

# FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24<sup>th</sup> February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

## PHYSICS

## TEST PAPER WITH ANSWER & SOLUTIONS

### SECTION-A

1. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is :

- (1) circular (2) elliptical  
(3) parabolic (4) straight line

2. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x ( $x \ll d$ ) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency : (m = mass of charged particle)

- (1)  $\left(\frac{2q^2}{\pi\epsilon_0 md^3}\right)^{\frac{1}{2}}$  (2)  $\left(\frac{\pi\epsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$   
(3)  $\left(\frac{q^2}{2\pi\epsilon_0 md^3}\right)^{\frac{1}{2}}$  (4)  $\left(\frac{2\pi\epsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$

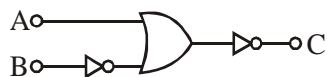
3. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules :

- (1) continuously lose their energy till it reaches wall.  
(2) are attracted by the walls of container.  
(3) continuously stick to the walls of container.  
(4) suffer change in momentum when impinge on the walls of container.

4. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains :

- (1) increase in size but no change in orientation.  
(2) have no relation with external magnetic field.  
(3) decrease in size and changes orientation.  
(4) may increase or decrease in size and change its orientation.

5.



The logic circuit shown above is equivalent to :

- (1)
- (2)
- (3)
- (4)

6. The period of oscillation of a simple pendulum

is  $T = 2\pi \sqrt{\frac{L}{g}}$ . Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be :

- (1) 1.13% (2) 1.03%  
(3) 1.33% (4) 1.30%

7. Given below are two statements :

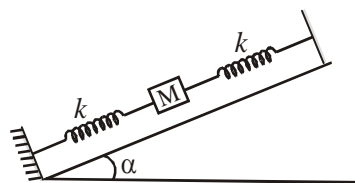
Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor  $\beta$  indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below :

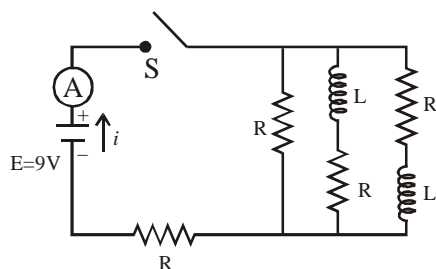
- (1) Statement I is false but Statement II is true  
(2) Both Statement I and Statement II are true  
(3) Both Statement I and Statement II are false  
(4) Statement I is true but Statement II is false

8. In the given figure, a body of mass  $M$  is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant  $k$ , the frequency of oscillation of given body is :



- (1)  $\frac{1}{2\pi} \sqrt{\frac{k}{2M}}$       (2)  $\frac{1}{2\pi} \sqrt{\frac{2k}{Mg \sin \alpha}}$   
 (3)  $\frac{1}{2\pi} \sqrt{\frac{2k}{M}}$       (4)  $\frac{1}{2\pi} \sqrt{\frac{k}{Mg \sin \alpha}}$

9. Figure shows a circuit that contains four identical resistors with resistance  $R = 2.0 \, \Omega$ , two identical inductors with inductance  $L = 2.0 \, \text{mH}$  and an ideal battery with  $\text{emf } E = 9 \, \text{V}$ . The current ' $i$ ' just after the switch ' $S$ ' is closed will be :

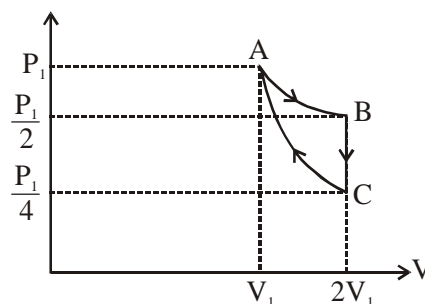


- (1) 2.25 A      (2) 3.0 A  
 (3) 3.37 A      (4) 9 A

10. The de Broglie wavelength of a proton and  $\alpha$ -particle are equal. The ratio of their velocities is :

- (1) 4 : 3      (2) 4 : 1      (3) 4 : 2      (4) 1 : 4

11. If one mole of an ideal gas at  $(P_1, V_1)$  is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B  $\rightarrow$  C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to :



