

Command Governor-Based Adaptive Control

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Even though an important prerequisite for an adaptive controller is the ability to adapt fast in the face of large system uncertainties, fast adaptation with high-gain learning rates can be a serious limitation due to the fact that it can possibly yield to high-frequency oscillations especially during the transient system response. Therefore, achieving fast adaptation with low-gain learning rates and predictable transient and steady-state system response is the challenge for the adaptive control theorist today. To that end, we introduce a novel command governor architecture for adaptive stabilization and command following. Specifically, the proposed command governor is a linear dynamical system which adjusts the trajectories of a given command in order to follow an ideal reference system both in transient time and steady-state without resorting to high-gain learning rates in the adaptation law. It is shown that by choosing the design parameter of the command governor, the controlled nonlinear uncertain dynamical system can approach a Hurwitz linear time-invariant dynamical system with \mathcal{L}_∞ input-output signals. This provides a systematic framework for verification and validation of adaptive control systems and the proposed command governor can be used in a complimentary way with many other approaches to adaptive control.

I. Introduction

While adaptive control has been used in numerous applications, the necessity of high-gain feedback for achieving fast adaptation can be a serious limitation of adaptive controllers^{1–4}. Specifically, in certain applications fast adaptation is required in order to achieve stringent stabilization or command following performance specifications in the face of large system uncertainties. In such situations, high-gain adaptive control is necessary for rapidly reducing the mismatch between the uncertain dynamical system and a given ideal reference system capturing a desired closed-loop dynamical system performance. However, update laws with high-gain learning rates possibly yield to high-frequency oscillations especially during the transient system response resulting in system instability for real applications^{5–7}.

Numerous adaptive control methods have been proposed in the past decades that deal with adaptive stabilization and command following of uncertain dynamical systems (see, for example, Refs. 4, 8–15 and references therein). Most of these approaches have averted the problem of high-gain control. Notable exceptions include Refs. 13–15. Specifically, the authors in Refs. 13, 14 use a low-pass filter that subverts high frequency oscillations that can occur due to high-gain adaptation. More recently, the authors in Ref. 15 present a high-gain adaptive control architecture that allows fast adaptation without hindering system robustness. Even though the high-gain adaptive control approaches documented in Refs. 13–15 are promising, they require the knowledge of a conservative upper bound on the unknown constant weights appearing in their uncertainty parametrization. While this conservative upper bound can be available for some specific applications, the actual upper bound may exceed its conservative estimate, for example, when an aircraft undergoes a sudden change in dynamics, such as might be due to reconfiguration, deployment of a payload,

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docking, or structural damage. In such circumstances, the transient and steady-state system performance of these controllers are no longer guaranteed, since it is not possible to redesign these adaptive controllers online with the new conservative upper bound. Furthermore, the system performance of these adaptive controllers in the presence of large system uncertainties may not be satisfactory as well, since both controllers converge to standard adaptive controllers as this upper bound on the unknown constant weights becomes arbitrarily large (see Section 2.1.2 in Ref. 14 and Section 4 in Ref. 15).

In this paper, we introduce a novel command governor architecture for adaptive stabilization and command following. Specifically, the proposed command governor is a linear dynamical system which adjusts the trajectories of a given command in order to follow an ideal reference system both in transient time and steady-state without resorting to high-gain learning rates in the adaptation law and without requiring a conservative upper bound on the unknown constant weights. It is shown that by properly choosing the design parameter of the command governor, the controlled nonlinear uncertain dynamical system approximates a Hurwitz linear time-invariant dynamical system with \mathcal{L}_∞ input-output signals. This provides a systematic framework for verification and validation of adaptive control systems and the proposed command governor can be used in a complimentary way with many other approaches to adaptive control. The proposed architecture is illustrated on a model of wing rock dynamics.

The notation used in this paper is fairly standard. Specifically, \mathbb{R} denotes the set of real numbers, \mathbb{R}^n denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n \times m}$ denotes the set of $n \times m$ real matrices, \mathbb{R}_+ (resp., $\overline{\mathbb{R}}_+$) denotes the set of positive (resp., nonnegative-definite) real numbers, $\mathbb{R}_+^{n \times n}$ (resp., $\overline{\mathbb{R}}_+^{n \times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}^{n \times n}$ denotes the set of $n \times n$ symmetric real matrices, $\mathbb{D}^{n \times n}$ denotes the $n \times n$ real matrices with diagonal scalar entries, $(\cdot)^T$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, $(\cdot)^+$ denotes the Moore-Penrose generalized inverse, and “ \triangleq ” denotes equality by definition. In addition, we write $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A , $\sigma_{\min}(A)$ (resp., $\sigma_{\max}(A)$) for the minimum (resp., maximum) singular value of the Hermitian matrix A , $\det(A)$ for the determinant of the Hermitian matrix A , $\text{tr}(\cdot)$ for the trace operator, $\text{vec}(\cdot)$ for the column stacking operator, $\mathcal{R}(\cdot)$ for the range space, A^L for the left inverse $(A^T A)^+ A^T$ of $A \in \mathbb{R}^{n \times m}$, P_A for the projection matrix AA^L of $A \in \mathbb{R}^{n \times m}$, $\|\cdot\|_2$ for the Euclidian norm, $\|\cdot\|_\infty$ for the infinity norm, and $\|\cdot\|_F$ for the Frobenius matrix norm. Furthermore, for a signal $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ defined for all $t \geq 0$, the truncated \mathcal{L}_∞ norm and the \mathcal{L}_∞ norm¹⁶ are defined as $\|x_\tau(t)\|_{\mathcal{L}_\infty} \triangleq \max_{1 \leq i \leq n} (\sup_{0 \leq t \leq \tau} |x_i(t)|)$ and $\|x(t)\|_{\mathcal{L}_\infty} \triangleq \max_{1 \leq i \leq n} (\sup_{t \geq 0} |x_i(t)|)$, respectively.

II. Problem Formulation

We begin by presenting a simple formulation of the adaptive control problem without loss of generality. Specifically, consider the nonlinear uncertain dynamical system given by

$$\dot{x}(t) = Ax(t) + B[u(t) + \delta(x(t))], \quad x(0) = x_0, \quad t \in \overline{\mathbb{R}}_+, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector available for feedback, $u(t) \in \mathbb{R}^m$ is the control input restricted to the class of admissible controls consisting of measurable functions, $\delta: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is an *uncertainty*, $A \in \mathbb{R}^{n \times n}$ is a known system matrix, and $B \in \mathbb{R}^{n \times m}$ is a known control input matrix such that $\det(B^T B) \neq 0$ and the pair (A, B) is controllable.

Assumption 1. The uncertainty in (1) is parameterized as

$$\delta(x) = W^T \sigma(x), \quad x \in \mathbb{R}^n, \quad (2)$$

where $W \in \mathbb{R}^{s \times m}$ is an *unknown* weight matrix and $\sigma : \mathbb{R}^n \rightarrow \mathbb{R}^s$ is a known basis function of the form $\sigma(x) = [\sigma_1(x), \sigma_2(x), \dots, \sigma_s(x)]^T$.

