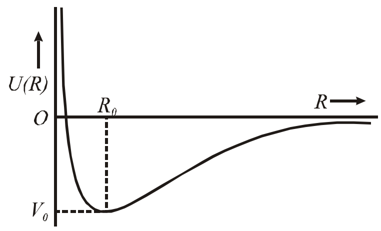
Chapter 9
Mechanical Properties of Solids

INTERATOMIC AND INTERMOLECULAR FORCE

The force between atoms of an element is called **interatomic force**. The force between molecules of a compound (or element) is called **intermolecular force**. These forces are electrical in nature. Depending on the distance between the atoms, this force may be attractive or repulsive in nature. These forces are responsible for the definite size or shape of a solid.

Detailed calculations as well as deductions from experiment show that the interaction between any isolated pairs of atoms or molecules may be represented by a curve that shows how the potential energy varies with separation between them as shown in the figure.

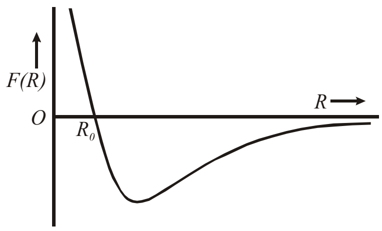


This curve describes the interatomic potential. The force between the atoms can be found from the potential energy by using the relation

$$F = -\frac{dU}{dr}$$

The resulting interatomic force curve is shown in figure.

Force is along the line joining the atoms or molecules, and is shown negative for attraction and positive for repulsion.



We see that as the distance decreases, the attractive force first increases and then decreases to zero at a separation where the potential energy is minimum.

For smaller distance force is repulsive, because at these distance the negative charge distribution associated with one atom begins to over lap with that associated with the neighboring atom.

The force of attraction between molecules may be written as

$$F_a = -\frac{a}{r^7}$$

The negative sign shows that F_a is a force of attraction and 'a' is a constant which depends on the kind of attractive force between the molecules and the structure of the molecules. The force of attraction results from the creation of induced dipole moment in one molecule by the neighbouring molecule.

When the molecules are brought closer there is a force of repulsion between them. It can be shown that the repulsive force is

$$F_r = +\frac{b}{r^9},$$

where b is a constant like a. The force of repulsion varies very rapidly. F_r is inversely proportional to the ninth power of distance between the molecules. The resultant force acting on the molecules is

$$F = -\frac{a}{r^7} + \frac{b}{r^9}$$

The distance between the molecules decides the sign of the resultant force.

ELASTICITY

*The property of the body by virtue of which it tends to regain its original shape and size after removing the deforming force is called **elasticity***. If the body regains its original shape and size completely, after the removal of deforming forces, then the body is said to be **perfectly elastic**.

- *The property of the body by virtue of which it tends to retain its deformed state after removing the deforming force is called **plasticity***. If the body does not have any tendency to recover its original shape and size, it is called **perfectly plastic**.

STRESS AND STRAIN

Stress :

When a deforming force is applied to a body, an internal restoring force comes into play.

*The restoring force per unit area is called **stress**.*

$$Stress = \frac{Restoring\ force}{Area}$$

Its **S.I. Unit** is Nm^{-2}

Stress is a **tensor** as its value changes when direction changes.

Types of Stress :

Longitudinal stress or tensile stress, volumetric stress and tangential stress are the types of stress.

Strain :

It is defined as the ratio of the change in shape or size to the original shape or size of the body.

$$Strain = \frac{Change\ in\ dimension}{Original\ dimension}$$

Strain has **no units or dimensions**.

Types of Strain :

Longitudinal strain : It is defined as the ratio of the change in length to the original length.

$$Longitudinal\ \ strain = \frac{\Delta\ell}{\ell}$$

Volume strain : It is the ratio of the change in volume to the original volume.

$$Volume\ \ strain = \frac{\Delta V}{V}$$

Shearing strain : It is the angular deformation produced in a body.

$$Shearing\ strain\ (\theta) = \frac{\Delta x}{h}$$

HOOKE'S LAW

It is the fundamental law of elasticity given by Robert Hooke in 1679. *It states that "the stress is directly proportional to strain provided the strain is small* ".

$$\text{i.e. } Stress \propto strain \Rightarrow \frac{stress}{strain} = \text{constant} = E$$

This proportionality constant is called **modulus of elasticity** (name given by Thomas Young) or **coefficient of elasticity (E)**. Since stress has same dimensions as that of pressure and strain is dimensionless. So the **dimensions** of E is same as, that of stress or pressure i.e. $[ML^{-1}T^{-2}]$

The modulus of elasticity depends on the material and on the nature of deformation. There are three type of deformations and therefore three **types of modulus of elasticity**.

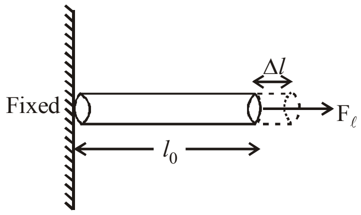
(i) ***Young's modulus (Y) :*** It measures the resistance of a solid to elongation.

(ii) ***Shear modulus*** (η) ***or modulus of rigidity :*** It measures the resistance to motion of the plane of a solid sliding part on each other.

(iii) ***Bulk modulus (B) :*** It measure the resistance that solid or liquid offer to their volume change.

The stress under which the system breaks is called breaking stress.

(i) **Young's modulus (Y) :** Let us consider a long bar (shown in fig.) of cross-sectional area A and length ℓ_0, which is clamped at one end. When we apply external force F_ℓ longitudinally along the bar, internal forces in bar resist distortion, but bar attains equilibrium in which its length is greater and in which external force is exactly balanced by internal forces. The bar is said to be stressed in this condition.



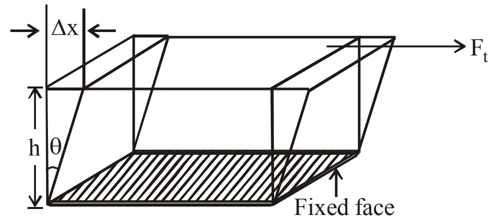
Now if the solid bar obey the Hooke' law, the Young's modulus, Y is defined as

$$Y = \frac{Tensile\ stress}{Tensile\ strain} = \frac{F_\ell / A}{\Delta\ell / \ell}$$

Where Δl is change in the length of bar, when we apply F_ℓ.

(ii) **Shear modulus (η) :** Shear modulus or modulus of rigidity η is

$$\eta = \frac{Shearing\ stress}{Shearing\ strain} = \frac{F_t / A}{\Delta x / h}$$

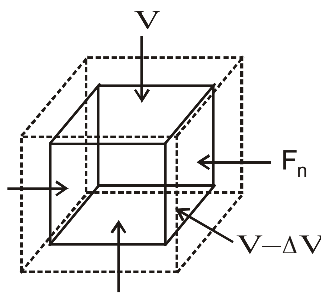


(a) As we see, there is no change in volume under this deformation, but shape changes.

(b) $\frac{\Delta x}{h} = \tan\theta \approx \theta$ (see the figure), where θ is shear angle.

(iii) **Bulk modulus (B) :** The Bulk modulus B is defined as

$$B = \frac{Volume\ stress}{Volume\ stain} = \frac{F_n / A}{\Delta V / V} = \frac{-\Delta P}{\Delta V / V}$$



Negative sign comes to make B positive, because with the increase of pressure, the volume of body decreases or vice versa.

The reciprocal of the Bulk modulus is called compressibility of material

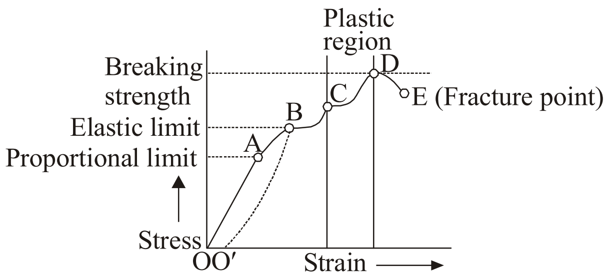
i.e., *Compressibility* = 1/*B*.

Poisson's Ratio (σ) :

Lateral strain/longitudinal strain = Poisson's ratio (σ).

The theoretical value of σ lies between –1 and 0.5 and practical value of σ lies between 0 and 0.5.

STRESS-STRAIN CURVE



(i) **Proportional limit :** The limit in which Hooke's law is valid i.e, stress is directly proportional to strain is called proportion limit. **Stress $\propto$ strain**

(ii) **Elastic limit :** It is a maximum stress upto which the body completely recovers its original state after the removal of the deforming forces.

(iii) **Yield point :** The point beyond elastic limit, at which the length of wire starts increasing without increasing stress, is defined as the yield point.

(iv) **Breaking point :** The position when the strain becomes so large that the wire breaks down at last, is called breaking point. At this position the stress acting in that wire is called breaking stress and strain is called breaking strain.

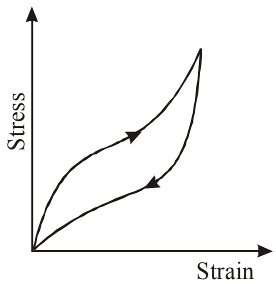
- Breaking stress is also known as the tensile strength.
- Metals with small plastic deformation are called brittle.
- Metals with large plastic deformation are called ductile.

Elastic fatigue : This is the phenomenon of a delay in recovering the original configuration by a body, if it had been subjected to stress for a longer time the body looses the property of elasticity temporarily.

Elastic relaxation time : It is the time delay in regaining the original shape after removal of deforming forces. Elastic relaxation time for gold, silver and phosphor bronze is negligible.

Elastic Hysteresis

When the stress applied on a body, is decreased to zero, the strain will not be reduced to zero immediately. For some substances (e.g.-vulcanized rubber), the strain lags behind the stress. This lagging of strain behind stress is called ***elastic hysteresis****.*



The stress-strain graph for increasing and decreasing load encloses a loop, as shown in figure. The area of the loop gives the energy dissipated during its deformation.

