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$\mathcal{L}_1$ Adaptive Control Used in Path Following of Surface Ships

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Abstract: In this paper, a cascaded control structure and the $\mathcal{L}_1$ adaptive control law are used to deal with the path following problem of surface ships. The guidance-based strategy separates the geometric task from the ship motion control system. The whole control problem is decoupled into two subsystems, namely the outer-loop guidance subsystem and the inner-loop control subsystem. The outer-loop subsystem only concentrates on the geometrical part of the system and generates a desired yaw rate which guarantees the convergence of the ship to any proper given path. The desired yaw rate is then regarded as the reference output of the inner-loop control subsystem. Considering about the model uncertainty, the $\mathcal{L}_1$ adaptive control strategy is used to deal with the inner-loop control subsystem. The $\mathcal{L}_1$ adaptive method has a fast adaptation rate and robust control performance, which can ensure the uniformly bounded transient and asymptotic tracking performances. Besides, the $\mathcal{L}_1$ theory is relative simple and easy for industrial implementation. The results show the effectiveness of the control law to the path following problem of ships.

Key Words: Guidance-based, Path-following, $\mathcal{L}_1$ adaptive

1 Introduction

Steering a surface ship along any admissible predefined path is an important issue in ship control community [1–7]. Especially in recent years, with the high speed development of the oceanic exploration and offshore applications, industry increases the demands for more accurate and robust control methods for ship motion control to any proper curve path. Typically, this goal can be divided into two categories, namely the Path-Following (PF) problem and Trajectory-Tracking (TT) problem [8]. The main difference between them is that the PF problem only considers the positional requirement while the TT problem also considers the time requirement at the same time. The PF control strategy regards the geometric demands as the primary task, and takes the dynamics aspect as a secondary objective, negligible if necessary [9].

The so-called Serrent-Frenet frame (SF frame) is often used to describe the PF problem for ships [5, 10–12]. In SF frame, the original PF control problem is turned into a regulation problem, thus makes it very easy to deal with. However, as pointed out in [8], the result in Serrent-Frenet frame is only local. To avoid this, the velocity of the origin of the SF frame can be considered as an extra input of the system. It can update its velocity along the path more flexibly according to the vessel's moving information, and gives a global valid performance [8, 13].

The main difficulties in surface ship control include the underactuated nature and nonlinearity [1, 14–16]. These controllers often attempt to cancel or compensate for high order nonlinearities, thus the control laws are usually very complicated. Other challenges such as the parameter uncertainty and robustness issues have drawn much attention in the control community [17–19]. In reality, there must exist some uncertainties due to disturbance forces and modeling errors, which makes the system very difficult to control. To solve this problem, control strategies such as high gain control, robust control and adaptive control are often used to reject the effects of the uncertainties [20–22].

This paper use a novel $\mathcal{L}_1$ adaptive control strategy to control the PF system. This control method was first developed by Cao and Hovakimyan[23, 24], they call it $\mathcal{L}_1$ adaptive control because that the stability analysis of this adaptive control law needs the concept of $\mathcal{L}_1$-norm. This control strategy has fast adaptive and converge properties in the presence of unknown high-frequency gain, time-varying unknown parameters and time-varying bounded disturbances without restricting the rate of their variation[25]. Besides, $\mathcal{L}_1$ adaptive control law has relative simple form, which make it possible for industrial application. These characters make it quite suitable for the ship motion control. The $\mathcal{L}_1$ adaptive control been used in many areas, especially in aircraft control area [6, 26–28]. Several experiment results show the effectiveness of this control strategy [29, 30]. However, the application of $\mathcal{L}_1$ theory to ship control is relatively rare. Breu and Fossen used this strategy to deal with the parameter resonance problem of ships [31].

Inspired by [8, 13], a cascaded control structure is used in this paper. The PF problem is decoupled into a inner-outer control loop structure. The outer-loop guidance subsystem only considers the kinematic and guidance information about the system, the inner-loop control subsystem considers the dynamics of the system. This structure actual decouples the space requirement and the time requirement. As to the PF problem in this paper, more attention is paid to the space requirement, the primary objective is to steer the ship to move along the given curve path.

