On the Diophantine Equation $(a + 2)^x - a^y = z^2$ where $a = n^2 - 1$ and a and n are non-negative integers

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Abstract

In this paper, we find all non-negative integer solutions (x, y, z) of the Diophantine equation $(a + 2)^x - a^y = z^2$ which have three solutions where a, n are non-negative integers with $a = n^2 - 1$. In the proof, we apply reasonably Catalan's conjecture and various theories concerning the congruence to obtain the solutions. The result reveals that the solutions are (0, 0, 0) or (1, 0, n) or (2, 2, 2n).

Keywords: Diophantine equation, Catalan's conjecture, Integer solution