Keep in Memory

1. Thermal stress $= \dfrac{F}{A} = Y\alpha\Delta T$, where α is the coefficient of linear expansion and ΔT is the change in temperature.
2. (i) The modulus of rigidity (η) for liquids is zero.
   (ii) For a given tensile force, the increase in length is inversely proportional to square of its diameter.
   (iii) The pressure required to stop volume expansion of a piece of metal is
   $$P = B\gamma dT,$$
   where γ = coefficient of volume expansion = 3α
   (iv) To compare elasticities of different materials, their identical small balls are made and they are dropped from same height on a hard floor. The ball which rises maximum after striking the floor, is most elastic. The order of elasticity of different materials on this basis is as follows :
   $$Y_{ivory} > Y_{steel} > Y_{rubber} > Y_{clay}$$
   (v) For the construction of rails, bridges, girders and machines, materials with high Young's modulus are used so that they may not get permanently deformed.
   For a spring, $F = \dfrac{YA\Delta L}{L} = kx.$
   Hence spring constant $k = \dfrac{YA}{L}$. Here $\Delta L = x$.
3. When equal force is applied on identical wires of different materials then the wire in which minimum elongation is produced is more elastic. For the same load, more elongation is produced in rubber than in steel wire, hence steel is more elastic than rubber.

Relation between Y, B, η and σ

(i) $Y = 3B(1-2\sigma)$ (ii) $Y = 2\eta(1+\sigma)$

(iii) $\sigma = \dfrac{3B-2\eta}{6B+2\eta}$ (iv) $\dfrac{9}{Y} = \dfrac{3}{\eta} + \dfrac{1}{B}$

Energy stored per unit volume in a strained body

Energy per unit volume $= \dfrac{1}{2}$ *stress × strain*

$= \dfrac{1}{2}$ modulus of elasticity × (strain)^2

$= \dfrac{1}{2}$ (stress)^2 /modulus of elasticity.

Work done in stretching a wire or work done per unit volume

$$= \frac{1}{2} \times stress \times strain = \frac{1}{2} \times load \times extension$$

Torsional rigidity of a cylinder :

(i) The torsional rigidity C of a cylinder is given by,

$$C = \frac{\pi \eta R^4}{2\ell}$$

where η = modulus of rigidity of material of cylinder, R = radius of cylinder, ℓ = length of cylinder

(ii) Restoring couple, $\tau = C\phi = \dfrac{\pi \eta R^4 \phi}{2\ell}$

(iii) Work done in twisting the cylinder through an angle ϕ, $W = \dfrac{\pi \eta R^4 \phi^2}{4\ell}$ joule $= \dfrac{1}{2} C \phi^2$

Cantilever :

A beam fixed at one end and loaded at the other end is called a cantilever.

(i) The depression y at a distance x from the fixed end is (when the weight of cantilever is ineffective)

$$y = \frac{mg}{YI}\left(\frac{\ell x^2}{2} - \frac{x^3}{6}\right)$$

where mg = load applied, ℓ = length of cantilever, I = geometrical moment of inertia of its cross section.

(ii) Maximum depression at free end of cantilever,

$$y_{max} = \delta = \frac{mg\ell^3}{3YI}$$

(a) For rectangular beam, $I = \dfrac{bd^3}{12}$

(b) For a circular cross section beam of radius r, $I = \dfrac{\pi r^4}{4}$

(iv) Depression produced in a beam supported at two ends and loaded at the middle, $\delta = \dfrac{mg\ell^3}{48YI}$

For rectangular beam, $\delta = \dfrac{mgl^3}{4Ybd^3}$

and for circular beam, $\delta = \dfrac{mgl^3}{12\pi r^4 Y}$

Keep in Memory

1. Wound spring possess elastic potential energy.
2. A material which can be drawn into wires is called **ductile** and a material which can be hammered into sheet is called **malleable**.
3. Ductility, brittleness, malleability, etc., are not elastic properties.
4. The substance which breaks just beyond the elastic limit is called **brittle**.
5. (i) Breaking force of a wire = breaking stress × area of cross section
   (ii) Breaking stress does not depend on the length of wire
   (iii) Breaking stress depends on the material of wire.

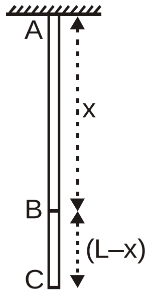
Example 1.

A uniform rod of mass m, length L, area of cross section A and Young's modulus Y hangs from a ceiling. Its elongation under its own weight will be

(a) $\dfrac{2\,mg\,L}{AY}$ (b) $\dfrac{mg\,L}{AY}$

(c) $\dfrac{mg\,L}{2\,AY}$ (d) zero

***Solution* (c)**



Mass of section BC of wire $= \dfrac{m}{L}(L-x)$;

Tension at B, $T = \dfrac{m}{L}(L-x)g$

Elongation of element dx at B,

$$d\ell = \frac{T}{A}\frac{dx}{Y} = \frac{m(L-x)g}{LAY}dx$$

Total elongation $= \int d\ell = \dfrac{mg}{LAY}\displaystyle\int_0^L (L-x)dx = \dfrac{mg\,L}{2YA}$

Example 2.

If the potential energy of the molecule is given by $U = \dfrac{A}{r^6} - \dfrac{B}{r^{12}}$. *Then at equilibrium position its potential energy is equal to*

(a) $-A^2/4\,B$ (b) $A^2/4\,B$

(c) $2\,A/B$ (d) $A/2\,B$

***Solution :* (b)**

$$F = -\frac{dU}{dr} = -\frac{d}{dr}\left[\frac{A}{r^6} - \frac{B}{r^{12}}\right] = \left[\frac{-A\,x\,6}{r^7} + \frac{12\,B}{r^{13}}\right]$$

In equilibrium position F = 0.

so, $\dfrac{6\,A}{r^7} = \dfrac{12\,B}{r^{13}}$ or $r^6 = \dfrac{2\,B}{A}$

$\therefore$ Potential energy at equilibrium position

$$U = \frac{A}{(2\,B/A)} - \frac{B}{(2\,B/A)^2} = \frac{A^2}{2\,B} - \frac{A^2}{4\,B} = \frac{A^2}{4\,B}$$

Example 3.

The normal density of gold is ρ and its bulk modulus is K. The increase in density of a lump of gold when a pressure P is applied uniformly on all sides is

(a) $K/\rho\,P$ (b) $P/\rho\,K$

(c) $\rho\,P/K$ (d) $\rho\,K/P$

***Solution :* (a)**

$$K = \frac{p}{\Delta V/V} \text{ or } \frac{\Delta V}{V} = \frac{p}{K};$$

Also $\rho = \dfrac{M}{V}$ and $\rho' = \dfrac{M}{V - \Delta V}$;

$$\therefore \frac{\rho'}{\rho} = \frac{V}{(V-\Delta V)} = \frac{1}{(1-\Delta V/V)} = \left(1 - \frac{\Delta V}{V}\right)^{-1}$$

$$\approx\left(1+\frac{\Delta V}{V}\right)=1+\frac{p}{K} \text{ or } \frac{\rho'}{\rho}-1=\frac{p}{K}$$

or $\rho'-\rho=\dfrac{p\rho}{K}$ $\qquad (\because \Delta V << V)$

Example 4.

A wire 3 m in length and 1 mm in diameter at 30ºC and kept in a low temperature at –170ºC and is stretched by hanging a weight of 10 kg at one end. Calculate the change in the length of the wire. Given $\alpha = 1.2 \times 10^{-5}/^{\circ}C$ and $Y = 2 \times 10^{11}\ N/m^2$. Take $g = 10\ m/s^2$.

Solution :

We know that,

$$\ell=\frac{FL}{YA}=\frac{10\times10\times3}{2\times10^{11}\times0.785\times10^{-6}}=1.91\times10^{-3}$$

(where $A = \pi r^2$) $= 0.785 \times 10^{-6}\ m^2$

Contraction in length $= -\alpha L \Delta T$

$= -(1.2 \times 10^{-5})(3)(-170-30)$

$= 7.2 \times 10^{-3}$ m

The resultant change in length

$= 7.2 \times 10^{-3} - 1.91 \times 10^{-3} = 5.29$ mm

Example 5.

A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, what is the percentage increase in volume?

Solution :

Here

$$\frac{\Delta V}{V}=\frac{\Delta(\pi r^2\ell)}{\pi r^2\ell}=\frac{r^2\Delta\ell+2r\ell\Delta r}{r^2\ell} \text{ or } \frac{\Delta V}{V}=\frac{\Delta\ell}{\ell}+2\frac{\Delta r}{r}$$

Now $\sigma=-\dfrac{\Delta r/r}{\Delta\ell/\ell}$ or $\dfrac{\Delta r}{r}=-\sigma\dfrac{\Delta\ell}{\ell}$;

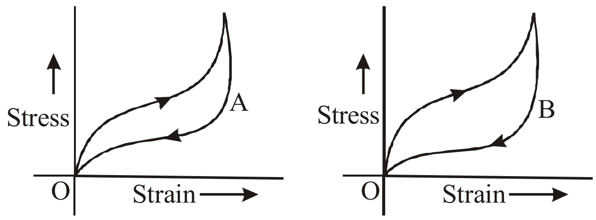
$\therefore\ \dfrac{\Delta r}{r}=-0.5\times(2\times10^{-3})=-1\times10^{-3}$

Further, $\dfrac{\Delta V}{V}=(2\times10^{-3})-2\times(1\times10^{-3})=0$

$\therefore$ % increase in volume is 0.

Example 6.

(a) *A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machine and floor, which of the two rubbers A and B would you prefer to use for the purpose ? Why ?*



(b) *Which of the two rubber materials would you choose for a car tyre ?*

Solution :

(a) Rubber B is preferred. The area of the hysteresis loop measures the amount of heat energy dissipated by the material. The area of loop B is more than A. So B can absorb more vibrations.

(b) To avoid excessive heating of car tyre, rubber A would be preferred over rubber B.

