12/04/2019
Evening

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

PART–A : PHYSICS

1. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to:

(1) $9F/(\pi r^2 YT)$

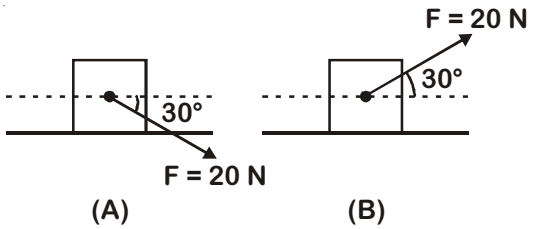
(2) $3F/(\pi r^2 YT)$

(3) $F/(3\pi r^2 YT)$

(4) $6F/(\pi r^2 YT)$

2. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be :

($g = 10\ ms^{-2}$)



(1) $0.4\ ms^{-2}$

(2) $3.2\ ms^{-2}$

(3) $0\ ms^{-2}$

(4) $0.8\ ms^{-2}$

3. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is

(1) 7/5

(2) 27/5

(3) 9/7

(4) 20/7

4. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to $V (V = GI_0)$ by connecting a series resistance R_V to it. Then,

(1) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(2) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(3) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

(4) $R_A R_V = G^2\left(\frac{(I_0 - I_g)}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

5. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be

(1) n^2 (2) $\frac{1}{n^2}$

(3) n (4) $\frac{1}{n}$

6. One kg of water, at 20°C, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to

[Specific heat of water = 4200 J/(kg °C),

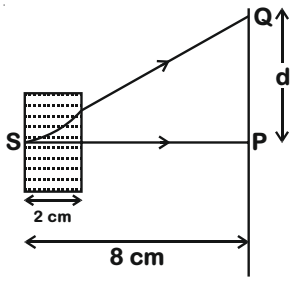
Latent heat of water = 2260 kJ/kg]

(1) 16 minutes (2) 3 minutes

(3) 22 minutes (4) 10 minutes

7. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5\times10^{-3}\ T)\,\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is

(electron's charge = 1.6 × 10^{-19} C, mass of electron = 9.1 × 10^{-31} kg)



(1) 11.65 cm (2) 12.87 cm

(3) 2.25 cm (4) 1.22 cm

8. A Carnot engine has an efficiency of $\frac{1}{6}$. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively,

(1) 99°C, 37°C (2) 37°C, 99°C

(3) 124°C, 62°C (4) 62°C, 124°C

9. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound?

[Given reference intensity of sound as 10^{-12} W/m^2]

(1) 30 cm (2) 40 cm

(3) 10 cm (4) 20 cm

10. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60°C to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly) :

(1) 1.80 (2) 5.33

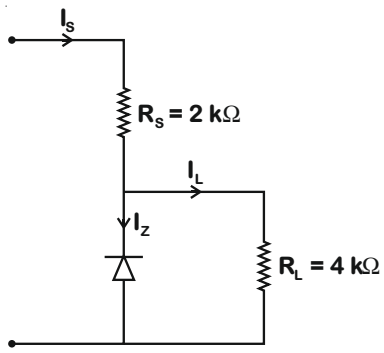
(3) 10.67 (4) 16.00

11. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio $\left(\frac{v_1}{v_2}\right)$ equals :

(1) $\frac{1}{27}$ (2) 9

(3) $\frac{1}{9}$ (4) 27

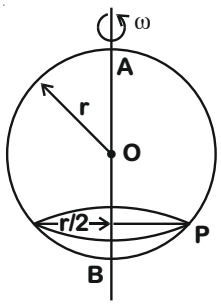
12. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?



