

# Master Thesis

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May 31, 2016

# 1 Preface

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## 2 Introduction

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## 3 Theory

### 3.1 Bridge Weigh-in-Motion

A Bridge Weigh-in-Motion system is based on measurements of a bridge's deformation. The BWIM system uses these measurements to calculate passing vehicles axle loads. There are different approaches to assembling such a system, but they typically consists of a strain gauge measuring the strain induced by passing vehicles, a axle detector used to find the vehicle speed and spacing of axles and a computer or data storage device. An algorithm then is able to use the data gathered from the axle detector and strain gauge to calculate axle loads [5].

#### 3.1.1 Moses' Algorithm

Moses' algorithm is based on the fact that a moving load along a bridge will set up stresses in proportion to the product of the value of the influence line and the axle load magnitude. The influence line being defined as the bending moment at the point of measurement due to a unit axle load crossing the bridge [5].

Moses' algorithm is built from the fact that a moving unit load on a bridge will induce stresses proportional to the product of the value of the influence line and the axle load magnitude.

Each individual girder's stress is related to moment:

$$\underbrace{\sigma_i}_{\text{stress in } i\text{'th girder}} = \frac{\text{bending moment } i\text{'th girder}}{\underbrace{\frac{M_i}{W_i}}_{\text{section modulus}}} \quad (1)$$

Expressing the moment in terms of strain gives

$$M_i = W_i \sigma_i = \underbrace{E}_{\text{Modulus of elasticity}} \times W_i \times \underbrace{\varepsilon_i}_{\text{strain in } i\text{'th girder}} \quad (2)$$

The sum of the individual girder moments is therefore:

$$M = \sum_{i=1}^N M_i = \sum_{i=1}^N E W_i \varepsilon_i = E W \sum_{i=1}^N \varepsilon_i \quad (3)$$

The sum of the girder strains is proportional to the gross bending moment. The total bending moment and the measured strain is thus directly related by  $EW$ . These constants can be calculated through the bridge's dimensions and material properties. However through measuring the effects of a known vehicle passing the bridge these constants can be derived.

Weigh in motion is an inverse type problem, the strain is measured and the cause of the strain is to be calculated. The theoretical bending moment corresponding to axle loads on the bridge at one strain sample, is given by:

$$M_k^T = \sum_{i=1}^N A_i I_{(k-C_i)} \quad (4)$$

$$C_i = (L_i \times f)/v \quad (5)$$

Where:

$N$  = the number of vehicle axles

$A_i$  = the weight of axle  $i$

$I_{k-C_i}$  = the influence line ordinate for axle  $i$  at sample  $k$

$L_i$  = the distance between axle  $i$  and the first axle in meters

$C_i$  = The number of strain samples corresponding to the axle distance  $L_i$

$f$  = the strain gauge's sampling frequency, in Hz

### 3.2 Influence lines

For a B-WIM system the influence line is defined as "the bending moment at the point of measurement due to a unit axle load moving along the bridge [4]". The influence line could be found through assembling a model of the bridge in any CAD or frame-program, this would however take a lot of time especially for more advanced bridge's. Depending on the support of the bridge the influence lines takes different theoretical forms, as seen in Figure 1. The true influence line for a bridge lie somewhere in between the simply supported and fixed version [4, p. 146]. Influence lines is a big source of error in a B-WIM system.

Znidaric and Baumgärter [4], did a study on the effect of choice of influence line. This study shows errors up to 10% for a short 2 m bridge span and errors of several hundred percent for a 32 m bridge span. This underlines the importance of using correct influence lines for a B-WIM system.

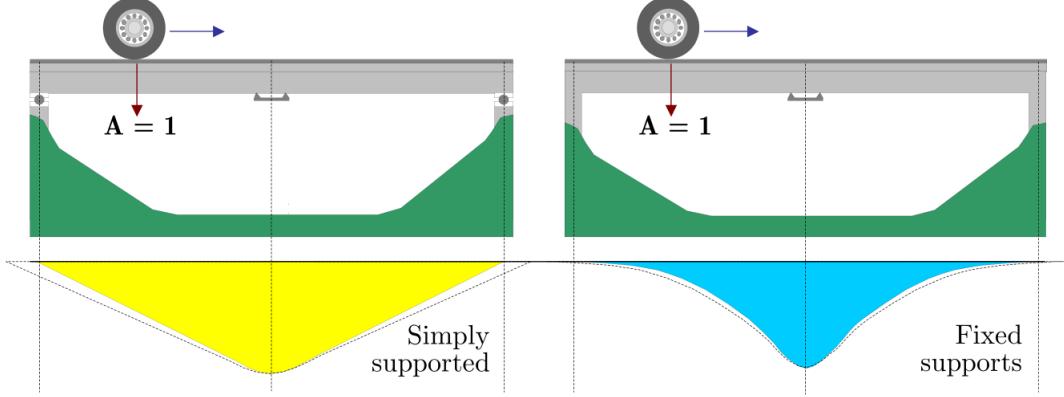


Figure 1: Influence lines for simply and fixed supported bridges, figure from [5]

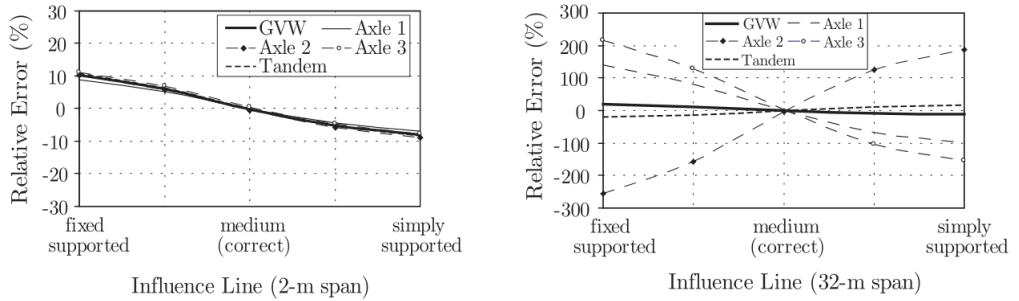


Figure 2: Errors of axle loads due to wrongly selected influence lines, figure from [5]

### 3.2.1 Matrix method

Quilligan [5] developed a 'matrix method' to calculate the influence line of a bridge through the measured strain induced by a vehicle. This method is derived from Moses', equation 6.

$$Error = \sum_{k=1}^K [\varepsilon_k^{measured} - \varepsilon_k^{theoretical}]^2 \quad (6)$$

Equation 6 were originally used to filter out the dynamic response of the bridge. The theoretical strain in this equation can be expressed as a product of axle loads and influence ordinates at sampling points, see equation 4, thus we can expand equation 6:

$$Error = \sum_{k=1}^K \left[ \varepsilon_k^{measured} - \left( \sum_{i=1}^N A_i I_{(k-C_i)} \right) \right]^2 \quad (7)$$

The set of influence ordinates  $I$  that minimizes  $Error$ , forms the wanted influence line.

$$\frac{\partial Error}{\partial I_R} = \frac{\partial \sum_{k=1}^K \left[ \varepsilon_k^{measured} - \left( \sum_{i=1}^N A_i I_{(k-C_i)} \right) \right]^2}{\partial I_R} \quad (8)$$

For a given number of known axle loads this equation comes down to a set of  $(K - C_n)$  number of linear equations. Rearranging the equations and writing them in matrix form leads to:

$$[Am]_{K-C_N, K-C_N} \{I\}_{K-C_N, 1} = \{M\}_{K-C_N, 1} \quad (9)$$

Where:

$\{M\}$  = a vector depending on axle weights and measured strain,  $M_{i,1} = \left( \sum_{j=1}^N A_j \varepsilon_{(i+C_j)} \right)$

$[Am]$  is a matrix depending only on the axle loads, defined by equation 10.

$$[Am] = \sum_{i=1}^N \sum_{j=i}^N [Am] + (A_i A_j [D]_{C_j - C_i}) \quad (10)$$

Which produces the upper triangle of the symmetric  $[Am]$ . Where:

$[D]_{C_j - C_i}$  = a matrix containing only one diagonal of ones, where the diagonal is placed with an offset,  $C_j - C_i$ , from the center matrix diagonal.

Solving equation 9 for the influence ordinate vector gives the influence line for the strain history. This can be done through inversion of the AxleMatrix (equation 10) or other numerical solutions like a Cholesky factorization.

### 3.2.2 Optimization

testing [2]. For this thesis one of the goals where to assess the accuracy of and optimization algorith for finding a bridges influence lines. To develope such an algorithm test strain signal where produced by a matlab script, and the algorithm developed was to find the influence line used to produce the strain signal. In theory using optimization to identify influence lines should work well, and indeed it did for these produced theoretical strain signals.

## 3.3 Finding the train's speed

## 3.4 The axle distances

## 3.5 Filtering and noise

All signals are subjected to noise, which can be defined as

unwanted disturbances superposed upon a useful signal that tend to obscure its information content [6]

Noise in a BWIM system can be intrinsic noise, that is noise generated inside a system, and extrinsic noise which is noise generated outside the system. A train approaching the BWIM sensors may be a source of extrinsic noise. Performing bridge weigh-in motion relies upon the information provided by the sensor signals. When distance between detected axles is to be found, noise is a source of distortion which may increase error of found distance, it may also make it difficult for the program to detect the desired peaks in the signal. Smoothing the signal is therefore completely necessary for a BWIM system. During the developement of the software for this thesis, several attempts on finding and using appropriate filters have been made. Matlab contains many such filter functions which can be used, such as a Butterworth and SGOLAY filters.

### 3.5.1 Noise smoothing through fourier transformation

The following quotation from Matlabs: Practical Introduction to Frequency-Domain Analysis, see [3], describes how frequency analysis can be done with Matlab. (badly written!!)

Frequency-domain analysis shows how a signal's energy is distributed over a range of frequencies. A signal can be converted between the time and frequency domains with a pair of mathematical operators called a transform. An example of this is the Fourier transform which decomposes a function into the sum of a number of sine wave frequency components. The 'spectrum' of frequency components is the frequency domain representation of the signal. The inverse Fourier transform converts the frequency domain function back to a time function.

Performing a fast fourier transformation in matlab on a vector signal, gives the opportunity to remove unwanted frequencies from the signal. When the signal is transformed into the frequency domain, setting all the frequencies above 30 Hz to zero and then transforming the signal back into the time domain would smooth a typical BWIM signal greatly.

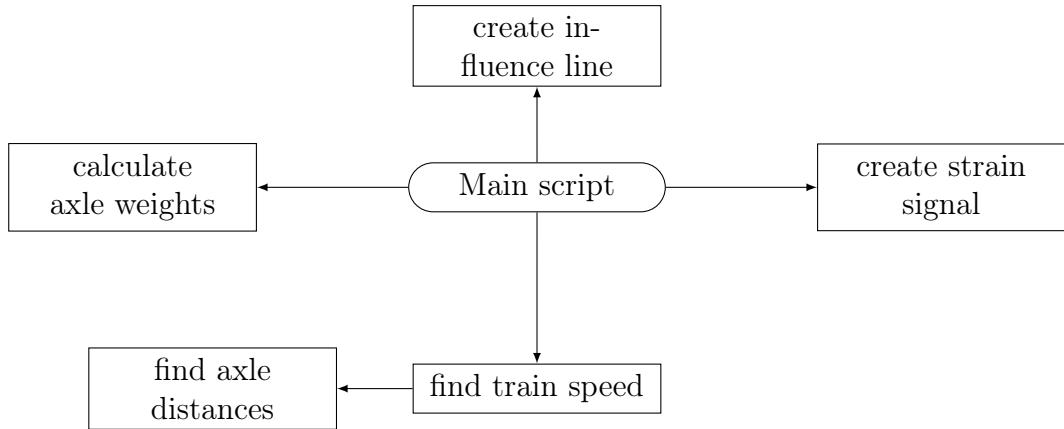
## 4 Method

### 4.1 Programming a BWIM system

Describe shortly how the BWIM system have been programmed. Keywords: This master project began by learning how a BWIM-system works, and to then create

a working model performing BWIM. To not make this a too big project this meant building a simple beam model of a bridge in Matlab, and simulate moving loads crossing it.

A simple flow diagram describing the intial BWIM program:



#### 4.1.1 Producing a strain signal

Through the theoretical moment influence lines of the beam, a strain signal can be built through the moment-strain relationship, found in equation3, for a given set of axle weights. A simple beam bridge model, as seen in figure 3, will not recreate a actual bridge strain signal but will be used to create a working BWIM system. The produced strain signal will differ from an actual strain signal mostly because of dynamics, from the train and bridge, and because actual boundary conditions of a bridge will differ from the boundary conditions of a simple beam model. The strain sensors will also produce noise distorting the signal. To make as good a signal as possible, some effort were placed into recreating the effect mentioned above. To add noise to the signal, white gaussian noise was included in the signal through Matlabs wgn function "<http://se.mathworks.com/help/comm/ref/wgn.html>".

This strain signal could then be used as a base to build the code for a BWIM system.

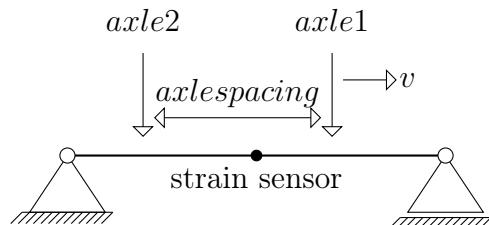


Figure 3: Beam model for developement

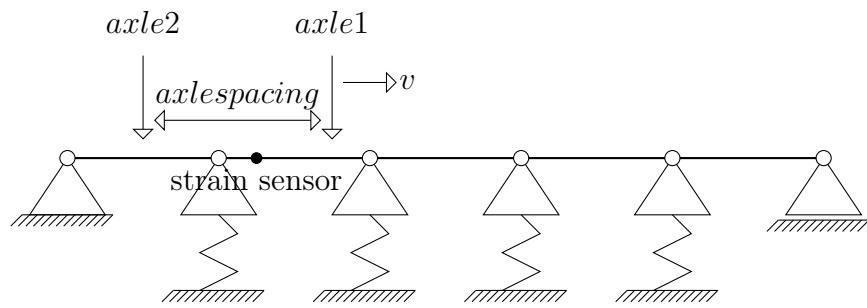


Figure 4: A more realistic beam bridge model

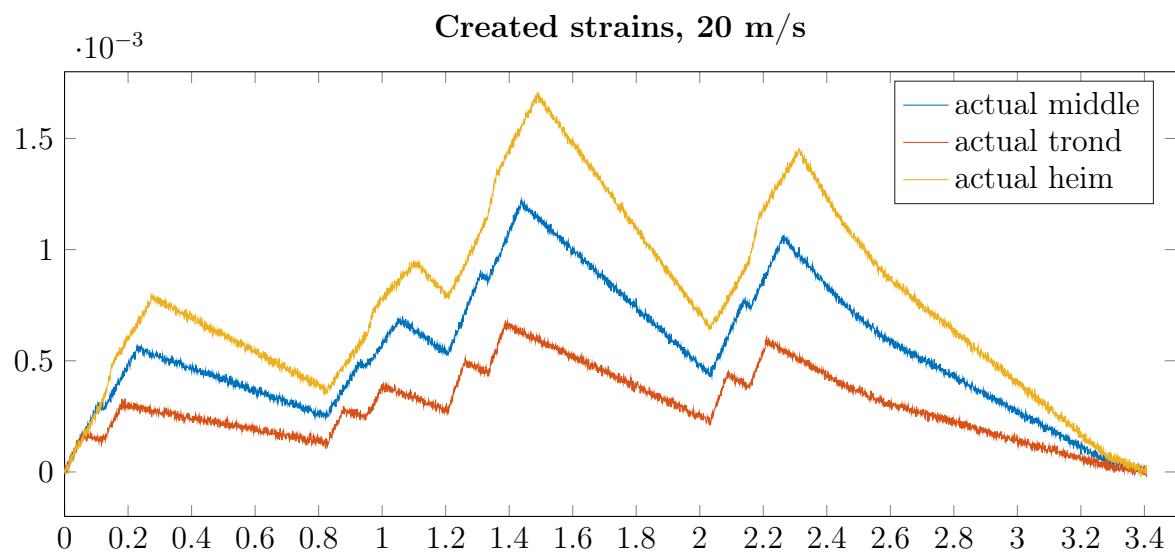


Figure 5: Strain signal created through beam model

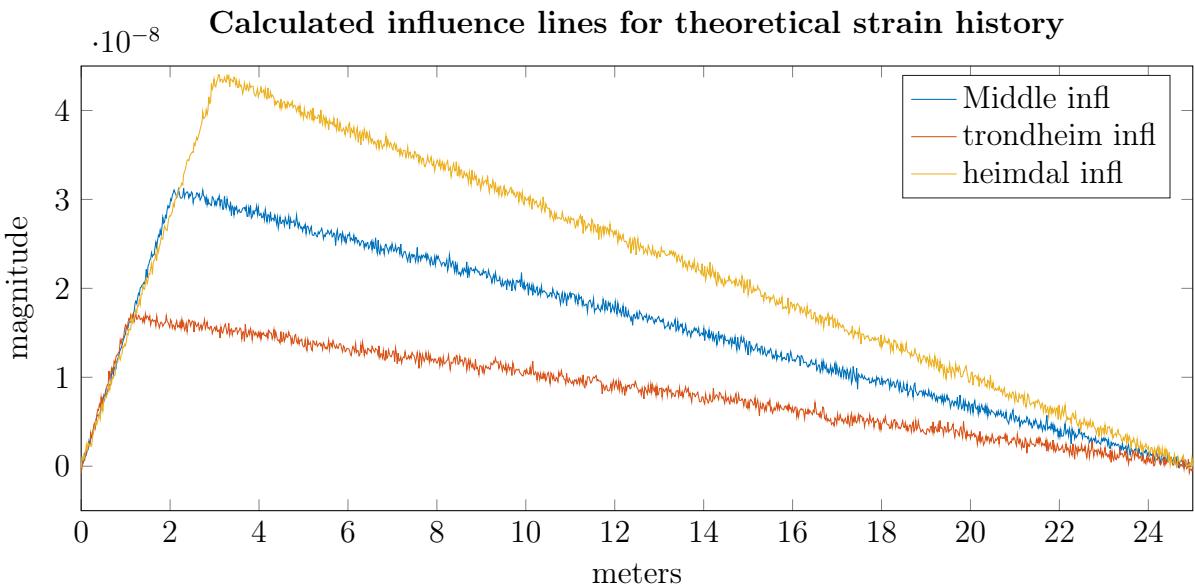


Figure 6: Influence line calculated from created strain

#### 4.1.2 Finding the speed of the train

Two working methods of finding the speed of a passing train were developed:

- By identifying peaks in the strain history for two different sensors, representing the same axle. The distance between the two sensors and the time difference between the found peaks should theoretically give a good estimate of the trains velocity.
- Through doing cross correlation between two sensors strain history. This involves finding the phase difference, or lag, between the signals. The known distance between the strain gauges should then along with a constant, based on distance between sensors, give a reliable estimate of train velocity. INSERT PLOT OF CORRELATION AND SHOW MATHEMATICAL EQUATION DESCRIBING CROSS CORRELATION.