The outer-loop guidance subsystem concentrates on guidance and geometrical information of the path, it generates a feasible angular rate commands for the inner-loop subsystem to track. The essential ideal of this strategy is to generate guidance laws which are at an ideal, dynamics-independent level. Especially in [8], they defined a concept of guidance-based path following, which generates a gener-

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ally valid guidance law unaffected by the particularities of any specific dynamics case. But quite different from their work, we use the heading angle rate rather than the heading angle to stabilize the guidance subsystem, which is more suitable for the inner-loop design. A Lyapunov function is derived to prove the stability of the outer-loop.

The inner-loop considers the dynamics properties of the system, and $\mathcal{L}_1$ adaptive control law is used to control this subsystem. The control part is simplified into a single-input-single-output (SISO) system, the rudder angle is the only input in this paper. Due to such simplification, the inherent underactuated property issue and nonlinearity are avoided. This simplified model is similar to the classical first-order Nomoto model [32], which has been successfully used in industry for years. However, the parameter uncertainty is considered here, the exact model of the control loop is assumed unknown. Instead, a state predictor equation and a proper adaptation law equation are used to design a adaptive control law, which can have a fast adaptation speed even with high model uncertainty. $\mathcal{L}_1$ theory has a theoretically rigorous foundation, thus the effectiveness under certain assumption can be guaranteed. The $\mathcal{L}_1$ control law is simple and robust, which makes it easy for industrial application.

The structure of this paper is as follows. In section 2 we introduce the guidance-based path following strategy in the outer-loop subsystem. The desired heading angle rate is derived in this part. The inner-loop control subsystem is described in section 3, where the essential conception about $\mathcal{L}_1$ adaptive control law is introduced. Section 4 is the results and conclusion.

2 Outer-loop subsystem

In this section, the discussion is restricted to the kinematics level and we derive a desired angle rate that can guarantee the convergence of the vessel to the path. This desired angle rate is the reference signal for the inner-loop control subsystem to trace. The error dynamics is proven to be stabilized by constructing a Lyapunov function. The origin ideal of this work can be traced back to [8, 13], although these approaches seem different, the essential ideals are similar.

To describe the PF problem, three coordinate frames are introduced in this problem, namely the stationary inertial frame O, path-fixed frame P and body-fixed frame B, with origin of O, P and B, respectively. The origin of frame P is attached to the path and with its x-basis pointing to the tangential direction. Unlike the traditional approach, where the dynamic equation are described in the B frame and the kinematic equation is described in the O frame. The equations of the ship motion are described in the frame P, whose origin moves along with the path according to the vessel. And it is also regarded as an extra input to the system. The bases of these three frames are expressed as $e_O = [e_{O1}, e_{O2}], e_P = [e_{P1}, e_{P2}], e_B = [e_{B1}, e_{B2}]$, respectively. The relationship between these three bases is:

$$e_\alpha = e_\beta R_{\beta\alpha} \tag{1}$$

where

$$R_{\beta\alpha} = \begin{pmatrix} \cos\theta_{\beta\alpha} & -\sin\theta_{\beta\alpha} \\ \sin\theta_{\beta\alpha} & \cos\theta_{\beta\alpha} \end{pmatrix} \tag{2}$$

$$\tag{3}$$

where $\alpha, \beta \in (\boldsymbol{O}, \boldsymbol{P}, \boldsymbol{B}), \theta_{\beta\alpha} \in (-\pi, \pi]$. $\theta_{\beta\alpha}$ is is the rotation angular from frame α to frame β, positive if the rotation is anti-clockwise. And $R_{\beta\alpha}$ is a unit orthogonal rotation matrix between frame α and frame β. It is obvious that $R_{\beta\alpha}^T = R_{\beta\alpha}^{-1} = R_{\alpha\beta}$.

