TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

PHYSICS

1. Two forces P and Q of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle is :

(1) 30° (2) 60° (3) 90° (4) 120°

Ans. **(4)**

Sol. $4F^2 + 9F^2 + 12F^2 \cos\theta = R^2$

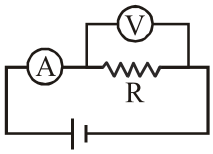
$4F^2 + 36F^2 + 24F^2 \cos\theta = 4R^2$

$4F^2 + 36F^2 + 24F^2 \cos\theta$

$= 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$

$\cos\theta = -\frac{12F^2}{24F^2} = -\frac{1}{2}$

2. The actual value of resistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is :



(1) 350Ω (2) 570Ω (3) 35 Ω (4) 600 Ω

Ans. **(2)**

Sol. $0.95\,R = \frac{RR_\upsilon}{R+R_\upsilon}$

$0.95 \times 30 = 0.05\,R_\upsilon$

$R_\upsilon = 19 \times 30 = 570\ \Omega$

3. An unknown metal of mass 192 g heated to a temperature of 100ºC was immersed into a brass calorimeter of mass 128 g containing 240 g of water a temperature of 8.4ºC Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5ºC (Specific heat of brass is 394 J kg^{-1} K^{-1})

(1) 1232 J kg^{-1} K^{-1} (2) 458 J kg^{-1} K^{-1}

(3) 654 J kg^{-1} K^{-1} (4) 916 J kg^{-1} K^{-1}

Ans. **(4)**

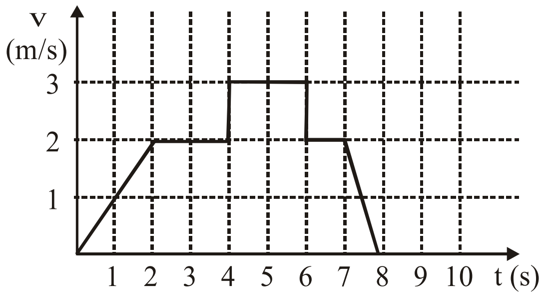
Sol. $192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4)$

$+ 240 \times 4200 \times (21.5 - 8.4)$

$\Rightarrow S = 916$

4. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ?



(1) 6 m (2) 9 m (3) 3 m (4) 10 m

Ans. **(2)**

S = Area under graph

$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9$ m

5. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is :

(1) 437.5 J (2) 637.5 J

(3) 740 J (4) 540 J

Ans. **(1)**

$L\frac{di}{dt} = 25$

$L \times \frac{15}{1} = 25$

$L = \frac{5}{3}H$

$\Delta E = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5$ J

6. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11V is connected across it is :

(1) 11×10^{-5} W (2) 11×10^{-4} W

(3) 11×10^{5} W (4) 11×10^{-3} W

Ans. (1)

$P = I^2R$

$4.4 = 4 \times 10^{-6} R$

$R = 1.1 \times 10^{6}\ \Omega$

$$P' = \frac{11^2}{R} = \frac{11^2}{1.1} \times 10^{-6} = 11 \times 10^{-5} W$$

7. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

(1) 4260 ± 80 cm³ (2) 4300 ± 80 cm³

(3) 4264.4 ± 81.0 cm³ (4) 4264 ± 81 cm³

Ans. (1)

$$\frac{\Delta V}{V} = 2\frac{\Delta d}{d} + \frac{\Delta h}{h} = 2\left(\frac{0.1}{12.6}\right) + \frac{0.1}{34.2}$$

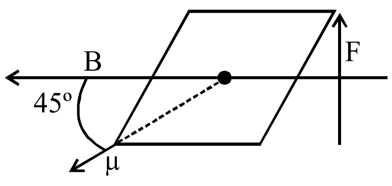
$$V = 12.6 \times \frac{\pi}{4} \times 314.2$$

8. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic neeedle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is :

(1) 3.6×10^{-5} N (2) 6.5×10^{-5} N

(3) 1.3×10^{-5} N (4) 1.8×10^{-5} N

Ans. (2)



$$\mu B \sin 45° = F\frac{\ell}{2}\sin 45°$$

$F = 2\mu B$

9. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot ?

(1) 2750 kHz (2) 2000 kHz

(3) 2250 kHz (4) 2900 kHz

Ans. (2)

$$f_{carrier} = \frac{250}{0.1} = 2500\,KHZ$$

∴ Range of signal = 2250 Hz to 2750 Hz

Now check all options : for 2000 KHZ

f_{mod} = 200 Hz

∴ Range = 1800 KHZ to 2200 KHZ

10. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then :

(1) $T_h = 0.5\ T_c$ (2) $T_h = 2\ T_c$

(3) $T_h = 1.5\ T_c$ (4) $T_h = T_c$

Ans. (4)

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

$$T_h = 2\pi\sqrt{\frac{mR^2}{(2\mu)B}}$$

$$T_C = 2\pi\sqrt{\frac{1/2mR^2}{\mu B}}$$

11. The electric field of a plane polarized electromagnetic wave in free space at time t= 0 is given by an expression

$$\vec{E}(x,y) = 10\hat{j}\ \cos\ [(6x + 8z)]$$

The magnetic field $\vec{B}$ (x, z, t) is given by : (c is the velocity of light)

(1) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos\left[\left(6x-8z+10ct\right)\right]$

(2) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos\left[\left(6x+8z-10ct\right)\right]$

(3) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos\left[\left(6x+8z-10ct\right)\right]$

(4) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos\left[\left(6x+8z+10ct\right)\right]$

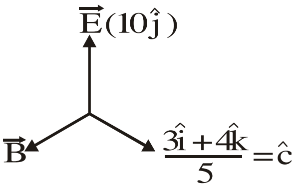
Ans. (2)

$\vec{E} = 10\hat{j}\cos\left[\left(6\hat{i}+8\hat{k}\right)\cdot\left(x\hat{i}+z\hat{k}\right)\right]$

$= 10\,\hat{j}\cos[\vec{K}\cdot\vec{r}]$

$\therefore\ \vec{K} = 6\hat{i}+8\hat{k}$; direction of waves travel.

i.e. direction of 'c'.



$\therefore$ Direction of $\hat{B}$ will be along

$\hat{C}\times\hat{E} = \dfrac{-4\hat{i}+3\hat{k}}{5}$

Mag. of $\vec{B}$ will be along $\hat{C}\times\hat{E} = \dfrac{-4\hat{i}+3\hat{k}}{5}$

Mag. of $\vec{B}$ = $\dfrac{E}{C} = \dfrac{10}{C}$

$\therefore\ \vec{B} = \dfrac{10}{C}\left(\dfrac{-4\hat{i}+3\hat{k}}{5}\right) = \dfrac{\left(-8\hat{i}+6\hat{k}\right)}{C}$

12. Condiser the nuclear fission

$Ne^{20} \rightarrow 2He^{4} + C^{12}$

Given that the binding energy/nucleon of Ne^{20}, He^{4} and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement :

(1) 8.3 MeV energy will be released

(2) energy of 12.4 MeV will be supplied

(3) energy of 11.9 MeV has to be supplied

(4) energy of 3.6 MeV will be released

Ans. (3)

$Ne^{20} \quad\rightarrow\quad 2He^{4} + C^{12}$

$8.03\times 20 \qquad 2\times 7.07\times 4 + 7.86\times 12$

$\therefore\ E_B = (BE)_{react} - (BE)_{product} = 9.72$ MeV

13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $\left(\vec{A}+\vec{B}\right)$ is 'n' times the magnitude of $\left(\vec{A}-\vec{B}\right)$. The angle between $\vec{A}$ and $\vec{B}$ is :

