JEE Main 2020 Paper

Date: **8th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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1. The maximum values of ${}^{19}C_p, {}^{20}C_q, {}^{21}C_r$ are a, b, c respectively. Then, the relation between a, b, c is

   a. $\frac{a}{22} = \frac{b}{42} = \frac{c}{11}$
   b. $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$
   c. $\frac{a}{22} = \frac{b}{11} = \frac{c}{42}$
   d. $\frac{a}{21} = \frac{b}{11} = \frac{c}{22}$

   Answer: (b)

   Solution:

   We know that, ${}^{n}C_r$ is maximum when $r = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n+1}{2} \text{ or } \frac{n-1}{2}, & n \text{ is odd} \end{cases}$

   Therefore, $\max\left({}^{19}C_p\right) = {}^{19}C_9 = a$

   $\max\left({}^{20}C_q\right) = {}^{20}C_{10} = b$

   $\max\left({}^{21}C_r\right) = {}^{21}C_{11} = c$

   $\therefore \frac{a}{{}^{19}C_9} = \frac{b}{\frac{20}{10} \times {}^{19}C_9} = \frac{c}{\frac{21}{11} \times \frac{20}{10} \times {}^{19}C_9}$

   $\frac{a}{1} = \frac{b}{2} = \frac{c}{\frac{42}{11}}$

   $\Rightarrow \frac{a}{11} = \frac{b}{22} = \frac{c}{42}.$

2. Let $P(A) = \frac{1}{3}, P(B) = \frac{1}{6}$ where A and B are independent events, then

   a. $P\left(\frac{A}{B}\right) = \frac{2}{3}$
   b. $P\left(\frac{A}{B'}\right) = \frac{5}{6}$
   c. $P\left(\frac{A}{B'}\right) = \frac{1}{3}$
   d. $P\left(\frac{A}{B}\right) = \frac{1}{6}$

   Answer: (c)

   Solution:

   If X and Y are independent events, then

   $$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X)P(Y)}{P(Y)} = P(X)$$

Therefore, $P\left(\frac{A}{B}\right) = P(A) = \frac{1}{3} \Longrightarrow P\left(\frac{A}{B'}\right) = P(A) = \frac{1}{3}$.

3. If $f(x) = \frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$, then inverse of $f(x)$ is

a. $\frac{1}{2}\log_8\left(\frac{1+x}{1-x}\right)$

b. $\frac{1}{2}\log_8\left(\frac{1-x}{1+x}\right)$

c. $\frac{1}{4}\log_8\left(\frac{1-x}{1+x}\right)$

d. $\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

Answer: (d)

Solution:

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}} = \frac{8^{4x} - 1}{8^{4x} + 1}$$

Put $y = \frac{8^{4x}-1}{8^{4x}+1}$

Applying componendo-dividendo on both sides

$$\frac{y+1}{y-1} = \frac{2 \times 8^{4x}}{-2}$$

$\frac{y+1}{y-1} = -8^{4x} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$

$\Rightarrow x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$

$f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$.

4. Roots of the equation $x^2 + bx + 45 = 0, b \in \boldsymbol{R}$ lies on the curve $|z+1| = 2\sqrt{10}$, where z is a complex number, then

a. $b^2 + b = 12$

b. $b^2 - b = 36$

c. $b^2 - b = 30$

d. $b^2 + b = 30$

Answer: (c)

Solution:

Given $x^2 + bx + 45 = 0, b \in \boldsymbol{R}$, let roots of the equation be $p \pm iq$

Then, sum of roots $= 2p = -b$

Product of roots $= p^2 + q^2 = 45$

As $p \pm iq$ lies on $|z+1| = 2\sqrt{10}$, we get

$(p+1)^2 + q^2 = 40$

$\Rightarrow p^2 + q^2 + 2p + 1 = 40$

$\Rightarrow 45 - b + 1 = 40$

$\Rightarrow b = 6$

$\Rightarrow b^2 - b = 30.$

5. Rolle's theorem is applicable on $f(x) = \ln\left(\frac{x^2+\alpha}{7x}\right)$ in $[3, 4]$. The value of $f''(c)$ is equal to

a. $\frac{1}{12}$

b. $\frac{-1}{12}$

c. $\frac{-1}{6}$

d. $\frac{1}{6}$

Answer: (a)

Solution:

Rolle's theorem is applicable on $f(x)$ in $[3, 4]$

$\Rightarrow f(3) = f(4)$

$$\Rightarrow \ln\left(\frac{9+\alpha}{21}\right) = \ln\left(\frac{16+\alpha}{28}\right)$$

$$\Rightarrow \frac{9+\alpha}{21} = \frac{16+\alpha}{28}$$

$\Rightarrow 36 + 4\alpha = 48 + 3\alpha$

$\Rightarrow \alpha = 12$

Now,

$$f(x) = \ln\left(\frac{x^2+12}{7x}\right) \Rightarrow f'(x) = \frac{7x}{x^2+12} \times \frac{7x \times 2x - (x^2+12) \times 7}{(7x)^2}$$

$$f'(x) = \frac{x^2 - 12}{x(x^2+12)}$$

$f'(c) = 0 \Rightarrow c = 2\sqrt{3}$

$$f''(x) = \frac{-x^4 + 48x^2 + 144}{x^2(x^2+12)^2}$$

$f''(c) = \frac{1}{12}.$

6. Let $f(x) = x\cos^{-1}(\sin(-|x|))$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then

a. $f'(0) = -\frac{\pi}{2}$

b. $f'(x)$ is not defined at $x = 0$

c. $f'(x)$ is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

d. $f'(x)$ is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$

Answer: (c)

Solution:

$f(x) = x\cos^{-1}(\sin(-|x|))$

$\Rightarrow f(x) = x\cos^{-1}(-\sin|x|)$

$\Rightarrow f(x) = x[\pi - \cos^{-1}(\sin|x|)]$

$\Rightarrow f(x) = x\left[\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right]$

$\Rightarrow f(x) = x\left(\frac{\pi}{2} + |x|\right)$

$$\Rightarrow f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right), & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right), & x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \left(\frac{\pi}{2} + 2x\right), & x \geq 0 \\ \left(\frac{\pi}{2} - 2x\right), & x < 0 \end{cases}$$

Therefore, $f'(x)$ is decreasing $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$.

7. Ellipse $2x^2 + y^2 = 1$ and $y = mx$ meet at a point P in the first quadrant. Normal to the ellipse at P meets x −axis at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and y −axis at $(0, \beta)$, then $|\beta|$ is

a. $\frac{2}{3}$

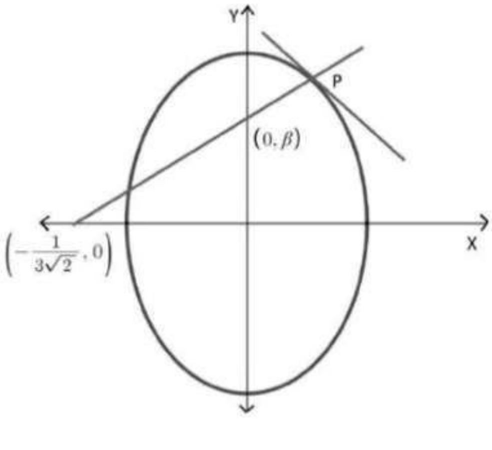
b. $\frac{2\sqrt{2}}{3}$

c. $\frac{\sqrt{2}}{3}$

d. $\frac{2}{\sqrt{3}}$

Answer: (c)

Solution:



Let $P \equiv (x_1, y_1)$

$2x^2 + y^2 = 1$ is given equation of ellipse.

$\Rightarrow 4x + 2yy' = 0$

$$\Rightarrow y'|_{(x_1,y_1)} = -\frac{2x_1}{y_1}$$

Therefore, slope of normal at $P(x_1, y_1)$ is $\frac{y_1}{2x_1}$

Equation of normal at $P(x_1, y_1)$ is

$$(y - y_1) = \frac{y_1}{2x_1}(x - x_1)$$

It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right)$

$$\Rightarrow -y_1 = \frac{y_1}{2x_1}\left(-\frac{1}{3\sqrt{2}} - x_1\right)$$

$$\Rightarrow x_1 = \frac{1}{3\sqrt{2}}$$

$\Rightarrow y_1 = \frac{2\sqrt{2}}{3}$ as P lies in first quadrant

Since $(0, \ \beta)$ lies on the normal of the ellipse at point P, hence we get

$$\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$$

8. If ABC is a triangle whose vertices are $A(1, \ -1)$, $B(0, 2)$, $C(x', \ y')$ and area of ΔABC is 5, and $C(x', \ y')$ lies on $3x + y - 4\lambda = 0$, then

a. $\lambda = 3$

b. $\lambda = 4$

c. $\lambda = -3$

d. $\lambda = 2$

Answer: (a)

Solution:

Area of triangle is

$$A = \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix} = \pm 5$$

$$-2(1 - x') + (y' + x') = \pm 10$$

$$-2 + 2x' + y' + x' = \pm 10$$

$$3x' + y' = 12 \text{ or } 3x' + y' = -8$$

$$\Rightarrow \lambda = 3 \text{ or } -2$$

9. Shortest distance between the lines $\frac{x-3}{1}=\frac{y-8}{4}=\frac{z-3}{22}$, $\frac{x+3}{1}=\frac{y+7}{1}=\frac{z-6}{7}$ is

a. $3\sqrt{30}$
b. $\sqrt{30}$
c. $2\sqrt{30}$
d. $4\sqrt{30}$

Answer: (a)

Solution:

$\overrightarrow{AB}=-3\hat{\imath}-7\hat{\jmath}+6\hat{k}-(3\hat{\imath}+8\hat{\jmath}+3\hat{k})=-6\hat{\imath}-15\hat{\jmath}+3\hat{k}$

$\vec{p}=\hat{\imath}+4\hat{\jmath}+22\hat{k}$

$\vec{q}=\hat{\imath}+\hat{\jmath}+7\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 4 & 22\\ 1 & 1 & 7\end{vmatrix}=6\hat{\imath}+15\hat{\jmath}-3\hat{k}$$

Shortest distance $=\frac{|\overrightarrow{AB}.(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}=\frac{|36+225+9|}{\sqrt{36+225+9}}=3\sqrt{30}.$

10. Let $\int \frac{\cos x}{\sin^3 x(1+\sin^6 x)^{\frac{2}{3}}}\,dx=f(x)(1+\sin^6 x)^{\frac{1}{\lambda}}+c$, then the value of $\lambda f\left(\frac{\pi}{3}\right)$ is

a. -4
b. -2
c. 8
d. 4

Answer: (b)

Solution:

Let $\sin x=t\Rightarrow \cos x\,dx=dt$

$$\therefore \int\frac{dt}{t^3(1+t^6)^{\frac{2}{3}}}=\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Let $1+\frac{1}{t^6}=u\Rightarrow -6t^{-7}dt=du$

$$\Rightarrow\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}=-\frac{1}{6}\int\frac{du}{u^{\frac{2}{3}}}=-\frac{3}{6}u^{\frac{1}{3}}+c=-\frac{1}{2}\left(1+\frac{1}{t^6}\right)^{\frac{1}{3}}+c$$

$$=-\frac{(1+\sin^6 x)^{\frac{1}{3}}}{2\sin^2 x}+c=f(x)(1+\sin^6 x)^{\frac{1}{\lambda}}$$

$\therefore \lambda=3$ and $f(x)=-\frac{1}{2\sin^2 x}$

$\Rightarrow \lambda f\left(\frac{\pi}{3}\right)=-2.$

11. If $y(x)$ is a solution of the differential equation $\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$, such that $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, then

a. $y\left(\frac{1}{\sqrt{2}}\right)=-\frac{1}{\sqrt{2}}$

b. $y\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{2}$

c. $y\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}$

d. $y\left(\frac{1}{\sqrt{2}}\right)=\frac{\sqrt{3}}{2}$

Answer: (c)

Solution:

$\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0$

$\Rightarrow \sin^{-1} y+\sin^{-1} x=c$

If $x=\frac{1}{2}, y=\frac{\sqrt{3}}{2}$ then,

$\sin^{-1}\frac{\sqrt{3}}{2}+\sin^{-1}\frac{1}{2}=c$

$\therefore \frac{\pi}{3}+\frac{\pi}{6}=c \Rightarrow c=\frac{\pi}{2}$

$\Rightarrow \sin^{-1} y=\frac{\pi}{2}-\ \sin^{-1} x=\cos^{-1} x$

$\therefore \sin^{-1} y=\cos^{-1}\frac{1}{\sqrt{2}}$

$\Longrightarrow \sin^{-1} y=\frac{\pi}{4}$

$\Longrightarrow y\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}$

12. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

a. e^{-2}

b. e^2

c. $e^{\frac{3}{7}}$

d. $e^{\frac{2}{7}}$

Answer: (a)

Solution:

Let $L=\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ [Intermediate form 1^{∞}]

$\therefore L = e^{\lim_{x\to 0} \frac{1}{x^2}\left(\frac{3x^2+2}{7x^2+2}-1\right)}$

$= e^{\lim_{x\to 0} \frac{1}{x^2}\left(-\frac{4x^2}{7x^2+2}\right)}$

$= e^{-2}$.

13. In a bag there are 5 red balls, 3 white balls and 4 black balls. Four balls are drawn from the bag. Find the number of ways in which at most 3 red balls are selected.

a. 360
b. 450
c. 490
d. 510

Answer: (c)

Solution:

Number of ways to select at most 3 red balls $= P(0 \text{ red balls}) + P(1 \text{ red ball}) + P(2 \text{ red balls}) + P(3 \text{ red balls})$

$= {}^7C_4 + {}^5C_1 \times {}^7C_3 + {}^5C_2 \times {}^7C_2 + {}^5C_3 \times {}^7C_1$

$= 35 + 175 + 210 + 70 = 490.$

14. Let $f(x) = [\sin(\tan^{-1} x) + \sin(\cot^{-1} x)]^2 - 1$ where $|x| > 1$ and $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1} f(x))$. If $y(\sqrt{3}) = \frac{\pi}{6}$ then $y(-\sqrt{3})$ is equal to

a. $\frac{5\pi}{6}$
b. $-\frac{\pi}{6}$
c. $\frac{2\pi}{3}$
d. $\frac{\pi}{3}$

Answer: (b)

Solution:

$f(x) = [\sin(\tan^{-1} x) + \sin(\cot^{-1} x)]^2 - 1$

$\Rightarrow f(x) = [\sin(\tan^{-1} x) + \cos(\tan^{-1} x)]^2 - 1$

$\Rightarrow f(x) = \sin(2\tan^{-1} x)$

Now, $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$

$\Rightarrow 2y = \sin^{-1}(f(x)) + c$

If $x = \sqrt{3}, y = \frac{\pi}{6}$

$\therefore \frac{\pi}{3} = \sin^{-1}(\sin(2\tan^{-1}\sqrt{3})) + c$

$\Rightarrow \frac{\pi}{3} = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + c$

$\Rightarrow \frac{\pi}{3} = \frac{\pi}{3} + c \Rightarrow c = 0$

$\Rightarrow 2y = \sin^{-1}\sin(2\tan^{-1}x)$

When $x = -\sqrt{3}$

$2y = \sin^{-1}\left(\sin\left(2\tan^{-1}(-\sqrt{3})\right)\right) = \sin^{-1}\left(\sin\left(-\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$

$\Rightarrow y = -\frac{\pi}{6}.$

15. The system of equation
$$\begin{aligned} 3x + 4y + 5z &= \mu \\ x + 2y + 3z &= 1 \\ 4x + 4y + 4z &= \delta \end{aligned}$$

is inconsistent, then $(\mu,\ \delta)$ can be

a. $(4,\ 6)$
b. $(1,\ 0)$
c. $(4,\ 3)$
d. $(3,\ 4)$

Answer: (c)

Solution:

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$$

$R_3 \to R_3 - 2R_1 + 2R_2$

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

For inconsistent system, one of $D_x,\ D_y,\ D_z$ should not be equal to 0

$$D_x = \begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix} \qquad D_y = \begin{vmatrix} 3 & \mu & 5 \\ 1 & 1 & 3 \\ 4 & \delta & 4 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix}$$

For inconsistent system, $2\mu \neq \delta + 2$

$\therefore$ The system will be inconsistent for $\mu = 4,\ \delta = 3$.

16. If volume of parallelepiped whose conterminous edges are $\vec{u} = \hat{\imath} + \hat{\jmath} + \lambda\,\hat{k}$, $\vec{v} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{w} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ is 1 cubic units. Then, the cosine of angle between $\vec{u}$ and $\vec{v}$ is

a. $\frac{5}{3\sqrt{10}}$
b. $\frac{5}{7}$
c. $\frac{7}{6\sqrt{3}}$
d. $\frac{7}{3\sqrt{3}}$

Answer: (c)

Solution:

Volume of parallelepiped $= [\vec{u}\ \ \vec{v}\ \ \vec{w}]$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \pm 1$$

$\Rightarrow \lambda = 2$ or 4

For $\lambda = 4$,

$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$

17. If $2^{1-x} + 2^{1+x}, f(x), 3^x + 3^{-x}$ are in $A.P.$ then the minimum value of $f(x)$ is

a. 4
b. 2
c. 3
d. 1

Answer: (c)

Solution:

$2^{1-x} + 2^{1+x}, f(x), 3^x + 3^{-x}$ are in A.P.

$$\therefore f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying A.M. $\geq$ G.M. inequality, we get

$$\frac{(3^x + 3^{-x})}{2} \geq \sqrt{3^x . 3^{-x}}$$

$$\Longrightarrow \frac{(3^x+3^{-x})}{2} \geq 1 \qquad \dots(1)$$

Also, Applying A.M. $\geq$ G.M. inequality, we get

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} . 2^{1-x}}$$

$$\Longrightarrow \frac{2^{1+x} + 2^{1-x}}{2} \geq 2 \quad \dots(2)$$

Adding (1) and (2), we get

$f(x) \geq 1 + 2 = 3$

Thus, minimum value of $f(x)$ is 3.