- (1)  $RT \left( \ln 2 - \frac{1}{2(\gamma - 1)} \right)$       (2)  $-\frac{RT}{2(\gamma - 1)}$   
 (3) 0      (4)  $RT \ln 2$

12. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be :

(1)  $10^{-3}$  nm                      (2)  $10^{-1}$  nm  
(3)  $10^{-2}$  nm                      (4)  $10^{-4}$  nm

13. Which of the following equations represents a travelling wave ?

(1)  $y = A \sin(15x - 2t)$   
(2)  $y = Ae^{-x^2}(vt + \theta)$   
(3)  $y = Ae^{x \cos(\omega t - \theta)}$   
(4)  $y = A \sin x \cos \omega t$

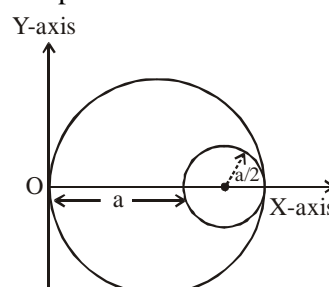
14. According to Bohr atom model, in which of the following transitions will the frequency be maximum ?

(1)  $n = 4$  to  $n = 3$                       (2)  $n = 2$  to  $n = 1$   
(3)  $n = 5$  to  $n = 4$                       (4)  $n = 3$  to  $n = 2$

15. If the source of light used in a Young's double slit experiment is changed from red to violet :

(1) consecutive fringe lines will come closer.  
(2) the central bright fringe will become a dark fringe.  
(3) the fringes will become brighter.  
(4) the intensity of minima will increase.

16. A circular hole of radius  $\left(\frac{a}{2}\right)$  is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be :



(1)  $\frac{1}{6}a$                       (2)  $\frac{10}{11}a$                       (3)  $\frac{5}{6}a$                       (4)  $\frac{2}{3}a$

Choose the correct answer from the options given below :

- (1) (a)-(vi), (b)-(iv), (c)-(i), (d)-(v)
- (2) (a)-(vi), (b)-(v), (c)-(i), (d)-(iv)
- (3) (a)-(ii), (b)-(iv), (c)-(vi), (d)-(iii)
- (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

17. Zener breakdown occurs in a  $p$ - $n$  junction having  $p$  and  $n$  both :

- (1) lightly doped and have wide depletion layer.
- (2) heavily doped and have narrow depletion layer.
- (3) lightly doped and have narrow depletion layer.
- (4) heavily doped and have wide depletion layer.

18. Match List - I with List - II.

- | List - I                          | List - II                             |
|-----------------------------------|---------------------------------------|
| (a) Source of microwave frequency | (i) Radioactive decay on nucleus      |
| (b) Source of infrared frequency  | (ii) Magnetron                        |
| (c) Source of Gamma Rays          | (iii) Inner shell electrons           |
| (d) Source of X-rays              | (iv) Vibration of atoms and molecules |
|                                   | (v) LASER                             |
|                                   | (vi) RC circuit                       |

19. A particle is projected with velocity  $v_0$  along  $x$ -axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin i.e.,  $ma = -\alpha x^2$ . The distance at which the particle stops :

- |   |   |
|---|---|
| (1) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$ | (2) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$   |
| (3) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$ | (4) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$ |

20. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator ?

(Use  $g = \frac{GM}{R^2} = 9.8 \text{ ms}^{-2}$  and radius of earth,  $R = 6400 \text{ km}$ .)

- (1) 49 N                                      (2) 48.83 N  
(3) 49.83 N                                  (4) 49.17 N

**SECTION-B**

1. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force  $F$ . The elongation, if its length and diameter is doubled and subjected to the same force will be \_\_\_\_\_ cm.

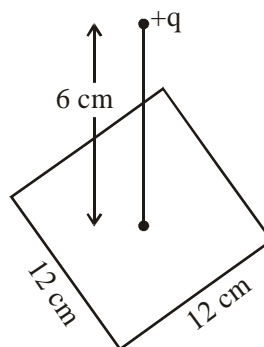
2. A cylindrical wire of radius 0.5 mm and conductivity  $5 \times 10^7 \text{ S/m}$  is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be  $x^3\pi \text{ mA}$ . The value of  $x$  is \_\_\_\_\_.

3. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is \_\_\_\_\_  $\times 10^{-1} \text{ kg m}^2$ .

4. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies  $(K.E.)_A : (K.E.)_B$  will be  $\frac{A}{1}$ , so the value of A will be \_\_\_\_.

5. The root mean square speed of molecules of a given mass of a gas at  $27^\circ\text{C}$  and 1 atmosphere pressure is  $200 \text{ ms}^{-1}$ . The root mean square speed of molecules of the gas at  $127^\circ\text{C}$  and 2 atmosphere pressure is  $\frac{x}{\sqrt{3}} \text{ ms}^{-1}$ . The value of x will be \_\_\_\_.

6. A point charge of  $+12 \mu\text{C}$  is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be \_\_\_\_  $\times 10^3 \text{ Nm}^2/\text{C}$ .



7. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is  $-5$  dB per km and cable length is 20 km. The power received at receiver is  $10^{-x}$  W. The value of  $x$  is \_\_\_\_\_.

$$[\text{Gain in dB} = 10 \log_{10} \left( \frac{P_0}{P_i} \right)]$$

8. A series LCR circuit is designed to resonate at an angular frequency  $\omega_0 = 10^5$  rad/s. The circuit draws 16 W power from 120 V source at resonance. The value of resistance 'R' in the circuit is \_\_\_\_  $\Omega$ .

9. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be \_\_\_\_ Hz. [Velocity of sound in air is 340 m/s.]

10. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be \_\_\_\_  $\times 10^{-2}$  cm.

= 1



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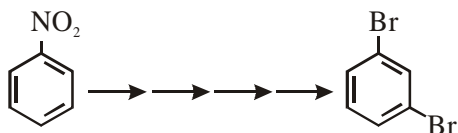
(Held On Wednesday 24<sup>th</sup> February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

## CHEMISTRY

## TEST PAPER WITH SOLUTION

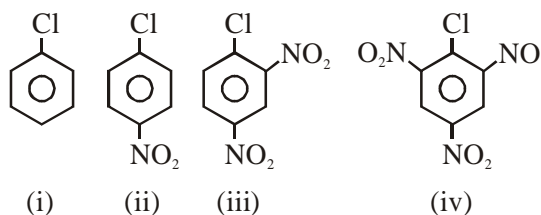
### SECTION-A

1. What is the correct sequence of reagents used for converting nitrobenzene into *m*-dibromobenzene ?



- (1)  $\xrightarrow{\text{NaNO}_2} / \xrightarrow{\text{HCl}} / \xrightarrow{\text{KBr}} / \xrightarrow{\text{H}^+}$   
 (2)  $\xrightarrow{\text{Br}_2/\text{Fe}} / \xrightarrow{\text{Sn/HCl}} / \xrightarrow{\text{NaNO}_2/\text{HCl}} / \xrightarrow{\text{CuBr/HBr}}$   
 (3)  $\xrightarrow{\text{Sn/HCl}} / \xrightarrow{\text{KBr}} / \xrightarrow{\text{Br}_2} / \xrightarrow{\text{H}^+}$   
 (4)  $\xrightarrow{\text{Sn/HCl}} / \xrightarrow{\text{Br}_2} / \xrightarrow{\text{NaNO}_2} / \xrightarrow{\text{NaBr}}$

3. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is :-



- (1) (iv) < (iii) < (ii) < (i)  
 (2) (iv) < (i) < (ii) < (iii)  
 (3) (iv) < (i) < (iii) < (ii)  
 (4) (i) < (ii) < (iii) < (iv)

2. Most suitable salt which can be used for efficient clotting of blood will be :-

- (1)  $\text{NaHCO}_3$  (2)  $\text{FeSO}_4$   
 (3)  $\text{Mg}(\text{HCO}_3)_2$  (4)  $\text{FeCl}_3$

4. According to Bohr's atomic theory :-

(A) Kinetic energy of electron is  $\propto \frac{Z^2}{n^2}$ .

(B) The product of velocity (v) of electron and principal quantum number (n), 'vn'  $\propto Z^2$ .