(1) $\sin^{-1}\left[\dfrac{n^2-1}{n^2+1}\right]$ (2) $\cos^{-1}\left[\dfrac{n-1}{n+1}\right]$

(3) $\cos^{-1}\left[\dfrac{n^2-1}{n^2+1}\right]$ (4) $\sin^{-1}\left[\dfrac{n-1}{n+1}\right]$

Ans. (3)

$|\vec{A}+\vec{B}| = 2a\cos\theta/2$ ___(1)

$|\vec{A}-\vec{B}| = 2a\cos\dfrac{(\pi-\theta)}{2} = 2a\sin\theta/2$ ___(2)

$\Rightarrow\ n\left(2a\cos\dfrac{\theta}{2}\right) = 2a\dfrac{\sin\theta}{2}$

$\Rightarrow\ \tan\dfrac{\theta}{2} = n$

14. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is :

(1) $\dfrac{7}{3}\pi$ (2) $\dfrac{3}{8}\pi$

(3) $\dfrac{4\pi}{3}$ (4) $\dfrac{8\pi}{3}$

Ans. (4)

$v = \omega\sqrt{A^2-x^2}$ ___(1)

$a = -\omega^2 x$ ___(2)

$|v| = |a|$ ___(3)

$\omega\sqrt{A^2-x^2} = \omega^2 x$

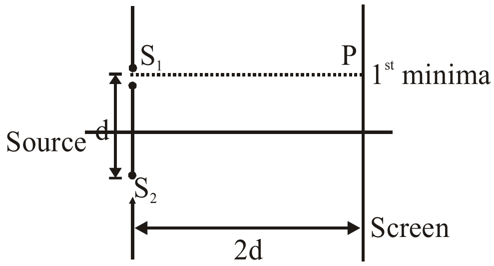
$A^2 - x^2 = \omega^2 x^2$

$5^2 - 4^2 = \omega^2(4^2)$

$\Rightarrow\ 3 = \omega\times 4$

$T = 2\pi/\omega$

15. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?



(1) $\frac{\lambda}{2(5-\sqrt{2})}$ (2) $\frac{\lambda}{(5-\sqrt{2})}$

(3) $\frac{\lambda}{(\sqrt{5}-2)}$ (4) $\frac{\lambda}{2(\sqrt{5}-2)}$

Ans. **(4)**

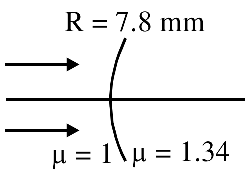
$$\sqrt{5}d - 2d = \frac{\lambda}{2}$$

16. The eye can be regarded as a single refracting surface . The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(1) 2 cm (2) 1 cm

(3) 3.1 cm (4) 4.0 cm

Ans. **(3)**



$$\frac{1.34}{V} - \frac{1}{\infty} = \frac{1.34-1}{7.8}$$

$\therefore$ V = 30.7 mm

17. Half mole of an ideal monoatomic gas is heated at constant pressure of 1atm from 20 ºC to 90ºC. Work done by gas is close to : (Gas constant R = 8.31 J /mol.K)

(1) 73 J (2) 291 J (3) 581 J (4) 146 J

Ans. **(2)**

$$WD = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70$$

18. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2.The work function of the metal is 5eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be : [1 eV = 1.6×10^{-19}J]

(1) 10^{10} and 5 eV (2) 10^{14} and 10 eV

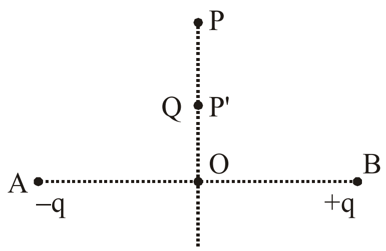
(3) 10^{12} and 5 eV (4) 10^{11} and 5 eV

Ans. **(4)**

$$I = \frac{nE}{At}$$

$$16 \times 10^{-3} = \left(\frac{n}{t}\right)_{\text{Photon}} \frac{10 \times 1.6 \times 10^{-19}}{10^{-4}} = 10^{12}$$

19. Charges –q and +q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences and electrostatic force F. If Q is now moved along the equatorial line to P' such that OP'=$\left(\frac{y}{3}\right)$, the force on Q will be close to : $\left(\frac{y}{3} >> 2a\right)$



(1) $\frac{F}{3}$ (2) 3F (3) 9F (4) 27F

Ans. (4)

Sol. Electric field of equitorial plane of dipole

$$= -\frac{K\vec{P}}{r^3}$$

$$\therefore \text{ At P, F} = -\frac{K\vec{P}}{r^3}Q.$$

$$\text{At P}^1\text{ , F}^1 = -\frac{K\vec{P}Q}{(r/3)^3} = 27F.$$

20. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11}m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is : (Take Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2})

(1) 1.4×10^5 m/s (2) 24×10^4 m/s

(3) 3.8×10^4 m/s (4) 2.8×10^5 m/s

Ans. (4)

By energy convervation between 0 & ∞.

$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV^2 = 0 + 0$$

[M is mass of star m is mass of meteroite)

$$\Rightarrow \quad v = \sqrt{\frac{4GM}{r}} = 2.8 \times 10^5 \text{m/s}$$

21. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

(1) 7 (2) 5 (3) 6 (4) 4

Ans. (1)

Sol. For closed organ pipe, resonate frequency is odd multiple of fundamental frequency.

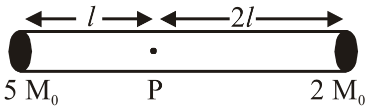
$\therefore$ $(2n + 1) f_0 \le 20,000$

(f_o is fundamental frequency = 1.5 KHz)

$\therefore$ n = 6

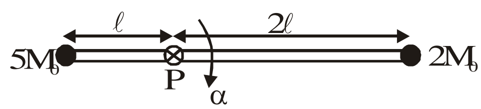
$\therefore$ Total number of overtone that can be heared is 7. (0 to 6).

22. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



(1) $\frac{g}{2l}$ (2) $\frac{7g}{3l}$ (3) $\frac{g}{13l}$ (4) $\frac{g}{3l}$

Ans. (3)



Applying torque equation about point P.

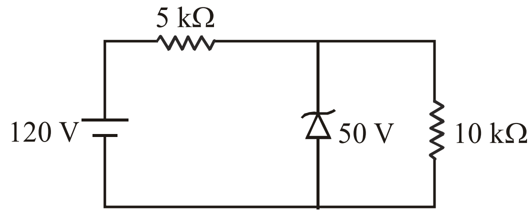
$2M_0 (2l) - 5 M_0 gl = I\alpha$

$I = 2M_0 (2l)^2 + 5M_0 l^2 = 13 M_0 l^2 d$

$$\therefore \quad \alpha = -\frac{M_0 g\ell}{13M_0\ell^2} \Rightarrow \alpha = -\frac{g}{13\ell}$$

$$\therefore \quad \alpha = \frac{g}{13\ell} \text{ anticlockwise}$$

23. For the circuit shown below, the current through the Zener diode is :



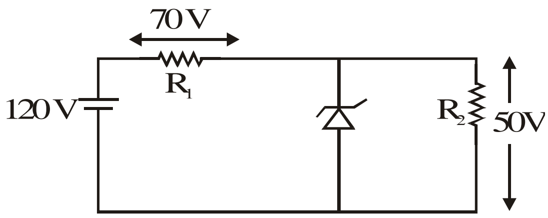
(1) 5 mA (2) Zero (3) 14 mA (4) 9 mA

Ans. (4)

Assuming zener diode doesnot undergo breakdown, current in circuit $= \frac{120}{15000} = 8\text{mA}$

$\therefore$ Voltage drop across diode = 80 V > 50 V.