18. Which of the following is tautology?

a. $(p \wedge (p \rightarrow q)) \rightarrow q$

b. $q \rightarrow p \wedge (p \rightarrow q)$

c. $(p \wedge (p \vee q))$

d. $(p \vee (p \wedge q)$

Answer: (a)

Solution:

$(p \wedge (p \rightarrow q)) \rightarrow q$

$= (p \wedge (\sim p \vee q)) \rightarrow q$

$= [(p \wedge \sim p) \vee (p \wedge q)] \rightarrow q$

$= (p \wedge q) \rightarrow q$

$= \sim (p \wedge q) \vee q$

$= \sim p \vee \sim q \vee q$

$= T$

19. A is a 3×3 matrix whose elements are from the set $\{-1, 0, 1\}$. Find the number of matrices A such that $tr(AA^T) = 3$ where $tr(A)$ is sum of diagonal elements of matrix A

a. 612

b. 572

c. 672

d. 682

Answer: (c)

Solution:

$tr(AA^T) = 3$

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

$tr(AA^T) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2 = 3$

So out of 9 elements (a_{ij})'s, 3 elements must be equal to 1 or -1 and rest elements must be 0.

So, the total possible cases will be

When there is $6(0's)$ and $3(1's)$ then the total possibilities is ${}^9C_6$

For $6(0's)$ and $3(-1's)$ total possibilities is ${}^9C_6$

For $6(0's), 2(1's)$ and $1(-1's)$ total possibilities is ${}^9C_6 \times 3$

For $6(0's), 1(1's)$ and $2(-1's)$ total possibilities is ${}^9C_6 \times 3$

$\therefore$ Total number of cases $= {}^9C_6 \times 8 = 672$.

20. Mean and standard deviation of 10 observations are 20 and 2 respectively. If p $(p \neq 0)$ is multiplied to each observation and then q $(q \neq 0)$ is subtracted from each of them, then new mean and standard deviation becomes half of it's original value. Then find q

a. 10 b. -20

c. -5 d. -10

Answer: (b)

Solution:

If mean $\bar{x}$ is multiplied by p and then q is subtracted from it,

then new mean $\bar{x}' = p\bar{x} - q$

$\therefore \bar{x}' = \frac{1}{2}\bar{x}$ and $\bar{x} = 10$

$\Rightarrow 10 = 20p - q \quad ...(1)$

If standard deviation is multiplied by p, new standard deviation (σ') is p times of the initial standard deviation (σ).

$\sigma' = |p|\sigma$

$$\Rightarrow \frac{1}{2}\sigma = |p|\sigma \Rightarrow |p| = \frac{1}{2}$$

If $p = \frac{1}{2}, q = 0$

If $p = -\frac{1}{2}, q = -20$.

21. Let P be a point on $x^2 = 4y$. The segment joining $A(0, -1)$ and P is divided by a point Q in the ratio $1:2$, then locus of point Q is

a. $9x^2 = 3y + 2$ b. $9x^2 = 12y + 8$

c. $9y^2 = 3x + 2$ d. $9y^2 = 12x + 8$

Answer: (b)

Solution:

Let point P be $(2t, t^2)$ and Q be (h, k).

$h = \frac{2t}{3}, k = \frac{-2 + t^2}{3}$

Now, eliminating t from the above equations we get:

$3k + 2 = \left(\frac{3h}{2}\right)^2$

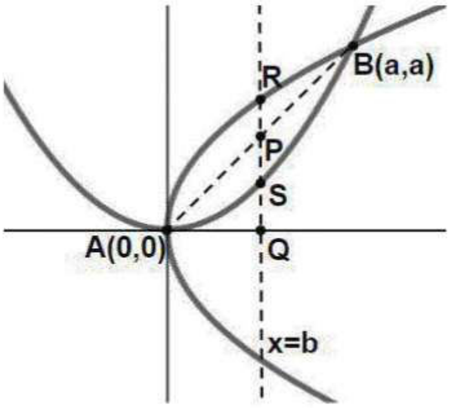
Replacing h and k by x and y, we get the locus of the curve as $9x^2 = 12y + 8$.

22. If the curves $y^2 = ax$ and $x^2 = ay$ intersect each other at A and B such that the area bounded by the curves is bisected by the line $x = b$ (given $a > b > 0$) and the area of triangle formed by the lines $AB, x = b$ and the x-axis is $\frac{1}{2}$. Then

a. $a^6 + 12a^3 + 4 = 0$
b. $a^6 + 12a^3 - 4 = 0$
c. $a^6 - 12a^3 + 4 = 0$
d. $a^6 - 12a^3 - 4 = 0$

Answer: (c)

Solution:



Given, $ar(\Delta APQ) = \frac{1}{2}$

$\Rightarrow \frac{1}{2} \times b \times b = \frac{1}{2}$

$\Rightarrow b = 1$

As per the question

$\Rightarrow \int_0^1 \left(\sqrt{ax} - \frac{x^2}{a}\right) dx = \frac{1}{2}\int_0^a \left(\sqrt{ax} - \frac{x^2}{a}\right) dx$

$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$

$\Rightarrow 2a\sqrt{a} - 1 = \frac{a^3}{2}$

$\Rightarrow 4a\sqrt{a} = 2 + a^3$

$\Rightarrow 16a^3 = 4 + a^6 + 4a^3$

$\Rightarrow a^6 - 12a^3 + 4 = 0.$

23. The sum $\sum_{k=1}^{20}(1 + 2 + 3 + \ldots + k)$ is __________.

Answer: (1540)

Solution:

$$= \sum_{k=1}^{20} \frac{k(k+1)}{2}$$

$$= \frac{1}{2}\sum_{k=1}^{20} k^2 + k$$

$$= \frac{1}{2}\left[\frac{20(21)(41)}{6} + \frac{20(21)}{2}\right]$$

$$= \frac{1}{2}[2870 + 210]$$

$= 1540.$

24. If $2x^2 + (a - 10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+$ has real roots, then minimum value of $'a'$ is equal to __________.

Answer: (8)

Solution:

$\because 2x^2 + (a - 10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+$ has real roots

$\Rightarrow D \geq 0$

$\Rightarrow (a - 10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \geq 0$

$\Rightarrow (a - 10)^2 - 4(33 - 4a) \geq 0$

$\Rightarrow a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, 4] \cup [8, \infty)$

Thus, minimum value of $'a'$ $\forall\, a \in \mathbf{Z}^+$ is 8.

25. If normal at P on the curve $y^2 - 3x^2 + y + 10 = 0$ passes through the point $\left(0, \frac{3}{2}\right)$ and the slope of tangent at P is n. The value of $|n|$ is equal to__________.

Answer: (4)

Solution:

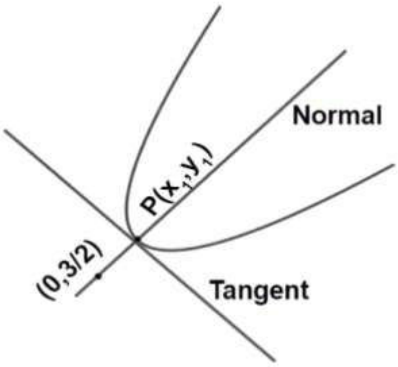
Let co-ordinate of P be (x_1, y_1)

Differentiating the curve w.r.t x

$2yy' - 6x + y' = 0$

Slope of tangent at P

$\Rightarrow \; y' = \dfrac{6x_1}{1 + 2y_1}$



$\because \; m_{\text{normal}} = \left(\dfrac{y_1 - \frac{3}{2}}{x_1 - 0}\right)$

$\because \; m_{\text{normal}} \times m_{\text{tangent}} = -1$

$\Rightarrow \dfrac{\frac{3}{2} - y_1}{-x_1} \times \dfrac{6x_1}{1+2y_1} = -1$

$\Rightarrow y_1 = 1$

$\Rightarrow x_1 = \pm 2$

Slope of tangent $= \pm \frac{12}{3} = \pm 4$

$\Rightarrow |n| = 4$

JEE Main 2020 Paper

Date: 8th January 2020

Time: 09:30 AM – 12:30 PM

Subject: Chemistry

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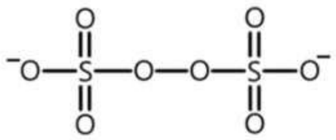
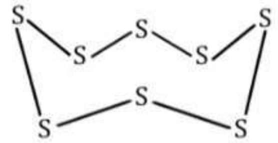
1. The number of S–O bonds in $S_2O_8^{2-}$ and number of S–S bonds in Rhombic sulphur, respectively, are:
   a. 8, 8
   b. 6, 8
   c. 2, 4
   d. 4, 2

Answer: a

Solution: Here, we have to count $S - O$ single bonds as well as $S = O$ in $S_2O_8^{2-}$, as each double bond also has one sigma bond. The structure of $S_2O_8^{2-}$ and S_8 is shown below:





2. Which of the following van der Waals forces are present in ethyl acetate liquid?
   a. H- bond, London forces
   b. Dipole-dipole interaction, H-bond
   c. Dipole-dipole interaction, London forces
   d. H-bond, dipole-dipole interaction, London forces

Answer: c

Solution: London dispersion forces (also called as induced dipole - induced dipole interactions), exist because of the generation of temporary polarity due to collision of particles and for this very reason, they are present in all molecules and inert gases as well.