These two methods both work very well for a theoretical signal, however when noise and dynamics are introduced as well as more complicated bridge boundary conditions identifying the peaks representing the same axles becomes complex. A method indentifying peaks, will have to adapt to each signal because the magnitude of noise and dynamics vary for the different sensors and train passings. These difficulties also causes problems for placing the found influence lines, this is discussed in section ??.

The speed of the train passings in this thesis was not attainable from NSB or Jernbaneverket, so the method of correlation will not be usable because of the undetermined constant the method requires.

Since neither of these methods were usable without calibration, an alternative way was developed. This method determined the speed by recreating the strain signal for various train velocities and minimizing the difference between measured and recreated signal. This method uses brute force, and its time consumption proved high. Due to this time consumption the velocities used furter in thesis were only based on reading from the middle sensor.

The importance of using the correct speed in a BWIM system becomes apparent when calculating influence line for a sensor. A wrongly determined speed will result in what looks like dynamic effects or an oscillating influence line, none of which should appear in a static influence line.

## 4.2 Finding influence lines

Describe how influence lines have been found from the given strain history from Lerelva Bridge. Keywords:

- Matrix method
- Optimization
- Speed

### 4.2.1 Matrix method

Describe the matrix method.

### 4.2.2 Optimization

Describe how optimization can be used to find optimal influence lines for the bridge.

## 4.3 System setup

To test the BWIM-program on actual data, we Gunnstein, Daniel set up a BWIM-system to gather strain data from actual train passings. The subject bridge were Lerelva-Bridge in Trondheim, figure 8, a typical Norwegian steel railway bridge. Three strain gauges, 3 mm 120 ohms from HBM, were placed by the support towards Trondheim on the first section of the longitudinal stringer, see figure 7b. The sensors were placed with 1 m spacing around the middle of the stringer section.

These strain gauges were connected to a National Instruments compactDAQ with module NI 9235 which produced an continuous data flow to a standard laptop, see figure 7a. A Kipor generator was brought for power.



(a) System setup from data gathering at Lerelva  
(b) Placement of strain gauges on stringer section

Figure 7: Instruments for aquiring strain data



Figure 8: Lerelva bridge with a train passing over

## 4.4 Testing

Keywords:

- Comparing calculated strain with measured strain
- Perform the same test with a influence line found through the matrix method

When performing BWIM with a influence line through the matrix method, the length of the strain signal will require a influence line of a certain length. The exact position where the train begins to influence the bridge is not known due to the special conditions of the Trondheim side support and the dynamic effects in the influence line. To use the found influence line as correctly as possible, it will be needed to place it in accordance with the provided strain signal.

The best form of testing the matrixmethod and the BWIM system would be to have several runs in both directions using the same known train, or locomotive. This would give the possibility of comparing calculated and known axle weights. This will not be possible due to insufficient data material.

### 4.4.1 Using calculated influence lines

To perform a standard BWIM calculation, the influence line needs to be aligned correctly with the strain signal, otherwise calculated Axle weights will be based on faulty calculations from solving the system

$A = I_m \setminus \varepsilon$  (NEEDS TO BE IN THEORY). The first peak of the strain signal, corresponding to the first axle of the train, should occur at the same location as the peak of the influence line which should be precisely at the strain-gauge/sensor location. Identifying the first peak of the strain signal is subject to noise which corrupts any reading of peaks in the raw strain signa. Therefore filtering of noise is needed to correctly identify the signals peaks. A trains axle spacings, as seen in B.1 which is the train type of the readings, may consist short spacings. If the axle spacing between two axles are short compared to the bridge, they both influence the signal simoultaneously and the peaks corresponding to the two axles thus lies very close to each other. The filtering can therefore not be to hard or soft, which may result in problems when trying to automate the procedure of identifying axles. To correctly align the strain signal and influence line, the matlab code used in this thesis first smooths the strain signal to a degree where the desired number of peaks are identifiable before using matlabs findpeaks (REFERENCE THIS) procedure to find the peak locations, like seen in figure 9.

When trying to place the influence line, it was found that axle detection needs to be very accurate for the calculation of axle loads. When using the method described above, with filtering to a degree where 8 peaks are found and to check how those peaks correspond with the known train's axle distances, proved to be

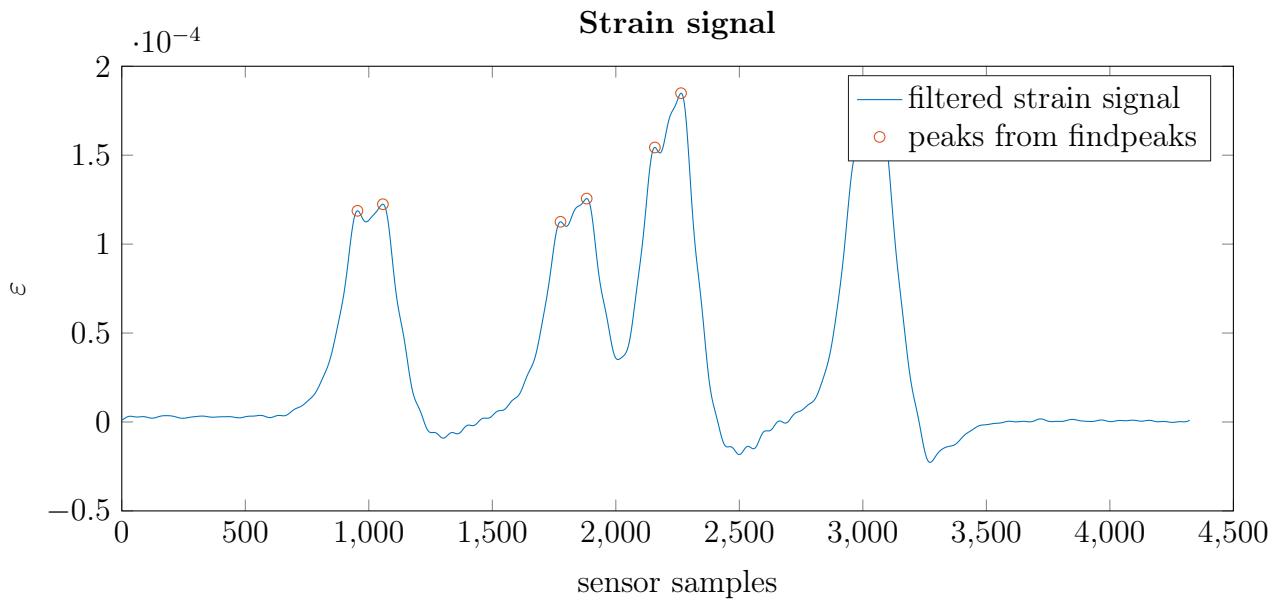


Figure 9: Axle peaks in strain signal

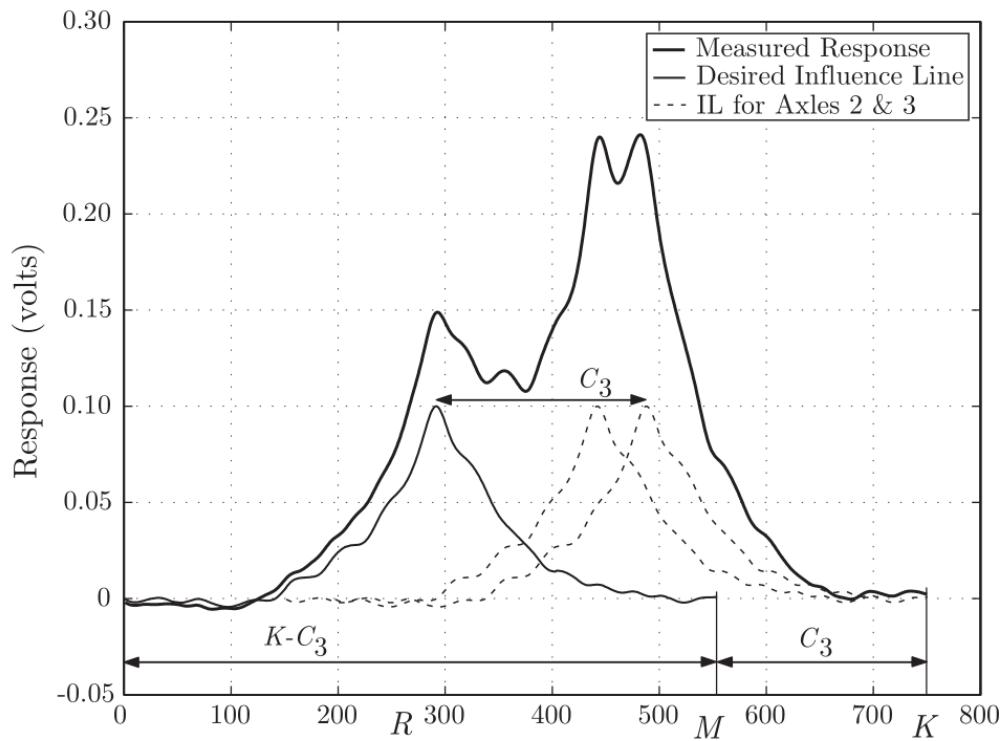


Figure 10: Placement of influence lines, influence line has been scaled.

accurate in some cases and very wrong in other cases. A wrongly found axle peak could for instance result in negative axle weights. A more general method which seems to place the influence better is to filter the signal to a degree where only the major peaks are found. It is believed that this peak is roughly the middle between closely spaced axles, or a bogie, on a train. Since the trains axles spacings are known, a successfully identified bogie location should place the influence line with a decent accuracy.

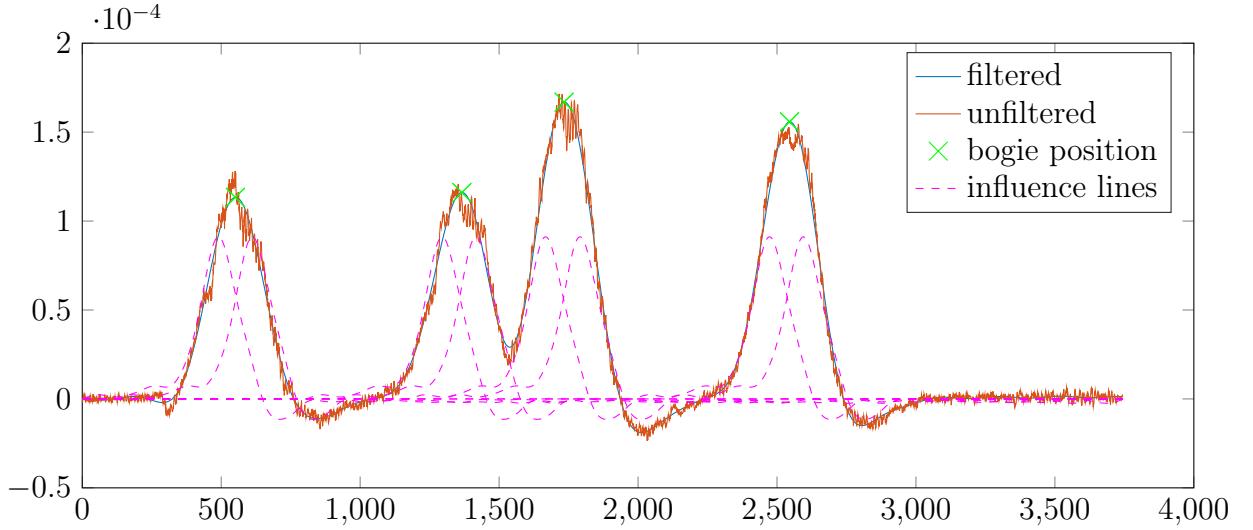


Figure 11: Placement of influence lines, based on bogie peaks in signal

(The noise level seem to vary according to sensor location.. Place in theory maybe!!)

#### 4.5 Calibrating sensors

The sensors described in system setup have not been calibrated. This would result in wrong end results. For instance, the calculated axle weights for a train would vary from sensor to sensor but the relationship between axle 1 for two different sensors would be approximately constant. Because of a lack of calibration trains, this calibration of the will not be exact. The only train passing where axle weights could be determined is the locomotive of the freight train B.2.

### 5 Analysis

This chapter will describe how the BWIM system performs. What works? Why? How? etc. The main focus should perhaps be placed on identifying the pros and

cons of the matrix method and optimization method.

Should include:

- Compare theoretical and calculated influence lines. Also include influence lines found through Abaqus.
- Check how influence lines found through matrix method and optimization reproduces the strain history
- Test obtained influence line by running the bwim routine on the hitherto unused freight train. (Depends on getting info about the train). Also Do this test on the other trains.

Lerelva bridge has been a subject of several phd students at NTNU and because of this there exist an accurate Abaqus model of the bridge. Two other student had master projects similar to mine, and I was lucky enough to "borrow" an influence line from them.

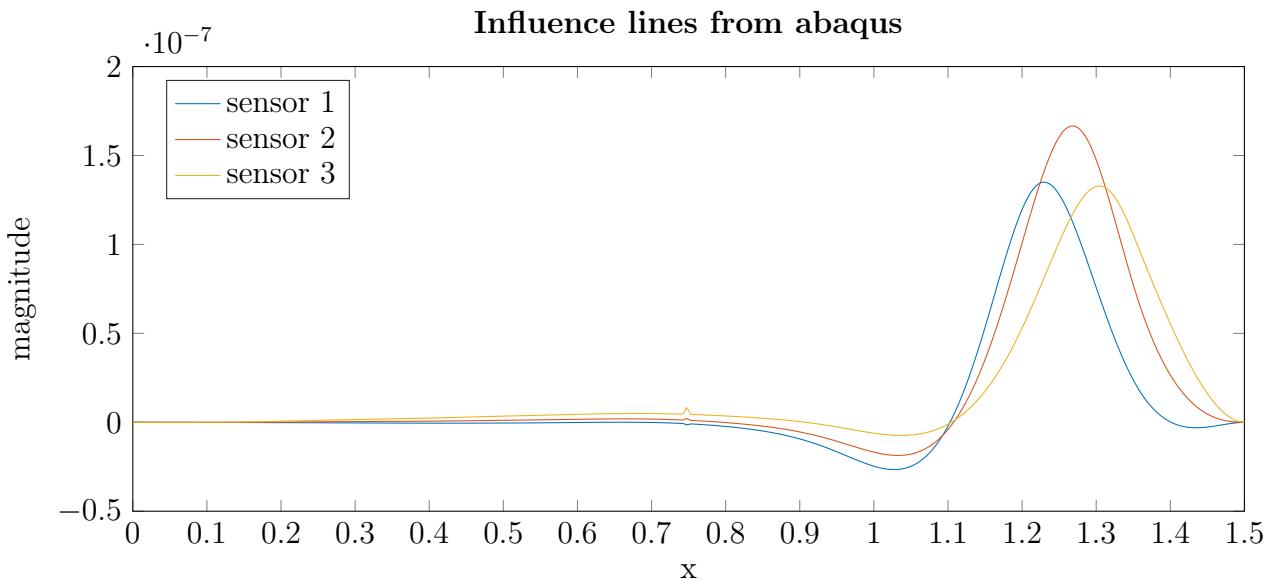


Figure 12: Influence line generated by abaqus

## 5.1 Strain data

The following figure 13, contains raw strain data for 6 different trains passing the Lerelva bridge. Each subfigure contains data from three different strain sensors placed as described in System setup, see section:4.3. Three of the trains comes from the north side; train 3, train 5 and train 7, and three from the south side;

train 4, 6 and 8. The strain signals all appear similar in form, except for train 7, figure 13e, which is a freight train. The other 5 trains are all of the same type, a NSB 92 type passenger train B.1.

The strain signals have different levels of peak height suggesting that the trains actual axle weights differ from what is found in table 5. This will throw off the magnitude of the resulting average influence line found through the matrix method. And this error in calculated influence line will inevitably be found again in the calculated axle weights .

To account for the different directions of the trains, the strain data for the trains going towards Trondheim has been reversed (correct word ?). This is not necessary for finding influence lines, but makes it easier placing the found influence lines in the same coordinate system.

Some of the signals were originally very long, due to not knowing exactly when the train would pass. This means cutting the signal into a vector containing the essential data. Ideally the goal was to identify exactly, or as closely as possible, the time the train entered the bridge. Due to noise and dynamic effects identifying this, proved a difficult process involving detection of peaks which lies close to peaks of noise. This proved possible to do for each individual signal, but a general method performing this for every signal was not within the authors capabilites. Therefore, to cut the signals as equally as possible the first and last major peaks of the signals were used as reference points for appending of samples before and after these peaks. For this method to prove exact, the speed of the train should be taken into consideration when appending sample points so that the influence lines of the signals gets an as equal length as possible.

The strain data from the freight train, figure 13e, is not used for finding the bridge's influence line because the train data is unknown. Axle weights for this train was not found, and guesswork of this data would be difficult. The locomotive data, and possibly the axle spacings of the train, would be the only data possible to find. Therefore this data will only be used for testing of influence lines.

