JEE Main 2020 Paper

Date of Exam: **8th January (Shift II)**
Time: **2:30 pm – 5:30 pm**
Subject: **Physics**

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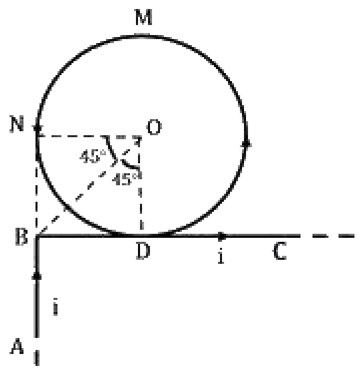
1. A very long wire ABDMNDC is shown in figure carrying current i. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R. AB, BC are tangential to circular turn at N and D. Magnetic field at the centre of circle is



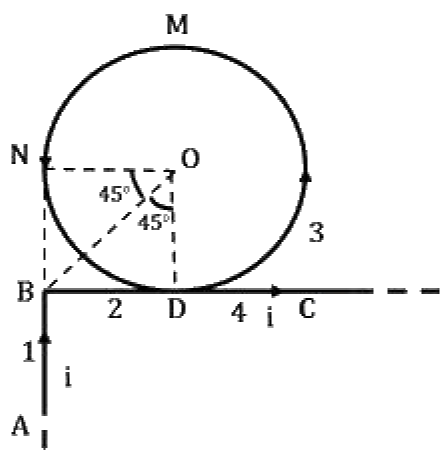
a. $\frac{\mu_0 i}{2\pi R}\left[\pi - \frac{1}{\sqrt{2}}\right]$

b. $\frac{\mu_0 i}{2\pi R}[\pi + 1]$

c. $\frac{\mu_0 i}{2R}$

d. $\frac{\mu_0 i}{2\pi R}\left[\pi + \frac{1}{\sqrt{2}}\right]$

Solution: (d)



To get magnetic field at O, we need to find magnetic field due to each current carrying part 1, 2, 3 and 4 individually.

Let's take total magnetic field as B_T, then

$\vec{B}_T = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$

Since 2 and 4 are parts of same wire, hence

$$\vec{B}_T = \frac{\mu_o i}{4\pi R}(\sin 90° - \sin 45°)(-\hat{k}) + \frac{\mu_o i}{2R}\hat{k} + \frac{\mu_o i}{4\pi R}(\sin 90° + \sin 45°)\hat{k}$$

$$= \frac{-\mu_o i}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right] + \frac{\mu_o i}{2R} + \frac{\mu_o i}{4\pi R}\left[1 + \frac{1}{\sqrt{2}}\right]\hat{k}$$

$$\vec{B}_T = \frac{\mu_o i}{4\pi R}\left[\sqrt{2} + 2\pi\right]\hat{k}$$

$$\vec{B}_T = \frac{\mu_o i}{2\pi R}\left[\frac{1}{\sqrt{2}} + \pi\right]\hat{k}$$

$\hat{k}$ denotes that direction of magnetic field is in the plane coming out of the plane of current.

2. A particle moves such that its position vector $\vec{r}(\mathrm{t}) = \cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}$, where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle?
a. $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$
b. $\vec{v}$ and $\vec{r}$ both are parallel to $\vec{r}$
c. $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin
d. $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

Solution :(d)

$$\vec{r} = \cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}$$
$$\vec{v} = \frac{d\vec{r}}{dt} = \omega[-\sin\omega t\,\hat{\imath} + \cos\omega t\,\hat{\jmath}]$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2[\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}]$$
$$\vec{a} = -\omega^2\vec{r}$$

Since there is negative sign in acceleration, this means that acceleration is in opposite direction of $\vec{r}$

For velocity direction we can take dot product of $\vec{v}$ and $\vec{r}$.

$$\vec{v}.\vec{r} = \omega(-\sin\omega t\,\hat{\imath} + \cos\omega t\,\hat{\jmath}).(\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath})$$
$$= \omega[-\sin\omega t\cos\omega t + \cos\omega t\sin\omega t] = 0$$

This implies that $\vec{v}$ is perpendicular to $\vec{r}$.

3. Consider two charged metallic spheres S_1 and S_2 of radii r_1 and r_2, respectively. The electric fields E_1(on S_1) and E_2(on S_2) on their surfaces are such that $\frac{E_1}{E_2} = \frac{r_1}{r_2}$. Then the ratio V_1(on S_1)/V_2(on S_2) of the electrostatic potentials on each sphere is

a. $\frac{r_1}{r_2}$
b. $\left(\frac{r_1}{r_2}\right)^2$
c. $\frac{r_2}{r_1}$
d. $\left(\frac{r_1}{r_2}\right)^3$

Solution: (b)

$$\frac{E_1}{E_2} = \frac{r_1}{r_2}$$

$$\frac{V_1}{V_2} = \frac{E_1 r_1}{E_2 r_2} = \frac{r_1}{r_2} \times \frac{r_1}{r_2} = \left(\frac{r_1}{r_2}\right)^2$$

4. A transverse wave travels on a taut steel wire with a velocity of V when tension in it is $2.06 \times 10^4\ N$. When the tension is changed to T, the velocity changed to $\frac{V}{2}$. The value of T is close to

a. $30.5 \times 10^4\ N$
b. $2.50 \times 10^4\ N$
c. $10.2 \times 10^2\ N$
d. $5.15 \times 10^3 N$

Solution :(d)

$$V \propto \sqrt{T}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{2V}{V} = \sqrt{\frac{2.06 \times 10^4}{T}}$$

$$\Rightarrow T = \frac{2.06 \times 10^4}{4}\ N = 5.15 \times 10^3\ N$$

5. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is

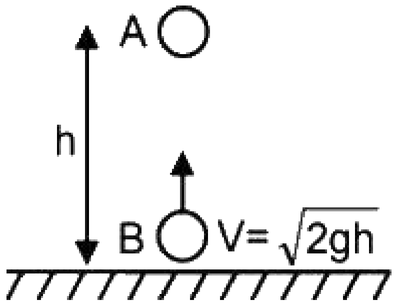
a. $\sqrt{\frac{3}{2}}$

b. $\sqrt{\frac{1}{2}}$

c. $\frac{1}{2}$

d. $\sqrt{\frac{3}{4}}$

Solution: (a)



Time taken for the collision $t_1 = \frac{h}{\sqrt{2gh}}$

After t_1

$$V_A = 0 - gt_1 = -\frac{\sqrt{gh}}{2}$$

And $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right]$

At the time of collision

$$\overrightarrow{P_i} = \overrightarrow{P_f}$$

$\Rightarrow m\overrightarrow{V_A} + m\overrightarrow{V_B} = 2\,m\overrightarrow{V_f}$

$$\Rightarrow -\sqrt{\frac{gh}{2}} + \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right] = 2\overrightarrow{V_f}$$

$$V_f = 0$$

And height from the ground $= h - \frac{1}{2}g\,t_1^2 = h - \frac{h}{4} = \frac{3h}{4}$

So, time taken to reach ground after collision $= \sqrt{2 \times \frac{\left(\frac{3h}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$

6. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is

a. 99 J
b. 90 J
c. 1 J
d. 100 J

Solution :(b)

For Carnot engine using as refrigerator

Work done on engine is given by

$W = Q_1 - Q_2$ ... (1)

where Q_1 is heat rejected to the reservoir at higher temperature and Q_2 is the heat absorbed from the reservoir at lower temperature.

It is given η = 1/10

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{9}{10} \; ..... (2)$$

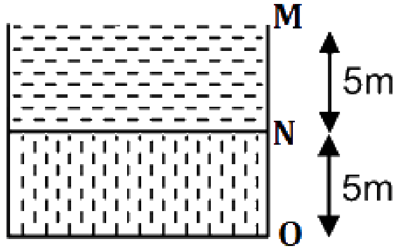
We are given, $W = 10$ J

Therefore, from equations (1) and (2),

$$Q_2 = \frac{10}{\frac{10}{9} - 1}$$

$$\Rightarrow Q_2 = 90 \text{ J}$$

7. Two liquids of density ρ_1 and ρ_2 ($\rho_2 = 2\rho_1$) are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of forces due to these liquids exerted on the upper part MN to that at the lower part NO is (Assume that the liquids are not mixing)



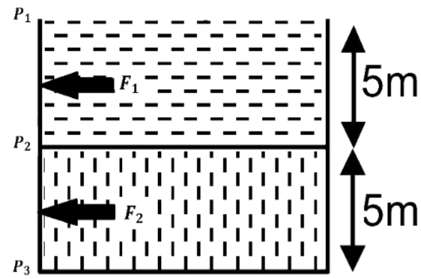
a. $\frac{2}{3}$

b. $\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{1}{3}$

Solution:(c)



The net force exerted on the wall by one type of liquid will be average value of pressure due to that liquid multiplied by the area of the wall.

Here, since the pressure due a liquid of uniform density varies linearly with depth, its average will be just the mean value of pressure at the top and pressure at the bottom.

