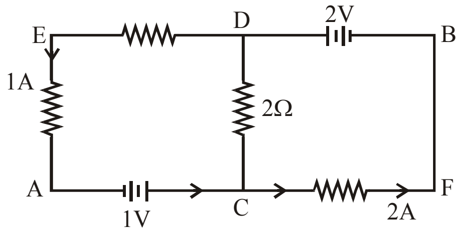
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Saturday 05th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is :



(1) +1V (2) – 1V (3) – 2V (4) +2V

2. A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant k = 4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will be energy stored in the capacitor be two times the initial energy stored?

(1) $l / 4$ (2) $l / 2$ (3) $l / 3$ (4) $2l / 3$

3. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge (+q) each, while 2, 4, 6, 8, 10 have charge (–q) each. The potential V and the electric field E at the centre of the circle are respectively:
(Take V = 0 at infinity)

(1) $V = \dfrac{10q}{4\pi\epsilon_0 R}; E = \dfrac{10q}{4\pi\epsilon_0 R^2}$

(2) $V = 0, E = \dfrac{10q}{4\pi\epsilon_0 R^2}$

(3) V = 0, E = 0

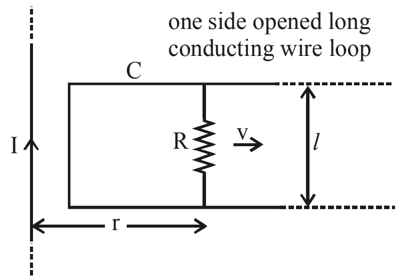
(4) $V = \dfrac{10q}{4\pi\epsilon_0 R}; E = 0$

4. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be :

(1) 0.5×10^2 Am^2 (2) 50×10^2 Am^2

(3) 500×10^2 Am^2 (4) 5×10^2 Am^2

5. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is :



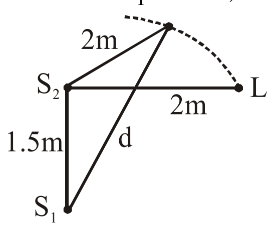
(1) $\frac{\mu_0}{\pi}\frac{Ivl}{Rr}$ (2) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$

(3) $\frac{2\mu_0}{\pi}\frac{Ivl}{Rr}$ (4) $\frac{\mu_0}{4\pi}\frac{Ivl}{Rr}$

6. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is : (h<<R, where R is the radius of the earth)

(1) $\frac{R^2\omega^2}{8g}$ (2) $\frac{R^2\omega^2}{4g}$ (3) $\frac{R^2\omega^2}{g}$ (4) $\frac{R^2\omega^2}{2g}$

7. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 1$ m, in phase. S_1 and S_2 are placed 1.5 m apart (see fig.) A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is :



(1) 12m (2) 3m (3) 5m (4) 2m

8. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is

(1) 36 km/hr (2) 24 km/hr

(3) 18 km/hr (4) 54 km/hr

9. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is:

(1) 326 (2) $\frac{1}{32}$ (3) 32 (4) 128

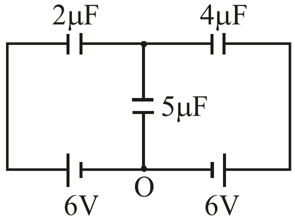
10. A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100s. the effective half life of the nucleus is close to:

(1) 9 sec (2) 55 sec
(3) 6 sec (4) 12 sec

11. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. the ratio $\frac{T_1}{T_2}$ will be :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{\sqrt{2}}$

12. In the circuit shown, charge on the 5 μF capacitor is :



(1) 5.45 μC (2) 16.36 μC
(3) 10.90 μC (4) 18.00 μC

13. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to :
(ignore viscosity of air)

(1) r (2) r^4 (3) r^3 (4) r^2

14. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is :

(1) $4\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2L_1}{(L_2+L_1)^2}$ (2) $2\sqrt{\alpha_1\alpha_2}$

(3) $\frac{\alpha_1+\alpha_2}{2}$ (4) $\frac{\alpha_1L_1+\alpha_2L_2}{L_1+L_2}$

15. The quantities $x=\frac{1}{\sqrt{\mu_0\in_0}}$, $y=\frac{E}{B}$ and $z=\frac{l}{CR}$ are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and $\in_0$, μ_0,-free space permittivity and permeability respectively. Then :

(1) Only x and y have the same dimension

(2) x, y and z have the same dimension

(3) Only x and z have the same dimension

(4) Only y and z have the same dimension

16. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6mA it produces a deflection of 2°, its figure of merit is close to :

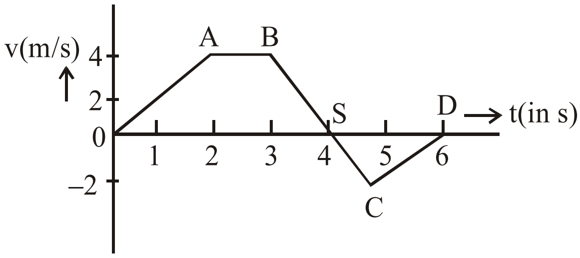
(1) 3×10^{-3} A/div.

