)(R - 1)$$

$$\Rightarrow (R^2 + R + 1)(R - 1)(2 - R) = (R - 1)$$

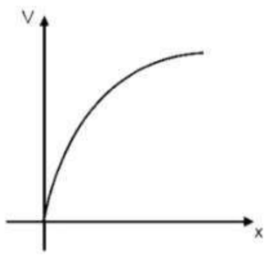
(using identity)

$$\Rightarrow (R^2 + R + 1)(2 - R) = 1$$

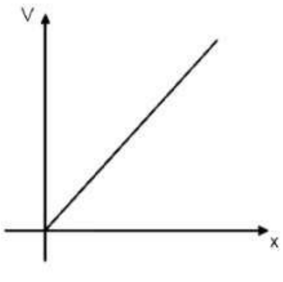
9. A charge particle of mass m and charge q is released from rest in uniform electric field. Its graph between velocity (v) and distance (x) will be :



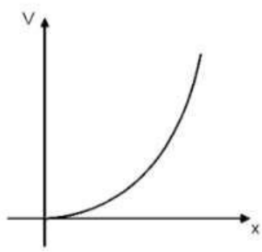
a.



b.



c.



d.

Solution. (b)

Applying work energy theorem,

$$qEx = \frac{1}{2}mv^2$$

$$v^2 \propto x$$

Hence, solution will be option (b)

10. In full scale deflection current in galvanometer of $100\ \Omega$ resistance is $1\ mA$. Resistance required in series to convern it into voltmeter of range $10\ V$.

a. $0.99\ K\Omega$
b. $9.9\ K\Omega$
c. $9.8\ K\Omega$
d. $10\ K\Omega$

Solution: (b)

$$V_0 = i_g R_g = 0.1\ v$$

$$v = 10\ V$$

$$R = R_g\left(\frac{v}{v_g} - 1\right)$$

$$= 100 \times 99 = 9.9\ K\Omega$$

11. n moles of He and $2n$ moles of O_2 are mixed in a container. Then $\left(\frac{Cp}{Cv}\right)_{mix}$ will be

a. $\frac{19}{23}$
b. $\frac{13}{19}$
c. $\frac{19}{13}$
d. $\frac{23}{19}$

Solution: (c)

Using formula

$$C_{v_{mix}} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

$= \frac{n\times\frac{3R}{2}+2n\times\frac{5R}{2}}{3n} = \frac{13R}{6}$ $(\because He$ is monoatomic and O_2 is diatomic)

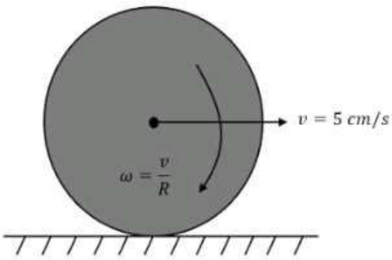
$$C_{p_{mix}} = C_{v_{mix}} + R = \frac{19R}{6}$$

$$\therefore \gamma_{mix} = \frac{C_{p_{mix}}}{C_{v_{mix}}} = \frac{19}{13}$$

12. A solid sphere of mass $m = 500\ gm$ is rolling without slipping on a horizontal surface. Find kinetic energy of the sphere if velocity of center of mass is $5\ cm/sec$.

a. $\frac{35}{4} \times 10^{-4}\ J$
b. $\frac{35}{2} \times 10^{-4}\ J$
c. $21 \times 10^{-4}\ J$
d. $70 \times 10^{-3}\ J$

Solution:(a)



Total K.E. = Translational K.E + Rotational K.E.

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

k is radius of gyration.

$$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{5}{100}\right)^2 \left(1 + \frac{2}{5}\right)$$

$$= \frac{35}{4} \times 10^{-4}\ J$$

13. Two square plates of side 'a' are arranged as shown in the figure. The minimum separation between plates is 'd' and one of the plates is inclined at small angle α with plane parallel to another plate. The capacitance of capacitor is (given α is very small

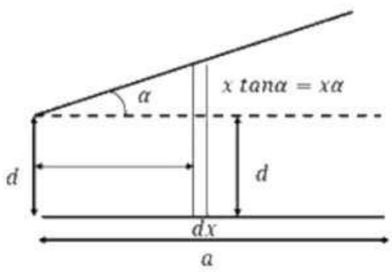
a. $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{d}\right)$

b. $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

c. $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{2d}\right)$

d. $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$

Solution:(b)



Let dC be the capacitance of the element of thickness dx

$$dc = \frac{\varepsilon_0 a dx}{d + \alpha x}$$

These are effectively in parallel combination

So,

$$C = \int dc$$

$$C = \int_0^a \frac{\varepsilon_0 a dx}{d + \alpha x}$$

$$\Rightarrow c = \frac{\varepsilon_0 a}{\alpha}[\ln(d + \alpha x)]_0^a$$

$$= \frac{\varepsilon_0 a}{\alpha}\left[\ln\left(1 + \frac{\alpha a}{d}\right)\right]$$

$$\approx \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

14. In YDSE path difference at a point on the screen is 8 λ . Find ratio of intensity at this point with maximum intensity.

a. 0.533
b. 0.853
c. 0.234
d. 0.123

Solution: (b)

In YDSE, the intensity at a point on the screen varies with the phase difference between the interfering light waves as:

$$I = I_0\, cos^2\left(\frac{\Delta\emptyset}{2}\right)$$

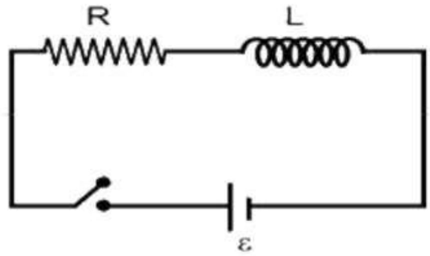
Here, $\Delta\emptyset$ = phase difference between the interfering waves

I_0 = maximum intensity on the screen

$$\frac{I}{I_0} = cos^2\left[\frac{\frac{2\pi}{\lambda}\times\Delta x}{2}\right] = \cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = 0.853$$

15. In the given circuit switch is closed at $t = 0$. The charge flown in time $t = T_c$(where T_c is time constant).



a. $\frac{\varepsilon L}{eR^2}$
b. $\frac{\varepsilon L}{4eR^2}$
c. $\frac{\varepsilon L(1-\frac{1}{e})}{eR^2}$
d. $\frac{\varepsilon L}{eR}$

Solution:(a)

This is standard $L-R$ growth of current circuit.

$$i(t)=\frac{\varepsilon}{R}e^{-\frac{t}{Tc}}$$

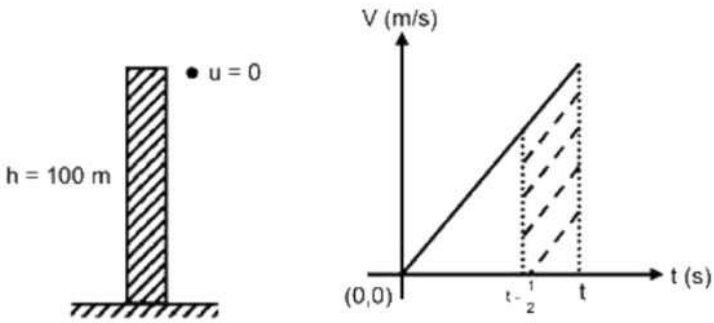
Substituting in the integral ,

$$q=\int_0^{Tc}(i)dt$$

$$=\frac{\varepsilon}{R}\left[t-\frac{e^{\frac{-t}{T_c}}}{\frac{-1}{T_c}}\right]_0^{T_c}$$

$$=\frac{\varepsilon L}{eR^2}$$

16. A particle is dropped from height $h=100\ m$, from surface of a planet. If in last $\frac{1}{2}\ s$ of its journey it covers $19\ m$, then the value of acceleration due to gravity in that planet is: (Assume the radius of planet to be much larger then $100\ m$)



a. $8\ m/s^2$
b. $7\ m/s^2$
c. $6\ m/s^2$
d. $5\ m/s^2$

Solution:(a)

Since the radius of planet is much larger than100 m, it's a uniformly accelerated motion.