A proper desired path is assumed to be predefined and parametrized in the frame O. The minimum radius of the osculating circle of the desired path is required larger than the minimum possible turning radius of the ship. While unlike [8], where an extra parameter ξ is used to parameterize the path, this paper uses the arc length parameter s to describe the path directly. Due to the fact that it is usually not easy to describe any given curve path the in the parameterized form of an extra parameter. While the arc length is the intrinsic parameter. It is especially convenient to discrete the path and store them according to the arc length by using computers. Besides, the arc length has intuitive physical meaning which makes it very convenient to describe the path, especially in a discrete form. We assume that the path can be described as :

$$\begin{cases} x = x(s) \\ y = y(s) \quad s \geq 0 \end{cases} \tag{4}$$

s is the arc length, so:

$$s = \int_0^s \sqrt{x'^2(l) + y'^2(l)} dl \tag{5}$$

Assume that θ_{PO} is the angle from Frame O to Frame P, then:

$$\theta_{PO} = \begin{cases} \arctan\frac{y'}{x'} & x' \geq 0 \\ -\frac{\pi}{2} & x' = 0, y' \leq 0 \\ \frac{\pi}{2} & x' = 0, y' \geq 0 \\ \pi + \arctan\frac{y'}{x'} & x' \leq 0, y' \geq 0 \\ -\pi + \arctan\frac{y'}{x'} & x' \leq 0, y' \leq 0 \end{cases} \tag{6}$$

where the prime denotes the derivative according to the arc length parameter s. x and y are the coordinates of the origin P of the frame P expressed in frame O.

The essential ideal of so-called *guidance-based* path following strategy in Breivik and Fossen's work is that the ship motion is represented by a *ideal particle*, it can acquire any speed or direction immediately regardless of the dynamics restriction [8]. And the target point restricted on the path for the ideal particle to track is called *path particle*, it updates its position according the ideal particle and is regarded as an extra input to this system. Naturally, the vessel is regarded as the ideal particle and the origin of frame P is regarded as the path particle. Then,

$$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} \tag{7}$$

If express $\overrightarrow{PB}$ in frame P, then

$$\overrightarrow{PB} = e_P \varepsilon \tag{8}$$

where $\varepsilon = [s, e]^T$ are the coordinates of the position of the vessel expressed in path-fixed frame *P*. They represent the deviation from the vessel to the path. Considering Eq.(7) and Eq.(8), then we have

$$e_P \varepsilon = \overrightarrow{OB} - \overrightarrow{OP} \tag{9}$$

The control objective is to make ε to converge to zero. Such that the vessel is restricted in the given path. This process can be turned into a set of ordinary equations called *error dynamics*, which can be obtained by taking the derivative of Eq.(9),

$$e_P \begin{bmatrix} \dot{s} \\ \dot{e} \end{bmatrix} + \dot{e}_P \begin{bmatrix} s \\ e \end{bmatrix} = \dot{\overrightarrow{OB}} - \dot{\overrightarrow{OP}} = \overrightarrow{V}_B - \overrightarrow{V}_P \tag{10}$$

where $\overrightarrow{V}_B$, $\overrightarrow{V}_P$ are the absolute velocity of the vessel and the path particle, respectively. They can also be expressed in frame P,

$$\overrightarrow{V}_B = e_P \begin{bmatrix} U\cos\theta_{WP} \\ U\sin\theta_{WP} \end{bmatrix} \tag{11}$$

$$\overrightarrow{V}_P = e_P \begin{bmatrix} V_P \\ 0 \end{bmatrix} \tag{12}$$

where U is the vessel's total velocity's value and θ_{WP} is the angle of the vessel's velocity in reference to the frame *P*.

The derivative to time of the basis e_P can also be expressed in the basis itself,

$$\dot{e}_P = e_P S_\chi \tag{13}$$

where the dot means the derivative of time, and

$$S_\chi = \begin{pmatrix} 0 & -\dot{\theta}_{PO} \\ \dot{\theta}_{PO} & 0 \end{pmatrix} \tag{14}$$

is an anti-symmetric matrix.

If substitute (11) and (12) into (10), then the error dynamics can be written as,

$$\begin{aligned} \begin{bmatrix} \dot{s} \\ \dot{e} \end{bmatrix} &= \begin{pmatrix} 0 & -\dot{\theta}_{PO} \\ \dot{\theta}_{PO} & 0 \end{pmatrix} \begin{bmatrix} s \\ e \end{bmatrix} \\ &+ \begin{bmatrix} U\cos\theta_{WP} \\ U\sin\theta_{WP} \end{bmatrix} + \begin{bmatrix} V_P \\ 0 \end{bmatrix} \end{aligned} \tag{15}$$

where $\dot{\theta}_{PO}$ is actually the angular rate of the frame P. According to Eq.(4), which can be expressed as,