(2) 333° A/div.

(3) 6×10^{-3} A/div.

(4) 666° A/div.

17. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6s is :



(1) 12m

(2) $\frac{49}{4}$ m

(3) 11 m

(4) $\frac{37}{3}$ m

18. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where v(t) is its instantaneous velocity. The instantaneous acceleration of the satellite is:

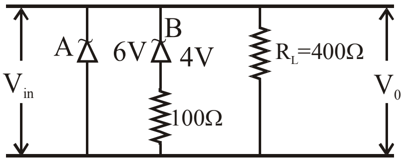
(1) $-\frac{2bv^3}{M(t)}$

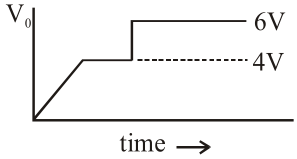
(2) $-\frac{bv^3}{2M(t)}$

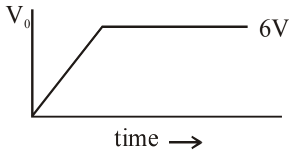
(3) $-bv^3(t)$

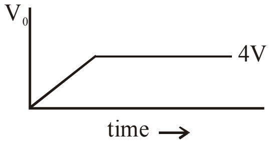
(4) $-\frac{bv^3}{M(t)}$

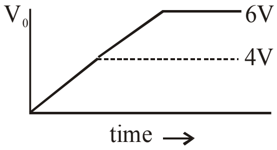
19. Two Zener diodes (A and B) having breakdown voltages of 6V and 4V respectively, are connected as shown in athe circuit below. The output voltage V_0 variation with input voltage linearly increasing with time, is given by : (V_{input} = 0V at t = 0) (figures are qualitative)



(1) 

(2) 

(3) 

(4) 

20. The correct match between the entries in column I and column II are :

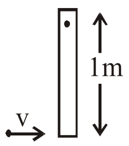
I **Radiation**	II **Wavelength**
(a) Microwave	(i) 100m
(b) Gamma rays	(ii) 10^{-15} m
(c) A.M. radio waves	(iii) 10^{-10} m
(d) X-rays	(iv) 10^{-3} m

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(2) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
(3) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)
(4) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

21. The surface of a metal is illuminated alternately with photons of energies E_1 = 4eV and E_2 = 2.5 eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is ______.

22. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is _____.
(Molar mass of N_2 gas 28 g)

23. A thin rod of mass 0.9 kg and length 1m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ______.



24. A body of mass 2kg is driven by an engine delivering a constant power 1J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) _______.

25. A prism of angle A = 1° has a refractive index μ = 1.5. A good estimate for the minimum angle of deviation (in degrees) is close to N/10. Value of N is ______.

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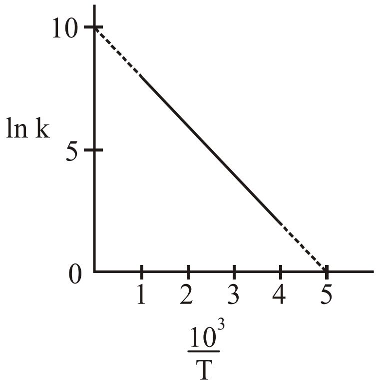
CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. The rate constant (k) of a reaction is measured at different temperatures (T), and the data are plotted in the given figure. The activation energy of the reaction in kJ mol^{-1} is :

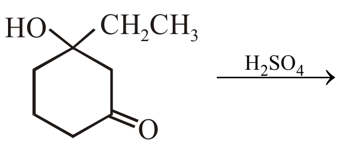
(R is gas constant)

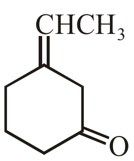


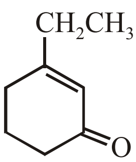
(1) 2R (2) R

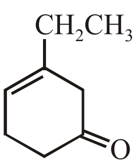
(3) 1/R (4) 2/R

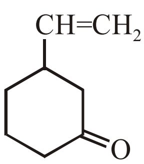
2. The major product of the following reaction is:



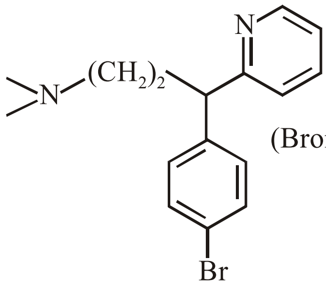
(1) 