Because of the presence of a permanent dipole, there will be dipole-dipole interactions present here.

There is no H that is directly attached to an oxygen atom, so H-bonding cannot be present.

3. Given, for H-atom

$$\bar{\nu} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Select the correct options regarding this formula for Balmer series:

A) $n_1 = 2$.
B) Ionization energy of H atom can be calculated from above formula.
C) $\lambda_{maximum}$ is for $n_2 = 3$.
D) If λ decreases then spectrum lines will converge.

a. A, B
b. C, D
c. A, C, D
d. A, B, C, D

Answer: c

Solution:

(A) is correct since the series studied in H-spectrum, including Balmer series, are de-excitation series or emission series. So, electrons get de-excited to n= 2 which means that $n_{lower} = 2$.

(B) It is possible to obtain I.E. from the formula above, but since the question has stated the formula for the Balmer series, n_{lower} has been fixed as 2. So, it is not possible to calculate I.E. from it. To calculate I.E., we'll have to put $n_{lower} = 1$, which isn't possible here.

(C) $\Delta E = hc/\lambda$

With n_{lower} fixed as 2, ΔE increases as n_{higher} is increased. So, the last line of the Balmer series, i.e. from infinity to n= 2, will have the maximum energy in the series and thus, the lowest wavelength. Similarly, the first line in the series, i.e. from n= 3 to n= 2 will have the lowest energy in the series and thus, the highest wavelength. Which makes this statement correct.

(D)As orbits with higher orbit number or those that are further away from the nucleus are considered, the energy gap in-between subsequent orbits decreases. Now, consider the following for example and with n_{lower} fixed as 2.

Energy of a photon released on transition from n= 100 to n= 2 will have similar energy to that of the photon that gets released on transition from n= 101 to n= 2, because energy of the 100th and the 101th orbit will be very close in value. That means they will also have very close values of wavelengths, which further implies that these two lines will be situated quite close to each other on the photographic plate.

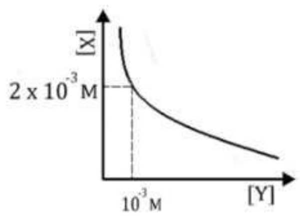
In a similar fashion, we can see that as the n_{higher} increases, the lines start to converge together. And since, increasing the n_{higher} will indeed lead to an increase in the energy of the photon released, it will end up releasing photons of shorter wavelengths. Combining these two statements we can easily see that as the wavelength decreases, the spectral lines start to converge.

4. The correct order of the first ionization energies of the following metals; Na, Mg, Al, Si in $kJmol^{-1}$, respectively is:
   a. 497, 737, 577, 786
   b. 497, 577, 737, 786
   c. 786, 739, 577, 497
   d. 739, 577, 786, 487

Answer: a

Solution:
The expected order is Na < Mg < Al < Si.
But the actual/experimental order turns out to be Na < Al < Mg < Si, because of the fully filled s-subshell of magnesium and the s^2p^1 configuration of Al which makes it relatively easy for Al to lose its outermost electron.

5. Select the correct stoichiometry and its K_{sp} value according to the given graph:



   a. $XY, K_{sp} = 2 \times 10^{-6}$
   b. $XY_2, K_{sp} = 4 \times 10^{-9}$
   c. $X_2Y, K_{sp} = 9 \times 10^{-9}$
   d. $XY_2, K_{sp} = 1 \times 10^{-9}$

Answer: a

Solution:

(a) $XY_{(s)} \rightleftharpoons X^+ + Y^-$

$K_{sp} = [X^+][Y^-] = 2 \times 10^{-3} \times 10^{-3} = 2 \times 10^{-6}$

(b) $XY_{2(s)} \rightleftharpoons X^{2+} + 2Y^-$

$K_{sp} = [X^{2+}][Y^-]^2 = 2 \times 10^{-3} \times 10^{-6} = 2 \times 10^{-9}$

(c) $X_2Y_{(s)} \rightleftharpoons 2X^+ + Y^{2-}$

$K_{sp} = [X^+]^2[Y^{2-}] = 4 \times 10^{-6} \times 10^{-3} = 4 \times 10^{-9}$

(d) $XY_{2(s)} \rightleftharpoons X^{2+} + 2Y^-$

$K_{sp} = [X^{2+}][Y^-]^2 = 2 \times 10^{-3} \times 10^{-6} = 2 \times 10^{-9}$

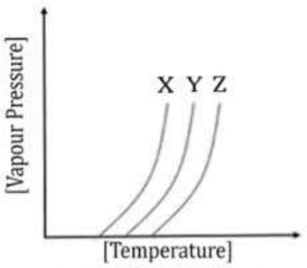
6. Which of the following complex exhibit facial-meridional geometrical isomerism?

a. $[Pt(NH_3)Cl_3]^-$
b. $[PtCl_2(NH_3)_2]$
c. $[Ni(CO)_4]$
d. $[Co(NO_2)_3(NH_3)_3]$

Answer: d

Solution: Facial and meridional geometrical isomerism is observed only in $[MA_3B_3]$ type complexes which is given in option d.

7.



A) Intermolecular force of attraction of X > Y
B) Intermolecular force of attraction of X < Y.
C) Intermolecular force of attraction of Z < X.

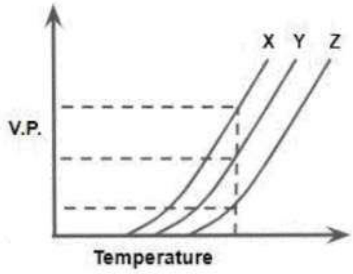
Select the correct option(s):

a. A and C
b. A and B
c. B only
d. B and C

Answer: c

Solution:

As shown in the plot below, for the same T, the vapour pressure of X is the highest and of Z is the lowest. Now, that means with the same average K.E. of X, Y and Z molecules, the X molecules are able to compensate their respective intermolecular forces better. So, X molecules have the highest vapour pressure. Which implies that the intermolecular forces in X are the weakest among the three. The opposite could be said for Z as well.



8. Rate of a reaction increases by 10^6 times when a reaction is carried out in presence of enzyme catalyst at the same temperature. Determine the change in activation energy.
   a. -6×2.303 RT
   b. $+6 \times 2.303$ RT
   c. $+6$RT
   d. -6RT

Answer: a

Solution:

$K_1 = Ae^{-E_{a1}/RT}$----(1)

$K_2 = Ae^{-E_{a2}/RT}$----(2)

Dividing equation 1 with equation 2, we get

$\frac{K_1}{K_2} = e^{(E_{a2}-E_{a1})/RT}$

$10^{-6} = e^{(E_{a2}-E_{a1})/RT}$

Taking $\log_e$ on both sides, we get

$\Delta E = E_{a2} - E_{a1} = -6 \times 2.303$ RT

9. Gypsum on heating at 393K produces:
   a. Dead burnt plaster
   b. Anhydrous $CaSO_4$
   c. $CaSO_4.\frac{1}{2}H_2O$
   d. $CaSO_4 \cdot 5H_2O$

Answer: c

Solution: $CaSO_4.2H_2O \xrightarrow{393K} CaSO_4.\frac{1}{2}H_2O$

10. Among the following, the least 3^{rd} ionization energy is for:
   a. Mn
   b. Co
   c. Fe
   d. Ni

Answer: c

Solution:
Consider an element E
$E^{2+} \rightarrow E^{3+}$ would be the 3rd I.E. of the element E.

Electronic configuration of Mn is $[Ar]4s^2 3d^5$

Electronic configuration of Co is $[Ar]4s^2 3d^7$

Electronic configuration of Fe is $[Ar]4s^2 3d^6$

Electronic configuration of Ni is $[Ar]4s^2 3d^8$

Electronic configuration of Mn^{2+} is $[Ar]3d^5$

Electronic configuration of Co^{2+} is $[Ar]3d^7$

Electronic configuration of Fe^{2+} is $[Ar]3d^6$

Electronic configuration of Ni^{2+} is $[Ar]3d^8$

As it is evident from the above configurations of the E^{2+} for the given elements, Fe^{2+} would require the least amount of energy for removal of electron as it has the configuration $3d^6\ 4s^0$. That means that its E^{3+} form is the most stable among the four elements provided in their respective E^{3+} states, i.e., when compared, the next electron removal will require least amount of energy.

11. Accurate measurement of concentration of NaOH can be performed by which of the following titration?
   a. NaOH in burette and oxalic acid in conical flask
   b. NaOH in burette and concentrated H_2SO_4 in conical flask
   c. NaOH in volumetric flask and concentrated H_2SO_4 in conical flask
   d. Oxalic acid in burette and NaOH in conical flask

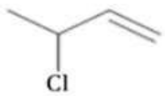
Answer: d

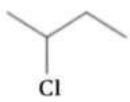
Solution: The standard solution is always kept in burette. The oxalic acid is a primary standard solution while H_2SO_4 is a secondary standard solution.

