FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Friday 26th February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTIONS

SECTION-A

1. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ, where $\frac{C}{V} = \lambda$?

(1) $[M^{-2}L^{-3}I^{2}T^{6}]$ (2) $[M^{-3}L^{-4}I^{3}T^{7}]$

(3) $[M^{-1}L^{-3}I^{-2}T^{-7}]$ (4) $[M^{-2}L^{-4}I^{3}T^{7}]$

2. The length of metallic wire is ℓ_1 when tension in it is T_1. It is ℓ_2 when the tension is T_2. The original length of the wire will be -

(1) $\frac{\ell_1 + \ell_2}{2}$ (2) $\frac{T_2\ell_1 + T_1\ell_2}{T_1 + T_2}$

(3) $\frac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$ (4) $\frac{T_1\ell_1 - T_2\ell_2}{T_2 - T_1}$

3. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5×10^{-4} Wb/m^2 and the angle of dip is 60°. The emf induced between the tips of the plane wings will be :-

(1) 108.25 mV (2) 54.125 mV

(3) 88.37 mV (4) 62.50 mV

4. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A ?

(1) 342 Hz (2) 345 Hz

(3) 335 Hz (4) 338 Hz

5. A particle executes S.H.M., the graph of velocity as a function of displacement is :-

(1) A circle (2) A parabola

(3) An ellipse (4) A helix

6. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by :-

(1) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$ (2) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

(3) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$ (4) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$

7. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be :-

(1) $\frac{2mgh}{I+2mr^2}$ (2) $\frac{2mgh}{I+mr^2}$

(3) $2gh$ (4) $\frac{2gh}{I+mr^2}$

8. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as $U = 3PV + 4$. The gas is :-

(1) Diatomic only
(2) Polyatomic only
(3) Either monoatomic or diatomic
(4) Monoatomic only

9. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is true but R is false
(2) Both A and R are true but R is NOT the correct explanation of A.
(3) Both A and R are true and R is the correct explanation of A
(4) A is false but R is true

10. Given below are two statements :

Statement I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

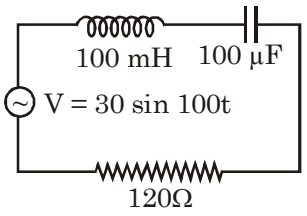
(4) Statement I is false but Statement II is true.

11. The recoil speed of a hydrogen atom after it emits a photon in going from n = 5 state to n = 1 state will be :-

(1) 4.17 m/s (2) 2.19 m/s

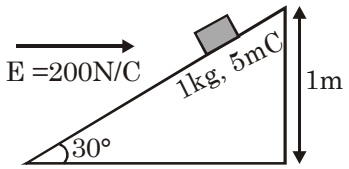
(3) 3.25 m/s (4) 4.34 m/s

12. Find the peak current and resonant frequency of the following circuit (as shown in figure).



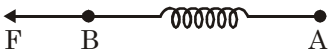
(1) 0.2 A and 50 Hz (2) 0.2 A and 100 Hz
(3) 2 A and 100 Hz (4) 2A and 50 Hz

13. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\frac{N}{C}$ as shown in the figure. A body of mass 1kg and charge 5 mC is allowed to slide down from rest at a height of 1m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.[g = 9.8 m/s^2, sin 30° = $\frac{1}{2}$; cos 30° = $\frac{\sqrt{3}}{2}$]