(2) 

(3) 

(4) 

3. The following molecule acts as an :



(Brompheniramine)

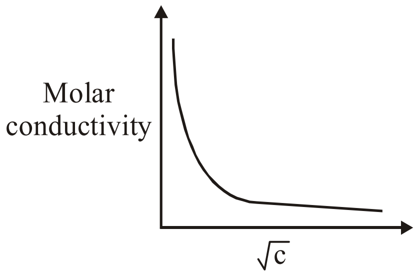
(1) Antiseptic (2) Anti-bacterial

(3) Anti-histamine (4) Anti-depressant

4. An element crystallises in a face-centred cubic (fcc) unit cell with cell edge a. The distance between the centres of two nearest octahedral voids in the crystal lattice is

(1) a (2) $\sqrt{2}a$ (3) $\frac{a}{\sqrt{2}}$ (4) $\frac{a}{2}$

5. The variation of molar conductivity with concentration of an electrolyte (X) in aqueous solution is shown in the given figure.



The electrolyte X is :

(1) CH_3COOH (2) KNO_3

(3) HCl (4) NaCl

6. The one that is NOT suitable for the removal of permanent hardness of water is :

(1) Treatment with sodium carbonate

(2) Calgon's method

(3) Clark's method

(4) Ion-exchange method

7. The correct statement about probability density (except at infinite distance from nucleus) is :

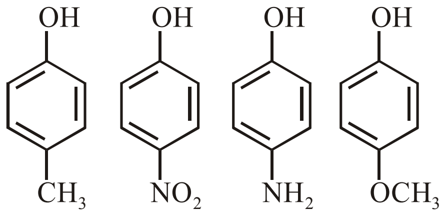
(1) It cn be negative for 2p orbital

(2) It can be zero for 3p orbital

(3) It can be zero for 1s orbital

(4) It can never be zero for 2s orbital

8. The increasing order of boiling points of the following compounds is :



I II III IV

(1) I < IV < III < II

(2) IV < I < II < III

(3) I < III < IV < II

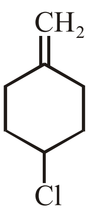
(4) III < I < II < IV

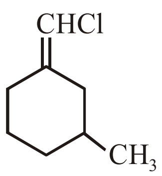
9. The compound that has the largest H–M–H bond angle (M=N, O, S, C), is :

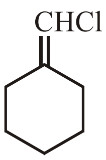
(1) H_2O (2) CH_4

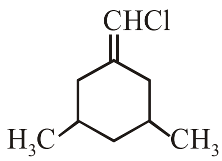
(3) NH_3 (4) H_2S

10. Among the following compounds, geometrical isomerism is exhibited by :

(1) 

(2) 

(3) 

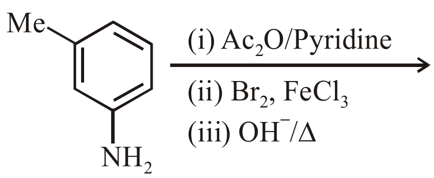
(4) 

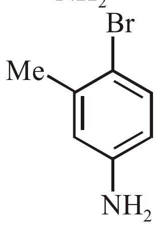
11. Which one of the following polymers is not obtained by condensation polymerisation?

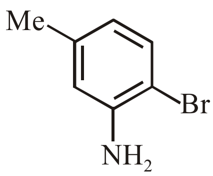
(1) Buna - N (2) Bakelite

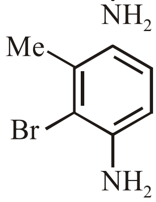
(3) Nylon 6 (4) Nylon 6, 6

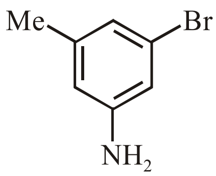
12. The final major product of the following reaction is :



(1) 

(2) 

(3) 

(4) 

13. Hydrogen peroxide, in the pure state, is :
(1) non-planar and almost colorless
(2) linear and almost colorless
(3) planar and blue in color
(4) linear and blue in color

14. Boron and silicon of very high purity can be obtained through :
(1) vapour phase refining
(2) electrolytic refining
(3) liquation
(4) zone refining

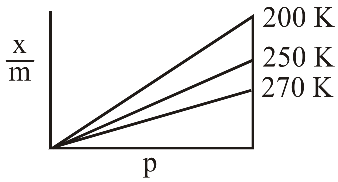
15. Lattice enthalpy and enthalpy of solution of NaCl are 788 kJ mol^{-1} and 4 kJ mol^{-1}, respectively. The hydration enthalpy of NaCl is :
(1) –780 kJ mol^{-1} (2) –784 kJ mol^{-1}
(3) 780 kJ mol^{-1} (4) 784 kJ mol^{-1}

