

# Jee Advanced 2019 Paper 1

## PHYSICS – JEE ADVANCED PAPER – 1

### SECTION – 1

1. A current carrying wire heats a metal rod. The wire provides a constant power  $P$  to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature ( $T$ ) in the metal rod changes with time ( $t$ ) as  $T(t) = T_0 (1 + \beta t^{1/4})$  where  $\beta$  is a constant with appropriate dimension of temperature. The heat capacity of metal is:

(a)  $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$       (b)  $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$       (c)  $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$       (d)  $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

2. In a capillary tube of radius 0.2 mm the water rises up to height of 7.5 cm with angle of contact equal to zero. If another capillary with same radius but of different material dipped in the same liquid. The height of water raised in capillary will be, if angle of contact becomes  $60^\circ$ .

(a) 7.5 cm      (b) 15 cm      (c) 3.75 cm      (d) 30 cm

3. A sample of  ${}_{19}\text{K}^{40}$  disintegrates into two nuclei Ca & Ar with decay constant  $\lambda_{\text{Ca}} = 4.5 \times 10^{-10} \text{ S}^{-1}$  and  $\lambda_{\text{Ar}} = 0.5 \times 10^{-10} \text{ S}^{-1}$  respectively. The time after which 99% of  ${}_{19}\text{K}^{40}$  gets decayed is:  
 (a)  $6.2 \times 10^9 \text{ sec}$       (b)  $9.2 \times 10^9 \text{ sec}$       (c)  $7.2 \times 10^9 \text{ sec}$       (d)  $4.2 \times 10^9 \text{ sec}$
4. Consider a spherical gaseous cloud of mass density  $\rho(r)$  in a free space where  $r$  is the radial distance from its centre. The gaseous cloud is made of particles of equal mass  $m$  moving in circular orbits about their common centre with the same kinetic energy  $K$ . The force acting on the particles is their mutual gravitational force. If  $\rho(r)$  is constant with time. The particle number density  $n(r) = \rho(r)/m$  is:  
 ( $g$  = universal gravitational constant)  
 (a)  $\frac{3K}{\pi r^2 m^2 G}$       (b)  $\frac{K}{2\pi r^2 m^2 G}$       (c)  $\frac{K}{\pi r^2 m^2 G}$       (d)  $\frac{K}{6\pi r^2 m^2 G}$

**SECTION – 2**

5. A thin spherical insulating shell of radius  $R$  carries a uniformly distributed charge such that the potential at its surface is  $V_0$ . A hole with small area  $\alpha 4\pi R^2$  ( $\alpha \ll 1$ ) is made in the shell without effecting the rest of the shell. Which one of the following is correct.
- (a) The magnitude of  $\vec{E}$  at a point located on a line passing through the hole and shell's centre on a distance  $2R$  from the centre of spherical shell will be reduced by  $\frac{\alpha V_0}{2R}$
- (b) Potential at the centre of shell is reduced by  $2\alpha V_0$ .
- (c) The magnitude of  $\vec{E}$  at the centre of shell reduced by  $\frac{\alpha V_0}{2R}$
- (d) The ratio of potential at the centre of the shell to that of the point at  $\frac{1}{2}R$  from centre towards the hole will be  $\frac{1-\alpha}{1-2\alpha}$

6. A charged shell of radius  $R$  carries a total charge  $Q$ . Given  $\phi$  as the flux of electric field through a closed cylindrical surface of height  $h$ , radius  $r$  & with its center same as that of the shell. Here center of cylinder is a point on the axis of the cylinder which is equidistant from its top & bottom surfaces. Which of the following are correct.

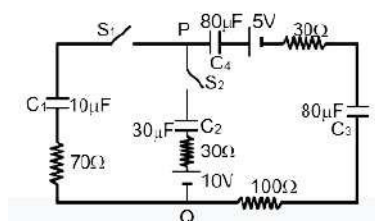
(a) If  $h > 2R$  &  $r > R$  then  $\phi = \frac{Q}{\epsilon_0}$