12. Arrange the following compounds in order of dehydrohalogenation (E_1) reaction:

A. 

B. 

C. 

D. 

a. C > B > D > A
b. C > D > B > A
c. B > C > D > A
d. A > B > C > D

Answer: b

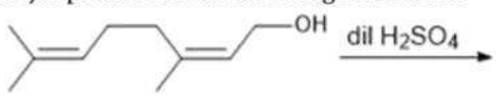
Solution: In E_1 mechanism, the rate determining step is formation of carbocation. So, stability of carbocation formed decides the rate.

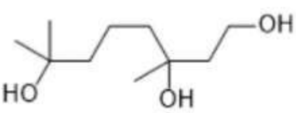
In option c, the cation formed is resonance stabilised.

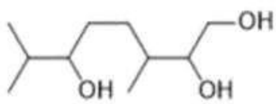
In option d, the cation formed is a 2^0 carbocation.

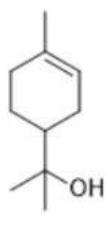
In option a and b, the carbocations formed are 1^0 but there is a chance of rearrangement in option b and after the rearrangement, the carbocation formed in option b will be allylic. So, the order of reaction is as follows:
C > D > B > A.

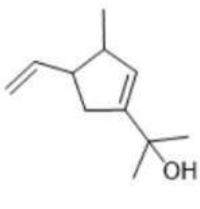
13. Major product in the following reaction is:



a. 

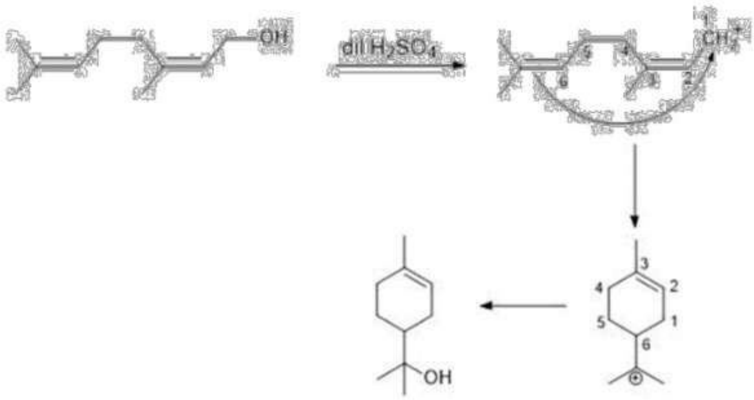
b. 

c. 

d. 

Answer: c

Solution:



14. Arrange the order of C—OH bond length in the following compounds:

Methanol	Phenol	p-Ethoxyphenol
(A)	(B)	(C)

a. A > B > C
b. A > C > B
c. C > B > A
d. B > C > A

Answer: b

Solution: In methanol, there is no resonance. In phenol, there is resonance. In p-Ethoxyphenol, there is resonance involved but the involvement of lone pair of oxygen in OH group is poor as compared with phenol due to the presence of lone pair oxygen in OCH_3 group which are also involved in resonance.

So, partial double bond character develops in C—OH bond of phenol and p-Ethoxyphenol but in case of p-Ethoxyphenol, resonance is poor as compared to phenol. So, bond length follows the order: A > C > B

15. Which of the following are "greenhouse gases"?

i. CO_2
ii. O_2
iii. O_3
iv. CFC
v. H_2O vapours

a. i, ii and iv
b. i, ii, iii and iv
c. i, iii and iv
d. i, iii, iv and v

Answer: d

Solution: CO_2, O_3, H_2O vapours and CFC's are green house gases.

16. Two liquids, isohexane and 3-methylpentane have boiling points 60 °C and 63 °C, respectively. They can be separated by:
    a. Simple distillation and isohexane comes out first
    b. Fractional distillation and isohexane comes out first.
    c. Simple distillation and 3-Methylpentane comes out first.
    d. Fractional distillation and 3-Methylpentane comes out first

Answer: b

Solution: When the difference between the B.P. of the two liquids is less than around 40 °C, fractional distillation is more efficient. The difference between the boiling points of isohexane and 3-methylpentane is only 3 degrees. So, fractional distillation is the best suitable method. Since, isohexane has a lower boiling point, it comes out first.

17. Which of the given statement is incorrect about glucose?
    a. Glucose exists in two crystalline forms α and β.
    b. Glucose gives Schiff's test.
    c. Penta acetate of glucose does not form oxime.
    d. Glucose forms oxime with hydroxylamine.

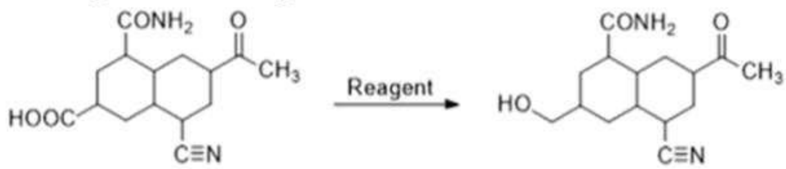
Answer: b

Solution: Glucose exists in two crystalline forms α and β which are anomers of each other.

Glucose does not react with Schiff's reagent because after the internal cyclisation, it forms either α-anomer or β-anomer. In these forms, free aldehydic group is not present.

Glucose forms open chain structure in aqueous solution which contains aldehyde at chain end. This aldehydic group reacts with NH_4OH to form oxime. On the other hand, glucose penta acetate being a cyclic structure even in aqueous form does not have terminal carbonyl group. Therefore it will not react with NH_4OH.

18. The reagent used for the given conversion is:



    a. H_2, Pd
    b. B_2H_6
    c. $NaBH_4$
    d. $LiAlH_4$

Answer: b

Solution: B_2H_6 does not reduce amide, carbonyl group and cyanide. It selectively reduces carboxylic acid to alcohol. So, for this conversion, it is the best suitable reagent.

19. 0.3 g $[ML_6]Cl_3$ of molar mass 267.46 g/mol is reacted with 0.125 M $AgNO_{3(aq)}$ solution, calculate volume of $AgNO_3$ required in mL.

Answer: 26.92

Solution: To react completely with one mole of $[ML_6]Cl_3$, 3 moles of $AgNO_3$ is required.

0.3 g $[ML_6]Cl_3$ means $\frac{0.3}{267.46}$moles of $[ML_6]Cl_3$.

So, moles of $AgNO_3$ required will be $\frac{0.3\times3}{267.46}$moles

To find the volume,

$$\frac{0.3\times3}{267.46} = 0.125\times V(L)$$
$$V(L) = 0.02692$$
$$V(mL) = 26.92$$

20. Given : $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$; $E^\circ = -1.23$ V

Calculate the electrode potential at pH= 5.

Answer: -0.93

Solution:

$2H_2O \rightarrow O_2 + 4H^+ + 4e^-$; $E^\circ = -1.23$ V

$E = -1.23 - \frac{0.0591}{4}\log[H^+]^4$

$= -1.23 + (0.0591 \times pH) = -1.23 + 0.0591 \times 5$

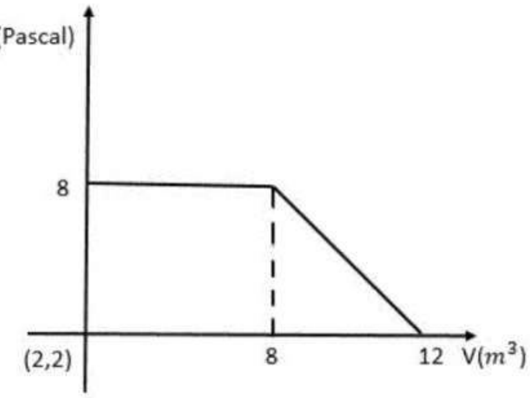
$= -1.23 + 0.2955 = -0.9345\ V = -0.93\ V$

21. Calculate the mass of $FeSO_4.7H_2O$, which must be added in 100 kg of wheat to get 10 ppm of Fe.

Answer: 4.96

Solution: 10 ppm of Fe means 10 g of Fe in 10^6 g of wheat. So, for 100 kg i.e., 10^5 g of wheat. Fe needed is 1 g. So, for 1 g of Fe, the mass of $FeSO_4.7H_2O$ required is $\frac{278}{56} = 4.96$ g.

22. A gas undergoes expansion according to the following graph. Calculate work done by the gas (in Joules).



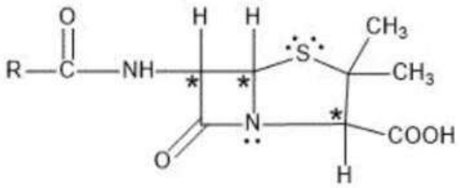
Answer: 48

Solution:
Work done by the gas
= The area under the curve
= (Area of the square) + (Area of the triangle)
= 48 J

23. The number of chiral centres in Penicillin is ____.

Answer: 3

Solution: The structure of penicillin is shown below:



So, the number of chiral centres= 3

JEE Main 2020 Paper

Date of Exam: **8th January (Shift I)**
Time: **9:30 am – 12:30 pm**
Subject: **Physics**

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1. A block of mass m is connected at one end of natural length l_o and spring constant k. The spring is fixed at its other end. The block is rotated with constant angular speed (ω) in gravity free space. The elongation in spring is

a. $\frac{l_o m\omega^2}{k-m\omega^2}$
b. $\frac{l_o m\omega^2}{k+m\omega^2}$
c. $\frac{l_o m\omega^2}{k-m\omega}$
d. $\frac{l_o m\omega^2}{k+m\omega}$

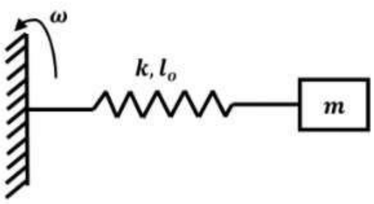
Solution: (a)

The centripetal force is provided by the spring force.