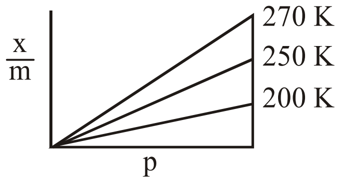
16. Reaction of ammonia with excess Cl_2 gives :
(1) NH_4Cl and N_2
(2) NCl_3 and NH_4Cl
(3) NH_4Cl and HCl
(4) NCl_3 and HCl

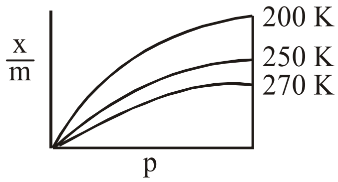
17. The correct order of the ionic radii of O^{2-}, N^{3-}, F^-, Mg^{2+}, Na^+ and Al^{3+} is :
(1) $Al^{3+} < Na^+ < Mg^{2+} < O^{2-} < F^- < N^{3-}$
(2) $N^{3-} < O^{2-} < F^- < Na^+ < Mg^{2+} < Al^{3+}$
(3) $Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$
(4) $N^{3-} < F^- < O^{2-} < Mg^{2+} < Na^+ < Al^{3+}$

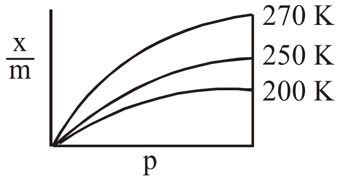
18. Consider the complex ions, *trans*-$[Co(en)_2Cl_2]^+$ (A) and *cis*-$[Co(en)_2Cl_2]^+$ (B). The correct statement regarding them is :
(1) both (A) and (B) can be optically active
(2) both (A) and (B) cannot be optically active
(3) (A) can be optically active, but (B) cannot be optically active
(4) (A) cannot be optically active, but (B) can be optically active

19. Adsorption of a gas follows Freundlich adsorption isotherm. If x is the mass of the gas adsorbed on mass m of the adsorbent, the correct plot of $\frac{x}{m}$ versus p is :

(1) 

(2) 

(3) 

(4) 

20. The major product formed in the following reaction is :

$$CH_3CH=CHCH(CH_3)_2 \xrightarrow{HBr}$$

(1) $CH_3\ CH_2\ CH_2\ C(Br)\ (CH_3)_2$
(2) $Br(CH_2)_3\ CH(CH_3)_2$
(3) $CH_3\ CH_2\ CH(Br)\ CH(CH_3)_2$
(4) $CH_3\ CH(Br)\ CH_2\ CH(CH_3)_2$

21. The number of chiral carbons present in sucrose is_______ .

22. For a dimerization reaction,

$2\ A(g) \rightarrow A_2(g)$

at 298 K, $\Delta U^\ominus$, = – 20kJ mol^{-1}, $\Delta S^\ominus$ = –30 J K^{-1} mol^{-1}, then the $\Delta G^\ominus$ will be _________J.

23. For a reaction $X + Y \rightleftharpoons 2Z$, 1.0 mol of X, 1.5 mol of Y and 0.5 mol of Z were taken in a 1 L vessel and allowed to react. At equilibrium, the concentration of Z was 1.0 mol L^{-1}. The equilibrium constant of the reaction is ______ $\frac{x}{15}$. The value of x is ________ .

X + Y = 2Z

24. The volume, in mL, of 0.02 M $K_2Cr_2O_7$ solution required to react with 0.288 g of ferrous oxalate in acidic medium is_______.
(Molar mass of Fe = 56 g mol^{-1})

25. Considering that $\Delta_0 > P$, the magnetic moment (in BM) of $[Ru(H_2O)_6]^{2+}$ would be________ .

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MATHEMATICS — TEST PAPER WITH SOLUTION

1. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to :

(1) 9 (2) –3

(3) –9 (4) 3

Sol. $x + y + 3z = 0$ .....(i)

$x + 3y + k^2z = 0$ .....(ii)

$3x + y + 3z = 0$ .....(iii)

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 9 + 3 + 3k^2 - 27 - k^2 - 3 = 0$

$\Rightarrow k^2 = 9$

(i) – (iii) $\Rightarrow -2x = 0 \Rightarrow x = 0$

Now from (i) $\Rightarrow y + 3z = 0$

$$\Rightarrow \frac{y}{z} = -3$$

$$x + \frac{y}{z} = -3$$

2. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to :

