FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Wednesday 17th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take g = 10 ms^{-2})

(1) 3.0 ms^{-1} (2) 3.50 ms^{-1}

(3) 2.0 ms^{-1} (4) 2.50 ms^{-1}

2. If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_P}{C_V}\right)$ then the value of β is :

(1) 1.02 (2) 1.2

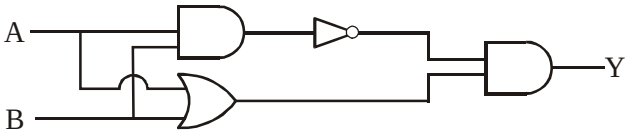
(3) 1.25 (4) 1.35

3. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take In 2 = 0.693)

(1) 0.69 × 10^2 kg s^{-1} (2) 3.3 × 10^2 kg s^{-1}

(3) 1.16 × 10^2 kg s^{-1} (4) 5.7 × 10^{-3} kg s^{-1}

4. Which one of the following will be the output of the given circuit ?



(1) NOR Gate (2) NAND Gate

(3) AND Gate (4) XOR Gate

5. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36% , the ratio of hydraulic stress to the corresponding hydraulic strain will be _____.

[Given : density of water is 1000 kg m^{-3} and g = 9.8 ms^{-2}.]

(1) 1.96 × 10^7 Nm^{-2} (2) 1.44 × 10^7 Nm^{-2}

(3) 2.26 × 10^9 Nm^{-2} (4) 1.44 × 10^9 Nm^{-2}

6. A geostationary satellite is orbiting around an arbitary planet 'P' at a height of 11R above the surface of 'P' , R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is_____.'P' has the time period of 24 hours.

(1) $6\sqrt{2}$ (2) $\frac{6}{\sqrt{2}}$ (3) 3 (4) 5

7. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

(1) $Y(x,t) = 0.03\ [\sin 5.1\ x - (0.2 \times 10^3)t]$

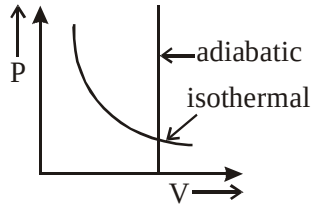
(2) $Y(x,t) = 0.06\ [\sin 5.1\ x - (1.5 \times 10^3)t]$

(3) $Y(x,t) = 0.06\ [\sin 0.8\ x - (0.5 \times 10^3)t]$

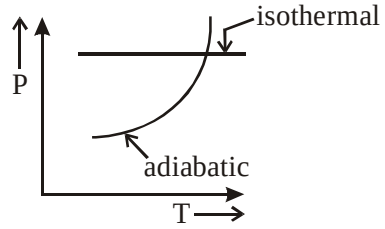
(4) $Y(x,t) = 0.03\ [\sin 5.1\ x - (1.5 \times 10^3)t]$

8. Which one is the correct option for the two different thermodynamic processes ?

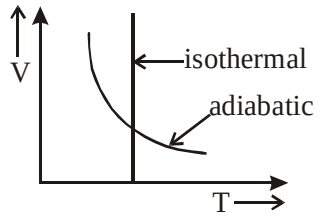
(a)



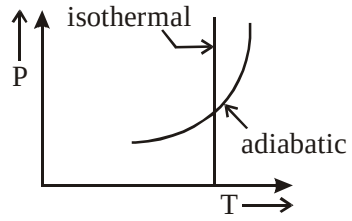
(b)



(c)



(d)



(1) (c) and (a) (2) (c) and (d)

(3) (a) only (4) (b) and (c)

9. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is x = 0 at t = 0 ; then its displacement after time (t = 1) is :

(1) $v_0 + g + F$

(2) $v_0 + \frac{g}{2} + \frac{F}{3}$

(3) $v_0 + \frac{g}{2} + F$

(4) $v_0 + 2g + 3F$

10. A carrier signal $C(t) = 25 \sin (2.512 \times 10^{10}\, t)$ is amplitude modulated by a message signal $m(t) = 5 \sin (1.57 \times 10^{8}\, t)$ and transmitted through an antenna.What will be the bandwidth of the modulated signal ?