The diode undergo breakdown.



$$\text{Current is } R_1 = \frac{70}{5000} = 14\text{mA}$$

$$\text{Current is } R_2 = \frac{50}{10000} = 5\text{mA}$$

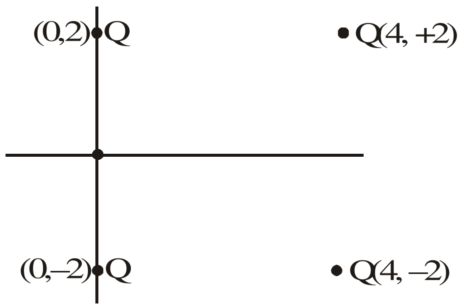
$\therefore$ Current through diode = 9mA

24. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be :

(1) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (2) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

(3) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{Q^2}{4\pi\varepsilon_0}$

Ans. (2)



Potential at origin = $\frac{KQ}{2}+\frac{KQ}{2}+\frac{KQ}{\sqrt{20}}+\frac{KQ}{\sqrt{20}}$

(Potential at ∞ = 0)

$= KQ\left(1+\frac{1}{\sqrt{5}}\right)$

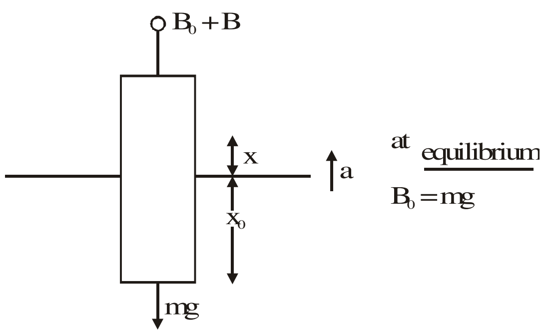
∴ Work required to put a fifth charge Q at origin is equal to $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

25. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω close to : (density of water = 10^3 kg / m^3)

(1) 5.00 rad s^{-1} (2) 1.25 rad s^{-1}

(3) 3.75 rad s^{-1} (4) 2.50 rad s^{-1}

Ans. (Bonus)



Extra Boyant force = δAxg

B_0 + B × mg = ma

B = ma

$a=\left(\frac{\delta Ag}{m}\right)^x$

$w^2=\frac{\delta Ag}{m}$

$w=\sqrt{\frac{10^3\times\pi(2.5)^2\times10^{-4}\times10}{310\times10^{-6}\times10^3}}$

$=\sqrt{63.30}$ = 7.95

26. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

(1) 692 pJ (2) 60 pJ

(3) 508 pJ (4) 560 pJ

Ans. (3)

Intial energy of capacitor

$U_i=\frac{1}{2}\frac{v^2}{c}$

$=\frac{1}{2}\times\frac{120\times120}{12}=600$ J

Since battery is disconnected so charge remain same.

Final energy of capacitor

$U_f=\frac{1}{2}\frac{v^2}{c}$

$=\frac{1}{2}\times\frac{120\times120}{12\times6.5}=92$

W + U_f = U_i

W = 508 J

27. Two kg of a monoatomic gas is at a pressure of 4 × 10^4 N/m^2 . The density of the gas is 8 kg /m^3. What is the order of energy of the gas due to its thermal motion ?

(1) 10^3 J (2) 10^5 J

(3) 10^6 J (4) 10^4 J

Ans. (4)

Thermal energy of N molecule

$= N\left(\frac{3}{2}kT\right)$

$$= \frac{N}{N_A}\frac{3}{2}RT$$

$$= \frac{3}{2}(nRT)$$

$$= \frac{3}{2}PV$$

$$= \frac{3}{2}P\left(\frac{m}{8}\right)$$

$$= \frac{3}{2}\times 4\times 10^4 \times \frac{2}{8}$$

$= 1.5 \times 10^4$

order will 10^4

28. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

(1) 15 J (2) 10 J (3) 12 J (4) 9 J

Ans. (1)

Work done = $\vec{F}\cdot\vec{d}$

= 12J

work energy theorem

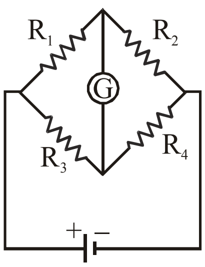
$w_{net} = \Delta K.E.$

$12 = K_f - 3$

$K_f = 15J$

29. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω, respectively.

Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be :



(1) Red, Green, Brown

(2) Brown, Blue, Brown

(3) Grey, Black, Brown

(4) Brown, Blue, Black

Ans. (2)

$R_1 = 32 \times 10 = 320$

for wheat stone bridge

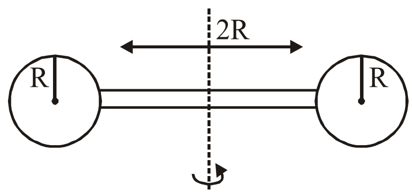
$$\Rightarrow \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$\frac{320}{R_3} = \frac{80}{40}$$

$R_3 = 160$

Brown Blue Brown

30. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(1) $\frac{152}{15}MR^2$ (2) $\frac{17}{15}MR^2$

(3) $\frac{137}{15}MR^2$ (4) $\frac{209}{15}MR^2$

Ans. (3)

For Ball

using parallel axis theorem.

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2$$

$$= \frac{22}{5}MR^2$$

2 Balls so $\frac{44}{5}MR^2$

Irod = for rod $\frac{M(2R)^2}{R} = \frac{MR^2}{3}$

$I_{system} = I_{Ball} + I_{rod}$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3}$$

$$= \frac{137}{15}MR^2$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thrusday 10th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

CHEMISTRY

1. An ideal gas undergoes isothermal compression from 5 m^3 against a constant external pressure of 4 Nm^{-2}. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J mol^{-1} K^{-1}, the temperature of Al increases by :

(1) $\frac{3}{2}$K (2) $\frac{2}{3}$K (3) 1 K (4) 2 K

Ans. **(2)**

Sol. Work done on isothermal irreversible for ideal gas

$= -P_{ext}(V_2 - V_1)$

$= -4\ N/m^2\ (1m^3 - 5m^3)$

= 16 Nm

Isothermal process for ideal gas

$\Delta U = 0$

q = –w

= –16 Nm

= – 16 J

Heat used to increase temperature of $A\ell$

$q = n\ C_m\ \Delta T$

$$16\ J = 1 \times 24\ \frac{J}{mol.K} \times \Delta T$$

$$\Delta T = \frac{2}{3}K$$

2. The 71st electron of an element X with an atomic number of 71 enters into the orbital :

(1) 4f (2) 6p (3) 6s (4) 5d

Ans. **(1)**

3. The number of 2-centre-2-electron and 3-centre-2-electron bonds in B_2H_6, respectively, are :

(1) 2 and 4 (2) 2 and 1

(3) 2 and 2 (4) 4 and 2

Ans. **(4)**

4. The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2 L of its 0.1 M aqueous solution is :

(1) 68.4 g (2) 17.1 g (3) 34.2 g (4) 136.8 g

Ans. **(1)**

Sol. $$\text{Molarity} = \frac{(n)_{solute}}{V_{solution}\ (\text{in lit})}$$

$$0.1 = \frac{wt./342}{2}$$

wt ($C_{12}H_{22}O_{11}$) = 68.4 gram

5. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is :