(1) $\frac{27}{16}$ (2) $\frac{1}{24}$

(3) $\frac{27}{32}$ (4) $\frac{3}{8}$

Sol. $7x^2 - 3x - 2 = 0$

$\alpha + \beta = \frac{3}{7}$ $\alpha\beta = \frac{-2}{7}$

$$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{\alpha + \beta - \alpha\beta(\alpha+\beta)}{1-\alpha^2-\beta^2+\alpha^2\beta^2}$$

$$= \frac{\frac{3}{7} + \frac{2}{7}\left(\frac{3}{7}\right)}{1-(\alpha+\beta)^2 + 2\alpha\beta + \alpha^2\beta^2} = \frac{27}{16}$$

3. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \ldots$ is 460, then x is equal to :

(1) $7^{46/21}$ (2) $7^{1/2}$

(3) e^2 (4) 7^2

Sol. $460 = \log_7 x \cdot (2 + 3 + 4 + ... + 20 + 21)$

$$\Rightarrow 460 = \log_7 x \cdot \left(\frac{21 \times 22}{2} - 1\right)$$

$\Rightarrow 460 = 230 \cdot \log_7 x$

$\Rightarrow \log_7 x = 2 \Rightarrow x = 49$

4. $$\lim_{x\to 0} \frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x} - 1\right)}{\sqrt{1+x^2+x^4}-1}$$

(1) does not exist. (2) is equal to $\sqrt{e}$.

(3) is equal to 0. (4) is equal to 1.

Sol. $$\lim_{x\to 0} \frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x} - 1\right)}{\sqrt{1+x^2+x^4}-1}$$

$$\because \lim_{x\to 0} \frac{\sqrt{1+x^2+x^4}-1}{x} \quad \left(\frac{0}{0} \text{ from}\right)$$

$$\lim_{x\to 0} \frac{(1+x^2+x^4)-1}{x(\sqrt{1+x^2+x^4}+1}$$

$$\lim_{x\to 0} \frac{x(1+x^2)}{\left(\sqrt{1+x^2+x^4}+1\right)} = 0$$

$$\text{So } \lim_{x\to 0} \frac{x\left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} - 1\right)}{\sqrt{1+x^2+x^4}-1} \quad \left(\frac{0}{0} \text{ from}\right)$$

$$\lim_{x\to 0} \frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} = 1$$

5. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is :

(1) $\frac{2}{13}(3^{50}-1)$ (2) $\frac{1}{26}(3^{50}-1)$

(3) $\frac{1}{13}(3^{50}-1)$ (4) $\frac{1}{26}(3^{49}-1)$

Sol. Let first term = a > 0

Common ratio = r > 0

$ar + ar^2 + ar^3 = 3$ ....(i)

$ar^5 + ar^6 + ar^7 = 243$ ....(ii)

$r^4(ar + ar^2 + ar^3) = 243$

$r^4(3) = 243 \Rightarrow r = 3$ as $r > 0$

from (1)

$3a + 9a + 27a = 3$

$a = \frac{1}{13}$

$S_{50} = \frac{a(r^{50}-1)}{(r-1)} = \frac{1}{26}(3^{50} - 1)$

6. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is :

(1) 2^{15} i (2) -2^{15}

(3) -2^{15} i (4) 6^5

Sol. $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} = \left(\frac{2\omega}{1-i}\right)^{30}$

$= \frac{2^{30}\cdot\omega^{30}}{\left((1-i)^2\right)^{30}}$

$= \frac{2^{30}\cdot 1}{(1+i^2-2i)^{15}}$

$= \frac{2^{30}}{-2^{15}\cdot i^{15}}$

$= -2^{15}i$

7. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is :

(1) $\frac{\sqrt{3}}{12}$ (2) $\frac{\sqrt{3}}{10}$

(3) $\frac{2\sqrt{3}}{5}$ (4) $\frac{2\sqrt{3}}{3}$

Sol. Let $f = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$f = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$f = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \frac{\theta}{2}$

$f = \frac{\tan^{-1}x}{2} \Rightarrow \frac{df}{dx} = \frac{1}{2(1+x^2)}$ ....(i)

Let $g = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1} x$

$g = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\right)$

$g = \tan^{-1}(\tan 2\theta) = 2\theta$

$g = 2\sin^{-1} x$

$\frac{dg}{dx} = \frac{2}{\sqrt{1-x^2}}$ ....(ii)

$\frac{df}{dg} = \frac{1}{2(1+x^2)}\,\frac{\sqrt{1-x^2}}{2}$

at $x = \frac{1}{2}$ $\left(\frac{df}{dg}\right)_{x=\frac{1}{2}} = \frac{\sqrt{3}}{10}$

8. The area (in sq. units) of the region $A = \{(x, y) : (x-1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where [t] denotes the greatest integer function, is :