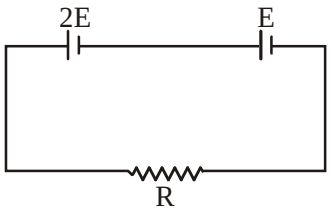
(1) 8 GHz

(2) 2.01 GHz

(3) 1987.5 MHz

(4) 50 MHz

11. Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is



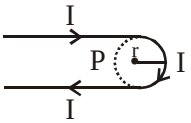
(1) $r_1 + r_2$

(2) $\frac{r_1}{2} - r_2$

(3) $\frac{r_1}{2} + r_2$

(4) $r_1 - r_2$

12. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle ?



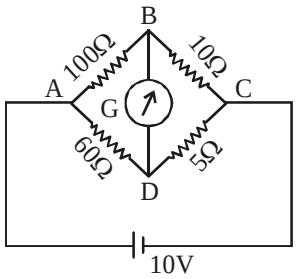
(1) $\frac{\mu_0 I}{4\pi r}(2-\pi)$

(2) $\frac{\mu_0 I}{4\pi r}(2+\pi)$

(3) $\frac{\mu_0 I}{2\pi r}(2+\pi)$

(4) $\frac{\mu_0 I}{2\pi r}(2-\pi)$

13. The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10V is maintained across AC.



(1) 2.44 μA (2) 2.44 mA
(3) 4.87 mA (4) 4.87 μA

14. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively.If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is

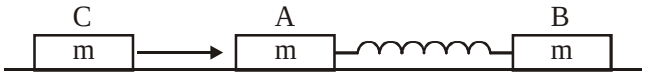
(1) $\frac{K_2}{K_1}$ (2) $\frac{K_1}{K_2}$ (3) $\sqrt{\frac{K_1}{K_2}}$ (4) $\sqrt{\frac{K_2}{K_1}}$

15. Match List-I with List-II

List-I	**List-II**
(a) Phase difference between current and voltage in a purely resistive AC circuit	(i) $\frac{\pi}{2}$; current leads voltage
(b) Phase difference between current and voltage in a pure inductive AC circuit	(ii) zero
(c) Phase difference between current and voltage in a pure capacitive AC circuit	(iii) $\frac{\pi}{2}$; current lags voltage
(d) Phase difference between current and voltage in an LCR series circuit	(iv) $\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below :
(1) (a)–(i),(b)–(iii),(c)–(iv),(d)–(ii)
(2) (a)–(ii),(b)–(iv),(c)–(iii),(d)–(i)
(3) (a)–(ii),(b)–(iii),(c)–(iv),(d)–(i)
(4) (a)–(ii),(b)–(iii),(c)–(i),(d)–(iv)

16. Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A.The maximum compression in the spring is



(1) $v\sqrt{\frac{M}{2K}}$ (2) $\sqrt{\frac{mv}{2K}}$

(3) $\sqrt{\frac{mv}{K}}$ (4) $\sqrt{\frac{m}{2K}}$

17. The atomic hydrogen emits a line spectrum consisting of various series.Which series of hydrogen atomic spectra is lying in the visible region ?

(1) Brackett series (2) Paschen series

(3) Lyman series (4) Balmer series

18. Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then

(1) $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$

(2) $v_1^2 + v_2^2 = \frac{2h}{m}[f_1 + f_2]$

(3) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(4) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

19. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved ?

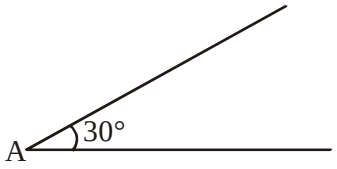
(1) Both, inductive reactance and current will be halved.

(2) Inductive reactance will be halved and current will be doubled.

(3) Inductive reactance will be doubled and current will be halved.

(4) Both, inducting reactance and current will be doubled.

20. A sphere of mass 2kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How low will the sphere take to return to the starting point A ?