(b) If  $h < \frac{8R}{5}$  &  $r = \frac{3R}{5}$  then  $\phi = 0$

(c) If  $h > 2R$  &  $r = \frac{4R}{5}$  then  $\phi = \frac{Q}{5\epsilon_0}$

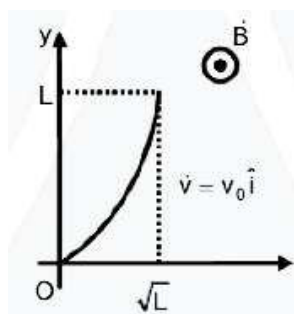
(d) If  $h > 2R$  &  $r = \frac{3R}{5}$  then  $\phi = \frac{Q}{5\epsilon_0}$

7. Which statements is/are correct:



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- (a) At time  $t = 0$ , the  $S_1$  is closed instantaneous current in the closed circuit will be 25 mA
- (b) The key  $S_1$  is kept closed for long time such that capacitors are fully charged. Now key  $S_2$  is closed at this time the instantaneous current across  $30\Omega$  resistor between P & Q will be 0.2A.
- (c) If key  $S_1$  is kept closed for long time such that capacitors are fully charged the voltage across  $C_1$  will be 4V.
- (d) If  $S_1$  is kept closed for long time such that capacitors are fully charged the voltage difference between P & Q will be 10V.
8. A galvanometer of resistance 10 ohm and maximum current of  $2\mu\text{A}$  is converted into voltmeter of range 100mV and when converted into ammeter then range is 1mA. When these voltmeter and ammeter are connected by a (ideal) battery in series with a resistance of  $R = 1000\Omega$ , then
- (a) Measured value of R is between  $978\Omega$  and  $996\Omega$
- (b) Resistance of voltmeter  $10^5\Omega$
- (c) Shunt resistance is  $20\text{m}\Omega$
- (d) If the ideal battery is replaced by non-ideal battery with internal resistance of  $5\Omega$  then R will be  $> 1000\Omega$
9. Conducting wire of parabolic shape, initially  $y = x^2$  is moving with velocity  $\vec{v} = v_0\hat{i}$  in a non-uniform magnetic field  $\vec{B} = B_0 \left( 1 + \left( \frac{y}{L} \right)^\beta \right) \hat{k}$  as shown in figure. If  $V_0$ ,  $B_0$ ,  $L$  &  $\beta$  are +ve constants &  $\Delta\phi$  is potential difference develop between the ends of wire, then correct statement(s) is/are



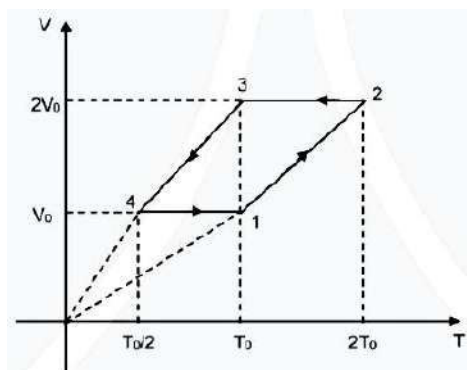
- (a)  $|\Delta\phi| = \frac{1}{2} B_0 V_0 L$  for  $\beta = 0$
- (b)  $|\Delta\phi| = \frac{4}{3} B_0 V_0 L$  for  $\beta = 2$
- (c)  $|\Delta\phi|$  is proportional to the length of wire projected on y-axis
- (d)  $|\Delta\phi|$  remains same if the parabolic wire is replaced by a straight wire,  $y = x$ , initially of length  $\sqrt{2}\ell$

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10. If in a hypothetical system if the angular momentum and mass are dimensionless. Then which of the following is true.

- |  |                                   |
|--|-----------------------------------|
| (a) The linear momentum varies as $L^{-1}$ | (b) The energy varies as $L^{-2}$ |
| (c) The power varies as $L^{-4}$           | (d) The force varies as $L^{-5}$  |

11.  $V - T$  diagram for  $n$  mol monoatomic gas is given below:



Choose the correct statement:

(a)  $\left| \frac{\Delta Q_{1 \rightarrow 2}}{\Delta Q_{3 \rightarrow 4}} \right| = \frac{1}{2}$

(b)  $\left| \frac{\Delta Q_{1 \rightarrow 2}}{\Delta Q_{2 \rightarrow 3}} \right| = \frac{5}{3}$

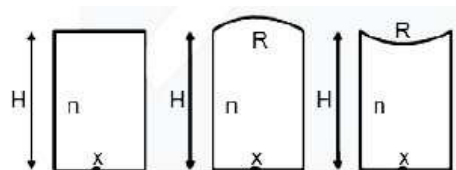
(c) Work done in cyclic process is  $\Delta W = \frac{nRT_0}{2}$

(d) There are only adiabatic and isochoric processes are involved.