(1) 7.5 mA (2) 1.5 mA

(3) 2.5 mA (4) 3.5 mA

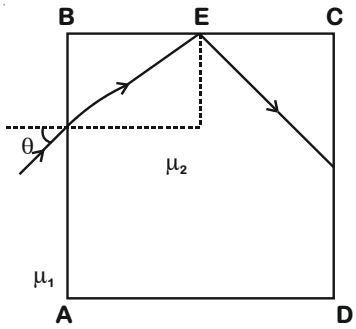
13. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to:



(1) $\frac{\left(g\sqrt{3}\right)}{r}$ (2) $\frac{2g}{r}$

(3) $\frac{2g}{\left(r\sqrt{3}\right)}$ (4) $\frac{\sqrt{3}g}{2r}$

14. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.



Then θ must satisfy:

(1) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$ (2) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$

(3) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (4) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

15. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0).

(1) $b^2\tau$ (2) $\frac{b^2\tau}{4}$

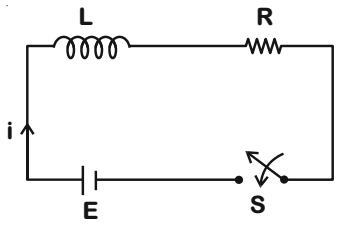
(3) $\frac{b^2\tau}{2}$ (4) $\frac{b^2\tau}{\sqrt{2}}$

16. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals:

(1) 10.0 m/s (2) 5.5 m/s

(3) 2.5 m/s (4) 15.0 m/s

17. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is:



(1) $\frac{2.7EL}{R^2}$ (2) $\frac{EL}{2.7R^2}$

(3) $\frac{7.3\,EL}{R^2}$ (4) $\frac{EL}{7.3\,R^2}$

18. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density)

(1) $\frac{R}{4}$ (2) $\frac{R}{3}$

(3) $\frac{R}{9}$ (4) $\frac{R}{2}$

19. In an amplitude modulator circuit, the carrier wave is given by,

C(t) = 4 sin (20000 πt) while modulating signal is given by, m(t) = 2 sin (2000 πt). The values of modulation index and lower side band frequency are :

(1) 0.5 and 10 kHz

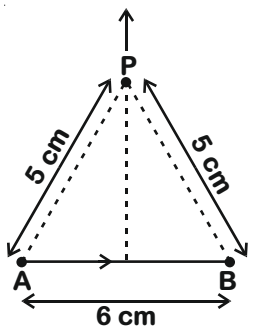
(2) 0.3 and 9 kHz

(3) 0.4 and 10 kHz

(4) 0.5 and 9 kHz

20. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure)

($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2})



(1) 2.5×10^{-5} T (2) 1.5×10^{-5} T

(3) 3.0×10^{-5} T (4) 2.0×10^{-5} T

21. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is :

(1) 3.5 Å

(2) 12.9 Å

(3) 9.7 Å

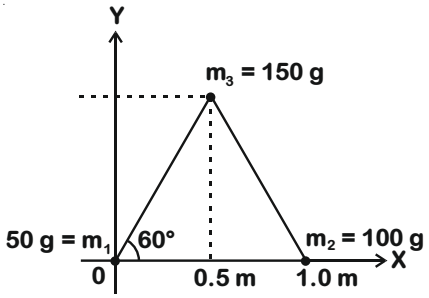
(4) 6.6 Å

22. Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then :

(1) $a = \frac{3R}{2^{1/4}}$ (2) $a = R/\sqrt{3}$

(3) $a = 2^{-1/4}R$ (4) $a = 8^{-1/4}R$

23. Three particles of masses, 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :



(1) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$ (2) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$

(3) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$ (4) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$

24. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :

(1) 9 : 8 (2) 3 : 8

(3) 8 : 1 (4) 1 : 8

25. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, l_1 = 30 cm and l_2 = 70 cm. Then, v is equal to

(1) 332 ms^{-1} (2) 384 ms^{-1}

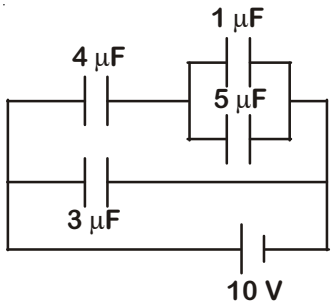
(3) 379 ms^{-1} (4) 338 ms^{-1}

26. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to

(1) $n_0\alpha^{-3/4}$ (2) $\sqrt{n_0}\,\alpha^{1/2}$

(3) $n\,\alpha^{1/4}$ (4) $n\,\alpha^{-3}$

27. In the given circuit, the charge on 4 μF capacitor will be



(1) 5.4 μC (2) 9.6 μC
(3) 13.4 μC (4) 24 μC

28. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct?