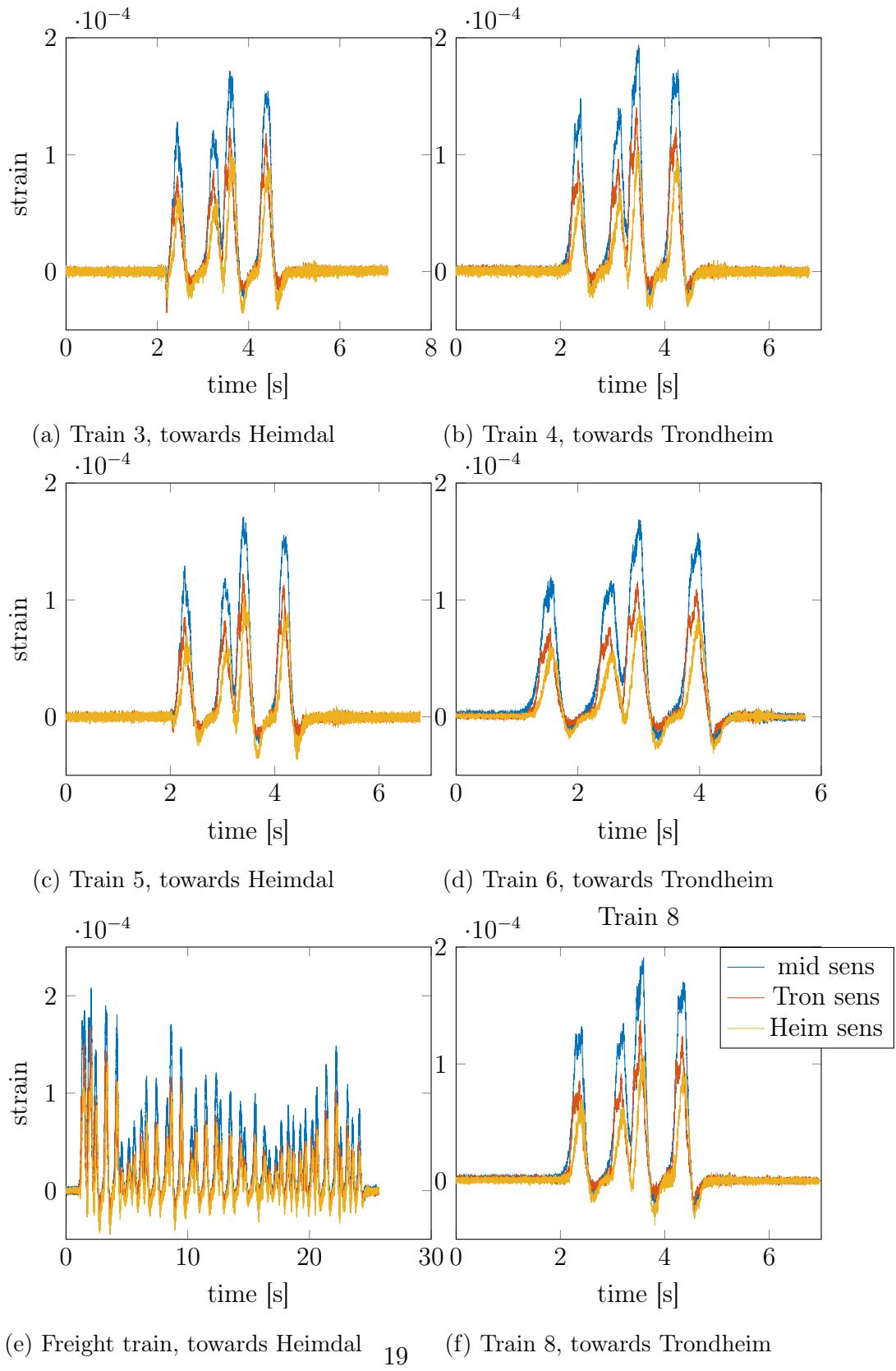


Figure 13: Strain data from Leirelva bridge

## 5.2 The matrix method

The matrix method creates an influence line for a specific strain history given a known train with known axle weights and velocity. Thus if the strain signal were recreated given with given parameters, the signal would be a almost exact replica of the measured strain signal, where the differences should originate from sensor noise. The found influence line would however be for this specific train and the passing's dynamic effects on the bridge, which is likely to vary from train to train. Therefore an averaging of a sufficient number of calculated influence lines should reduce or eliminate the dynamic effects from the influence line.

The analysis of the matrix method is based on 4 different train passings, and 3 sensor readings on each passing. The trains in these measurements is of the same type (not entirely shure!!) but the exact weight is not known. The weight of each axle were approximated by distributing carriage and locomotive gross weight. Passengers in the passing trains were not accounted for, and may lead to some deviation from ideal results.

To include effects from the train approach to the bridge, additional samples have been included from the strain signal before and after the first and last peak found in the signal. This will recreate the strain signal to a more complete degree (needs to be shown in a figure), but the found influence line usually displays more dynamic effects which is unwanted properties in a static influence line.

TODO:

- Show the found influence lines for some sensors
- discuss the plots
- reproduce strain signal, and compare with measured signal
- soutshow averaged influence line, and perform the same tests
- show interpolation of this averaged influence line
- perform the same test with this interpolated influence line
- the alternative should also be done, interpolate each found influence line and average them, then reproduce the strain signal, and find difference through comparison.

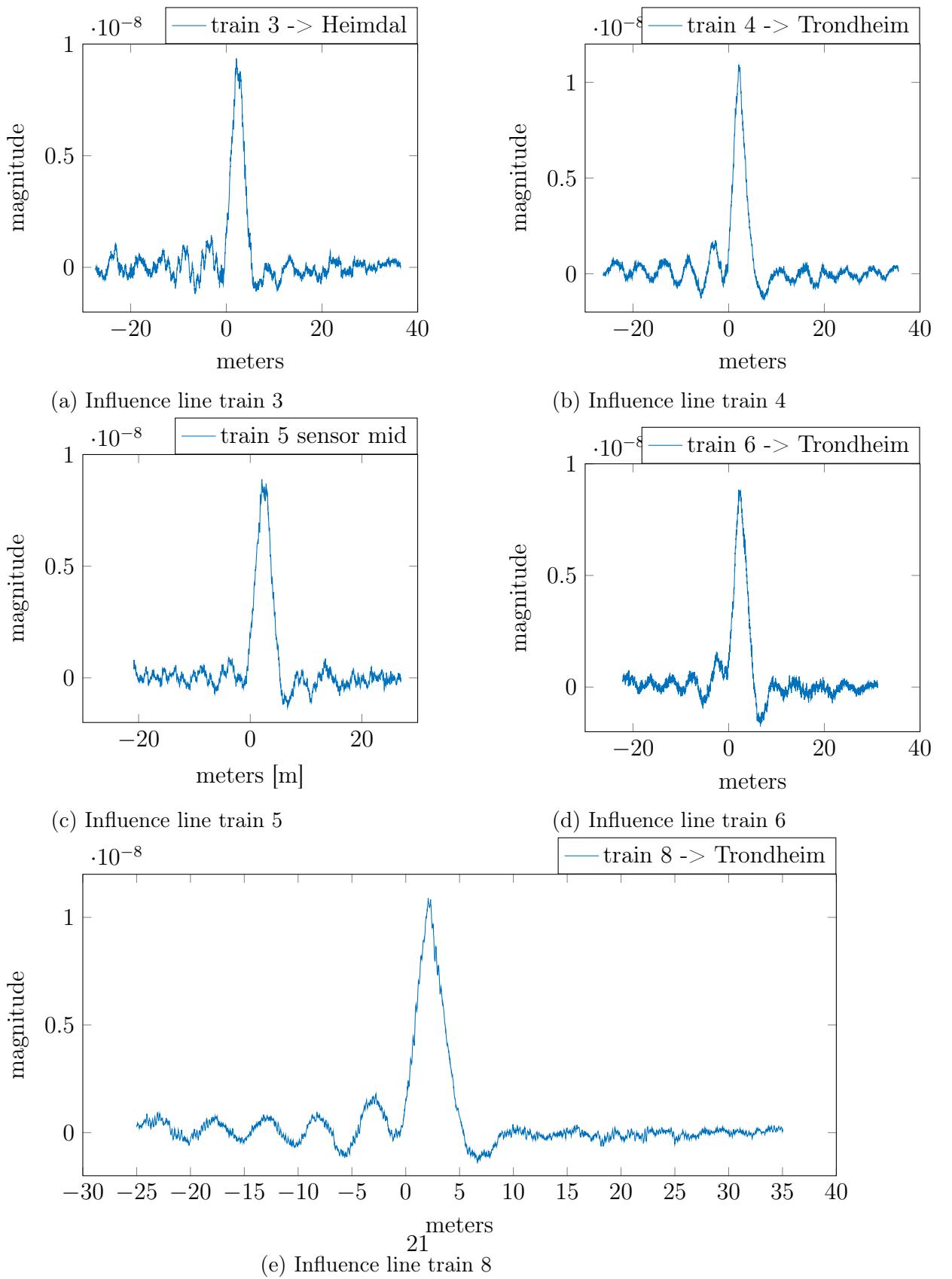


Figure 14: Influence lines found through the matrix method, for the middle sensor

As plainly seen in figure 14 there is big differences between the found influence lines. The trains are all of the same type meaning that the magnitudes of the influence lines, which should be the mostly dependent on axle weights, ought to be similar for all train passings. However as discussed in 5.1, the different magnitudes could be explained with the unknown values of axle weights. When the plots are laid on top of each other, as in figure 19, it is clearly visible that there is some variation in peak magnitude. Especially train 4 and 8 have a higher maximum peak magnitude than the others.

### 5.2.1 Accuracy of the matrix method through recreating the strain signal

One way of examining the accuracy of the matrix method is to recreate the strain signals by using the calculated influence lines. The following three subfigures show recreated signals for a single train passage, but for different sensor locations.

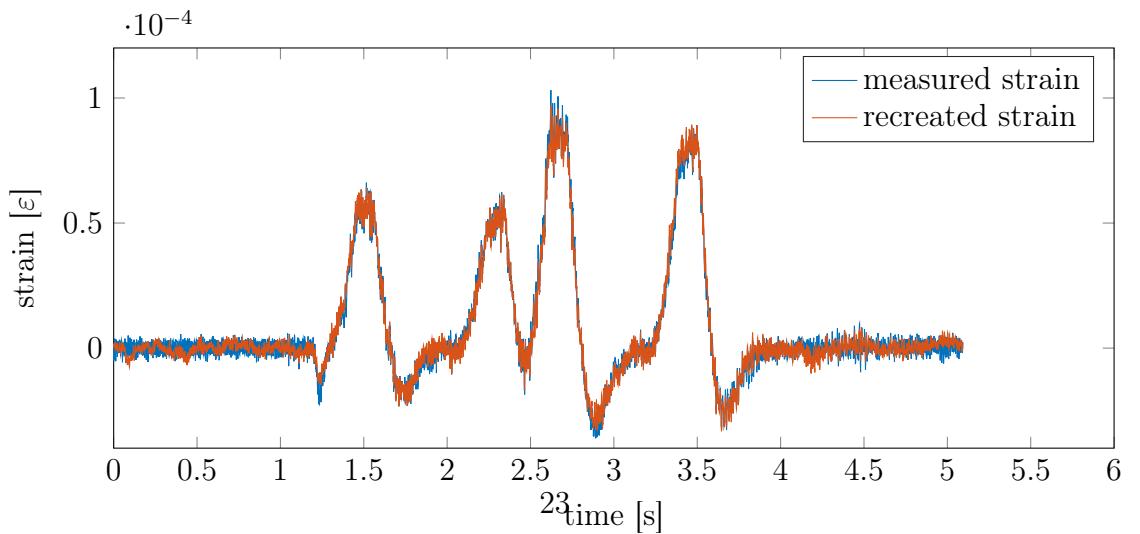
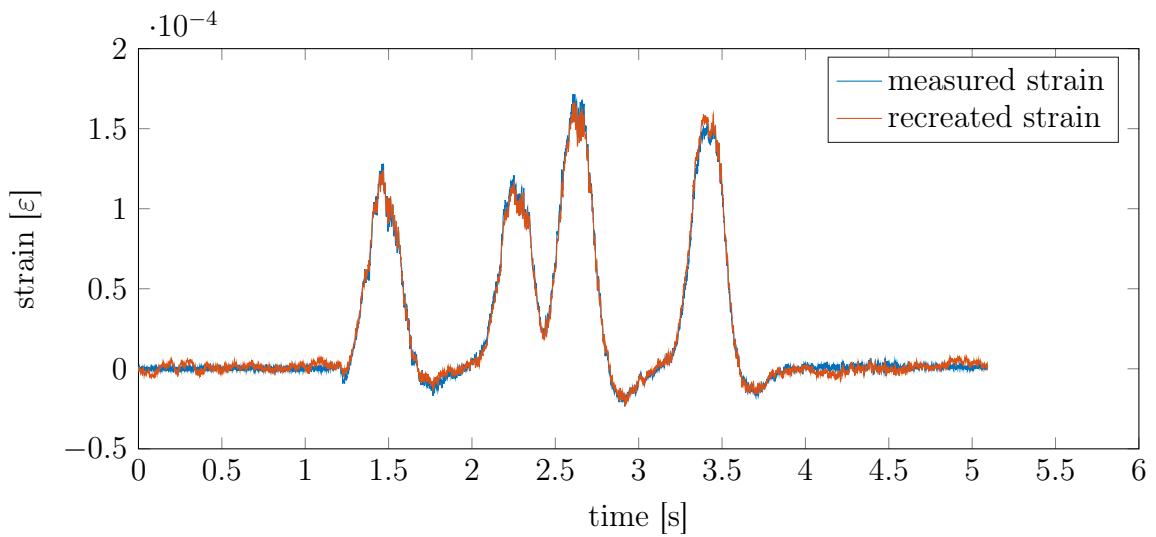
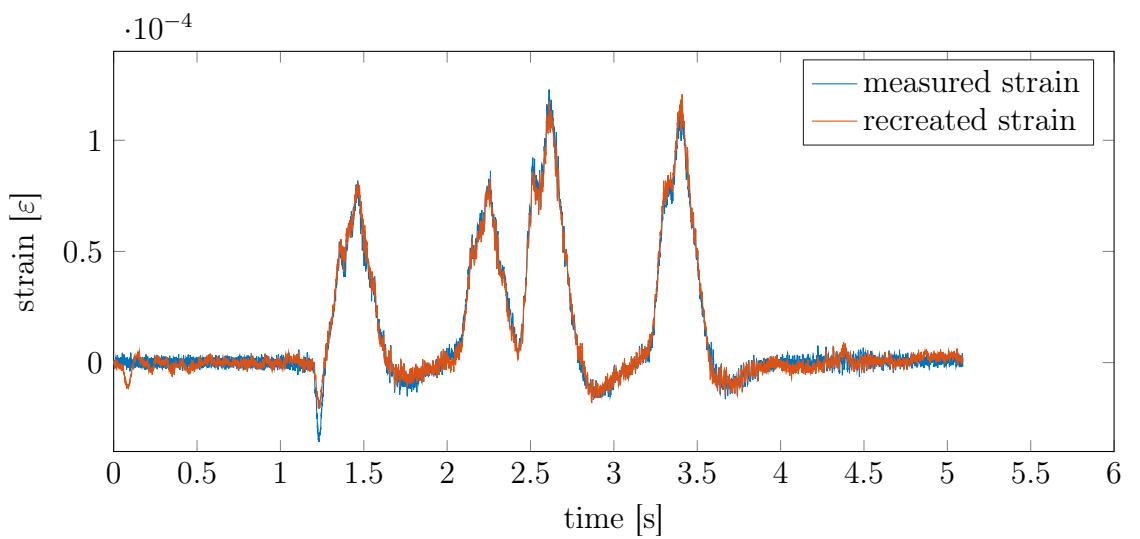


Figure 15: Recreated strain signals for train 3

Figure ?? shows a strain signal from the sensor closest to Trondheim along with a signal recreated using the found influence line for that sensor. To identify and compare errors the following equation 15, performing least square error, will be used.

$$Error = \sum (\varepsilon_{meas} - \varepsilon_{calc})^2 \quad (11)$$

The recreated strain signals, see figure 15, illustrates the accuracy of the matrix method.

Error table			
	Trondheim sensor	middle sensor	Heimdal sensor
Sum original strain	total = 0.004293487978238	total = 0.013095734854378	total = 0.001689786792932
train 3	Error = $3.8684 \cdot 10^{-8}$	Error = $3.9936 \cdot 10^{-8}$	Error = $3.0987 \cdot 10^{-8}$
train 4	Error = $3.7437 \cdot 10^{-8}$	Error = $2.6794 \cdot 10^{-8}$	Error = $2.4965 \cdot 10^{-8}$
train 5	Error = $3.2433 \cdot 10^{-8}$	Error = $4.8605 \cdot 10^{-8}$	Error = $3.5970 \cdot 10^{-8}$
train 6	Error = $5.3555 \cdot 10^{-8}$	Error = $2.8783 \cdot 10^{-8}$	Error = $2.4336 \cdot 10^{-8}$
train 8	Error = $3.6865 \cdot 10^{-8}$	Error = $2.3341 \cdot 10^{-8}$	Error = $2.6944 \cdot 10^{-8}$
average	Error = $3.9795 \cdot 10^{-8}$	Error = $3.3492 \cdot 10^{-8}$	Error = $2.8640 \cdot 10^{-8}$

Table 1: Errors of the recreated strain signals found in 15, rounded to four decimals

$$(3.9936 + 2.6794 + 4.8605 + 2.8783 + 2.3341)/5 \quad (3.0987 + 2.4965 + 3.5970 + 2.4336 + 2.6944)/5$$

$3.8684 + 3.7437 + 3.2433 + 5.3555 + 3.6865 + 1.2353$  Train 5 is clearly distinctive, in regards of how it oscillates in plot 14c and how it recreates the strain with error as in table 1. There are several possible causes for these deviating results, among them wrongly determined velocity of the train and that the trains speed amplifies dynamic effects in the bridge and sensor.

Error table for 600 samples before and after, for smoothed and unsmoothed signal. Also excluding train 5 which is found to show large dynamic effects. Filtering removes frequencies above 10 Hz.

The differences between the unfiltered and filtered errors, tables 2 and 3 respectively, are clear but not unexpected. They show that the filtering does not distort

Error table, filtered signals			
	Trondheim sensor	middle sensor	Heimdal sensor
train 3	$Error = 9.0579 \cdot 10^{-8}$	$Error = 8.8553 \cdot 10^{-8}$	$Error = 8.2682 \cdot 10^{-8}$
train 4	$Error = 5.8366 \cdot 10^{-8}$	$Error = 9.8604 \cdot 10^{-8}$	$Error = 6.0953 \cdot 10^{-8}$
train 6	$Error = 4.9018 \cdot 10^{-8}$	$Error = 9.2035 \cdot 10^{-8}$	$Error = 4.5365 \cdot 10^{-8}$
train 8	$Error = 5.4430 \cdot 10^{-8}$	$Error = 9.5485 \cdot 10^{-8}$	$Error = 6.5290 \cdot 10^{-8}$
average	$Error = 6.3098 \cdot 10^{-8}$	$Error = 9.3669 \cdot 10^{-8}$	$Error = 6.357 \cdot 10^{-8}$

Table 2: Errors of the recreated strain signals with original signal filtered for noise, rounded to four decimals

the error to an amount which destroys the accuracy of the influence line. To really compare the methods of filtering however the found influence lines should be used to calculate axle weights. Averaging of the influence lines gives the following plots.