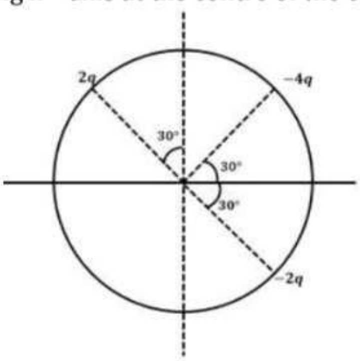
$m\omega^2(l_o + x) = kx$

$\left(\frac{l_o}{x} + 1\right) = \frac{k}{m\omega^2}$

$x = \frac{l_o m\omega^2}{k-m\omega^2}$



2. Three charges are placed on the circumference of a circle of radius d as shown in the figure. The electric field along x −axis at the centre of the circle is



a. $\frac{q}{4\pi\varepsilon_0 d^2}$

b. $\frac{q\sqrt{3}}{4\pi\varepsilon_0 d^2}$

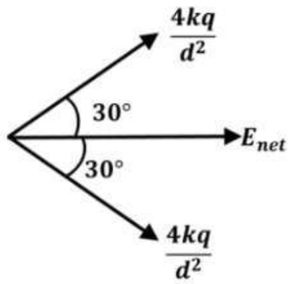
c. $\frac{q\sqrt{3}}{\pi\varepsilon_0 d^2}$

d. $\frac{q\sqrt{3}}{2\pi\varepsilon_0 d^2}$

Solution: (c)

Applying superposition principle,

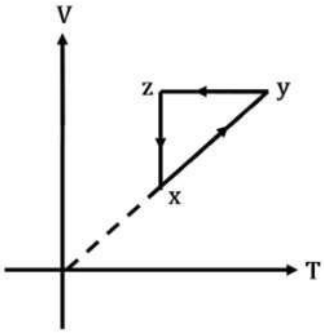
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$



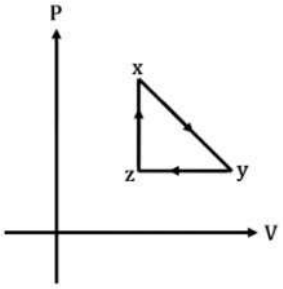
By symmetry, net electric field along the x-axis.

$$\left|\vec{E}_{net}\right| = \frac{4kq}{d^2} \times 2\cos 30° = \frac{q\sqrt{3}}{\pi\varepsilon_0 d^2}$$

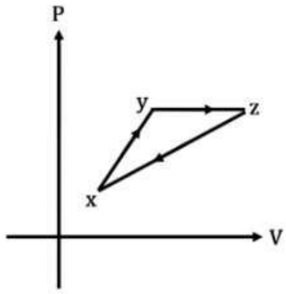
3. Choose the correct $P - V$ graph of an ideal gas for the given $V - T$ graph.



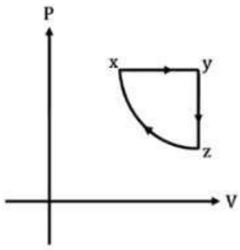
a.



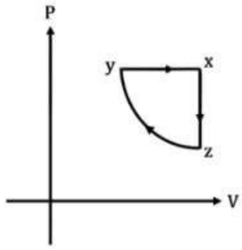
b.



c.



d.



Solution: (a)

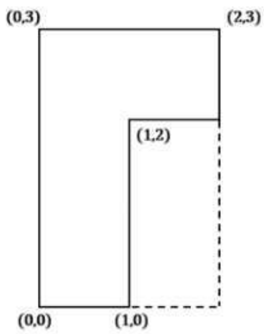
For the given $V - T$ graph

For the process $x \rightarrow y$; $V \propto T$; P =constant

For the process $y \rightarrow z$; V =constant

Only 'a' satisfies these two conditions.

4. Find the co-ordinates of center of mass of the lamina shown in the figure below.



a. (0.75, 1.75)

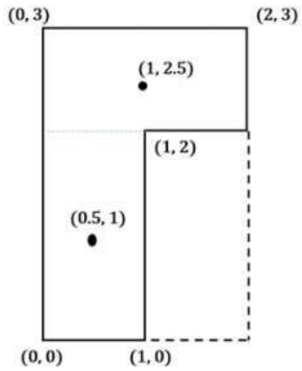
b. (0.75, 1.5)

c. (0.5, 1.75)

d. (0.5, 1.5)

Solution: (a)

The Lamina can be divided into two parts having equal mass m each.

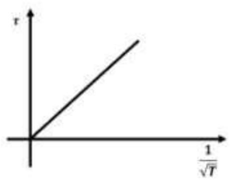


$$\vec{r}_{cm} = \frac{m \times \left(\frac{\hat{i}}{2} + \hat{j}\right) + m \times \left(\hat{i} + \frac{5\hat{j}}{2}\right)}{2m}$$

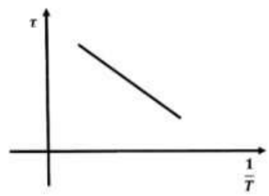
$$\vec{r}_{cm} = \frac{3}{4}\hat{i} + \frac{7}{4}\hat{j}$$

5. Which graph correctly represents the variation between relaxation time (τ) of gas molecules with absolute temperature (T) of the gas?

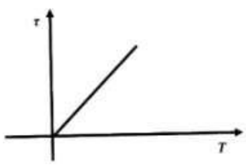
a.



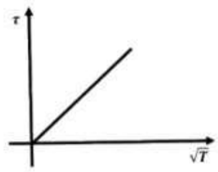
b.



c.



d.

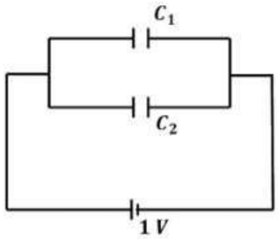


Solution: (a)

$\tau \propto \frac{1}{\sqrt{T}}$

6. If two capacitors C_1 and C_2 are connected in a parallel combination then the equivalent capacitance is 10 μF. If both the capacitors are connected across a 1 V battery, then energy stored in C_2 is 4 times of that in C_1. The equivalent capacitance if they are connected in series is
   a. 1.6 μF
   b. 16 μF
   c. 4 μF
   d. $\frac{1}{4}$ μF

Solution: (a)



Given that,

$C_1 + C_2 = 10\ \mu F$ .....(i)

$4\left(\frac{1}{2}C_1V^2\right) = \frac{1}{2}C_2V^2$

$\Rightarrow 4C_1 = C_2$ ...(ii)

From equations (i) and (ii)

$C_1 = 2\ \mu F$

$C_2 = 8\ \mu F$

If they are in series

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.6\ \mu F$$

7. A rod of mass $4m$ and length L is hinged at its midpoint. A ball of mass m is moving in the plane of the rod with a speed v. It strikes the end at an angle of 45^0 and sticks to it. The angular velocity of system after collision is

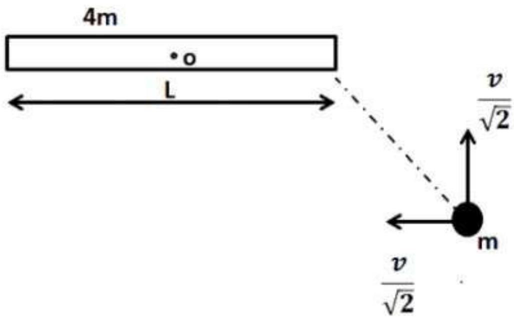
a. $\frac{3\sqrt{2}v}{7L}$

b. $\frac{\sqrt{2}v}{7L}$

c. $\frac{\sqrt{2}v}{3L}$

d. $\frac{3v}{7L}$

Solution: (a)



There is no external torque on the system about the hinge point.