(1) 0.60 s (2) 0.52 s
(3) 0.57 s (4) 0.80 s

SECTION-B

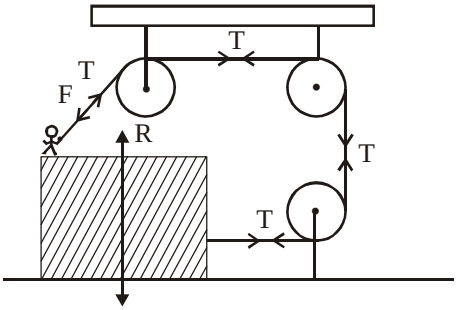
1. The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3m is E. The electric field intensity produced by the radiation coming from 60 W at the same distance is $\sqrt{\frac{x}{5}}E$. Where the value of x =_____.

2. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be _____. (Round off to the Nearest Integer)

[Take g = 10 ms^{-2}]

3. A boy of mass 4 kg is standing on a piece of wood having mass 5kg . If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is _____N.(Round off to the Nearest Integer)

[Take g = 10 ms^{-2}]



4. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm. Then the thickness of oleic acid layer will be $x \times 10^{-14}$ m.

Where x is_____.

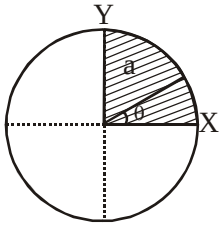
5. A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals r_n vary with $n^{1/\alpha}$, where α is_____.

6. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3 \frac{N}{C}$. The flux of this field through a rectangular surface area 0.4 m^2 parallel to the Y – Z plane is _____Nm^2C^{-1}.

7. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is _____.

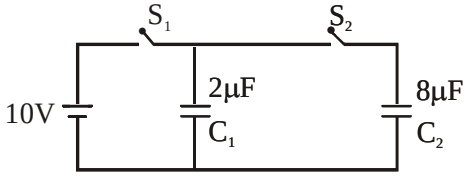
(Round off to the Nearest Integer)

[a is an area as shown in the figure]



8. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2^{rd}}{3}$ of the distance of the object from the surface .The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. the value of 'x' is_____.

9. A 2 μF capacitor C_1 is first charged to a potential difference of 10 V using a battery.Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8μF. The charge in C_2 on equilibrium condition is_____μC. (Round off to the Nearest Integer)



10. Seawater at a frequency f = 9 × 10^2 Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\ \Omega m$. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t)=V_0 \sin(2\pi ft)$. Then the conduction current density becomes 10^x times the displacement current density after time t = $\frac{1}{800}$ s. The value of x is _____

$$(\text{Given}: \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2})$$

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CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. Fructose is an example of :-
(1) Pyranose
(2) Ketohexose
(3) Aldohexose
(4) Heptose

2. The set of elements that differ in mutual relationship from those of the other sets is :
(1) Li – Mg (2) B – Si
(3) Be – Al (4) Li – Na

3. The functional groups that are responsible for the ion-exchange property of cation and anion exchange resins, respectively, are :
(1) $-SO_3H$ and $-NH_2$
(2) $-SO_3H$ and –COOH
(3) $-NH_2$ and –COOH
(4) $-NH_2$ and $-SO_3H$

4. Match List-I and List-II :

	List-I		**List-II**
(a)	Haematite	(i)	$Al_2O_3.xH_2O$
(b)	Bauxite	(ii)	Fe_2O_3
(c)	Magnetite	(iii)	$CuCO_3.Cu(OH)_2$
(d)	Malachite	(iv)	Fe_3O_4

Choose the correct answer from the options given below :
(1) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)
(2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
(3) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)
(4) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

5. The correct pair(s) of the ambident nucleophiles is (are) :
(A) AgCN/KCN
(B) RCOOAg/RCOOK
(C) $AgNO_2/KNO_2$
(D) AgI/KI
(1) (B) and (C) only
(2) (A) only
(3) (A) and (C) only
(4) (B) only

6. The set that represents the pair of neutral oxides of nitrogen is :
(1) NO and N_2O (2) N_2O and N_2O_3
(3) N_2O and NO_2 (4) NO and NO_2

7. Match List-I with List-II :

	List-I		**List-II**
(a)	$[Co(NH_3)_6]$ $[Cr(CN)_6]$	(i)	Linkage isomerism
(b)	$[Co(NH_3)_3$ $(NO_2)_3]$	(ii)	Solvate isomerism
(c)	$[Cr(H_2O)_6]Cl_3$	(iii)	Co-ordination isomerism
(d)	*cis*-$[CrCl_2(ox)_2]^{3-}$	(iv)	Optical isomerism