So, Trapezium's area

$$s = \frac{g(\left[t - \frac{1}{2} + t\right]}{2} \times \frac{1}{2} = 19 \qquad \text{(i)}$$

$$\frac{1}{2} g t^2 = 100 \qquad \text{(ii)}$$

Solving equations (i) and (ii), we get

$$g = 8\ m/s^2$$

17. Coming Soon

18. A simple pendulum of length 25.0 cm makes 40 oscillation in 50 sec. If resolution of stopwatch is 1 sec, then accuracy of g is (in %)
    a. 1.2
    b. 3.2
    c. 4.4
    d. 5.4

Solution:(c)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$g = L.\left(\frac{2\pi}{T}\right)^2$$

$$\frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta L}{L}$$

$$2\left(\frac{1}{50}\right) + \frac{0.1}{25} = 4.4\%$$

19. An electron is moving initially with velocity $v_o\hat{\imath} + v_o\hat{\jmath}$ in uniform electric field $\vec{E} = -E_0\hat{k}$. If initial wavelength of electron is λ_0 and mass of electron is m, find wavelength of electron as a function of time.

a. $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2}{m^2v_0^2}t^2}}$

b. $\frac{2\lambda_0}{\sqrt{1+\frac{e^2E_0^2}{m^2v_0^2}t^2}}$

c. $\frac{\lambda_0 m v_0}{eE_0 t}$

d. $\frac{2\lambda_0 m v_0}{eE_0 t}$

Solution:(b)

Momentum of an electron

$p = mv = \frac{h}{\lambda}$

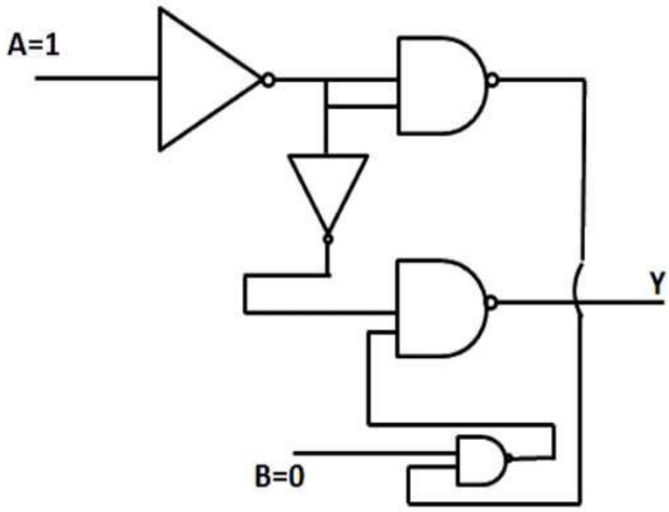
Initially $m(\sqrt{2}v_0) = \frac{h}{\lambda_0}$

Velocity as a function of time=$v_0\hat{\imath} + v_0\hat{\jmath} + \frac{eE_0}{m}t\hat{k}$

So, wavelength $\lambda = \frac{h}{m\sqrt{2v_0^2+\frac{e^2E_0^2}{m^2}t^2}}$

$$\Rightarrow \lambda = \frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2}{m^2v_0^2}t^2}}$$

20. Output at terminal Y of given logic circuit.



a. 1
b. 0
c. Can't determine
d. Oscillating between 0 and 1

Solution: b.

$$Y = \overline{\overline{AB}.A}$$

$$= \overline{\overline{AB}} + \bar{A}$$

$$= AB + \bar{A}$$

$$Y = 0 + 0 = 0$$

21. In H–spectrum wavelength of 1^{st} line of Balmer series is $\lambda = 6561\text{Å}$. Find out wavelength of 2^{nd} line of same series in nm.

Solution: (486)

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}.$$

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{36}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\lambda_2 = \frac{20}{27} \times 6561 \text{ Å} = 4860 \text{ Å}$$

22. An EMW is travelling along z-axis is $\vec{B} = 5 \times 10^{-8}\hat{j}\ T$, $c = 3 \times 10^8\ m/s$ and frequency of wave is 25 Hz , then electric field in $\frac{volt}{m}$.

Solution:(15)

$$\frac{E}{B} = c$$

$$E = B \times c$$

Given,
$\vec{B} = 5 \times 10^{-8}\hat{j}\ T$ and $C = 3 \times 10^8\ m/s$

$$E = 15\ \frac{volt}{m}.$$

23. There are three containers C_1, C_2 and C_3 filled with same material at different constant temperature. When we mix them in different quantity (volume) then we get some final temperature as shown in the table. Then find the value of final temperature θ as shown in the table.

C_1	C_2	C_3	$t(^0C)$
$1\,l$	$2\,l$	0	60
0	$1\,l$	$2\,l$	30
$2\,l$	0	$1\,l$	60
$1\,l$	$1\,l$	$1\,l$	θ

Solution: (50)

Since, all the containers have same material, specific heat capacity is the same for all.

$$V_1\theta_1 + 2\theta_2 = (V_1 + V_2)\theta_f$$

$$1\theta_1 + 1\theta_2 = (1 + 2)60$$
$$\theta_1 + 2\theta_2 = 180$$
$$0 \times \theta_1 + 1 \times \theta_2 + 2 \times \theta_3 = (1 + 2)30$$

$$\theta_2 + 2\theta_3 = 180$$
$$\theta_1 + 2\theta_2 + \theta_3 = (1+1+1)\theta$$

From equation. (1)+(2)+(3)

$$3\theta_1 + 3\theta_2 + 3\theta_3 = 450$$

Where, $\theta_1 + \theta_2 + \theta_3 = 150$

From (4) equation 150=30

So, $\theta = 50^0C$

24. An asteroid of mass $m(m << m_E)$ is approaching with a velocity 12 km/s when it is at distance of $9R$ from the surface of earth (where R is radius of earth). When it reaches at the surface of Earth, its velocity is (Nearest Integer) in km/s.

Solution: (16)

Taking, asteroid and earth as an isolated system conserving total energy.

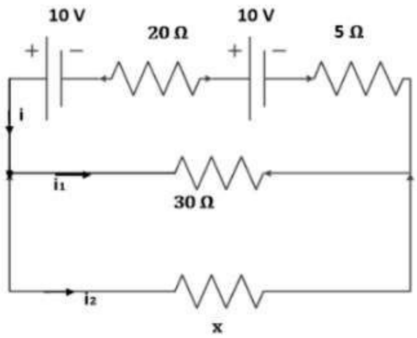
$$KE_i + PE_i = KE_f + PE_f$$
$$\frac{1}{2}mu_0^2 + \left(-\frac{GMm}{10R}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$
$$v^2 = u_0^2 + \frac{2GM}{R}\left[1 - \frac{1}{10}\right]$$
$$v = \sqrt{u_0^2 + \frac{9}{5}\frac{GM}{R}}$$

Since, escape velocity from surface of earth is $11.2\frac{km}{sec^2} = \sqrt{\frac{2GM}{R}}$

$$= \sqrt{12^2 + \frac{9}{5}\frac{(11.2)^2}{2}}$$

$$= \sqrt{256.9} \approx 16\ km/s.$$

25. Two batteries (connected in series) of same emf 10 V of internal resistances 20 Ω and 5 Ω are connected to a load resistance of 30 Ω. Now an unknown resistance x is connected in parallel to the load resistance. Find value of x so that potential drop of battery having internal resistance 20 Ω becomes zero.