(1) $R^2 = 4 h_1 h_2$ (2) $R^2 = 16 h_1 h_2$
(3) $R^2 = 2 h_1 h_2$ (4) $R^2 = h_1 h_2$

29. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

(1) 30 J (2) 35 J
(3) 25 J (4) 40 J

30. A plane electromagnetic wave having a frequency ν = 23.9 GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave?

(1) $\vec{} = 0\sin(0.5\times10^3 x + 1.5\times10^{11} t)\hat{k}$

(2) $\vec{B} = 2\times10^{7}\sin(0.5\times10^3 z + 1.5\times10^{11} t)\hat{i}$

(3) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^3 z - 1.5\times10^{11} t)\hat{i}$

(4) $\vec{B}$ 2 $10^{-7}\sin(1.5\ 10^2 x\ 0.5\ 10^{11} t)\hat{j}$

PART–B : CHEMISTRY

1. The INCORRECT statement is :
(1) Lithium is least reactive with water among the alkali metals.
(2) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2.
(3) Lithium is the strongest reducing agent among the alkali metals.
(4) LiCl crystallises from aqueous solution as $LiCl.2H_2O$.

2. The pair that has similar atomic radii is :

(1) Mo and W (2) Ti and Hf
(3) Sc and Ni (4) Mn and Re

3. The C–C bond length is maximum in :

(1) graphite (2) C_{60}

(3) diamond (4) C_{70}

4. The decreasing order of electrical conductivity of the following aqueous solutions is:

0.1 M Formic acid (A),

0.1 M Acetic acid (B),

0.1 M Benzoic acid (C),

(1) A > B > C (2) A > C > B

(3) C > B > A (4) C > A > B

5. The compound used in the treatment of lead poisoning is :

(1) EDTA (2) desferrioxime B

(3) Cis-platin (4) D-penicillamine

6. An 'Assertion' and a 'Reason' are given below. Choose the correct answer from the following options :

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R) : Even though the intermediate carbocation is stabilized by loosely held π-electrons, the cleavage is difficult because of strong bonding.

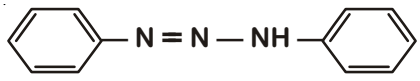
(1) Both (A) and (R) are correct statements and (R) is the correct explanation of (A).

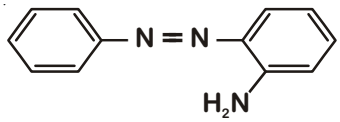
(2) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A).

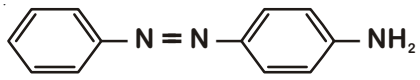
(3) Both (A) and (R) are wrong statements.

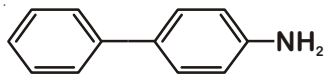
(4) (A) is a correct statement but (R) is a wrong statement.

7. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives :

(1) 

(2) 

(3) 

(4) 

8. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains :

(1) 22 g of carbon and 3 g of hydrogen

(2) 24 g of carbon and 1 g of hydrogen

(3) 20 g of carbon and 5 g of hydrogen

(4) 18 g of carbon and 7 g of hydrogen

9. In which one of the following equilibria, $K_P \neq K_C$?

(1) $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$

(2) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$

(3) $NO_2(g) + SO_2(g) \rightleftharpoons NO(g) + SO_3(g)$

(4) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

10. Among the following, the INCORRECT statement about colloids is

(1) They can scatter light.