Next, consider the ideal reference system capturing a desired closed-loop dynamical system performance given by

$$\dot{x}_r(t) = A_r x_r(t) + B_r c(t), \quad x_r(0) = x_{r0}, \quad t \in \overline{\mathbb{R}}_+, \quad (3)$$

where $x_r(t) \in \mathbb{R}^n$ is the reference state vector, $c(t) \in \mathbb{R}^m$ is a bounded command for tracking (or $c(t) \equiv 0$ for stabilization), $A_r \in \mathbb{R}^{n \times n}$ is the Hurwitz reference system matrix, and $B_r \in \mathbb{R}^{n \times m}$ is the command input matrix. The objective of the adaptive control problem is to construct a feedback control law $u(t)$ such that the state vector $x(t)$ asymptotically follows the reference state vector $x_r(t)$ subject to Assumption 1.

For the purpose of solving the adaptive control problem, consider the feedback control law given by

$$u(t) = u_n(t) + u_a(t), \quad (4)$$

where $u_n(t)$ and $u_a(t)$ are the nominal feedback control law and the adaptive feedback control law, respectively. Let the nominal feedback control law be given by

$$u_n(t) = K_1 x(t) + K_2 c(t), \quad (5)$$

where $K_1 \in \mathbb{R}^{m \times n}$ and $K_2 \in \mathbb{R}^{m \times m}$ are the nominal feedback and the nominal feedforward gains, respectively, such that $A_r = A + BK_1$, $B_r = BK_2$, and $\det(K_2) \neq 0$ hold. Now, using (4) and (5) in (1) with Assumption 1 yields

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B[K_1 x(t) + K_2 c(t) + u_a(t) + \delta(x(t))] \\ &= [A + BK_1]x(t) + BK_2 c(t) + B[u_a(t) + W^T \sigma(x(t))] \\ &= A_r x(t) + B_r c(t) + B[u_a(t) + W^T \sigma(x(t))]. \end{aligned} \quad (6)$$

Next, let the adaptive feedback control law be given by

$$u_a(t) = -\hat{W}^T(t) \sigma(x(t)), \quad (7)$$

where $\hat{W}(t) \in \mathbb{R}^{s \times m}$ is the estimate of W satisfying the weight update law

$$\dot{\hat{W}}(t) = \Gamma \sigma(x(t)) e^T(t) P B, \quad \hat{W}(0) = \hat{W}_0, \quad t \in \overline{\mathbb{R}}_+, \quad (8)$$

where $\Gamma \in \mathbb{R}_+^{s \times s} \cap \mathbb{S}^{s \times s}$ is the learning rate matrix, $e(t) \triangleq x(t) - x_r(t)$ is the system error state vector, and $P \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ is a solution of the Lyapunov equation

$$0 = A_r^T P + P A_r + R, \quad (9)$$

where $R \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ can be viewed as an additional learning rate. Since A_r is Hurwitz, it follows from converse Lyapunov theory¹⁷ that there exists a unique P satisfying (9) for a given R .

Now, using (7) in (6) yields

$$\begin{aligned}\dot{x}(t) &= A_r x(t) + B_r c(t) + B[-\hat{W}^T(t)\sigma(x(t)) + W^T\sigma(x(t))] \\ &= A_r x(t) + B_r c(t) - B\tilde{W}^T(t)\sigma(x(t)),\end{aligned}\quad (10)$$

and the system error dynamics is given by using (3) and (10) as

$$\dot{e}(t) = A_r e(t) - B\tilde{W}^T(t)\sigma(x(t)), \quad e(0) = e_0, \quad t \in \bar{\mathbb{R}}_+, \quad (11)$$

where $\tilde{W}(t) \triangleq \hat{W}(t) - W$ and $e_0 \triangleq x_0 - x_{r0}$.

The weight update law given by (8) can be derived using Lyapunov analysis by considering the Lyapunov function candidate

$$\mathcal{V}(e, \tilde{W}) = e^T P e + \text{tr } \tilde{W}^T \Gamma^{-1} \tilde{W}. \quad (12)$$

Note that $\mathcal{V}(0, 0) = 0$ and $\mathcal{V}(e, \tilde{W}) > 0$ for all $(e, \tilde{W}) \neq (0, 0, 0)$. Now, differentiating (12) yields

$$\dot{\mathcal{V}}(e(t), \tilde{W}(t)) = -e^T(t) R e(t) - 2e^T(t) P B \tilde{W}^T(t) \sigma(x(t)) + 2\text{tr } \tilde{W}^T(t) \Gamma^{-1} \dot{\tilde{W}}(t), \quad (13)$$

where using (8) in (13) results in

$$\dot{\mathcal{V}}(e(t), \tilde{W}(t)) = -e^T(t) R e(t) \leq 0, \quad t \in \bar{\mathbb{R}}_+, \quad (14)$$

which guarantees that the system error state vector $e(t)$ and the weight error $\tilde{W}(t)$ are Lyapunov stable, and hence, are bounded for all $t \in \bar{\mathbb{R}}_+$. Since $\sigma(x(t))$ is bounded for all $t \in \bar{\mathbb{R}}_+$, it follows from (11) that $\dot{e}(t)$ is bounded, and hence, $\ddot{\mathcal{V}}(e(t), \tilde{W}(t))$ is bounded for all $t \in \bar{\mathbb{R}}_+$. Now, it follows from Barbalat's lemma¹⁶ that

$$\lim_{t \rightarrow \infty} \dot{\mathcal{V}}(e(t), \tilde{W}(t)) = 0, \quad (15)$$

which consequently shows that $e(t) \rightarrow 0$ as $t \rightarrow \infty$.

For the case when the nonlinear uncertain dynamical system given by (1) includes bounded exogenous disturbances, measurement noise, and/or the uncertainty in (1) cannot be perfectly parameterized, then Assumption 1 can be relaxed by considering

$$\delta(t, x) = W(t)^T \sigma(x) + \varepsilon(t, x), \quad x \in \mathcal{D}_x, \quad (16)$$

where $W(t) \in \mathbb{R}^{s \times m}$ is an *unknown* time-varying weight matrix satisfying $\|W(t)\|_F \leq w$ and $\|\dot{W}(t)\|_F \leq \dot{w}$ with $w \in \mathbb{R}_+$ and $\dot{w} \in \mathbb{R}_+$ being unknown scalars, $\sigma : \mathcal{D}_x \rightarrow \mathbb{R}^s$ is a known basis function of the form $\sigma(x) = [1, \sigma_1(x), \sigma_2(x), \dots, \sigma_{s-1}(x)]^T$, $\varepsilon : \bar{\mathbb{R}}_+ \times \mathcal{D}_x \rightarrow \mathbb{R}^m$ is the system modeling error satisfying $\|\varepsilon(t, x)\|_2 \leq \epsilon$ with $\epsilon \in \mathbb{R}_+$ being an unknown scalar, and \mathcal{D}_x is a compact subset of \mathbb{R}^n . In this case, the weight update law given by (8) can be replaced by

$$\dot{\hat{W}}(t) = \Gamma \text{Proj}[\hat{W}(t), \sigma(x(t))e^T(t)PB], \quad \hat{W}(0) = \hat{W}_0, \quad t \in \bar{\mathbb{R}}_+, \quad (17)$$

with $\Gamma = \gamma I_s$, $\gamma \in \mathbb{R}_+$, to guarantee the uniform boundedness of the system error state vector $e(t)$ and the weight error $\tilde{W}(t)$, where Proj denotes the projection operator¹⁸.

Remark 1. Even though the above analysis shows that the state vector $x(t)$ asymptotically converges

to the reference state vector $x_r(t)$ (in steady-state), $x(t)$ can be far different from $x_r(t)$ in transient time. High-gain learning rates can be used in (8) in order to achieve fast adaptation and to minimize the distance between $x(t)$ and $x_r(t)$ in transient time. However, update law with high-gain learning rates possibly yield to high-frequency oscillations especially during the transient system response resulting in system instability for real applications⁵⁻⁷.

