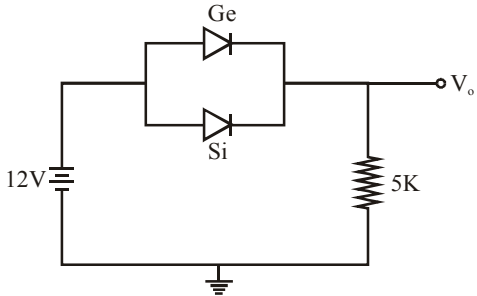


TEST PAPER OF JEE(MAIN) EXAMINATION – 2019**(Held On Wednesday 09th JANUARY, 2019) TIME : 2 : 30 PM To 05 : 30 PM****PHYSICS**

1. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be :
 (1) 90° (2) 45° (3) 75° (4) 60°
2. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^\circ \leq \theta \leq 30^\circ$ is:
 (1) 320 (2) 641 (3) 321 (4) 640
3. At a given instant, say $t = 0$, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t} . [If the half-life of A is m_1 , the half-life of B is :
 (1) $\frac{\ln 2}{2}$ (2) $2\ln 2$ (3) $\frac{\ln 2}{4}$ (4) $4\ln 2$
4. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by : (assume that the Ge diode has large breakdown voltage)
- 
- (1) 0.6 V (2) 0.8 V (3) 0.4 V (4) 0.2 V

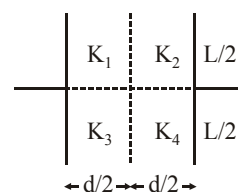
- | | |
|--|---|
| <p>5. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :</p> <p>(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77</p> | <p>8. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to :</p> <p>(1) 753 Hz (2) 500 Hz
(3) 333 Hz (4) 666 Hz</p> |
| <p>6. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about :</p> <p>[Take R = 8.3 J/ K mole]</p> <p>(1) 10 kJ (2) 0.9 kJ (3) 6 kJ (4) 14 kJ</p> | <p>9. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accommodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3 \times 10^8$ m/s, $h = 6.6 \times 10^{-34}$ J-s)</p> <p>(1) 3.75×10^6 (2) 4.87×10^5
(3) 3.86×10^6 (4) 6.25×10^5</p> |
| <p>7. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about $x = 0$. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :</p> <p>(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$ (3) $\frac{A}{\sqrt{2}}$ (4) A</p> | |

- 10.** Two point charges $q_1(\sqrt{10} \mu\text{C})$ and $q_2(-25 \mu\text{C})$ are placed on the x-axis at $x = 1 \text{ m}$ and $x = 4 \text{ m}$ respectively. The electric field (in V/m) at a point $y = 3 \text{ m}$ on y-axis is,

$$\left[\text{take } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2} \right]$$

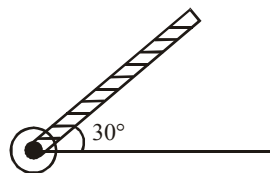
- (1) $(-63\hat{i} + 27\hat{j}) \times 10^2$ (2) $(81\hat{i} - 81\hat{j}) \times 10^2$
 (3) $(63\hat{i} - 27\hat{j}) \times 10^2$ (4) $(-81\hat{i} + 81\hat{j}) \times 10^2$

- 11.** A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be :



$$\begin{aligned} (1) \quad &= \frac{(K_1 + K_2)(K_3 + K_4)}{1 + 2 + 3 + 4} \\ &= \frac{(K_1 + K_2)(K_3 + K_4)}{1 + 2 + 3 + 4} \\ &= \frac{(K_1 + K_4)(K_2 + K_3)}{1 + 2 + 3 + 4} \\ K &= \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4} \end{aligned}$$

12. A rod of length 50cm is pivoted at one end. It is raised such that it makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10\text{ms}^{-2}$)



- (1) $\sqrt{30}$ (2) $\sqrt{\frac{30}{2}}$ (3) $\frac{\sqrt{30}}{2}$ (4) $\frac{\sqrt{20}}{3}$

- 13.** One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. $R \frac{B_L}{B_C}$ will be :

(1) $\frac{1}{N}$ (2) N^2 (3) $\frac{1}{N^2}$ (4) N

- 14.** The energy required to take a satellite to a height ' h ' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2 . The value of h for which E_1 and E_2 are equal, is:

(1) 1.28×10^4 km (2) 6.4×10^3 km
(3) 3.2×10^3 km (4) 1.6×10^3 km

- 15.** The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then :

(1) $U_E = \frac{U_B}{2}$ (2) $U_E < U_B$
(3) $U_E = U_B$ (4) $U_E > U_B$

- 16.** A series AC circuit containing an inductor (20 mH), a capacitor (120 μ F) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is :
- (1) 2.26×10^3 J (2) 3.39×10^3 J
 (3) 5.65×10^2 J (4) 5.17×10^2 J

- 18.** The magnetic field associated with a light wave is given, at the origin, by
 $B = B_0 [\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons ?
 ($c = 3 \times 10^8 \text{ms}^{-1}$, $h = 6.6 \times 10^{-34} \text{J-s}$)
- (1) 7.72 eV (2) 8.52 eV
 (3) 12.5 eV (4) 6.82 eV

- 17.** Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to :

- (1) $\sqrt{\frac{Gh}{c^3}}$ (2) $\sqrt{\frac{hc^5}{G}}$
 (3) $\sqrt{\frac{c^3}{Gh}}$ (4) $\sqrt{\frac{Gh}{c^5}}$

19. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2} e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :

(1) $\frac{a}{2} \log \left(1 - \frac{Q}{2\pi a A} \right)$ (2) $a \log \left(1 - \frac{Q}{2\pi a A} \right)$
 (3) $a \log \left(\frac{1}{1 - \frac{Q}{2\pi a A}} \right)$ (4) $\frac{a}{2} \log \left(\frac{1}{1 - \frac{Q}{2\pi a A}} \right)$

20. Two Carnot engines A and B are operated in series. The first one, A, receives heat at $T_1 (= 600 \text{ K})$ and rejects to a reservoir at temperature T_2 . The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at $T_3 (= 400 \text{ K})$. Calculate the temperature T_2 if the work outputs of the two engines are equal :
- (1) 400 K (2) 600 K (3) 500 K (4) 300 K

21. A carbon resistance has a following colour code. What is the value of the resistance ?



- (1) $1.64 \text{ M}\Omega \pm 5\%$ (2) $530 \text{ k}\Omega \pm 5\%$
 (3) $64 \text{ k}\Omega \pm 10\%$ (4) $5.3 \text{ M}\Omega \pm 5\%$
22. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?
- (1) 850 J (2) 900 J
 (3) 950 J (4) 875 J

23. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$$x = a \cos \omega t$$

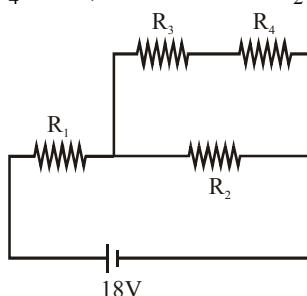
$$y = a \sin \omega t$$

$$\text{and } z = a \omega t$$

The speed of the particle is :

- (1) $a\omega$ (2) $\sqrt{3} a\omega$
 (3) $\sqrt{2} a\omega$ (4) $2a\omega$

24. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400 \Omega$, $R_3 = 100 \Omega$ and $R_4 = 500 \Omega$ and the reading of an ideal voltmeter across R_4 is 5V, then the value R_2 will be :



- (1) 300Ω (2) 230Ω
 (3) 450Ω (4) 550Ω

25. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10 \text{ ms}^{-2}$)
 (1) 200 N (2) 100 N (3) 140 N (4) 70 N

26. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed ' v ' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then ' v ' is equal to

- (1) $\frac{a_1 + a_2}{2} t$ (2) $\sqrt{2a_1 a_2} t$
 (3) $\frac{2a_1 a_2}{a_1 + a_2} t$ (4) $\sqrt{a_1 a_2} t$

- 27.** A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be :

(1) 25 A (2) 50 A
(3) 35 A (4) 45 A

- 28.** The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to :
- (1) 9.6 m (2) 4.8 m (3) 2.9 m (4) 6.0 m

- 29.** The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is :

(1) 5.755 mm (2) 5.725 mm
(3) 5.740 mm (4) 5.950 mm

- 30.** A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron $=1.6 \times 10^{-19}\text{C}$)