$$\dot{\theta}_{PO} = k\dot{s} \tag{16}$$

where a $\dot{s}$ is also the tangent velocity of the frame P. It is obvious that $\dot{s} = V_P$, which is the update law of the arc parameter. And k is the curvature of the path, where

$$k = \frac{y''x' - x''y'}{x'^2 + y'^2} \tag{17}$$

In this outer-loop subsystem, V_p and $\dot{\theta}_{wp} = r_c$ are regarded as the inputs to this pure kinematic level motion. Then the whole kinematic equations can be written as:

$$\dot{s} = -\dot{\theta}_{po} e + U\cos\theta_{wp} + V_P \tag{18}$$

$$\dot{e} = \dot{\theta}_{po} s + U\sin\theta_{wp} \tag{19}$$

$$\dot{\theta}_{wp} = r_c \tag{20}$$

The control objective is to make:

$$\lim_{t\to\infty} \begin{bmatrix} s \\ e \\ \theta_{wp} - \delta_\theta \end{bmatrix} = \vec{0} \tag{21}$$

where

$$\delta_\theta = sin^{-1}\left(\frac{-e}{\sqrt{|e|^2 + d_0^2}}\right) \tag{22}$$

is the reference angle for θ_{WP}. The positive constant d_0 is called *look-ahead distance*.

The most important task for this guidance subsystem is to design proper control laws for V_p and r_c, such that Eq.(21) holds. V_p is a virtual input which can be obtained exactly according to the following update law, and r_c is the desired heading angle rate for the inner-loop control subsystem to track. The control laws of V_p and r_c are defined as follows.

Theorem 1. *The error dynamics are defined as Eq.(18-20). If the kinematic subsystem's input V_p and r_c are set as:*

$$V_p = -U\cos\theta_{wp} - \lambda s \tag{23}$$

$$\begin{aligned} r_c &= \dot{\delta}_\theta - k_1(\theta_{wp} - \delta_\theta) \\ &+ \frac{-c_1 e U \sin\theta_{wp} + c_1 e U \sin\delta_\theta}{c_2(\theta_{wp} - \delta_\theta)} \end{aligned} \tag{24}$$

where λ, k_1, c_1 and c_2 are all positive constants. Then, the error dynamics are stabilized, which means the following holds in the domain $\mathcal{D}$,

$$\lim_{t\to\infty} [s, e, \theta_{wp} - \delta_\theta]^T = \vec{0} \tag{25}$$

the domain $\mathcal{D}$ is defined as:

$$\mathcal{D}(s, e, \theta_{wp}) = \begin{cases} |s(t)| \le d \\ |e(t)| \le d \\ |\theta_{wp}| \le \sqrt{\frac{c_1}{c_2} d^2} + \sup(\delta_\theta(t)) \le \frac{\pi}{2} \end{cases} \tag{26}$$

where d is a positive constant.

Proof. Firstly, we construct a positive function $V(t)$,

$$V(t) = \frac{1}{2}c_1 s^2 + \frac{1}{2}c_1 e^2 + \frac{1}{2}c_2(\theta_{wp} - \delta_\theta)^2 \ge 0 \tag{27}$$

Then, take the derivative to time t of $V(t)$,

$$\dot{V} = c_1 s\dot{s} + c_1 e\dot{e} + c_2(\theta_{WP} - \delta_\theta)(\dot{\theta}_{WP} - \dot{\delta}_\theta) \tag{28}$$

Considering about Eq.(18-20), then we have,

$$\begin{aligned} \dot{V} &= c_1 s(-\dot{\theta}_{PO} e + U\cos\theta_{WP} - V_P) + c_1 e(\dot{\theta}_{PO} s + U\sin\theta_{WP}) \\ &+ c_2(\theta_{WP} - \delta_\theta)(\dot{\theta}_{WP} - \dot{\delta}_\theta) \end{aligned} \tag{29}$$

if substitute Eq.(23,25) into Eq.(29), then

$$\dot{V} = -c_1\lambda s^2 - \frac{c_1 U}{\sqrt{|e|^2+d^2}}e^2 - k_1 c_2(\theta_{WP}-\delta_\theta)^2 \leq 0 \quad (30)$$