Solution: (30)

If V_1 and V_2 are terminal voltage across the two batteries.

$$V_1 = 0$$

$$V_1 = \varepsilon_1 - i.r_1$$

$$0 = 10 - i \times 20$$

$$i = 0.5\ A$$

$$V_2 = 10 - 0.5 \times 5$$

$$V_2 = 7.5\ V$$

$$0.5 = \frac{7.5}{30} + \frac{7.5}{x}$$

$$\frac{7.5}{x} = 0.25$$

$$x = 30\ \Omega$$

JEE Main 2020 Paper

Date: 8th January 2020

Time: 02:30 PM – 05:30 PM

Subject: Chemistry

---

1. Correct bond energy order of the following is:
   a. C-Cl > C-Br > C-I > C-F
   b. C-F < C-Cl < C-Br < C-I
   c. C-F > C-Cl > C-Br > C-I
   d. C-I < C-Br < C-F < C-Cl

Answer: c

Solution: In C − F there is 2p-2p overlapping involved, in C − Cl the overlapping involved is 2p-3p whereas for C − Br and C − I the overlappings involved are 2p-4p and 2p-5p, respectively. The bond length for the various type of overlappings can be given as:

2p-2p < 2p-3p < 2p-4p < 2p-5p.

As we know that Bond energy $\alpha \frac{1}{\text{Bond length}}$

The order of bond energy comes out: C-F > C-Cl > C-Br > C-I

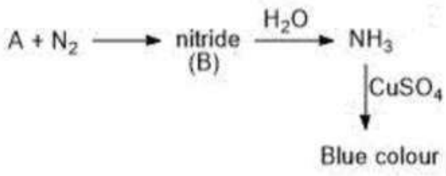
2. Determine Bohr's radius of Li^{2+} ion for n = 2. Given (Bohr's radius of H-atom = a_0)
   a. $\frac{3a_0}{4}$
   b. $\frac{4a_0}{3}$
   c. $\frac{a_0}{3}$
   d. $\frac{16a_0}{9}$

Answer: b

Solution: The formula for Bohr's radius for any unielectronic species is: $r = \frac{a_0 n^2}{z}$

for Li^{2+} : $r = \frac{a_0 2^2}{3} = \frac{4a_0}{3}$

3. Given the following reaction sequence:



A and B are respectively;

a. Mg, Mg_3N_2
b. Na, Na_3N
c. Mg, $Mg(NO_3)_2$
d. Na, $NaNO_3$

Answer: a

Solution: As it is provided in the question that nitride is being formed so the option c and d can be eliminated. Amongst Mg and Na we already know that Mg can only form nitride so the correct choice is option a.

$$3Mg + N_2 \rightarrow Mg_3N_2 \xrightarrow{H_2O} Mg(OH)_2 + NH_3$$

4. Correct order of the magnetic moment(spin only) for the following complexes is:

A. $[Pd(PPh_3)_2Cl_2]$
B. $[Ni(CO)_4]$
C. $[Ni(CN)_4]^{2-}$
D. $[Ni(H_2O)_6]^{2+}$

a. A=B=C<D
b. A<B<C<D
c. A>B>C>D
d. A=B>C>D

Answer: a

Solution: $[Pd(PPh_3)_2Cl_2]$: Here Pd is in +2 oxidation state and configuration of Pd^{2+} is $[Kr]4d^8$. As the CFSE value for Pd is very high so all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CO)_4]$: Here Ni is in 0 oxidation state and configuration of Ni is $[Ar]3d^84s^2$. As here the ligand is carbonyl which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CN)_4]^{2-}$: Here Ni is in +2 oxidation state and configuration of Ni^{2+} is $[Ar]3d^8$. As here the ligand is cyanide which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(H_2O)_6]^{2+}$: Here Ni is in +2 oxidation state and configuration of Ni^{2+} is $[Ar]3d^8$. As here the ligand is water which is a weak field ligand, the electrons will not be paired and there are two unpaired electrons in this complex hence magnetic moment for this complex will be $\sqrt{8}$ BM.

So the order of magnetic moment is A=B=C<D.

5. Determine the total number of neutrons in three isotopes of hydrogen.

a. 1
b. 2
c. 3
d. 4

Answer: c

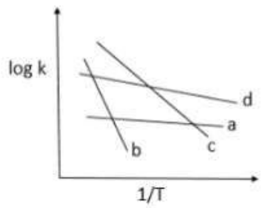
Solution: Number of neutrons in protium = 0

Number of neutrons in deuterium = 1

Number of neutrons in tritium = 2

So, total number of neutrons = 3

6.



Compare E_a (activation energy) for a, b, c and d.

a. $E_b > E_c > E_d > E_a$
b. $E_a > E_d > E_c > E_b$
c. $E_c > E_b > E_a > E_d$
d. $E_d > E_a > E_b > E_c$

Answer: a

Solution:

To avoid confusion, in this question we'll be denoting activation energy by E_x

$K = Ae^{-E_x/RT}$

$\log K = \log A - \frac{E_x}{2.303RT}$ -----(1)

Here, the graph given in the question is of a straight line and we know that the equation of straight line is

$y = mx + c$ ----(2)

Comparing equation 1 with 2 we get,

$\text{Slope} = \frac{-E_x}{2.303R}$

So, from the graph we can conclude that the line with the most negative slope will have the maximum activation energy value.

$E_b > E_c > E_d > E_a$

7. Which of the following exhibit both Frenkel and Schottky defect?

a. AgBr
b. KCl
c. CsCl
d. ZnS

Answer: a

Solution: The radius ratio for AgBr is intermediate. Thus, it shows both Frenkel and Schottky defects.

8. Given:

$$\begin{array}{l} P_4 + NaOH + H_2O \longrightarrow PH_3 + A \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \downarrow HCl \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad B \end{array}$$

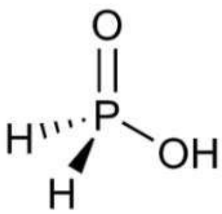
Basicity of B is:

a. 1
b. 2
c. 3
d. 4

Answer: a

Solution: $P_4 + NaOH + H_2O \rightarrow PH_3 + \boxed{NaH_2PO_2 \xrightarrow{HCl} H_3PO_2 + NaCl}$

Here the product B which is mentioned in the question is H_3PO_2. The structure of H_3PO_2 can be given as:



As only 1 Hydrogen atom is attached to the oxygen, its basicity is one.

9. Which reactions do not occurs in the blast furnace in the metallurgy of Fe?

A. $CaO + SiO_2 \rightarrow CaSiO_3$

B. $Fe_2O_3 + CO \rightarrow Fe_3O_4 + CO_2$

C. $FeO + SiO_2 \rightarrow FeSiO_3$

D. $FeO \xrightarrow{\Delta} Fe + \frac{1}{2}O_2$

a. A and B

b. A, B and C

c. C and D

d. A, B, C, D

Answer: c

Solution: In metallurgy of iron, CaO is used as flux which is used to remove the impurities of SiO_2 , $CaO + SiO_2 \rightarrow CaSiO_3$.