(1) $2.0 \times 10^{-24} \text{ kg}$
(2) $1.6 \times 10^{-19} \text{ kg}$
(3) $1.6 \times 10^{-27} \text{ kg}$
(4) $9.1 \times 10^{-31} \text{ kg}$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 2 : 30 PM To 05 : 30 PM

CHEMISTRY

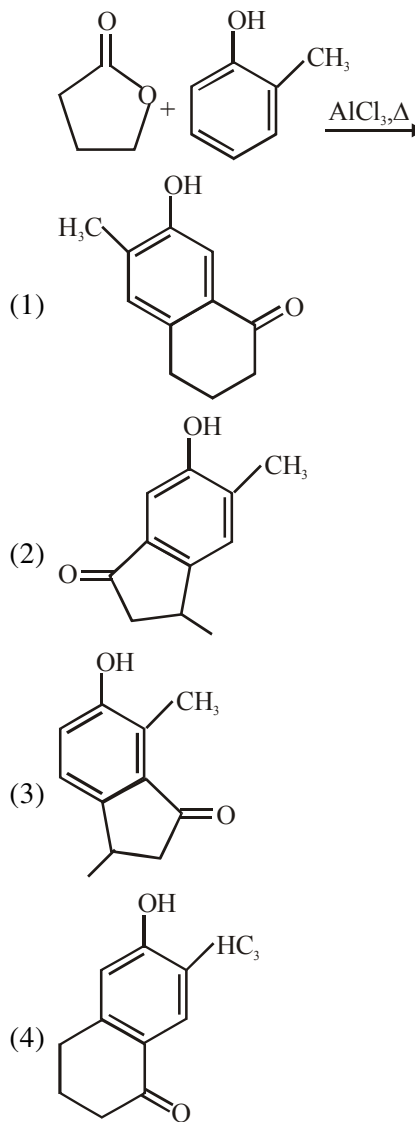
1. Good reducing nature of H_3PO_2 is attributed to the presence of:
- (1) One P-OH bond (2) One P-H bond
(3) Two P-H bonds (4) Two P-OH bonds

2. The complex that has highest crystal field splitting energy (Δ), is :
- (1) $\text{K}_3[\text{Co}(\text{CN})_6]$
(2) $[\text{Co}(\text{NH}_3)_5(\text{H}_2\text{O})]\text{Cl}_3$
(3) $\text{K}_2[\text{CoCl}_4]$
(4) $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$

3. The metal that forms nitride by reacting directly with N_2 of air, is :
- (1) K (2) Cs (3) Li (4) Rb

4. In which of the following processes, the bond order has increased and paramagnetic character has changed to diamagnetic ?
- (1) $\text{N}_2 \rightarrow \text{N}_2^+$ (2) $\text{NO} \rightarrow \text{NO}^+$
(3) $\text{O}_2 \rightarrow \text{O}_2^{2-}$ (4) $\text{O}_2 \rightarrow \text{O}_2^+$

5. The major product of the following reaction is:



6. The transition element that has lowest enthalpy of atomisation, is :

- (1) Zn
- (2) Cu
- (3) V
- (4) Fe

7. Which of the following combination of statements is true regarding the interpretation of the atomic orbitals ?

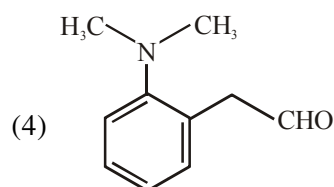
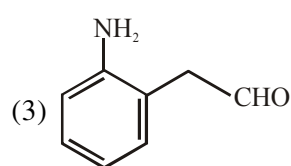
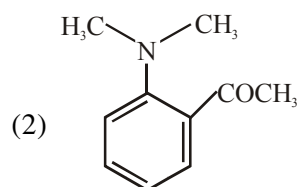
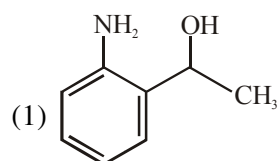
- (a) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.
- (b) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number.
- (c) According to wave mechanics, the ground state angular momentum is h equal to $\frac{h}{2\pi}$.
- (d) The plot of ψ Vs r for various azimuthal quantum numbers, shows peak shifting towards higher r value.