So the function $V(t)$ can be selected as the Lyapunov function of the error dynamics of Eq.(18-20). If assume that $|s(t)| \leq d$ and $|s(t)| \leq d$, and by properly selecting the values of d, c_1 and c_2, such that the $|\theta_{WP}| \leq \sqrt{\frac{c_1}{c_2}d^2} + \sup(\delta_\theta(t)) \leq \frac{\pi}{2}$ can also hold, then in the region $\mathcal{D}$, the error dynamics are guaranteed to be stabilized. □

The kinematic level subsystem is stabilized by the input V_p and r_c according to Eq.(25) and Eq.(25). r_c is regarded as a reference output for the ship to track. In this section, only the kinematics level is considered. While in the following section, the inner-loop control subsystem will be considered, where the control objective is to generate a robust control law to trace the reference signal such that the vessel can be steered to the path.

3 Inner-loop control

The discussion above is only restricted in the kinematic level, and actually decouples the space and time requirements. The outer-loop subsystem gives a pure guidance level command, which can guarantee the convergence of the ship to the path. While the dynamics and rudder angle are left to be determined in the inner-loop subsystem. The control objective for the inner-loop control subsystem is to give an adaptive control law, such that the output of the subsystem can track the desired yaw rate command even there exists large uncertainty in the system.

In this section, the dynamics level properties are considered. This inner-loop control part is simplified as an SISO system, where the rudder angle δ is the only input and the angle rate $\dot{\theta}_{wp}$ is the only output.

This abstraction avoids the nonlinearity and the underactuated properties, thus makes the problem much easier. We can use a first-order transfer function to describe the relationship between δ and $\dot{\theta}_{wp}$. This simplification makes it very similar with the first order Nomoto model, which has been used in industry for many years. However, different from the Nomoto model, the output $\dot{\theta}_{wp}$ is described in the path frame P. That is,

$$\begin{aligned} \theta_{wp}(s) &= H(s)\delta(s) \quad (31)\\ H(s) &= \frac{K_n}{T_n s+1} \quad (32) \end{aligned}$$

where $\theta_{wp}(s)$ and $\delta(s)$ are the Laplace transforms of $\theta_{wp}(t)$ and $\delta(t)$ respectively. $H(s)$ is the first-order transfer function from δ to $\dot{\theta}_{wp}$, and K_n and T_n are the corresponding parameter. We only assume the proper characteristic of $H(s)$, the exact value of K_n and T_n need not be known. In fact, due to different navigation states and time-varying environment disturbances, the exact value of K_n and T_n are changing with time. By using $\mathcal{L}_1$ adaptive control strategy, these parameter uncertainties are abstracted by an adaptive estimate $\hat{\sigma}(t)$.

Since r_c stabilizes the subsystem $\mathcal{G}_e$, the control objective for subsystem $\mathcal{G}_p$ is to design an adaptive output feedback controller δ such that the output $r = \dot{\theta}_{wp}$ tracks the reference output r_c following a desired reference model, i.e.

$$r(s) \approx M(s) r_c(s) \quad (33)$$

For simplicity we consider a first order system, by setting

$$M(s) = \frac{m}{s+m}, m > 0 \quad (34)$$

Also, the system in Eq.(31) and Eq.(32) can be rewritten as following:

$$\begin{aligned} \dot{\theta}_{wp}(s) &= M(\delta(s)+\sigma(s)) \quad (35)\\ \sigma &= \frac{(H(s)-M(s))\delta(s)}{M(s)} \quad (36) \end{aligned}$$

here the reference output of the inner-loop subsystem is actually filtered by a low pass filter. This procedure is important because the guidance system may disturbed by noises and tends to give a high frequency reference signal r_c, thus $M(s)$ can be regarded as a shape function to tune the control subsystem to track a more reasonable reference signal. The standard procedure of the $\mathcal{L}_1$ adaptive control strategy can be divided into four steps for simplicity. More detailed information can be obtained in [13].