CONCEPT MAP

Young's modulus of elasticity

$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F\Delta l}{Al} = \frac{Mg\Delta l}{\pi r^2 l}$

Bulk modulus or volume modulus of elasticity

$B = \frac{\text{hydraulic stress}}{\text{volume strain}}$

$B = -\frac{\Delta P}{\Delta V / V}; \frac{1}{B} =$ compressibility

Rigidity or shear modulus of elasticity

$\eta = \frac{\text{tangential stress}}{\text{shearing strain}}$

$\eta = \frac{\sigma_s}{\varepsilon_s} = \frac{F/A}{\theta}$

Types of modulus of elasticity

Stress

Restoring force per unit area i.e., stress $= \frac{F}{A}$

Hooke's laws

Within the elastic limit stress is directly proportional to strain i.e. stress $\propto$ strain or, stress $= E \times$ strain

Strain

Ratio of change in configuration to the original configuration

Strain $= \frac{\text{change in configuration}}{\text{original configuration}}$

Types of stress

MECHANICAL PROPERTIES OF SOLIDS

Types of strain

(i) Longitudinal stress

$= \frac{\text{deforming force applied normal}}{\text{area}}$

$= \frac{F_n}{A}$

(ii) Volumetric stress

$= \frac{\text{force changing volume}}{\text{area}} = \frac{F_v}{A}$

(iii) Shearing or tangential stress

$= \frac{\text{tangential force}}{\text{area}} = \frac{F_t}{A}$

Elasticity

Property of materials by virtue of which it regain its original configuration after the removal of deforming force.

(i) Longitudinal strain $= \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l_0}$

(ii) Volumetric strain $\varepsilon_v = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V_0}$

(iii) Shearing strain = angular displacement of the plane perpendicular to the fixed surface

Elastic potential energy in a stretched wire

$= \frac{1}{2}$ stress $\times$ strain $\times$ volume of the wire

Relation between Y, B, η and σ

(i) $Y = 3B(1 - 2\sigma)$

(ii) $Y = 2\eta(1 + \sigma)$

(iii) $\sigma = \frac{3B - 2\eta}{2\eta + 6B}$

(iv) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$ or $Y = \frac{9B\eta}{\eta + 3B}$

Poisson's ratio (σ)

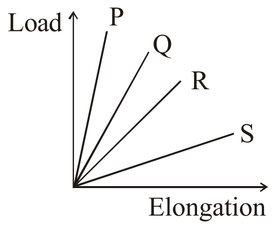
$= \frac{\text{lateral strain }(\beta)}{\text{longitudinal strain }(\alpha)} = \frac{\Delta d / d}{\Delta l / l}$

Value of σ lies between 0 and 0.5

EXERCISE - 1
Conceptual Questions

1. Elastomers are the materials which
(a) are not elastic at all
(b) have very small elastic range
(c) do not obey Hooke's law
(d) None of these

2. The load versus elongation graph for four wires is shown. The thinnest wire is



(a) *P* (b) *Q*
(c) *R* (d) *S*

3. Which of the following affects the elasticity of a substance?
(a) hammering and annealing
(b) change in temperature
(c) impurity in substance
(d) All of these

4. Which of the following has no dimensions ?
(a) strain (b) angular velocity
(c) momentum (d) angular momentum

5. Minimum and maximum values of Possion's ratio for a metal lies between
(a) $-\infty$ to $+\infty$ (b) 0 to 1
(c) $-\infty$ to 1 (d) 0 to 0.5

6. In solids interatomic forces are
(a) totally repulsive (b) totally attractive
(c) both (a) and (b) (d) None of these

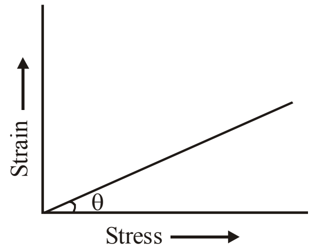
7. Which one of the following is not a unit of Young's modulus ?
(a) Nm^{-1} (b) Nm^{-2}
(d) dyne cm^{-2} (d) mega pascal

8. Two rods A and B of the same material and length have their radii r_1 and r_2 respectively. When they are rigidly fixed at one end and twisted by the same couple applied at the other end, the ratio

$$\left(\frac{\text{Angle of twist at the end of A}}{\text{Angle of twist at the end of B}}\right) \text{ is}$$

(a) r_1^2 / r_2^2 (b) r_1^3 / r_2^3
(c) r_2^4 / r_1^4 (d) r_1^4 / r_2^4

9. The value of tan (90 – θ) in the graph gives



(a) Young's modulus of elasticity
(b) compressibility
(c) shear strain
(d) tensile strength

10. The length of a metal is ℓ_1 when the tension in it is T_1 and is ℓ_2 when the tension is T_2. The original length of the wire is

(a) $\dfrac{\ell_1 + \ell_2}{2}$ (b) $\dfrac{\ell_1 T_2 + \ell_2 T_1}{T_1 + T_2}$

(c) $\dfrac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$ (d) $\sqrt{T_1 T_2 \ell_1 \ell_2}$

11. Uniform rod of mass m, length ℓ, area of cross-section A has Young's modulus Y. If it is hanged vertically, elongation under its own weight will be

(a) $\dfrac{mg\ell}{2AY}$ (b) $\dfrac{2mg\ell}{AY}$

(c) $\dfrac{mg\ell}{AY}$ (d) $\dfrac{mgY}{A\ell}$

12. Which one of the following affects the elasticity of a substance ?
(a) Change in temperature
(b) Hammering and annealing
(c) Impurity in substance
(d) All of the above

13. According to Hooke's law of elasticity, if stress is increased, then the ratio of stress to strain
(a) becomes zero (b) remains constant
(c) decreases (d) increases

14. The length of an iron wire is *L* and area of corss-section is A. The increase in length is *l* on applying the force F on its two ends. Which of the statement is correct?
(a) Increase in length is inversely proportional to its length
(b) Increase in length is proportional to area of cross-section
(c) Increase in length is inversely proportional to area of cross-section
(d) Increase in length is proportional to Young's modulus

15. A and B are two wires. The radius of A is twice that of B. They are stretched by the same load. Then the stress on B is

(a) equal to that on A (b) four times that on A
(c) two times that on A (d) half that on A

16. Hooke's law defines

(a) stress
(b) strain
(c) modulus of elasticity
(d) elastic limit

17. In case of steel wire (or a metal wire), the limit is reached when

(a) the wire just break
(b) the load is more than the weight of wire
(c) elongation is inversely proportional to the tension
(d) None of these

18. A steel ring of radius r and cross sectional area A is fitted onto a wooden disc of radius R (R > r). If the Young's modulus of steel is Y, then the force with which the steel ring is expanded is

(a) A Y (R/r) (b) A Y (R – r)/r
(c) (Y/A)[(R – r)/r] (d) Y r/A R

19. Which of the following relation is true ?

(a) $3Y = K(1-\sigma)$ (b) $K = \frac{9\eta Y}{Y+\eta}$

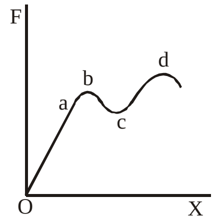
(c) $\sigma = (6K+\eta)Y$ (d) $\sigma = \frac{05.Y-\eta}{\eta}$

20. For a constant hydraulic stress on an object, the fractional change in the object volume $\left(\frac{\Delta V}{V}\right)$ and its bulk modulus (B) are related as

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$

(c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto B^{-2}$

21. The diagram shown below represents the applied forces per unit area with the corresponding change X (per unit length) produced in a thin wire of uniform cross section in the curve shown. The region in which the wire behaves like a liquid is



(a) ab
(b) bc
(c) cd
(d) Oa

22. A steel wire is suspended vertically from a rigid support. When loaded with a weight in air, it extends by ℓ_a and when the weight is immersed completely in water, the extension is reduced to ℓ_w. Then the relative density of material of the weight is

(a) ℓ_a / ℓ_w (b) $\frac{\ell_a}{\ell_a - \ell_w}$

(c) $\ell_w / (\ell_a - \ell_w)$ (d) ℓ_w / ℓ_a

23. When an elastic material with Young's modulus Y is subjected to stretching stress S, elastic energy stored per unit volume of the material is

(a) YS / 2 (b) $S^2Y / 2$
(c) $S^2 / 2Y$ (d) S / 2Y

24. The ratio of shearing stress to the corresponding shearing strain is called

(a) bulk modulus (b) Young's modulus
(c) modulus of rigidity (d) None of these

25. The Young's modulus of a perfectly rigid body is

(a) unity
(b) zero
(c) infinity
(d) some finite non-zero constant

EXERCISE - 2
Applied Questions

1. Two wires of same material and length but cross-sections in the ratio 1 : 2 are used to suspend the same loads. The extensions in them will be in the ratio

(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

2. A body of mass 10 kg is attached to a wire of radius 3 cm. It's breaking stress is 4.8×10^7 Nm^{-2}, the area of cross-section of the wire is 10^{-6} m^2. What is the maximum angular velocity with which it can be rotated in the horizontal circle ?