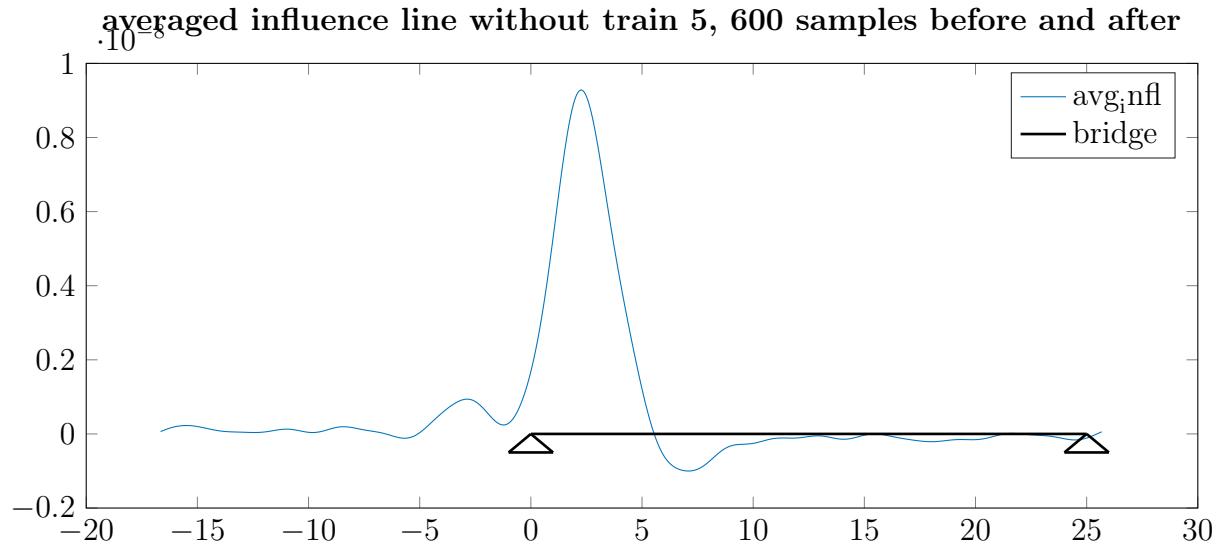


Figure 16: Influence line, filtered after averaging

Error table, w/o filtering			
	Trondheim sensor	middle sensor	Heimdal sensor
train 3	$Error = 5.9214 \cdot 10^{-8}$	$Error = 6.3572 \cdot 10^{-8}$	$Error = 3.9315 \cdot 10^{-8}$
train 4	$Error = 3.3116 \cdot 10^{-8}$	$Error = 7.1066 \cdot 10^{-8}$	$Error = 3.1627 \cdot 10^{-8}$
train 6	$Error = 3.3947 \cdot 10^{-8}$	$Error = 6.9690 \cdot 10^{-8}$	$Error = 2.9709 \cdot 10^{-8}$
train 8	$Error = 2.8439 \cdot 10^{-8}$	$Error = 6.6137 \cdot 10^{-8}$	$Error = 3.6453 \cdot 10^{-8}$
average	$Error = 3.8679 \cdot 10^{-8}$	$Error = 6.7616 \cdot 10^{-8}$	$Error = 3.4276 \cdot 10^{-8}$

Table 3: Error table w/o filtering

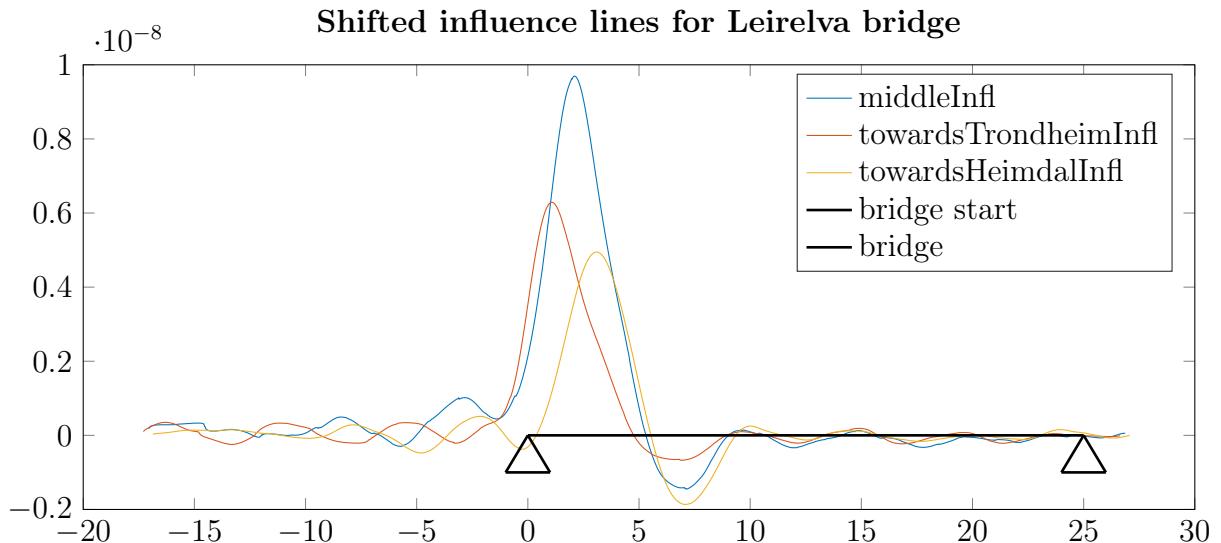


Figure 17: Influence line, filtered strain signals

### 5.2.2 Dynamic effects

The dynamic effects can clearly be seen in the plots for the various train passings. They appear as oscillations in the plots, and are more visible in the low magnitude areas of the influence line. These oscillations vary from train to train making it clear that the dynamic effects depends on the train. The varying influencing factors may be train speed and weight. In the source code producing these influence lines

an assumption of train weight has been made, which makes all train axles equal in weight. What is interesting is the effects of an approaching train, which clearly induces oscillations in the bridge even though the train is as far as 40 meters away from the beginning of the bridge. The differences between the dynamic effects for the train passings may relate to velocity, axle weights and train acceleration (there may be more causes).

A wrongly estimated speed These dynamic effects are unwanted in the static influence line. In theory, averaging enough influence lines should reduce these effects enough to get usable data. This thesis does not contain enough train passings to achieve this. Also as mentioned before, one of the trains have amplified dynamic effects which throw off the results somewhat when performing averaging. Therefore excluding the results from train 5 as in would be reasonable. Figure 21 shows the average of the different influence lines where train 5 has been excluded. This plot still contain dynamic effects, which will need to be removed, but the amplitude of the oscillations have been visibly reduced.

Wrongly determined train velocity is a cause of oscillating influence lines, and can easily be mistaken for dynamic effects. Figure 18 is an example of an influence line determined from a wrongly set speed.

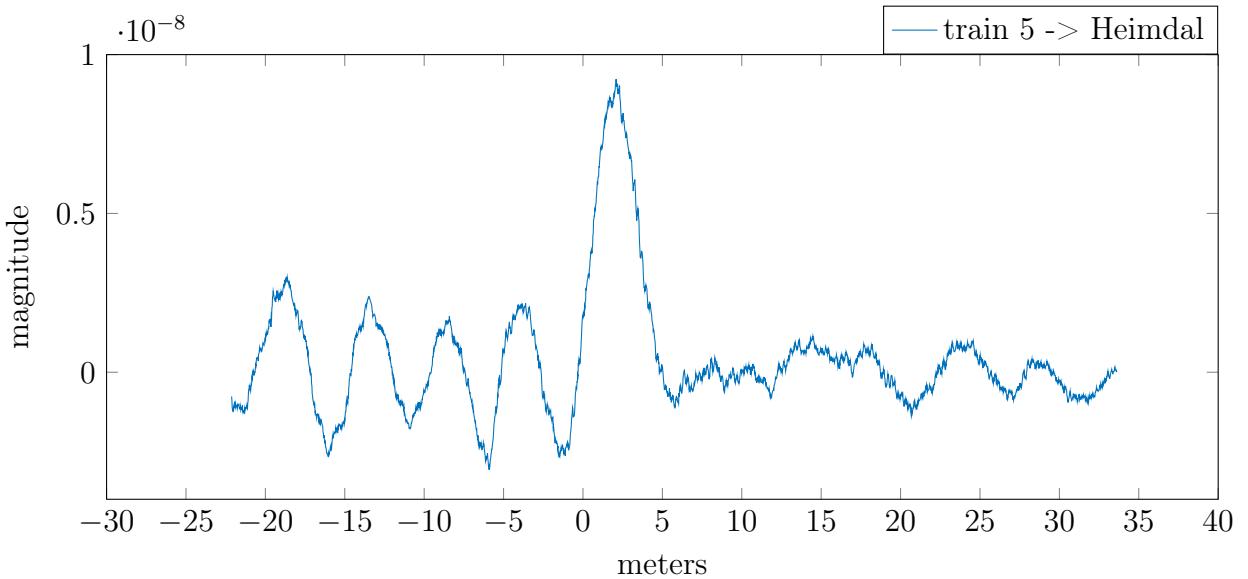


Figure 18: Influence line train 5

Shortening the number of samples in the case without train 5, reduces the dynamic effects further, and a filtering of higher frequencies gives a more usable influence line as seen in 22. A general formula for identifying influence lines with too much oscillation should be developed. One way could be to use table 1 and

exclude the trains which dominates error, or that differs most from the other trains.

The support towards Trondheim is of a special nature, it is connected to a very little bridge spanning perhaps 2 meters which cars may pass under. This may affect the train's entry and cause dynamic effects. It also provides a problem when deciding what should be part of the final influence line, what influences the sensor? One way to do it would be to simply cut the influence line at the samples corresponding to the bridge, however that does not seem likely to be a very good solution. Another way would be to smooth the influence line to the point where the entry part becomes integrated with the major influence line peak, which would result in a greatly distorted peak and is therefore not a good solution.

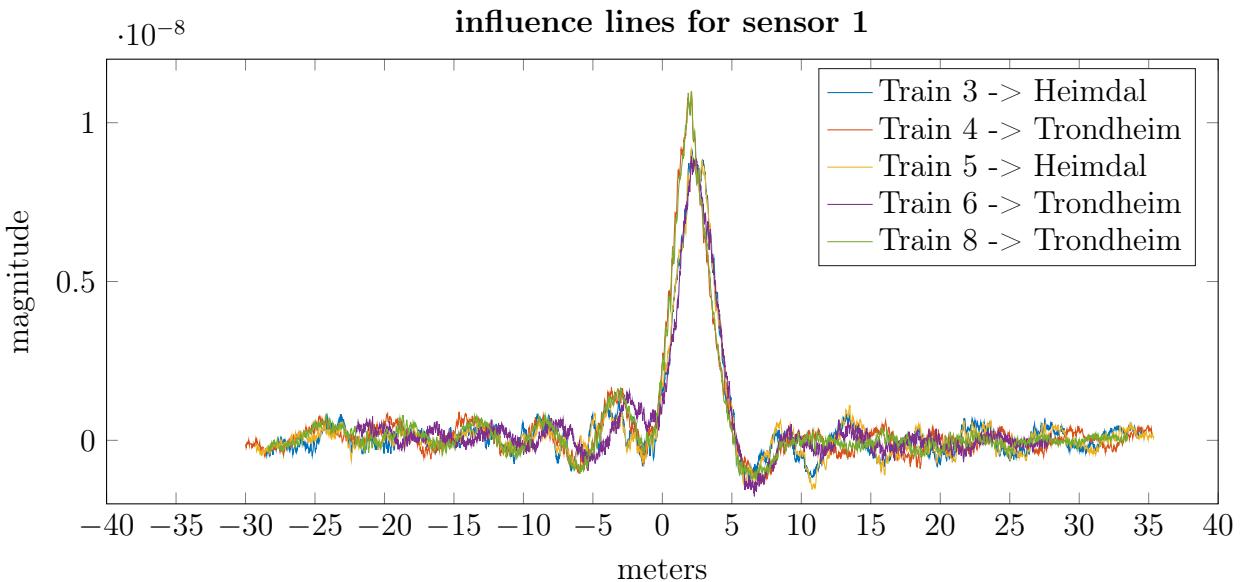


Figure 19: Influence lines from figure:14 on top of each other for sensor 1

Figure 19 shows all influence lines in one figure, which highlights the differences and similarities between the figure.

Clearly two of the influence lines, train 4 and train 8 has a maximum peak magnitude which differs from the others. These two trains both travel the bridge in the same direction, which could be a cause for differing magnitudes, however train 6 ,which also travels the same direction, does not follow this trend and in fact aligns with the other peaks of train 3 and 5. Based on this it can be assumed that direction of the train should not affect the magnitude of the maximum peak.

Another hypothesis for the differing peak heights, could be that the trains differ greatly in actual axle weights. A heavy train would cause higher values of measured strain and the measured strain is what the matrix method uses to find the influence lines. The values of axle weights used to produce the influence lines

of the bridge are fixed at the values of a empty train. A quick study of equation 9, shows that increasing the values measured strain also would increase the values of the influence line. This is therefore a likely cause of differing magnitudes. (Look into possibility of identifying this effect in calculated axle weights !!! )

The average of these influence lines will likely have a maximal peak magnitude somewhere between the peaks of train 3,5 and 6 and train 4 and 8. This would cause problems when calculating the axle weights, the axle weights of train 3, 5 and 6 would be underestimated, and the axle weights of train 4 and 8 would be overestimated. This effect can be seen in the tables 12 and 8, showing calculated axle weights using this averaged influence line. The equivalent of figure 19 for sensors 2 and 3 can be found in appendix 35 and 36. These collection of influence lines also display a differing in magnitudes of the influence lines, with some differences. For sensor 2 train 4 and 8 still has higher maximum values, but train 8 produces a lower value of maximum compared with train 4. For sensor 3 the efect of differing magnitudes are almost invisible, for this sensor all trains seems to produce similar values except for train 6. This is also visible in table 12, where the axle weights for sensor 3 shows train 6 having the highest total value.

Another factor which could be the source of these effects are the velocity of the trains. A wrongly determined velocity causes oscillations in the influence lines as discussed previously, and maybe this also could cause differing maximum peak values. It may also be that different velocities could cause differing entry effects, which would provide

Determined velocities for the trains.

train	3	4	5	6	8
velocity [meterpersecond]	20.99	21.7276	21.4857	16.83	20.591465

Table 4: Table of determined train velocities

In the execution of the matrix method, the train velocity affects the distance between the

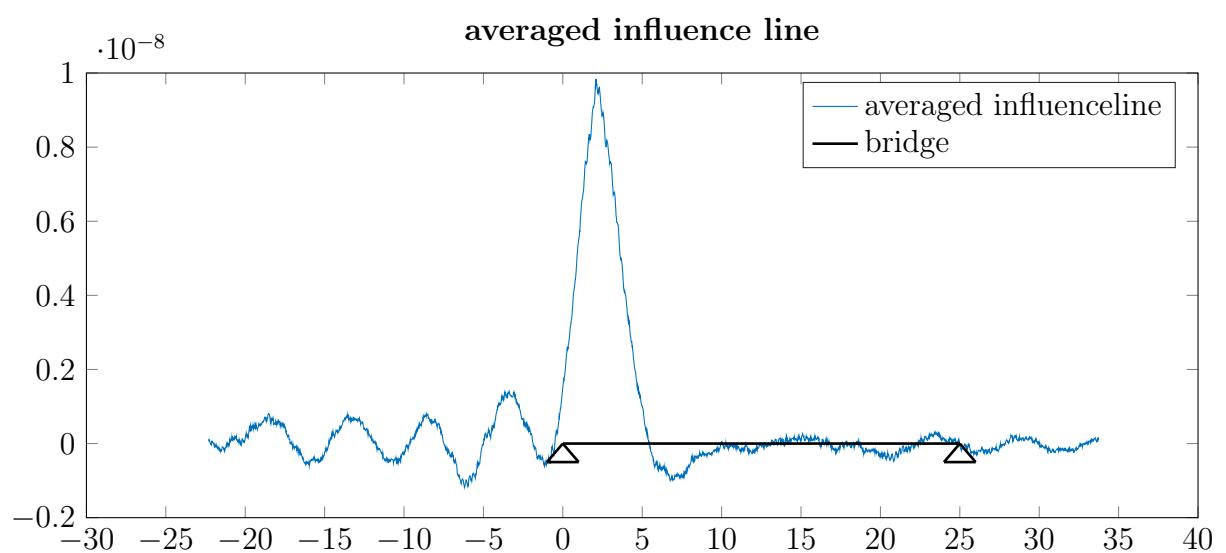


Figure 20: Averaged of the 5 trains

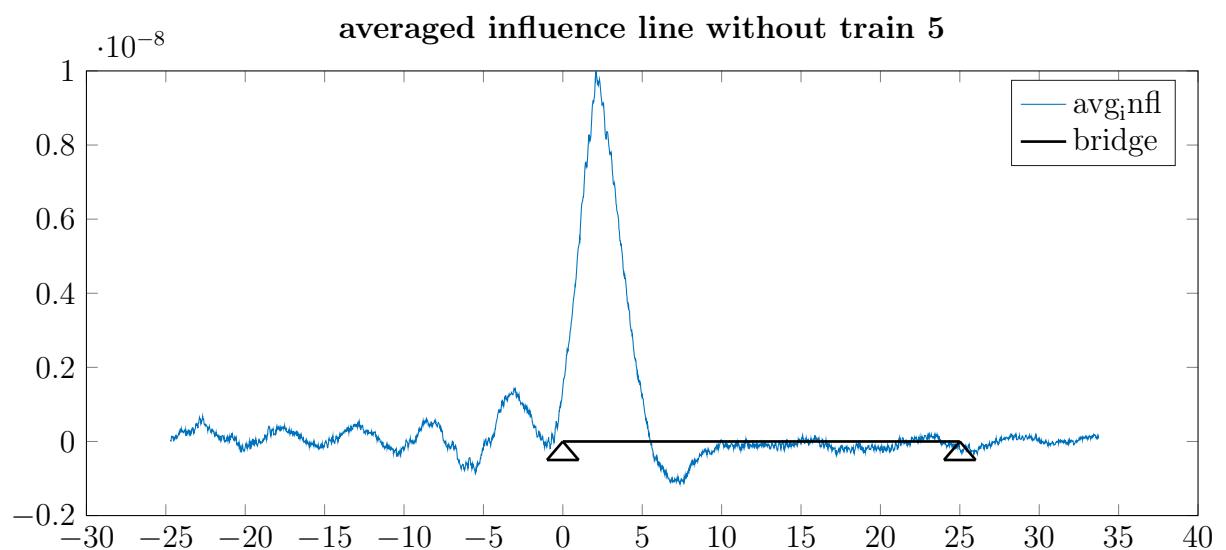


Figure 21: Averaged influence line without train 5

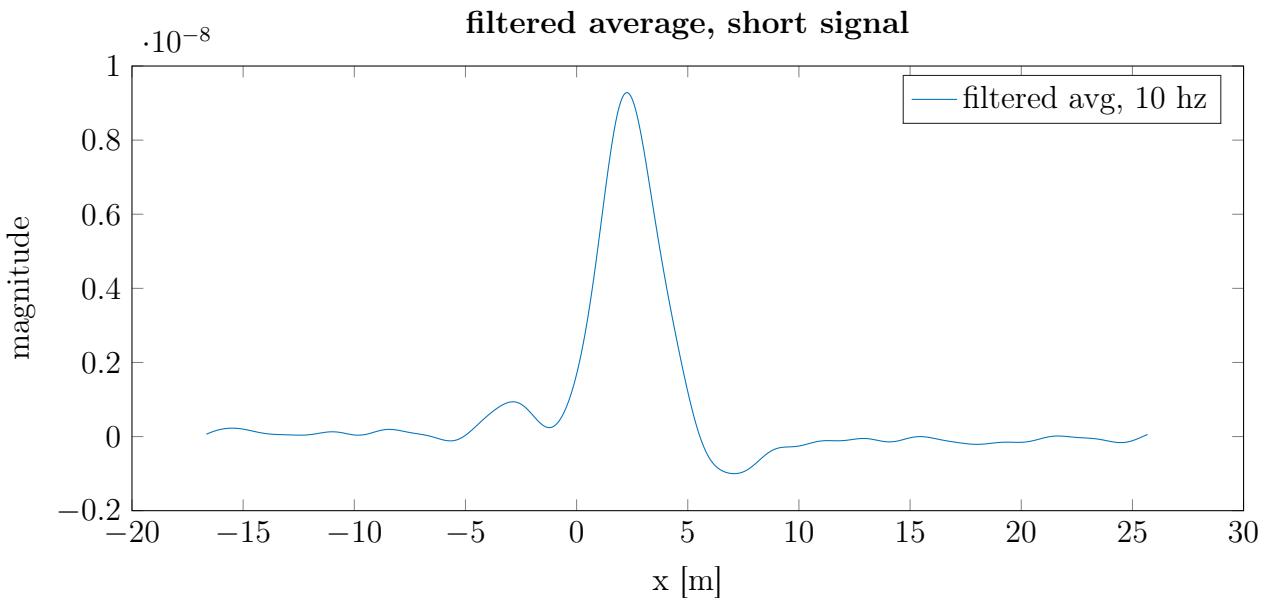


Figure 22: Shorter averaged influence line for the middle sensor without train 5

A possible way to place the found influence line is shown in figure 20, which places the influence line in the assumed position on the bridge. The maximum magnitude of the influence line should be found at the sensor location, thusly the average influence line has been placed in the coordinate system of the bridge accordingly. There is however the problem of noise, which makes identifying the actual max peak difficult.