Choose the correct answer from the options given below :
(1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
(2) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
(3) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)
(4) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

8. Primary, secondary and tertiary amines can be separated using :-
(1) Para-Toluene sulphonyl chloride
(2) Chloroform and KOH
(3) Benzene sulphonic acid
(4) Acetyl amide

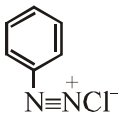
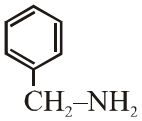
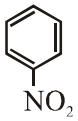
9. The common positive oxidation states for an element with atomic number 24, are :
(1) +2 to +6 (2) +1 and +3 to +6
(3) +1 and +3 (4) +1 to +6

10. Match List-I with List-II :

	List-I Chemical Compound		**List-II Used as**
(a)	Sucralose	(i)	Synthetic detergent
(b)	Glyceryl ester of stearic acid	(ii)	Artificial sweetener
(c)	Sodium benzoate	(iii)	Antiseptic
(d)	Bithionol	(iv)	Food preservative

Choose the correct match :
(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

11. Given below are two statements :
Statement-I : 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol.
Statement-II : n-alkanes can be easily oxidised to corresponding alcohol with $KMnO_4$.
Choose the correct option :
(1) Both statement I and statement II are correct
(2) Both statement I and statement II are incorrect
(3) Statement I is correct but Statement II is incorrect
(4) Statement I is incorrect but Statement II is correct

12. Nitrogen can be estimated by Kjeldahl's method for which of the following compound ?
(1) 
(2) 
(3) 
(4) 

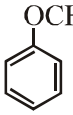
13. Amongst the following, the linear species is :
(1) NO_2 (2) Cl_2O
(3) O_3 (4) N_3^-

14. $C_{12}H_{22}O_{11}+H_2O \xrightarrow{\text{Enzyme A}} C_6H_{12}O_6+C_6H_{12}O_6$
Sucrose — Glucose — Fructose

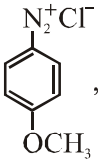
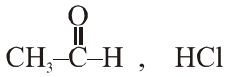
$C_6H_{12}O_6 \xrightarrow{\text{Enzyme B}} 2C_2H_5OH+2CO_2$
Glucose

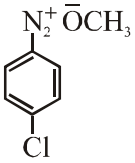
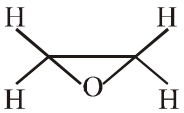
In the above reactions, the enzyme A and enzyme B respectively are :-
(1) Amylase and Invertase
(2) Invertase and Amylase
(3) Invertase and Zymase
(4) Zymase and Invertase

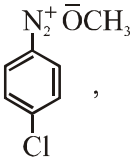
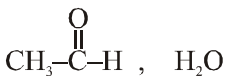
15. One of the by-products formed during the recovery of NH_3 from Solvay process is :
(1) $Ca(OH)_2$ (2) $NaHCO_3$
(3) $CaCl_2$ (4) NH_4Cl

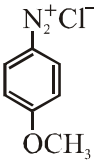
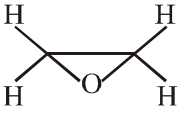
16. $C_7H_7N_2OCl+C_2H_5OH \rightarrow$  $+ N_2+$"X"+"Y"
(A)

In the above reaction, the structural formula of (A), "X" and "Y" respectively are :

(1)  ,  , HCl

(2)  ,  , HCl

(3)  ,  , H_2O

(4)  ,  , H_2O

17. For the coagulation of a negative sol, the species below, that has the highest flocculating power is :
(1) SO_4^{2-} (2) Ba^{2+} (3) Na^+ (4) PO_4^{3-}

18. Which of the following statement(s) is (are) incorrect reason for eutrophication ?
(A) excess usage of fertilisers
(B) excess usage of detergents
(C) dense plant population in water bodies
(D) lack of nutrients in water bodies that prevent plant growth
Choose the most appropriate answer from the options given below :
(1) (A) only (2) (C) only
(3) (B) and (D) only (4) (D) only