So,

$$P_1 = 0$$

$$P_2 = \rho g \times 5$$

$$P_3 = 5\rho g + 2\rho g \times 5$$

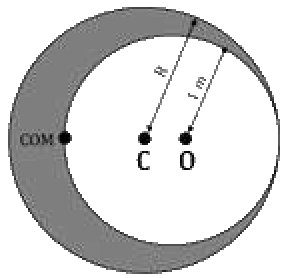
$$F_1 = \left(\frac{P_1 + P_2}{2}\right) A$$

$$F_2 = \left(\frac{P_2 + P_3}{2}\right) A$$

So,

$$\frac{F_1}{F_2} = \frac{1}{4}$$

8. As shown in figure, when a spherical cavity (centered at O) of radius 1 m is cut out of a uniform sphere of radius R (centered at C), the center of mass of remaining (shaded) part of sphere is shown by COM, i.e. on the surface of the cavity. R can be determined by the equation



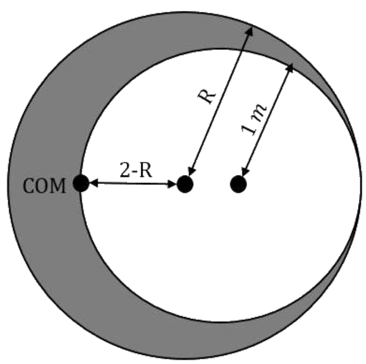
a. $(R^2 + R + 1)(2 - R) = 1$

b. $(R^2 - R - 1)(2 - R) = 1$

c. $(R^2 - R + 1)(2 - R) = 1$

d. $(R^2 + R - 1)(2 - R) = 1$

Solution (a)



Let M be the mass of the sphere and M' be the mass of the cavity.

Mass of the remaining part of the sphere = $M - M'$

Mass moments of the cavity and the remaining part of sphere about the original COM should add up to zero.

$$(M - M')(2 - R) - M'(R - 1) = 0$$

(Mass of the cavity to be taken negative)

$$\Rightarrow \frac{4}{3}\pi(R^3 - 1^3)\rho g\ (2-R) = \frac{4}{3}\pi(1)^3\rho g(R-1)$$

$$\Rightarrow (R^3 - 1^3)(2-R) = \ (1^3)(R-1)$$

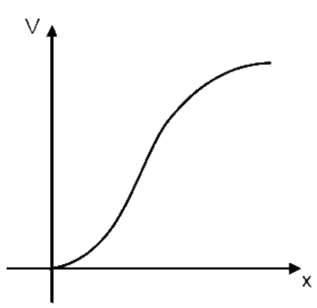
$$\Rightarrow (R^2 + R + 1)(R-1)(2-R) = \ (R-1)$$

(using identity)

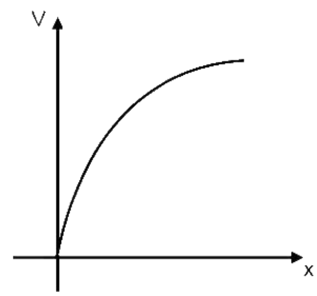
$$\Rightarrow (R^2 + R + 1)(2-R) = \ 1$$

9. A particle of mass m and charge q is released from rest in uniform electric field. If there is no other force on the particle, the dependence of its speed V on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

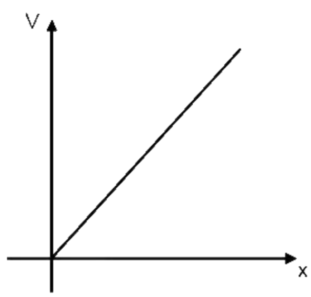
a.



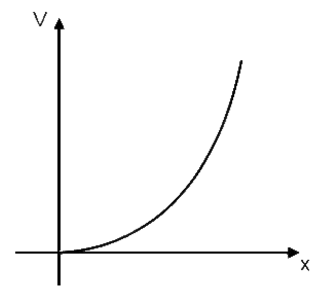
b.



c.



d.



Solution. (b)

Applying work energy theorem,

$$qEx = \frac{1}{2}mv^2$$

$$v^2 \propto x$$

Hence, solution will be option (b)

10. A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into voltmeter giving full scale deflection for a potential difference of 10 V? In full scale deflection, current in galvanometer of resistance is 1 mA. Resistance required in series to convert it into voltmeter of range 10 V.

a. 7.9 $k\Omega$
b. 9.9 $k\Omega$
c. 8.9 $k\Omega$
d. 10 $k\Omega$

Solution: (b)

$$V_0 = i_g R_g = 0.1\ v$$

$$v = 10\ V$$

$$R = R_g\left(\frac{v}{v_g} - 1\right)$$

$$= 100 \times 99 = 9.9\ k\Omega$$

11. Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its $\left(\frac{Cp}{Cv}\right)$ value will be

a. $\frac{67}{45}$
b. $\frac{40}{27}$
c. $\frac{19}{13}$
d. $\frac{23}{15}$

Solution: (c)

Using formula

$$C_{v_{mix}} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

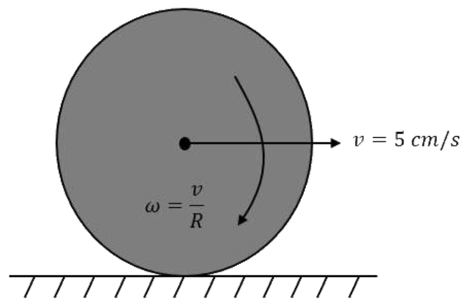
$$C_{v_{mix}} = \frac{n\times\frac{3R}{2} + 2n\times\frac{5R}{2}}{3n} = \frac{13R}{6} \quad (\because He \text{ is monoatomic and } O_2 \text{ is diatomic})$$

$$C_{p_{mix}} = C_{v_{mix}} + R = \frac{19R}{6}$$

$$\therefore \gamma_{mix} = \frac{C_{p_{mix}}}{C_{v_{mix}}} = \frac{19}{13}$$

12. A uniform sphere of mass 500 gm rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5 cm/sec. Its kinetic energy is
    a. $8.75 \times 10^{-4}\, J$
    b. $6.25 \times 10^{-4}\, J$
    c. $8.75 \times 10^{-3}\, J$
    d. $1.13 \times 10^{-3}\, J$

Solution :(a)



Total K.E. = Translational K.E + Rotational K.E.

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

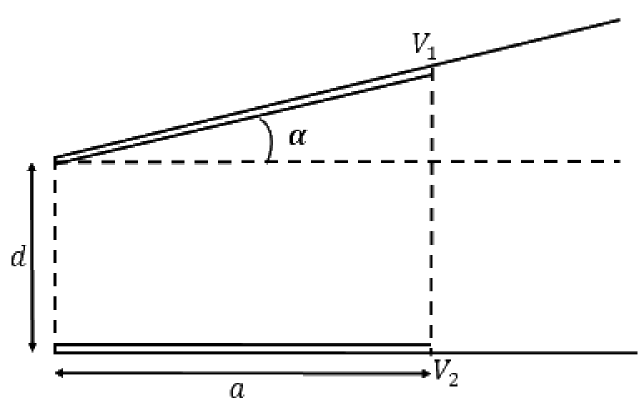
$$= \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

k is radius of gyration.

$$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{5}{100}\right)^2 \left(1 + \frac{2}{5}\right)$$

$$= 8.75 \times 10^{-4}\, J$$

13. A capacitor is made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to



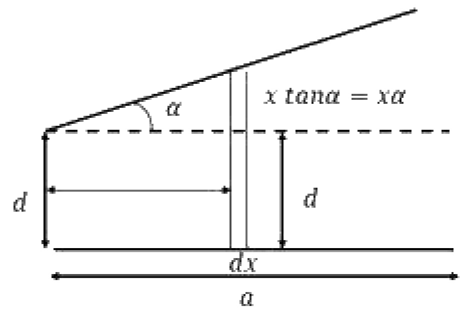
a. $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

b. $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$

c. $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$

d. $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$

Solution :(a)



Let dC be the capacitance of the element of thickness dx

$$dc = \frac{\varepsilon_0 a dx}{d + \alpha x}$$

These are effectively in parallel combination

So,

$$C = \int dc$$

$$C = \int_0^a \frac{\varepsilon_0 a dx}{d + \alpha x}$$

$$\Rightarrow C = \frac{\varepsilon_0 a}{\alpha}\left[\ln(d + \alpha x)\right]_0^a$$

$$C = \frac{\varepsilon_0 a}{\alpha}\left[\ln\left(1 + \frac{\alpha a}{d}\right)\right]$$

$$C \approx \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

14. In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}th$ of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is

a. 0.568

b. 0.853

c. 0.672

d. 0.760

Solution: (b)

In YDSE, the intensity at a point on the screen varies with the phase difference between the interfering light waves as:

$$I = I_0\, cos^2\left(\frac{\Delta\emptyset}{2}\right)$$

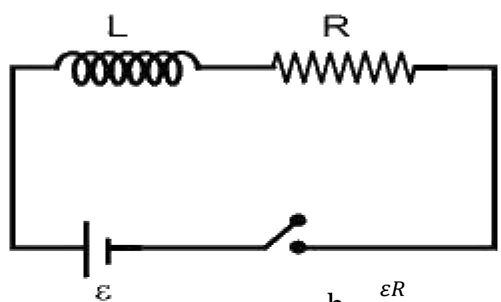
Here, $\Delta\emptyset$ = phase difference between the interfering waves

I_0 = maximum intensity on the screen

$$\frac{I}{I_0} = cos^2\left[\frac{\frac{2\pi}{\lambda}\times\Delta x}{2}\right] = \cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = 0.853$$