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12. Apparent depth for point object  $x$  in all three cases are  $H_1$ ,  $H_2$  &  $H_3$  respectively when seen from below given  $H = 30$  cm,  $n = 1.5$  &  $R = 3$  m, then



(a)  $0.8 < H_2 - H_1 < 0.9$

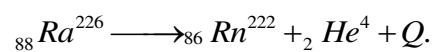
(b)  $H_2 > H_1$

(c)  $H_2 > H_3$

(d)  $H_3 > H_1$

## SECTION – 3

13. Consider the following nuclear fission reaction



In this fission reaction. Kinetic energy of  $\alpha$ -particle emitted is 4.44 MeV. Find the energy emitted as  $\gamma$  – radiation in keV in this reaction.

$$m({}_{88}\text{Ra}^{226}) = 226.005 \text{ amu}$$

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$$m({}_{86}\text{Rn}^{222}) = 222.000 \text{ amu}$$

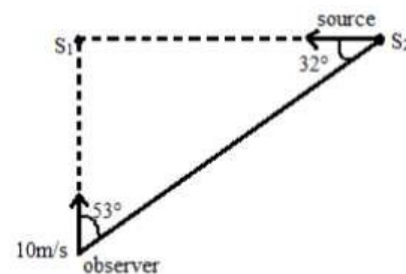
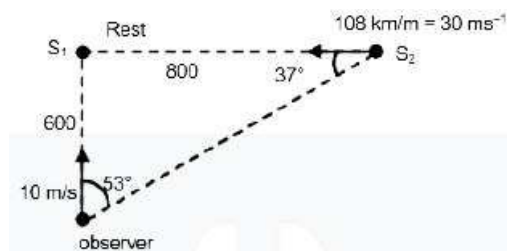
$$m({}_2\text{He}^4) = 4.000 \text{ amu}$$

14.  $N$  dielectrics are introduced in series in a capacitor of thickness  $D$ . Each dielectric have width  $d = D/N$  & dielectric constant of  $m^{\text{th}}$  dielectric is given by  $K_m = K(1 + m/N)$ : [ $N \gg 10^3$ , Area of plates =  $A$ ]

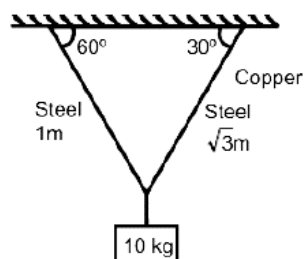
Net capacitance is given by  $\frac{K\epsilon_0 A}{\alpha D \ln 2}$ . Find value of  $\alpha$ .

15. If at angle  $\theta$  the light takes maximum time to travel in optical fiber. Then the maximum time is  $x \times 10^{-8}$ , calculate x.

16. The source  $S_1$  is at rest. The observer and the source  $S_2$  are moving towards  $S_1$  as shown in figure. The beats observed by the observer if both sources have frequency 120 Hz and speed of sound 330 m/s is



17. A weight of 100 N is suspended by two wires made by steel and copper as shown in figure length of steel wire is 1 m and copper wire is  $\sqrt{3}m$ . Find ratio of change in length of copper wire ( $\Delta\ell_o$ ) to change in length of steel wire ( $\Delta\ell_s$ ). Given Young's modulus:  $Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$ ,  $Y_{\text{copper}} = 1 \times 10^{11} \text{ N/m}^2$ .



18. An optical bench, to measure the focal length of lens, is 1.5 m long and on the bench marks are with spacing  $\frac{1}{4}$  cm. Now a lens is placed at 75 cm and pin type object is placed at 45 cm marks on the bench. If its image is formed at 135 cm find maximum possible error in calculation of focal length.