19. Choose the correct statement regarding the formation of carbocations A and B given :-

$CH_3-CH_2-CH=CH_2+HBr \rightarrow CH_3-CH_2-CH_2-\overset{+}{C}H_2+Br^-$ "A"

$CH_3-CH_2-CH=CH_2+HBr \rightarrow CH_3-CH_2-\overset{+}{C}H-CH_3+Br^-$ "B"

(1) Carbocation B is more stable and formed relatively at faster rate
(2) Carbocation A is more stable and formed relatively at slow rate
(3) Carbocation B is more stable and formed relatively at slow rate
(4) Carbocation A is more stable and formed relatively at faster rate

20. During which of the following processes, does entropy decrease ?

(A) Freezing of water to ice at 0°C

(B) Freezing of water to ice at –10°C

(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

(D) Adsorption of CO(g) and lead surface

(E) Dissolution of NaCl in water

SECTION-B

1. A KCl solution of conductivity 0.14 S m^{-1} shows a resistance of 4.19 Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to 1.03 Ω. The conductivity of the HCl solution is ____× 10^{-2} S m^{-1}. (Round off to the Nearest Integer).

2. On complete reaction of $FeCl_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is ____.

(Round off to the Nearest Integer).

3. The reaction $2A + B_2 \rightarrow 2AB$ is an elementary reaction.

For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of _____. (Round off to the Nearest Integer).

4. The total number of C–C sigma bond/s in mesityl oxide ($C_6H_{10}O$) is_____. (Round off to the Nearest Integer).

5. A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation of 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is_____ u. (Round off to the Nearest Integer).

[Density of water = 1.0 g cm^{-3}]

6. In the ground state of atomic Fe(Z = 26), the spin-only magnetic moment is _____ $\times 10^{-1}$ BM. (Round off to the Nearest Integer).

[Given : $\sqrt{3}$ = 1.73, $\sqrt{2}$ = 1.41]

7. The number of chlorine atoms in 20 mL of chlorine gas at STP is_____10^{21}. (Round off to the Nearest Integer).

[Assume chlorine is an ideal gas at STP

R = 0.083 L bar mol^{-1} K^{-1}, N_A = 6.023 $\times 10^{23}$]

8. KBr is doped with 10^{-5} mole percent of $SrBr_2$. The number of cationic vacancies in 1 g of KBr crystal is _____10^{14}. (Round off to the Nearest Integer).

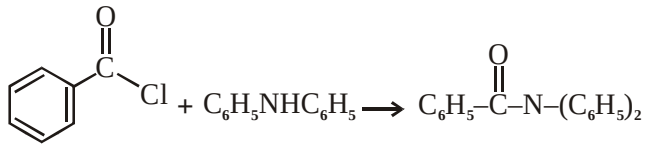
[Atomic Mass : K : 39.1 u, Br : 79.9 u,

N_A = 6.023 × 10^{23}]

9. Consider the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$. The temperature at which K_C = 20.4 and K_P = 600.1, is_____K. (Round off to the Nearest Integer).

[Assume all gases are ideal and R = 0.0831 L bar K^{-1} mol^{-1}]

10.



0.140g 0.388g 0.210g

Consider the above reaction. The percentage yield of amide product is _____. (Round off to the Nearest Integer).

(Given : Atomic mass : C : 12.0 u, H : 1.0u, N : 14.0 u, O : 16.0 u, Cl : 35.5 u)

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Wednesday 17th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f : R \to R$ be defined as $f(x) = e^{-x}\sin x$. If $F : [0, 1] \to R$ is a differentiable function such that $F(x) = \int_0^x f(t)\,dt$, then the value of $\int_0^1 (F'(x) + f(x))e^x dx$ lies in the interval

(1) $\left[\frac{327}{360}, \frac{329}{360}\right]$ (2) $\left[\frac{330}{360}, \frac{331}{360}\right]$

(3) $\left[\frac{331}{360}, \frac{334}{360}\right]$ (4) $\left[\frac{335}{360}, \frac{336}{360}\right]$

2. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where α, β, γ are integers and $[x]$ denotes the greatest integer less than or equal to x, then the value of $\alpha + \beta + \gamma$ is equal to :