15. As shown in figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at $t = 0$. The total charge that flows from thebattery, between $t = 0$ and $t = t_c$ (t_c is the time constant of the circuit) is



a. $\frac{\varepsilon L}{eR^2}$

b. $\frac{\varepsilon R}{eL^2}$

c. $\frac{\varepsilon L}{R^2}(1-\frac{1}{e})$

d. $\frac{\varepsilon L}{R^2}$

Solution :(a)

This is standard $L - R$ growth of current circuit.

$$i(t) = \frac{\varepsilon}{R}e^{-\frac{t}{T_c}}$$

Substituting in the integral

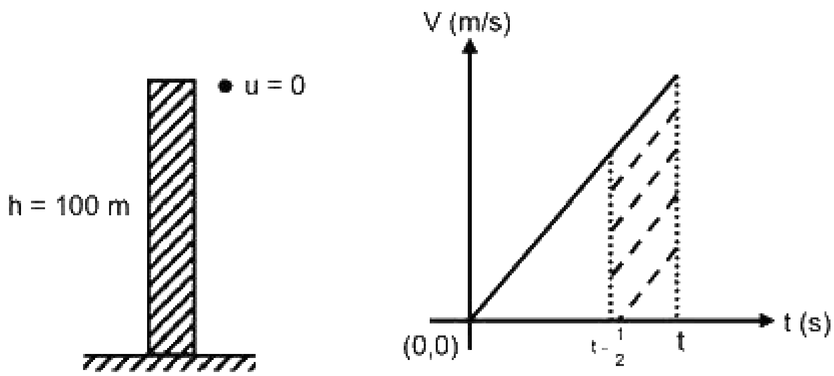
$$q = \int_0^{T_c} (i)dt$$

$$= \frac{\varepsilon}{R}\left[t - \frac{e^{\frac{-t}{T_c}}}{\frac{-1}{T_c}}\right]_0^{T_c}$$

$$= \frac{\varepsilon L}{eR^2}$$

16. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is

Solution: $g = 8\, m/s^2$



Since the radius of planet is much larger than100 m, it's a uniformly accelerated motion.

So, Trapezium's area

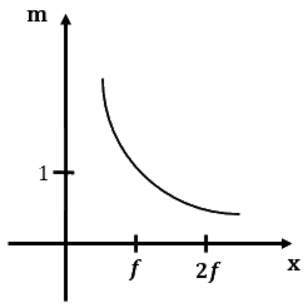
$$s = \frac{g(\left[t-\frac{1}{2}+t\right]}{2} \times \frac{1}{2} = 19 \qquad \text{(i)}$$

$$\frac{1}{2}gt^2 = 100 \qquad \text{(ii)}$$

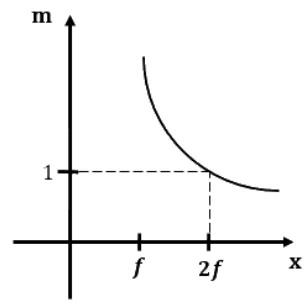
Solving equations (i) and (ii), we get

$$g = 8\, m/s^2$$

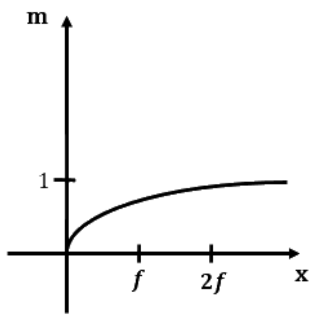
17. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by
(Graphs are drawn schematically and are not to scale)



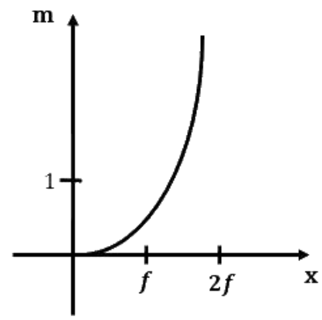
a.



b.



c.



d.

Solution: (b)

Magnitude of linear magnification, m= $\left|-\frac{v}{x}\right|$

From mirror formula,

$$\frac{1}{v}+\frac{1}{x}=\frac{1}{f}$$

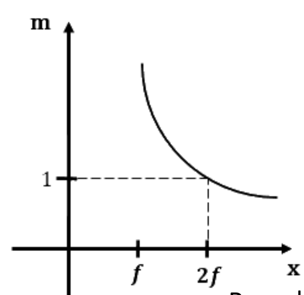
Multiplying the whole equation by u, we get

$$\frac{v}{x}=\frac{f}{x-f}$$

Using the above equation in magnification formula,

$$m=\left|-\frac{v}{x}\right|=\left|\frac{f}{f-x}\right|$$

For $x=2f$, $\quad m=1$



Thus option (b) is correct

18. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1 sec resolution measures the time taken for 40 oscillations to be 50 sec. The accuracy in g

a. 5.40% b. 3.40%
c. 4.40% d. 2.4%

Solution :(c)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$g = L.\left(\frac{2\pi}{T}\right)^2$$

$$\frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta L}{L}$$

$$2\left(\frac{1}{50}\right) + \frac{0.1}{25} = 4.40\%$$

19. An electron (mass m) with initial velocity $\vec{v} = v_o\hat{\imath} + v_o\hat{\jmath}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wavelength at time t is given by

a. $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$ b. $\frac{\lambda_0\sqrt{2}}{\sqrt{1+\frac{e^2E^2t^2}{m^2v_0^2}}}$

c. $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{2m^2v_0^2}}}$ d. $\frac{\lambda_0}{\sqrt{2+\frac{e^2E^2t^2}{m^2v_0^2}}}$

Solution :(a)

Momentum of an electron, $p = mv = \frac{h}{\lambda}$

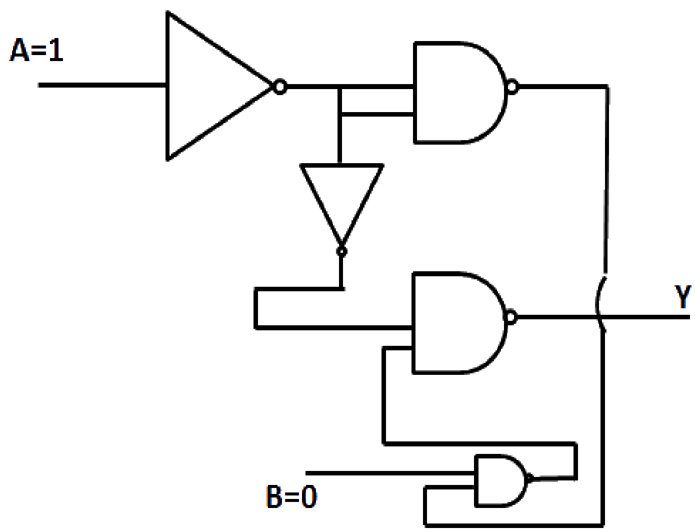
Initially $m(\sqrt{2}v_0) = \frac{h}{\lambda_0}$

Velocity as a function of time=$v_0\hat{\imath} + v_0\hat{\jmath} + \frac{eE_0}{m}t\hat{k}$

So, wavelength $\lambda = \frac{h}{m\sqrt{2v_0^2+\frac{e^2E_0^2}{m^2}t^2}}$

$$\Rightarrow \lambda = \frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2}{m^2v_0^2}t^2}}$$

20. In the given circuit, value of Y is



a. 1
b. 0
c. Will not execute
d. Toggles between 0 and 1

Solution: (b)

$$Y = \overline{\overline{AB}.A}$$

$$= \overline{\overline{AB}} + \bar{A}$$

$$= AB + \bar{A}$$

$$Y = 0 + 0 = 0$$

21. The first member of Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is

Solution: (486 nm)

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{36}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\lambda_2 = \frac{20}{27} \times 6561 \text{ Å} = 486 \text{ nm}$$

22. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B} = 5 \times 10^{-8}\hat{j}\ T$. The corresponding electric field is given by $\vec{E}$ is (speed of light $c = 3 \times 10^8\ m/s$)

a. $1.66 \times 10^{-16}\hat{\imath}$ V/m
b. $-1.66 \times 10^{-16}\hat{\imath}$ V/m
c. $15\hat{\imath}$ V/m
d. $-15\hat{\imath}$ V/m

Solution: (c)

$$\frac{|E|}{|B|} = c$$

$$|E| = |B| \times c$$

Given,
$\vec{B} = 5 \times 10^{-8}\hat{j}\ T$ and $C = 3 \times 10^8 \frac{m}{s}$

Therefore, $|E| = 15\ \frac{volt}{m}$

Direction of $\vec{E}$ will be perpendicular to $\vec{B}$ and $\vec{E} \times \vec{B}$ gives direction of propagation of wave which is in z-direction ($\hat{k}$). Therefore, $\vec{E}$ will be along positive x-direction ($\hat{\imath}$).
Hence, $\vec{E} = 15\hat{\imath}$ V/m

23. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process)

C_1	C_2	C_3	$T(^0C)$
$1\ l$	$2\ l$	--	60
--	$1\ l$	$2\ l$	30
$2\ l$	--	$1\ l$	60
$1\ l$	$1\ l$	$1\ l$	θ

The value of θ (in °C to the nearest integer) is

Solution: (50)

Since, all the containers have same material, specific heat capacity is the same for all.

$$V_1\theta_1 + V_2\theta_2 = (V_1+V_2)\theta_f$$

From second row of table,

$$1\theta_1 + 2\theta_2 = (1+2)60$$

$\theta_1 + 2\theta_2 = 180$…………… (1)

From third row of table,

$$0 \times \theta_1 + 1 \times \theta_2 + 2 \times \theta_3 = (1+2)30$$

$\theta_2 + 2\theta_3 = 90$…………… (2)

From fourth row of table,

$$2\theta_1 + \theta_3 = (1+2)60$$

$2\theta_1 + \theta_3 = 180$…………… (3)

From fifth row of table,

$$\theta_1 + \theta_2 + \theta_3 = (1+1+1)\theta$$

$\theta_1 + \theta_2 + \theta_3 = 3\theta$………… (4)

From equation. (1)+ (2) + (3)

$$3\theta_1 + 3\theta_2 + 3\theta_3 = 450$$

Thus, $\theta_1 + \theta_2 + \theta_3 = 150$

From (4) equation $150 = 3\theta$

So, $\theta = 50^0C$

24. An asteroid is moving directly towards the centre of the earth. When at a distance of 10R (R is the radius of the earth) from the earth's centre, it has a speed of 12 km/s. Neglecting the effect of earth's atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 12 km/s)? Give your answer to the nearest integer in km/s

Solution: (16)

Taking, asteroid and earth as an isolated system conserving total energy.