III. Command Governor

This section introduces a novel command governor architecture to the adaptive control problem described in the previous section. Specifically, let the command $c(t)$ be given by

$$c(t) = c_d(t) + G\eta(t), \quad (18)$$

where $c_d(t) \in \mathbb{R}^m$ is a bounded command for tracking (or $c_d(t) \equiv 0$ for stabilization) and $G\eta(t) \in \mathbb{R}^m$ is the command governor signal with $G \in \mathbb{R}^{m \times n}$ being the matrix defined by

$$G \triangleq K_2^{-1}B^L = K_2^{-1}(B^T B)^{-1}B^T, \quad (19)$$

and $\eta(t) \in \mathbb{R}^n$ being the command governor output generated by

$$\dot{\xi}(t) = -\lambda\xi(t) + \lambda e(t), \quad \xi(0) = 0, \quad t \in \overline{\mathbb{R}}_+, \quad (20)$$

$$\eta(t) = \lambda\xi(t) + (A_r - \lambda I_n)e(t), \quad (21)$$

where $\xi(t) \in \mathbb{R}^n$ is the command governor state vector and $\lambda \in \mathbb{R}_+$ is the command governor gain.

It is important to note that the command $c(t)$ was assumed to be bounded in the previous section, since it was representing a bounded command for tracking (i.e., $c(t) = c_d(t)$ and $G\eta(t) \equiv 0$). In this section, however, $c(t)$ used in the reference system (3) and the nominal feedback control law (5) consists of the bounded command for tracking $c_d(t)$ and the command governor signal $G\eta(t)$. Hence, the Lyapunov analysis presented in the previous section does not hold in this case, since we cannot *a priori* assume the boundedness of the command $c(t)$. The stability analysis of the proposed command governor-based adaptive control architecture is given in the next section.

Finally, note that the addition of the command governor signal $G\eta(t)$ to the command for tracking $c_d(t)$ in (18) does not change the system error dynamics given by (11), and hence, the weight update law (8) for $\hat{W}(t)$ remains the same. In this case, however, (3) and (10) change to

$$\dot{x}_r(t) = A_r x_r(t) + B_r c_d(t) + P_B \eta(t), \quad (22)$$

$$\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_B \eta(t) - B\tilde{W}^T(t)\sigma(x(t)), \quad (23)$$

where $P_B = BB^L = B(B^T B)^{-1}B^T$. Even though this implies the modification of the reference system with the signal $P_B \eta(t)$, as we see later, by properly choosing the command governor gain λ it is possible to suppress the effect of $B\tilde{W}^T(t)\sigma(x(t))$ in (23) through $P_B \eta(t)$, and hence, it is possible to track the ideal reference system given by (3) (with $c(t)$ replaced by $c_d(t)$).

A block diagram showing the proposed command governor-based adaptive control architecture is given in Figure 1.

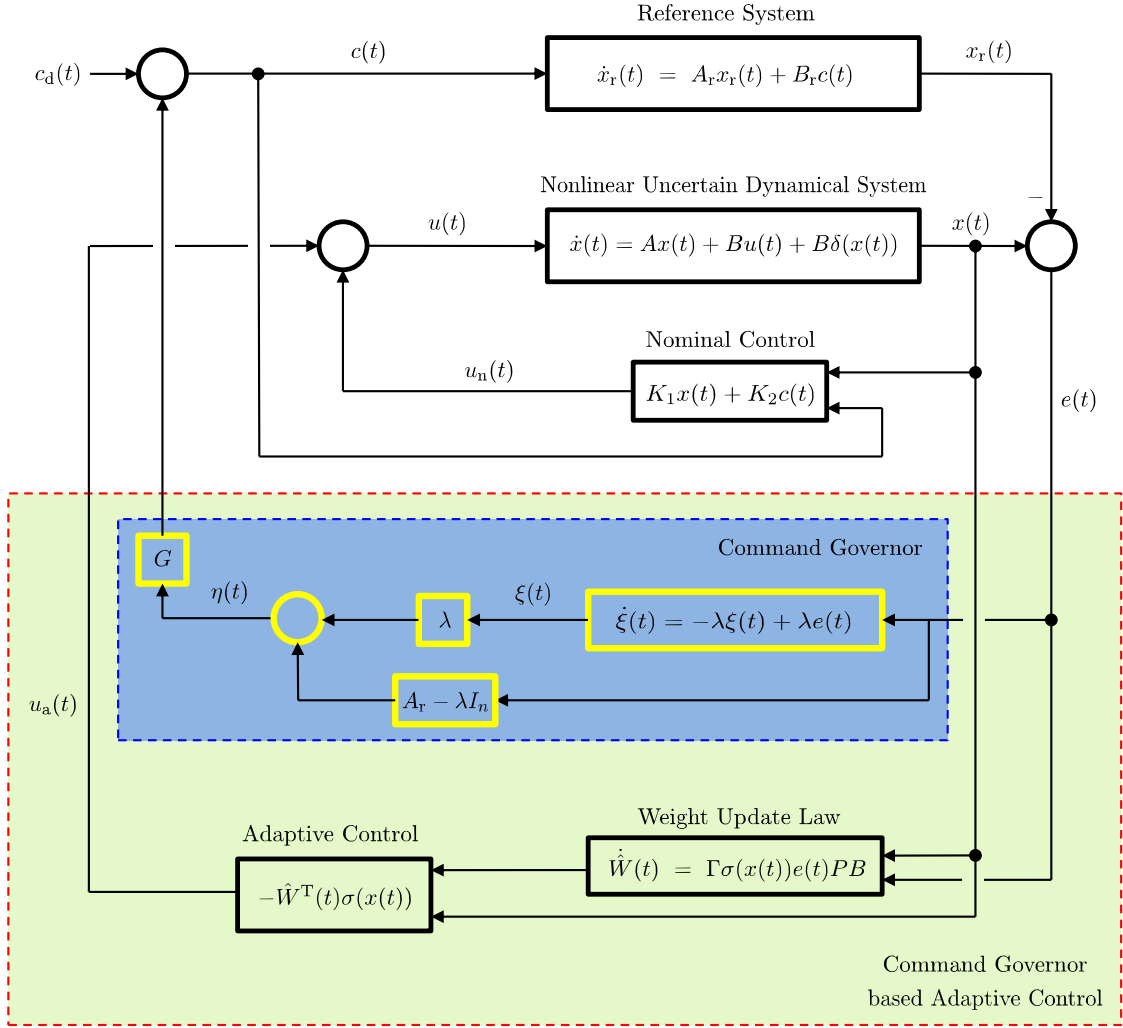


Figure 1. Visualization of the proposed command governor based adaptive control architecture.

IV. Stability Analysis

This section presents the stability analysis for the command governor-based adaptive control architecture. For this purpose, the system error dynamics and the weight update error dynamics can be given by, respectively, (11),

$$\dot{\tilde{W}}(t) = \Gamma \sigma(x(t)) e^T(t) P B, \quad \tilde{W}(0) = \tilde{W}_0, \quad t \in \overline{\mathbb{R}}_+, \quad (24)$$

where $\tilde{W}_0 \triangleq \hat{W}_0 - W$. For the following theorem presenting the main result of this section, we assume that the choice of R in (9) satisfies $R = R_0 + \gamma \lambda I_n$, where $R_0 \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ and $\gamma \in \mathbb{R}_+$ is an arbitrary constant that can be chosen to be sufficiently small. Therefore, this assumption is technical and does not place restrictions on the selection of R .