Also here Fe_2O_3 is reduced by CO to Fe_3O_4 which is further reduced to FeO which is further reduced to Fe.

$$3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$$

$$Fe_3O_4 + CO \rightarrow 3FeO + CO_2$$

$$FeO + CO \rightarrow Fe + CO_2$$

10. Correct order of radius of the elements C, O, F, Cl, Br, is;

a. $Br > Cl > C > O > F$

b. $Br < Cl < C < O < F$

c. $Cl < C < O < F < Br$

d. $C > F > O > Br > Cl$

Answer: a

Solution:

Across the period size decreases, so the order that follows is: $C > O > N > F$

Down the group size increases and the order is: $Br > Cl > F$

Change in size down the group is much more significant as compared to across the period.

So, the overall order of radius of elements is: $Br > Cl > C > O > F$.

11. Among the following, which will show geometrical isomerism?

A. $[Ni(NH_3)_5Cl]^+$

B. $[Ni(NH_3)_4ClBr]$

C. $[Ni(NH_3)_3Cl]^+$

D. $[Ni(NH_3)_2(NO_2)_2]$

a. B, D

b. A , B

c. A , B and C

d. A, B, C and D

Answer: a

Solution: The complexes of type Ma_4bc and Ma_2b_2 can show geometrical isomerism provided Ma_2b_2 is square planar. The compound given in B is Ma_4bc type and compound in D is Ma_2b_2 type also in D, Ni is surrounded with strong field ligands which will result in dsp^2 hybridisation and hence square planar geometry.

12. **Assertion:** pH of water increases on increasing temperature.

Reason: $H_2O \rightarrow H^+ + OH^-$ is an exothermic process

a. Both assertion and reason are correct and reason is correct explanation of assertion.
b. Both assertion and reason are correct and reason is not correct explanation of assertion.
c. Assertion is true and reason is false.
d. Both assertion and reason are incorrect.

Answer: d

Solution: $H_2O \rightarrow H^+ + OH^-$ is an endothermic process. On increasing the temperature the value of K_w increases which will result in decrease in pK_w . So we can say that pH of water will decrease on increasing temperature because pH for water $=\frac{1}{2}pK_w$.

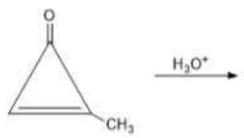
13. **Assertion:** It has been found that for hydrogenation reaction the catalytic activity increases from group-5 to group-11 metals with maximum activity being shown by group 7-9 elements of the periodic table.
Reason: For 7-9 group elements adsorption rate is maximum.

a. Both assertion and reason are correct and reason is correct explanation of assertion.
b. Both assertion and reason are correct and reason is not correct explanation of assertion.
c. Assertion is true & reason is false.
d. Both are incorrect

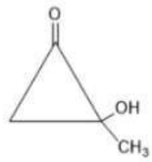
Answer: a

Solution: Group 7-9 elements of the periodic table show variable valencies so they have maximum activity because of the increase in adsorption rate.

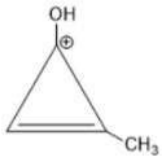
14. The major product of the following reactions is



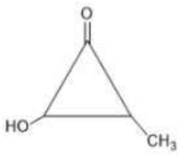
a.



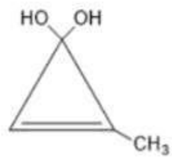
b.



c.

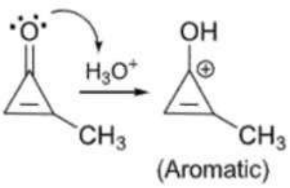


d.

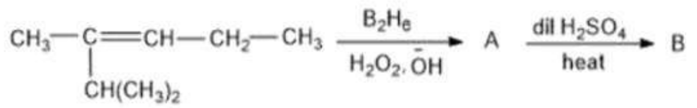


Answer: b

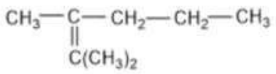
Solution:



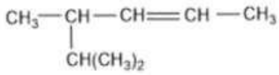
15. Find the final major product of the following reactions?



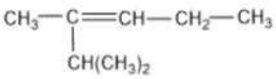
a.



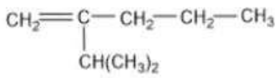
b.



c.

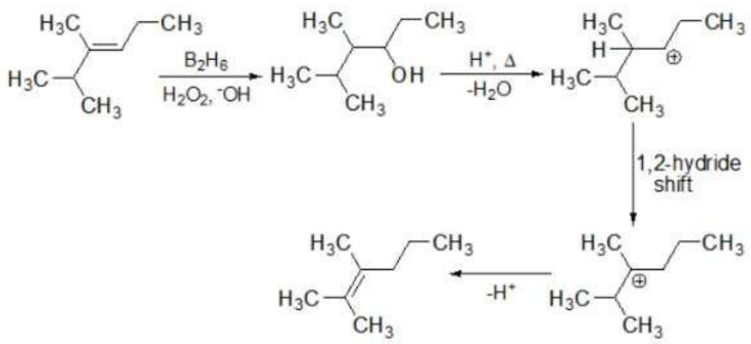


d.



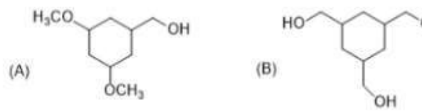
Answer: a

Solution:

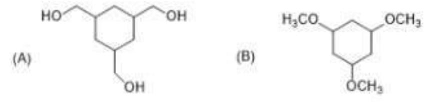


16. There are two compounds A and B of molecular formula $C_9H_{18}O_3$. A has higher boiling point than B. What are the possible structures of A and B?

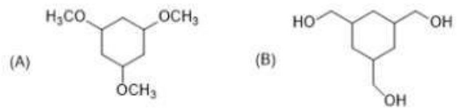
a.



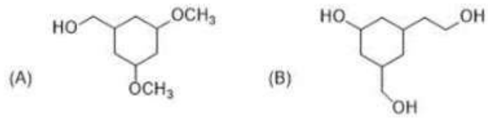
b.



c.



d.

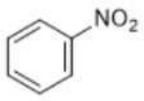


Answer: b

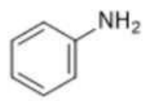
Solution: In option b compound A has extensive inter-molecular hydrogen bonding because of the 3 $-OH$ groups while in compound B there are $-OCH_3$ groups present and no inter-molecular hydrogen bonding is possible.

17. Kjeldahl method cannot be used for :

a.

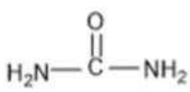


b.



c. $CH_3-CH_2-CH_2-C\equiv N$

d.

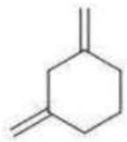


Answer: a

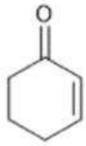
Solution: Kjeldahl method cannot be used for the estimation of nitrogen in the compounds in which nitrogen is involved in nitro, diazo groups or is present in the ring, as nitrogen atom can't be converted to ammonium sulphate under the reaction conditions.

18. A compound X adds 2 hydrogen molecules on hydrogenation. The compound X also gives 3-oxohexanedioic acid on oxidative ozonolysis. The compound 'X' is:

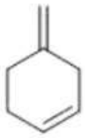
a.