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mu_0^2 + \left(-\frac{GMm}{10R}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$

$$v^2 = u_0^2 + \frac{2GM}{R}\left[1 - \frac{1}{10}\right]$$

$$v = \sqrt{u_0^2 + \frac{9}{5}\frac{GM}{R}}$$

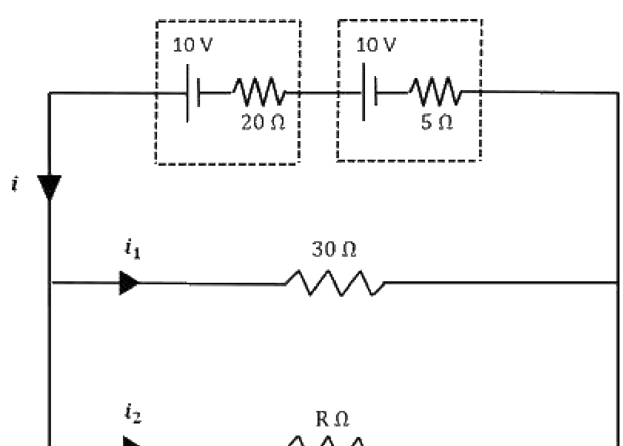
Since, escape velocity from surface of earth is $11.2\frac{km}{sec^2} = \sqrt{\frac{2GM}{R}}$

$$= \sqrt{12^2 + \frac{9}{5}\frac{(11.2)^2}{2}}$$

$$= \sqrt{256.9} \approx 16\ km/s$$

25. The series combination of two batteries both of the same emf 10 V, but different internal resistance of 20 Ω and 5 Ω, is connected to the parallel combination of two resistors 30 Ω and R Ω. The voltage difference across the battery of internal resistance 20 Ω is zero, the value of R (in Ω) is

Solution: (30)



If V_1 and V_2 are terminal voltage across the two batteries.

$$V_1 = 0$$

$$V_1 = \varepsilon_1 - i.r_1$$

$$0 = 10 - i \times 20$$

$$i = 0.5\ A$$

$$V_2 = 10 - 0.5 \times 5$$

$$V_2 = 7.5\ V$$

$$0.5 = \frac{7.5}{30} + \frac{7.5}{x}$$

$$\frac{7.5}{x} = 0.25$$

$$x = 30\ \Omega$$

JEE Main 2020 Paper

Date: 8th January 2020

Time: 02:30 PM – 05:30 PM

Subject: Chemistry

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1. Arrange the following bonds according to their average bond energies in descending order:
C-Cl, C-Br, C-F, C-I
   a. C-Cl > C-Br > C-I > C-F
   b. C-Br>C-I>C-Cl>C-F
   c. C-I>C-Br>C-Cl>C-F
   d. C-F>C-Cl>C-Br>C-I

Answer: d

Solution: In $\mathrm{C-F}$ there is 2p-2p overlapping involved, in $\mathrm{C-Cl}$ the overlapping involved is 2p-3p whereas for $\mathrm{C-Br}$ and $\mathrm{C-I}$ the overlappings involved are 2p-4p and 2p-5p, respectively. The bond length for the various type of overlappings can be given as:

2p-2p < 2p-3p < 2p-4p < 2p-5p.

As we know that Bond energy $\alpha \frac{1}{\text{Bond length}}$

The order of bond energy comes out: C-F > C-Cl > C-Br > C-I

2. The radius of second Bohr orbit, in terms of the Bohr radius, a_0 , in Li^{2+} is:
   a. $\frac{2a_0}{3}$
   b. $\frac{4a_0}{9}$
   c. $\frac{4a_0}{3}$
   d. $\frac{2a_0}{9}$

Answer: c

Solution: The formula for Bohr's radius for any one electron species is: $\mathrm{r} = \frac{\mathrm{a_0 n^2}}{\mathrm{z}}$

for $\mathrm{Li^{2+}}$: $\mathrm{r} = \frac{\mathrm{a_0 2^2}}{3} = \frac{4\mathrm{a_0}}{3}$

3. A metal (A) on heating in nitrogen gas gives compound B. B on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a dark blue-violet coloured solution. A and B respectively, are :

   a. Na and Na_3N
   b. Mg and Mg_3N_2
   c. Mg, $Mg(NO_3)_2$
   d. Na, $NaNO_3$

Answer: b

Solution: As it is provided in the question that nitride is being formed so the option c and d can be eliminated. Amongst Mg and Na we already know that Mg can only form nitride so the correct choice is option a.

$$\mathrm{3Mg + N_2 \rightarrow Mg_3N_2 \xrightarrow{H_2O} Mg(OH)_2 + NH_3}$$

4. The correct order of the calculated spin-only magnetic moments of complexes A to D is:

A. $Ni(CO)_4$
B. $[Ni(H_2O)_6]^{2+}$
C. $Na_2[Ni(CN)_4]$
D. $PdCl_2(PPh_3)_2$

a. (C) < (D) < (B) < (A)
b. (A) ≈ (C) ≈ (D) < (B)
c. (A) ≈ (C)<(B) ≈ (D)
d. (C) ≈ (D) < (B) < (A)

Answer: b

Solution: $[Pd(PPh_3)_2Cl_2]$: Here Pd is in +2 oxidation state and configuration of Pd^{2+} is $[Kr]4d^8$. As the CFSE value for Pd is very high so all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CO)_4]$: Here Ni is in 0 oxidation state and configuration of Ni is $[Ar]3d^84s^2$. As here the ligand is carbonyl which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CN)_4]^{2-}$: Here Ni is in +2 oxidation state and configuration of Ni^{2+} is $[Ar]3d^8$. As here the ligand is cyanide which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(H_2O)_6]^{2+}$: Here Ni is in +2 oxidation state and configuration of Ni^{2+} is $[Ar]3d^8$. As here the ligand is water which is a weak field ligand, the electrons will not be paired and there are two unpaired electrons in this complex hence magnetic moment for this complex will be $\sqrt{8}$ BM.

So the order of magnetic moment is A=B=C<D.

5. Hydrogen has three isotopes (A), (B) and (C). If the number of neutron(s) in (A), (B) and (C) respectively, are (x), (y) and (z), the sum of (x), (y) and (z) is:

a. 4
b. 1
c. 3
d. 2

Answer: c

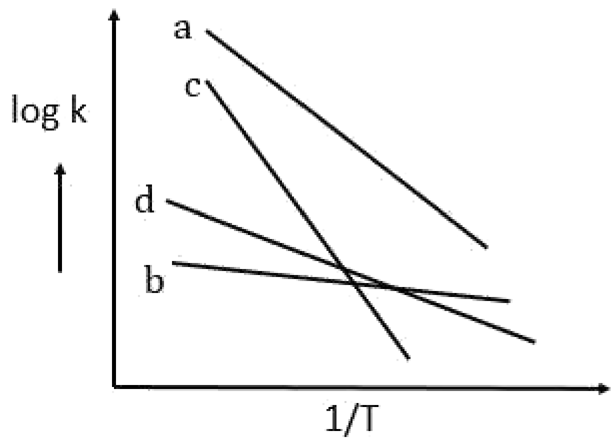
Solution: Number of neutrons in protium = 0

Number of neutrons in deuterium = 1

Number of neutrons in tritium = 2

So, total number of neutrons = 3

6. Consider the following plots of rate constant versus $\frac{1}{T}$ for four different reactions. Which of the following orders is correct for the activation energies of these reactions?



a. $E_a > E_c > E_d > E_b$
b. $E_c > E_a > E_d > E_b$
c. $E_b > E_d > E_c > E_a$
d. $E > E_a > E_d > E_c$

Answer: b

Solution:

To avoid confusion, in this question we'll be denoting activation energy by E_x

$K = Ae^{-E_x/RT}$

$\log K = \log A - \frac{E_x}{2.303RT}$ -----(1)

Here, the graph given in the question is of a straight line and we know that the equation of straight line is
$y = mx + c$ ----(2)
Comparing equation 1 with 2 we get,

$\text{Slope} = \frac{-E_x}{2.303R}$

So, from the graph we can conclude that the line with the most negative slope will have the maximum activation energy value.