(a) 1 rad sec^{-1} (b) 2 rad sec^{-1}
(c) 4 rad sec^{-1} (d) 8 rad sec^{-1}

3. The Young's modulus of brass and steel are respectively 10^{10} N/m^2. and 2×10^{10} N/m^2. A brass wire and a steel wire of the same length are extended by 1 mm under the same force, the radii of brass and steel wires are R_B and R_S respectively. Then

(a) $R_S = \sqrt{2}\, R_B$ (b) $R_S = R_B / \sqrt{2}$
(c) $R_S = 4R_B$ (d) $R_S = R_B / 4$

4. A steel wire of length 20 cm and uniform cross-section 1 mm^2 is tied rigidly at both the ends. The temperature of the wire is altered from 40ºC to 20ºC. Coefficient of linear expansion for steel $\alpha = 1.1 \times 10^{-5}$/ºC and Y for steel is 2.0×10^{11} N/m^2. The change in tension of the wire is

(a) 2.2×10^6 newton (b) 16 newton
(c) 8 newton (d) 44 newton

5. A cube is subjected to a uniform volume compression. If the side of the cube decreases by 2% the bulk strain is
(a) 0.02 (b) 0.03
(c) 0.04 (d) 0.06

6. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
(a) 0.2 J (b) 10 J
(c) 20 J (d) 0.1 J

7. A metal rod of Young's modulus 2×10^{10} N m^{-2} undergoes an elastic strain of 0.06%. The energy per unit volume stored in J m^{-3} is
(a) 3600 (b) 7200
(c) 10800 (d) 14400

8. A force of 10^3 newton, stretches the length of a hanging wire by 1 millimetre. The force required to stretch a wire of same material and length but having four times the diameter by 1 millimetre is
(a) 4×10^3 N (b) 16×10^3 N
(c) $\frac{1}{4} \times 10^3$ N (d) $\frac{1}{16} \times 10^3$ N

9. A 2 m long rod of radius 1 cm which is fixed from one end is given a twist of 0.8 radian. The shear strain developed will be
(a) 0.002 (b) 0.004
(c) 0.008 (d) 0.016

10. There are two wire of same material and same length while the diameter of second wire is two times the diameter of first wire, then the ratio of extension produced in the wires by applying same load will be
(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) 4 : 1

11. For a given material, the Young's modulus is 2. 4 times that of rigidity modulus. Its Poisson's ratio is
(a) 2.4 (b) 1.2
(c) 0.4 (d) 0.2

12. A cube at temperature 0ºC is compressed equally from all sides by an external pressure P. By what amount should its temperature be raised to bring it back to the size it had before the external pressure was applied. The bulk modulus of the material of the cube is B and the coefficient of linear expansion is α.
(a) $P/B\alpha$ (b) $P/3B\alpha$
(c) $3\pi\alpha/B$ (d) $3B/P$

13. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmosphere will be
(a) 0.4 cm^3 (b) 4×10^{-5} cm^3
(c) 0.025 cm^3 (d) 0.004 cm^3

14. For the same cross-sectional area and for a given load, the ratio of depressions for the beam of a square cross-section and circular cross-section is
(a) $3 : \pi$ (b) $\pi : 3$
(c) $1 : \pi$ (d) $\pi : 1$

15. A massive stone pillar 20 m high and of uniform cross-section rests on a rigid base and supports a vertical load of 5.0×10^5 N at its upper end. If the compressive stress in the pillar is not to exceed 1.6×10^6 N m^{-2}, what is the minimum cross-sectional area of the pillar? Density of the stone $= 2.5 \times 10^3$ kg m^{-3}. (Take g = 10 N kg^{-1})
(a) 0.15 m^2 (b) 0.25 m^2
(c) 0.35 m^2 (d) 0.45 m^2

16. A circular tube of mean radius 8 cm and thickness 0.04 cm is melted up and recast into a solid rod of the same length. The ratio of the torsional rigidities of the circular tube and the solid rod is
(a) $\frac{(8.02)^4-(7.98)^4}{(0.8)^4}$ (b) $\frac{(8.02)^2-(7.98)^2}{(0.8)^2}$
(c) $\frac{(0.8)^2}{(8.02)^4-(7.98)^4}$ (d) $\frac{(0.8)^2}{(8.02)^3-(7.98)^2}$

17. From a steel wire of density ρ is suspended a brass block of density ρ_b. The extension of steel wire comes to e. If the brass block is now fully immersed in a liquid of density ρ_l, the extension becomes e'. The ratio $\frac{e}{e'}$ will be
(a) $\frac{\rho_b}{\rho_b-\rho_l}$ (b) $\frac{\rho_b-\rho_l}{\rho_b}$
(c) $\frac{\rho_b-\rho}{\rho_l-\rho}$ (d) $\frac{\rho_l}{\rho_b-\rho_l}$

18. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and W_1 weight is suspended from looser end. If A is area of cross-section of the wire, the stress in the wire at a height $\frac{L}{4}$ from the upper end is
(a) $\frac{W_1+W}{a}$ (b) $\frac{W_1+3W/4}{a}$
(c) $\frac{W_1+W/4}{a}$ (d) $\frac{4W_1+3W}{a}$

19. A beam of metal supported at the two edges is loaded at the centre. The depression at the centre is proportional to
(a) Y^2 (b) Y
(c) 1/Y (d) $1/Y^2$

20. An iron rod of length 2m and cross-sectional area of 50 mm^2 stretched by 0.5 mm, when a mass of 250 kg is hung from its lower end. Young's modulus of iron rod is
(a) 19.6×10^{20} N/m^2 (b) 19.6×10^{18} N/m^2
(c) 19.6×10^{10} N/m^2 (d) 19.6×10^{15} N/m^2

21. A wire fixed at the upper end stretches by length ℓ by applying a force F. The work done in stretching is
(a) $2F\ell$ (b) $F\ell$
(c) $\frac{F}{2\ell}$ (d) $\frac{F\ell}{2}$

22. A metalic rod of length l and cross-sectional area A is made of a material of Young modulus Y. If the rod is elongated by an amount y, then the work done is proportional to

(a) y (b) $\frac{1}{y}$

(c) y^2 (d) $\frac{1}{y^2}$

23. On stretching a wire, the elastic energy stored per unit volume is

(a) $Fl/2AL$ (b) $FA/2L$

(c) $FL/2A$ (d) $FL/2$

24. When a pressure of 100 atmosphere is applied on a spherical ball, then its volume reduces to 0.01%. The bulk modulus of the material of the rubber in dyne/cm^2 is

(a) 10×10^{12} (b) 100×10^{12}

(c) 1×10^{12} (d) 10×10^{12}

25. What per cent of length of wire increases by applying a stress of 1 kg weight/mm^2 on it?
($Y = 1 \times 10^{11}$ N/m^2 and 1 kg weight = 9.8 newton)

(a) 0.0067% (b) 0.0098%

(c) 0.0088% (d) 0.0078%

26. K is the force constant of a spring. The work done in increasing its extension from l_1 to l_2 will be

(a) $K(l_2 - l_1)$ (b) $\frac{K}{2}(l_2 + l_1)$

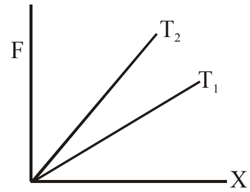
(c) $K(l_2^2 - l_1^2)$ (d) $\frac{K}{2}(l_2^2 - l_1^2)$

27. If a rubber ball is taken at the depth of 200 m in a pool, its volume decreases by 0.1%. If the density of the water is 1×10^3 kg/m^3 and g = 10m/s^2, then the volume elasticity in N/m^2 will be

(a) 10^8 (b) 2×10^8

(c) 10^9 (d) 2×10^9

28. The diagram below shows the change in the length X of a thin uniform wire caused by the application of stress F at two different temperatures T_1 and T_2. The variation shown suggests that



(a) $T_1 > T_2$

(b) $T_1 < T_2$

(c) $T_2 > T_1$

(d) $T_1 \geq T_2$

29. A material has poisson's ratio 0.50. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(a) 0.6 (b) 0.4

(c) 0.2 (d) Zero

30. A 5 metre long wire is fixed to the ceiling. A weight of 10 kg is hung at the lower end and is 1 metre above the floor. The wire was elongated by 1 mm. The energy stored in the wire due to stretching is

(a) zero (b) 0.05 joule

(c) 100 joule (d) 500 joule

31. Two wires A and B are of the same material. Their lengths are in the ratio of 1 : 2 and the diameter are in the ratio 2 : 1. If they are pulled by the same force, then increase in length will be in the ratio of

(a) 2 : 1 (b) 1 : 4

(c) 1 : 8 (d) 8 : 1

32. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(a) $4F$ (b) $6F$

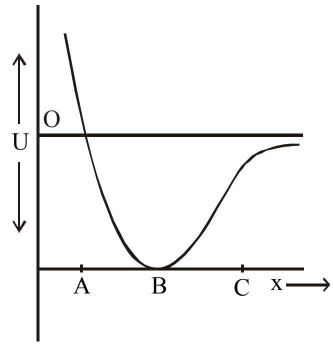
(c) $9F$ (d) F

33. A rubber cord catapult has cross-sectional area 25 mm^2 and initial length of rubber cord is 10 cm. It is stretched to 5 cm and then released to project a missile of mass 5 gm. Taking $Y_{rubber} = 5 \times 10^8$ N/m^2. Velocity of projected missile is

(a) 20 ms^{-1} (b) 100 ms^{-1}

(c) 250 ms^{-1} (d) 200 ms^{-1}

34. The potential energy U between two atoms in a diatomic molecules as a function of the distance x between atoms has been shown in the figure. The atoms are



(a) attracted when x lies between A and B and are repelled when x lies between B and C

(b) attracted when x lies between B and C and are repelled when x lies between A and B

(c) are attracted when they reach B from C

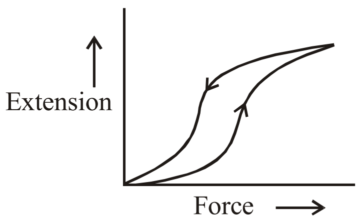
(d) are repelled when they reach B from A

35. The diagram shows a force - extension graph for a rubber band. Consider the following statements :

I. It will be easier to compress this rubber than expand it

II. Rubber does not return to its original length after it is stretched

III. The rubber band will get heated if it is stretched and released



Which of these can be deduced from the graph?

(a) III only (b) II and III

(c) I and III (d) I only

36. A rod of length l and radius r is joined to a rod of length $l/2$ and radius $r/2$ of same material. The free end of small rod is fixed to a rigid base and the free end of larger rod is given a twist of θ°, the twist angle at the joint will be

(a) θ/4 (b) θ/2
(c) 5θ/6 (d) 8θ/48

37. To break a wire, a force of 10^6 N/m^2 is required. If the density of the material is 3×10^3 kg/m^3, then the length of the wire which will break by its own weight will be

(a) 34 m (b) 30 m
(c) 300 m (d) 3 m

38. The upper end of a wire of diameter 12mm and length 1m is clamped and its other end is twisted through an angle of 30°. The angle of shear is

(a) 18° (b) 0.18°
(c) 36° (d) 0.36°

39. A steel wire of uniform cross-section of 1mm^2 is heated upto 50°C and clamped rigidly at its ends. If temperature of wire falls to 40°C, change in tension in the wire is (coefficient of linear expansion of steel is 1.1×10^{-5}/°C and Young's modulus of elasticity of steel is 2×10^{11} N/m^2)

(a) 22 N (b) 44 N
(c) 88 N (d) 88×10^6 N

40. In Searle's experiment to find Young's modulus the diameter of wire is measured as d = 0.05cm, length of wire is ℓ = 125cm and when a weight, m = 20.0 kg is put, extension in wire was found to be 0.100 cm. Find maximum permissible error in Young's modulus (Y). Use : $Y = \dfrac{mg\ell}{(\pi/4)\,d^2 x}$.