(C) Frequency of revolution of electron in an orbit is  $\propto \frac{Z^3}{n^3}$ .

(D) Coulombic force of attraction on the electron is  $\propto \frac{Z^3}{n^4}$ .

Choose the most appropriate answer from the options given below :

- (1) (C) Only
- (2) (A) Only
- (3) (A), (C) and (D) only
- (4) (A) and (D) only

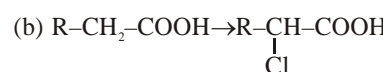
5. Match list - I and List - II.

**List-I**

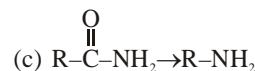
**List-II**



(i)  $\text{Br}_2/\text{NaOH}$



(ii)  $\text{H}_2/\text{Pd}-\text{BaSO}_4$



(iii)  $\text{Zn(Hg)/Conc.HCl}$



(iv)  $\text{Cl}_2/\text{Red P, H}_2\text{O}$

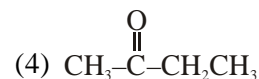
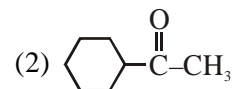
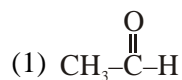
Choose the correct answer from the options given below :

- (1) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)
- (2) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)
- (3) (a)–(ii), (b)–(iv), (c)–(i), (d)–(iii)
- (4) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

6. The calculated magnetic moments (spin only value) for species  $[\text{FeCl}_4]^{2-}$ ,  $[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}$  and  $\text{MnO}_4^{2-}$  respectively are :

- (1) 5.82, 0 and 0 BM
- (2) 4.90, 0 and 1.73 BM
- (3) 5.92, 4.90 and 0 BM
- (4) 4.90, 0 and 2.83 BM

8. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of  $\text{HgSO}_4$  and  $\text{H}_2\text{SO}_4$  ?



7. Match List-I with List-II :

List-I (Salt)		List-II (Flame colour wavelength)	
(a)	LiCl	(i)	455.5 nm
(b)	NaCl	(ii)	670.8 nm
(c)	RbCl	(iii)	780.0 nm
(d)	CsCl	(iv)	589.2 nm

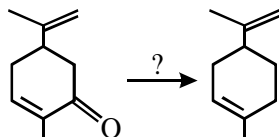
Choose the correct answer from the options given below :

- (1) (a)–(iv), (b)–(ii), (c)–(iii), (d)–(i)  
 (2) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)  
 (3) (a)–(i), (b)–(iv), (c)–(ii), (d)–(iii)  
 (4) (a)–(ii), (b)–(iv), (c)–(iii), (d)–(i)

9. In polymer Buna-S: 'S' stands for :-

- (1) Sulphonation                      (2) Strength  
 (3) Sulphur                              (4) Styrene

10.



Which of the following reagent is suitable for the preparation of the product in the above reaction ?

- (1)  $\text{NaBH}_4$
- (2)  $\text{NH}_2\text{-NH}_2 / \text{C}_2\text{H}_5\text{ONa}^\ominus$
- (3)  $\text{Ni/H}_2$
- (4) Red P +  $\text{Cl}_2$

11. Match List-I and List-II.

**List-I**

- (a) Valium
- (b) Morphine
- (c) Norethindrone
- (d) Vitamin  $\text{B}_{12}$

**List-II**

- (i) Antifertility drug
- (ii) Pernicious anaemia
- (iii) Analgesic
- (iv) Tranquilizer

- (1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)
- (2) (a)–(iv), (b)–(iii), (c)–(i), (d)–(ii)
- (3) (a)–(ii), (b)–(iv), (c)–(iii), (d)–(i)
- (4) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

12. Match List-I with List-II.

**List-I  
(Metal)**

- (a) Aluminium
- (b) Iron
- (c) Copper
- (d) Zinc

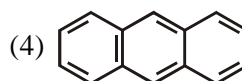
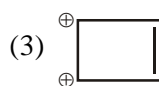
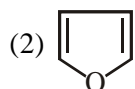
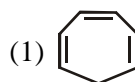
**List-II  
(Ores)**