(2) The range of diameters of colloidal particles is between 1 and 1000 nm.

(3) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration.

(4) They are larger than small molecules and have high molar mass.

11. The primary pollutant that leads to photochemical smog is

(1) Nitrogen oxides (2) Sulphur dioxide

(3) Ozone (4) Acrolein

12. The INCORRECT match in the following is

(1) $\Delta G° = 0, K = 1$ (2) $\Delta G° < 0, K < 1$

(3) $\Delta G° > 0, K < 1$ (4) $\Delta G° < 0, K > 1$

13. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation,

$2N_2O_5(g) \rightarrow 4\ NO_2(g) + O_2(g)$.

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is

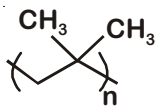
(1) 1.667×10^{-2} mol L^{-1} min^{-1}

(2) 4.167×10^{-3} mol L^{-1} min^{-1}

(3) 8.333×10^{-3} mol L^{-1} min^{-1}

(4) 2.083×10^{-3} mol L^{-1} min^{-1}

14. The correct name of the following polymer is



(1) Polyisobutane (2) Polyisoprene

(3) Polyisobutylene (4) Polytert-butylene

15. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27°C. The osmotic pressure of the solution is

(R = 0.08206 L atm K^{-1} mol^{-1})

(1) 1.64 atm (2) 2.46 atm

(3) 8.2 atm (4) 4.92 atm

16. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are, respectively

(1) 4 : 2 : 3 (2) 4 : 2 : 1

(3) 8 : 1 : 6 (4) 1 : 2 : 4

17. In comparison to boron, berylium has

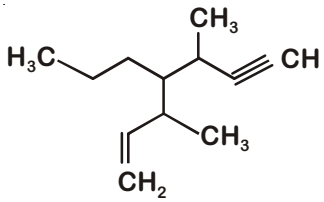
(1) Greater nuclear charge and lesser first ionisation enthalpy.

(2) Greater nuclear charge and greater first ionisation enthalpy.

(3) Lesser nuclear charge and greater first ionisation enthalpy.

(4) Lesser nuclear charge and lesser first ionisation enthalpy.

18. The IUPAC name for the following compound is



(1) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene

(2) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

(3) 3,5-dimethyl-4-propylhept-1-en-6-yne

(4) 3,5-dimethyl-4-propylhept-6-en-1-yne

19. The molar solubility of $Cd(OH)_2$ is 1.84×10^{-5} M in water. The expected solubility of $Cd(OH)_2$ in a buffer solution of pH = 12 is:

(1) 1.84×10^{-9} M (2) 6.23×10^{-11} M

(3) $\frac{2.49}{1.84} \times 10^{-9}$ M (4) 2.49×10^{-10} M

20. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively are:

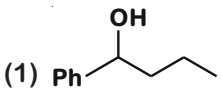
(1) $Ca(HCO_3)_2$ and $Ca(OH)_2$

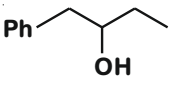
(2) $Mg(HCO_3)_2$ and $Mg(OH)_2$

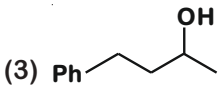
(3) $Mg(HCO_3)_2$ and $MgCO_3$

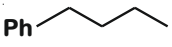
(4) $Ca(HCO_3)_2$ and CaO

21. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is:

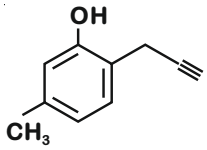
(1) 

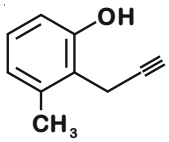
(2) 

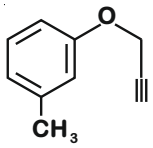
(3) 

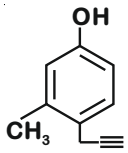
(4) 

22. What will be the major product when m-cresol is reacted with propargyl bromide ($HC \equiv C-CH_2Br$) in presence of K_2CO_3 in acetone ?