(1) 0.92 s (2) 0.46 s
(3) 2.3 s (4) 1.3 s

14. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B wil be :-



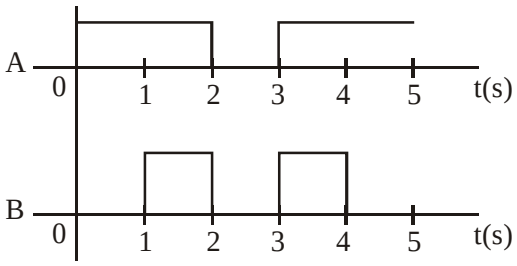
(1) $\frac{Ma - F}{M}$

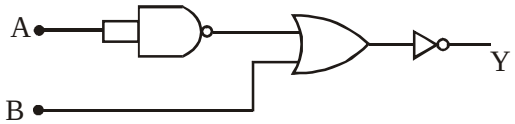
(2) $\frac{MF}{F + Ma}$

(3) $\frac{F + Ma}{M}$

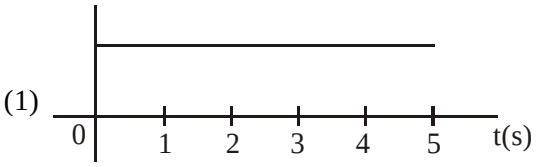
(4) $\frac{F - Ma}{M}$

15. Draw the output signal Y in the given combination of gates :-

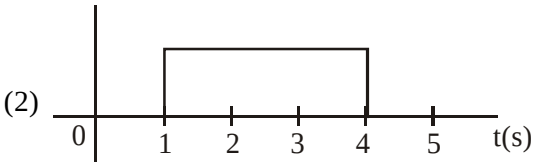




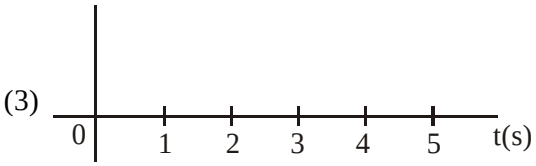
(1)



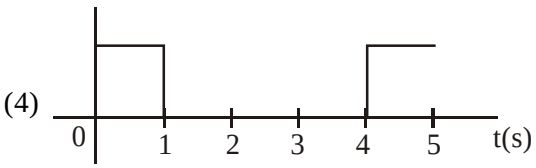
(2)



(3)



(4)



16. A radioactive sample is undergoing α decay. At any time t_1, its activity is A and another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample ?

(1) $\frac{\ell n5}{t_2 - t_1}$ (2) $\frac{t_1 - t_2}{\ell n5}$

(3) $\frac{t_2 - t_1}{\ell n5}$ (4) $\frac{\ell n(t_2 + t_1)}{2}$

17. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be :-

(1) $\frac{a_1 + a_2}{a_2}$ (2) $\frac{a_2}{a_1}$ (3) $\frac{a_1}{a_2}$ (4) $\frac{a_1 + a_2}{a_1}$

18. Given below are two statements :

Statement I : A second's pendulum has a time period of 1 second.

Statement II : It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false.

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true.

19. A wire of 1Ω has a length of 1m. It is stetched till its length increases by 25%. The percentage change in resistance to the neartest integer is :-

(1) 56% (2) 25% (3) 12.5% (4) 76%

20. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

(1) $\vec{b} = \vec{a} + 2\vec{c}$ (2) $\vec{b} = 2\vec{a} + \vec{c}$

(3) $\vec{b} = \vec{a} - 2(\vec{a}.\vec{c})\vec{c}$ (4) $\vec{b} = \vec{a} - \vec{c}$

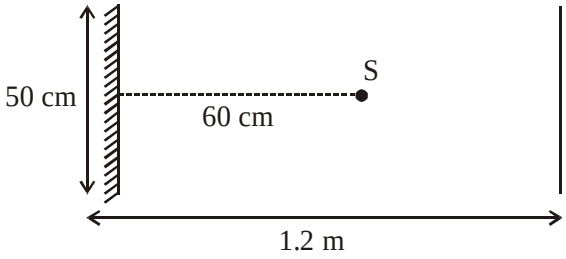
SECTION-B

1. The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = KT^{2/3}$. The workdone when temperature changes by 90 K will be xR. The value of x is [R = universal gas constant]

2. If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are .....

3. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is x : y. The value of x is .....

4. A point source of light S, placed at a distance 60 cm infront of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks infront of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is ..... cm.

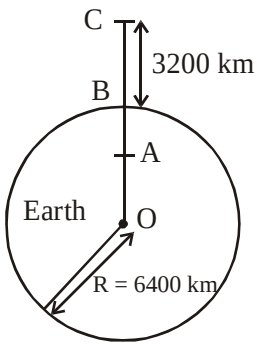


5. A particle executes S.H.M. with amplitude 'a' and time period V. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of x is .....

6. 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is ..... times that of a smaller drop.

7. Time period of a simple pendulum is T. The time taken to complete 5/8 oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is .....

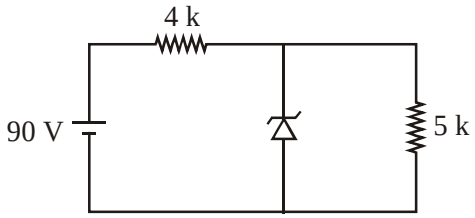
8. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is .....