(1) (b), (c) (2) (a), (d) (3) (a), (b) (4) (a), (c)

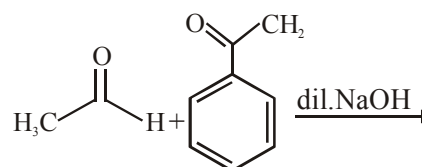
8. The tests performed on compound X and their inferences are:

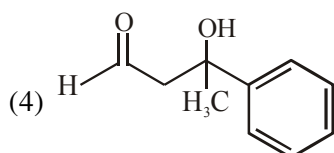
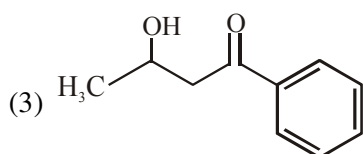
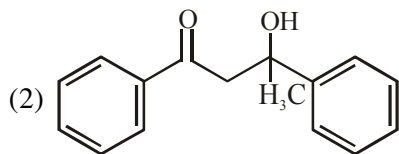
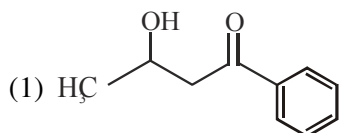
Test	Inference
(a) 2,4 - DNP test	Coloured precipitate
(b) Iodoform test	Yellow precipitate
(c) Azo-dye test	No dye formation

Compound 'X' is:



9. The major product formed in the following reaction is:



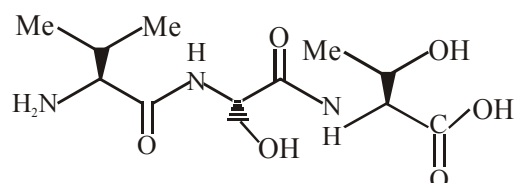


10. For the reaction, $2A + B \rightarrow \text{products}$, when the concentrations of A and B both were doubled, the rate of the reaction increased from $0.3 \text{ mol L}^{-1}\text{s}^{-1}$ to $2.4 \text{ mol L}^{-1}\text{s}^{-1}$. When the concentration of A alone is doubled, the rate increased from $0.3 \text{ mol L}^{-1}\text{s}^{-1}$ to $0.6 \text{ mol L}^{-1}\text{s}^{-1}$

Which one of the following statements is correct ?

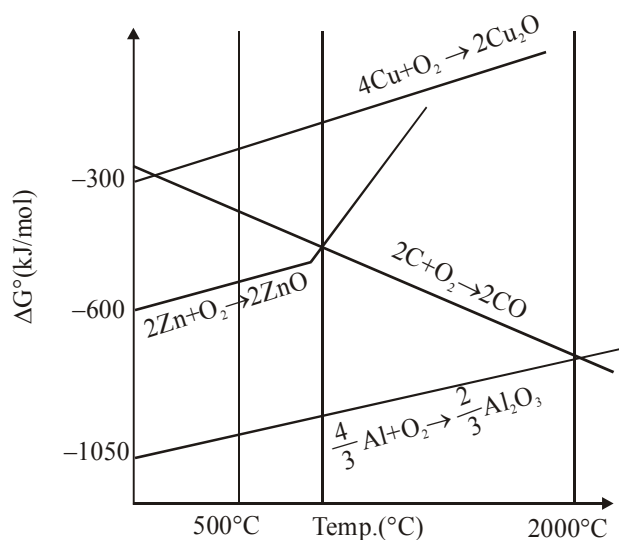
- (1) Order of the reaction with respect to B is 2
- (2) Order of the reaction with respect to A is 2
- (3) Total order of the reaction is 4
- (4) Order of the reaction with respect to B is 1

11. The correct sequence of amino acids present in the tripeptide given below is :



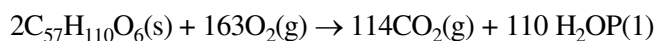
- (1) Leu - Ser - Thr
- (2) Thr - Ser - Leu
- (3) Thr - Ser - Val
- (4) Val - Ser - Thr

12. The correct statement regarding the given Ellingham diagram is:



- (1) At 800°C , Cu can be used for the extraction of Zn from ZnO
- (2) At 500°C , coke can be used for the extraction of Zn from ZnO
- (3) Coke cannot be used for the extraction of Cu from Ca_2O .
- (4) At 1400°C , Al can be used for the extraction of Zn from ZnO