(1) 

(2) 

(3) 

(4) 

23. Among the following, the energy of 2s orbital is lowest in:

(1) Li (2) K

(3) H (4) Na

24. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z respectively are:

(1) K_2MnO_4, $KMnO_4$ and Cl_2

(2) K_3MnO_4, K_2MnO_4 and Cl_2

(3) K_2MnO_4, $KMnO_4$ and SO_2

(4) KMnO , K MnO and Cl

25. The correct statement is

(1) Pig iron is obtained from cast iron.

(2) Leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate.

(3) The blistered appearance of copper during the metallurgical process is due to the evolution of CO_2.

(4) The Hall-Heroult process is used for the production of aluminium and iron.

26. Which of the given statements is INCORRECT about glycogen?

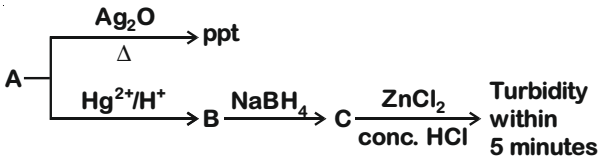
(1) It is present in some yeast and fungi.

(2) It is a straight chain polymer similar to amylose.

(3) It is present in animal cells.

(4) Only α-linkages are present in the molecule.

27. Consider the following reactions :



'A' is

(1) $CH_3 - C \equiv CH$

(2) $CH_3 - C \equiv C - CH_3$

(3) $CH_2 = CH_2$

(4) $CH \equiv CH$

28. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are (en = ethane-1, 2-diamine)

(1) 3 and 3

(2) 5 and 3

(3) 5 and 6

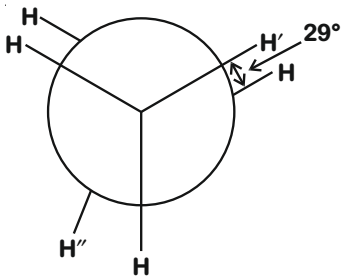
(4) 6 and 6

29. Which one of the following is likely to give a precipitate with $AgNO_3$ solution?

(1) $CH_2 = CH - Cl$ (2) $CHCl_3$

(3) CCl_4 (4) $(CH_3)_3CCl$

30. In the following skew conformation of ethane, H′ – C – C – H″ dihedral angle is



(1) 120° (2) 58°

(3) 151° (4) 149°

PART–C : MATHEMATICS

1. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?

(1) $\left(\sqrt{2}, 2\right)$

(2) $\left(2, 2\sqrt{2}\right)$

(3) $\left(1, 2\sqrt{2}\right)$

(4) $\left(2, \sqrt{2}\right)$

2. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is:

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

(2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(3) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

(4) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

3. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ is :

(1) $\frac{1}{2}$

(2) $\frac{2}{3}$

(3) 2

(4) 1

4. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

(1) $\left(\frac{5}{2},1\right)$ (2) $\left(-\frac{5}{2},1\right)$

(3) $\left(\frac{5}{2},-1\right)$ (4) $\left(-\frac{5}{2},-1\right)$

5. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 48 (2) 24

(3) $4\sqrt{3}$ (4) $2\sqrt{6}$

6. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 24 (2) 27

(3) 25 (4) 28

7. If $a_1, a_2, a_3, \ldots\ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

(1) 150

(2) 280

(3) 200

(4) 120

8. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is :

(1) $15(3+\sqrt{3})$ (2) $15(1+\sqrt{3})$

(3) $15(3-\sqrt{3})$ (4) $15(5-\sqrt{3})$

9. If [x] denotes the greatest integer $\leq x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$ $[\cot\theta]x + y = 0$

(1) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(2) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(3) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(4) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

10. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0, \text{ is :}$$

(1) $\frac{\pi}{18}$

(2) $\frac{7\pi}{36}$

(3) $\frac{7\pi}{24}$

(4) $\frac{\pi}{9}$

11. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$

(3) $p \wedge q$ (4) $q \Rightarrow \sim p$

12. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is :