Theorem 1. Consider the nonlinear uncertain dynamical system given by (1) subject to Assumption 1, the reference system given by (3) with the command given by (18), the feedback control law given by (4) along with (5), (7), and (8), and the command governor given by (20) and (21). Then, the solution

$(e(t), \tilde{W}(t), \xi(t))$ of the closed-loop dynamical system given by (11), (20), and (24) is Lyapunov stable for all $(e_0, \tilde{W}_0, 0) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^n$ and $t \in \overline{\mathbb{R}}_+$, and $\lim_{t \rightarrow \infty} e(t) = 0$, $\lim_{t \rightarrow \infty} \xi(t) = 0$, $\lim_{t \rightarrow \infty} \eta(t) = 0$, and $\lim_{t \rightarrow \infty} (c(t) - c_d(t)) = 0$. For $t \in \overline{\mathbb{R}}_+$, in addition, the system error state vector, the weight update error dynamics, and the command governor dynamics satisfy the transient performance bounds given by

$$\|e(t)\|_{\mathcal{L}_\infty} \leq \left[(\lambda_{\max}(P)\|e_0\|_2^2 + \|\Gamma^{-1}\|_F \|\tilde{W}_0\|_F^2) / \lambda_{\min}(P) \right]^{\frac{1}{2}}, \quad (25)$$

$$\|\text{vec}(\tilde{W}(t))\|_{\mathcal{L}_\infty} \leq \left[(\lambda_{\max}(P)\|e_0\|_2^2 + \|\Gamma^{-1}\|_F \|\tilde{W}_0\|_F^2) / \lambda_{\min}(\Gamma^{-1}) \right]^{\frac{1}{2}}, \quad (26)$$

$$\|\xi(t)\|_{\mathcal{L}_\infty} \leq \left[(\lambda_{\max}(P)\|e_0\|_2^2 + \|\Gamma^{-1}\|_F \|\tilde{W}_0\|_F^2) / \gamma \right]^{\frac{1}{2}}. \quad (27)$$

Proof. To show Lyapunov stability of the closed-loop dynamical system (11), (20), and (24), consider the Lyapunov function candidate

$$\mathcal{V}(e, \tilde{W}, \xi) = e^T P e + \text{tr } \tilde{W}^T \Gamma^{-1} \tilde{W} + \gamma \xi^T \xi, \quad (28)$$

and note that

$$\mathcal{V}(0, 0, 0) = 0. \quad (29)$$

Since $P \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$, $\Gamma \in \mathbb{R}_+^{s \times s} \cap \mathbb{S}^{s \times s}$, and $\gamma \in \mathbb{R}_+$,

$$\mathcal{V}(e, \tilde{W}, \xi) > 0, \quad (e, \tilde{W}, \xi) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^n \setminus (0, 0, 0). \quad (30)$$

In addition, $\mathcal{V}(e, \tilde{W}, \xi)$ is radially unbounded.

Differentiating (28) along the closed-loop trajectories of (11), (20), and (24) yields

$$\dot{\mathcal{V}}(e(t), \tilde{W}(t), \xi(t)) = -e^T(t) R e(t) - 2\gamma \lambda \xi^T(t) \xi(t) + 2\gamma \lambda \xi^T(t) e(t). \quad (31)$$

Since $R = R_0 + \gamma \Lambda I_n$ holds, then (31) can be rewritten as

$$\dot{\mathcal{V}}(e(t), \tilde{W}(t), \xi(t)) = -e^T(t) R_0 e(t) - \gamma \lambda \xi^T(t) \xi(t) - \gamma \lambda (e^T(t) e(t) - 2\xi^T(t) e(t) + \xi^T(t) \xi(t)), \quad (32)$$

which yields

$$\dot{\mathcal{V}}(e(t), \tilde{W}(t), \xi(t)) = -e^T(t) R_0 e(t) - \gamma \lambda \xi^T(t) \xi(t) - \gamma \lambda \|e(t) - \xi(t)\|_2^2 \leq 0, \quad t \in \overline{\mathbb{R}}_+. \quad (33)$$

Hence, the closed-loop dynamical system given by (11), (20), and (24) is Lyapunov stable for all $(e_0, \tilde{W}_0, 0) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^n$ and $t \in \overline{\mathbb{R}}_+$.

Next, since $\sigma(x(t))$ is bounded for all $t \in \overline{\mathbb{R}}_+$, it follows from (11) that $\dot{e}(t)$ is bounded. Furthermore, since $\dot{\xi}(t)$ is also bounded, then $\ddot{\mathcal{V}}(e(t), \tilde{W}(t), \xi(t))$ is bounded for all $t \in \overline{\mathbb{R}}_+$. Now, it follows from Barbalat's lemma¹⁶ that

$$\lim_{t \rightarrow \infty} \dot{\mathcal{V}}(e(t), \tilde{W}(t), \xi(t)) = 0, \quad (34)$$

which consequently shows that

$$\lim_{t \rightarrow \infty} e(t) = 0, \quad (35)$$

$$\lim_{t \rightarrow \infty} \xi(t) = 0. \quad (36)$$

As a direct consequence,

$$\lim_{t \rightarrow \infty} \eta(t) = 0, \quad (37)$$

$$\lim_{t \rightarrow \infty} (c(t) - c_d(t)) = 0, \quad (38)$$

hold.

Finally, since $\dot{\mathcal{V}}(e(t), \tilde{W}, \xi(t)) \leq 0$ for $t \in \overline{\mathbb{R}}_+$, this implies that

$$\mathcal{V}(e(t), \tilde{W}, \xi(t)) \leq \mathcal{V}(e_0, \tilde{W}_0, 0). \quad (39)$$

Using the inequalities

$$\mathcal{V}(e(t), \tilde{W}, \xi(t)) \geq \lambda_{\min}(P) \|e(t)\|_2^2, \quad (40)$$

$$\mathcal{V}(e_0, \tilde{W}_0, 0) \leq \lambda_{\max}(P) \|e_0\|_2^2 + \|\Gamma^{-1}\|_F \|\tilde{W}_0\|_F^2, \quad (41)$$

in (39) results in

$$\|e(t)\|_2 \leq \left[(\lambda_{\max}(P) \|e_0\|_2^2 + \|\Gamma^{-1}\|_F \|\tilde{W}_0\|_F^2) / \lambda_{\min}(P) \right]^{\frac{1}{2}}. \quad (42)$$

Since $\|\cdot\|_\infty \leq \|\cdot\|_2$, and this bound is uniform, then (42) yields

$$\|e_\tau(t)\|_{\mathcal{L}_\infty} \leq \left[(\lambda_{\max}(P) \|e_0\|_2^2 + \|\Gamma^{-1}\|_F \|\tilde{W}_0\|_F^2) / \lambda_{\min}(P) \right]^{\frac{1}{2}}, \quad (43)$$

and hence, (25) is a direct consequence of (43) due to the fact that (43) holds uniformly in τ . Similarly, using the inequalities

$$\mathcal{V}(e(t), \tilde{W}, \xi(t)) \geq \lambda_{\min}(\Gamma^{-1}) \|\tilde{W}(t)\|_F^2, \quad (44)$$

$$\mathcal{V}(e(t), \tilde{W}, \xi(t)) \geq \gamma \|\xi(t)\|_2^2, \quad (45)$$

and (41) in (39), noting $\|\tilde{W}(t)\|_F^2 = \|\text{vec}(\tilde{W}(t))\|_2^2$, and repeating the above analysis yields (26) and (27). This completes the proof. \blacksquare

Theorem 1 highlights not only the stability but also the transient and steady-state performance guarantees of the closed-loop dynamical system given by (11), (20), and (24). A similar result can be obtained for the case when the nonlinear uncertain dynamical system given by (1) includes bounded exogenous disturbances, measurement noise, and/or the uncertainty in (1) cannot be perfectly parameterized by using the weight update law (17) instead of (8) (see, for example, Theorem 4.1 of Ref. 15).