13. For the following reaction, the mass of water produced from 445 g of $C_{57}H_{110}O_6$ is :

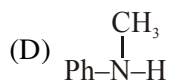
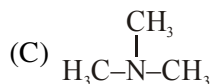
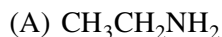


- (1) 495 g (2) 490 g (3) 890 g (4) 445 g

14. The correct match between Item I and Item II is :

Item I	Item II
(A) Benzaldehyde	(P) Mobile phase
(B) Alumina	(Q) Adsorbent
(C) Acetonitrile	(R) Adsorbate
(1) (A) \rightarrow (Q); (B) \rightarrow (R); (C) \rightarrow (P)	
(2) (A) \rightarrow (P); (B) \rightarrow (R); (C) \rightarrow (Q)	
(3) (A) \rightarrow (Q); (B) \rightarrow (P); (C) \rightarrow (R)	
(4) (A) \rightarrow (R); (B) \rightarrow (Q); (C) \rightarrow (P)	

15. The increasing basicity order of the following compounds is :



- (1) (D)<(C)<(A)<(B) (2) (A)<(B)<(D)<(C)
(3) (A)<(B)<(C)< (D) (4) (D)<(C)<(B)<(A)

16. For coagulation of arsenious sulphide sol, which one of the following salt solution will be most effective ?

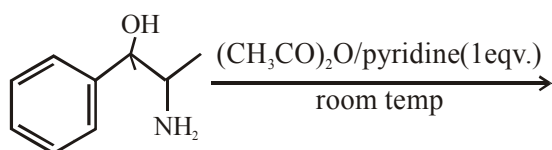
- (1) AlCl_3 (2) NaCl
(3) BaCl_2 (4) Na_3PO_4

17. At 100°C, copper (Cu) has FCC unit cell structure with cell edge length of x Å. What is the approximate density of Cu (in g cm^{-3}) at this temperature ?

[Atomic Mass of Cu = 63.55u]

- (1) $\frac{105}{x^3}$ (2) $\frac{211}{x^3}$ (3) $\frac{205}{x^3}$ (4) $\frac{422}{x^3}$

18. The major product obtained in the following reaction is :



- (1)
- (2)
- (3)
- (4)

19. Which of the following conditions in drinking water causes methemoglobinemia ?

- (1) > 50ppm of lead
 (2) > 100 ppm of sulphate
 (3) > 50 ppm of chloride
 (4) > 50 ppm of nitrate

20. Homoleptic octahedral complexes of a metal ion M^{3+} with three monodentate ligands and $\text{L}_1, \text{L}_2, \text{L}_3$ absorb wavelengths in the region of green, blue and red respectively. The increasing order of the ligand strength is :

- (1) $\text{L}_2 < \text{L}_1 < \text{L}_3$ (2) $\text{L}_3 < \text{L}_2 < \text{L}_1$
 (3) $\text{L}_3 < \text{L}_1 < \text{L}_2$ (4) $\text{L}_1 < \text{L}_2 < \text{L}_3$

21. The product formed in the reaction of cumene with O_2 followed by treatment with dil. HCl are :

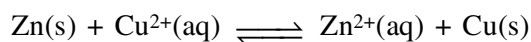
- (1) and
 (2) and $\text{CH}_3 - \text{OH}$
 (3) and
 (4) and

22. The temporary hardness of water is due to :-
 (1) $\text{Ca}(\text{HCO}_3)_2$ (2) NaCl
 (3) Na_2SO_4 (4) CaCl_2

23. The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is :
 (Specific heat of water liquid and water vapour are $4.2 \text{ kJ K}^{-1} \text{ kg}^{-1}$ and $2.0 \text{ kJ K}^{-1} \text{ kg}^{-1}$; heat of liquid fusion and vapourisation of water are 344 kJ kg^{-1} and 2491 kJ kg^{-1} , respectively).
 ($\log 273 = 2.436$, $\log 373 = 2.572$, $\log 383 = 2.583$)
 (1) $7.90 \text{ kJ kg}^{-1} \text{ K}^{-1}$ (2) $2.64 \text{ kJ kg}^{-1} \text{ K}^{-1}$
 (3) $8.49 \text{ kJ kg}^{-1} \text{ K}^{-1}$ (4) $4.26 \text{ kJ kg}^{-1} \text{ K}^{-1}$

24. The pH of rain water, is approximately :
 (1) 6.5 (2) 7.5 (3) 5.6 (4) 7.0

25. If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction



at 300 K is approximately.