Filtering the signals so that a singular smooth maximum peak can be identified. This could distort the actual signal, but will be an interesting approach.

This placement of the first peak and it's influence line also decides the following placement of the influence line which are decided by the axle spacings found by the BWIM system. Therefore a wrongly placed first axle would provide increasing error for each axle found. Alternatively each influence line could be placed according to the identified axles or bogies. Since the axle distances is known for the signals in this thesis identifying the first axle or bogie and placing the rest

### 5.2.3 Influence lines from filtered strain

### 5.2.4 Influence lines from unfiltered strain

## 5.3 Optimized influence lines

Perform the same procedures as for the matrix method

## 5.4 Differences between the methods

Compare the optimized influence lines and the matrix method influence lines. This should be done in a thorough manner.

## 5.5 Problems

- Big problem with identifying exactly when train enters and leaves the bridge. This results in guesswork when placing influence line in a coordinate system. Where does the bridge begin and end in the influence line.. The only definite certainty seems to be placing the index of the maximum magnitude of the influence line in the correct position according to the measuring sensor's location.
- This could be problematic when using the found influence lines
- These problems have been reduced, now the biggest problem is placing the peak of the influence line as well as possible. Possibly performing a smoothing and then finding position of peak could give a better estimate of sensorloc at influence line.. currently the max value of influence line is placed at sensorloc.

### 5.5.1 Calculate the axle weights

The axle weights used to calculate the influence lines are as in table 5, which values correspond to an empty train. The system setup described in section 4.3, gives us three different locations for measuring strain and so we have three different influence lines generated by the BWIM program. When calculating the axle weights corresponding to each train, we will then have three different estimates of the axle weights. However, since we do not know the exact weights of the trains, estimating correctness of the BWIM will be speculative.

Axle	1	2	3	4	5	6	7	8
Axle weight [kg]	9500	9500	9500	9500	14575	14575	14575	14575
sum	38000				58300			
sum total	96300							

Table 5: Table of axle weights used to perform BWIM

Figures 24 and 23 are the influence lines used to calculate the axle weights in the tables 8 and 12. By studying the different influence lines, it is clear that the

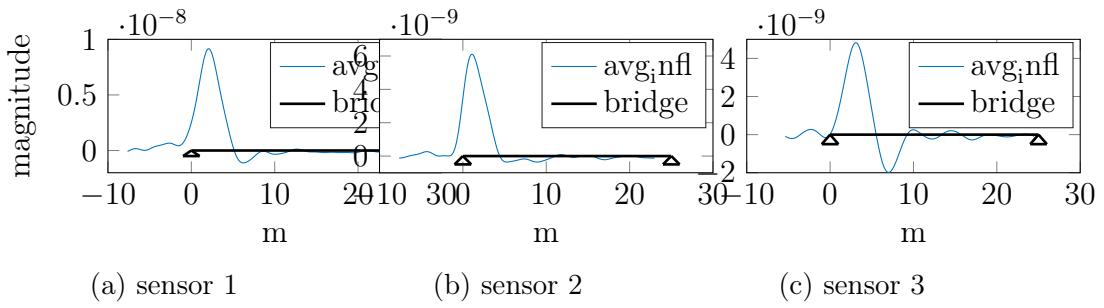


Figure 23: averaged influence lines used to calculate axle weights

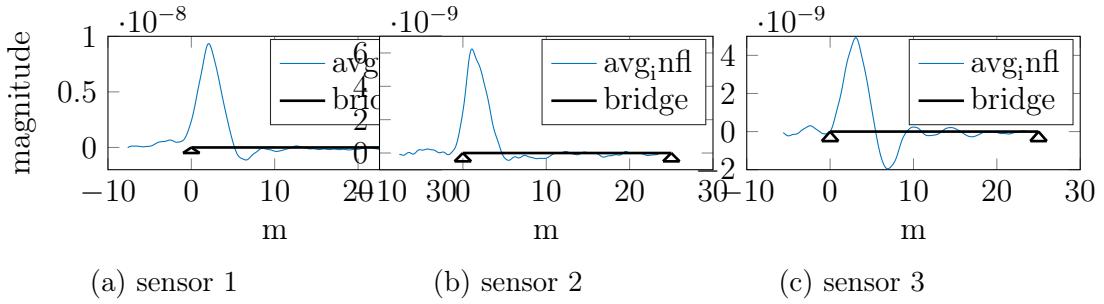


Figure 24: averaged influence lines, based on filtered strains, used to calculate axle weights

visible differences between the two variants are minimal. This can also be seen in their respected tables. The influence lines produced by filtered strain seems to produce a influence line of slightly lower magnitude, which when used to calculate axle weights results in slightly higher values.

As seen in tables 8 and 12, there are differences between the calculated axle weights for each sensor. These are errors which may have one or more reasons.

- the sensors may not be correctly calibrated, resulting in differences in axle weights from sensor to sensor. This can be controlled by simply calculating the ratio between the same axles for different sensors.
  - The initially set axle weights remain constant for each train resulting in influence lines of incorrect magnitude. The averaged influence line would in case of a too low cal

As table ?? shows, there clearly is some error in the calculated axle weights. Especially sensor 2 and 3 which generally gives very low estimates. This trend hold for minimal and extended influence lines as well which means that the sensors have not been calibrated. By looking at the same measurement for a specific axle for one sensor and comparing with the same calculated axle weight for another sensor

	sensor 1					sensor 2					sensor 3				
	trains and their axle weights for sensors														
axle	train 3	train 4	train 5	train 6	train 8	train 3	train 4	train 5	train 6	train 8	train 3	train 4	train 5	train 6	train 8
1	8563	10689	8578	11156	10617	9006	11278	9570	11195	11233	8341	8763	8402	9111	8532
2	9343	10379	9170	10237	10284	7814	8628	7491	8176	8295	9581	9820	9440	10945	10043
3	8709	10294	8817	11353	10353	9521	11446	9983	11940	11668	8837	8563	9203	8752	8320
4	9057	9868	8451	10400	10285	8214	8586	7351	8336	8697	9073	9626	8547	10479	10262
sum car	35672	41230	35016	43146	41539	34555	39938	34395	39647	39893	35832	36772	35592	39287	37157
5	13392	15615	13546	15879	14865	14904	18402	15379	17434	17489	14064	14462	14660	13515	13475
6	14581	14893	13859	15985	16313	13059	13336	11674	13391	14079	16116	15509	15121	18038	17548
7	11303	15097	11479	15656	14380	13238	17679	13678	17332	17278	12374	13561	12595	13615	12636
8	14184	12962	13616	13549	14350	12496	10913	11196	10910	12026	14788	13792	13933	16003	15501
sum loc	53460	58567	52500	61069	59908	53697	60330	51927	59067	60872	57342	57324	56309	61171	59160
sum tot	89132	99797	87516	104215	101447	788252	100268	86322	98714	100765	93174	94096	91901	100458	96317

Table 6: Table of axle weights for averaged influence lines, all trains

it is possible to calculate the ratio between them. If this ratio also holds for the other axles, the relationship between sensor 1 and 2 is almost constant which in the authors opinion shows the uncalibrated nature of the sensors. If at least one trains axle weights were known, it would be possible to scale the sensor readings to the show the correct values. This would in theory make the table above show the correct results.

### 5.5.2 Calibration of sensors

As seen in section 5.5.1, a calibration of the sensors is necessary for correct calculations of a trains axle weights. In normal circumstances this could be done in the following manner.

1. Have a train, of which the known properties are velocity, axle spacings and axle weights, perform one or more runs in both directions.
2. The obtained strain signals from these passings are used along with the Influence line to calculate the axle weights for at least one sensor.
3. The resulting axle weights should have a constant ratio between the same axles for different sensors.

	sensor 1					sensor 2					sensor 3				
	trains and their axle weights for sensors														
axle	train 3	train 4	train 5	train 6	train 8	train 3	train 4	train 5	train 6	train 8	train 3	train 4	train 5	train 6	train 8
1	8819	10971	8837	11301	10858	9788	12060	10353	11531	11859	8042	8436	8093	8916	8241
2	9106	10086	8932	10052	10046	6847	7536	6423	7435	7367	9835	10100	9715	11083	10263
3	8940	10522	9055	11488	10561	10343	12185	10765	12282	12338	8625	8317	8994	8561	8081
4	8822	9620	8207	10252	10066	7231	7577	6313	7563	7740	9376	9928	8867	10687	10522
sum car	35687	41199	35031	43093	41531	34209	39358	33854	38811	39304	35878	36781	35669	39247	37107
5	13772	16046	13938	16095	15283	16131	19592	16561	17926	18582	13572	13946	14169	13138	12950
6	14255	14494	13517	15729	15949	11548	11597	10066	12196	12487	16510	15931	15561	18235	17885
7	11696	15513	11883	15874	14771	14504	18803	14944	17770	18271	11909	13092	12127	13310	12179
8	13866	12567	13280	13332	13990	11051	9210	9625	9748	10496	15178	14230	14359	16217	15850
Sum loc	53589	58620	52618	61030	59993	53234	59202	51196	57640	59836	57169	57199	56216	60900	58864
Sum tot	89276	99819	87649	104123	101524	87443	98560	85050	96451	99140	93047	93980	91885	100147	95971

Table 7: Table of axle weights for averaged influence lines, where strains have been filtered, all trains

	trains and their axle weights for sensors															
	sensor 1	sensor 2	sensor 3													
trains and their axle weights for sensors																

Table 8: Table of axle weights for averaged influence lines, where strains have been filtered, all trains

4. The axle weights are scaled to equal the known values. These scalars are the calibrating constants for the sensors.
5. The scalars obtained could be used directly on the signal data, but the only part of the BWIM which directly requires this scaling for correct results are the calculated axle weights.

Due to limited field data, the only train which is usable for calibrating the sensors is the freight train. The freight train has one constant which the other trains do not have, a locomotive which will have axle weights approximately equal the given properties of the locomotive as listed in B.2. The calibration performed in this project was performed thusly:

1. Identify the first 2 major peaks of the signal corresponding to the 6 axles of

the locomotive and cut the signal accordingly.

2. Find the speed of the train as well as possible.
3. Use the influence line found through the other trains to calculate the axle weights for each sensor.
4. Find the scalar giving correct results for each sensor.

The strain signal however proved difficult or impossible to cut correctly, due to the length of the locomotive and the width of the influence line the next axle after the locomotive also influenced the signal, which would affect the results. The best suited sensor for this task proved to be the sensor closes to the support on the side towards Trondheim. Figure ?? shows the first peaks of the strain signal for this sensor, where the first two major peaks corresponds to the axles of the locomotive. The red circles named first cutting point and first boigie over, shows a possible cut of the signal could be made. The third and fourth major peak indicates axles of the vagons. The two first peaks should ideally have had the same level of magnitude, the fact that they do not shows that the first and second set of axles influence the sensor at the same time. The second point also is raised above the first point, which also is the case for the next peak corresponding to vagon axles. A safe signal could based on this not be found to perform calibration with, as it would provide a error. Therefore this a calibration of the sensors have not been achieved in this project.

This way of calibration is prone to error. The speed of the freight train is unknown, and the methods described in the thesis for obtaining the speed will not work as well as for the other trains, where all the axle spacings are known. So if a successful identification of the locomotive's axles could be found in the strain signal for at least one sensor, the system could be calibrated. However, due to the width of influence around the sensor, there may not be a set of peaks in the strain signal which influenced only by the locomotive's axles. Also, the method used so far to determine the speed has been based on the difference between the strain history produced by calculated influence lines, so determining the speed of the freight train will produce a probable error. The length of the locomotive from axle 1 to axle 6 is 12.2 meters, and the distance from the locomotive's last axle to the next axle of the train is unknown. Sources of error are many

- The program does not know the exact axle weights, which are required for an exact influence line.
- There might be errors in the calculated speed of the influence line. When the speed is wrong the influence line typically gains additional peaks and curves, which could be confused with dynamic effects.

- The noise levels vary for the different sensors and trains. This difference from train passing to train passing will cause error when smoothing influence lines.
- There is not enough data gathered to produce a general averaged influence line.
- Error in the placement of found influence lines, which is a likely cause of error due to filtering.

## 6 Conclusion and summary

## References

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- [5] Michael Quilligan. “Bridge Weigh-in Motion : Development of a 2-D multi-vehicle algorithm”. NR 20140805. PhD thesis. KTH, Civil and Architectural Engineering, 2003, pp. viii, 144.
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## A Dynamics

A train traversing a railway bridge creates actions in longitudinal, lateral, and vertical directions. Braking and traction from a passing train causes longitudinal forces Rocking, or rotations around an axis parallel to the longitudinal axis of the bridge, and vertical dynamic forces are created by structure-track-vehicle conditions and interactions.

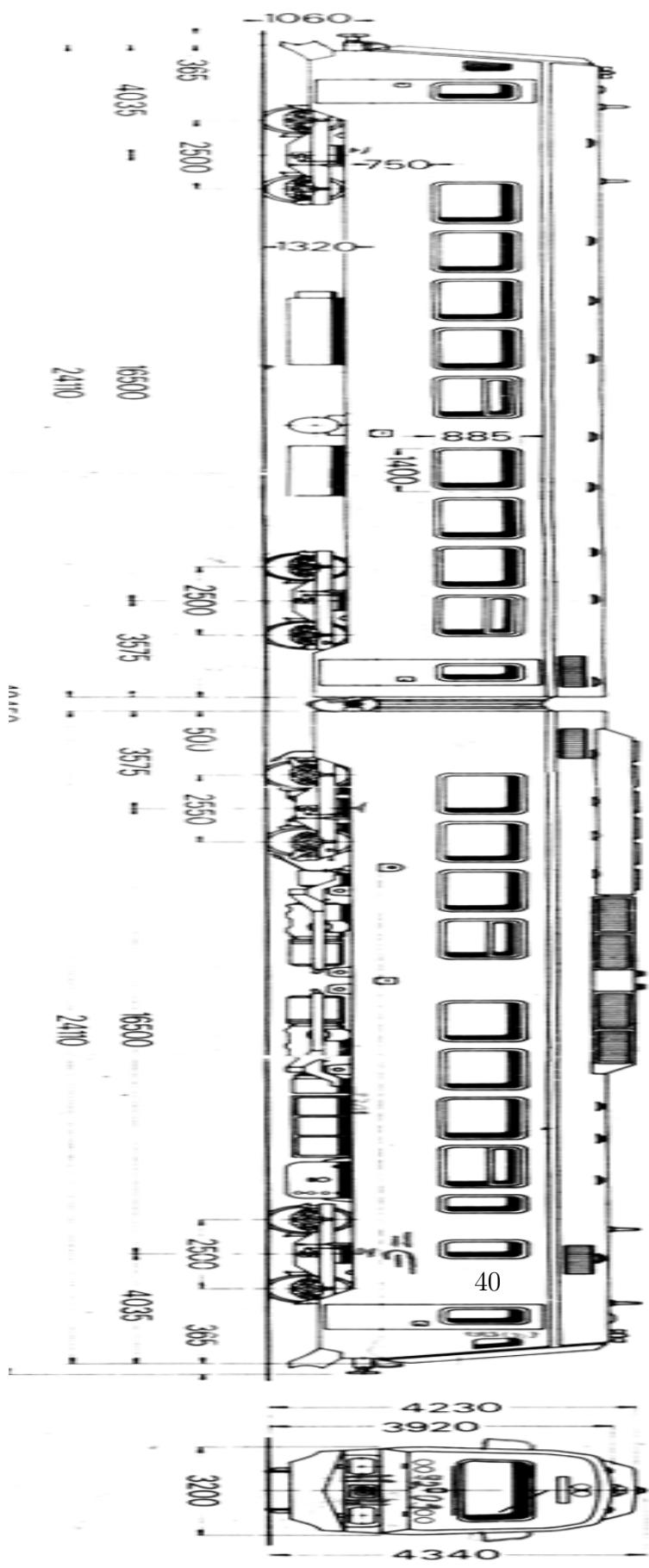
## A.1 Rocking and vertical dynamic forces

Lateral rocking of moving vehicles provide amplification of vertical wheel loads. This amplification increases the stresses in the members supporting the track. Superstructure-vehicle interaction creates a vertical dynamic amplification of moving loads, which will result in vibrations causing additional stresses in members supporting the track. The unloaded simply supported beam frequency  $\omega_1 = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{m}}$ , provides a basic indicator of superstructure vertical dynamic response.