- (i) Siderite
- (ii) Calamine
- (iii) Kaolinite
- (iv) Malachite

Choose the correct answer from the options given below :

- (1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)
- (2) (a)–(ii), (b)–(iv), (c)–(i), (d)–(iii)
- (3) (a)–(i), (b)–(ii), (c)–(iii), (d)–(iv)
- (4) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

13. Which one of the following compounds is non-aromatic ?



14. What is the correct order of the following elements with respect to their density ?

- (1)  $\text{Cr} < \text{Zn} < \text{Co} < \text{Cu} < \text{Fe}$
- (2)  $\text{Zn} < \text{Cu} < \text{Co} < \text{Fe} < \text{Cr}$
- (3)  $\text{Zn} < \text{Cr} < \text{Fe} < \text{Co} < \text{Cu}$
- (4)  $\text{Cr} < \text{Fe} < \text{Co} < \text{Cu} < \text{Zn}$

15. Given below are two statements :-

**Statement I :** The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

**Statement II :** The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Statement I is false but Statement II is true
- (2) Both Statement I and Statement II are true
- (3) Statement I is true but Statement II is false
- (4) Both Statement I and Statement II are false

16. The **incorrect** statement among the following is :-

- (1)  $\text{VO}_2^+$  is a reducing agent
- (2)  $\text{Cr}_2\text{O}_3$  is an amphoteric oxide
- (3)  $\text{RuO}_4$  is an oxidizing agent
- (4) Red colour of ruby is due to the presence of  $\text{Co}^{3+}$

17. The correct shape and I-I-I bond angles respectively in  $\text{I}_3^-$  ion are :-

- (1) Distorted trigonal planar;  $135^\circ$  and  $90^\circ$
- (2) T-shaped;  $180^\circ$  and  $90^\circ$
- (3) Trigonal planar;  $120^\circ$
- (4) Linear;  $180^\circ$

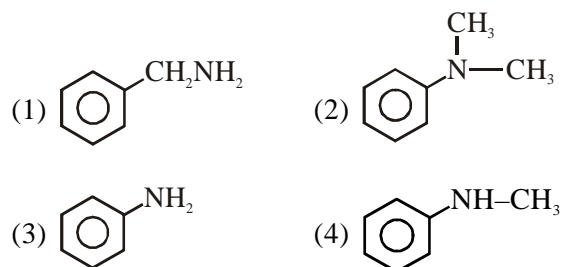
18. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A :** Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

**Reason R :** Hydrogen is the lightest element. In the light of the above statements, choose the correct answer from the options given below :

- (1) **A** is true but **R** is false
- (2) Both **A** and **R** are true and **R** is the correct explanation of **A**
- (3) **A** is false but **R** is true
- (4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

19. The diazonium salt of which of the following compounds will form a coloured dye on reaction with  $\beta$ -Naphthol in  $\text{NaOH}$  ?

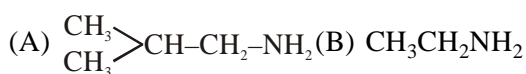


20. The correct set from the following in which both pairs are in correct order of melting point is :-

- (1)  $\text{LiF} > \text{LiCl}$  ;  $\text{MgO} > \text{NaCl}$
- (2)  $\text{LiCl} > \text{LiF}$  ;  $\text{NaCl} > \text{MgO}$
- (3)  $\text{LiF} > \text{LiCl}$  ;  $\text{NaCl} > \text{MgO}$
- (4)  $\text{LiCl} > \text{LiF}$  ;  $\text{MgO} > \text{NaCl}$

### SECTION-B

1. The total number of amines among the following which can be synthesized by Gabriel synthesis is \_\_\_\_\_.



2. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is \_\_\_\_\_.

- (A)  $\alpha$ -sulphur
- (B)  $\beta$ -sulphur
- (C)  $\text{S}_2$ -form

3. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of  $\text{O}_2$  for complete oxidation and produces 4 times its own volume of  $\text{CO}_2$  is  $\text{C}_x\text{H}_y$ . The value of y is \_\_\_\_\_.