9. 1 mole of rigid diatomic gas performs a work of Q/5 when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$, The value of x is .....

[K = universal gas constant]

10. The zener diode has a V_z = 30 V. The current passing through the diode for the following circuit is ..... mA.



FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Friday 26th February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Which of the following forms of hydrogen emits low energy β^- particles?

(1) Deuterium $^{2}_{1}H$ (2) Tritium $^{3}_{1}H$

(3) Protium $^{1}_{1}H$ (4) Proton H^+

2. Given below are two statements :one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : In $T\ell I_3$, isomorphous to CsI_3, the metal is present in +1 oxidation state.

Reason R : $T\ell$ metal has fourteen f electrons in the electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) **A** is correct but **R** is not correct

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(3) **A** is not correct but **R** is correct

(4) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

3. Match List-I with List-II

List-I	**List-II**
(a) Sucrose	(i) β-D-Galactose and β-D-Glucose
(b) Lactose	(ii) α-D-Glucose and β-D-Fructose
(c) Maltose	(iii) α-D-Glucose and α-D-Glucose

Choose the correct answer from the options given below :

Options :

(1) (a) → (i), (b) → (iii), (c) → (ii)

(2) (a) → (iii), (b) → (i), (c) → (iii)

(3) (a) → (ii), (b) → (i), (c) → (iii)

(4) (a) → (iii), (b) → (ii), (c) → (i)

4. A. Phenyl methanamine

B. N,N-Dimethylaniline

C. N-Methyl aniline

D. Benzenamine

Choose the correct order of basic nature of the above amines.

(1) A > C > B > D (2) D > C > B > A

(3) D > B > C > A (4) A > B > C > D

5. The correct order of electron gain enthalpy is

(1) S > Se > Te > O (2) Te > Se > S > O

(3) O > S > Se > Te (4) S > O > Se > Te

6. In $\overset{1}{CH_2}=\overset{2}{C}=\overset{3}{C}H-\overset{4}{C}H_3$ molecule, the hybridization of carbon 1,2,3 and 4 respectively are :

(1) sp^3, sp, sp^3, sp^3 (2) sp^2, sp^2, sp^2, sp^3

(3) sp^2, sp, sp^2, sp^3 (4) sp^2, sp^3, sp^2, sp^3

7. Seliwanoff test and Xanthoproteic test are used for the identification of _____and _____ respectively

(1) Aldoses, ketoses (2) Proteins, ketoses

(3) Ketoses, proteins (4) Ketoses, aldoses

8. 2,4-DNP test can be used to identify :

(1) Amine (2) Aldehyde

(3) Ether (4) Halogens

9. Ceric ammonium nitrate and $CHCl_3$ / alc. KOH are used for the identification of functional groups present in _____ and ___ respectively.

(1) Alcohol, phenol (2) Amine, alcohol

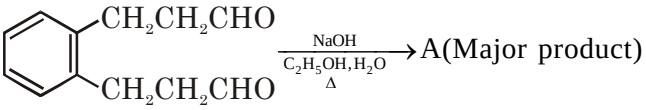
(3) Alcohol, amine (4) Amine, phenol

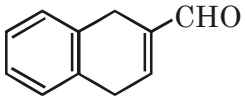
10. Which pair of oxides is acidic in nature?

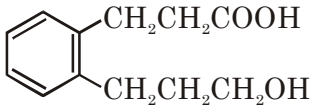
(1) B_2O_3, CaO (2) B_2O_3, SiO_2

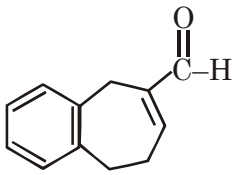
(3) N_2O, BaO (4) CaO, SiO_2

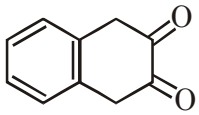
11. Identify A in the given chemical reaction,



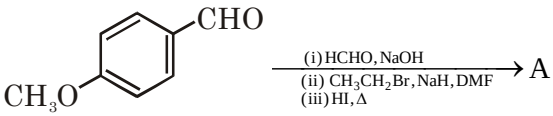
(1) 