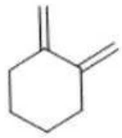
b.



c.

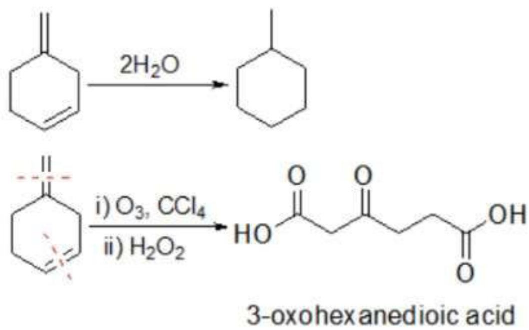


d.



Answer: c

Solution:

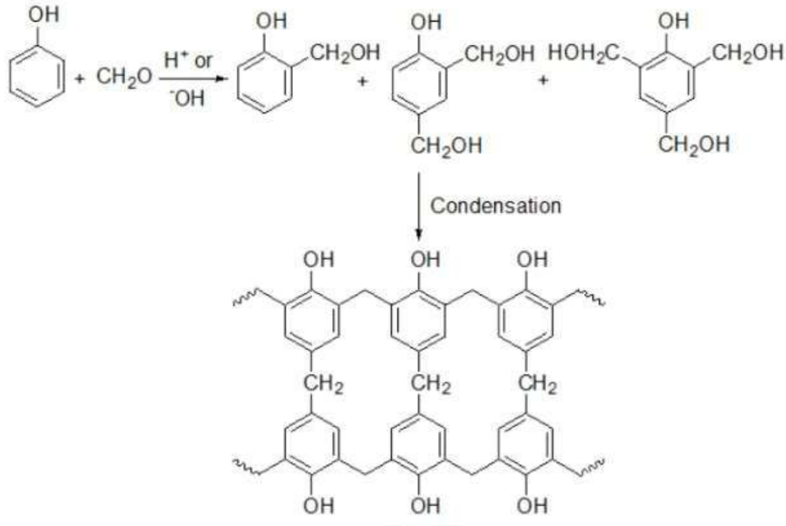


19. Formation of Bakelite follows :

a. Electrophilic substitution followed by condensation.
b. Nucleophilic addition followed by dehydration.
c. Electrophilic addition followed by dehydration.
d. Hydration followed by condensation.

Answer: a

Solution: Bakelite is a condensation polymer of phenol and formaldehyde.



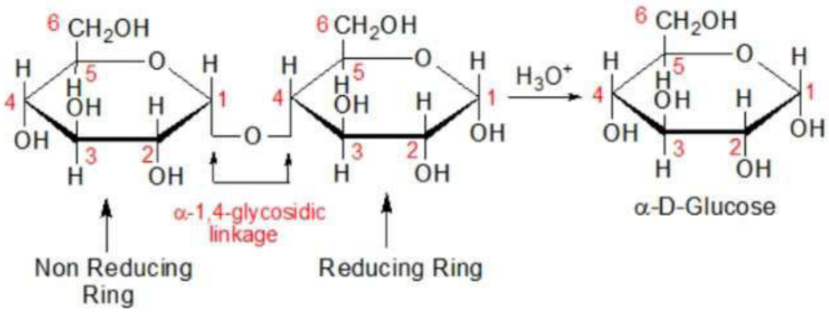
Bakelite

20. Products formed by hydrolysis of maltose are:

a. α-D-Glucose, α-D-Glucose
b. α-D-Glucose, β-D-Glucose
c. α-D-Galactose, β-D-Glucose
d. β-D-Galactose, α-D-Glucose

Answer: a

Solution:



Maltose is formed by the glycosidic linkage between C-1 of one α-D-Glucose unit to the C-4 of another α-D-Glucose.

21. Temperature of 4 moles of gas increases from 300 K to 500 K find 'C_v', if ΔU = 5000 J.

Answer: 6.25

Solution:
$\Delta U = nC_v\Delta T$
$5000 = 4 \times Cv\ (500 - 300)$
$Cv = 6.25\ JK^{-1}\ mol^{-1}$

22. Given : $E^0_{Sn^{2+}/Sn}$ = -0.14 V ; $E^0_{Pb^{2+}/Pb}$ =-0.13 V. Determine $\frac{[Sn^{2+}]}{[Pb^{2+}]}$ at equilibrium
For the cell reaction $Sn|Sn^{2+}||Pb^{2+}|Pb$
Take $\frac{2.303RT}{F}$=0.06 V, and $\sqrt[3]{10} = 2.154$

Answer: 2.15

Solution:

Anodic half: $Sn \rightarrow Sn^{2+} + 2e^-$

Cathodic half: $Pb^{2+} + 2e^- \rightarrow Pb$

Net reaction: $Sn + Pb^{2+} \rightarrow Pb + Sn^{2+}$

$E^0_{cell} = E^0_{cathode} - E^0_{anode}$

$E^0_{cell} = 0.01\ V$

$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log Q$

At equilibrium state $E_{cell} = 0$

So,

$0 = 0.01 - \frac{0.06}{2}\log\frac{[Sn^{2+}]}{[Pb^{2+}]}$

$0.01 = \frac{0.06}{2}\log\frac{[Sn^{2+}]}{[Pb^{2+}]}$

$\log\frac{[Sn^{2+}]}{[Pb^{2+}]} = \frac{1}{3}$

$\frac{[Sn^{2+}]}{[Pb^{2+}]} = 10^{\frac{1}{3}} = 2.154$

23. Given following reaction,
$NaClO_3 + Fe \rightarrow O_2 + FeO + NaCl$
In the above reaction 492 L of O_2 is obtained at 1 atm & 300 K temperature. Determine mass of $NaClO_3$ required (in kg). (R = 0.082 L atm mol^{-1} K^{-1})

Answer: 2.13

Solution:

Mol of $NaClO_3$ = mol of O_2

Mol of $O_2 = \frac{PV}{RT} = \frac{1\times492}{0.082\times300}$ = 20 mol

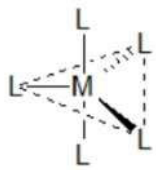
Molar mass of $NaClO_3$ is 106.5

So, mass = 20×106.5 = 2130 g = 2.13 Kg

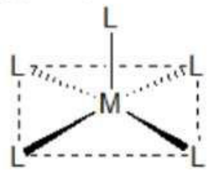
24. Complex $[ML_5]$ can exhibit trigonal bipyramidal and square pyramidal geometry. Determine total number of 180^0, 90^0 & 120^0 L-M-L bond angles

Answer: 20

Solution:



For trigonal bipyramidal geometry
Total number of 180^0 L-M-L bond angles = 1
Total number of 90^0 L-M-L bond angles = 6
Total number of 120^0 L-M-L bond angles = 3
Total = 10

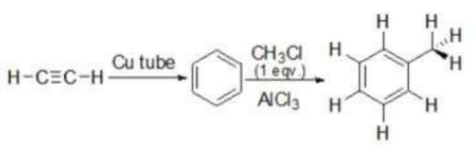


For square pyramidal geometry
Total number of 180^0 L-M-L bond angles = 2
Total number of 90^0 L-M-L bond angles = 8
Total number of 120^0 L-M-L bond angles = 0
Total = 10
Total for both the structures = 20

25. How many atoms lie in the same plane in the major product (C)?
$A \xrightarrow{\text{Cu tube}} B \xrightarrow{CH_3Cl(1\text{ eq.}),\ AlCl_3} C$
(Where A is the alkyne of lowest molecular mass).