Note that, Theorem 1 shows that $\lim_{t \rightarrow \infty} \eta(t) = 0$, and hence, the modified reference system in (22) converges to the ideal reference system

$$\dot{x}_r(t) = A_r x_r(t) + B_r c_d(t), \quad (46)$$

as $t \rightarrow \infty$.

Remark 2. Consider the case when $e_0 = 0$. Then, (25) implies

$$\|e(t)\|_{\mathcal{L}_\infty} \leq \|\tilde{W}_0\|_F \left[\frac{\|\Gamma^{-1}\|_F}{\lambda_{\min}(P)} \right]^{\frac{1}{2}}. \quad (47)$$

Note that, the distance between $x(t)$ and $x_r(t)$ can be made arbitrarily small in transient time if high-gain learning rates can be used in (8). However, as we see in the next section, by properly choosing the command governor gain λ it is possible to follow the ideal reference system (46) in transient time without requiring high-gain learning rates and assuming $e_0 = 0$, that is, the effect of $B\tilde{W}^T(t)\sigma(x(t))$ in (23) can be suppressed through $P_B\eta(t)$ as noted earlier, and hence, (23) approximates a Hurwitz linear time-invariant dynamical system having the form of the ideal reference system.

V. Shaping the Transient System Response

This section shows that the controlled nonlinear uncertain dynamical system (1) *approximates* a Hurwitz linear time-invariant dynamical system having the form of the ideal reference system (46) with \mathcal{L}_∞ input-output signals for a high command governor gain λ . The following theorem presents the main result of this section.

Theorem 2. Consider the nonlinear uncertain dynamical system given by (1) subject to Assumption 1, the reference system given by (3) with the command given by (18), the feedback control law given by (4) along with (5), (7), and (8), and the command governor given by (20) and (21). Then, for a sufficiently high command governor gain λ , (1) approximates

$$\dot{x}(t) = A_r x(t) + B_r c_d(t), \quad (48)$$

$$z(t) = C_r x(t), \quad (49)$$

where $A_r = A + BK_1$ is Hurwitz, $B_r = BK_2$, $C_r = I_n$, $c_d(\cdot) \in \mathcal{L}_\infty$, and $z(\cdot) \in \mathcal{L}_\infty$.

Proof. Note that the command governor system given by (20) and (21) can be written in Laplace domain as

$$\mathcal{G}_{\eta \rightarrow e}(s) = \lambda(s + \lambda)^{-1} \lambda I_n + (A_r - \lambda I_n), \quad (50)$$

where s denotes the Laplace variable and $\mathcal{G}_{\eta \rightarrow e}(s) \triangleq e(s)/\eta(s)$. We can rewrite (50) as

$$\mathcal{G}_{g \rightarrow e}(s) = \frac{\lambda^2}{s + \lambda} I_n + \frac{s + \lambda}{s + \lambda} (A_r - \lambda I_n) = \frac{(s + \lambda)A_r - \lambda s I_n}{s + \lambda} = -\frac{s}{\frac{1}{\lambda}s + 1} I_n + A_r, \quad (51)$$

where (51), for a sufficiently high λ , implies $\mathcal{G}_{\eta \rightarrow e}(s) \approx -sI_n + A_r$, or equivalently,

$$\eta(t) \approx -\dot{e}(t) + A_r e(t). \quad (52)$$

As a direct consequence of Theorem 1, $e(\cdot) \in \mathcal{L}_\infty$ and $\dot{e}(\cdot) \in \mathcal{L}_\infty$ in (52). To prove that (1) approximates

(48) and (49) for a sufficiently high λ , we can rewrite (23) as

$$\begin{aligned}\dot{x}(t) &= A_r x(t) + B_r c_d(t) + P_B \eta(t) - B \tilde{W}^T(t) \sigma(x(t)) \\ &= A_r x(t) + B_r c_d(t) + B \left[(B^T B)^{-1} B^T \eta(t) - \tilde{W}^T(t) \sigma(x(t)) \right].\end{aligned}\quad (53)$$

Furthermore, note that (11) can be written as

$$-B \tilde{W}^T(t) \sigma(x(t)) = \dot{e}(t) - A_r e(t) \quad \Rightarrow \quad -\tilde{W}^T(t) \sigma(x(t)) = (B^T B)^{-1} B^T (\dot{e}(t) - A_r e(t)). \quad (54)$$

Now, using (54) in (53) yields

$$\begin{aligned}\dot{x}(t) &= A_r x(t) + B_r c_d(t) + B (B^T B)^{-1} B^T \left[\eta(t) + (\dot{e}(t) - A_r e(t)) \right] \\ &= A_r x(t) + B_r c_d(t) + P_B \left[\eta(t) + (\dot{e}(t) - A_r e(t)) \right],\end{aligned}\quad (55)$$

and hence, for a sufficiently high λ , (55) along with (52) approximates (48) and (49). ■

Remark 3. Since Theorem 2 shows that the controlled nonlinear uncertain dynamical system (1) approximates a Hurwitz linear time-invariant dynamical system capturing a desired closed-loop dynamical system performance for a high command governor gain λ , the learning rate matrix Γ for (8) can be chosen to be sufficiently small. However, it should be also noted from (52) that a *very* high command governor gain λ can amplify the measurement noise possibly exists in the state error vector of a real physical system. Hence, for real applications, λ should be high enough (and hence, Γ should be small enough) to *approximately* guarantee that (1) behaves as (46) in transient system response, but should *not* be *very* large in order to avoid measurement noise amplification.

VI. Illustrative Example

In order to illustrate the proposed command governor-based adaptive control architecture, consider the nonlinear dynamical system representing a controlled wing rock dynamics model given by

$$\dot{x}_1(t) = x_2(t), \quad x_1(0) = 0, \quad t \in \mathbb{R}_+, \quad (56)$$

$$\dot{x}_2(t) = u(t) + \delta(x(t)), \quad x_2(0) = 0, \quad t \in \mathbb{R}_+, \quad (57)$$

where x_1 represents the roll angle in radians and x_2 represents the roll rate in radians per second. In (57), $\delta(x)$ representing an uncertainty of the form

$$\delta(x) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 |x_1| x_2 + \alpha_4 |x_2| x_2 + \alpha_5 x_1^3, \quad (58)$$

where α_i , $i = 1, \dots, 5$, are unknown parameters that are derived from the aircraft aerodynamic coefficients¹⁹. For our numerical example, we set $\alpha_1 = 0.1414$, $\alpha_2 = 0.5504$, $\alpha_3 = -0.0624$, $\alpha_4 = 0.0095$, and $\alpha_5 = 0.0215$. We choose $K_1 = [-0.16, -0.57]$ and $K_2 = 0.16$ for the nominal controller design that yields to a reference system with a natural frequency of $\omega_n = 0.40$ rad/s and a damping ratio $\zeta = 0.707$. For the standard adaptive controller design (i.e., $c(t) = c_d(t)$), $\sigma(x) = [x_1, x_2, |x_1| x_2, |x_2| x_2, x_1^3]^T$ is chosen as the basis function and we set $R = I_2$. For the proposed command governor based-adaptive control architecture (see Figure 1), we use the same basis function and R . Here, our aim is to track a given roll angle command $c_d(t)$.