(a) 6.3% (b) 5.3%
(c) 2.3% (d) 1%

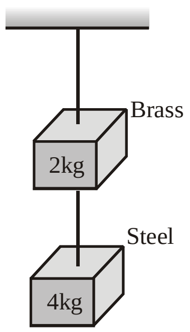
41. If the ratio of lengths, radii and Young's modulus of steel and brass wires shown in the figure are a, b, and c, respectively. The ratio between the increase in lengths of brass and steel wires would be

(a) $\dfrac{b^2 a}{2c}$

(b) $\dfrac{bc}{2a^2}$

(c) $\dfrac{ba^2}{2c}$

(d) $\dfrac{a}{2b^2 c}$



42. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is

(a) 0.01 (b) 0.06
(c) 0.02 (d) 0.03

43. When a 4 kg mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches by 2 cms. The work required to be done by an external agent in stretching this spring by 5 cms will be (g = 9.8 m/sec^2)

(a) 4.900 joule (b) 2.450 joule
(c) 0.495 joule (d) 0.245 joule

44. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ?

(a) Length = 100 cm, diameter = 1 mm
(b) Length = 200 cm, diameter = 2 mm
(c) Length = 300 cm, diameter = 3 mm
(d) Length = 50 cm, diameter = 0.5 mm

45. A steel rod of radius R = 10 mm and length L= 100 cm is stretched along its length by a force $F = 6.28 \times 10^4$ N. If the Young's modulus of steel is $Y = 2 \times 10^{11}$ N/m^2, the percentage elongation in the length of the rod is :

(a) 0.100 (b) 0.314
(c) 2.015 (d) 1.549

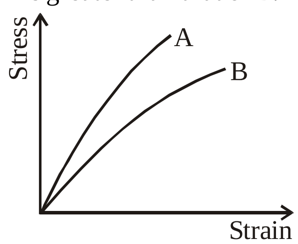
DIRECTIONS for Qs. 46 to 50 : These are Assertion-Reason type questions. Each of these question contains two statements: Statement-1 (Assertion) and Statement-2 (Reason). Answer these questions from the following four options.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement -1
(b) Statement-1 is True, Statement -2 is True; Statement-2 is NOT a correct explanation for Statement - 1
(c) Statement-1 is True, Statement- 2 is False
(d) Statement-1 is False, Statement -2 is True

46. **Statement-1** Identical springs of steel and copper are equally stretched. More work will be done on the steel spring.
Statement-2 Steel is more elastic than copper.

47. **Statement-1** Stress is the internal force per unit area of a body.
Statement-2 Rubber is less elastic than steel.

48. **Statement 1 :** The stress-strain graphs are shown in the figure for two materials A and B are shown in figure. Young's modulus of A is greater than that of B.



Statement 2 : The Young's modules for small strain is,
$Y = \dfrac{\text{stress}}{\text{strain}}$ = slope of linear portion, of graph; and slope of A is more than slope that of B.

49. **Statement 1:** Young's modulus for a perfectly plastic body is zero.
Statement 2: For a perfectly plastic body, restoring force is zero.

50. **Statement 1:** Strain causes the stress in an elastic body.
Statement 2: An elastic rubber is more plastic in nature.

EXERCISE - 3
Exemplar & Past Years NEET/AIPMT Questions

Exemplar Questions

1. Modulus of rigidity of ideal liquids is
(a) infinity
(b) zero
(c) unity
(d) some finite small non-zero constant value

2. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will
(a) be double (b) be half
(c) be four times (d) remain same

3. The temperature of a wire is doubled. The Young's modulus of elasticity
(a) will also double (b) will become four times
(c) will remain same (d) will decrease

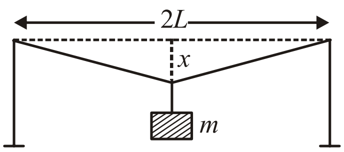
4. A spring is stretched by applying a load to its free end. The strain produced in the spring is
(a) volumetric
(b) shear
(c) longitudinal and shear
(d) longitudinal

5. A rigid bar of mass M is supported symmetrically by three wires each of length l. Those at each end are of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to

(a) Ycopper / Yiron (b) $\sqrt{\dfrac{Y_{\text{iron}}}{Y_{\text{copper}}}}$

(c) $\dfrac{Y_{\text{iron}}^2}{Y_{\text{copper}}^2}$ (d) $\dfrac{Y_{\text{iron}}}{Y_{\text{copper}}}$

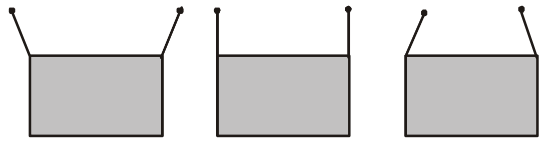
6. A mild steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars (figure). A mass m is suspended from the mid-point of the wire. Strain in the wire is



(a) $\dfrac{x^2}{2L^2}$ (b) $\dfrac{x}{L}$

(c) $\dfrac{x^2}{L}$ (d) $\dfrac{x^2}{2L}$

7. A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports (figure). It can be done in one of the following three ways;


(a) (b) (c)

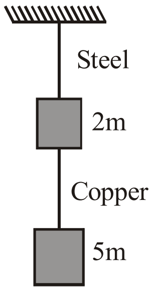
The tension in the strings will be
(a) the same in all cases (b) least in (a)
(c) least in (b) (d) least in (c)

8. Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods.
(a) Both the rods will elongate but there shall be no perceptible change in shape
(b) The steel rod will elongate and change shape but the rubber rod will only elongate
(c) The steel rod will elongate without any perceptible change in shape, but the rubber rod will elongate and the shape of the bottom edge will change to an ellipse
(d) The steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre

NEET/AIPMT (2013-2017) Questions

9. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ? ***[2013]***
(a) Length = 100 cm, diameter = 1 mm
(b) Length = 200 cm, diameter = 2 mm
(c) Length = 300 cm, diameter = 3 mm
(d) Length = 50 cm, diameter = 0.5 mm

10. If the ratio of diameters, lengths and Young's modulus of steel and copper wires shown in the figure are p, q and s respectively, then the corresponding ratio of increase in their lengths would be ***[NEET Kar. 2013]***



(a) $\frac{7q}{(5sp)}$

(b) $\frac{5q}{(7sp^2)}$

(c) $\frac{7q}{(5sp^2)}$

(d) $\frac{2q}{(5sp)}$

11. Copper of fixed volume 'V; is drawn into wire of length 'l'. When this wire is subjected to a constant force 'F', the extension produced in the wire is 'Δl'. Which of the following graphs is a straight line? ***[2014]***

(a) Δl versus $\frac{1}{l}$ (b) Δl versus l^2

(c) Δl versus $\frac{1}{l^2}$ (d) Δl versus l

12. The approximate depth of an ocean is 2700 m. The compressibility of water is 45.4×10^{-11} Pa^{-1} and density of water is 10^3 kg/m^3. What fractional compression of water will be obtained at the bottom of the ocean ? ***[2015]***

(a) 1.0×10^{-2} (b) 1.2×10^{-2}

(c) 1.4×10^{-2} (d) 0.8×10^{-2}

13. The Young's modulus of steel is twice that of brass. Two wires of same length and of same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of : ***[2015 RS]***

(a) 2 : 1 (b) 4 : 1

(c) 1 : 1 (d) 1 : 2

14. The bulk modulus of a spherical object is 'B'. If it is subjected to uniform pressure 'p', the fractional decrease in radius is ***[2017]***

(a) $\frac{B}{3p}$ (b) $\frac{3p}{B}$

(c) $\frac{p}{3B}$ (d) $\frac{p}{B}$

Hints & Solutions

EXERCISE - 1

1. (c) 2. (b) 3. (d) 4. (a) 5. (d)
6. (c) 7. (a)

8. (c) Couple per unit angle of twist, $C = \frac{\pi \eta r^4}{2\ell}$

$\therefore$ Couple $\tau = C\theta = \frac{\pi \eta r^4 \theta}{2\ell}$

Here η, ℓ, C & τ are same. So, $r^4\theta$ = constant

$\therefore \frac{\theta_1}{\theta_2} = \left(\frac{r_2^4}{r_1^4}\right)$

9. (a) $\tan(90-\theta) = \frac{\text{stress}}{\text{strain}}$

10. (c) If ℓ is the original length of wire, then change in length of first wire, $\Delta\ell_1 = (\ell_1 - \ell)$

change in length of second wire, $\Delta\ell_2 = (\ell_2 - \ell)$

Now, $Y = \frac{T_1}{A} \times \frac{\ell}{\Delta\ell_1} = \frac{T_2}{A} \times \frac{\ell}{\Delta\ell_2}$

or $\frac{T_1}{\Delta\ell_1} = \frac{T_2}{\Delta\ell_2}$ or $\frac{T_1}{\ell_1 - \ell} = \frac{T_2}{\ell_2 - \ell}$

or $T_1\ell_2 - T_1\ell = T_2\ell_1 - \ell T_2$ or $\ell = \frac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$

11. (c) $Y = \frac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \frac{F\ell}{YA} = \frac{mg\ell}{YA}$

12. (d) The elasticity of a material depends upon the temperature of the material. Hammering & annealing reduces elastic property of a substance.

13. (b) The ratio of stress to strain is always constant. If stress is increased, strain will also increase so that their ratio remains constant.

14. (c) $l = \frac{FL}{YA} \Rightarrow l \propto \frac{1}{A}$

15. (b) Stress $= \frac{\text{Force}}{\text{Area}}$ $\therefore$ Stress $\propto \frac{1}{\pi r^2}$

$\frac{S_B}{S_A} = \left(\frac{r_A}{r_B}\right)^2 = (2)^2 \Rightarrow S_B = 4S_A$

16. (c)

17. (d) According to Hooke's Law, within the elastic limits stress is directly proportional to strain.