4. The volume occupied by 4.75 g of acetylene gas at  $50^\circ\text{C}$  and 740 mmHg pressure is \_\_\_\_\_ L. (Rounded off to the nearest integer)  
[Given  $R = 0.0826 \text{ L atm K}^{-1} \text{ mol}^{-1}$ ]

5.  $\text{C}_6\text{H}_6$  freezes at  $5.5^\circ\text{C}$ . The temperature at which a solution 10 g of  $\text{C}_4\text{H}_{10}$  in 200 g of  $\text{C}_6\text{H}_6$  freeze is \_\_\_\_\_  $^\circ\text{C}$ . (The molal freezing point depression constant of  $\text{C}_6\text{H}_6$  is  $5.12^\circ\text{C/m}$ .)
6. The magnitude of the change in oxidising power of the  $\text{MnO}_4^- / \text{Mn}^{2+}$  couple is  $x \times 10^{-4} \text{ V}$ , if the  $\text{H}^+$  concentration is decreased from 1 M to  $10^{-4} \text{ M}$  at  $25^\circ\text{C}$ . (Assume concentration of  $\text{MnO}_4^-$  and  $\text{Mn}^{2+}$  to be same on change in  $\text{H}^+$  concentration). The value of  $x$  is \_\_\_\_\_. (Rounded off to the nearest integer)
- [Given :  $\frac{2.303 RT}{F} = 0.059$ ]
7. The solubility product of  $\text{PbI}_2$  is  $8.0 \times 10^{-9}$ . The solubility of lead iodide in 0.1 molar solution of lead nitrate is  $x \times 10^{-6} \text{ mol/L}$ . The value of  $x$  is \_\_\_\_\_. (Rounded off to the nearest integer)
- [Given :  $\sqrt{2} = 1.41$ ]

8. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is  $f$ . The

value of  $\log_{10}\left(\frac{1}{f}\right)$  is \_\_\_\_\_  $\times 10^{-2}$ . (Rounded

off to the nearest integer)

[Assume :  $\ln 10 = 2.303$ ,  $\ln 2 = 0.693$ ]

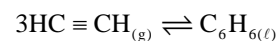
9. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is \_\_\_\_\_  $\times 10^{-2}$ .

**Official Ans. by NTA (243)**

M = 98

M = 135

10. Assuming ideal behaviour, the magnitude of  $\log K$  for the following reaction at 25°C is  $x \times 10^{-1}$ . The value of  $x$  is \_\_\_\_\_. (Integer answer)



[Given:  $\Delta_f G^\circ(\text{HC} \equiv \text{CH}) = -2.04 \times 10^5 \text{ J mol}^{-1}$ ;  $\Delta_f G^\circ(\text{C}_6\text{H}_6) = -1.24 \times 10^5 \text{ J mol}^{-1}$ ;  $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ ]



# FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24<sup>th</sup> February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

## MATHEMATICS

### SECTION-A

## TEST PAPER WITH SOLUTION

1. For the statements p and q, consider the following compound statements :

(a)  $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b)  $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.

(2) (a) and (b) both are tautologies.

(3) (a) is a tautology but not (b).

(4) (b) is a tautology but not (a).

3. The vector equation of the plane passing through the intersection of the planes  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$  and  $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$ , and the point (1, 0, 2) is :

(1)  $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

(2)  $\vec{r} \cdot (3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(3)  $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(4)  $\vec{r} \cdot (\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

2. Let a, b  $\in$  R. If the mirror image of the point P(a, 6, 9) with respect to the line

$$\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9} \text{ is } (20, b, -a-9), \text{ then } |a+b|$$

is equal to :

(1) 88

(2) 86

(3) 84

(4) 90

4. If P is a point on the parabola  $y = x^2 + 4$  which is closest to the straight line  $y = 4x - 1$ , then the co-ordinates of P are :

(1) (3, 13)                      (2) (1, 5)  
(3) (-2, 8)                      (4) (2, 8)

5. The angle of elevation of a jet plane from a point A on the ground is  $60^\circ$ . After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to  $30^\circ$ . If the jet plane is flying at a constant height, then its height is :

(1)  $1800\sqrt{3}$  m                      (2)  $3600\sqrt{3}$  m  
(3)  $2400\sqrt{3}$  m                      (4)  $1200\sqrt{3}$  m