- State Predictor:

$$\begin{aligned} \dot{\hat{\theta}}_{wp}(t) &= -m\hat{\theta}_{wp}(t) + m(u_1(t)+\hat{\sigma}(t)) \quad (37)\\ \hat{\theta}_{wp}(0) &= 0 \quad (38) \end{aligned}$$

  where the hat means the predict value of corresponding state. $\hat{\sigma}$ is the prediction of the uniformly rate bounded unknown input disturbance σ defined in Eq.(36), and the estimation of $\hat{\sigma}(t)$ is governed by the following adaptation law:

- Adaptive Law:

$$\dot{\hat{\sigma}}(t) = \Gamma_c \text{Proj}(\hat{\sigma}(t), -\tilde{\theta}_{wp}(t)), \hat{\sigma}(0) = 0 \quad (39)$$

  where $\tilde{\theta}_{wp}(t) = \hat{\theta}_{wp}(t) - \theta_{wp}(t)$ is the error signal between the output of the system and the state predictor. $\Gamma_c \in \mathcal{R}^+$ is the adaptation rate subject to a computable lower bound. The project operator is defined as follows [**?**]:

$$\text{Proj}(p,y) = \begin{cases} y & \text{if } \mathcal{F}(p) \leq 0 \\ y & \text{if } \mathcal{F}(p) \geq 0 \text{ and} \\ & \frac{\partial \mathcal{F}}{\partial p}(p) y \leq 0 \\ y - \frac{\mathcal{F}(p)\frac{\partial \mathcal{F}}{\partial p}(p)y}{\|\frac{\partial \mathcal{F}}{\partial p}(p)\|^2} & \text{others} \end{cases}$$

  where

$$\mathcal{F}(p) = \frac{2}{\varepsilon}[\sum_{i=1}^{l} |\frac{p_i-\rho_i}{\sigma_i}|^q - 1 + \varepsilon] \quad (40)$$

- Control Law:

$$u_1(s) = C(s)(r_c(s) - \hat{\sigma}(s)) \quad (41)$$

  where C(s) is a strictly proper system with $C(0) = 1$. Here we only consider the simpliest choice of a first order low-pass filter:

$$C(s) = \frac{\omega}{s+\omega} \quad (42)$$

- $\mathcal{L}_1$-gain stability requirement:
  $C(s)$ and $M(s)$ need to be ensure that,

$$F(s) = \frac{H(s)M(s)}{C(s)H(s) + (1 - C(s))M(s)} \tag{43}$$

is stable and

$$\| G(s) \|_{\mathcal{L}_1} L_z < 1 \tag{44}$$

where

$$G(s) = F(s)(1 - C(s)) \tag{45}$$
$$z(s) = H^{-1}(s)d(s) \tag{46}$$

where $d(t)$ denotes the environment disturbances. L_z is the Lipschitz constant of $z(t)$ with respect to $\theta_{wp}(t)$. The$\mathcal{L}_1$-norm of an SISO system is defined as follows: *Definition:* The $\mathcal{L}_1$ gain of a stable proper stable SISO system is defined $\| N(s) \|_{\mathcal{L}_1} = \int_0^\infty |n(t)|dt$, where $n(t)$ is the impulse response of $N(s)$.
In this paper, the environment disturbances are not considered, we only consider the uncertainty involved in $H(s)$, so $d(s)$ is assumed to be zero. Thus Eq.(44) always holds. The detailed information about the stability issues can refer to [33].

4 Simulation Results

A 3-DOF (surge-sway-yaw) nonlinear model of a mariner ship is used to evaluate the performance of the derived $\mathcal{L}_1$ adaptive control strategy. This model is a quite comprehensive and can be regarded as a substitution to the real experiments with considerable accuracy. This nonlinear model is only used for simulation, it is regarded as a black box to the controller designer, only the input and output of the ship motion system are accessible. More detailed information about the nonlinear model can be found in [34].

The simulation of the process is conducted by using Matlab and Simulink. The fourth-order Runge-Kutta method with a time interval of 0.1s is used for the time domain simulation. The rudder satuation and rate limits ($|\delta| \leqslant 20°$ and $|\dot{\delta}| \leqslant 5°/s$) are considered in the feedback design. The revolution speed, and the velocity is around.

The system parameter are chosen as $\lambda = 0.3, c_1 = 0.0035, c_2 = 400, k_1 = 0.3$. The look-ahead distance $d_0 = 500(m)$. The filter models are selected as $M(s) = \frac{0.2}{s+0.2}, C(s) = \frac{0.4}{s+0.4}$. The adaptive gain is selected as $\Gamma_c = 0.3$.

To demonstrate the efficiency of the $\mathcal{L}_1$ adaptive control strategy, three simulation cases are designed in these paper. All the parameters and disturbance models are kept the same values on these three cases.