Figures 2–5 show the performance of the standard adaptive controller for two different learning rate

matrix choices and for two different roll angle commands. As these figures imply, a satisfactory closed-loop dynamical system response could not be achieved with the standard adaptive controller. Specifically, low-gain learning rate matrix $\Gamma = 0.01I_5$ in Figures 2 and 3 is not sufficient to achieve fast adaptation. Furthermore, high-gain learning rate matrix $\Gamma = 100I_5$ in Figures 4 and 5 leads to high-frequency oscillations. It is also important to note that the system performance changes significantly as the amplitude of the roll angle command changes. Therefore, it is not possible to obtain a predictable command tracking performance especially during the transient system response.

Figures 6–11 show the performance of the proposed command governor based-adaptive control architecture with a low-gain learning rate matrix $\Gamma = 0.01I_5$. In particular, we use $\lambda = 0$ in Figures 6 and 7. It can be observed from these figures that the closed-loop dynamical system response is better when they are compared with the figures associated with the standard adaptive controller.

In Figures 8 and 9, the command governor gain λ is increased from 0 to 0.5. As expected from the theory, the command governor shapes the transient system response better due to the increment in the command governor gain, and hence, the closed-loop dynamical system response depicted in these figures are much better than the ones with the standard adaptive controller. Furthermore, notice that the command tracking performances presented in these two figures are (almost) identical, even though the roll angle commands are different in amplitude.

Next, we further increased the command governor gain λ from 0.5 to 5 in Figures 10 and 11. For two different roll angle commands, we can able to obtain a satisfactory closed-loop dynamical system response which is clearly superior as compared to the ones with the standard adaptive controller. In order to show that the command tracking performances depicted in Figures 10 and 11 are identical to the desired closed-loop dynamical system performance, we also present Figure 12 showing the system performance of the nominal controller for the case when $\delta(x) \equiv 0$. Obviously, the command tracking performances presented in Figures 10, 11, and 12 are (almost) the same as expected from the presented theory. This illustrates the point that the controlled nonlinear dynamical system tracking performance behaves linearly (i.e., predictable).

It is of importance to note that for the roll angle command having a low amplitude (see Figures 6, 8, and 10), the adaptive control history is close to zero. However, for the roll angle command having a high amplitude (see Figures 7, 9, and 11), the adaptive control history is dominant when it is compared with the nominal control history. This result is expected since the proposed command governor shapes the transient response.

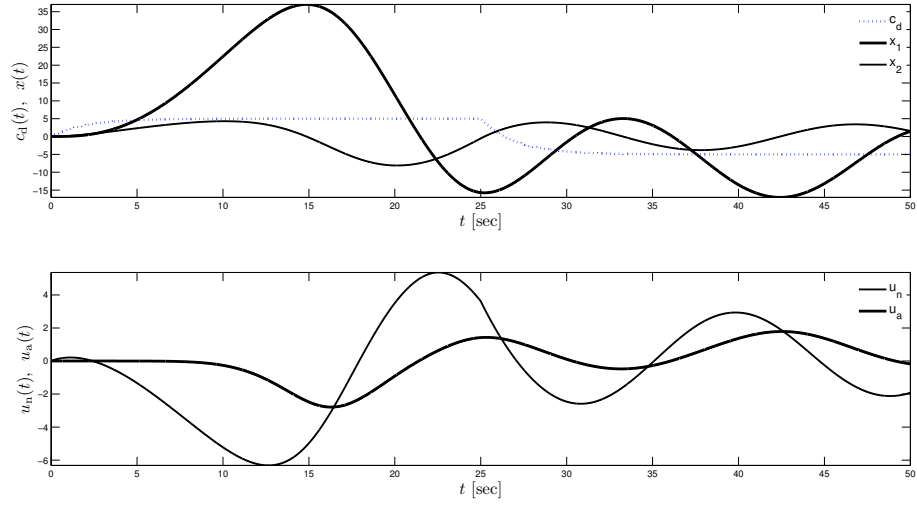


Figure 2. Standard adaptive control performance with $\Gamma = 0.01I_5$ for a given filtered square-wave roll angle command having a low amplitude ($c_d(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

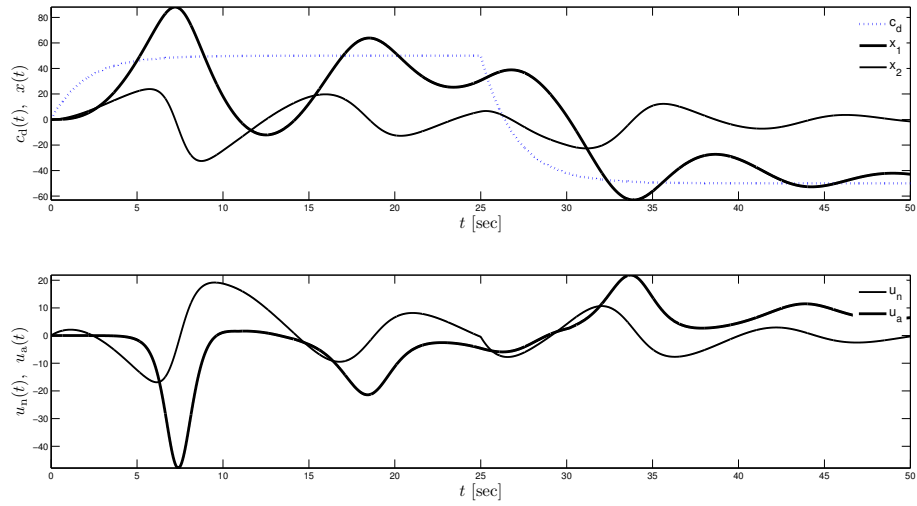


Figure 3. Standard adaptive control performance with $\Gamma = 0.01I_5$ for a given filtered square-wave roll angle command having a high amplitude ($c_d(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

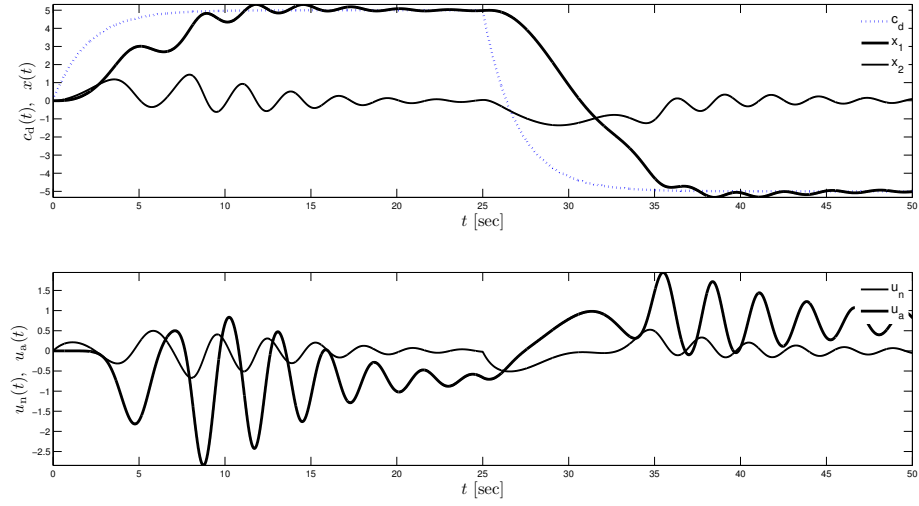


Figure 4. Standard adaptive control performance with $\Gamma = 100I_5$ for a given filtered square-wave roll angle command having a low amplitude ($c_d(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