## B Trains

## B.1 NSB92



## B.2 Freight train - EL14

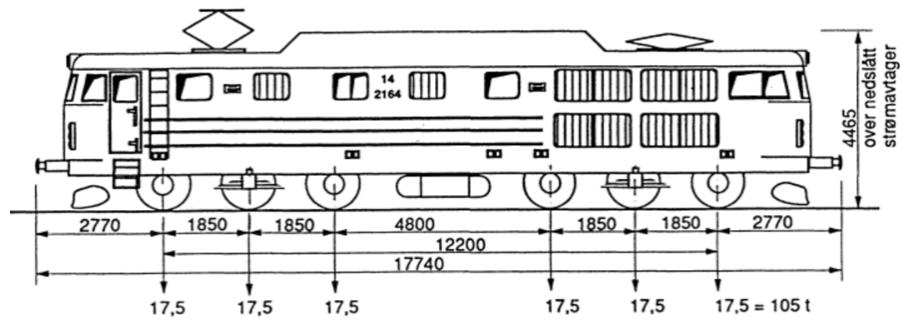


Figure 25: Axle distances and weights for a EL14 locomotive

## C Figures

### C.1 Recreated strain signals

### C.2 Influence lines all sensors

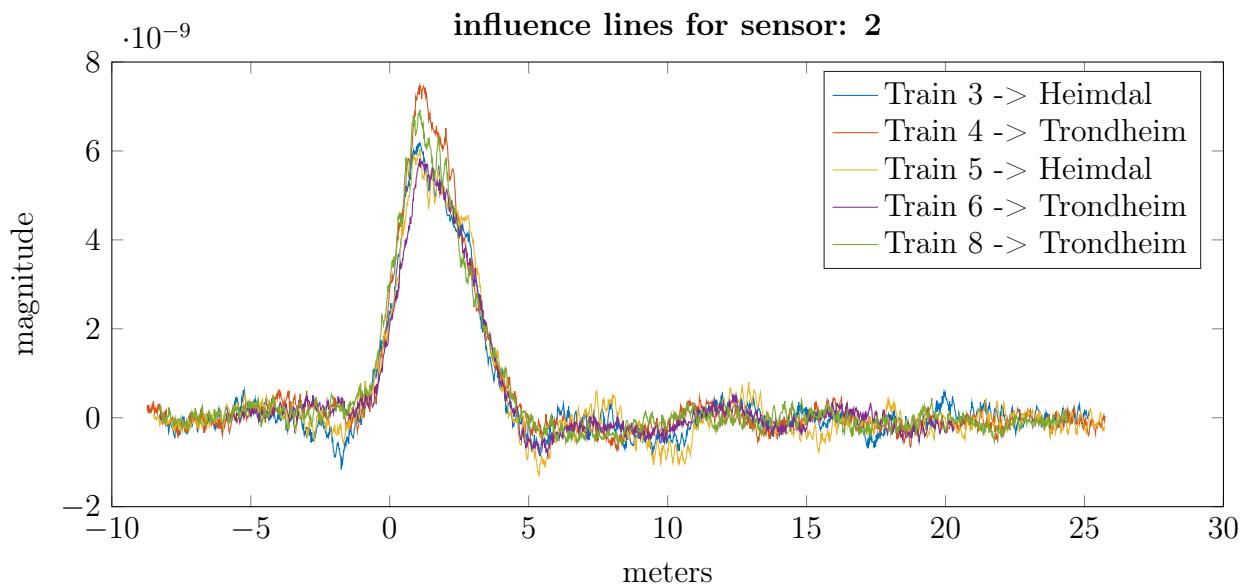


Figure 35: Influence lines from figure:14 on top of each other for sensor 2

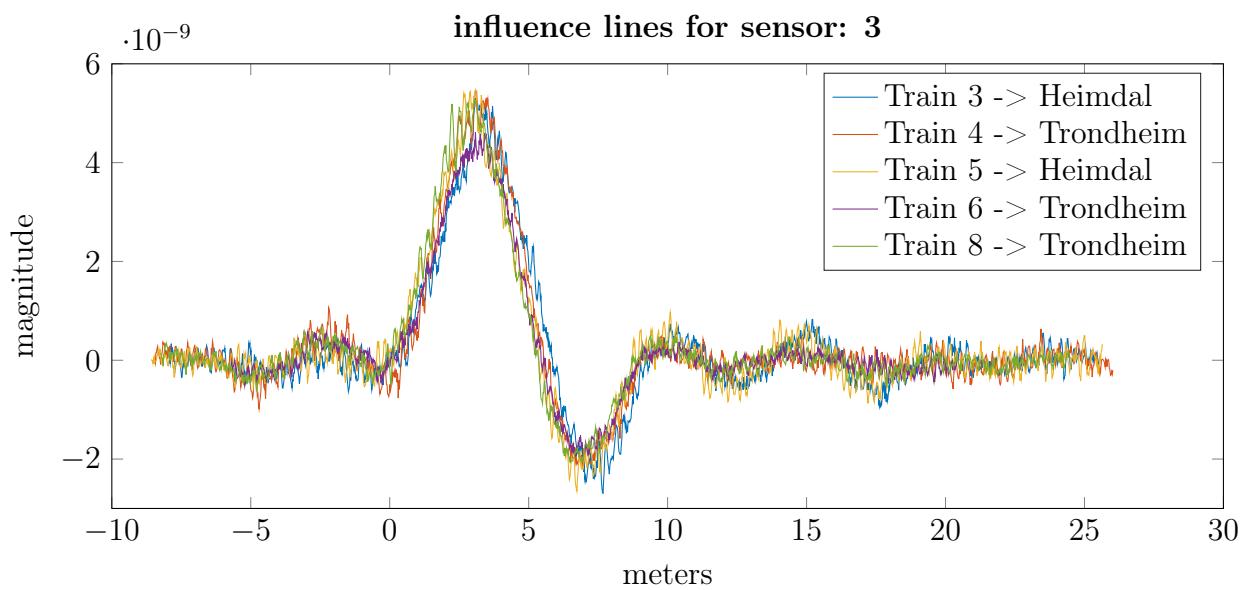


Figure 36: Influence lines from figure:14 on top of each other for sensor 3

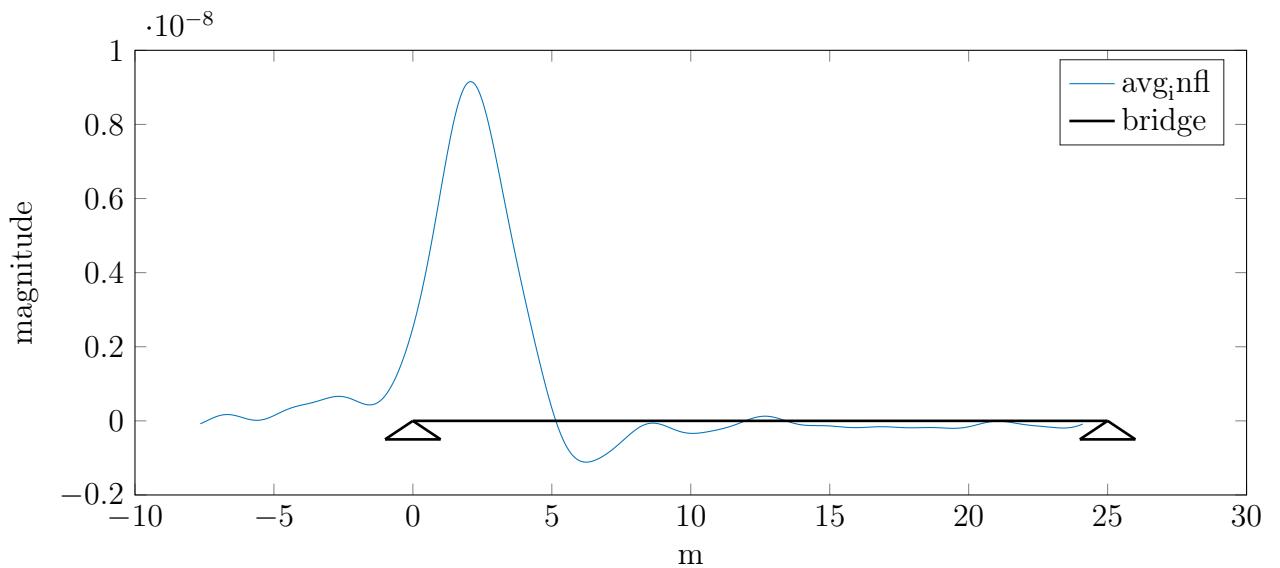


Figure 37: Averaged influence line for sensor 2

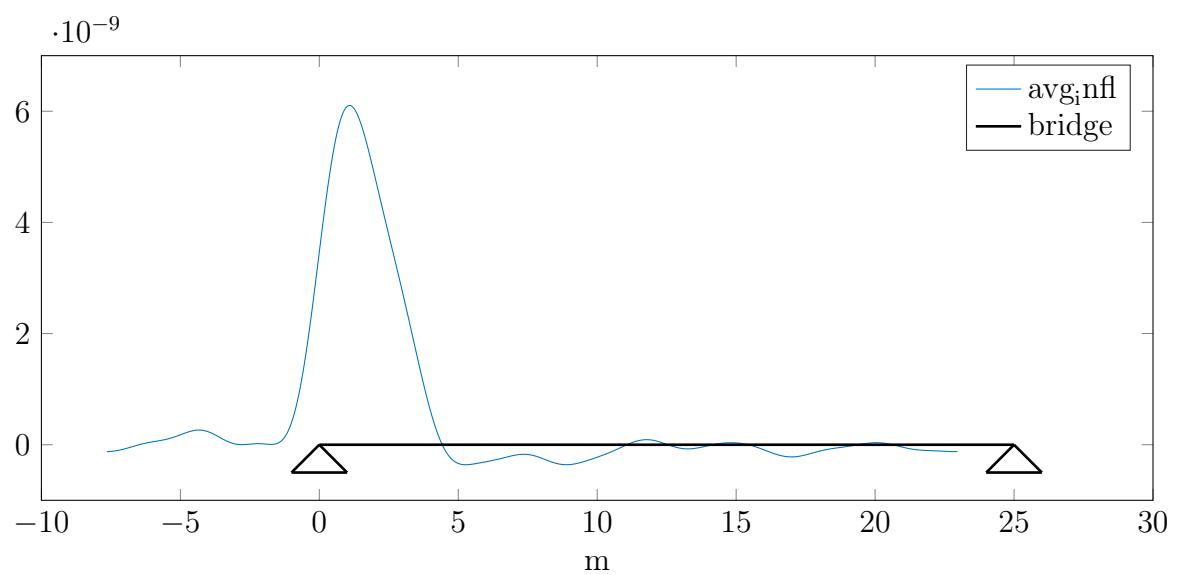


Figure 38: Averaged influence line for sensor 2

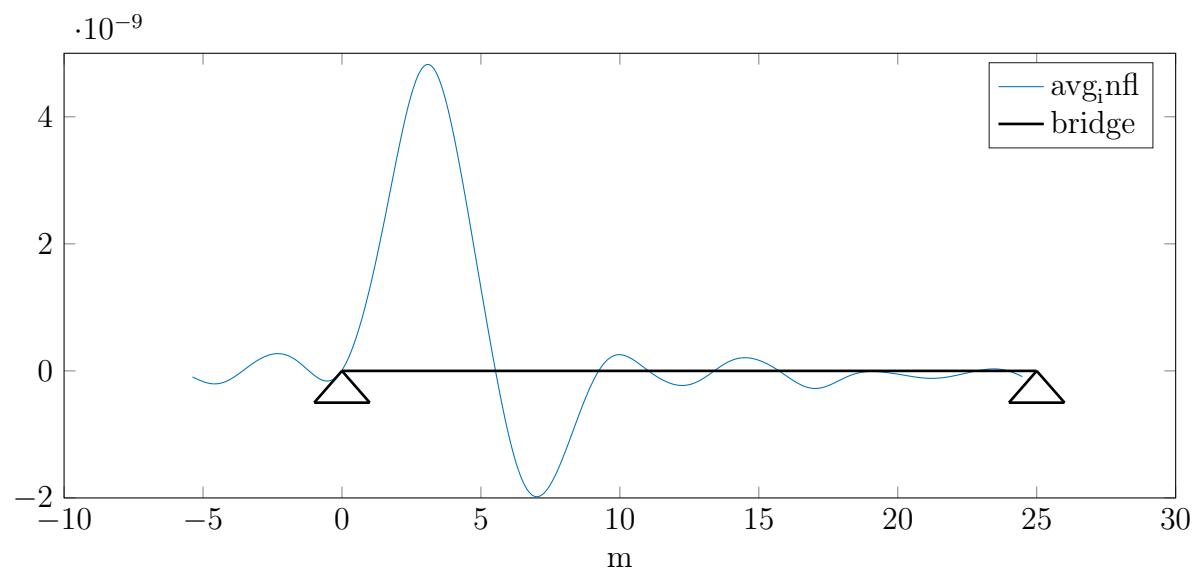


Figure 39: Averaged influence line for sensor 3

## D Calculated axle weights

### D.1 Unfiltered strain

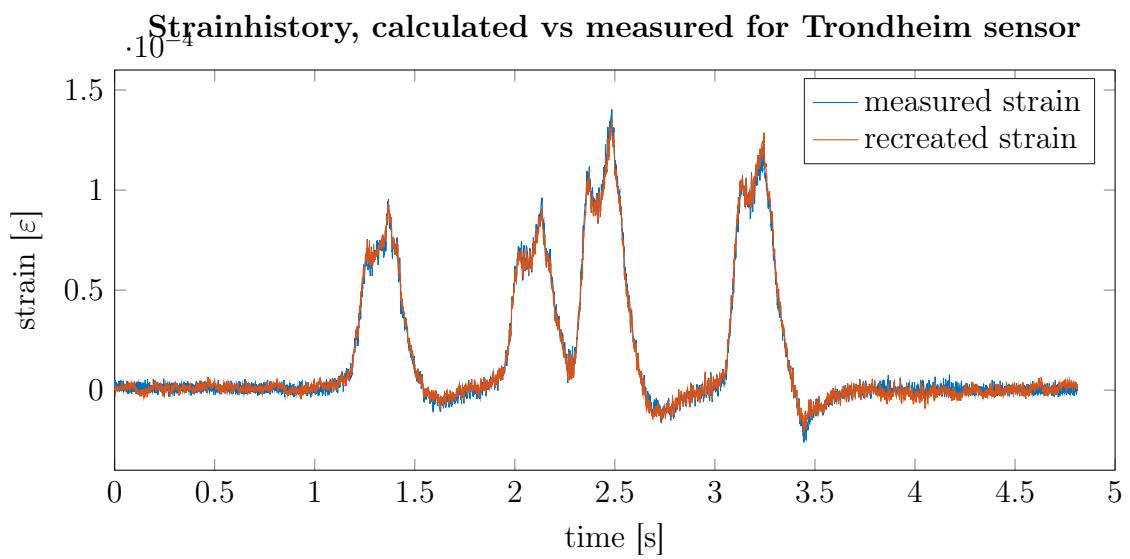
axle	sensor 1				sensor 2				sensor 3			
	trains and their axle weights for sensors											
	train 3	train 4	train 6	train 8	train 3	train 4	train 6	train 8	train 3	train 4	train 6	train 8
1	8721	10984	11155	10755	6151	8109	8211	8056	6736	7279	7094	6960
2	9106	9975	10208	10154	6744	7299	7072	7235	5726	5872	6772	6237
3	8791	10211	10960	10091	6658	8159	8652	8174	6860	6747	6623	6368
4	8444	9359	10340	10037	6413	6581	7003	7060	4519	4952	6137	5717
sum car	35062	40529	42663	41037	25966	30148	30938	30525	23841	24850	26626	25282
5	13511	15291	15949	14452	10354	12860	12698	12029	10217	10025	10142	9252
6	14238	14945	15802	16555	11069	11591	11474	12387	10005	10122	11362	11809
7	10909	13789	15030	13248	8675	11665	12444	11464	9507	10152	10277	9291
8	13517	13084	13267	14338	10516	9954	9502	10769	8838	8519	9413	9807
sum loc	52175	57109	60048	58593	40614	46070	46118	46649	38567	38818	41194	40159
sum tot	87237	97638	102711	99630	66580	76218	77056	77174	62408	63668	67820	65441

Table 9: Table of axle weights for short influence lines

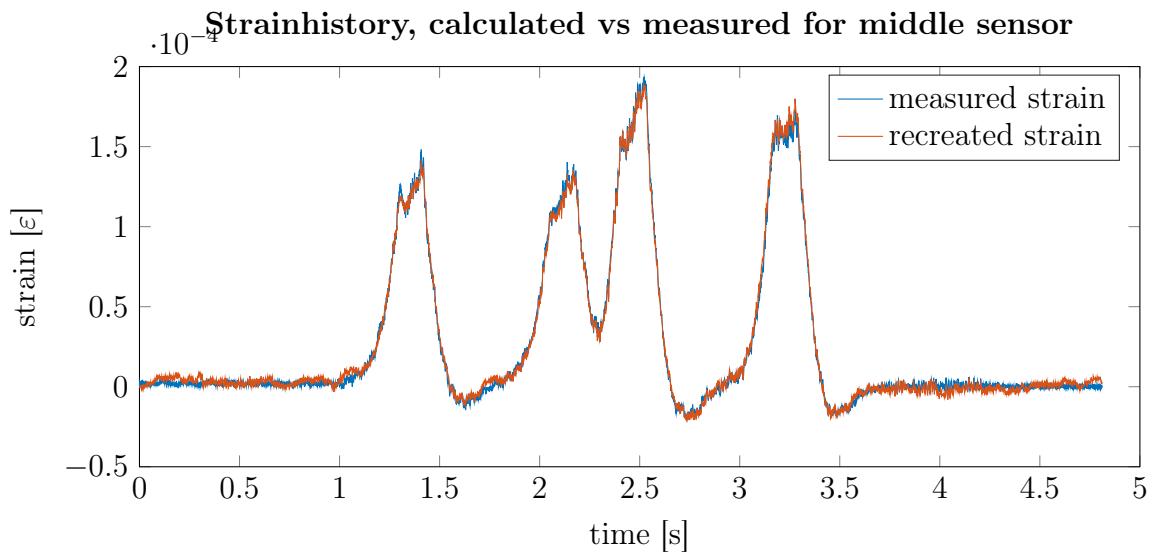
### D.2 FilteredStrain

	sensor 1				sensor 2				sensor 3			
axle	trains and their axle weights for sensors											
1	8896	11072	11098	10751	6342	8176	8235	8075	6829	7343	7153	7044
2	8385	9145	9480	9480	6202	6756	6581	6763	5303	5434	6389	5796
3	9249	10688	10901	10574	6944	8376	8573	8425	7144	7022	6732	6702
4	8012	8945	9645	9549	6079	6328	6526	6702	4238	4693	5757	5393
sum car	34542	39850	41124	40354	25567	29636	29915	29965	23514	24492	26031	24935
5	14008	15758	15818	15033	10717	13195	12755	12405	10647	10437	10426	9787
6	13357	14129	14881	15585	10384	10893	10790	11637	9291	9447	10824	11035
7	12235	14976	15334	14668	9547	12519	12699	12419	10207	10772	10617	10061
8	13120	12924	12755	13885	10215	9735	9127	10388	8540	8307	9063	9500
sum loc	52720	57787	58788	59171	40863	46342	45371	46849	38685	38963	40930	40383
sum tot	87262	97637	99912	99525	66430	75978	75286	76814	62199	63455	66961	65318