(1) $x + y + 4 = 0$ (2) $2x - y + 2 = 0$

(3) $x - 2y + 16 = 0$ (4) $x - y + 4 = 0$

13. The general solution of the differential equation $(y^2 - x^3)\,dx - xydy = 0$ $(x \neq 0)$ is :

(where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$ (2) $y^2 - 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$ (4) $y^2 + 2x^2 + cx^3 = 0$

14. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point :

(1) (1, –4, 1) (2) (2, –4, 1)

(3) (1, 4, –1) (4) (2, 4, 1)

15. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to :

(1) [1, 4] (2) R

(3) [2, 6] (4) [3, 7]

16. Let $\alpha \in R$ and the three vectors

$\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and

$\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set

$S = \{\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$\}$

(1) contains exactly two numbers only one of which is positive

(2) is singleton

(3) contains exactly two positive numbers

(4) is empty

17. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is :

(1) $\frac{1}{3}$

(2) 3

(3) $\frac{1}{\sqrt{3}}$

(4) $\sqrt{3}$

18. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral

$\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx =$

$A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B(x) are respectively :

(1) $x - \alpha$ and $\log_e|\cos(x - \alpha)|$

(2) $x + \alpha$ and $\log_e|\sin(x - \alpha)|$

(3) $x - \alpha$ and $\log_e|\sin(x - \alpha)|$

(4) $x + \alpha$ and $\log_e|\sin(x + \alpha)|$

19. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + \ldots\ldots + (20^2)\,{}^{20}C_{20} = A(2^{\beta})$, then the ordered pair (A, β) is equal to :

(1) (420, 19)

(2) (380, 19)

(3) (420, 18)

(4) (380, 18)

20. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2, 3). Then the centroid of this triangle is :

(1) $\left(\frac{1}{3}, 2\right)$ (2) $\left(\frac{1}{3}, \frac{5}{3}\right)$

(3) $\left(\frac{1}{3}, 1\right)$ (4) $\left(1, \frac{7}{3}\right)$

21. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :

(1) 0 (2) $\alpha\gamma$

(3) $\beta\gamma$ (4) $\alpha\beta$

22. $\lim\limits_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is :

(1) 3

(2) 6

(3) 1

(4) 2

23. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is :

(1) $\left(\sqrt{3} + 1\right)x + \left(\sqrt{3} - 1\right)y = 8\sqrt{2}$

(2) $\left(\sqrt{3} - 1\right)x + \left(\sqrt{3} + 1\right)y = 8\sqrt{2}$

(3) $\sqrt{3}x + y = 8$

(4) $x + \sqrt{3}y = 8$

24. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to :

(1) –108 (2) –36

(3) –72 (4) 36

25. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (2, 3) (2) (1, 5)

(3) (3, 5) (4) (3, 10)

26. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/ loss (in Rs.) of the person is :

(1) $\frac{1}{2}$ loss (2) 2 gain

(3) $\frac{1}{2}$ gain (4) $\frac{1}{4}$ loss

27. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

(1) $B \cap C \neq \phi$

(2) $(C \cup A) \cap (C \cup B) = C$

(3) If $(A - C) \subseteq B$, then $A \subseteq B$

(4) If $(A - B) \subseteq C$, then $A \subseteq C$

28. A value of α such that

$$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right) \text{ is :}$$

(1) $\frac{1}{2}$ (2) -2

(3) $-\frac{1}{2}$ (4) 2

29. Let $z \in C$ with $Im(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then :

(1) n = 20 and Re(z) = 10

(2) n = 20 and Re(z) = –10

(3) n = 40 and Re(z) = –10

30. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|$, $x \in R$. If f(x) attains maximum value at α and g(x) attains minimum value at β, then

$$\lim_{x \to -\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8} \text{ is equal to :}$$

(1) 1/2

(2) –1/2

(3) –3/2

(4) 3/2

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