(1) $H_2 + I_2 \rightarrow 2HI$ (2) $H_2 + F_2 \rightarrow 2HF$

(3) $H_2 + Cl_2 \rightarrow 2HCI$ (4) $H_2 + Br_2 \rightarrow 2HBr$

Ans. **(1)**

6. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of:

(1) sodium ion-ammonia complex

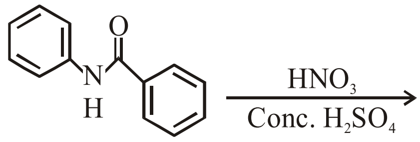
(2) sodamide

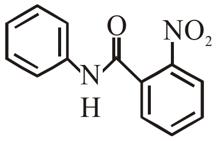
(3) sodium-ammonia complex

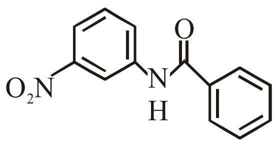
(4) ammoniated electrons

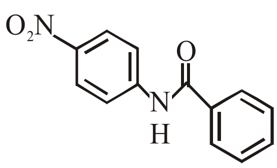
Ans. **(4)**

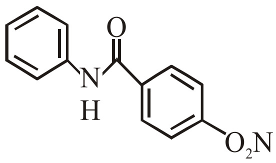
7. What will be the major product in the following mononitation reaction ?



(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

8. In the cell Pt(s)|H_2(g, 1bar|HCl(aq)|Ag(s)|Pt(s) the cell potential is 0.92 when a 10^{-6} molal HCl solution is used. THe standard electrode potential of (AgCl/Ag,Cl^-) electrode is :

$$\left\{\text{given}, \frac{2.303RT}{F} = 0.06\text{V at } 298\text{K}\right\}$$

(1) 0.20 V (2) 0.76 V (3) 0.40 V (4) 0.94 V

Ans. **(1)**

Sol. $Pt(s)|H_2(g, 1\,bar)|HCl(aq)|AgCl(s)|Ag(s)|Pt(s)$

10^{-6} m

Anode: $H_2 \longrightarrow 2H^+ + 2e \times 1$

Cathode : $e^- + AgCl(s) \longrightarrow Ag(s) + Cl^-(aq) \times 2$

$H_2(g)l + AgCl(s) \longrightarrow 2H^+ + 2Ag(s) + 2Cl^-(aq)$

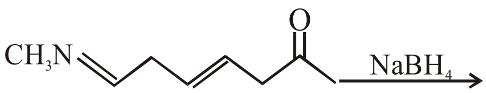
$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log_{10}\left((H^+)^2 \cdot (Cl^-)^2\right)$$

$$.925 = \left(E^0_{H_2/H^+} + E^0_{AgCl/Ag,\,Cl^-}\right) - \frac{0.06}{2}\log_{10}\left((10^{-6})^2(10^{-6})^2\right)$$

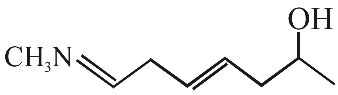
$$.92 = 0 + E^0_{AgCl/Ag,Cl^-} - 0.03\log_{10}(10^{-6})^4$$

$$E^0_{AgCl}/Ag,\,Cl^- = .92 + .03 \times -24 = 0.2\text{ V}$$

9. The major product of the following recation is:



(1)



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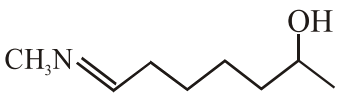

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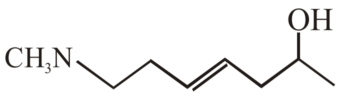

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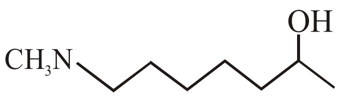


Ans. **(3)**

10. The pair that contains two P–H bonds in each of the oxoacids is :

(1) H_3PO_2 nad $H_4P_2O_5$

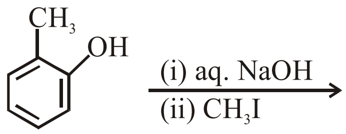
(2) $H_4P_2O_5$ and $H_4P_2O_6$

(3) H_3PO_3 and H_3PO_2

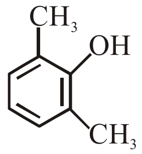
(4) $H_4P_2O_5$ nad H_3PO_3

Ans. **(1)**

11. The major product of the following reaction is:



(1)



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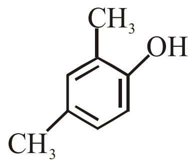

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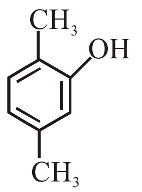

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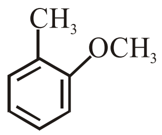


Ans. **(4)**

12. The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. The metal ion is :

(1) Fe^{2+} (2) Co^{2+} (3) Mn^{2+} (4) Ni^{2+}

Ans. **(2)**

13. A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms :

(1) hcp lattice-A, $\frac{2}{3}$ Tetrachedral voids-B

(2) hcp lattice-B, $\frac{1}{3}$ Tetrachedral voids-A

(3) hcp lattice-B, $\frac{2}{3}$ Tetrachedral voids-A

(4) hcp lattice-A $\frac{1}{3}$ Tetrachedral voids-B

Ans. **(2)**

Sol. A_2B_3 has HCP lattice

If A form HCP, then $\frac{3}{4}^{th}$ of THV must occupied by B to form A_2B_3

If B form HCP, then $\frac{1}{3}^{th}$ of THV must occupied by A to form A_2B_3

14. The reaction that is NOT involved in the ozone layer depletion mechanism is the stratosphere is:

(1) $HOCl(g) \xrightarrow{h\upsilon} \dot{O}H(g) + \dot{C}l(g)$

(2) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$

(3) $CH_4 + 2O_3 \rightarrow 3CH_2= O + 3H_2OP$

(4) $Cl\dot{O}(g) + O(g) \rightarrow \dot{C}l(g) + O_2(g)$

Ans. (3)

15. The process with negative entropy change is :

(1) Dissolution of iodine in water

(2) Synthesis of ammonia from N_2 and H_2

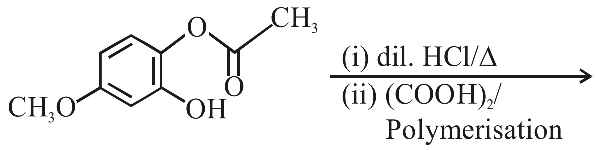
(3) Dissolution of $CaSO_4(s)$ to $CaO(s)$ and $SO_3(g)$

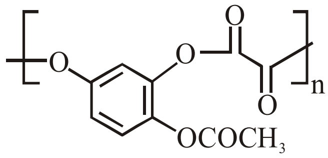
(4) Subimation of dry ice

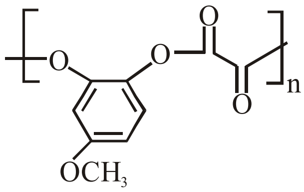
Ans. (2)

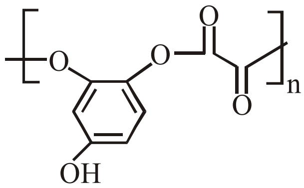
Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$; $\Delta n_g < 0$

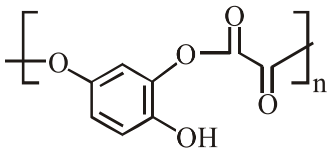
16. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

Ans. (3)

17. A reaction of cobalt(III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A (violet coloured) B (green coloured). A can show optial actively, B is optically inactive. What type of isomers does A and B represent ?