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## SECTION – 1

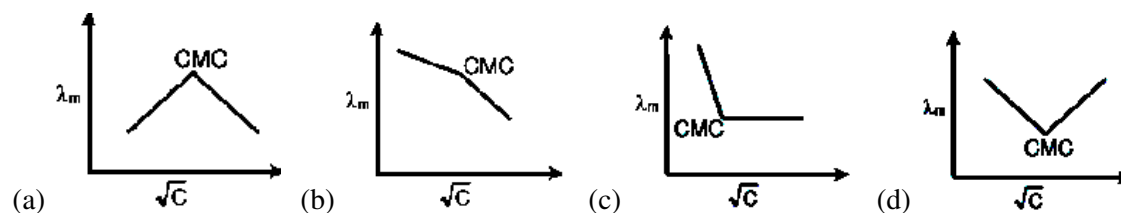
1. Which of the following set represent correct formula for Malachite, Magnetite, Calamine & Cryolite?

- (a)  $CuCO_3, Fe_2O_3, ZnO, Al_2O_3$       (b)  $CuCO_3, Cu(OH)_2, Fe_3O_4, ZnCO_3, Na_3AlF_6$   
 (c)  $CuCO_3, Fe_3O_4, ZnCO_3, Al_2O_3$       (d)  $CuCO_3, Cu(OH)_2, Fe_2O_3, ZnCO_3, Na_3AlF_6$

2. Find the correct acidic strength order:

- (i)  $HC \equiv C - COOH$     (ii)  $H_2C = CH - COOH$     (iii)     (iv)  $CH_3 - CH_2 - COOH$   
 (a)  $i > ii > iv > iii$     (b)  $i > ii > iii > iv$     (c)  $iii > ii > i > iv$     (d)  $iii > i > iv > ii$

3. Sodium stearate is a strong electrolyte. Which of the following plot is correct regarding its conductance:



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4. Which green coloured compound of chromium is formed in borax bead test?

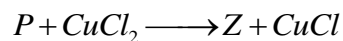
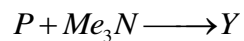
- (a)  $Cr(BO_2)_3$                       (b)  $Cr_2O_3$                       (c)  $CrB$                       (d)  $CrBO_3$

### SECTION – 2

5. Choose the reaction, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation:

- (a)  $2C_{(g)} + 3H_{2(g)} \rightarrow C_2H_{6(g)}$                       (b)  $\frac{3}{2}O_{2(g)} \rightarrow O_{3(g)}$   
(c)  $\frac{1}{8}S_{8(s)} + O_{2(g)} \rightarrow SO_{2(g)}$                       (d)  $2H_{2(g)} + O_{2(g)} \rightarrow 2H_2O(\ell)$

6. A Tin – chloride 'P' gives following reaction (unbalanced reaction)



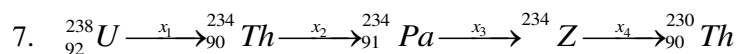
Then which of the following is/are correct.

- (a) Y contains co-ordinate bond                      (b) X is  $sp^3$  hybridised.

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(c) Oxidation state of Sn is X is +1.

(d) X contain lone pair on central atom.



$x_1, x_2, x_3, x_4$ , are either particles or radiation. Then

(a)  $x_1$  is deflected toward negatively charged plate.

(b)  $x_2$  is  $\beta$ -particle.

(c)  $x_3$  is  $\gamma$ -radiation.

(d) z is isotope of  ${}^{238}\text{U}$

8. Fusion of  $\text{MnO}_2$  along with  $\text{KOH}$  and  $\text{O}_2$  forms X. Electrolytic oxidation of X yields Y. X undergoes disproportionation reaction in acidic medium to  $\text{MnO}_2$  and Y. The Manganese in X and Y is in the form W & Z respectively, then

(a) W & Z are coloured

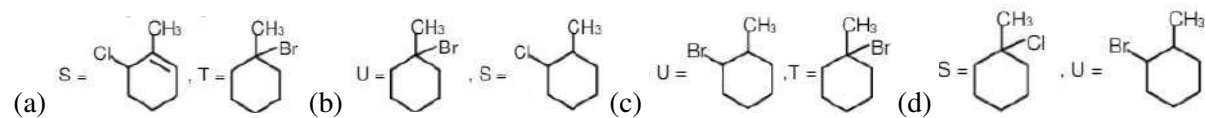
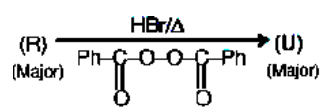
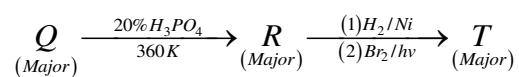
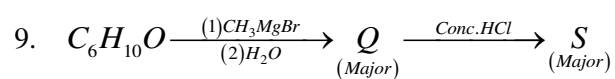
(b) W is diamagnetic and Z is paramagnetic

(c) Both W & Z are tetrahedral in shape

(d) Both W & Z involve  $p\pi$ - $d\pi$  bonding for  $\pi$  bond



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10. Which of the following are true.