Answer: 13

Solution:



JEE Main 2020 Paper

Date: **8th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

1. Solution set of $3^x(3^x-1)+2=|3^x-1|+|3^x-2|$ contains
   a. exactly one element
   b. at least four elements
   c. two elements
   d. infinite elements

   Answer: (a)

   Solution:

   $3^x(3^x-1)+2=|3^x-1|+|3^x-2|$

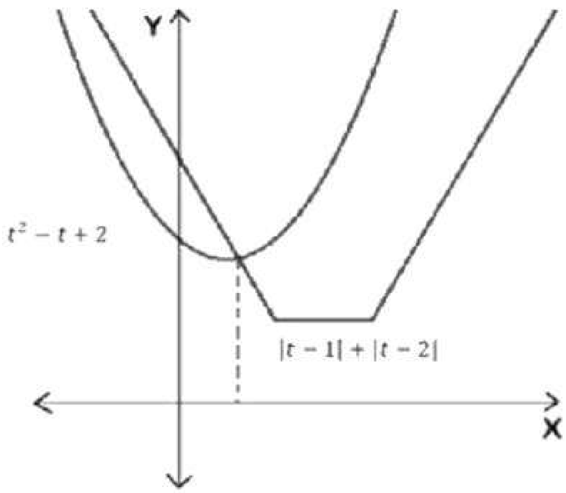
   Let $3^x=t$

   $t(t-1)+2=|t-1|+|t-2|$

   $\Rightarrow t^2-t+2=|t-1|+|t-2|$

   We plot t^2-t+2 and $|t-1|+|t-2|$

   As 3^x is always positive, therefore only positive values of t will be the solution.



   Therefore, we have only one solution.

2. Which of the following is a tautology?
   a. $\sim(p\wedge\sim q)\rightarrow(p\vee q)$
   b. $(\sim p\vee q)\rightarrow(p\vee q)$
   c. $\sim(p\vee\sim q)\rightarrow(p\vee q)$
   d. $\sim(p\vee\sim q)\rightarrow(p\wedge q)$

   Answer: (c)

   Solution:

$\sim(p \vee \sim q) \rightarrow (p \vee q)$

$= (p \vee \sim q) \vee (p \vee q)$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$

3. If a hyperbola has vertices $(\pm 6, \ 0)$ and $P(10, \ 16)$ lies on it, then the equation of normal at P is

a. $2x + 5y = 10$
b. $2x + 5y = 100$
c. $2x - 5y = 100$
d. $5x + 2y = 100$

Answer: (b)

Solution:

Vertex of hyperbola is $(\pm a, 0) \equiv (\pm 6, 0) \Rightarrow a = 6$

Let the equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{36} - \frac{y^2}{b^2} = 1$

As $P(10, 16)$ lies on the hyperbola.

$\frac{100}{36} - \frac{256}{b^2} = 1$

$\Rightarrow \frac{64}{36} = \frac{256}{b^2} \Rightarrow b^2 = 144$

Equation of hyperbola becomes $\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$

$\Rightarrow \frac{36x}{10} + \frac{144y}{16} = 180$

$\Rightarrow \frac{x}{50} + \frac{y}{20} = 1$

$\Rightarrow 2x + 5y = 100$

4. If $y = mx + c$ is a tangent to the circle $(x - 3)^2 + y^2 = 1$ and also perpendicular to the tangent to the circle $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then

a. $c^2 - 6c - 7 = 0$
b. $c^2 - 6c + 7 = 0$
c. $c^2 + 6c - 7 = 0$
d. $c^2 + 6c + 7 = 0$

Answer: (d)

Solution:

For circle, $x^2 + y^2 = 1$

$2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$

Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -1$

$\Rightarrow$Slope of tangent to $(x-3)^2 + y^2 = 1$ is $1 \Rightarrow m = 1$

Tangent to $(x-3)^2 + y^2 = 1$ is $y = x + c$

Perpendicular distance of tangent $y = x + c$ from centre $(3, 0)$ is equal to radius $= 1$

$\left|\frac{3+c}{\sqrt{2}}\right| = 1$

$\Rightarrow c + 3 = \pm\sqrt{2}$

$\Rightarrow c^2 + 6c + 9 = 2$

$\Rightarrow c^2 + 6c + 7 = 0$

5. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c}$ is non-zero vector and $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$, $\vec{a}.\vec{c} = 0$ then $\vec{b}.\vec{c}$ is equal to

a. $\frac{1}{2}$

b. $-\frac{1}{3}$

c. $-\frac{1}{2}$

d. $\frac{1}{3}$

Answer: (c)

Solution:

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$

$\Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\Rightarrow -(\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\Rightarrow -4\vec{c} = 6(\hat{\imath} - \hat{\jmath} + \hat{k}) - 4(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$\therefore \vec{b}.\vec{c} = -\frac{1}{2}.$

6. If the coefficient of x^4 and x^2 in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$ is α and β ,then $\alpha-\beta$ is equal to

a. 48
b. -60
c. 60
d. -132

Answer: (d)

Solution:

$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$

$=2[\,{}^6C_0x^6+{}^6C_2x^4(x^2-1)+{}^6C_4x^2(x^2-1)^2+{}^6C_6(x^2-1)^3]$

$=2[32x^6-48x^4+18x^2-1]$

$\Rightarrow \alpha=-96, \qquad \beta=36$

$\Rightarrow \alpha-\beta=-132$

7. Differential equation of $x^2=4b(y+b)$, where b is a parameter, is

a. $x\left(\frac{dy}{dx}\right)^2=2y\left(\frac{dy}{dx}\right)+x$
b. $x\left(\frac{dy}{dx}\right)^2=2y\left(\frac{dy}{dx}\right)+x^2$
c. $x\left(\frac{dy}{dx}\right)^2=y\left(\frac{dy}{dx}\right)+x^2$
d. $x\left(\frac{dy}{dx}\right)^2=y\left(\frac{dy}{dx}\right)+2x^2$

Answer: (a)

Solution:

$x^2=4b(y+b)$ ... (1)

Differentiating both the sides w.r.t. x, we get

$\Rightarrow 2x=4by'$

$\Rightarrow b=\frac{x}{2y'}$

Putting the value of b in (1), we get

$\Rightarrow x^2=\frac{2x}{y'}\left(y+\frac{x}{2y'}\right)$

$\Rightarrow x^2=\frac{2xy}{y'}+\frac{x^2}{y'^2}$

$\Rightarrow xy'^2=2yy'+x$

$\Rightarrow x\left(\frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right) + x$

8. Image of point $(1, 2, 3)$ w.r.t a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$ then which of the following points lie on this plane

a. $(1,\ 1,\ -1)$
b. $(-1,\ -1,\ 1)$
c. $(-1,\ 1,\ -1)$
d. $(-1,\ -1,\ -1)$

Answer: (a)

Solution:

Image of point $P(1, 2, 3)$ w.r.t. a plane $ax + by + cz + d = 0$ is $Q\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$

Direction ratios of PQ: $-\frac{10}{3}, -\frac{10}{3}, -\frac{10}{3} = 1, 1, 1$

Direction ratios of normal to plane is $1, 1, 1$

Mid-point of PQ lies on the plane

$\therefore$ The mid-point of $PQ = \left(-\frac{2}{3},\ \frac{1}{3},\ \frac{4}{3}\right)$

$\therefore$ Equation of plane is $x + \frac{2}{3} + y - \frac{1}{3} + z - \frac{4}{3} = 0$

$\Rightarrow x + y + z = 1$

$(1,\ 1,\ -1)$ satisfies the equation of the plane.