(1) Geometrical isomers

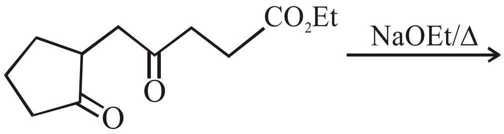
(2) Ionisation isomers]

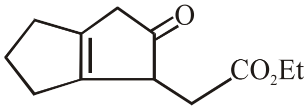
(3) Coordination isomers

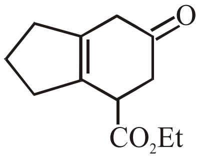
(4) Linkage isomers

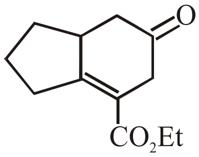
Ans. (1)

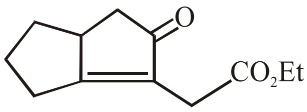
18. The major product obtained in the following reaction is :



(1) 

(2) 

(3) 

(4) 

Ans. (4)

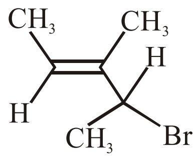
19. Which of the following tests cannot be used for identifying amino acids ?

(1) Biuret test (2) Xanthoproteic test

(3) Barfoed test (4) Ninhydrin test

Ans. (3)

20. What is the IUPAC name of the following compound ?



(1) 3-Bromo-1, 2-dimethylbut-1-ene]

(2) 4-Bromo-3-methylpent-2-ene

(3) 2-Bromo-3-methylpent-3-ene

(4) 3-Bromo-3-methyl-1, 2-dimethylprop-1-ene

Ans. (2)

21. Which is the most suitable reagent for the following transformation ?

$$CH_3–CH=CH–CH_2–\underset{}{\overset{OH}{\overset{|}{C}}}H–CH_3 \longrightarrow$$

$$CH_3–CH=CH–CH_2CO_2H$$

(1) alkaline $KMnO_4$ (2) $I_2/NaOH$

(3) Tollen's reagent (4) CrO_2/CS_2

Ans. (2)

22. The correct match between item 'I' and item 'II' is :

Item 'I' (compound)	Item 'II' (reagent)
(A) Lysine	(P) 1-naphthol
(B) Furfural	(Q) ninhydrin
(C) Benzyl alcohol	(R) $KMnO_4$
(D) Styrene	(S) Ceric ammonium nitrate

(1) (A)→(Q), (B)→(P), (C)→(S), (D)→(R)

(2) (A)→(Q), (B)→(R), (C)→(S), (D)→(P)

(3) (A)→(Q), (B)→(P), (C)→(R), (D)→(S)

(4) (A)→(R), (B)→(P), (C)→(Q), (D)→(S)

Ans. (1)

23. In the reaction of oxalate with permaganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is :

(1) 10 (2) 2 (3) 1 (4) 5

Ans. (3)

Sol. $$2\overset{+7}{M}nO_4 + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2\overset{+2}{M}n^{2+} + 10CO_2 + 8H_2O$$

10 e^- trans for 10 molecules of CO_2 so per molecule of CO_2 transfer of e^- is '1'

24. 5.1g NH_4SH is introduced in 3.0 L evacuated flask at 327°C. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_p of the reaction at 327°C is (R = 0.082 L atm $mol^{-1}K^{-1}$, Molar mass of S = 32 g $mol^{/01}$, molar mass of N = 14g mol^{-1})

(1) 1×10^{-4} atm^2 (2) 4.9×10^{-3} atm^2

(3) 0.242 atm^2 (4) 0.242×10^{-4} atm^2

Ans. (3)

$$NH_4SH(s) \rightleftharpoons NH_3(g) + H_2S(g)$$

Sol. $n = \frac{5.1}{51} = .1\,\text{mole}$ 0 0

$.1(-1-\alpha)$ $.1\alpha$ $.1\alpha$

$\alpha = 30\% = .3$

so number of moles at equilibrium

.1 (1 – .3) .1 × .3 .1 × .3

= .07 =.03 =.03

Now use PV = nRT at equilibrium

$P_{total} \times 3$ lit = (.03 + .03) × .082 × 600

P_{total} = .984 atm

At equilibrium

$$P_{NH_3} = P_{H_2S} = \frac{P_{total}}{2} = .492$$

So $k_p = P_{NH_3} \cdot P_{H_2S} = (.492)\,(.492)$

$k_p = .242$ atm^2

25. The electrolytes usually used in the electroplating of gold and silver, respectively, are :

(1) $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$

(2) $[Au(CN)_2]^-$ and $[Ag\ CI_2]^-$

(3) $[Au(NH_3)_2]^+$ and $[Ag(CN)_2]^-$

(4) $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$

Ans. (4)

26. Elevation in the boiling point for 1 molal solution of glucose is 2 K. The depression in the freezing point of 2 molal solutions of glucose in the same solvent is 2 K. The relation between K_b and K_f is:

(1) $K_b = 0.5\ K_f$ (2) $K_b = 2\ K_f$

(3) $K_b = 1.5\ K_f$ (4) $K_b = K_f$

Ans. (2)

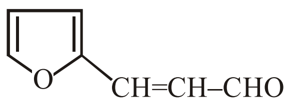
Sol. Ans.(2)

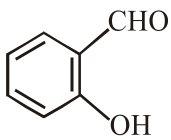
$$\frac{\Delta T_b}{\Delta T_f} = \frac{i.m \times k_b}{i \times m \times k_f}$$

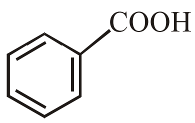
$$\frac{2}{2} = \frac{1 \times 1 \times k_b}{1 \times 2 \times k_f}$$

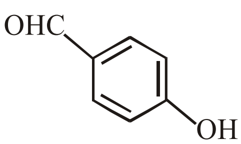
$k_b = 2k_f$

27. An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula C_6H_7N. The structure of 'A' is :

(1) 

(2) 

(3) 

(4) 

Ans. (3)

28. The ground state energy of hydrogen atom is –13.6 eV. The energy of second excited state He^+ ion in eV is :

(1) –6.04 (2) –27.2 (3) –54.4 (4) –3.4

Ans. (1)

Sol. $(E)_{n^{th}} = (E_{GND})_H \cdot \frac{Z^2}{n^2}$

$$E_{3^{rd}}(He^+) = (-13.6eV) \cdot \frac{2^2}{3^2} = -6.04\ eV$$

29. For an elementary chemical reaction,

$A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$, the expression for $\frac{d[A]}{dt}$ is :

(1) $2k_1[A_2]-k_{-1}[A]^2$ (2) $k_1[A_2]-k_{-1}[A]^2$

(3) $2k_1[A_2]-2k_{-1}[A]^2$ (4) $k_1[A_2]+k_{-1}[A]^2$

Ans. (3)