- (a) Monosachharides can not be hydrolysed to give polyhydroxy aldehydes and ketones.
- (b) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose
- (c) Oxidation of glucose with bromine water gives glutamic acid.
- (d) The two six membered hemiacetal form of D(+) glucose are anomers.

11. Identify the option where all four molecules possess permanent dipole moment at room temperature.

- |   |  |
|---|--|
| (a) $BF$ , $O_3$ , $SF_6$ , $XeF_6$         | (b) $BeCl_2$ , $CO_2$ , $BCl_3$ , $CHCl_3$ |
| (c) $SO_2$ , $C_6H_5Cl$ , $H_2Se$ , $BrF_5$ | (d) $NO_2$ , $NH_3$ , $POCl_3$ , $CH_3Cl$  |

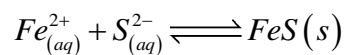
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12. Which of the following is/are correct regarding root mean square speed ( $U_{\text{rms}}$ ) & average translation K.E. ( $E_{\text{av}}$ ) of molecule in a gas at equilibrium.
- (a)  $E_{\text{av}}$  is doubled when its temperature is increased 4 times
  - (b)  $U_{\text{rms}}$  is inversely proportional to the square root of its molecular mass
  - (c)  $E_{\text{av}}$  at a given temperature doesn't depend on its molecular mass
  - (d)  $U_{\text{rms}}$  is doubled when its temperature is increased 4 times

### SECTION – 3

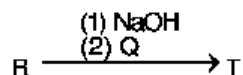
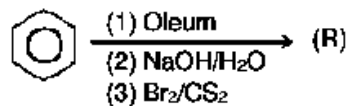
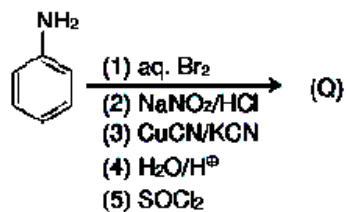
13.  $\text{XeF}_4 + \text{O}_2\text{F}_2 \longrightarrow \text{product}$ . The total number of lone pairs on the xenon containing product is: (l)

14. For the following reaction, equilibrium constant  $K_c$  at 298 K is  $1.6 \times 10^{17}$



When equal volume of 0.06 M  $Fe^{+2}$  and 0.2 M  $S^{-2}$  solution are mixed, then equilibrium concentration of  $Fe^{+2}$  is found to be  $Y \times 10^{-17}$  M. Y is:

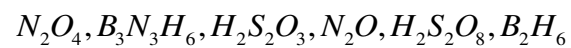
15.



Number of atoms of Br in compound 'T'



16. Which of the following compounds contain bond between same type of atoms.



17.  $A + B + C \rightarrow \text{Product}$

Ex. No	[A]	[B]	[C]	Rate of reaction
1.	0.2	0.1	0.1	$6 \times 10^{-5}$
2.	0.2	0.2	0.1	$6 \times 10^{-5}$
3.	0.2	0.1	0.2	$1.2 \times 10^{-4}$

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4.	0.3	0.1	0.1	$9 \times 10^{-5}$
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When  $[A] = 0.15$

$[B] = 0.25$

$[C] = 0.15$

Rate of reaction is  $Y \times 10^{-5}$  M/s Find Y.

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18. On dissolving 0.5 g of non-volatile, non-ionic solute to 39 g of benzene, its vapour pressure decreases from 650 mm of Hg to 640 mm of Hg. The depression of freezing point of benzene (in K) upon addition of the solute is \_\_\_\_\_.