Three cases are demonstrated in this section. Case 1 demonstrates the classical track keeping problem, the desired path is a straight line. As seen in Fig. 1, the control law with and without adaptation are illustrated in the diagram. When the adaptation is OFF, we mean that the rudder command angle only takes the heading command $r_c(s)$ into consideration, and the adaptation part is neglected, that is $\delta(s) = C(s)r_c(s)$. As shown in Fig. 1, when the adaptation is OFF, the system has a faster response speed, but it has larger overshoot, and the vessel has an unnecessary rudder input. While when the $\mathcal{L}_1$ adaptive control is on, the ship converges to the path in a very natural way. It has very small overshoot, even when the initial position is far from the path. The ship converges to the path in a fast speed and the generated path is much smoother.

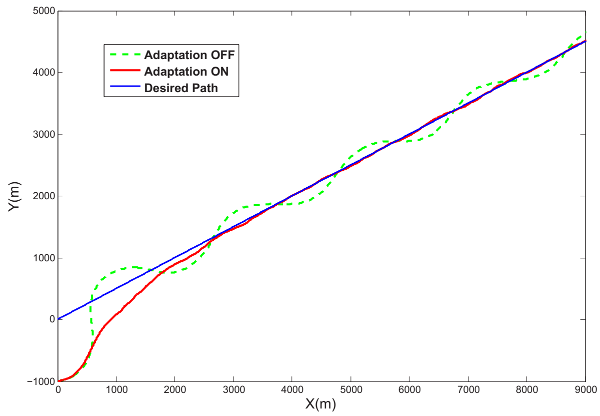

Figure 1: Track keeping (Case 1).

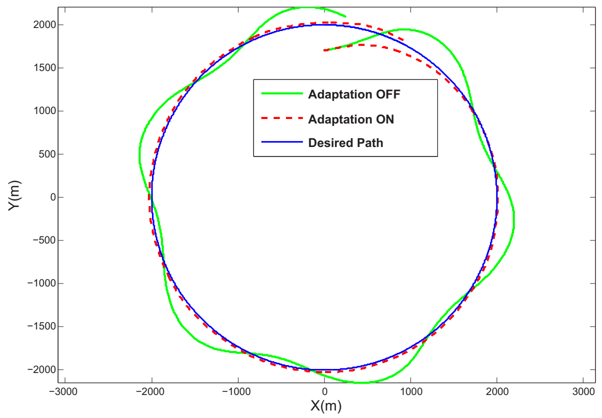

Figure 2: Rotational motion (Case 2).

Case 2 describes the rotational motion of the ship, the desired path is a circle with a radius of around 2000 meters, which is larger than the minimum possible turning radius of the ship. As shown in Fig. 2, in this curve path case, the performance is relatively bad when the adaptation is OFF, it has large deviations from the path. However,when the adaptation is ON, the control performance can guarantee the fast convergent speed and considerable tracking accuracy. There is a small steady-state error as shown in the Fig. 2. According to [33], the steady-state error can be reduced by suitably adding the adaptive gain Γ_c.

Case 3 demonstrates the tracking performance to an arbitrarily curve path. It is a general case, thus we give a detailed analysis to its performances. Fig. 3 shows that the initial position of the ship is around kilometers from the path. The trajectory of the ship with $\mathcal{L}_1$ adaptation control is quite smooth. There are only small overshoot under this control law. It has a relatively larger deviation when the path has a larger curvature. However, this deviation can be restricted in 50 meters in this case. Mostly, the deviation can be less than 10 meters according to Fig. 4, which are the diagrams of tangential tracking error s and cross tracking error e.