Sol. Ans.(3)

$$A_2 \underset{K_{-1}}{\overset{K_1}{\rightleftharpoons}} 2A$$

$$\frac{d[A]}{dt} = 2k_1[A_2] - 2k_{-1}[A]^2$$

30. Haemoglobin and gold sol are examples of :

(1) negatively charged sols

(2) positively charged sols]

(3) negatively and positively charged sols, respectively

(4) positively and negatively charged sols, respectively

Ans. (4)

Sol. Ans.(4)

Haemoglobin $\longrightarrow$ positive sol

Ag – sol $\longrightarrow$ negative sol

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

1. Let $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I[z] respectively denote the real and imaginary parts of z, then :

(1) R(z) > 0 and I(z) > 0

(2) R(z) < 0 and I(z) > 0

(3) R(z) = –3

(4) I(z) = 0

Ans. **(4)**

Sol. $z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$

$z = \left(e^{i\pi/6}\right)^5 + \left(e^{-i\pi/6}\right)^5$

$= e^{i5\pi/6} + e^{-i5\pi/6}$

$= \cos\frac{5\pi}{6} + i\frac{\sin 5\pi}{6} + \cos\left(\frac{-5\pi}{6}\right) + i\sin\left(\frac{-5\pi}{6}\right)$

$= 2\cos\frac{5\pi}{6} < 0$

I(z) = 0 and Re(z) < 0

Option (4)

2. Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for i = 1,2,...., 10 and S be the set of pairs (r,k), r k∈N (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S, is :

(1) Infinitely many (2) 4

(3) 10 (4) 2

Ans. **(1)**

Sol. Apply

$C_3 \to C_3 - C_2$

$C_2 \to C_2 - C_1$

We get D = 0

Option (1)

3. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is :

(1) $\sqrt{5}$ (2) 4

(3) $2\sqrt{2}$ (4) 3

Ans. **(2)**

Sol. $x^2\left({}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)$

$x^2\left[{}^{10}C_r(x)^{\frac{10-r}{2}}(\lambda)^r(x)^{-2r}\right]$

$x^2\left[{}^{10}C_r\,\lambda^r\,x^{\frac{10-5r}{2}}\right]$

∴ r = 2

Hence, ${}^{10}C_2\,\lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

Option (2)

4. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\ldots\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

(1) $\frac{1}{256}$ (2) $\frac{1}{2}$

(3) $\frac{1}{512}$ (4) $\frac{1}{1024}$

Ans. **(3)**

Sol. $2\sin\frac{\pi}{2^{10}}\cos\frac{\pi}{2^{10}}\ldots\ldots\cos\frac{\pi}{2^2}$

$\frac{1}{2^9}\sin\frac{\pi}{2} = \frac{1}{512}$

Option (3)

5. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi+5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$ (4) $\frac{3}{20}(4\pi-3)$

Ans. (4)

Sol. $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4}$

$$= \int_{\frac{-\pi}{2}}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^{0} \frac{dx}{-1-1+4}$$

$$+\int_{0}^{1} \frac{dx}{0+0+4} + \int_{1}^{\frac{\pi}{2}} \frac{dx}{1+0+4}$$

$$\int_{\frac{-\pi}{2}}^{-1} \frac{dx}{1} + \int_{-1}^{0} \frac{dx}{2} + \int_{0}^{1} \frac{dx}{4} + \int_{1}^{\frac{\pi}{2}} \frac{dx}{5}$$

$$\left(-1+\frac{\pi}{2}\right)+\frac{1}{2}(0+1)+\frac{1}{4}+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$-1+\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{\pi}{2}+\frac{\pi}{10}$$

$$\frac{-20+10+5-4}{20}+\frac{6\pi}{10}$$

$$\frac{-9}{20}+\frac{3\pi}{5}$$

Option (4)

6. If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

(1) 6 (2) 5

(3) 4 (4) 3

Ans. (2)

Sol. $1 - {}^{n}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$

$n_{min} = 5 \Rightarrow$ Option (2)

7. If mean and standard deviation of 5 observations x_1, x_2, x_3,x_4,x_5 are 10 and 3, respectively, then the variance of 6 observations x_1,x_2, ...,x_5 and –50 is equal to :

(1) 582.5 (2) 507.5

(3) 586.5 (4) 509.5

Ans. (2)

Sol. $\bar{x}=10 \Rightarrow \sum_{i=1}^{5} x_i = 50$

$$S.D. = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 8$$

$$\Rightarrow \sum_{i=1}^{5} (x_i)^2 = 109$$

$$\text{variance} = \frac{\sum_{i=1}^{5} (x_i)^2 + (-50)^2}{6} - \left(\sum_{i=1}^{5} \frac{x_i - 50}{6}\right)$$

$= 507.5$

Option (2)

8. The length of the chord of the parabola $x^2 = 4y$ having equation $x-\sqrt{2}y+4\sqrt{2}=0$ is :

(1) $2\sqrt{11}$ (2) $3\sqrt{2}$

(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

Ans. (3)

Sol. $x^2 = 4y$

$x-\sqrt{2}y+4\sqrt{2}=0$

Solving together we get

$$x^2=4\left(\frac{x+4\sqrt{2}}{\sqrt{2}}\right)$$

$$\sqrt{2}x^2+4x+16\sqrt{2}$$

$$\sqrt{2}x^2-4x-16\sqrt{2}=0$$

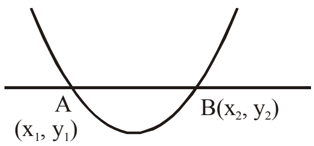
$x_1 + x_2 = 2\sqrt{2}$; $x_1x_2 = \dfrac{-16\sqrt{2}}{\sqrt{2}} = -16$

Similarly,

$$\left(\sqrt{2}y-4\sqrt{2}\right)^2=4y$$

$2y^2 + 32 - 16y = 4y$

$2y^2 - 20y + 32 = 0$ ⟶ $y_1 + y_2 = 10$, $y_1 y_2 = 16$



$\ell_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{\left(2\sqrt{2}\right)^2+64+(10)^2-4(16)}$

$=\sqrt{8+64+100-64}$

$=\sqrt{108}=6\sqrt{3}$

Option (3)

9. Let $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ where $b > 0$. Then the minimum value of $\dfrac{\det(A)}{b}$ is :

(1) $\sqrt{3}$ (2) $-\sqrt{3}$

(3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

Ans. **(4)**

Sol. $A = \begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ $(b > 0)$

$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1\ (b^2 - b^2 - 1)$

$|A| = 2(b^2 + 2) - b^2 - 1$

$|A| = b^2 + 3$

$\dfrac{|A|}{b}=b+\dfrac{3}{b}$ $\Rightarrow$ $\dfrac{b+\frac{3}{b}}{2}\ge\sqrt{3}$

$b+\dfrac{3}{b}\ge 2\sqrt{3}$

Option (4)

10. The tangent to the curve, $y=xe^{x^2}$ passing through the point (1,e) also passes through the point :

(1) $\left(\dfrac{4}{3},2e\right)$ (2) (2,3e)

(3) $\left(\dfrac{5}{3},2e\right)$ (4) (3,6e)

Ans. **(1)**

Sol. $y=xe^{x^2}$

$\left.\dfrac{dy}{dx}\right|_{(1,e)}=\left.\left(e\cdot e^{x^2}\cdot 2x+e^{x^2}\right)\right|_{(1,e)} = 2\cdot e + e = 3e$