18. (b) Let T be the tension in the ring, then

$Y = \frac{T.2\pi r}{A.2\pi(R-r)} = \frac{Tr}{A(R-r)}$ $\therefore T = \frac{YA(R-r)}{r}$

19. (d) $Y = 2\eta(1+\sigma) \Rightarrow \sigma = \frac{0.5Y - \eta}{\eta}$

20. (b) $B = \frac{\Delta p}{\Delta V/V} \Rightarrow \frac{1}{B} \propto \frac{\Delta V}{V}$ [Δp = constant]

21. (b) The wire starts behaving like a liquid at point b. It behaves like a viscous liquid in the region bc of the graph.

22. (b) Let V be the volume of the load and ρ its relative density

So, $Y = \frac{FL}{A\ell_a} = \frac{V\rho g L}{A\ell_a}$ .....(1)

When the load is immersed in the liquid, then

$Y = \frac{F'L}{A\ell_w} = \frac{(V\rho g - V \times 1 \times g)L}{A\ell_w}$ ....(2)

($\because$ Now net weight = weight – upthrust)
From eqs. (1) and (2), we get

$\frac{\rho}{\ell_a} = \frac{(\rho-1)}{\ell_w}$ or $\rho = \frac{\ell_a}{(\ell_a - \ell_w)}$

23. (c) Energy stored per unit volume

$= \frac{1}{2} \times \text{stress} \times \text{strain}$

$= \frac{1}{2} \times \text{stress} \times (\text{stress / Young's modulus})$

$= \frac{1}{2} \times (\text{stress})^2 / (\text{Young's modulus}) = \frac{S^2}{2Y}$

24. (c)

25. (c) For a perfectly rigid body strain produced is zero for the given force applied, so
Y = stress/strain = ∞

EXERCISE - 2

1. (b) Let W newton be the load suspended. Then

$Y = \frac{(W/A_1)}{(\ell_1/L)} = \frac{WL}{A_1\ell_1}$ ...(1)

and $Y = \frac{(W/A_2)}{(\ell_2/L)} = \frac{WL}{A_2\ell_2}$ ....(2)

Dividing equation (1) by equation (2), we get

$1 = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{A_2}{A_1}\right) = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{2}{1}\right)$

$\therefore \frac{\ell_1}{\ell_2} = \frac{2}{1}$ or $\ell_1 : \ell_2 = 2 : 1$

2. (c) Given that F/A = 4.8×10^7 Nm^{-2}
$\therefore$ $F = 4.8 \times 10^7 \times A$ or

$\frac{mv^2}{r} = 4.8 \times 10^7 \times 10^{-6} = 48$

or $\frac{mr^2\omega^2}{r} = 48$ or $\omega^2 = \frac{48}{mr}$

$\omega = \sqrt{\left(\frac{48}{10 \times 0.3}\right)} = \sqrt{16} = 4$ rad / sec

3. (b) We know that $Y = FL/\pi r^2 \ell$ or $r^2 = FL/(Y\pi\ell)$

$\therefore \; R_B^2 = FL/(Y_B \pi \ell)$ and $R_S^2 = FL/(Y_S \pi \ell)$

or $\dfrac{R_B^2}{R_S^2} = \dfrac{Y_S}{Y_B} = \dfrac{2\times 10^{10}}{10^{10}} = 2$

or $R_B^2 = 2R_S^2$ or $R_B = \sqrt{2}\, R_S$ $\quad\therefore$

$R_S = R_B/\sqrt{2}$

4. (d) $F = YA\alpha t = (2.0 \times 10^{11})(10^{-6})(1.1 \times 10^{-5})(20)$
$= 44$ newton

5. (d)

6. (d) Elastic energy $= \dfrac{1}{2} \times F \times x$
$F = 200$ N, $x = 1$ mm $= 10^{-3}$ m

$\therefore \; E = \dfrac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1$ J

7. (a) $U/\text{volume} = \dfrac{1}{2} Y \times \text{strain}^2 = 3600 \text{ J m}^{-3}$
[Strain $= 0.06 \times 10^{-2}$]

8. (b) $F = Y \times A \times \dfrac{l}{L} \Rightarrow F \propto r^2$ (Y, l and and L are constant)
If diameter is made four times then force required will be 16 times, i.e., 16×10^3 N

9. (b) $\phi = \dfrac{r\theta}{\ell} = \dfrac{1}{100} \times \dfrac{0.8}{2} = 0.004$ radian

10. (d) $\because$ Both wires are same materials so both will have same Young's modulus, and let it be Y.

$Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F}{A.(\Delta L/L)}$, F = applied force

A = area of cross-section of wire

Now, $Y_1 = Y_2 \Rightarrow \dfrac{FL}{(A_1)(\Delta L_1)} = \dfrac{FL}{(A_2)(\Delta L_2)}$

Since load and length are same for both

$\Rightarrow r_1^2 \Delta L_1 = r_2^2 \Delta L_2$, $\left(\dfrac{\Delta L_1}{\Delta L_2}\right) = \left(\dfrac{r_2}{r_1}\right)^2 = 4$

$\Delta L_1 : \Delta L_2 = 4:1$

11. (d) $Y = 2\eta(1+\sigma)$

$2.4\eta = 2\eta(1+\sigma) \Rightarrow 1.2 = 1+\sigma \Rightarrow \sigma = 0.2$

12. (b) Bulk modulus $B = \dfrac{-P}{(\Delta V/V)} = \dfrac{-PV}{\Delta V}$ ....(1)

and $\Delta V = \gamma V \Delta T = 3\alpha.V.T$ or $\dfrac{-V}{\Delta V} = \dfrac{1}{3\alpha.T.}$ ...(2)

From eqs. (1) and (2), $B = P/(3\alpha.T)$ or $T = \dfrac{P}{3\alpha B}$

13. (a) $K = \dfrac{1}{B} = \dfrac{\Delta V/V}{P}$. Here, $P = 100$ atm,
$K = 4 \times 10^{-5}$ and $V = 100 \text{ cm}^3$.
Hence, $\Delta V = 0.4 \text{ cm}^3$

14. (a) $\delta = \dfrac{W\ell^3}{3YI}$, where W = load, ℓ = length of beam and I is geometrical moment of inertia for rectangular beam,

$I = \dfrac{bd^3}{12}$ where b = breadth and d = depth

For square beam b = d

$\therefore \; I_1 = \dfrac{b^4}{12}$

For a beam of circular cross-section, $I_2 = \left(\dfrac{\pi r^4}{4}\right)$

$\therefore \; \delta_1 = \dfrac{W\ell^3 \times 12}{3Yb^4} = \dfrac{4W\ell^3}{Yb^4}$ (for sq. cross section)

and $\delta_2 = \dfrac{W\ell^3}{3Y(\pi r^4/4)} = \dfrac{4W\ell^3}{3Y(\pi r^4)}$
(for circular cross-section)

Now $\dfrac{\delta_1}{\delta_2} = \dfrac{3\pi r^4}{b^4} = \dfrac{3\pi r^4}{(\pi r^2)^2} = \dfrac{3}{\pi}$

($\because b^2 = \pi r^2$ i.e., they have same cross-sectional area)

15. (d) $\dfrac{\text{Weight of load + Weight of puller}}{\text{area}} = \text{Compressive stress}$

$\Rightarrow \dfrac{20A \times d \times 10 + 5 \times 10^5}{\text{area}} = 1.6 \times 10^6$

Where d is the density.

$\dfrac{20A \times 2.5 \times 10^3 \times 10 + 5 \times 10^5}{A} = 1.6 \times 10^6$

ie, $5 \times 10^5 = 1.1 \times 10^6 A$ or $A = 0.45 \text{m}^2$

16. (a) $C_1 = \dfrac{\pi\eta(r_2^4 - r_1^4)}{2\ell}$, $C_2 = \dfrac{\pi\eta r^4}{2\ell}$

Initial volume = Final volume

$\therefore \; \pi[r_2^2 - r_1^2]\ell\rho = \pi r^2 \ell \rho$

$\Rightarrow r^2 = r_2^2 - r_1^2 \Rightarrow r^2 = (r_2 + r_1)(r_2 - r_1)$

$\Rightarrow r^2 = (8.02 + 7.98)(8.02 - 7.98)$

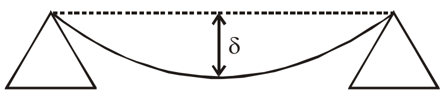
$\Rightarrow r^2 = 16 \times 0.04 = 0.64 \text{ cm} \Rightarrow r = 0.8 \text{ cm}$

$\therefore \dfrac{C_1}{C_2} = \dfrac{r_2^4 - r_1^4}{r^4} = \dfrac{[8.02]^4 - [7.98]^4}{[0.8]^4}$

17. (a) Weights without and with liquid proportional to ρ_b and $\rho_b - \rho_l$.

18. (b)

19. (c)



For a beam, the depression at the centre is given by,

$\delta = \left(\dfrac{fL}{4Ybd^3}\right)$

[f, L, b, d are constants for a particular beam]

i.e. $\delta \propto \dfrac{1}{Y}$

20. (c) $Y = \dfrac{F/A}{\Delta\ell/\ell} = \dfrac{\dfrac{250\times 9.8}{50\times 10^{-6}}}{\dfrac{0.5\times 10^{-3}}{2}} = \dfrac{250\times 9.8}{50\times 10^{-6}}\times\dfrac{2}{0.5\times 10^{-3}}$

$\Rightarrow 19.6\times 10^{10}\ \mathrm{N/m^2}$

21. (d) Work done by constant force in displacing the object by a distance ℓ.
= change in potential energy

$= \dfrac{1}{2}\times$ stress $\times$ strain $\times$volume

$= \dfrac{1}{2}\times\dfrac{F}{A}\times\dfrac{\ell}{L}\times A\times L = \dfrac{F\ell}{2}$

22. (c) Volume V = cross sectional A × length l or $V = Al$

Strain $= \dfrac{\text{Elongation}}{\text{Original length}} = \dfrac{Y}{l}$

Young's modulus $Y = \dfrac{\text{Stress}}{\text{Strain}}$

Work done, W $= \dfrac{1}{2}\times$stress × strain × volume

$W = \dfrac{1}{2}\times Y\times(\text{strain})^2\times Al$

$= \dfrac{1}{2}\times Y\times\left(\dfrac{y}{l}\right)^2\times Al = \dfrac{1}{2}\left(\dfrac{YA}{l}\right)y^2 \Rightarrow W\propto y^2$

23. (a) Energy stored per unit volume $= \dfrac{1}{2}\left(\dfrac{F}{A}\right)\left(\dfrac{l}{L}\right) = -\dfrac{Fl}{2AL}$

24. (c) $K = \dfrac{100}{0.01/100} = 10^6$ atm
$= 10^{11}\ \mathrm{N/m^2} = 10^{12}\ \mathrm{dyne/cm^2}$

25. (b) Stress = 1 kg wt/mm^2 = 9.8 N/mm^2
= 9.8 × 10^6 N/m^2.