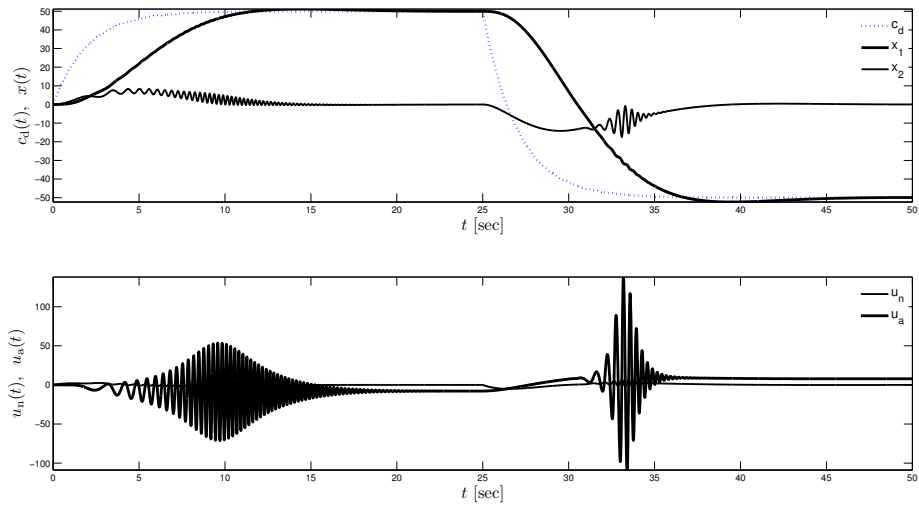


Figure 5. Standard adaptive control performance with $\Gamma = 100I_5$ for a given filtered square-wave roll angle command having a high amplitude ($c_d(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

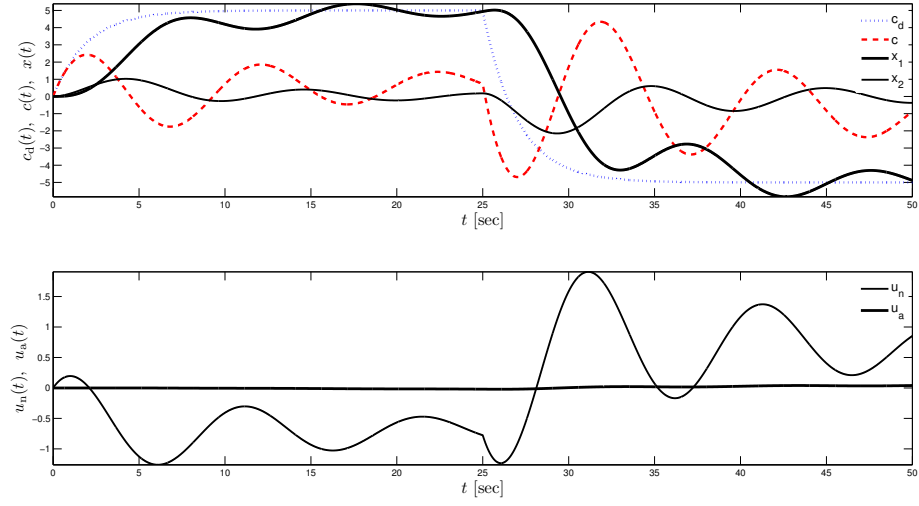


Figure 6. Command governor-based adaptive control performance with $\Gamma = 0.01I_5$ and $\lambda = 0$ for a given filtered square-wave roll angle command having a low amplitude ($c_d(t)$, $c(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

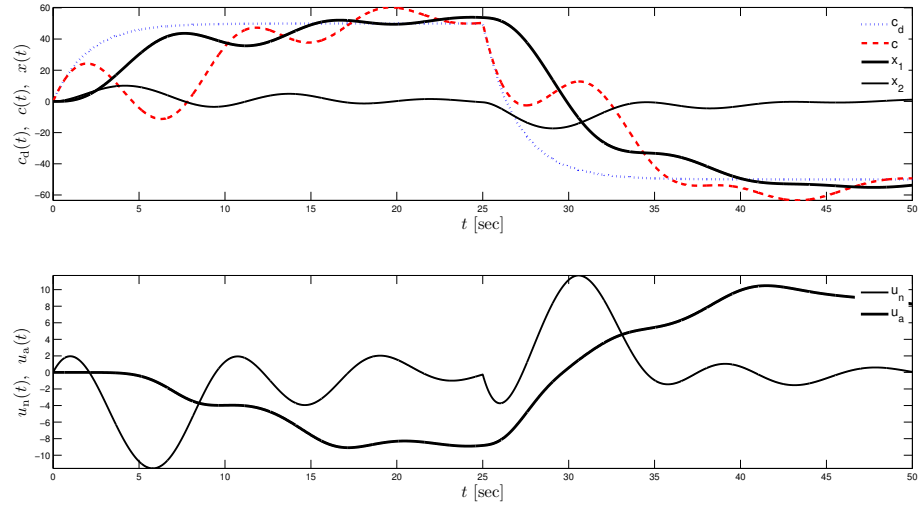


Figure 7. Command governor-based adaptive control performance with $\Gamma = 0.01I_5$ and $\lambda = 0$ for a given filtered square-wave roll angle command having a high amplitude ($c_d(t)$, $c(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

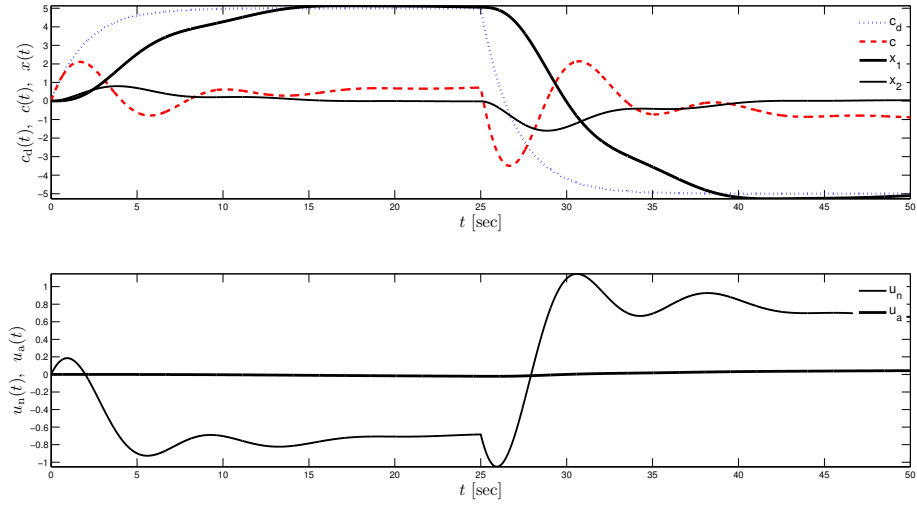


Figure 8. Command governor-based adaptive control performance with $\Gamma = 0.01I_5$ and $\lambda = 0.5$ for a given filtered square-wave roll angle command having a low amplitude ($c_d(t)$, $c(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