T : $y - e = 3e\ (x - 1)$

$y = 3ex - 3e + e$

$y = (3e)x - 2e$

$\left(\dfrac{4}{3}, 2e\right)$ lies on it

Option (1)

11. The number of values of $\theta\in(0,\pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)\ y + 2z = 0$

has a non-trivial solution, is :

(1) One (2) Three

(3) Four (4) Two

Ans. **(4)**

Sol. $\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$

$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta)$
$+ 7(-\cos 2\theta - 4\sin 3\theta) = 0$
$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta$
$- 28\sin 3\theta = 0$
$14 - 7\sin 3\theta - 14\cos 2\theta = 0$
$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$
$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$
$7\sin\theta\,[-3 + 4\sin^2\theta + 4\sin\theta] = 0$
$\sin\theta,$
$4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$
$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\sin\theta = \frac{-3}{2}$; $\sin\theta = \frac{1}{2}$

Hence, 2 solutions in $(0, \pi)$
Option (4)

12. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then $f'(1/2)$ is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$

(3) $\frac{18}{25}$ (4) $\frac{4}{5}$

Ans. **(2)**

Sol. $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$ $f'\left(\frac{1}{2}\right) = ?$

Differentiate w.r.t. 'x'
$f(x) = 2x + 0 - x^2 f(x)$

$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$

$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$

$$f'\left(\frac{1}{2}\right) = \frac{2 - 2\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{\left(\frac{3}{2}\right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

Option (2)

13. Let $f : (-1,1) \to R$ be a function defined by $f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

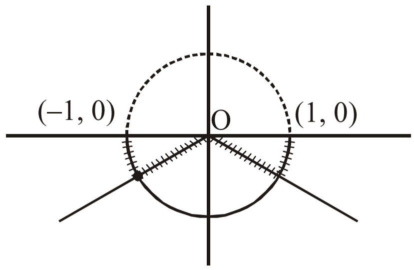
(1) Three elements (2) One element

(3) Five elements (4) Two elements

Ans. **(1)**

Sol. $f : (-1, 1) \to R$

$f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$



Non-derivable at 3 points in (–1, 1)
Option (1)

14. Let $S = \left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

Ans. **(4)**

Sol. $\frac{y^2}{1+r}-\frac{x^2}{1-r}=1$

for r > 1, $\frac{y^2}{1+r}+\frac{x^2}{r-1}=1$

$e=\sqrt{1-\left(\frac{r-1}{r+1}\right)}$

$=\sqrt{\frac{(r+1)-(r-1)}{(r+1)}}$

$=\sqrt{\frac{2}{r+1}}=\sqrt{\frac{2}{r+1}}$

Option (4)

15. If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to :

(1) $2^{25}-1$ (2) $(25)^2$ (3) 2^{25} (4) 2^{24}

Ans. **(3)**

Sol. $\sum_{r=0}^{25}{}^{50}C_r\cdot{}^{50-r}C_{25-r}$

$=\sum_{r=0}^{25}\frac{50!}{r!\,(50-r)!}\times\frac{(50-r)!}{(25)!\,(25-r)!}$

$=\sum_{r=0}^{25}\frac{50!}{25!\,25!}\times\frac{25!}{(25-r)!\,(r!)}$

$={}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r=\left(2^{25}\right)\,{}^{50}C_{25}$

$\therefore$ $K=2^{25}$

Option (3)

16. Let N be the set of natural numbers and two functions f and g be defined as f,g : $N\to N$

such that : $f(n)=\begin{cases}\frac{n+1}{2} & \text{if n is odd}\\ \frac{n}{2} & \text{if n is even}\end{cases}$

and $g(n)=n-(-1)^n$. The fog is :

(1) Both one-one and onto

(2) One-one but not onto

(3) Neither one-one nor onto

(4) onto but not one-one

Ans. **(4)**

Sol. $f(x)=\begin{cases}\frac{n+1}{2} & \text{n is odd}\\ n/2 & \text{n is even}\end{cases}$

$g(x)=n-(-1)^n\begin{cases}n+1\ ; & \text{n is odd}\\ n-1\ ; & \text{n is even}\end{cases}$

$f(g(n))=\begin{cases}\frac{n}{2}; & \text{n is even}\\ \frac{n+1}{2}; & \text{n is odd}\end{cases}$

$\therefore$ many one but onto

Option (4)

17. The values of λ such that sum of the squares of the roots of the quadratic equation, $x^2+(3-\lambda)x+2=\lambda$ has the least value is :

(1) 2 (2) $\frac{4}{9}$

(3) $\frac{15}{8}$ (4) 1

Ans. **(1)**

Sol. $\alpha+\beta=\lambda-3$

$\alpha\beta=2-\lambda$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(\lambda-3)^2-2(2-\lambda)$

$=\lambda^2+9-6\lambda-4+2\lambda$

$=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

$\therefore$ $\lambda=2$

Option (1)

18. Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

(1) Fourth

(2) Second

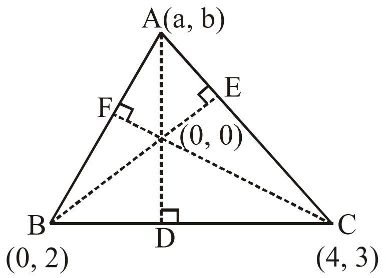
(3) Third

(4) First

Ans. **(2)**

Sol. $m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right)\times\left(\frac{b-0}{a-0}\right) = -1$

$\Rightarrow$ b + 4a = 0 ……(i)



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{(b-2)}{a-0}\right)\times\left(\frac{3}{4}\right) = -1$

$\Rightarrow$ 3b – 6 = –4a $\Rightarrow$ 4a + 3b = 6 ……(ii)

From (i) and (ii)

$a = \frac{-3}{4}$, b = 3

$\therefore$ II[nd] quadrant.

Option (2)

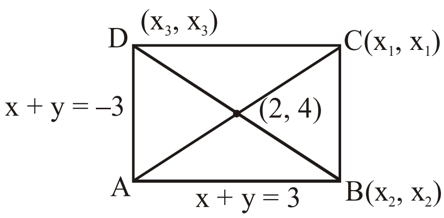
19. Two sides of a parallelogram are along the lines, x + y = 3 and x – y + 3 = 0. If its diagonals intersect at (2,4), then one of its vertex is :

(1) (2,6) (2) (2,1)

(3) (3,5) (4) (3,6)

Ans. (4)

Sol.