6. If  $n \geq 2$  is a positive integer, then the sum of the series  ${}^{n+1}C_2 + 2({}^nC_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$  is:

(1)  $\frac{n(n-1)(2n+1)}{6}$                       (2)  $\frac{n(n+1)(2n+1)}{6}$   
(3)  $\frac{n(2n+1)(3n+1)}{6}$                       (4)  $\frac{n(n+1)^2(n+2)}{12}$

7. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be defined as,

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let  $A = \{x \in \mathbf{R} : f \text{ is increasing}\}$ . Then  $A$  is equal to :

- (1)  $(-\infty, -5) \cup (4, \infty)$
- (2)  $(-5, \infty)$
- (3)  $(-\infty, -5) \cup (-4, \infty)$
- (4)  $(-5, -4) \cup (4, \infty)$

8. Let  $f$  be a twice differentiable function defined on  $\mathbf{R}$  such that  $f(0) = 1$ ,  $f'(0) = 2$  and  $f'(x) \neq 0$  for

all  $x \in \mathbf{R}$ . If  $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$ , for all  $x \in \mathbf{R}$ , then

the value of  $f(1)$  lies in the interval:

- (1)  $(9, 12)$
- (2)  $(6, 9)$
- (3)  $(0, 3)$
- (4)  $(3, 6)$

9. For which of the following curves, the line  $x + \sqrt{3}y = 2\sqrt{3}$  is the tangent at the point

$$\left( \frac{3\sqrt{3}}{2}, \frac{1}{2} \right) ?$$

- (1)  $x^2 + y^2 = 7$
- (2)  $y^2 = \frac{1}{6\sqrt{3}}x$
- (3)  $2x^2 - 18y^2 = 9$
- (4)  $x^2 + 9y^2 = 9$

10. The value of the integral,  $\int_1^3 [x^2 - 2x - 2] dx$ ,

where  $[x]$  denotes the greatest integer less than or equal to  $x$ , is :

- (1)  $-\sqrt{2} - \sqrt{3} + 1$  (2)  $-\sqrt{2} - \sqrt{3} - 1$   
(3)  $-5$  (4)  $-4$

11. A possible value of  $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$  is :

- (1)  $\frac{1}{\sqrt{7}}$  (2)  $2\sqrt{2} - 1$   
(3)  $\sqrt{7} - 1$  (4)  $\frac{1}{2\sqrt{2}}$

12. The negative of the statement  $\sim p \wedge (p \vee q)$  is

- (1)  $\sim p \vee q$  (2)  $p \vee \sim q$   
(3)  $\sim p \wedge q$  (4)  $p \wedge \sim q$

13. If the curve  $y = ax^2 + bx + c$ ,  $x \in \mathbb{R}$ , passes through the point  $(1, 2)$  and the tangent line to this curve at origin is  $y = x$ , then the possible values of  $a$ ,  $b$ ,  $c$  are :

- (1)  $a = \frac{1}{2}$ ,  $b = \frac{1}{2}$ ,  $c = 1$   
(2)  $a = 1$ ,  $b = 0$ ,  $c = 1$   
(3)  $a = 1$ ,  $b = 1$ ,  $c = 0$   
(4)  $a = -1$ ,  $b = 1$ ,  $c = 1$

14. The area of the region :

$R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$  is :

- (1)  $11\sqrt{3}$  square units  
(2)  $12\sqrt{3}$  square units  
(3)  $9\sqrt{3}$  square units  
(4)  $6\sqrt{3}$  square units

15. If a curve  $y = f(x)$  passes through the point

$(1, 2)$  and satisfies  $x \frac{dy}{dx} + y = bx^4$ , then for what

value of  $b$ ,  $\int_1^2 f(x) dx = \frac{62}{5}$  ?