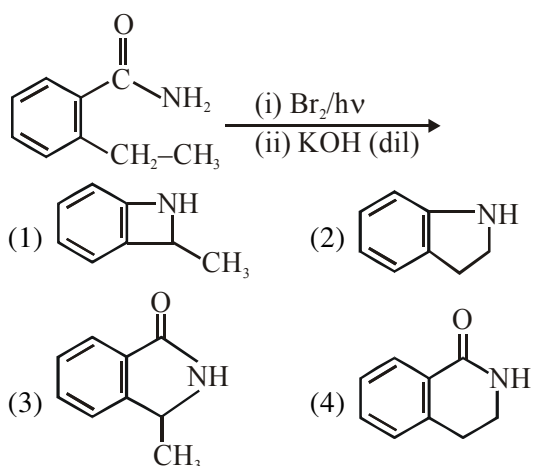
($R = 8 \text{ JK}^{-1} \text{ mol}^{-1}$, $F = 96000 \text{ C mol}^{-1}$)

- (1) e^{160} (2) e^{320}
 (3) e^{-160} (4) e^{-80}

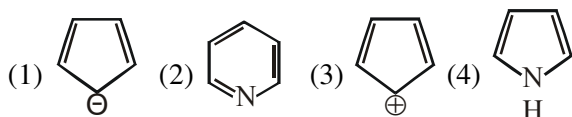
26. A solution containing 62 g ethylene glycol in 250 g water is cooled to -10°C . If K_f for water is $1.86 \text{ K kg mol}^{-1}$, the amount of water (in g) separated as ice is :
 (1) 32 (2) 48 (3) 16 (4) 64

27. When the first electron gain enthalpy ($\Delta_{\text{eg}}H$) of oxygen is -141 kJ/mol , its second electron gain enthalpy is :
 (1) almost the same as that of the first
 (2) negative, but less negative than the first
 (3) a positive value
 (4) a more negative value than the first

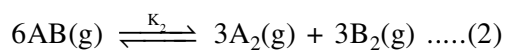
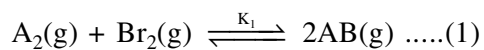
28. The major product of the following reaction is :



29. Which of the following compounds is not aromatic ?



30. Consider the following reversible chemical reactions :



The relation between K_1 and K_2 is :

(1) $K_2 = K_1^3$ (2) $K_2 = K_1^{-3}$

(3) $K_1 K_2 = 3$ (4) $K_1 K_2 = \frac{1}{3}$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019**(Held On Wednesday 09th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM****M A T H E M A T I C S**

1. Let f be a differentiable function from \mathbb{R} to \mathbb{R} such that $|f(x) - f(y)| \leq 2|x - y|^{\frac{3}{2}}$, for all $x, y \in \mathbb{R}$. If $f(0) = 1$ then $\int_0^1 f^2(x) dx$ is equal to
 (1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1
2. If $\int_0^{\frac{\pi}{3}} \frac{\tan \theta}{\sqrt{2k \sec \theta}} d\theta = 1 - \frac{1}{\sqrt{2}}$, ($k > 0$), then the value of k is :
 (1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1
3. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is
 (1) 12 (2) 15 (3) 10 (4) 14
4. For each $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x . Then $\lim_{x \rightarrow 0^-} \frac{x([x] + |x|) \sin [x]}{|x|}$ is equal to
 (1) $-\sin 1$ (2) 0 (3) 1 (4) $\sin 1$
5. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval:
 (1) (4, 5) (2) (3, 4) (3) (5, 6) (4) $(-5, -4)$

6. If

$$A = \begin{bmatrix} e^t & e^{-t} \cos t & e^{-t} \sin t \\ e^t & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ e^t & 2e^{-t} \sin t & -2e^{-t} \cos t \end{bmatrix}$$

Then A is-

- (1) Invertible only if $t = \frac{\pi}{2}$
- (2) not invertible for any $t \in \mathbb{R}$
- (3) invertible for all $t \in \mathbb{R}$
- (4) invertible only if $t = \pi$

7. The area of the region

$$A = \{(x, y) : 0 \leq y \leq x|x| + 1 \text{ and } -1 \leq x \leq 1\}$$

in sq. units, is :