Solving x + y = 3 and x – y = –3 $\Rightarrow$ A(0, 3)

$\frac{x_1+0}{2} = 2$; $x_i = 4$ similarly $y_1 = 5$

C $\Rightarrow$ (4, 5)

Now equation of BC is x – y = –1

and equation of CD is x + y = 9

Solving x + y = 9 and x – y = –3

Point D is (3, 6)

Option (4)

20. Let $\vec{\alpha} = (\lambda-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda-2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

(1) –3 (2) 4

(3) 3 (4) –4

Ans. (4)

Sol. $\vec{\alpha} = (\lambda-2)\vec{\alpha} + \vec{b}$

$\vec{\beta} = (4\lambda-2)\vec{\alpha} + 3\vec{b}$

$\frac{\lambda-2}{4\lambda-2} = \frac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\boxed{\lambda = -4}$

$\therefore$ Option (4)

21. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

Ans. (3)

Sol. $\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1))\right)$

$\cot\left(\sum_{n=1}^{19}\cot^{-1}(n^2+n+1)\right) = \cot\left(\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right)$

$\sum_{n=1}^{19}(\tan^{-1}(n+1) - \tan^{-1}n)$

$\cot(\tan^{-1}20 - \tan^{-1}1) = \frac{\cot A\cot\beta + 1}{\cot\beta - \cot A}$

(Where tanA=20 , tanB=1) $\frac{1\left(\frac{1}{20}\right)+1}{1-\frac{1}{20}} = \frac{21}{19}$

$\therefore$ Option (3)

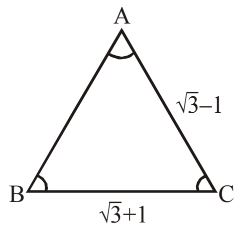
22. With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is :

(1) 7 : 1 (2) 5 : 3

(3) 9 : 7 (4) 3 : 1

Ans. **(1)**

Sol. A + B = 120°



$$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$=\frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30°)\ =\ \frac{1}{\sqrt{3}}.\sqrt{3}=1$$

$$\frac{A-B}{2}=45° \quad\Rightarrow\quad A-B=90°$$

$$A+B=120°$$

$$2A = 210°$$

$$A = 105°$$

$$B = 15°$$

$\therefore$ Option (1)

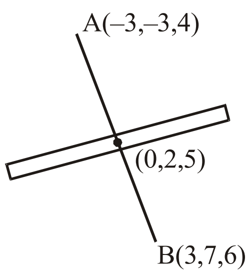
23. The plane which bisects the line segment joining the points (–3,–3,4) and (3,7,6) at right angles, passes through which one of the following points ?

(1) (4, –1,7) (2) (4,1,–2)

(3) (–2,3,5) (4) (2,1,3)

Ans. **(2)**

Sol.



$$\vec{n}=3\hat{i}+5\hat{j}+\hat{k}$$

p : 3(x – 0) + 5 (y – 2) + 1 (z – 5) = 0

3x + 5y + z = 15

$\therefore$ Option (2)

24. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

(1) $(P \wedge Q) \vee (\sim R)$

(2) $(\sim P) \wedge (\sim Q \wedge R)$

(3) $(\sim P) \vee (Q \wedge R)$

(4) $P \vee (\sim Q \wedge R)$

Ans. **(4)**

Sol. It is obvious

$\therefore$ Option (4)

25. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane, x + y + z = 2 ?

(1) $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

(2) $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$

(3) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

(4) $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$

Ans. **(3)**

Sol. General point on the given line is

$x = 2\lambda + 4$

$y = 2\lambda + 5$

$z = \lambda + 3$

Solving with plane,

$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$5\lambda + 12 = 2$

$5\lambda = -10$

$$\boxed{\lambda=-2}$$

$\therefore$ Option (3)

26. Let f be a differentiable function such that

$$f'(x)=7-\frac{3}{4}\frac{f(x)}{x},(x>0) \text{ and } f(1) \neq 4.$$

Then $\lim\limits_{x\to0^+} x f\left(\frac{1}{x}\right)$:

(1) Exists and equals 4

(2) Does not exist

(3) Exist and equals 0

(4) Exists and equals $\frac{4}{7}$

Ans. **(1)**

Sol. $f'(x)=7-\frac{3}{4}\frac{f(x)}{x} \quad (x>0)$

Given $f(1) \neq 4$ $\quad \lim\limits_{x\to0^+} xf\left(\frac{1}{x}\right) = ?$

$\frac{dy}{dx}+\frac{3}{4}\frac{y}{x}=7$ (This is LDE)

IF = $e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\ln|x|}=x^{\frac{3}{4}}$

$y.x^{\frac{3}{4}}=\int 7.x^{\frac{3}{4}}dx$

$y.x^{\frac{3}{4}}=7.\frac{x^{\frac{7}{4}}}{\frac{7}{4}}+C$

f(x) = $4x+C.x^{-\frac{3}{4}}$

$f\left(\frac{1}{x}\right)=\frac{4}{x}+C.x^{\frac{3}{4}}$

$\lim\limits_{x\to0^+} xf\left(\frac{1}{x}\right)=\lim\limits_{x\to0^+}\left(4+C.x^{\frac{7}{4}}\right)=4$

$\therefore$ Option (1)

27. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2},7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

(1) $\frac{1}{2}$ $\quad$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$

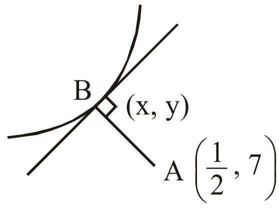
(3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ $\quad$ (4) $\frac{\sqrt{5}}{6}$

Ans. **(3)**

Sol. $y - x^{3/2} = 7 \ (x \geq 0)$

$\frac{dy}{dx}=\frac{3}{2}x^{1/2}$

$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right)=-1$



$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right)=-1$

$\frac{3}{2}\cdot x^2=\frac{1}{2}-x$

$3x^2 = 1 - 2x$

$3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$(x + 1)(3x - 1) = 0$

$\therefore$ x = –1 (rejected)

$x = \frac{1}{3}$

$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$

$\ell_{AB} = \sqrt{\left(\frac{1}{2}-\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3}=\sqrt{\frac{1}{36}+\frac{1}{27}}$

$=\sqrt{\frac{3+4}{9\times12}}$

$=\sqrt{\frac{7}{108}}=\frac{1}{6}\sqrt{\frac{7}{3}}$

Option (3)

28. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

(1) $-4x^3 - 1$ (2) $4x^3 + 1$

(3) $-2x^3 - 1$ (4) $-2x^3 + 1$

Ans. **(1)**

Sol. $\int x^5 \cdot e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$

Put $x^3 = t$

$3x^2\, dx = dt$

$\int x^3 \cdot e^{-4x^3} \cdot x^2 dx$

$\frac{1}{3}\int t \cdot e^{-4t}\, dt$

$\frac{1}{3}\left[t \cdot \frac{e^{-4t}}{-4} - \int \frac{e^{-4t}}{-4} dt\right]$

$-\frac{e^{-4t}}{48}[4t+1]+c$

$\frac{-e^{-4x^3}}{48}[4x^3+1]+c$

$\therefore$ $f(x) = -1 - 4x^3$

Option (1)

(From the given options (1) is most suitable)

29. The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy\, dy = 0$ which passes through (1,1) is :

(1) A circle with centre on the y-axis

(2) A circle with centre on the x-axis

(3) An ellipse with major axis along the y-axis

(4) A hyperbola with transverse axis along the x-axis

Ans. **(2)**

Sol. $(x^2 - y^2)\, dx + 2xy\, dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Solving we get,

$\int \frac{2v}{v^2+1} dv = \int -\frac{dx}{x}$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$\boxed{y^2 + x^2 = 2x}$

$\therefore$ Option (2)

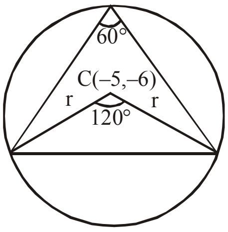
30. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(1) 20 (2) 25

(3) 13 (4) –25

Ans. (2)

Sol. $3\left(\frac{1}{2}r^2.\sin 120°\right) = 27\sqrt{3}$



$$\frac{r^2}{2}\frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$$

$$r^2 = \frac{108}{3} = 36$$

Radius = $\sqrt{25+36-C} = \sqrt{36}$

$\boxed{C = 25}$

$\therefore$ Option (2)