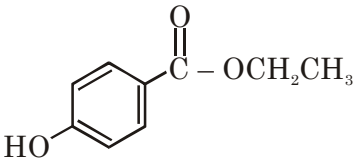
(2) 

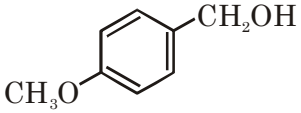
(3) 

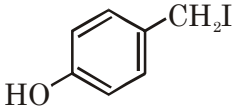
(4) 

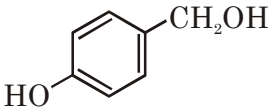
12. Identify A in the following chemical reaction



(1) 

(2) 

(3) 

(4) 

13. Calgon is used for water treatment. Which of the following statement is NOT true about Calgon?

(1) Calgon contains the 2nd most abundant element by weight in the Earth's crust.

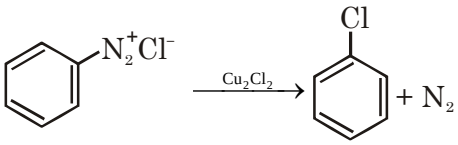
(2) It is polymeric compound and is water soluble.

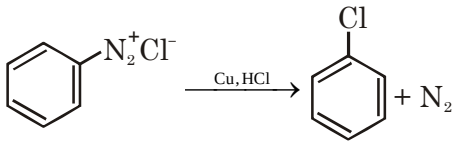
(3) It is also known as Graham's salt

(4) It does not remove Ca^{2+} ion by precipitation.

14. Match List-I with List-II

List-I

(a) 

(b) 

(c) $2CH_3CH_2Cl + 2Na \xrightarrow{Ether} C_2H_5–C_2H_5 + 2NaCl$

(d) $2C_6H_5Cl + 2Na \xrightarrow{Ether} C_6H_5–C_6H_5 + 2NaCl$

List-II

(i) Wurtz reaction

(ii) Sandmeyer reaction

(iii) Fittig reaction

(iv) Gatterman reaction

Choose the correct answer from the options given below :

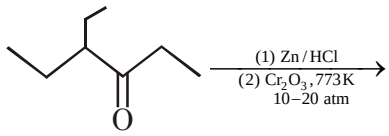
(1) (a) → (iii), (b) → (i), (c) → (iv), (d) → (ii)

(2) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

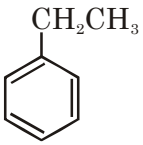
(3) (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)

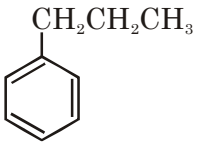
(4) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

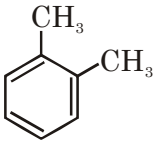
15.

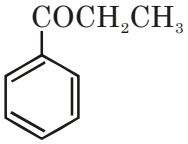


considering the above reaction, the major product among the following is :

(1) 

(2) 

(3) 

(4) 

16. Match List-I with List-II.

	List-I (Molecule)		**List-II (Bond order)**
(a)	Ne_2	(i)	1
(b)	N_2	(ii)	2
(c)	F_2	(iii)	0
(d)	O_2	(iv)	3

Choose the correct answer from the options given below :

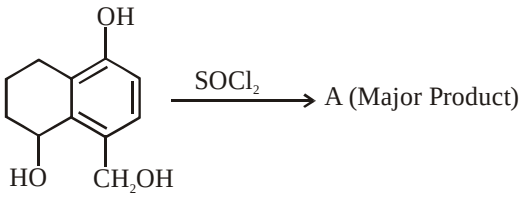
(1) (a)→(iii), (b)→(iv), (c)→(i), (d)→(ii)

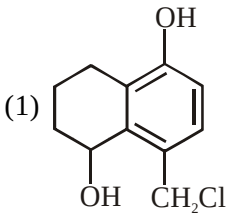
(2) (a)→(i), (b)→(ii), (c)→(iii), (d)→(iv)

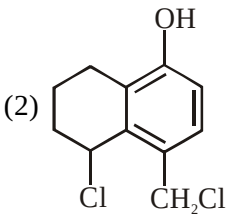
(3) (a)→(ii), (b)→(i), (c)→(iv), (d)→(iii)

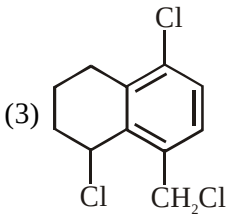
(4) (a)→(iv), (b)→(iii), (c)→(ii), (d)→(i)

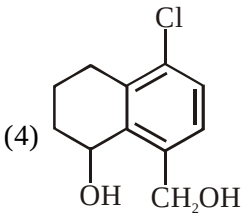
17. Identify A in the given reaction.