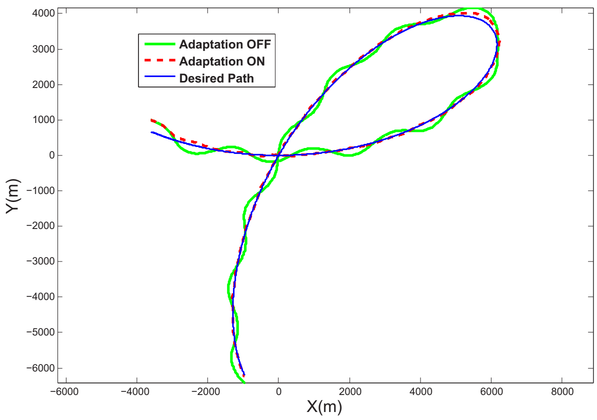

Figure 3: Arbitrary motion (Case 3)

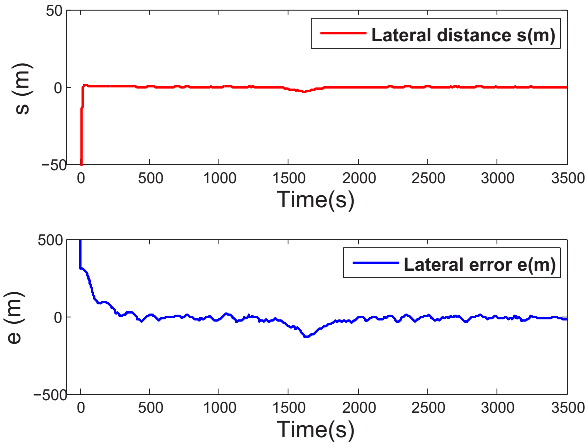

Figure 4: Tangential and cross error (Case 3).

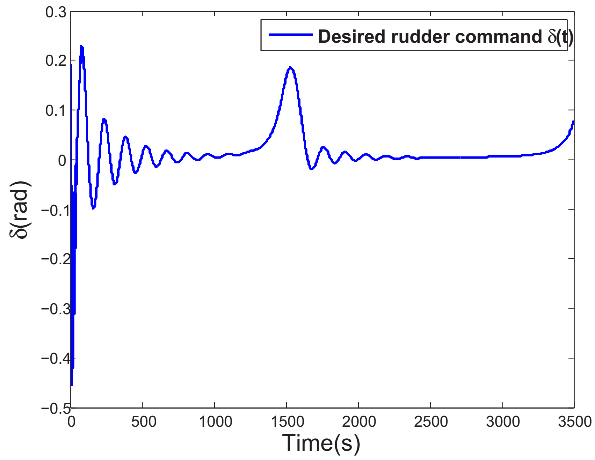

Figure 5: Rudder command (Case 3)

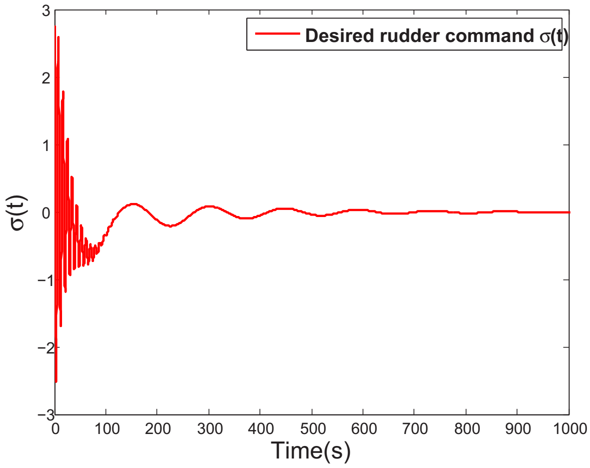

Figure 6: $\hat{\sigma}(t)$ (Case 3)

Fig. 5 shows the desired rudder input when the adaptation is ON. As shown, the desired rudder angle is mild for most of the time, except for the beginning time. At the beginning, there is chattering phenomenon due the the model uncertainty. In some extreme situation, the high frequency chattering should be filtered by low pass filter to give a practical rudder command.

Fig. 6 describes the prediction value of the time-varying input disturbance $\hat{\sigma}(t)$. At the beginning, the value of $\hat{\sigma}(t)$ also has a high frequency chattering. However, after about 100 seconds, the value of $\hat{\sigma}(t)$ is very mild and converges to near zero. It shows the very fast adaptation of the $\mathcal{L}_1$ adaptive control methods, which guarantees the good performances of the path following system.

5 Conclusion

This paper introduces the $\mathcal{L}_1$ adaptive control method to the ship path following control problems. This control strategy has a quite performance even when there exists large model uncertainty and environmental disturbances. This paper takes a comprehensive marina ship model for simulation. The results show that this control law have relatively good performance in tracking keeping, rotational motion and arbitrary motion cases.

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