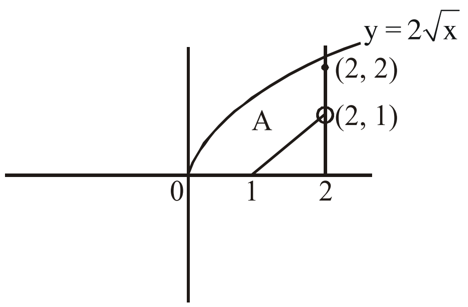
(1) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2} - 1$

(3) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (4) $\frac{4}{3}\sqrt{2} + 1$

Sol. $(x-1)[x] \le y \le 2\sqrt{x}$, $\boxed{0 \le x \le 2}$

Draw $y = 2\sqrt{x} \Rightarrow y^2 = 4x$ $\boxed{x \ge 0}$

$$y = (x-1)[x] = \begin{cases} 0 & , 0 \le x < 1 \\ x-1 & , 1 \le x < 2 \\ 2 & , x = 2 \end{cases}$$



$$A = \int_0^2 2\sqrt{x}\, dx - \frac{1}{2} 1 \cdot 1$$

$$A = 2 \cdot \left[\frac{x^{3/2}}{(3/2)}\right]_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

9. If the length of the chord of the circle, $x^2 + y^2 = r^2$ $(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to :

(1) $\frac{9}{5}$ (2) $\frac{12}{5}$

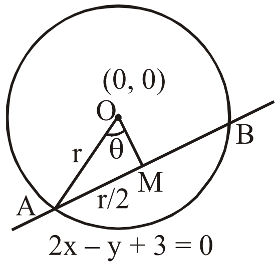
(3) 12 (4) $\frac{24}{5}$

Sol. Let chord

AB = r

$\because$ ΔAOM is right angled triangle

$\therefore$ $OM = \frac{r\sqrt{3}}{2}$ = perpendicular distance of line AB from (0,0)



$$\frac{r\sqrt{3}}{2} = \left|\frac{3}{\sqrt{5}}\right|$$

$$r^2 = \frac{12}{5}$$

10. If x = 1 is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then :

(1) x = 1 is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

(2) x = 1 is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.

(3) x = 1 and $x = -\frac{2}{3}$ are local minima of f.

(4) x = 1 and $x = -\frac{2}{3}$ are local maxima of f.

Sol. $f(x) = (3x^2 + ax - 2 - a)e^x$

$f'(x) = (3x^2 + ax - 2 - a)e^x + e^x(6x + a)$

$= e^x(3x^2 + x(6 + a) - 2)$

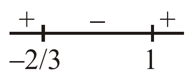
$f'(x) = 0$ at $x = 1$

$\Rightarrow 3 + (6 + a) - 2 = 0$

$a = -7$

$f'(x) = e^x(3x^2 - x - 2)$

$= e^x(x - 1)(3x + 2)$



x = 1 is point of local minima

$x = \frac{-2}{3}$ is point of local maxima

11. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

(1) $2x^2 - 20x + 19 = 0$
(2) $x^2 - 10x + 19 = 0$
(3) $x^2 - 10x + 18 = 0$
(4) $x^2 - 20x + 18 = 0$

Sol. Mean = 5

$$\frac{3+5+7+a+b}{5} = 5$$

$a + b = 10$ ....(i)

$$\text{S.d.} = 2 \Rightarrow \sqrt{\frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{5}} = 2$$

$(3-5)^2 + (5-5)^2 + (7-5)^2 + (a-5)^2 + (b-5)^2 = 20$
$\Rightarrow 4 + 0 + 4 + (a - 5)^2 + (b - 5)^2 = 20$
$a^2 + b^2 - 10(a + b) + 50 = 12$
$(a + b)^2 - 2ab - 100 + 50 = 12$
$ab = 19$ ....(ii)
Equation is $x^2 - 10x + 19 = 0$

12. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$$ is equal to :

(1) 0 (2) $y(a - b)$
(3) $y(b - a)$ (4) $y(a - c)$

Sol. $a + x = b + y = c + z + 1$

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \quad C_3 \to C_3 - C_1$$

$$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \quad R_3 \to R_3 - R_1,\ R_2 \to R_2 - R_1$$

$$\begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$= (-y)[(y - x)(c - a) - (b - a)(z - x)]$
$= (-y)[(a - b)(c - a) + (a - b)(a - c - 1)]$
$= (-y)[(a - b)(c - a) + (a - b)(a - c) + b - a)$
$= -y(b - a) = y(a - b)$

13. If $\int \frac{\cos\theta}{5 + 7\sin\theta - 2\cos^2\theta} d\theta = A \log_e |B(\theta)| + C$, where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be :

(1) $\frac{2\sin\theta + 1}{5(\sin\theta + 3)}$ (2) $\frac{2\sin\theta + 1}{\sin\theta + 3}$