$E_c > E_a > E_d > E_b$

7. Which of the following compounds is likely to show both Frenkel and Schottky defects in its crystalline form?

a. ZnS
b. CsCl
c. KBr
d. AgBr

Answer: d

Solution: The radius ratio for AgBr is intermediate. Thus, it shows both Frenkel and Schottky defects.

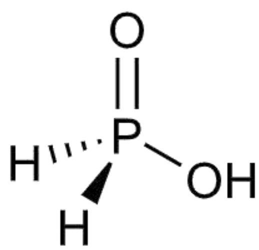
8. White phosphorus on reaction with concentrated NaOH solution in an inert atmosphere of CO_2 gives phosphine and compound (X). (X) on acidification with HCl gives compound (Y). The basicity of compound (Y) is:

a. 4
b. 2
c. 3
d. 1

Answer: d

Solution: $P_4 + NaOH + H_2O \rightarrow PH_3 + \boxed{NaH_2PO_2 \xrightarrow{HCl} H_3PO_2 + NaCl}$

Here the product B which is mentioned in the question is H_3PO_2. The structure of H_3PO_2 can be given as:



As only 1 Hydrogen atom is attached to the oxygen, its basicity is one.

9. Among the reactions (a) – (d), the reaction(s) that does/do not occur in the blast furnace during the extraction of iron is/are:

A. $CaO + SiO_2 \rightarrow CaSiO_3$
B. $3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$
C. $FeO + SiO_2 \rightarrow FeSiO_3$
D. $FeO \rightarrow Fe + \frac{1}{2} O_2$

a. A
b. D
c. C and D
d. A and D

Answer: c

Solution: In metallurgy of iron, CaO is used as flux which is used to remove the impurities of SiO_2,
$CaO + SiO_2 \rightarrow CaSiO_3$.

Also here Fe_2O_3 is reduced by CO to Fe_3O_4 which is further reduced to FeO which is further reduced to Fe.

$3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

$Fe_3O_4 + CO \rightarrow 3FeO + CO_2$

$FeO + CO \rightarrow Fe + CO_2$

10. The increasing order of the atomic radii of the following elements is:

A. C B. O C. F D. Cl E. Br

a. B<C<D<A<E
b. C<B<A<D<E
c. A<B<C<D<E
d. D<C<B<A<E l

Answer: a

Solution:

Across the period size decreases, so the order that follows is: C > O >N> F

Down the group size increases and the order is: Br > Cl > F

Change in size down the group is much more significant as compared to across the period.

So, the overall order of radius of elements is: Br > Cl > C > O > F.

11. Among (a) – (d), the complexes that can display geometrical isomerism are:

A. $[Pt(NH_3)_3Cl]^+$
B. $[Pt(NH_3)Cl_5]^-$
C. $[Pt(NH_3)_2Cl(NO_2)]$
D. $[Pt(NH_3)_4ClBr]^{2+}$

a. A and B
b. D and A
c. C and D
d. B and C

Answer: a

Solution: The complexes of type Ma_4bc and Ma_2b_2 can show geometrical isomerism provided Ma_2b_2 is square planar. The compound given in B is Ma_4bc type and compound in D is Ma_2b_2 type also in D, Ni is surrounded with strong field ligands which will result in dsp^2 hybridisation and hence square planar geometry.

12. For the following Assertion and Reason, the correct option is:

Assertion: The pH of water increases with increase in temperature.

Reason: The dissociation of water into H^+ and OH^- an exothermic reaction.

a. Both assertion and reason are false.
b. Assertion is not true, but reason is true.
c. Both assertion and reason are true and the reason is the correct explanation for the assertion.
d. Both assertion and reason are true, but the reason is not the correct explanation for the assertion.

Answer: d

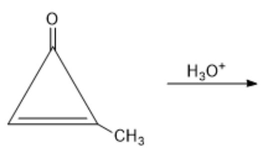
Solution: $H_2O \rightarrow H^+ + OH^-$ is an endothermic process. On increasing the temperature the value of K_w increases which will result in decrease in pK_w . So we can say that pH of water will decrease on increasing temperature because pH for water $=\frac{1}{2}pK_w$.

13. For the following Assertion and Reason, the correct option is:
Assertion: For hydrogenation reactions, the catalytic activity increases from group-5 to group-11 metals with maximum activity shown by group 7-9 elements
Reason: The reactants are most strongly adsorbed on group 7-9 elements
a. Both assertion and reason are false.
b. The assertion is true, but the reason is false.
c. Both assertion and reason are true, but the reason is not the correct explanation of assertion
d. Both assertion and reason are true and the reason is the correct explanation of assertion

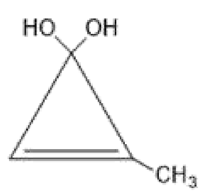
Answer: a

Solution: Group 7-9 elements of the periodic table show variable valencies so they have maximum activity because of the increase in adsorption rate.

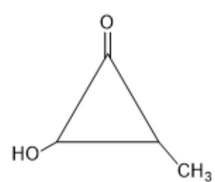
14. The major product of the following reactions is:



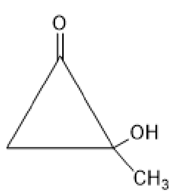
a.



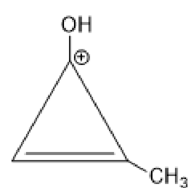
b.



c.

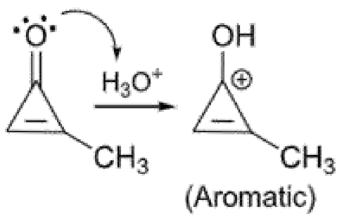


d.

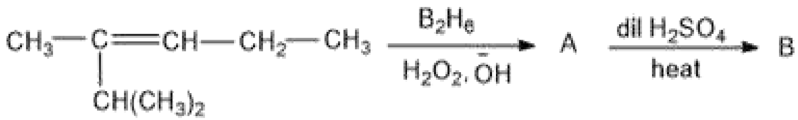


Answer: d

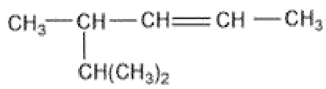
Solution:



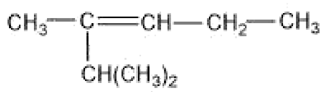
15. Find The major product [B] of the following sequence of reactions is:



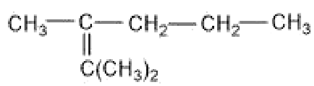
a.



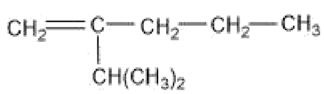
b.



c.

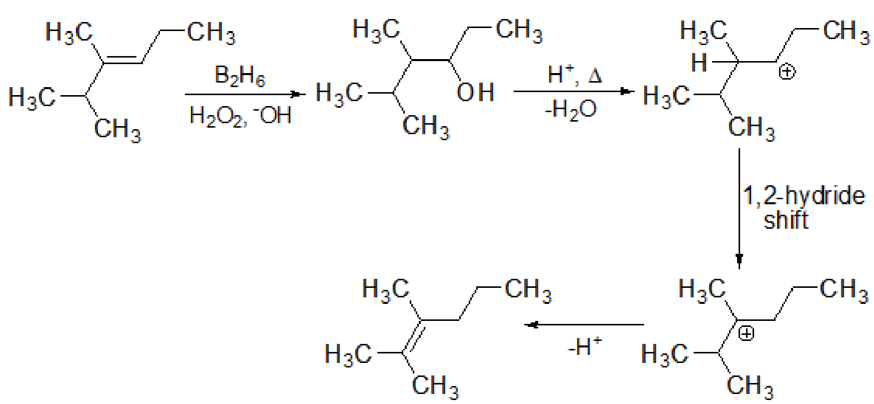


d.



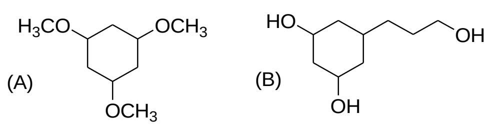
Answer: c

Solution:

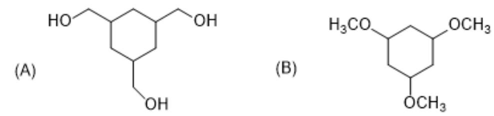


16. Among the following compounds A and B with molecular formula $C_9H_{18}O_3$, A is having higher boiling point than B. The possible structures of A and B are:

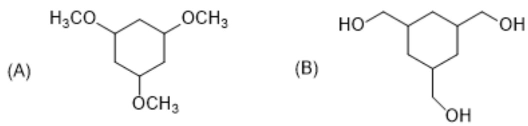
a.



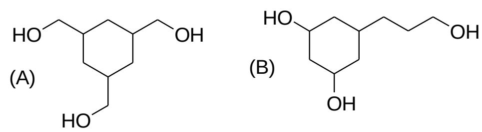
b.



c.



d.

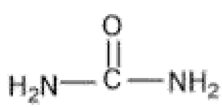


Answer: b

Solution: In option b compound A has extensive inter-molecular hydrogen bonding because of the 3 $-$OH groups while in compound B there are $-OCH_3$ groups present and no inter-molecular hydrogen bonding is possible.

17. Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?

a. $CH_3CH_2 - C \equiv N$
b. $C_6H_5NH_2$
c. $C_6H_5NO_2$
d.



Answer: c

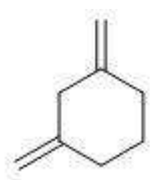
Solution: Kjeldahl method cannot be used for the estimation of nitrogen in the compounds in which nitrogen is involved in nitro, diazo groups or is present in the ring, as nitrogen atom can't be converted to ammonium sulphate under the reaction conditions.