(1) 

(2) 

(3) 

(4) 

18. Match List-I with List-II.

	List-I		List-II
(a)	Siderite	(i)	Cu
(b)	Calamine	(ii)	Ca
(c)	Malachite	(iii)	Fe
(d)	Cryolite	(iv)	Al
		(v)	Zn

Choose the correct answer from the options given below :

(1) (a)→(iii), (b)→(i), (c)→(v), (d)→(ii)
(2) (a)→(i), (b)→(ii), (c)→(v), (d)→(iii)
(3) (a)→(iii), (b)→(v), (c)→(i), (d)→(iv)
(4) (a)→(i), (b)→(ii), (c)→(iii), (d)→(iv)

19. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is :

(1) Positive
(2) Sometimes positive and sometimes negative
(3) Neutral
(4) Negative

20. Match List-I with List-II.

	List-I		List-II
(a)	Sodium Carbonate	(i)	Deacon
(b)	Titanium	(ii)	Castner-Kellner
(c)	Chlorine	(iii)	Van-Arkel
(d)	Sodium hydroxide	(iv)	Solvay

Choose the correct answer from the options given below :

(1) (a)→(iv), (b)→(iii), (c)→(i), (d)→(ii)
(2) (a)→(i), (b)→(iii), (c)→(iv), (d)→(ii)
(3) (a)→(iv), (b)→(i), (c)→(ii), (d)→(iii)
(4) (a)→(iii), (b)→(ii), (c)→(i), (d)→(iv)

SECTION-II

1. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is _____g. (Rounded off to the nearest integer)

[Given : Atomic weight in g mol^{-1} – Na : 23 ; N : 14 ; O : 16]

2. Emf of the following cell at 298 K in V is $x \times 10^{-2}$. $Zn|Zn^{2+}$ (0.1 M)$||Ag^+$ (0.01 M)| Ag

The value of x is _____. (Rounded off to the nearest integer)

[Given : $E^0_{Zn^{2+}/Zn} = -0.76V$; $E^0_{Ag^+/Ag} = +0.80V$; $\frac{2.303RT}{F} = 0.059$]

3. When 12.2 g of benzoic acid is dissolved in 100 g of water, the freezing point of solution was found to be –0.93°C ($K_f(H_2O)$ = 1.86K kg mol^{-1}). The number (n) of benzoic acid molecules associated (assuming 100% association) is ____.

4. The average S–F bond energy in kJ mol^{-1} of SF_6 is _____.(Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of SF_6(g), S(g) and F(g) are –1100, 275 and 80 kJ mol^{-1} respectively.]

5. A ball weighing 10 g is moving with a velocity of 90 ms^{-1}. If the uncertainty in its velocity is 5%, then the uncertainty in its position is_____$\times 10^{-33}$ m. (Rounded off to the nearest integer)

[Given : h = 6.63 × 10^{-34} Js]

6. The number of octahedral voids per lattice site in a lattice is _____.(Rounded off to the nearest integer)

7. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is _____.

8. The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is _____.[ox = oxalate]

9. If the activation energy of a reaction is 80.9 kJ mol^{-1}, the fraction of molecules at 700 K, having enough energy to react to form products is e^{-x}. The value of x is _____.
(Rounded off to the nearest integer)
[Use R = 8.31 J K^{-1} mol^{-1}]

10. The pH of ammonium phosphate solution, if pK_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is _____.

FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Friday 26th February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is

(1) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$ (2) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

(3) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (4) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

2. Let A = {1, 2, 3, ..., 10} and $f : A \to A$ be defined as $f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$

Then the number of possible functions $g : A \to A$ such that $gof = f$ is

(1) 10^5 (2) ${}^{10}C_5$ (3) 5^5 (4) 5!

3. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$$

If $f(x)$ is continuous on R, then a + b equals:

(1) –3 (2) –1 (3) 3 (4) 1

4. For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to

(1) 1 (2) –1 (3) $\frac{1}{2}$ (4) 0

5. A natural number has prime factorization given by $n = 2^x3^y5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :
(1) 11 (2) 6 (3) 6x (4) 12

6. Let $f(x) = \sin^{-1}x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

(2) $(-\infty, -2] \cup [-1, \infty)$

(3) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

(4) $(-\infty, -1] \cup [2, \infty)$

7. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :
(1) An isosceles triangle with base equal to 2r.
(2) An equilateral triangle of height $\frac{2r}{3}$.
(3) An equilateral triangle having each of its side of length $\sqrt{3}r$.
(4) A right angle triangle having two of its sides of length 2r and r.

8. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

(1) 142 (2) 68 (3) 136 (4) 102

9. Let $F_1(A,B,C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

(1) F_1 and F_2 both are tautologies

(2) F_1 is a tautology but F_2 is not a tautology

(3) F_1 is not tautology but F_2 is a tautology

(4) Both F_1 and F_2 are not tautologies

10. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $\frac{18}{35}$ (2) $-\frac{4}{3}$ (3) $-\frac{18}{19}$ (4) $-\frac{18}{11}$

11. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to :

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

12. Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$,

where a, b and c are real constants. Then the system of equations :

(1) has a unique solution when $5a = 2b + c$

(2) has infinite number of solutions when $5a = 2b + c$

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

13. If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of

$$(a+b)-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)-\left(\frac{a^4+b^4}{4}\right)+\ldots$$

is :

(1) $\log_e 2$ (2) $e^2 - 1$

(3) e (4) $\log_e\left(\frac{e}{2}\right)$

14. The sum of the series $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(2) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$

(3) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$

(4) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

15. Let $f(x)$ be a differentiable function at $x = a$ with $f'(a) = 2$ and $f(a) = 4$. Then $\lim\limits_{x\to a}\dfrac{xf(a)-af(x)}{x-a}$ equals :

(1) $2a + 4$ (2) $4 - 2a$

(3) $2a - 4$ (4) $a + 4$

16. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points P, A and B lie on :

(1) a straight line (2) a hyperbola

(3) an ellipse (4) a parabola

17. If the mirror image of the point (1, 3, 5) with respect to the plane $4x - 5y + 2z = 8$ is (α,β, γ), then $5(\alpha + \beta + \gamma)$ equals :

(1) 47 (2) 43 (3) 39 (4) 41

18. Let $f(x)=\int_0^x e^t f(t)dt+e^x$ be a differentiable function for all $x \in R$. Then $f(x)$ equals :

(1) $2e^{(e^x-1)}-1$

(2) $e^{e^x}-1$

(3) $2e^{e^x}-1$

(4) $e^{(e^x-1)}$

19. Let A_1 be the area of the region bounded by the curves y = sinx, y = cosx and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sinx, y = cosx, x-axis and $x=\frac{\pi}{2}$ in the first quadrant.

Then,

(1) $A_1 : A_2 = 1:\sqrt{2}$ and $A_1 + A_2 = 1$

(2) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(3) $2A_1 = A_2$ and $A_1 + A_2 = 1+\sqrt{2}$

(4) $A_1 : A_2 = 1:2$ and $A_1 + A_2 = 1$

20. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$ (2) $\frac{1}{7}$ (3) $\frac{3}{7}$ (4) $\frac{4}{7}$

SECTION B

1. Let z be those complex numbers which satisfy $|z + 5| \le 4$ and $z(1+i)+\bar{z}(1-i) \ge -10, i = \sqrt{-1}$. If the maximum value of $|z + 1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _____.

2. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $\left(4, -2\sqrt{2}\right)$, and given that $a - 2\sqrt{2}\, b = 3$, then $(a^2 + b^2 + ab)$ is equal to _____.

3. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of p_n^2 is _____.

4. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, for m, n $\geq$ 1 and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx = \alpha I_{m,n}$, $\alpha \in R$, then α equals _____.

5. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to _____.

6. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is _____.

7. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is _____.

8. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval (a, a + 1). Then, |a| is equal to _____.

9. Let $X_1, X_2, ..., X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i - \alpha) = 36$ and $\sum_{i=1}^{18}(X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is _____.

10. If the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 3 & 0 & -1\end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to _____.