(1) 5 (2) 10

(3)  $\frac{62}{5}$  (4)  $\frac{31}{5}$

16. Let  $f(x)$  be a differentiable function defined on  $[0, 2]$  such that  $f'(x) = f'(2 - x)$  for all  $x \in (0, 2)$ ,

$f(0) = 1$  and  $f(2) = e^2$ . Then the value of  $\int_0^2 f(x) dx$

is :

(1)  $1 - e^2$

(2)  $1 + e^2$

(3)  $2(1 - e^2)$

(4)  $2(1 + e^2)$

17. Let  $A$  and  $B$  be  $3 \times 3$  real matrices such that  $A$  is symmetric matrix and  $B$  is skew-symmetric matrix. Then the system of linear equations  $(A^2B^2 - B^2A^2)X = O$ , where  $X$  is a  $3 \times 1$  column matrix of unknown variables and  $O$  is a  $3 \times 1$  null matrix, has :

(1) no solution

(2) exactly two solutions

(3) infinitely many solutions

(4) a unique solution

18. Let  $a, b, c$  be in arithmetic progression. Let the centroid of the triangle with vertices  $(a, c)$ ,  $(2, b)$  and  $(a, b)$  be  $\left(\frac{10}{3}, \frac{7}{3}\right)$ . If  $\alpha, \beta$  are the roots of the equation  $ax^2 + bx + 1 = 0$ , then the value of  $\alpha^2 + \beta^2 - \alpha\beta$  is :
- (1)  $\frac{71}{256}$  (2)  $\frac{69}{256}$   
 (3)  $-\frac{69}{256}$  (4)  $-\frac{71}{256}$

19. For the system of linear equations :  
 $x - 2y = 1$ ,  $x - y + kz = -2$ ,  $ky + 4z = 6$ ,  $k \in \mathbb{R}$ ,  
 consider the following statements :  
 (A) The system has unique solution if  $k \neq 2$ ,  
 $k \neq -2$ .  
 (B) The system has unique solution if  $k = -2$ .  
 (C) The system has unique solution if  $k = 2$ .  
 (D) The system has no-solution if  $k = 2$ .  
 (E) The system has infinite number of solutions  
 if  $k \neq -2$ .  
 Which of the following statements are correct ?  
 (1) (C) and (D) only  
 (2) (B) and (E) only  
 (3) (A) and (E) only  
 (4) (A) and (D) only

20. The probability that two randomly selected subsets of the set  $\{1, 2, 3, 4, 5\}$  have exactly two elements in their intersection, is :
- (1)  $\frac{65}{2^7}$  (2)  $\frac{65}{2^8}$   
 (3)  $\frac{135}{2^9}$  (4)  $\frac{35}{2^7}$

### SECTION-B

1. For integers  $n$  and  $r$ , let  $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$
- The maximum value of  $k$  for which the sum  
 $\sum_{i=0}^k \binom{10}{i} \binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+1-i}$  exists, is  
 equal to \_\_\_\_\_.

2. Let  $\lambda$  be an integer. If the shortest distance between the lines  $x - \lambda = 2y - 1 = -2z$  and  $x = y + 2\lambda = z - \lambda$  is  $\frac{\sqrt{7}}{2\sqrt{2}}$ , then the value of  $|\lambda|$  is \_\_\_\_\_.
3. If  $a + \alpha = 1$ ,  $b + \beta = 2$  and  $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ,  $x \neq 0$ , then the value of expression  $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$  is \_\_\_\_\_.
4. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (-5, 0). If the locus of the point P is a circle of radius r, then  $4r^2$  is equal to \_\_\_\_\_.

5. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle  $(x - 2)^2 + (y - 3)^2 = 25$  at the point (5, 7) is A, then 24A is equal to\_\_\_\_\_.
6. If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is \_\_\_\_\_.
7. The sum of first four terms of a geometric progression (G.P.) is  $\frac{65}{12}$  and the sum of their respective reciprocals is  $\frac{65}{18}$ . If the product of first three terms of the G.P. is 1, and the third term is  $\alpha$ , then  $2\alpha$  is \_\_\_\_\_.
8. The students  $S_1, S_2, \dots, S_{10}$  are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is \_\_\_\_\_.



9. Let  $i = \sqrt{-1}$ . If  $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$ , and  $n = [k]$  be the greatest integral part of  $|k|$ . Then  $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$  is equal to \_\_\_\_\_.

10. The number of the real roots of the equation  $(x+1)^2 + |x-5| = \frac{27}{4}$  is \_\_\_\_\_.