18. An unsaturated hydrocarbon absorbs two hydrogen molecules on catalytic hydrogenation, and also gives following reaction;

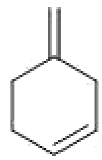
$X \xrightarrow{O_3,Zn/H_2O} A \xrightarrow{[Ag(NH_3)_2]^+} B$ (3-oxohexanedicarboxylic acid)

X will be:

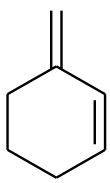
a.



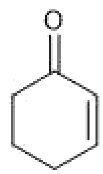
b.



c.

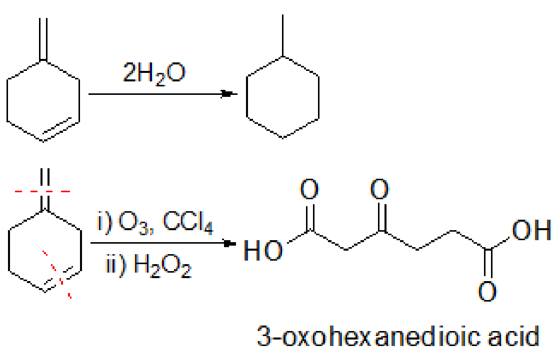


d.



Answer: b

Solution:

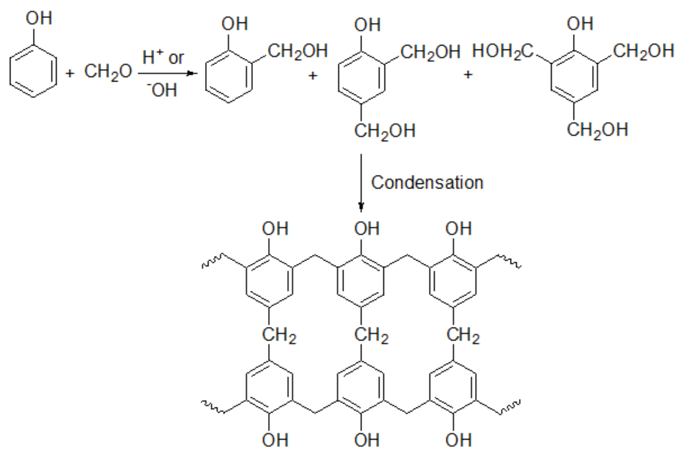


19. Preparation of Bakelite proceeds via reactions :

a. Electrophilic substitution and dehydration.
b. Electrophilic addition and dehydration.
c. Condensation and elimination
d. Nucleophilic addition and dehydration

Answer: a

Solution: Bakelite is a condensation polymer of phenol and formaldehyde.

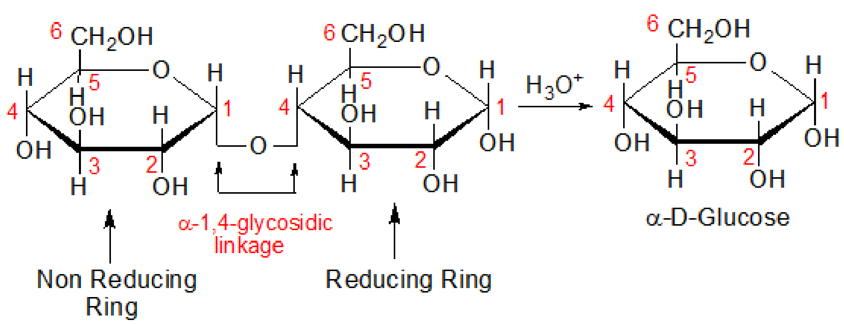


20. Two monomers of maltose are:

a. α-D-Glucose and α-D-Galactose
b. α-D-Glucose and α-D-Glucose
c. α-D-Glucose and α-D-Fructose
d. α-D-Glucose and β-D-Glucose

Answer: b

Solution:



Maltose is formed by the glycosidic linkage between C-1 of one α-D-Glucose unit to the C-4 of another α-D-Glucose.

21. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500 K changes its internal energy by 5000 J. The molar heat capacity at constant volume is _____. .

Answer: 6.25

Solution:

$\Delta U = nC_v\Delta T$

$5000 = 4 \times Cv\ (500 - 300)$

$Cv = 6.25\ JK^{-1}\ mol^{-1}$

22. For an electrochemical cell

$$Sn(s)|Sn^{2+}(aq., 1\ M)||\ Pb^{2+}(aq., 1M)|Pb(s)$$

the ratio $\frac{[Sn^{2+}]}{[Pb^{2+}]}$ when this cell attains equilibrium is ______.

(Given : $E^{o}_{Sn^{2+}/Sn}$ = -0.14 V ; $E^{o}_{Pb^{2+}/Pb}$ =-0.13 V, $\frac{2.303RT}{F}$ =0.06 V).

Answer: 2.15

Solution:

Anodic half: $Sn \rightarrow Sn^{2+} + 2e^{-}$

Cathodic half: $Pb^{2+} + 2e^{-} \rightarrow Pb$

Net reaction: $Sn + Pb^{2+} \rightarrow Pb + Sn^{2+}$

$E^0_{cell} = E^0_{cathode} - E^0_{anode}$

$E^0_{cell} = 0.01\ V$

$E_{cell} = E^0_{cell} - \frac{0.06}{2} \log Q$

At equilibrium state $E_{cell} = 0$

So,

$0 = 0.01 - \frac{0.06}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$

$0.01 = \frac{0.06}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$

$\log \frac{[Sn^{2+}]}{[Pb^{2+}]} = \frac{1}{3}$

$\frac{[Sn^{2+}]}{[Pb^{2+}]} = 10^{\frac{1}{3}} = 2.154$

23. $NaClO_3$ is used, even in spacecrafts, to produce O_2. The daily consumption of pure O_2 by a person is 492 L at 1 atm, 300 K. How much amount of $NaClO_3$, in grams, is required to produce O_2 for the daily consumption of a person at 1 atm, 300 K ?
$NaClO_3(s) + Fe(s) \rightarrow O_2(g) + FeO(s) + NaCl(s)$
$R = 0.082$ L atm mol^{-1} K^{-1}

Answer: 2.13

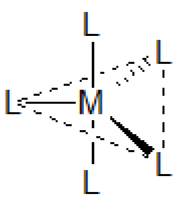
Solution:

Mol of $NaClO_3$ = mol of O_2
Mol of $O_2 = \frac{PV}{RT} = \frac{1\times492}{0.082\times300}$ = 20 mol
Molar mass of $NaClO_3$ is 106.5
So, mass = 20×106.5 = 2130 g = 2.13 kg

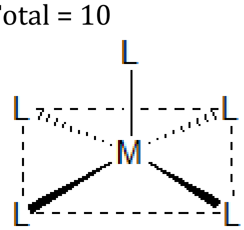
24. Complexes $[ML_5]$ of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal and geometries, respectively. The sum of the 90^0, 120^0 and 180^0 L-M-L angles in the two complexes is _____.

Answer: 20

Solution:



For trigonal bipyramidal geometry
Total number of 180^0 L-M-L bond angles = 1
Total number of 90^0 L-M-L bond angles = 6
Total number of 120^0 L-M-L bond angles = 3
Total = 10



For square pyramidal geometry
Total number of 180^0 L-M-L bond angles = 2
Total number of 90^0 L-M-L bond angles = 8
Total number of 120^0 L-M-L bond angles = 0
Total = 10
Total for both the structures = 20

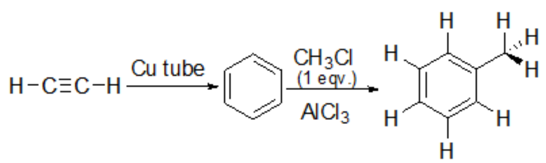
25. In the following sequence of reactions, the maximum number of atoms present in molecule 'C' in one plane is _____.

$$A \xrightarrow{\text{Red hot/Cu tube}} B \xrightarrow{CH_3Cl(1\text{ eq.}),\ \text{anhy.}AlCl_3} C$$

(Where A is a lowest molecular weight alkyne).

Answer: 13

Solution:



JEE Main 2020 Paper

Date of Exam: **8th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

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1. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is

   a. 0.10 b. 0.20
   c. 0.01 d. 0.02

   Answer: (a)

   Solution:

   $P(A \text{ or } B) = P(A \cup B) = \frac{1}{2}$

   $P(\text{exactly one of } A \text{ or } B) = \frac{2}{5}$

   $$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = \frac{2}{5}$$

   $$\Rightarrow P(A \cup B) - P(A \cap B) = \frac{2}{5}$$

   $$\Rightarrow P(A \cap B) = \frac{1}{10}$$

2. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S:

   a. is a singleton.
   b. is an empty set.
   c. contains at least four elements
   d. contains exactly two elements.