So,

$$\overrightarrow{L_1} = \overrightarrow{L_f}$$

$$\frac{mv}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4}\right] \times \omega$$

$$\omega = \frac{6\,V}{7\sqrt{2}L} = \frac{3\sqrt{2}V}{7L}$$

8. Two photons of energy 4 eV and 4.5 eV are incident on two metals A and B respectively. The maximum kinetic energy for an ejected electron is T_A for A, and $T_B = T_A - 1.5\ eV$ for the metal B. The relation between the de-Broglie wavelengths of the ejected electron of A and B are $\lambda_B = 2\lambda_B$. The work function of the metal B is

a. 3 eV

b. 1.5 eV

c. 4.5 eV

d. 4 eV

Solution: (d)

$$\lambda = \frac{h}{\sqrt{2(KE)m_B}} = \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{KE_B}}{\sqrt{KE_A}}$$

$$\frac{1}{2} = \sqrt{\frac{T_A - 1.5}{T_A}}$$

$T_A = 2\ eV$

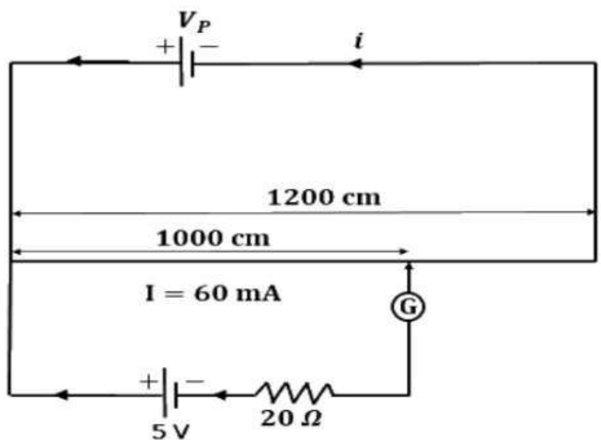
$KE_B = 2 - 1.5 = 0.5\ eV$

$\emptyset_B = 4.5 - 0.5 = 4\ eV$

9. There is a potentiometer wire of length 1200 cm and a 60 mA current is flowing in it. A battery of emf $5\ V$ and internal resistance of 20 Ω is balanced on this potentiometer wire with a balancing length1000 cm. The resistance of the potentiometer wire is

   a. 80 Ω
   b. 100 Ω
   c. 120 Ω
   d. 60 Ω

Solution: (b)



Assume the terminal voltage of the primary battery as V_p. As long as this potentiometer is operating on balanced length, V_p will remain constant.

As we know, potential gradient = $\frac{5}{1000} = \frac{V_p}{1200}$

$V_p = 6$ V

And $R_p = \frac{V_p}{i} = \frac{6}{60\times10^{-3}} = 100\ \Omega$

10. A telescope has a magnification equal to 5 and the length of its tube is 60 cm. The focal length of its eye piece is

a. 10 cm
b. 20 cm
c. 30 cm
d. 40 cm

Solution: (a)

$$m = \frac{f_0}{f_e} = 5$$

$$\Rightarrow f_0 = 5f_e$$

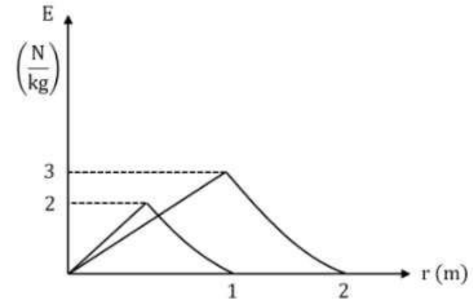
$$f_0 + f_e = 5f_e + f_e = 6f_e = \text{length of the tube}$$

$$\Rightarrow 6f_e = 60 \text{ cm}$$

$$\Rightarrow f_e = 10 \text{ cm}$$

11. Two spherical bodies of mass m_1 & m_2 have radii 1 m & 2 m respectively. The graph of the gravitational field of the two bodies with their radial distance is shown below. The value of $\frac{m_1}{m_2}$ is

a. $\frac{1}{6}$
b. $\frac{1}{3}$
c. $\frac{1}{2}$
d. $\frac{1}{4}$



Solution: (a)

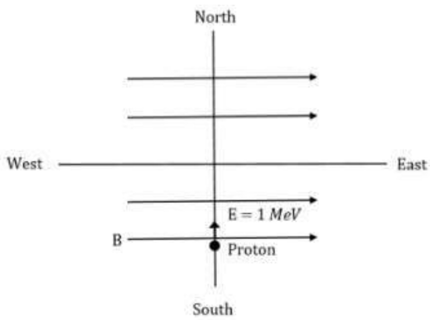
Gravitation field will be maximum at the surface of a sphere. Therefore,

$\frac{Gm_2}{2^2} = 3$ & $\frac{Gm_1}{1^2} = 2$

$\Rightarrow \frac{m_2}{m_1} \times \frac{1}{4} = \frac{3}{2}$

$\Rightarrow \frac{m_1}{m_2} = \frac{1}{6}$

12. When a proton of $KE = 1.0\,MeV$ moving towards North enters a magnetic field (directed along East), it accelerates with an acceleration, $a = 10^{12}\,m/s^2$. The magnitude of the magnetic field is



a. 0.71 mT
b. 7.1 mT
c. 71 mT
d. 710 mT

Solution: (a)

$K.E = 1 \times 10^6\ eV = 1.6 \times 10^{-13} J$

$= \frac{1}{2} m_e v^2$

Where m_e is the mass of the electron = $1.6 \times 10^{-27} kg$

$\Rightarrow 1.6 \times 10^{-13} = \frac{1}{2} \times\ 1.6 \times 10^{-27} \times v^2$

$\therefore v = \sqrt{2} \times 10^7 m/s$

$Bqv = m_e a$

$\therefore B = \frac{1.6 \times 10^{-27} \times 10^{12}}{1.6 \times 10^{-19} \times \sqrt{2} \times 10^7}$

$= 0.71 \times 10^{-3} T = 0.71\ mT$

13. If the electric field around a surface is given by $\left|\vec{E}\right| = \frac{Q_{in}}{\varepsilon_0|A|}$ where A is the normal area of surface and Q_{in} is the charge enclosed by the surface. This relation of Gauss' law is valid when

a. the surface is equipotential.
b. the magnitude of the electric field is constant.
c. the magnitude of the electric field is constant and the surface is equipotential.
d. for all the Gaussian surfaces.

Solution: (c)

The magnitude of the electric field is constant and the electric field must be along the area vector i.e. the surface is equipotential.

14. The stopping potential depends on the Planks constant(h), the current (I), the universal gravitational constant (G) and the speed of light (C). Choose the correct option for the dimension of the stopping potential (V)

a. $h^1 I^{-1} G^1 C^5$
b. $h^{-1} I^1 G^6$
c. $h^0 I^1 G^1 C^6$
d. $h^0 I^{-1} G^{-1} C^5$

Solution: (d)

$V = K(h)^a (I)^b (G)^c (C)^d$ Unit of stopping potential is (V) Volt.

We know $[h] = ML^2T^{-1}$

$[I] = A$

$[G] = M^{-1}L^3T^{-2}$

$[C] = LT^{-1}$

$[V] = ML^2T^{-3}A^{-1}$

$ML^2T^{-3}A^{-1} = (ML^2T^{-1})^a (A)^b \, (M^{-1}L^3T^{-2})^c (LT^{-1})^d$

$ML^2T^{-3}A^{-1} = M^{a-c}L^{2a+3c+d}T^{-a-2c-d}A^b$

$a - c = 1$

$2a + 3c + d = 2$

$-a - 2c - d = -3$

$b = -1$

On solving,

$c = -1$

$a = 0$

$d = 5$

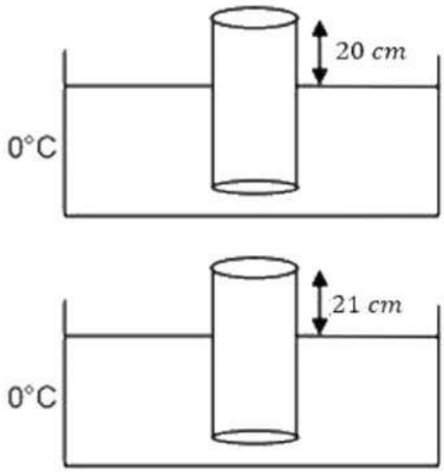
$b = -1$

$V = K(h)^0 (I)^{-1} (G)^{-1} (C)^5$

15. A cylinder of height 1 m is floating in water at 0^{o} C with 20 cm height in air. Now the temperature of water is raised to 4^{o} C, the height of the cylinder in air becomes 21 cm. The ratio of density of water at 4^{o} C to that at 0^{o} C is (Consider expansion of the cylinder is negligible)

a. 1.01
b. 1.03
c. 2.01
d. 1.04

Solution: (a)



Since the cylinder is in equilibrium, it's weight is balanced by the Buoyant force.

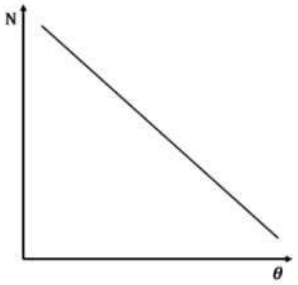
$mg = A(80)(\rho_{0^{o}C})g$

$mg = A(79)(\rho_{4^{o}C})g$

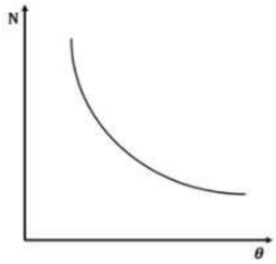
$\frac{\rho_{4^{o}C}}{\rho_{0^{o}C}} = \frac{80}{79} = 1.01$

16. Number of the α-particle deflected in Rutherford's α- particle scattering experiment varies with the angle of deflection. The graph between the two is best represented by

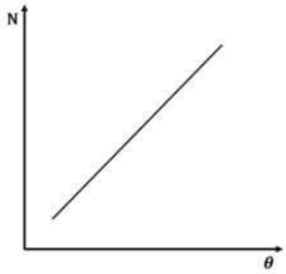
a.