(3) $\frac{5(\sin\theta + 3)}{2\sin\theta + 1}$ (4) $\frac{5(2\sin\theta + 1)}{\sin\theta + 3}$

Sol. $$\int \frac{\cos\theta\ d\theta}{5 + 7\sin\theta - 2\cos^2\theta}$$

$$\int \frac{\cos\theta\ d\theta}{3 + 7\sin\theta + 2\sin^2\theta} \quad \boxed{\begin{matrix}\sin\theta = t \\ \cos\theta d\theta = dt\end{matrix}}$$

$$\int \frac{dt}{2t^2 + 7t + 3} = \int \frac{dt}{(2t+1)(t+3)}$$

$$= \frac{1}{5}\int\left(\frac{2}{2t+1} - \frac{1}{t+3}\right)dt$$

$$= \frac{1}{5} ln\left|\frac{2t+1}{t+3}\right| + C$$

$$= \frac{1}{5} ln\left|\frac{2\sin\theta+1}{\sin\theta+3}\right| + C$$

$$A = \frac{1}{5} \text{ and } B(\theta) = \frac{2\sin\theta + 1}{\sin\theta + 3}$$

14. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

(1) $5m = 4$ (2) $4c^2 = 369$

(3) $c^2 = 369$ (4) $8m + 5 = 0$

Sol. $y = mx + c$ is tangent to

$\frac{x^2}{100} - \frac{y^2}{64} = 1$ and $x^2 + y^2 = 36$

$c^2 = 100\,m^2 - 64 \mid c^2 = 36\,(1 + m^2)$

$\Rightarrow 100\,m^2 - 64 = 36 + 36m^2$

$m^2 = \frac{100}{64} \Rightarrow m = \pm\frac{10}{8}$

$c^2 = 36\left(1 + \frac{100}{64}\right) = \frac{36 \times 164}{64}$

$4c^2 = 369$

15. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is :

(1) 1500 (2) 2255

(3) 3000 (4) 2250

Sol.

A	B	C
$\boxed{5}$	$\boxed{5}$	$\boxed{5}$
1	2	2
2	1	2
2	2	1
1	1	3
1	3	1
3	1	1

Total number of selection

$= ({}^5C_1\ {}^5C_2\ {}^5C_2)\cdot 3 + ({}^5C_1\ {}^5C_1\ {}^5C_3)\cdot 3$

$= 5 \cdot 10 \cdot 10 \cdot 3 + 5 \cdot 5 \cdot 10 \cdot 3$

$= 2250$

16. If for some $\alpha \in R$, the lines

$L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :

(1) $(-2, 10, 2)$ (2) $(10, 2, 2)$

(3) $(10, -2, -2)$ (4) $(2, -10, -2)$

Sol. $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point $A(-1, 2, 1)$ $B(-2, -1, -1)$

$\because L_1$ and L_2 are coplanar

$$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$$

$\alpha = -4$

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options $(2, -10, -2)$ lies on L_2

17. Let $y = y(x)$ be the solution of the differential equation $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to :

(1) $\sqrt{2} - 2$ (2) $\frac{1}{\sqrt{2}} - 1$

(3) $2 - \sqrt{2}$ (4) $2 + \sqrt{2}$

Sol. $\cos x \dfrac{dy}{dx} + 2y \sin x = \sin 2x$

$\dfrac{dy}{dx} + \dfrac{2\sin x}{\cos x} y = 2 \sin x$

I.F. $= e^{\int 2\frac{\sin x}{\cos x}dx}$

$= e^{2 \ln \sec x} = \sec^2 x$

$y \cdot \sec^2 x = \int 2\sin x \cdot \sec^2 x dx$

$y \sec^2 x = 2 \int \tan x \sec x \, dx$

$y \sec^2 x = 2 \sec x + c$

At $x = \dfrac{\pi}{3}$, $y = 0$

$\Rightarrow 0 = 2 \sec \dfrac{\pi}{3} + C \Rightarrow C = -4$

$\boxed{y \sec^2 x = 2\sec x - 4}$

Put $x = \dfrac{\pi}{4}$

$y \cdot 2 = 2\sqrt{2} - 4$

$y = \sqrt{2} - 2$

18. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0) ?