(1) 0 (2) 20 (3) 25 (4) 10

3. Let y = y(x) be the solution of the differential equation $\cos x\,(3\sin x + \cos x + 3)dy = (1 + y\sin x\,(3\sin x + \cos x + 3))dx$, $0 \le x \le \frac{\pi}{2}$, y(0) = 0. Then , $y\left(\frac{\pi}{3}\right)$ is equal to:

(1) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (2) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

(3) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$ (4) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

4. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to :

(1) 1124 (2) 1324 (3) 1024 (4) 924

5. The value of $\lim\limits_{n\to\infty}\frac{[r]+[2r]+.....+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to :

(1) $\frac{r}{2}$ (2) r (3) 2r (4) 0

6. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for $x \in [-1, 1]$, and [x] denotes the greatest integer less than or equal to x, is :

(1) 2 (2) 0
(3) 4 (4) Infinite

7. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

8. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is :

(1) 3 (2) 4 (3) 2 (4) 5

9. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \le \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \text{Re}((1-i)z) \ge 1\}$

$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

(1) is a singleton

(2) has exactly two elements

(3) has infinitely many elements

(4) has exactly three elements

10. If the curve $y = y(x)$ is the solution of the differential equation

$2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}\,dx$, $x > 0$ which passes through the point $\left(1, 1 - \frac{4}{3}\log_e 2\right)$, then the value of $y(16)$ is equal to :

(1) $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$ (2) $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

(3) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$ (4) $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

11. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :

(1) 364 (2) 240 (3) 333 (4) 360

12. If x, y, z are in arithmetic progression with common difference d, $x \neq 3d$, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is

(1) 72 (2) 12 (3) 36 (4) 6

13. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, x, y $\in$ R, x > 0, be such that $|\overrightarrow{PQ}| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, z $\in$ R, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to

(1) 7 (2) 9 (3) 2 (4) 1

14. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

(1) 11 : 4 (2) 9 : 4 (3) 3 :1 (4) 2 : 1

15. Consider the function $f : R \to R$ defined by $f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x|, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then f is :

(1) monotonic on $(-\infty, 0) \cup (0, \infty)$

(2) not monotonic on $(-\infty, 0)$ and $(0, \infty)$

(3) monotonic on $(0, \infty)$ only

(4) monotonic on $(-\infty, 0)$ only

16. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to :

(1) 11 (2) 14 (3) 16 (4) 20

17. The value of the limit $\lim\limits_{\theta \to 0} \frac{\tan\left(\pi \cos^2\theta\right)}{\sin\left(2\pi \sin^2\theta\right)}$ is equal to :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) 0 (4) $\frac{1}{4}$

18. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3, 4) meet x-axis and y-axis at point P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to

(1) $\frac{529}{64}$ (2) $\frac{125}{72}$ (3) $\frac{625}{72}$ (4) $\frac{585}{66}$

19. If the Boolean expression $(p \wedge q) \circledast (p \otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \vee$ (3) $\vee, \rightarrow$ (4) $\wedge, \rightarrow$

20. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to :

(1) 20 (2) 19 (3) 18 (4) 21

SECTION-B

1. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant

$$\begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$$

is equal to :

2. Let $f : [-1, 1] \to R$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where $a, b, c \in R$ such that $f(-1) = 2$, $f'(-1) = 1$ and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1, 1]$, then the least value of α is equal to _____.

3. Let $f : [-3, 1] \to R$ be given as

$$f(x) = \begin{cases} \min\{(x+6), x^2\}, & -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\}, & 0 \le x \le 1. \end{cases}$$

If the area bounded by $y = f(x)$ and x-axis is A, then the value of 6A is equal to _____.

4. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \ne \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin.If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to :

5. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to _____.

6. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x+\frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _____.

7. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B=\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to _____.

8. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i}+2\hat{j}-\hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to _____.

9. Let $I_n = \int_1^e x^{19}\left(\log|x|\right)^n dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equal to _____.

10. Let P be an arbitrary point having sum of the squares of the distance from the planes $x + y + z = 0$, $lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $l - n$ is equal to _____.