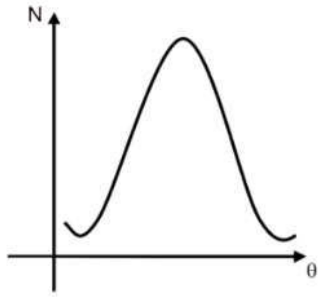
b.



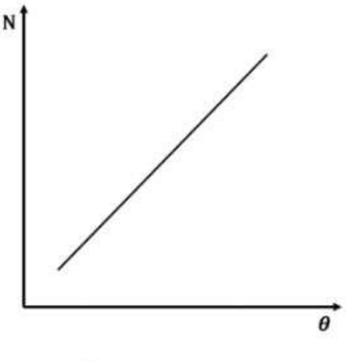
c.



d.



Solution: (c)



$N \propto \frac{1}{\sin^4(\theta/2)}$

17. If relative permittivity and relative permeability of a medium are 3 and $\frac{4}{3}$ respectively, the critical angle for this medium is

a. 45^o
b. 60^o
c. 30^0
d. 15^0

Solution: (c)

If the speed of light in the given medium is V then,

$$V = \frac{1}{\sqrt{\mu\epsilon}}$$

We know that, $\quad n = \frac{c}{v}$

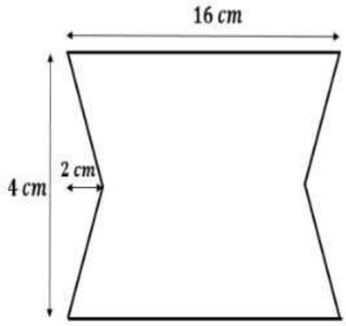
$$n = \sqrt{\mu_r \epsilon_r} = 2$$

$$\sin\theta_c = \frac{1}{2}$$

$$\theta_c = 30^o$$

18. The given loop is kept in a uniform magnetic field perpendicular to the plane of the loop. The field changes from 1000 *Gauss* to 500 *Gauss* in 5 seconds. The average induced emf in the loop is

a. $56\ \mu V$
b. $28\ \mu V$
c. $30\ \mu V$
d. $48\ \mu V$



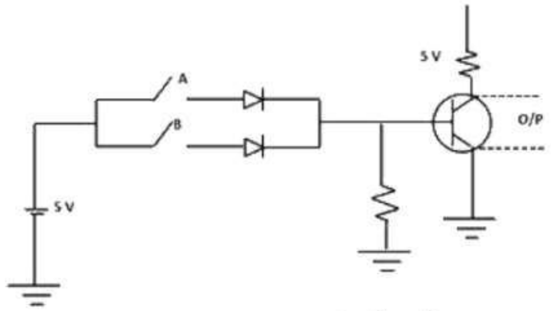
Solution: (a)

$$\epsilon = \left|-\frac{d\phi}{dt}\right| = \left|-\frac{AdB}{dt}\right|$$

$$= (16 \times 4 - 4 \times 2)\frac{(1000 - 500)}{5} \times 10^{-4} \times 10^{-4}$$

$$= 56 \times \frac{500}{5} \times 10^{-8} = 56 \times 10^{-6}\ V$$

19. Choose the correct Boolean expression for the given circuit diagram:



a. A.B

b. $\bar{A} + \bar{B}$

c. $A + B$

d. $\bar{A}.\bar{B}$

Solution: (d)
First part of figure shown OR gate and second part of figure shown NOT gate.
So, $Y = \overline{A + B} = \bar{A}.\bar{B}$

20. A Solid sphere of density $\rho = \rho_o\left(1 - \frac{r^2}{R^2}\right)$, $0 < r \leq R$ just floats in a liquid, then the density of the liquid is (r is the distance from the centre of the sphere)

a. $\frac{2}{5}\rho_o$

b. $\frac{5}{2}\rho_o$

c. $\frac{3}{5}\rho_o$

d. ρ_o

Solution: (a)

Let the mass of the sphere be m and the density of the liquid be ρ_L

$$\rho = \rho_o\left(1 - \frac{r^2}{R^2}\right), 0 < r \leq R$$

Since the sphere is floating in the liquid, buoyancy force (F_B) due to liquid will balance the weight of the sphere.

$$F_B = mg$$

$$\rho_L \frac{4}{3}\pi R^3 g = \int \rho(4\pi r^2 dr)g$$

$$\rho_L \frac{4}{3}\pi R^3 = \int \rho_o\left(1 - \frac{r^2}{R^2}\right)4\pi r^2 dr$$

$$\rho_L \frac{4}{3}\pi R^3 = \int_0^R \rho_o 4\pi\left(r^2 - \frac{r^4}{R^2}\right)dr = \rho_o 4\pi\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R$$

$$\rho_L = \frac{2}{5}\rho_o$$

21. Two masses each of mass $0.10\ kg$ are moving with velocities $3\ m/s$ along x −axis and $5\ m/s$ along y −axis respectively. After an elastic collision one of the mass moves with velocity $4\hat{\imath} + 4\hat{\jmath}\ m/s$. If the energy of the other mass after the collision is $\frac{x}{10}$, then x is

Solution: (1)

Mass of each object, $m_1 = m_2 = 0.1\ kg$
Initial velocity of 1st object, $u_1 = 5\ m/s$
Initial velocity of 2nd object, $u_2 = 3\ m/s$
Final velocity of 1st object, $V_1 = 4\hat{\imath} + 4\hat{\jmath}\ m/s = \sqrt{4^2 + 4^2} = 16\sqrt{2}\ m/s$

For elastic collision, kinetic energy remains conserved

$$Initial\ kinetic\ energy\ (K_i) = Final\ kinetic\ energy(K_f)$$

$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}m(5)^2 + \frac{1}{2}m(3)^2 = \frac{1}{2}m(16\sqrt{2})^2 + \frac{1}{2}mV_2^2$$

$$V_2 = \sqrt{2}\ m/s$$

Kinetic energy of second object $= \frac{1}{2}mV_2^2 = \frac{1}{2} \times 0.1 \times \sqrt{2}^2 = \frac{1}{10}$

$$\Rightarrow x = 1$$

22. A plano-convex lens of radius of curvature 30 cm and refractive index 1.5 is kept in air. Find its focal length (in cm).

Solution: (60 cm)

Applying Lens makers' formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$R_1 = \infty$$

$$R_2 = -30\ \text{cm}$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-30}\right)$$

$$\frac{1}{f} = \frac{0.5}{30}$$

$$f = 60 \text{ cm}$$

23. The position of two particles *A* and *B* as a function of time are given by $X_A = -3t^2 + 8t + c$ and $Y_B = 10 - 8t^3$. The velocity of *B* with respect to *A* at t = 1 is $\sqrt{v}$. Find v.

Solution: (580 m/s)

$$X_A = -3t^2 + 8t + c$$

$$\overrightarrow{v_A} = (-6t + 8)\hat{\imath}$$

$$= 2\hat{\imath}$$

$$Y_B = 10 - 8t^3$$

$$\overrightarrow{v_B} = -24t^2\hat{\jmath}$$

$$\left|\overrightarrow{v_{B/A}}\right| = |\overrightarrow{v_B} - \overrightarrow{v_A}| = |-24\hat{\jmath} - 2\hat{\imath}|$$

$$v = \sqrt{24^2 + 2^2}$$

$$v = 580 \text{ m/s}$$

24. An open organ pipe of length 1 m contains a gas whose density is twice the density of the atmosphere at STP. Find the difference between its fundamental and second harmonic frequencies if the speed of sound in atmosphere is 300 m/s.

Solution: (105.75 Hz)

$$V = \sqrt{\frac{B}{\rho}}$$

$$\frac{V_{pipe}}{V_{air}} = \frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}} = \frac{1}{\sqrt{2}}$$

$$V_{pipe} = \frac{V_{air}}{\sqrt{2}}$$

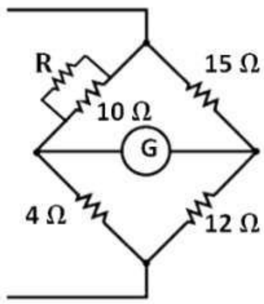
$$f_n = \frac{(n+1)}{2l} V_{pipe}$$

$$f_1 - f_0 = \frac{v_{pipe}}{2l} = \frac{300}{2\sqrt{2}} = 105.75 \text{ Hz (if } \sqrt{2} = 1.41)$$

$$= 106.05 \text{ Hz (if } \sqrt{2} = 1.414)$$

25. Four resistors of resistance 15 Ω, 12 Ω, 4 Ω and 10 Ω are connected in cyclic order to form a wheat stone bridge. The resistance (in Ω) that should be connected in parallel across the 10 Ω resistor to balance the wheat stone bridge is

Solution: (10 Ω)



$$\frac{10\,R}{10+R} \times 12 = 15 \times 4 \Rightarrow R = 10\,\Omega$$