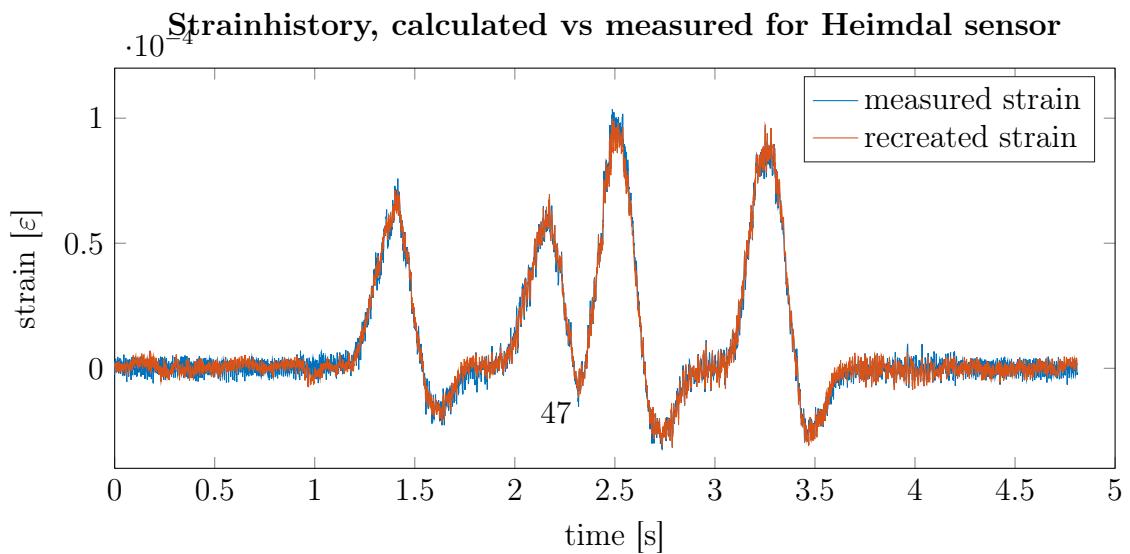
Table 10: Table of axle weights for long influence lines



(a) Recreated strain, Trondheim sensor, train4

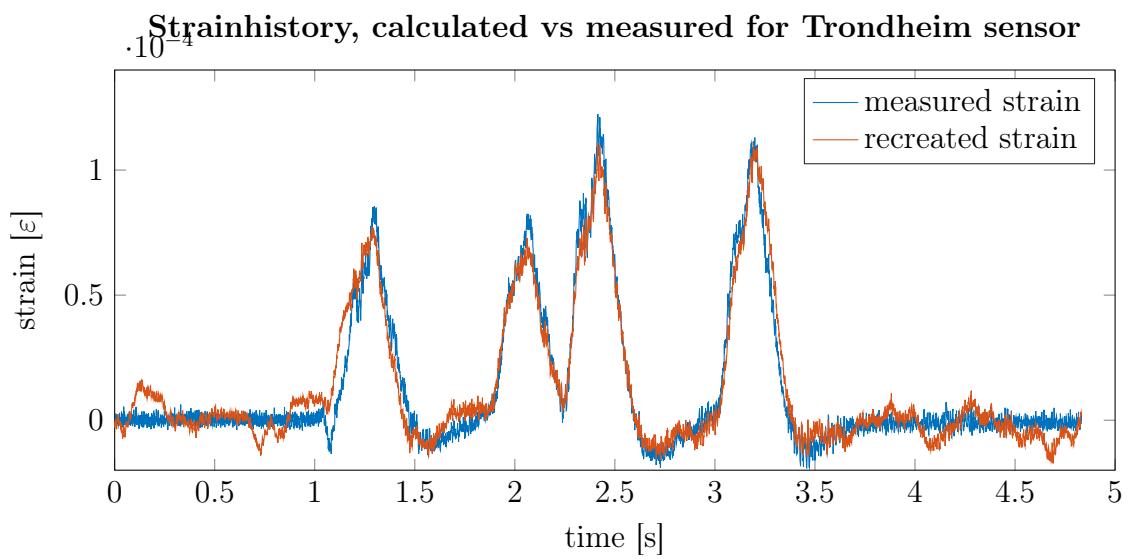


(b) Recreated strain, middle sensor, train4

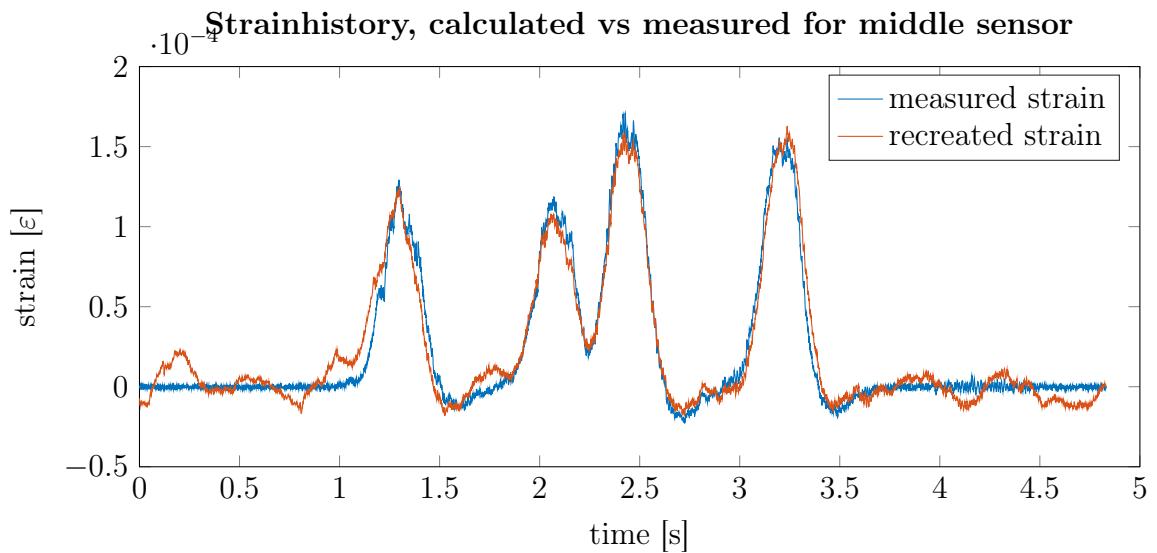


(c) Recreated strain, Heimdal sensor, train4

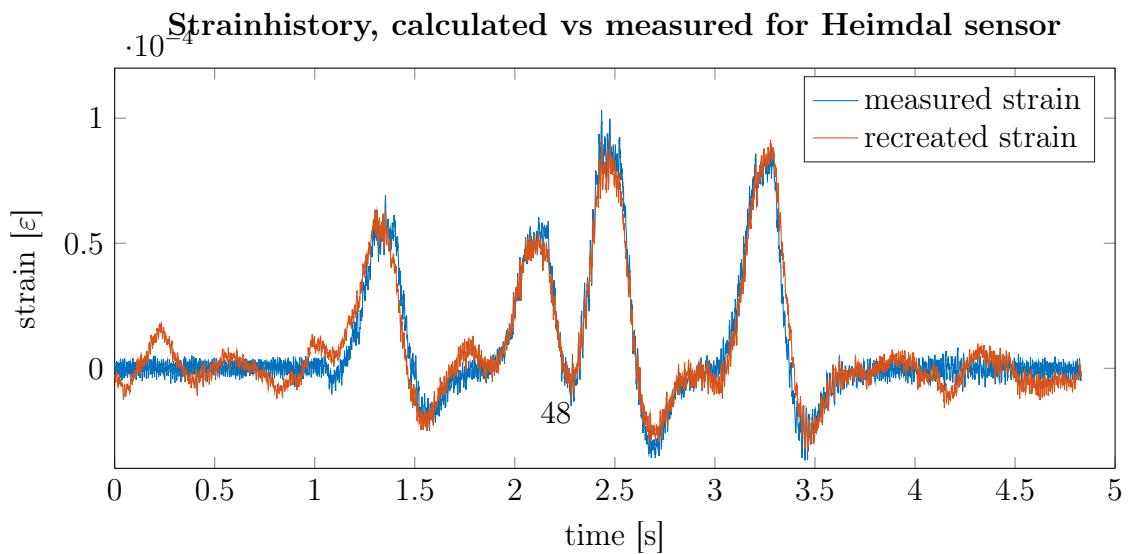
Figure 26: Recreated strain signals for train 4



(a) Recreated strain, Trondheim sensor, train5

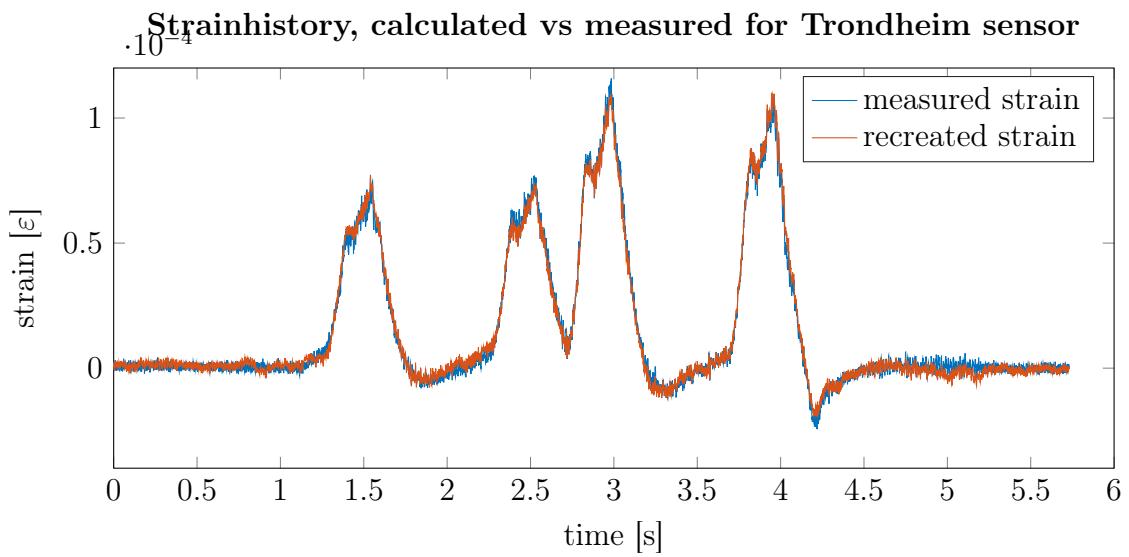


(b) Recreated strain, middle sensor, train5

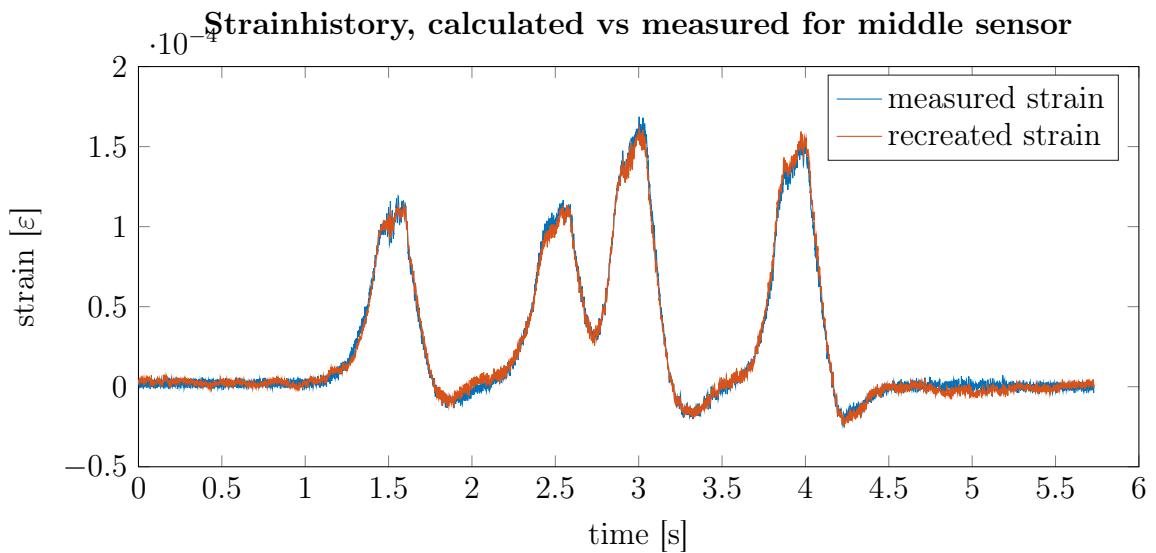


(c) Recreated strain, Heimdal sensor, train5

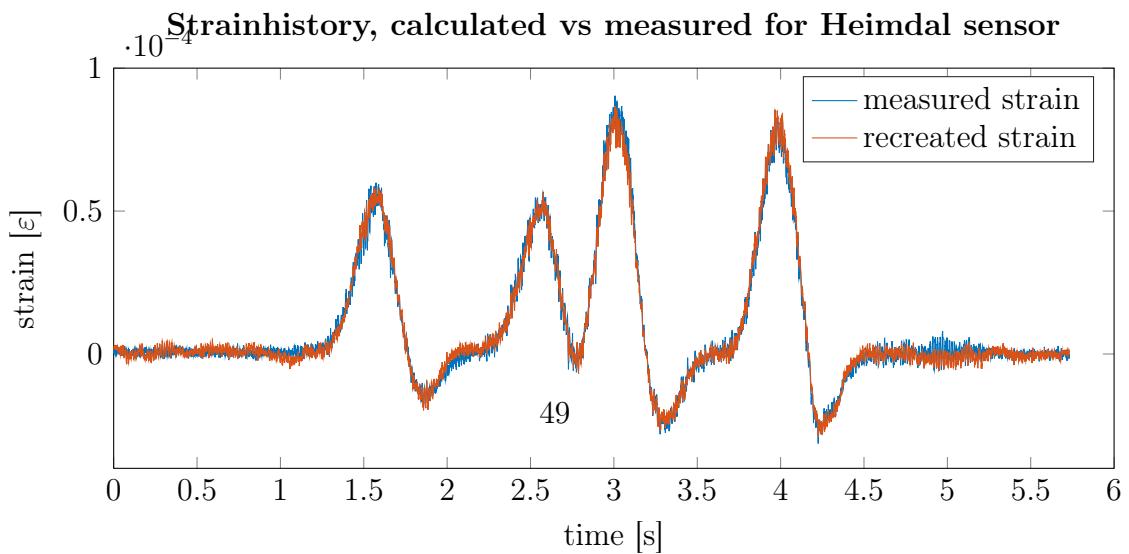
Figure 27: Recreated strain signals for train 5



(a) Recreated strain, Trondheim sensor, train6

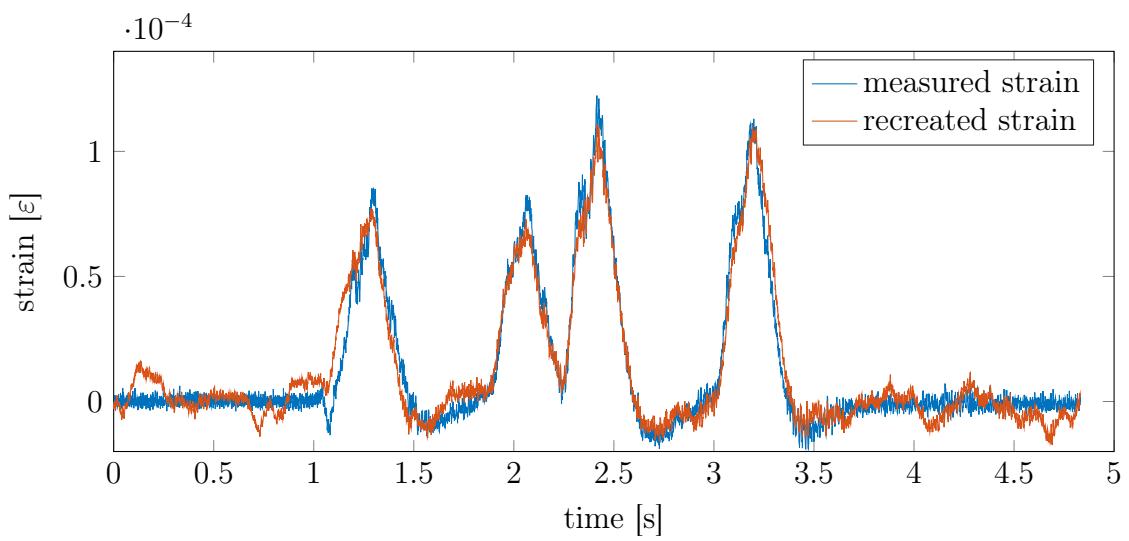


(b) Recreated strain, middle sensor, train6

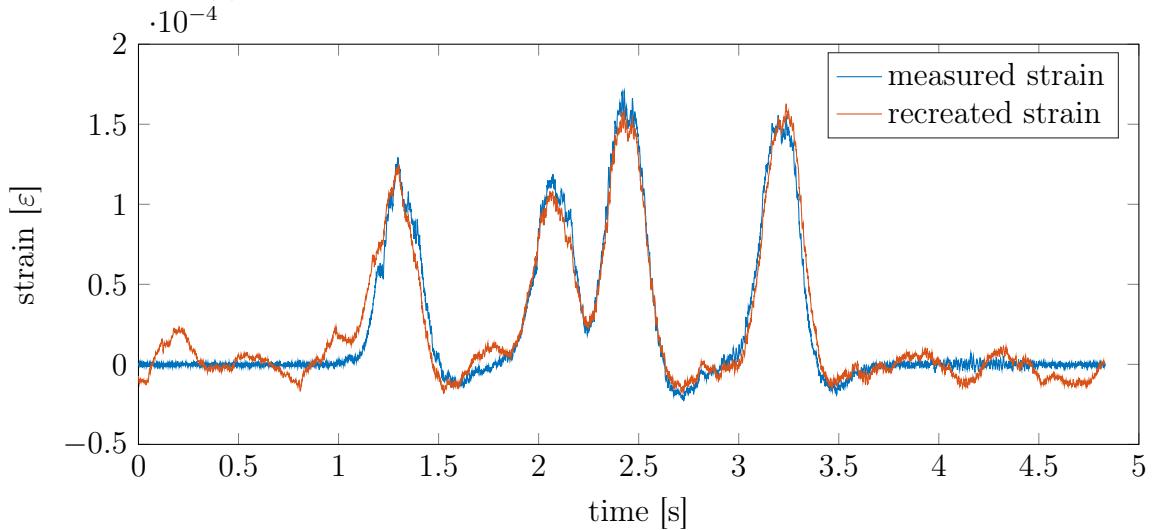


(c) Recreated strain, Heimdal sensor, train6

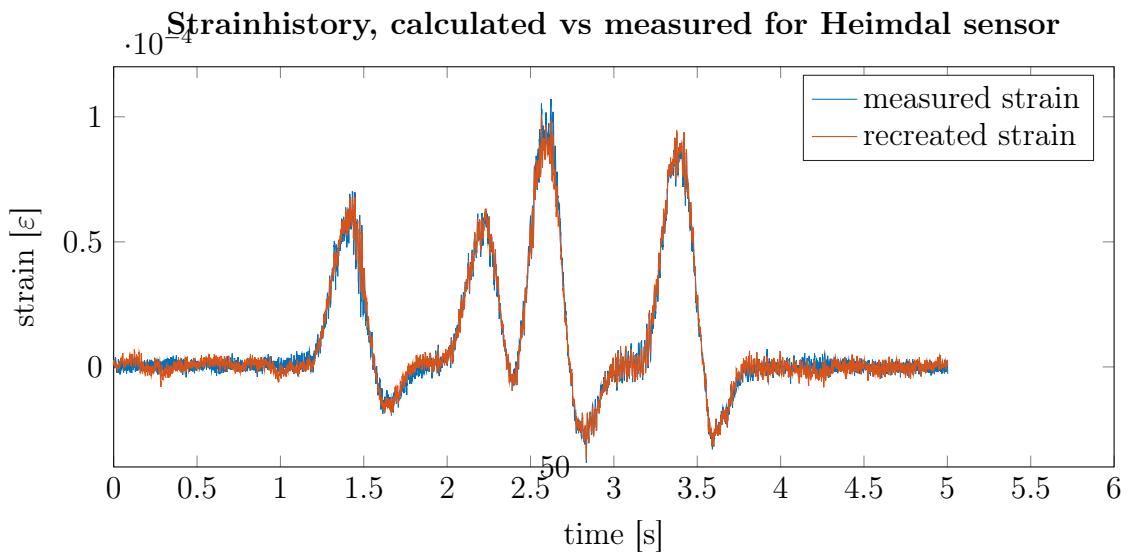
Figure 28: Recreated strain signals for train 6