9. $\lim\limits_{x\to 0} \dfrac{\int_0^x t\sin 10t\, dt}{x}$ is equal to

a. 10
b. 0
c. 1
d. 5

Answer: (b)

Solution:

$\lim\limits_{x\to 0} \dfrac{\int_0^x t\sin 10t\, dt}{x}$

Applying L'Hospital's Rule:

$= \lim\limits_{x\to 0} \dfrac{x\sin 10x}{1} = 0$

10. Let P be the set of points $(x,\ y)$ such that $(x^2 \le y \le -2x + 3)$. Then area bounded by points in P is

a. $\frac{16}{3}$

b. $\frac{29}{3}$

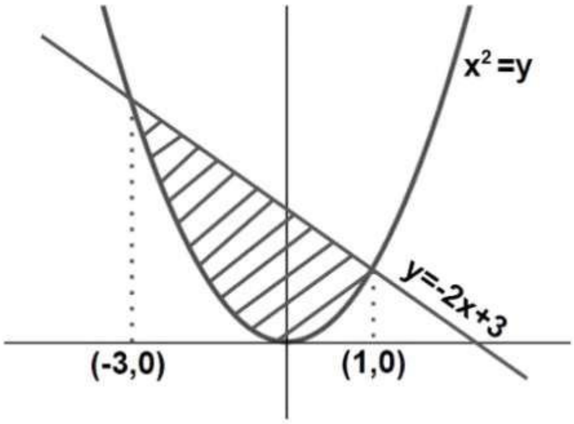
c. $\frac{20}{3}$

d. $\frac{32}{3}$

Answer: (d)

Solution:

We have $x^2 \leq y \leq -2x + 3$



For point of intersection of two curves -

$x^2 + 2x - 3 = 0$

$\Rightarrow x = -3, 1$

$\Rightarrow$ Area $= \int_{-3}^{1}\left((-2x+3) - x^2\right)dx$

$= \left[-x^2 + 3x - \frac{x^3}{3}\right]_{-3}^{1} = \frac{32}{3}$ sq. units.

11. If $f(x) = \frac{x[x]}{x^2+1} : (1, 3) \to \boldsymbol{R}$, then the range of $f(x)$ is (where $[.]$ denotes greatest integer function)

a. $\left(0, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{7}{5}\right]$

b. $\left(\frac{2}{5}, 1\right) \cup \left(1, \frac{4}{5}\right]$

c. $\left(0, \frac{1}{3}\right) \cup \left(\frac{2}{5}, \frac{4}{5}\right]$

d. $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

Answer: (d)

Solution:

$f(x) = \frac{x[x]}{x^2+1}$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{x^2+1} : 1 < x < 2 \\ \frac{2x}{x^2+1} : 2 \leq x < 3 \end{cases}$$

$\Rightarrow$Range of $f(x)$ is $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$.

12. If $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then the value of $10A^{-1}$ is
    a. $A - 4I$
    b. $A - 6I$
    c. $6I - A$
    d. $4I - A$

Answer: (b)

Solution:

$A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$

$A^{-1} = -\frac{1}{10}\begin{bmatrix} 4 & -2 \\ -9 & 2 \end{bmatrix}$

$\Rightarrow 10A^{-1} = \begin{bmatrix} -4 & 2 \\ 9 & -2 \end{bmatrix}$

$\Rightarrow 10A^{-1} = A - 6I$

13. For 20 observations mean and variance is given as 10 and 4, later it was observed that by mistake 9 was taken in place of 11, then the correct variance is
    a. 3.98
    b. 3.99
    c. 4.01
    d. 4.02

Answer: (b)

Solution:

Mean $= 10 \Rightarrow \frac{\sum x_i}{20} = 10 \Rightarrow \sum x_i = 200$

Variance $= 4 \Rightarrow \frac{\sum x\_i^2}{20} - 100 = 4 \Rightarrow \sum {x_i}^2 = 2080$

New mean $= \frac{200-9+11}{20} = \frac{202}{20} = 10.1$

New variance $= \frac{2080-81+121}{20} - (10.1)^2$

$= 106 - 102.01$

$= 3.99$

14. The correct option for the system of linear equations
    $\lambda x + 2y + 2z = 5$
    $2\lambda x + 3y + 5z = 8$
    $4x + \lambda y + 6z = 10$
    a. Infinite solutions when $\lambda = 2$
    b. Infinite solutions when $\lambda = 8$

c. No solutions when $\lambda = 2$
d. No solutions when $\lambda = 8$

Answer: (c)

Solution:

$$D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24$$

$\Rightarrow D = -\lambda^2 - 6\lambda + 16$

Now, $D = 0$

$\Rightarrow \lambda^2 + 6\lambda - 16 = 0$

$\Rightarrow \lambda = -8$ or 2

For $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix} = 40 + 4 - 28 \neq 0$$

$\therefore$ Equations have no solution for $\lambda = 2$.

15. In an A. P. if $T_{10} = \frac{1}{20}; T_{20} = \frac{1}{10}$, then the sum of first 200 terms is

a. $100\frac{1}{2}$
b. $101\frac{1}{2}$
c. $201\frac{1}{2}$
d. $301\frac{1}{2}$

Answer: (a)

Solution:

$T_{10} = \frac{1}{20}, T_{20} = \frac{1}{10}$

$T_{20} - T_{10} = 10d$

$\Rightarrow \frac{1}{20} = 10d$

$d = \frac{1}{200}$

$\therefore a = \frac{1}{200}$

$S_{200} = \frac{200}{2}\left[2\left(\frac{1}{100}\right) + 199\left(\frac{1}{200}\right)\right]$

$= 100\frac{1}{2}$

16. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$ and $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$, $b = \sum_{k=0}^{100}\alpha^{3k}$ where a and b are the roots of the quadratic equation then the quadratic equation will be

a. $x^2 - 102x + 101 = 0$
b. $x^2 + 102x + 100 = 0$
c. $x^2 - 101x + 100 = 0$
d. $x^2 + 101x + 100 = 0$

Answer: (a)

Solution:

$$\alpha = \frac{-1+i\sqrt{3}}{2} = \omega$$

$$a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$$

$$\Rightarrow a(1+\alpha)[1+\alpha^2+\alpha^4+\cdots.+\alpha^{200}]$$

$$\Rightarrow a = (1+\alpha)\left[\frac{1-(\alpha^2)^{101}}{1-\alpha^2}\right]$$

$$\Rightarrow a = \left[\frac{1-(\omega^2)^{101}}{1-\omega}\right] = \left[\frac{1-\omega}{1-\omega}\right] = 1$$

$$b = \sum_{k=0}^{100}\alpha^{3k} = 1+\alpha^3+\alpha^6+\cdots..+\alpha^{300}$$

$$\Rightarrow b = 1+\omega^3+\omega^6+\ldots.+\omega^{300}$$

$$\Rightarrow b = 101$$

Required equation is $x^2 - 102x + 101 = 0$

17. If $f(x)$ is a three-degree polynomial for which $f'(-1) = 0, f''(1) = 0, f(-1) = 10, f(1) = 6$ then the local minima of $f(x)$ will be at

a. $x = -1$
b. $x = 1$
c. $x = 2$
d. $x = 3$

Answer: (d)

Solution:

Let the polynomial be

$$f(x) = ax^3 + bx^2 + cx + d$$

$$\Rightarrow f'(x) = 3ax^2 + 2bx + c$$

$$\Rightarrow f''(x) = 6ax + 2b$$

$$f''(1) = 0 \Rightarrow 6a + 2b = 0 \Rightarrow b = -3a$$

$$f'(-1) = 0 \Rightarrow 3a - 2b + c = 0$$

$\Rightarrow c = -9a$

$f(-1) = 10 \Rightarrow -a + b - c + d = 10$

$\Rightarrow -a - 3a + 9a + d = 10$

$d = -5a + 10$

$f(1) = 6 \Rightarrow a + b + c + d = 6$

$\Rightarrow a - 3a - 9a - 5a + 10 = 6$

$\Rightarrow a = \frac{1}{4}$

$\therefore f'(x) = \frac{3}{4}x^2 - \frac{6}{4}x - \frac{9}{4} = \frac{3}{4}(x^2 - 2x - 3)$

For $f'(x) = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = 3, -1$

Minima exists at $x = 3$

18. Let $I = \int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$ then

a. $\frac{1}{9} < I^2 < \frac{1}{8}$

b. $\frac{1}{9} < I < \frac{1}{8}$

c. $\frac{1}{3} < I^2 < \frac{1}{2}$

d. $\frac{1}{3} < I < \frac{1}{2}$

Answer: (a)

Solution:

$f(x) = \frac{1}{\sqrt{2x^3-9x^2+12x+4}}$

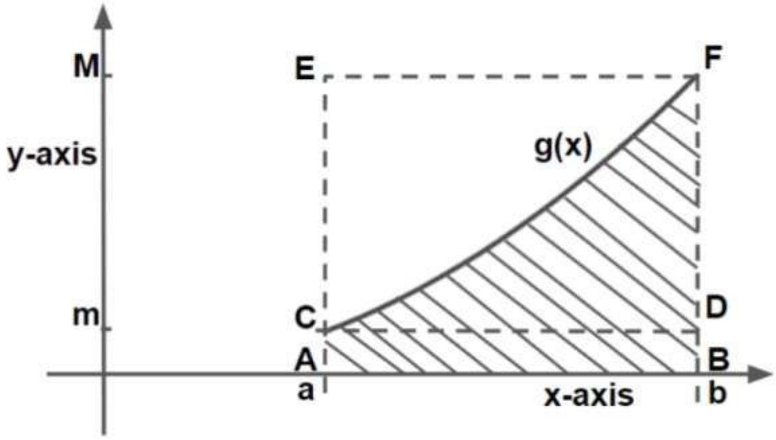
Differentiating w.r.t x

$$f'(x) = -\frac{1}{2} \times \frac{(6x^2-18x+12)}{(2x^3-9x^2+12x+4)^{3/2}}$$

$$= \frac{-6(x-1)(x-2)}{2(2x^3-9x^2+12x+4)^{3/2}}$$

Here $f(x)$ is increasing in $(1,2)$

At $x = 1,\ f(1) = \frac{1}{3}$ and $x = 2,\ f(2) = \frac{1}{\sqrt{8}}$



Let $g(x)$be a function such that it is increasing in (a, b) and $m \leq g(x) \leq M$, then

$\text{ar}(ABCD) < \int_a^b g(x)\,dx < \text{ar}(ABEF)$

$m(b-a) < \int_a^b g(x)\,dx < M(b-a)$

Thus, $\frac{1}{3} < \int_1^2 f(x)dx \ < \frac{1}{\sqrt{8}}$

$\frac{1}{3} < I \ < \frac{1}{\sqrt{8}},$

or $\frac{1}{9} < I^2 < \frac{1}{8}$

19. Normal at $(2,\ 2)$ to curve $x^2 + 2xy - 3y^2 = 0$ is L. Then perpendicular distance from origin to line L is

a. $2\sqrt{2}$
b. 4
c. $4\sqrt{2}$
d. 2

Answer: (a)

Solution:

Given curve: $x^2 + 2xy - 3y^2 = 0$

$\Rightarrow x^2 + 3xy - xy - 3y^2 = 0$

$\Rightarrow (x+3y)(x-y) = 0$

Equating we get,

$x + 3y = 0$ or $x - y = 0$

$(2,\ 2)$ lies on $x - y = 0$

$\therefore$ Equation of normal will be $x + y = \lambda$

It passes through $(2, 2)$

$\therefore \lambda = 4$

$L : x + y = 4$

Distance of L from the origin $= \left|\frac{-4}{\sqrt{2}}\right| = 2\sqrt{2}$

20. If A and B are two events such that $P(\text{exactly one}) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{2}$ then $P(A \cap B)$ is

a. $\frac{1}{8}$

b. $\frac{1}{10}$

c. $\frac{1}{12}$

d. $\frac{2}{9}$

Answer: (b)

Solution:

$P(\text{exactly one of } A \text{ or } B) = \frac{2}{5}$

$$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cup B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cap B) = \frac{1}{10}$$

21. The number of four-letter words that can be made from the letters of word "EXAMINATION" is

Answer: (2454)

Solution:

Word "EXAMINATION" consists of $2A, 2I, 2N, E, X, M, T, O$

Case I: All different letters are selected

Number of words formed $= {}^{8}C_{4} \times 4! = 1680$

Case II: 2 letters are same and 2 are different

Number of words formed $= {}^{3}C_{1} \times {}^{7}C_{2} \times \frac{4!}{2!} = 756$

Case III: 2 pair of letters are same

Number of words formed $= {}^{3}C_{2} \times \frac{4!}{2! \times 2!} = 18$

Total number of words formed $= 1680 + 756 + 18 = 2454$

22. Line $y = mx$ intersects the curve $y^2 = x$ at point P. The tangent to $y^2 = x$ at P intersects x −axis at Q. If area $\Delta OPQ = 4$, find m, $(m > 0)$

Answer: (0.5)

Solution:

Let the co-ordinates of P be $(t^2,\ t)$

Equation of tangent at $P(t^2,\ t)$ is $y - t = \frac{1}{2t}(x - t^2)$

Therefore, co-ordinates of Q will be $(-t^2,\ 0)$

Area of $\Delta OPQ = 4$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = \pm 4$$

$\Rightarrow t^3 = 8 \Rightarrow t = \pm 2 \Rightarrow t = 2 \quad \text{as}\ t > 0$

$$m = \frac{1}{t} = \frac{1}{2}$$

23. $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to

Answer: (504)

Solution:

$$\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$$

$$= \frac{1}{4}\sum_{n=1}^{7}(2n^3 + 3n^2 + n)$$

$$= \frac{1}{4}\left[2\sum_{n=1}^{7} n^3 + 3\sum_{n=1}^{7} n^2 + \sum_{n=1}^{7} n\right]$$

$$= \frac{1}{4}\left[2 \times \left(\frac{7 \times 8}{2}\right)^2 + 3 \times \frac{7 \times 8 \times 15}{6} + \frac{7 \times 8}{2}\right]$$

$$= \frac{1}{4}[2 \times 784 + 420 + 28] = 504$$

24. Let $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, where $\alpha, \beta \in \left(0,\ \frac{\pi}{2}\right)$. Then $\tan(\alpha + 2\beta)$ is equal to

Answer: (1)

Solution:

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7} \Rightarrow \frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7} \Rightarrow \tan\alpha = \frac{1}{7}$$

$$\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}} \Rightarrow \frac{\sqrt{2}sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}} \Rightarrow \sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}} = 1$$