Project INF5631- Biological model for infection

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Biological model

In this project I will look at a biological model which will describe how an infection will spread i a population. I will look at both linear and non-linear equations. For the non-linear equation I will use Picard and Newton. Here it is intresting to compare these methods. I will also compare FDM and FEM.

The model:

$$u_t = \nabla \cdot \alpha(u)\nabla u + f(u) \tag{1}$$

In a biological model, we will often use $f(u) = u(1-u)$.

1D model

If we look at this problem i 1D, we can use:

$$\alpha(u) = \beta_0 + \beta_1 u' \tag{2}$$

And

$$f(u) = ru(1 - \frac{u}{M}), u \to M \tag{3}$$

This will give us the equation

$$u_t = ((\beta_0 + \beta_1 u')u')' + ru(1 - \frac{u}{M}) \tag{4}$$

1 Linear equation

Here I will set $\beta_1 = 0$ and let $M \to \infty$ This will give me the equation

$$u_t = \beta_0 u'' + ru \tag{5}$$

Use FDM on this,plot and verify the method

Solution

2 Non-linear equation with Picard

- replace $\beta_1 u', ru(1-\frac{u}{M})$ with $\beta_1 u\_', ru(1-\frac{u_-}{M})$
- 1 iteration per timestep
- n iterations per timestep
- Enough with one iteration?

3 Non-linear equation with Newton

- FDM + Newton
- Newton on PDE + FDM

4 Non-linear equation with variational formula

- Picard on PDE level
- FEM variational formula
- Find element matrix and vector
- Assemble to linear system
- Compare formula with FDM

5 Non-linear: Newton+FEM

- FEM
  - Variational formulation, not calculate the element matrix
- FEM on CN discret eq + Newton(=Jacobian+F_i)
- Newton on PDE-level + FEM discrete + Jacobian and F_i

6 2D Picard on PDE-level

- Picard on PDE-level
- Variational formula
- FEnics