- (1) $\frac{2}{3}$
- (2) $\frac{1}{3}$
- (3) 2
- (4) $\frac{4}{3}$

8. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then $\arg z$ is equal to:

- (1) $\frac{\pi}{4}$
- (2) $\frac{\pi}{3}$
- (3) 0
- (4) $\frac{\pi}{6}$

9. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of \vec{b} on \vec{a} is \vec{a} . If $\vec{a} + \vec{b}$ is perpendicular to \vec{c} , then $|\vec{b}|$ is equal to:

- (1) $\sqrt{22}$
- (2) 4
- (3) $\sqrt{32}$
- (4) 6

10. Let A(4, -4) and B(9, 6) be points on the parabola, $y^2 + 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB , is:

- (1) $31\frac{3}{4}$
- (2) 32
- (3) $30\frac{1}{2}$
- (4) $31\frac{1}{4}$

11. The logical statement

$[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)]$ is equivalent to:

- (1) $(p \wedge r) \wedge \sim q$ (2) $(\sim p \wedge \sim q) \wedge r$
 (3) $\sim p \vee r$ (4) $(p \wedge \sim q) \vee r$

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

- (1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

13. If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is

- (1) 2 (2) 1
 (3) 3 (4) 4

14. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is:

- (1) $x + 2y - 2z = 0$ (2) $x - 2y + z = 0$
 (3) $5x + 2y - 4z = 0$ (4) $3x + 2y - 3z = 0$

15. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at $(1,1)$, then the equation of its third side is :
- (1) $122y - 26x - 1675 = 0$
(2) $26x + 61y + 1675 = 0$
(3) $122y + 26x + 1675 = 0$
(4) $26x - 122y - 1675 = 0$
16. If $x = 3 \tan t$ and $y = 3 \sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is:
- (1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$
17. If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to:
- (1) π (2) 7π (3) 0 (4) 10
18. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then:
- (1) $cc' + a + a' = 0$
(2) $aa' + c + c' = 0$
(3) $ab' + bc' + 1 = 0$
(4) $bb' + cc' + 1 = 0$
19. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is :
- (1) 2 (2) 5 (3) 3 (4) 4

20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{3}$ (4) 2

21. Let $A = \{x \in \mathbb{R} : x \text{ is not a positive integer}\}$

Define a function $f : A \rightarrow \mathbb{R}$ as $f(x) = \frac{2x}{x-1}$ then f is

- (1) injective but not surjective
 (2) not injective
 (3) surjective but not injective
 (4) neither injective nor surjective

22. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx, (x \geq 0)$ and $f(0) = 0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

23. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x-4)^2 + (y-7)^2 = 36$ intersect at two distinct points, then:

(1) $0 < r < 1$ (2) $1 < r < 11$
 (3) $r > 11$ (4) $r = 11$

24. Let S be the set of all triangles in the xy -plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is:

(1) 9 (2) 18 (3) 32 (4) 36

25. The sum of the following series

$1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9}$
 $+ \frac{15(1^2 + 2^2 + \dots + 5^2)}{11} + \dots$ up to 15 terms, is:
 (1) 7820 (2) 7830 (3) 7520 (4) 7510

26. Let a , b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:

- (1) $\frac{1}{2}$ (2) 4
(3) 2 (4) $\frac{7}{13}$

27. If the system of linear equations

$$x - 4y + 7z = g$$

$$3y - 5z = h$$

$$-2x + 5y - 9z = k$$

is consistent, then :

- (1) $g + h + k = 0$
(2) $2g + h + k = 0$
(3) $g + h + 2k = 0$
(4) $g + 2h + k = 0$

28. Let $f: [0, 1] \rightarrow \mathbb{R}$ be such that $f(xy) = f(x)f(y)$ for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the

differential equation, $\frac{dy}{dx} = f(x)$ with

$y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is equal to

- (1) 4 (2) 3 (3) 5 (4) 2

29. A data consists of n observations:

x_1, x_2, \dots, x_n . If $\sum_{i=1}^n (x_i + 1)^2 = 9n$ and

$\sum_{i=1}^n (x_i - 1)^2 = 5n$, then the standard deviation of this data is :

- (1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to :
- (1) 250 (2) 374 (3) 372 (4) 375