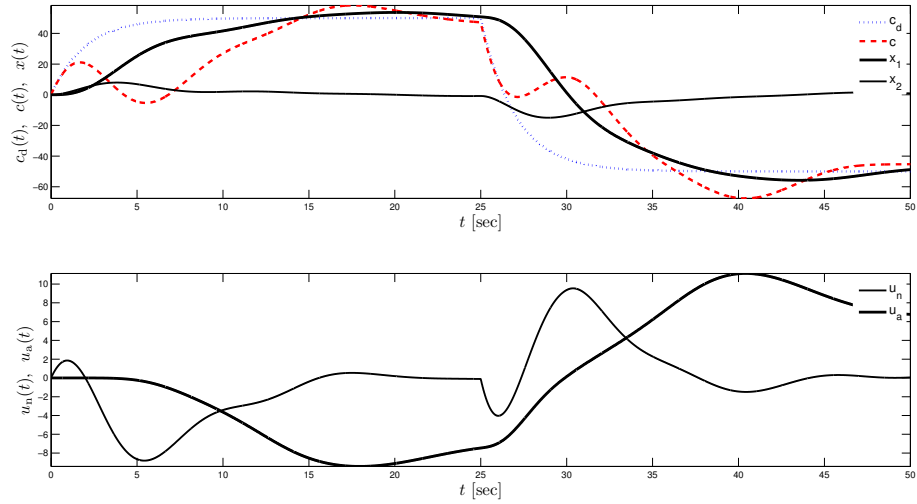


Figure 9. Command governor-based adaptive control performance with $\Gamma = 0.01I_5$ and $\lambda = 0.5$ for a given filtered square-wave roll angle command having a high amplitude ($c_d(t)$, $c(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

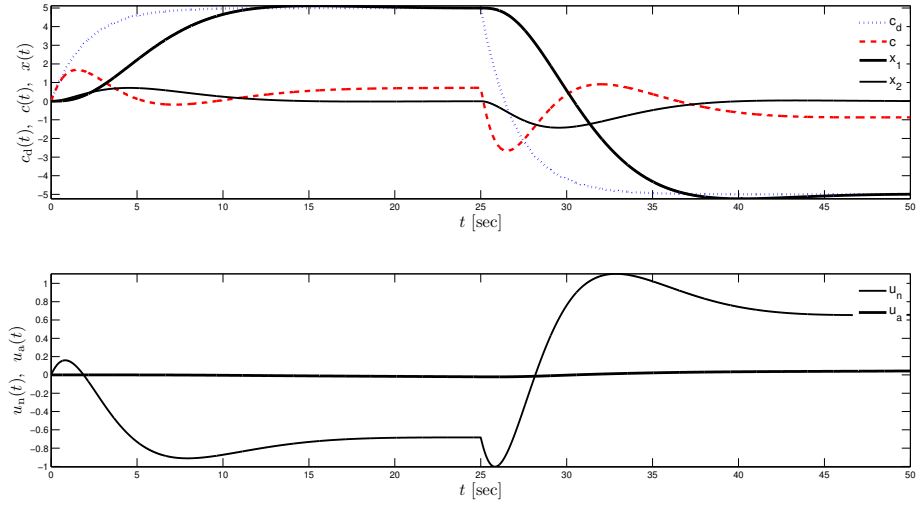


Figure 10. Command governor-based adaptive control performance with $\Gamma = 0.01I_5$ and $\lambda = 5$ for a given filtered square-wave roll angle command having a low amplitude ($c_d(t)$, $c(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

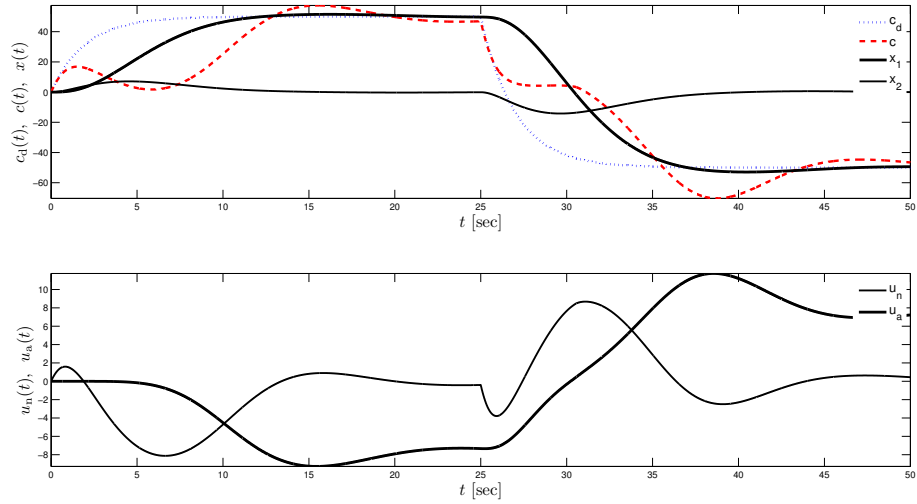


Figure 11. Command governor-based adaptive control performance with $\Gamma = 0.01I_5$ and $\lambda = 5$ for a given filtered square-wave roll angle command having a high amplitude ($c_d(t)$, $c(t)$, $x_1(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second).

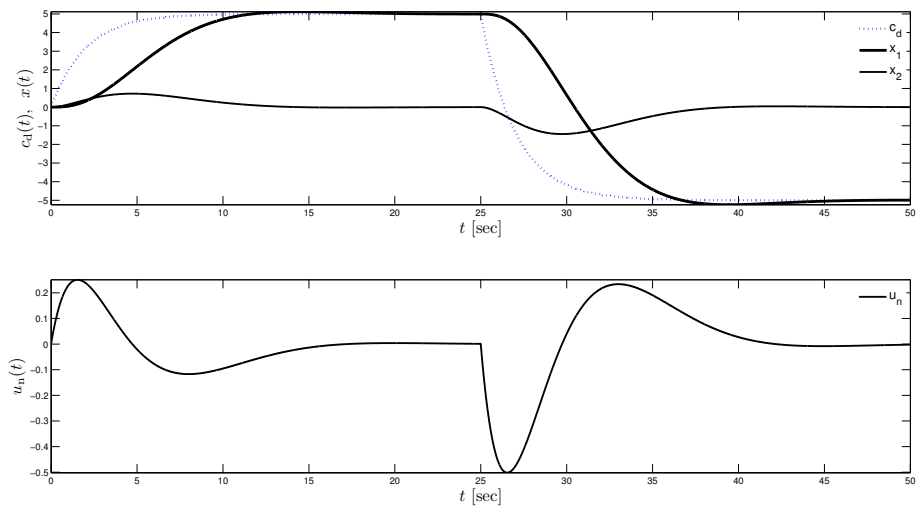


Figure 12. Nominal control performance for a given filtered square-wave roll angle command for the case $\delta(x) \equiv 0$ ($c_d(t)$, $x_1(t)$, and $u_n(t)$ in degrees, and $x_2(t)$ in degrees per second).

VII. Conclusion

In this paper, we present a novel command governor-based adaptive control architecture that allows fast adaptation using low-gain learning rates. Specifically, we first analyze the transient and steady-state performance guarantees of this framework. Then, we show that the controlled nonlinear uncertain dynamical system can approach a given ideal reference system both in transient time and steady-state by choosing the command governor gain. The numerical example on a model of wing rock dynamics illustrates the superior performance of the proposed architecture in comparison with a standard adaptive control law.

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