$Y = 1\times 10^{11}\ \mathrm{N/m^2},\quad \dfrac{\Delta\ell}{\ell}\times 100 = ?$

$Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{\text{Stress}}{\Delta\ell/\ell}$

$\therefore\ \dfrac{\Delta\ell}{\ell} = \dfrac{\text{Stress}}{Y} = \dfrac{9.8\times 10^6}{1\times 10^{11}}$

$\dfrac{\Delta\ell}{\ell}\times 100 = 9.8\times 10^{-11}\times 100\times 10^6$

$= 9.8\times 10^{-3} = 0.0098\ \%$

26. (d) At extension l_1, the stored energy $= \dfrac{1}{2}Kl_1^2$

At extension l_2, the stored energy $= \dfrac{1}{2}Kl_2^2$

Work done in increasing its extension from l_1 to l_2

$= \dfrac{1}{2}K(l_2^2 - l_1^2)$

27. (d) $K = \dfrac{\Delta P}{\Delta V/V} = \dfrac{h\rho g}{\Delta V/V} = \dfrac{200\times 10^3\times 10}{0.1/100} = 2\times 10^9$

28. (a) When same stress is applied at two different temperatures, the increase in length is more at higher temperature. Thus $T_1 > T_2$.

29. (d)

30. (b) $W = \dfrac{1}{2}\times F\times l = \dfrac{1}{2}mgl$

$= \dfrac{1}{2}\times 10\times 10\times 1\times 10^{-3} = 0.05J$

31. (c) We know that Young's modulus

$Y = \dfrac{F}{\pi r^2}\times\dfrac{L}{\ell}$

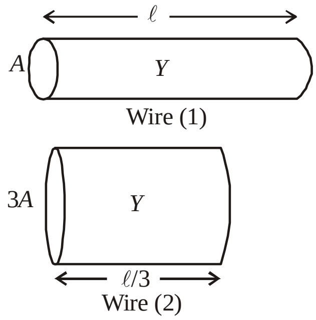
Since Y, F are same for both the wires, we have,

$\dfrac{1}{r_1^2}\dfrac{L_1}{\ell_1} = \dfrac{1}{r_2^2}\dfrac{L_2}{\ell_2}$ or, $\dfrac{\ell_1}{\ell_2} = \dfrac{r_2^2\times L_1}{r_1^2\times L_2} = \dfrac{(D_2/2)^2\times L_1}{(D_1/2)^2\times L_2}$

or, $\dfrac{\ell_1}{\ell_2} = \dfrac{D_2^2\times L_1}{D_1^2\times L_2} = \dfrac{D_2^2}{(2D_2)^2}\times\dfrac{L_2}{2L_2} = \dfrac{1}{8}$

So, $\ell_1 : \ell_2 = 1:8$

32. (c)



As shown in the figure, the wires will have the same Young's modulus (same material) and the length of the wire of area of cross-section 3A will be $\ell/3$ (same volume as wire 1).

For wire 1, $Y = \dfrac{F/A}{\Delta x/\ell}$ ...(i)

For wire 2, $Y = \dfrac{F'/3A}{\Delta x/(\ell/3)}$ ...(ii)

From (i) and (ii), $\dfrac{F}{A}\times\dfrac{\ell}{\Delta x} = \dfrac{F'}{3A}\times\dfrac{\ell}{3\Delta x} \Rightarrow F' = 9F$

33. (c) Young's modulus of rubber, Y_{rubber}

$= \dfrac{F}{A}\times\dfrac{\ell}{\Delta\ell} \Rightarrow F = YA.\dfrac{\Delta\ell}{\ell}$

On putting the values from question,

$F = \dfrac{5\times 10^8\times 25\times 10^{-6}\times 5\times 10^{-2}}{10\times 10^{-2}}$

$= 25\times 25\times 10^{2-1} = 6250\mathrm{N}$

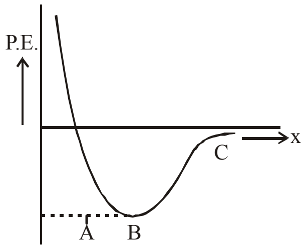
kinetic energy = potential energy of rubber

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}F\Delta\ell$

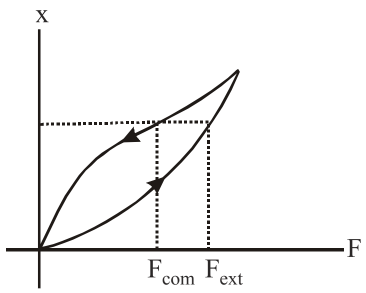
$v = \sqrt{\dfrac{F\Delta\ell}{m}} = \sqrt{\dfrac{6250\times 5\times 10^{-2}}{5\times 10^{-3}}} = \sqrt{62500}$

= 25 × 10 = 250 m/s

34. (b) The atoms when brought from infinity are attracted due to interatomic electrostatic force of attraction. At point B, the potential energy is minimum and force of attraction is maximum. But if we bring atoms closer than x = B, force of repulsion between two nuclei starts and P.E. increases.



35. (c)

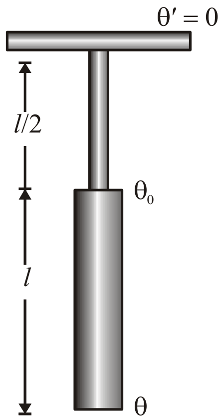


From the figure, it is clear that
$F_{com} < F_{ext.}$

36. (d) $r = C.\theta = \dfrac{\pi\eta r^4\theta}{2L} = \text{Constant}$

$$\Rightarrow \frac{\pi\eta r^4(\theta-\theta_0)}{2l} = \frac{\pi\eta(r/2)^4(\theta_0-\theta')}{2(l/2)}$$

$$\Rightarrow \frac{(\theta-\theta_0)}{2} = \frac{\theta_0}{16} \Rightarrow \theta_0 = \frac{8}{9}\theta$$



37. (a) $L = \dfrac{S}{dg} = \dfrac{10^6}{3\times10^3\times10} = \dfrac{100}{3} = 34\,m$

38. (b) $r\theta = \ell\phi \Rightarrow \phi = \dfrac{r\theta}{\ell} = \dfrac{6\text{mm}\times30°}{1\text{m}} = 0.18°$

39. (a) $F = YA\dfrac{\Delta\ell}{\ell} = YA\alpha\Delta\theta$

$= 2\times10^{11}\times(1\times10^{-6})\times1.1\times10^{-5}\times(10) = 22\text{ N}$

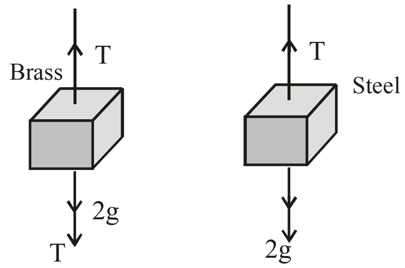
40. (a) $Y = \dfrac{mg\ell}{(\pi/4)\,d^2x}$ .... (1)

$$\left(\frac{dY}{Y}\right)_{max} = \frac{\Delta m}{m} + \frac{\Delta\ell}{\ell} + 2\frac{\Delta d}{d} + \frac{\Delta x}{x}$$

$m = 20.0\text{ kg} \Rightarrow \Delta m = 0.1\text{ kg}$
$\ell = 125\text{cm} \Rightarrow \Delta\ell = 1\text{ cm.}$
$d = 0.050\text{ cm.} \Rightarrow \Delta d = 0.001\text{ cm}$
$x = 0.100\text{ cm.} \Rightarrow \Delta x = 0.001\text{ cm.}$

$$\left(\frac{dY}{Y}\right)_{max} = \left(\begin{array}{l}\dfrac{0.1\text{kg}}{20.0\text{kg}} + \dfrac{1\text{cm}}{125\text{cm}} \\ +2\times\dfrac{0.001\text{cm}}{0.05\text{cm}} + \dfrac{0.001\text{cm}}{0.100\text{cm}}\end{array}\right)\times100\% = 6.3\%$$

41. (d) Given, $\dfrac{l_1}{l_2} = a,\ \dfrac{r_1}{r_2} = b,\ \dfrac{Y_1}{Y_2} = c$



Let Young's modulus of steel be Y_1, and that of brass be Y_2

$\therefore\ Y_1 = \dfrac{F_1 l_1}{A_1\Delta l_1}$ ...(i)

and $Y_2 = \dfrac{F_2 l_2}{A_2\Delta l_2}$ ...(ii)

Dividing equation (i) by equation (ii), we get

$\dfrac{Y_1}{Y_2} = \dfrac{F_1A_2l_1\Delta l_2}{F_2A_1l_2\Delta l_1}$ ...(iii)

Force on steel wire from free body diagram
$T = F_1 = (2g)$ Newton
Force on brass wire from free body diagram
$F_2 = T_1 = T + 2g = 4g$ Newton
Now putting the value of F_1, F_2 in equation (iii), we get

$$\frac{Y_1}{Y_2} = \left(\frac{2g}{4g}\right)\left(\frac{\pi r_2^2}{\pi r_1^2}\right)\cdot\left[\frac{l_1}{l_2}\right]\cdot\left(\frac{\Delta l_2}{\Delta l_1}\right) = \frac{1}{2}\left(\frac{1}{b^2}\right).a\left(\frac{\Delta l_2}{\Delta l_1}\right)$$

42. (d) If side of the cube is L then $V = L^3 \Rightarrow \dfrac{dV}{V} = 3\dfrac{dL}{L}$

$\therefore$ % change in volume = 3 × (% change in length)
= 3 × 1% = 3%

$\therefore$ Bulk strain $\dfrac{\Delta V}{V} = 0.03$

43. (b) $K = \dfrac{F}{x} = \dfrac{4\times9.8}{2\times10^{-2}} = 19.6\times10^2$

Work done $= \dfrac{1}{2}19.6\times10^2\times(0.05)^2 = 2.45\,J$

44. (d) $F = \dfrac{YA}{L}\times\ell$

So, extension, $\ell \propto \dfrac{L}{A} \propto \dfrac{L}{D^2}$ [$\because$ F and Y are constant]

$\ell_1 \propto \dfrac{100}{1^2} \propto 100$ and $\ell_2 \propto \dfrac{200}{2^2} \propto 50$

$\ell_3 \propto \dfrac{300}{3^2} \propto \dfrac{100}{3}$ and $\ell_4 \propto \dfrac{50}{\frac{1}{4}} \propto 200$

The ratio of $\dfrac{L}{D^2}$ is maximum for case (d).
Hence, option (d) is correct.