   Answer: (a)

   Solution:

   $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$

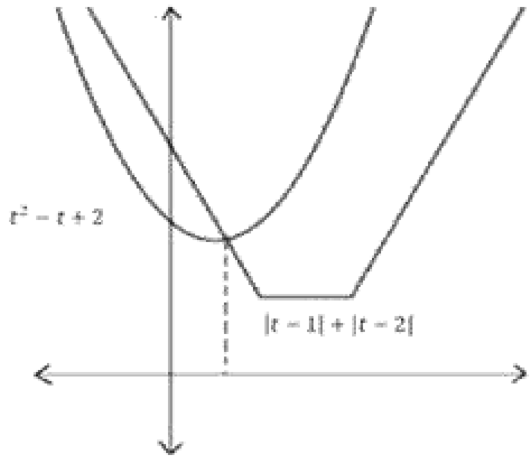
   Let $3^x = t$

   $t(t - 1) + 2 = |t - 1| + |t - 2|$

   $\Rightarrow t^2 - t + 2 = |t - 1| + |t - 2|$

We plot $t^2 - t + 2$ and $|t-1| + |t-2|$

As 3^x is always positive, therefore only positive values of t will be the solution.



Therefore, we have only one solution.

3. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:

a. 4.01
b. 3.99
c. 3.98
d. 4.02

Answer: (b)

Solution:

Mean $= 10 \Rightarrow \frac{\sum x_i}{20} = 10 \Rightarrow \sum x_i = 200$

Variance $= 4 \Rightarrow \frac{\sum {x_i}^2}{20} - 100 = 4 \Rightarrow \sum {x_i}^2 = 2080$

New mean $= \frac{200-9+11}{20} = \frac{202}{20} = 10.1$

New variance $= \frac{2080-81+121}{20} - (10.1)^2$

$= 106 - 102.01$

$= 3.99$

4. Let $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c}.\vec{a} = 0$ then $\vec{c}.\vec{b}$ is equal to:

a. $\frac{1}{2}$
b. $-\frac{3}{2}$
c. $-\frac{1}{2}$
d. -1

Answer: (c)

Solution:

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$

$\Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\Rightarrow -(\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\Rightarrow -4\vec{c} = 6(\hat{\imath} - \hat{\jmath} + \hat{k}) - 4(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$\therefore \vec{b}.\vec{c} = -\frac{1}{2}.$

5. Let $f: (1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{x^2+1}$, where $[x]$ denotes the greatest integer $\leq$ x. Then the range of f is:

a. $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$
b. $\left(\frac{2}{5}, \frac{4}{5}\right]$
c. $\left(\frac{3}{5}, \frac{4}{5}\right)$
d. $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

Answer: (d)

Solution:

$f(x) = \frac{x[x]}{x^2+1}$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{x^2+1} : 1 < x < 2 \\ \frac{2x}{x^2+1} : 2 \le x < 3 \end{cases}$$

$\Rightarrow$ Range of $f(x)$ is $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$.

6. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$, then:

a. $\alpha + \beta = -30$
b. $\alpha - \beta = -132$
c. $\alpha + \beta = 60$
d. $\alpha - \beta = 60$

Answer: (b)

Solution:

$\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$

$= 2[\,{}^6C_0 x^6 + {}^6C_2 x^4(x^2 - 1) + {}^6C_4 x^2(x^2 - 1)^2 + {}^6C_6(x^2 - 1)^3]$

$= 2[32x^6 - 48x^4 + 18x^2 - 1]$

$\Rightarrow \alpha = -96, \qquad \beta = 36$

$\Rightarrow \alpha - \beta = -132$

7. If a hyperbola passes through the point $P(10, 16)$ and it has vertices at $(\pm 6, 0)$, then the equation of the normal at P is:

a. $3x + 4y = 94$
b. $x + 2y = 42$
c. $2x + 5y = 100$
d. $x + 3y = 58$

Answer: (c)

Solution:

Vertex of hyperbola is $(\pm a, 0) \equiv (\pm 6, 0) \Rightarrow a = 6$

Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{36} - \frac{y^2}{b^2} = 1$

As $P(10, 16)$ lies on the parabola.

$\frac{100}{36} - \frac{256}{b^2} = 1$

$\Rightarrow \frac{64}{36} = \frac{256}{b^2} \Rightarrow b^2 = 144$

Equation of hyperbola becomes $\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$

$\Rightarrow \frac{36x}{10} + \frac{144y}{16} = 180$

$\Rightarrow \frac{x}{50} + \frac{y}{20} = 1 \Rightarrow 2x + 5y = 100$

8. $\lim\limits_{x \to 0} \frac{\int_0^x t \sin(10t)\, dt}{x}$ is equal to:

a. 0

b. $\frac{1}{10}$

c. $-\frac{1}{10}$

d. $-\frac{1}{5}$

Answer: (a)

Solution:

$$\lim_{x \to 0} \frac{\int_0^x t \sin 10t\, dt}{x}$$

Applying L'Hospital's Rule:

$$= \lim_{x \to 0} \frac{x \sin 10x}{1} = 0$$

9. If a line, $y = mx + c$ is a tangent to the circle, $(x-3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then:

a. $c^2 + 7c + 6 = 0$

b. $c^2 - 6c + 7 = 0$

c. $c^2 - 7c + 6 = 0$

d. $c^2 + 6c + 7 = 0$

Answer: (d)

Solution:

For circle, $x^2 + y^2 = 1$

$2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$

Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -1$

$\Rightarrow$Slope of tangent to $(x-3)^2 + y^2 = 1$ is $1 \Rightarrow m = 1$

Tangent to $(x-3)^2 + y^2 = 1$ is $y = x + c$

Perpendicular distance of tangent $y = x + c$ from centre $(3, 0)$ is equal to radius $= 1$

$$\left|\frac{3+c}{\sqrt{2}}\right| = 1$$

$\Rightarrow c + 3 = \pm\sqrt{2}$

$\Rightarrow c^2 + 6c + 9 = 2$

$\Rightarrow c^2 + 6c + 7 = 0$

10. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation:

a. $x^2 + 101x + 100 = 0$
b. $x^2 + 102x + 101 = 0$
c. $x^2 - 102x + 101 = 0$
d. $x^2 - 101x + 100 = 0$

Answer: (c)

Solution:

$\alpha = \frac{-1+i\sqrt{3}}{2} = \omega$

$$a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$$

$\Rightarrow a = (1+\alpha)[1 + \alpha^2 + \alpha^4 + \cdots .+\alpha^{200}]$

$$\Rightarrow a = (1+\alpha)\left[\frac{1-(\alpha^2)^{101}}{1-\alpha^2}\right]$$

$$\Rightarrow a = \left[\frac{1-(\omega^2)^{101}}{1-\omega}\right] = \left[\frac{1-\omega}{1-\omega}\right] = 1$$

$$b = \sum_{k=0}^{100} \alpha^{3k} = 1 + \alpha^3 + \alpha^6 + \cdots ..+\alpha^{300}$$

$\Rightarrow b = 1 + \omega^3 + \omega^6 + \ldots . + \omega^{300}$

$\Rightarrow b = 101$

Required equation is $x^2 - 102x + 101 = 0$

11. The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane?

a. $(1, -1, 1)$

b. $(-1, -1, 1)$

c. $(1, 1, 1)$

d. $(-1, -1, -1)$

Answer: (a)

Solution:

Image of point $P(1, 2, 3)$ w.r.t. a plane $ax + by + cz + d = 0$ is $Q\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$

Direction ratios of PQ: $-\frac{10}{3}, -\frac{10}{3}, -\frac{10}{3} = 1, 1, 1$

Direction ratios of normal to plane is $1, 1, 1$

Mid-point of PQ lies on the plane

$\therefore$ The mid-point of $PQ = \left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

$\therefore$ Equation of plane is $x + \frac{2}{3} + y - \frac{1}{3} + z - \frac{4}{3} = 0$

$\Rightarrow x + y + z = 1$

$(1, -1, 1)$ satisfies the equation of the plane.

12. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point $(2, 2)$ is:

a. 2

b. $2\sqrt{2}$

c. $4\sqrt{2}$

d. $\sqrt{2}$

Answer: (b)

Solution:

Given curve: $x^2 + 2xy - 3y^2 = 0$

$\Rightarrow x^2 + 3xy - xy - 3y^2 = 0$

$\Rightarrow (x + 3y)(x - y) = 0$

Equating we get,

$x + 3y = 0$ or $x - y = 0$

$(2, 2)$ lies on $x - y = 0$

$\therefore$ Equation of normal will be $x + y = \lambda$

It passes through $(2, 2)$

$\therefore \lambda = 4$

$L : x + y = 4$

Distance of L from the origin $= \left|\frac{-4}{\sqrt{2}}\right| = 2\sqrt{2}$

13. Which of the following statements is a tautology?
 a. $\sim(p \wedge \sim q) \rightarrow (p \vee q)$
 b. $(\sim p \vee \sim q) \rightarrow (p \wedge q)$
 c. $p \vee (\sim q) \rightarrow (p \wedge q)$
 d. $\sim(p \vee \sim q) \rightarrow (p \vee q)$

Answer: (d)

Solution:

$\sim(p \vee \sim q) \rightarrow (p \vee q)$

$= (p \vee \sim q) \vee (p \vee q)$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$

14. If $\mathrm{I} = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then:
 a. $\frac{1}{6} < \mathrm{I}^2 < \frac{1}{2}$
 b. $\frac{1}{8} < \mathrm{I}^2 < \frac{1}{4}$
 c. $\frac{1}{9} < \mathrm{I}^2 < \frac{1}{8}$
 d. $\frac{1}{16} < \mathrm{I}^2 < \frac{1}{9}$

Answer: (c)

Solution:

Let $f(x) = \frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

$$f'(x) = \frac{-(6x^2 - 18x + 12)}{2(2x^3 - 9x^2 + 12x + 4)^{3/2}} = \frac{-3(x-1)(x-2)}{(2x^3 - 9x^2 + 12x + 4)^{3/2}}$$

$\Rightarrow f_{min} = f(1)$ and $f_{max} = f(2)$

$$f(1) = \frac{1}{\sqrt{2 - 9 + 12 + 4}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$f(2) = \frac{1}{\sqrt{16 - 36 + 24 + 4}} = \frac{1}{\sqrt{8}}$$

$$\frac{1}{3} < \text{I} < \frac{1}{\sqrt{8}}$$

$$\Rightarrow \frac{1}{9} < \text{I}^2 < \frac{1}{8}$$

15. If $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$ and $\text{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $10A^{-1}$ is equal to:

a. $6\text{I} - A$
b. $A - 6\text{I}$
c. $4\text{I} - A$
d. $A - 4\text{I}$

Answer: (b)

Solution:

$$A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{10}\begin{bmatrix} 4 & -2 \\ -9 & 2 \end{bmatrix}$$

$$\Rightarrow 10A^{-1} = \begin{bmatrix} -4 & 2 \\ 9 & -2 \end{bmatrix} \Rightarrow 10A^{-1} = A - 6\text{I}$$

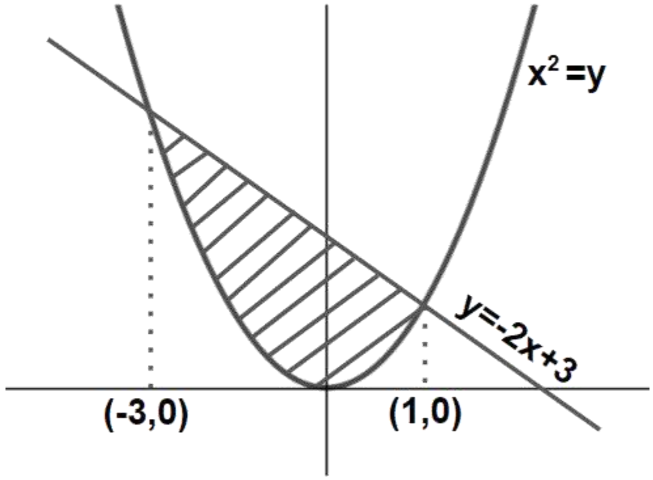
16. The area (in sq. units) of the region $\{(x, y) \in \boldsymbol{R^2} : x^2 \le y \le 3 - 2x\}$, is:

a. $\frac{31}{3}$
b. $\frac{32}{3}$
c. $\frac{29}{3}$
d. $\frac{34}{3}$

Answer: (b)

Solution:

We have $x^2 \le y \le -2x + 3$



For point of intersection of two curves

$x^2 + 2x - 3 = 0$

$\Rightarrow x = -3, 1$

$\Rightarrow$ Area $= \int_{-3}^{1}\left((-2x+3) - x^2\right)dx$

$= \left[-x^2 + 3x - \frac{x^3}{3}\right]_{-3}^{1} = \frac{32}{3}$ sq. units.

17. Let S be the set of all functions $f: [0,1] \rightarrow \mathbf{R}$, which are continuous on $[0, 1]$ and differentiable on $(0, 1)$. Then for every f in S, there exists a $c \in (0,1)$, depending on f, such that:

a. $\frac{f(1)-f(c)}{1-c} = f'(c)$

b. $|f(c) - f(1)| < |f'(c)|$

c. $|f(c) + f(1)| < (1 + \text{c})|f'(c)|$

d. $|f(c) - f(1)| < (1 - c)|f'(c)|$

Answer: (Bonus)

Solution:

S is set of all functions.

If we consider a constant function, then option 2, 3 and 4 are incorrect.

For option 1:

$$\frac{f(1) - f(c)}{1 - c} = f'(c)$$

This may not be true for $f(x) = x^2$

None of the option are correct

18. The differential equation of the family of curves, $x^2 = 4b(y+b), b \in \boldsymbol{R}$, is:
    a. $xy'' = y'$
    b. $x(y')^2 = x + 2yy'$
    c. $x(y')^2 = x - 2yy'$
    d. $x(y')^2 = 2yy' - x$

Answer: (b)

Solution:

$x^2 = 4b(y+b)$ ... (1)

Differentiating both the sides w.r.t. x, we get

$\Rightarrow 2x = 4by'$

$\Rightarrow b = \frac{x}{2y'}$

Putting the value of b in (1), we get

$\Rightarrow x^2 = \frac{2x}{y'}\left(y + \frac{x}{2y'}\right)$

$\Rightarrow x^2 = \frac{2xy}{y'} + \frac{x^2}{y'^2}$

$\Rightarrow x(y')^2 = 2yy' + x$

19. The system of linear equations
    $\lambda x + 2y + 2z = 5$
    $2\lambda x + 3y + 5z = 8$
    $4x + \lambda y + 6z = 10$ has:
    a. no solution when $\lambda = 2$
    b. infinitely many solutions when $\lambda = 2$
    c. no solution when $\lambda = 8$
    d. a unique solution when $\lambda = -8$

Answer: (a)

Solution:

$$D = \begin{bmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{bmatrix} = 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24$$

$\Longrightarrow D = -\lambda^2 - 6\lambda + 16$

Now, $D = 0$

$\Rightarrow \lambda^2 + 6\lambda - 16 = 0$

$\Rightarrow \lambda = -8 \text{ or } 2$

For $\lambda = 2$

$$D_1 = \begin{bmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{bmatrix} = 40 + 4 - 28 \neq 0$$

$\therefore$ Equations have no solution for $\lambda = 2$.

20. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is:

a. $50\frac{1}{4}$

b. 100

c. 50

d. $100\frac{1}{2}$

Answer: (d)

Solution:

$T_{10} = \frac{1}{20}, T_{20} = \frac{1}{10}$

$T_{20} - T_{10} = 10d$

$\Rightarrow \frac{1}{20} = 10d$

$d = \frac{1}{200}$

$\therefore a = \frac{1}{200}$

$S_{200} = \frac{200}{2}\left[2\left(\frac{1}{100}\right) + 199\left(\frac{1}{200}\right)\right]$

$= 100\frac{1}{2}$

21. Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x –axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to ________ .

Answer: (0.5)

Solution:

Let the co-ordinates of P be $(t^2,\ t)$

Equation of tangent at $P(t^2,\ t)$ is $y - t = \frac{1}{2t}(x - t^2)$

Therefore, co-ordinates of Q will be $(-t^2,\ 0)$

Area of $\Delta OPQ = 4$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = \pm 4$$

$\Rightarrow t^3 = \pm 8 \Rightarrow t = \pm 2 \Rightarrow t = 2 \quad \text{as } t > 0$

$$m = \frac{1}{t} = \frac{1}{2}$$

22. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10, f(1) = -6, f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then the local minima at $x =$_____

Answer: (3)

Solution:

Let the polynomial be

$f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow f'(x) = 3ax^2 + 2bx + c$

$\Rightarrow f''(x) = 6ax + 2b$

$f''(1) = 0 \Rightarrow 6a + 2b = 0 \Rightarrow b = -3a$

$f'(-1) = 0 \Rightarrow 3a - 2b + c = 0$

$\Rightarrow c = -9a$

$f(-1) = 10 \Rightarrow -a + b - c + d = 10$

$\Rightarrow -a - 3a + 9a + d = 10$

$d = -5a + 10$

$f(1) = 6 \Rightarrow a + b + c + d = 6$

$\Rightarrow a - 3a - 9a - 5a + 10 = 6$

$$\Rightarrow a = \frac{1}{4}$$

$$\therefore f'(x) = \frac{3}{4}x^2 - \frac{6}{4}x - \frac{9}{4} = \frac{3}{4}(x^2 - 2x - 3)$$

For $f'(x) = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = 3, -1$

Minima exists at $x = 3$

23. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to ________ .

Answer: (1)

Solution:

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7} \Rightarrow \frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7} \Rightarrow \tan\alpha = \frac{1}{7}$$

$$\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}} \Rightarrow \frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}} \Rightarrow \sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}} = 1$$

24. The number of 4 letter words (with or without meaning) that can be made from the eleven letters of the word "EXAMINATION" is ________ .

Answer: (2454)

Solution:

Word "EXAMINATION" consists of $2A, 2I, 2N, E, X, M, T, O$

Case I: All different letters are selected

Number of words formed $= {}^8C_4 \times 4! = 1680$

Case II: 2 letters are same and 2 are different

Number of words formed $= {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$

Case III: 2 pair of letters are same

Number of words formed $= {}^3C_2 \times \frac{4!}{2!\times 2!} = 18$

Total number of words formed $= 1680 + 756 + 18 = 2454$

25. The sum, $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to ________.

Answer: (504)

Solution:

$$\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$$

$$= \frac{1}{4} \sum_{n=1}^{7} (2n^3 + 3n^2 + n)$$

$$= \frac{1}{4}\left[2\sum_{n=1}^{7} n^3 + 3\sum_{n=1}^{7} n^2 + \sum_{n=1}^{7} n\right]$$

$$= \frac{1}{4}\left[2 \times \left(\frac{7 \times 8}{2}\right)^2 + 3 \times \frac{7 \times 8 \times 15}{6} + \frac{7 \times 8}{2}\right]$$

$$= \frac{1}{4}[2 \times 784 + 420 + 28] = 504$$