(1) (2, 2) (2) (–2, 6)
(3) (–2, 4) (4) (2, 6)

Sol. $x^4e^y + 2\sqrt{y+1} = 3$

d.w.r. to x

$x^4 e^y y' + e^y 4x^3 + \dfrac{2y'}{2\sqrt{y+1}} = 0$

at P(1, 0)

$y'_P + 4 + y'_P = 0$

$\Rightarrow y'_P = -2$

Tangent at P(1, 0) is

$y - 0 = -2 (x - 1)$

$2x + y = 2$

(–2, 6) lies on it

19. The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is:

(1) a contradiction
(2) equivalent to $(p \wedge q) \vee (\sim q)$
(3) a tautology
(4) equivalent to $(p \vee q) \wedge (\sim p)$

Sol.

p	q	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to p \vee q$	$(p \to (q \to p)) \to (p \to (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

20. If $L = \sin^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$ and

$M = \cos^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$, then :

(1) $M = \dfrac{1}{2\sqrt{2}} + \dfrac{1}{2}\cos\dfrac{\pi}{8}$

(2) $L = \dfrac{1}{4\sqrt{2}} - \dfrac{1}{4}\cos\dfrac{\pi}{8}$

(3) $M = \dfrac{1}{4\sqrt{2}} + \dfrac{1}{4}\cos\dfrac{\pi}{8}$

(4) $L = -\dfrac{1}{2\sqrt{2}} + \dfrac{1}{2}\cos\dfrac{\pi}{8}$

Sol. $L = \sin^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$

$\left(\because \sin^2\theta = \dfrac{1-\cos 2\theta}{2}\right)$

$\Rightarrow L = \left(\dfrac{1-\cos(\pi/8)}{2}\right) - \left(\dfrac{1-\cos(\pi/4)}{2}\right)$

$L = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{8}\right)\right]$

$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$M = \frac{1+\cos(\pi/8)}{2} - \frac{1-\cos(\pi/4)}{2}$

$M = \frac{1}{2}\cos\left(\frac{\pi}{8}\right) + \frac{1}{2\sqrt{2}}$

21. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _______.

Sol. $P(H) = \frac{1}{2}$

$P(\bar{H}) = \frac{1}{2}$

Let total 'n' bomb are required to destroy the target

$1 - {}^nC_n\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$

$1 - \frac{1}{2^n} - \frac{n}{2^n} \geq \frac{99}{100}$

$\frac{1}{100} \geq \frac{n+1}{2^n}$

Now check for value of n

$\boxed{n = 11}$

22. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is _______.

Sol. $C = \{f : A \to B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$

Case-I : If $f(x) = 2\ \forall\ x \in A$ then number of function = 1

Case-II : If $f(x) = 2$ for exactly two elements then total number of many-one function = ${}^3C_2\ {}^3C_1 = 9$

Case-III : If $f(x) = 2$ for exactly one element then total number of many-one functions = ${}^3C_1\ {}^3C_1 = 9$

Total = 19

23. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ______.

Sol. $(1 + x + x^2 + x^3)^6 = \left((1+x)(1+x^2)\right)^6$

$= (1 + x)^6 (1 + x^2)^6$

$= \sum_{r=0}^{6} {}^6C_r x^r \sum_{t=0}^{6} {}^6C_t x^{2t}$

$= \sum_{r=0}^{6}\sum_{t=0}^{6} {}^6C_r\, {}^6C_t\, x^{r+2t}$

For coefficient of $x^4 \Rightarrow r + 2t = 4$

r	t
0	2
2	1
4	0

Coefficient of x^4

$= {}^6C_0\ {}^6C_2 + {}^6C_2\ {}^6C_1 + {}^6C_4\ {}^6C_0$

$= 120$

24. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to________.

Sol. $y = x^2 - 3x + 2$

At x-axis $y = 0 = x^2 - 3x + 2$

$x = 1, 2$

$\frac{dy}{dx} = 2x - 3$

A(1, 0) B(2, 0)

$\left(\frac{dy}{dx}\right)_{x=1} = -1$ and $\left(\frac{dy}{dx}\right)_{x=2} = 1$

$x + y = a \Rightarrow \frac{dy}{dx} = -1$ So A(1, 0) lies on it

$\Rightarrow 1 + 0 = a \Rightarrow \boxed{a = 1}$

$x - y = b \Rightarrow \frac{dy}{dx} = 1$ So B(2, 0) lies on it

$2 - 0 = b \Rightarrow \boxed{b = 2}$

$\frac{a}{b} = 0.50$

25. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is __________.

Sol. Projection of $\vec{b}$ on $\vec{a}$ = projection of $\vec{c}$ on $\vec{a}$

$\Rightarrow \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \Rightarrow \vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a}$

$\because$ $\vec{b}$ is perpendicular to $\vec{c}$ $\Rightarrow \vec{b} \cdot \vec{c} = 0$

Let $|\vec{a} + \vec{b} - \vec{c}| = k$

Square both sides

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a} \cdot \vec{b} - 2\vec{a} \cdot \vec{c} - 2\vec{b} \cdot \vec{c}$

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 = 36$

$k = 6 = |\vec{a} + \vec{b} - \vec{c}|$