[Given data: Molar mass & molar freezing point depression of benzene is  $78 \text{ g mol}^{-1}$  &  $5.12 \text{ K Kg mol}^{-1}$ ]



**SECTION – 1**

1. Let  $M = \begin{bmatrix} \sin^4 \theta & -1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} = \alpha I + \beta M^{-1}$ ,

where  $\alpha = \alpha(\theta)$  and  $\beta = \beta(\theta)$  are real number, and I is the  $2 \times 2$  identity matrix. If

$\alpha^*$  is the minimum of the set  $\{\alpha(\theta) : \theta \in [0, 2\pi]\}$  and

$\beta^*$  is the minimum of the set  $\{\beta(\theta) : \theta \in [0, 2\pi]\}$

then the value of  $\alpha^* + \beta^*$  is

(a)  $-\frac{37}{16}$

(b)  $-\frac{29}{16}$

(c)  $-\frac{31}{16}$

(d)  $-\frac{17}{16}$

2. A line  $y = mx + 1$  intersects the circle  $(x - 3)^2 + (y + 2)^2 = 25$  at the points P and Q. If the midpoint of the line segment PQ has x-coordinate  $-\frac{3}{5}$ , then which one of the following options is correct?
- (a)  $6 \leq m < 8$       (b)  $2 \leq m < 4$       (c)  $4 \leq m < 6$       (d)  $-3 \leq m < -1$

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3. Let  $S$  be the set of all complex numbers  $z$  satisfying  $|z - 2 + i| \geq \sqrt{5}$ . If the complex number  $z_0$  is such that

$\frac{1}{|z_0 - 1|}$  is the maximum of the set  $\left\{ \frac{1}{|z - 1|} : z \in S \right\}$ , then the principle argument of  $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$  is

(a)  $\frac{\pi}{4}$

(b)  $-\frac{\pi}{2}$

(c)  $\frac{3\pi}{4}$

(d)  $\frac{\pi}{2}$

4. The area of the region is

- (a)  $8\log_e 2 - \frac{14}{3}$       (b)  $16\log_e 2 - \frac{14}{3}$       (c)  $16\log_e 2 - 6$       (d)  $8\log_e 2 - \frac{7}{3}$

**SECTION – 2**

1. There are three bags  $B_1$ ,  $B_2$  and  $B_3$ . The bag  $B_1$  contains 5 red and 5 green balls,  $B_2$  contains 3 red and 5 green balls, and  $B_3$  contains 5 red and 3 green balls, Bags  $B_1$ ,  $B_2$  and  $B_3$  have probabilities  $\frac{3}{10}$ ,  $\frac{3}{10}$  and  $\frac{4}{10}$  respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

- (a) Probability that the selected bag is  $B_3$  and the chosen ball is green equals  $\frac{3}{10}$
- (b) Probability that the chosen ball is green equals  $\frac{39}{80}$
- (c) Probability that the chosen ball is green, given that the selected bag is  $B_3$ , equals  $\frac{3}{8}$
- (d) Probability that the selected bag is  $B_3$ , given that the chosen balls is green, equals  $\frac{5}{13}$

2. Define the collections  $\{E_1, E_2, E_3, \dots\}$  of ellipses and  $\{R_1, R_2, R_3, \dots\}$  of rectangles as follows:

$$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1;$$

$R_1$ : rectangle of largest area, with sides parallel to the axes, inscribed in  $E_1$ ;

$E_n$ : Ellipse  $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$  of largest area inscribed in  $R_{n-1}, n > 1$ ;

$R_n$ : rectangle of largest area, with sides parallel to the axes, inscribed in  $E_n, n > 1$ .

Then which of the following options is/are correct?

- (a) The eccentricities of  $E_{18}$  and  $E_{19}$  are NOT equal

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(b) The distance of a focus from the centre in  $E_9$  is  $\frac{\sqrt{5}}{32}$

(c) The length of latus rectum of  $E_9$  is  $\frac{1}{6}$

(d)  $\sum_{n=1}^N (\text{area of } R_n) < 24$ , for each positive integer  $N$

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3. Let  $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$  and  $\text{adj}M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$  where a and b are real numbers. Which of the following

options is/are correct?

(a)  $a + b = 3$

(b)  $\det(\text{adj}M^2) = 81$

(c)  $(\text{adj}M)^{-1} + \text{adj}M^{-1} = -M$

(d) If  $M \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ , then  $\alpha - \beta + \gamma = 3$

4. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \leq x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \leq x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \geq 3 \end{cases}$$

Then which of the following options is/are correct?