45. (a) Percentage elongation in the wire $= \dfrac{F\ell}{AY}$

$$= \frac{6.28\times10^4\times(1)}{\pi(0.01)^2\times2\times10^{11}} = \frac{1}{1000}$$

46. (a) Work done $= \dfrac{1}{2}\times$ Stress $\times$ Strain $= \dfrac{1}{2}\times Y\times(\text{Strain})^2$
Since, elasticity of steel is more than copper, hence more work has to be done in order to stretch the steel.

47. (b) Stress is defined as internal force (restoring force) per unit area of a body. Also, rubber is less elastic than steel, because restoring force is less for rubber than steel.

48. (d) 49. (d) 50. (a)

EXERCISE - 3

Exemplar Questions

1. (b) As liquid is ideal so no. frictional force exists hence, tangential forces are zero so there is no stress developed.

2. (d) As we know that,

$$\text{Breaking stress} = \frac{\text{Breaking force}}{\text{Area of cross-section}} \quad ..(i)$$

When length of the wire changes (or by reducing half) area of cross-section remains same.
Hence, breaking force will be same because breaking stress does not depend on length.

3. (d) As we know that, length of a wire when the temperature increased

$L_t = L_0(1+\alpha\Delta T)$
where ΔT is change in the temperature.
L_0 is original length,
α is coefficient of linear expansion and
L_t is length at temperature T.
Now, $\Delta L = L_t - L_0 = L_0\alpha\Delta T$

Now, Young's modulus $(Y) = \dfrac{\text{Stress}}{\text{Strain}}$

$$= \frac{FL_0}{A\times\Delta L} = \frac{FL_0}{AL_0\alpha\Delta T} \propto \frac{1}{\Delta T}$$

As, $Y \propto \dfrac{1}{\Delta T}$

So, if temperature increass ΔT increases, hence Young's modulus of elasticity (Y) decreases.

4. (c) When a spring is stretched by applying a load to its free end. Clearly the length and shape of the spring changes. So strain produced when change in length corresponds to longitudinal strain and change in shape corresponds to shearing strain.

5. (b) As we know that,
The Young's modulus

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L} = \frac{F}{A}\times\frac{L}{\Delta L}$$

$$= \frac{F}{\pi(D/2)^2}\times\frac{L}{\Delta L} = \frac{4FL}{\pi D^2\Delta L}$$

$$D^2 = \frac{4FL}{\pi\Delta LY} \Rightarrow D = \sqrt{\frac{4FL}{\pi\Delta LY}}$$

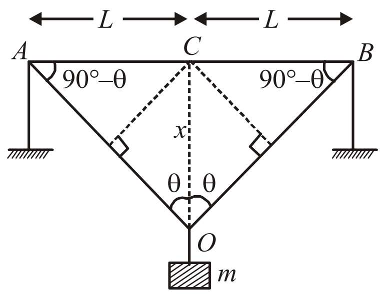
If F and $\dfrac{L}{\Delta L}$ are constants.

So, $D \propto \sqrt{\dfrac{1}{Y}}$

Hence, we can find ratio as

$$\frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$$

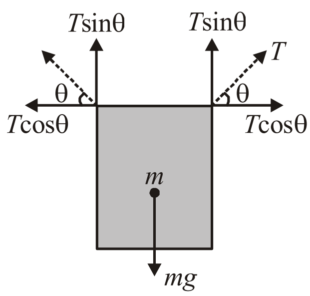
6. (a) Consider the given diagram



So, change in length

$$\Delta L = (A_0 + B_0) - (AC + CB)$$
$$= 2BO - 2AC$$
$$= 2\,[BO - AC] \quad (\because\ AO = BO, AC = CB)$$
$$= 2[(x^2+L^2)^{1/2} - L]$$
$$= 2L\left[\left(1+\frac{x^2}{L^2}\right)^{1/2} - 1\right]$$
$$\Delta L \approx 2L\left[1+\frac{1}{2}\frac{x^2}{L^2} - 1\right] = \frac{x^2}{L} \quad [\because x << L]$$
$$\text{Strain} = \frac{\Delta L}{2L} = \frac{x^2/L}{2L} = \frac{x^2}{2L^2}$$

7. (c) Let us consider the free body diagram of the rectangular frame



Net forces acting on frame will be zero.
So, Balancing vertical forces
$2T\sin\theta - mg = 0$ [T is tension in the string]
$2T\sin\theta = mg$ ...(i)
Total horizontal force
$= T\cos\theta - T\cos\theta = 0$

Now from Eq. (i), $T = \frac{mg}{2\sin\theta}$

$$T_{\max} = \frac{mg}{2\sin\theta_{\min}}$$

As mg is constant then

$$T \propto \frac{1}{\sin\theta}$$

$\sin\theta_{\min} = 0 \Rightarrow \theta_{\min} = 0$

No option matches with $\theta = 0°$

$$T_{\min} = \frac{mg}{2\sin\theta_{\max}} \qquad (\text{since, } \sin\theta_{\max} = 1)$$

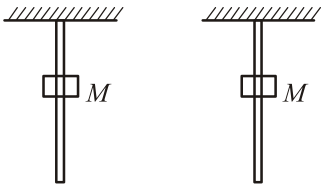
$\sin\theta_{\max} = 1 \Rightarrow \theta = 90°$

So tension is all three cases are different rejects option (a)

For minimum tension θ must be 90° i.e. $\sin\theta = 1$

Hence, tension is the least for the case (b).

8. (d) A mass M is attached at the centre or midpoint of rod of rubber and steel. As the mass is attached to both the rods, both rod will be elongated as shown in figures but due to different elastic properties of material rubber changes shape also.



(Rubber) (Steel)

As the Young's modulus of rigidity for steel, is larger than rubber, so strain $\frac{\Delta L}{L}$ for rubber is larger than steel for same stress.

NEET/AIPMT (2013-2017) Questions

9. (d) $F = \frac{YA}{L} \times \ell$ So, extension, $\ell \propto \frac{L}{A} \propto \frac{L}{D^2}$

[$\because$ F and Y are constant]

$$\ell_1 \propto \frac{100}{1^2} \propto 100 \text{ and } \ell_2 \propto \frac{200}{2^2} \propto 50$$

$$\ell_3 \propto \frac{300}{3^2} \propto \frac{100}{3} \text{ and } \ell_4 \propto \frac{50}{\frac{1}{4}} \propto 200$$

The ratio of $\frac{L}{D^2}$ is maximum for case (d).

Hence, option (d) is correct.

10. (c) From formula,

Increase in length $\Delta L = \frac{FL}{AY} = \frac{4FL}{\pi D^2 Y}$

$$\frac{\Delta L_S}{\Delta L_C} = \frac{F_S}{F_C}\left(\frac{D_C}{D_S}\right)^2 \frac{Y_C}{Y_S}\frac{L_S}{L_C} = \frac{7}{5} \times \left(\frac{1}{p}\right)^2 \left(\frac{1}{s}\right) q$$

$$= \frac{7q}{(5sp^2)}$$

11. (b) As $Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} \Rightarrow \Delta l = \frac{Fl}{AY}$

But $V = Al$ so $A = \frac{V}{l}$

Therefore $\Delta l = \frac{Fl^2}{VY} \propto l^2$

Hence graph of Δl versus l^2 will give a straight line.

12. (b) Compressibility of water,

$K = 45.4 \times 10^{-11}\ Pa^{-1}$

density of water $P = 10^3\ kg/m^3$

depth of ocean, $h = 2700$ m

We have to find $\frac{\Delta V}{V} = ?$

As we know, compressibility,

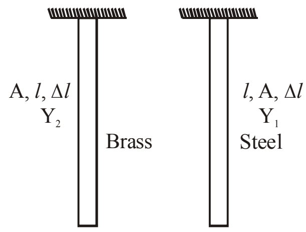
$$K = \frac{1}{B} = \frac{(\Delta V/V)}{P} \quad (P = \rho gh)$$

So, $(\Delta V/V) = K\rho gh$

$= 45.4 \times 10^{-11} \times 10^3 \times 10 \times 2700 = 1.2258 \times 10^{-2}$

13. (a) Young's modulus $Y = \frac{W}{A}.\frac{l}{\Delta l}$

$$\frac{W_1}{Y_1} = \frac{W_2}{Y_2} \quad [\because A, l, \Delta l \text{ same for both brass and steel}]$$



$$\frac{W_1}{W_2} = \frac{Y_1}{Y_2} = 2 \quad [Y_{steel}/Y_{brass} = 2 \text{ given}]$$

14. (c) Bulk modulus is given by

$$B = \frac{P}{\left(\frac{\Delta V}{V}\right)} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{P}{B}$$

$$3\frac{\Delta R}{R} = \frac{P}{B} \quad (\text{here, } \frac{\Delta R}{R} = \text{fractional decreases in radius})$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{P}{3B}$$