(a)  $f'$  has a local maximum at  $x = 1$

(b)  $f$  is onto

(c)  $f$  is increasing on  $(-\infty, 0)$

(d)  $f'$  is NOT differentiable at  $x = 1$



5. Let  $\alpha$  and  $\beta$  be the roots of  $x^2 - x - 1 = 0$ , with  $\alpha > \beta$ . For all positive integers  $n$ , define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1$$

$$\text{and } b_n = a_{n-1} + a_{n+1}, n \geq 2.$$

Then which of the following options is/are correct?

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(a)  $a_1 + a_2 + a_3 + \dots + a_n = a_{n+2} - 1$  for all  $n \geq 1$

(b)  $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(c)  $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(d)  $b_n = \alpha^n + \beta^n$  for all  $n \geq 1$

6. Let  $\Gamma$  denote a curve  $y = y(x)$  which is in the first quadrant and let the point  $(1, 0)$  lie on it. Let the tangent to  $\Gamma$  at a point  $P$  intersect the  $y$ -axis at  $Y_P$ . If  $PY_P$  has length 1 for each point  $P$  on  $\Gamma$ , then which of the following is options is/are correct?

(a)  $y = \log_e \left( \frac{1 + \sqrt{1 - x^2}}{x} \right) - \sqrt{1 - x^2}$

(b)  $xy' - \sqrt{1 - x^2} = 0$

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(c)  $y - \log_e \left( \frac{1 + \sqrt{1 - x^2}}{x} \right) + \sqrt{1 - x^2}$

(d)  $xy' + \sqrt{1 - x^2} = 0$

7. In a non-right-angle triangle  $\Delta PQR$ , let  $p, q, r$  denote the lengths of the sides opposite to the angles at  $P, Q, R$  respectively. The median from  $R$  meets the side  $PQ$  at  $S$ , the perpendicular from  $P$  meets the side  $QR$  at  $E$ , and  $RS$  and  $PE$  intersect at  $O$ . If  $p = \sqrt{3}$ ,  $q = 1$ , and the radius of the circumcircle of the  $\Delta PQR$  equals 1, then which of the following options is/are correct?

(a) Area of  $\Delta SOE = \frac{\sqrt{3}}{12}$

(b) Radius of incircle of  $\Delta PQR = \frac{\sqrt{3}}{2}(2 - \sqrt{3})$

(c) Length of  $RS = \frac{\sqrt{7}}{2}$

(d) Length of  $OE = \frac{1}{6}$

8. Let  $L_1$  and  $L_2$  denotes the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$$

$$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \mathbb{R}$$

respectively. If  $L_3$  is a line which is perpendicular to both  $L_1$  and  $L_2$  and cuts both of them, then which of the following options describe(s)  $L_3$ ?

(a)  $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(b)  $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(c)  $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(d)  $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$



**SECTION – 3**

1. If  $I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)}$

Then  $27I^2$  equals \_\_\_\_\_

2. Let the point B be the reflection of the point A(2, 3) with respect to the line  $8x - 6y - 23 = 0$ . Let  $\Gamma_A$  and  $\Gamma_B$  be circle of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circle  $\Gamma_A$  and  $\Gamma_B$  such that both the circle are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is \_\_\_\_\_



3. Let  $AP(a; d)$  denote the set of all the terms of an infinite arithmetic progression with first term  $a$  and common difference  $d > 0$ . If  $AP(1; 3) \cap AP(2; 5) \cap AP(3; 7) = AP(a; d)$  then  $a + d$  equals) \_\_\_\_\_

4. Let  $S$  be the sample space of all  $3 \times 3$  matrices with entries from the set  $\{0, 1\}$ . Let the events  $E_1$  and  $E_2$  be given by

$$E_1 = \{A \in S : \det A = 0\} \text{ and}$$

$$E_2 = \{A \in S : \text{sum of entries of } A \text{ is } 7\}.$$

If a matrix is chosen at random from  $S$ , then the conditional probability  $P(E_1|E_2)$  equals .....

5. Three lines are given by

and

Let the lines cut the plane at the points A, B and C respectively. If the area of the triangle ABC is  $\Delta$  then the value of  $(6\Delta)^2$  equals \_\_\_\_\_

6. Let  $\omega \neq 1$  be a cube root of unity. Then the minimum of the set

$$\left\{ \left| a + b\omega + c\omega^2 \right|^2 : a, b, c \text{ distinct non-zero integers} \right\}$$

equals \_\_\_\_\_