(a) Recreated strain, Trondheim sensor, train8

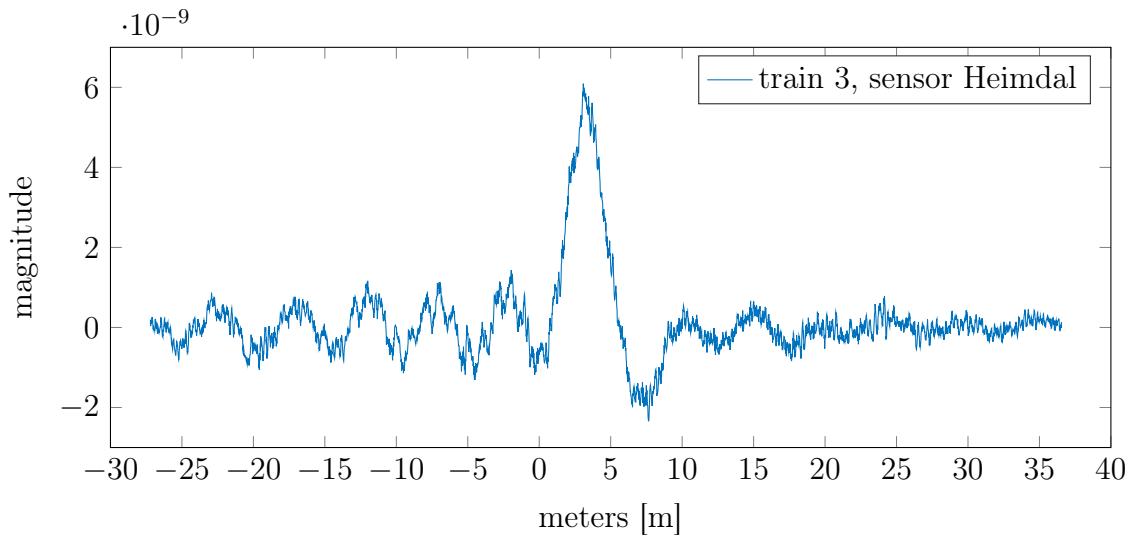


(b) Recreated strain, middle sensor, train8

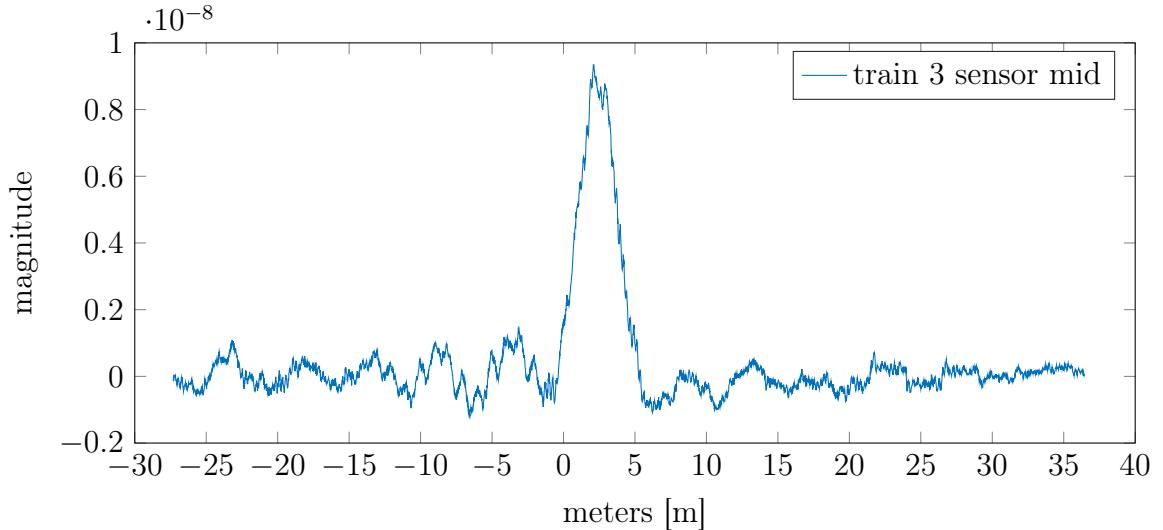


(c) Recreated strain, Heimdal sensor, train8

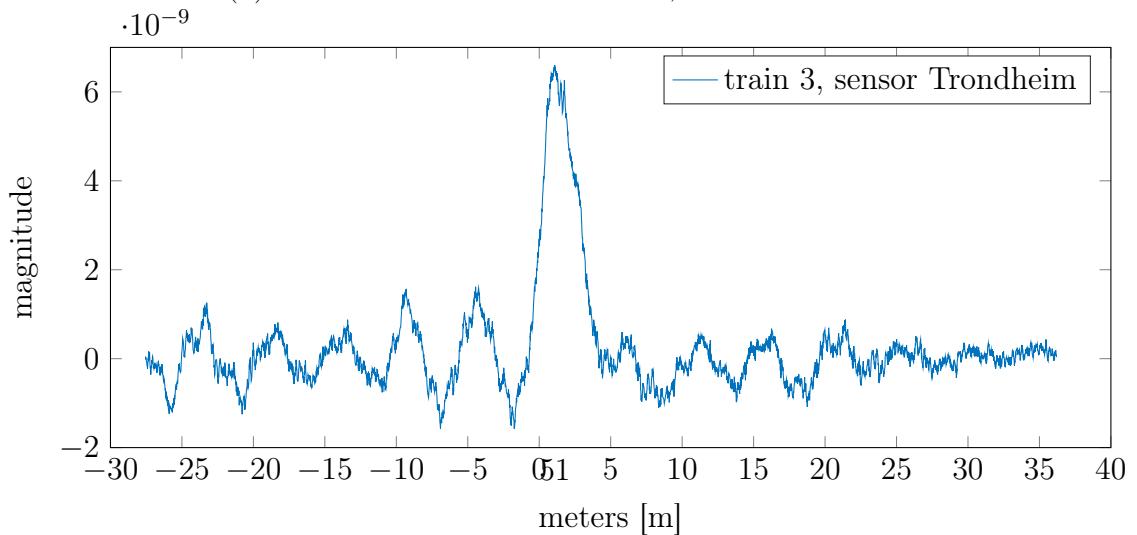
Figure 29: Recreated strain signals for train 8



(a) Influence line for sensor towards Heimdal, train 3

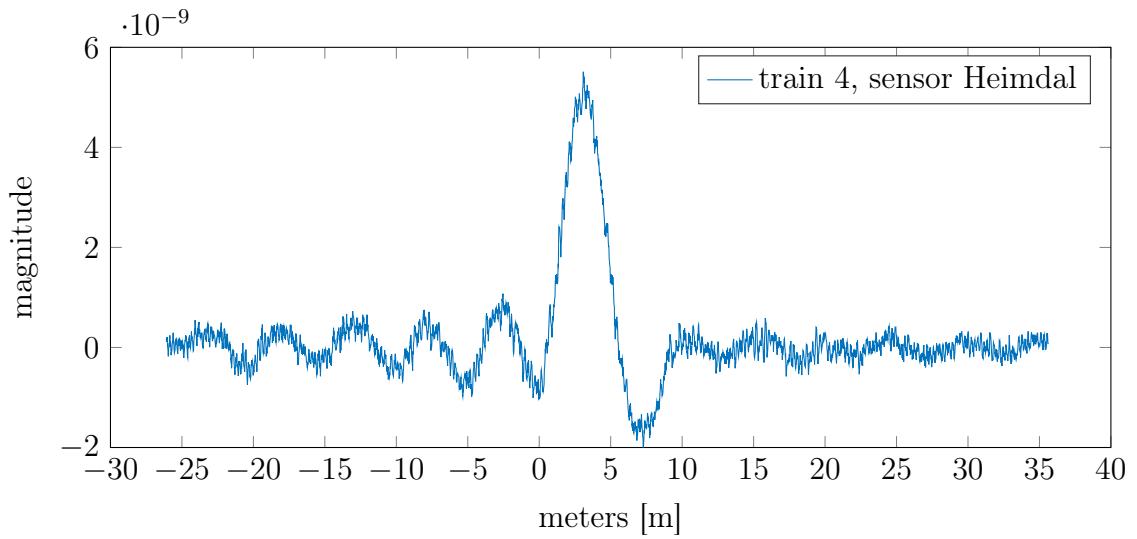


(b) Influence line for middle sensor, train 3

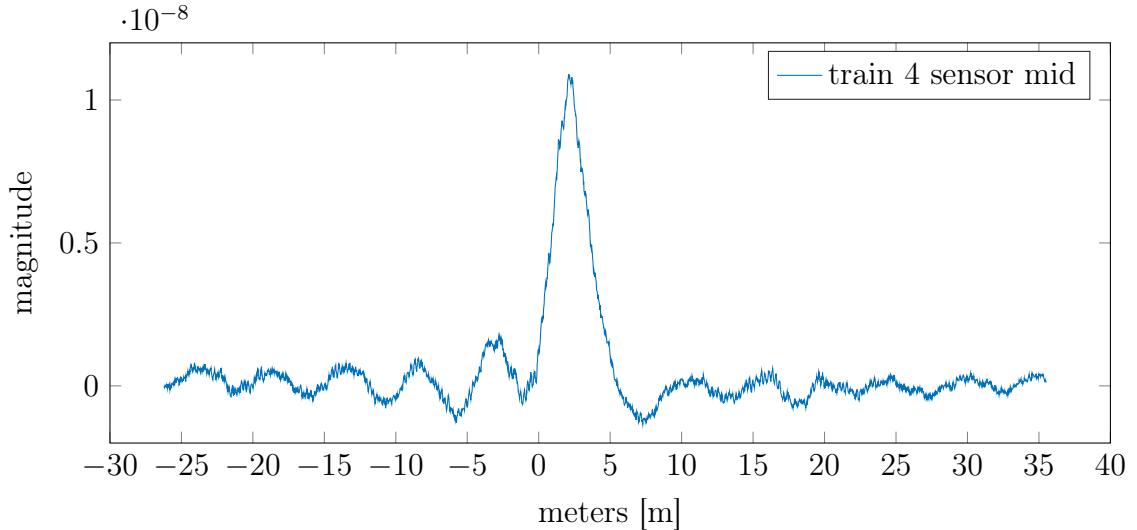


(c) Influence line for sensor closest Trondheim, train 3

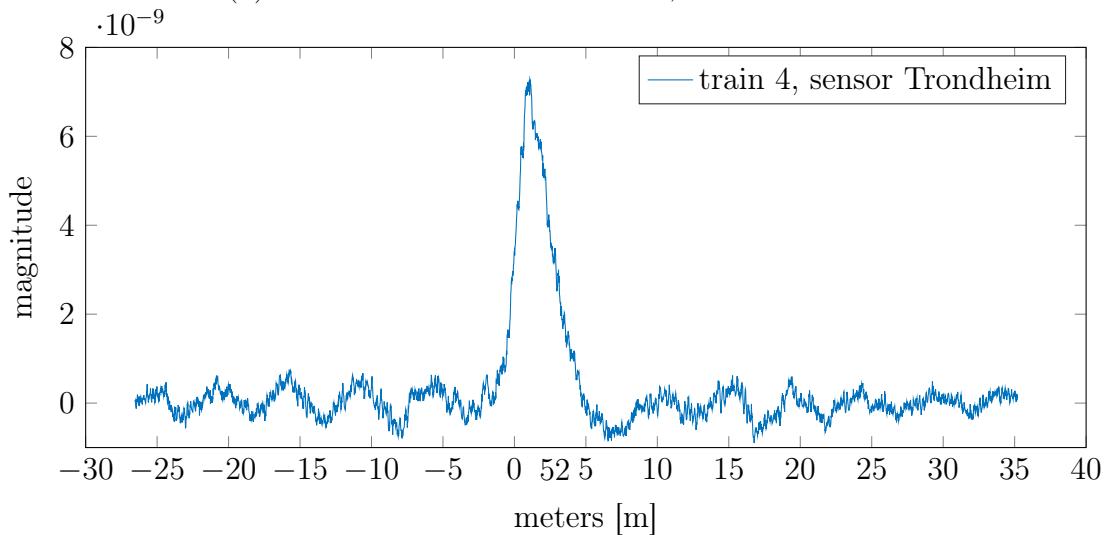
Figure 30: Influence lines train 3



(a) Influence line for sensor towards Heimdal, train 4

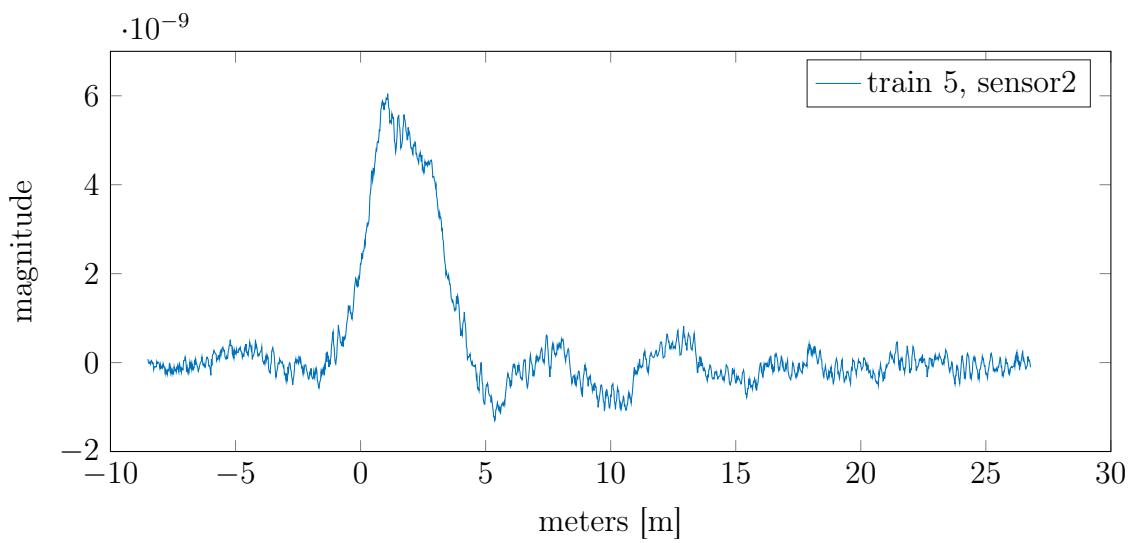


(b) Influence line for middle sensor, train 4

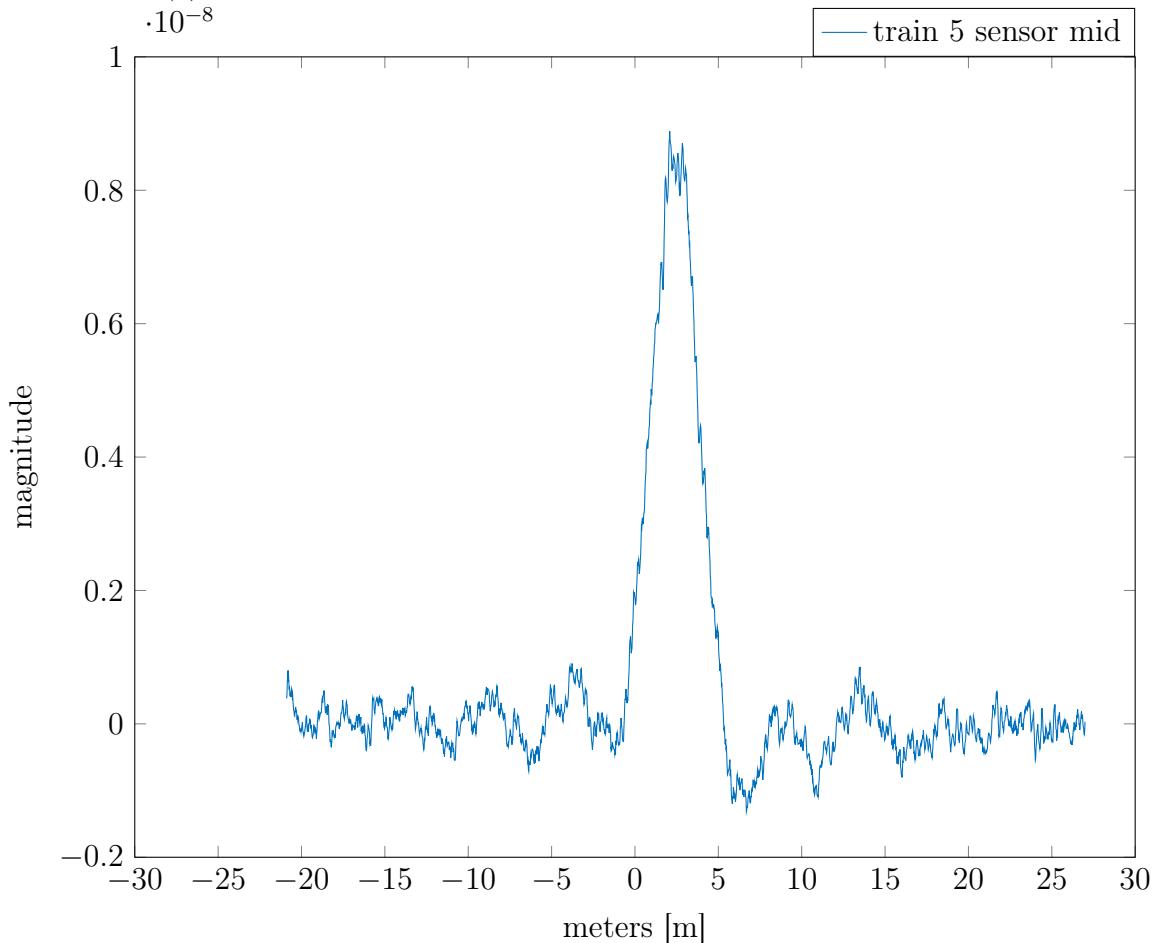


(c) Influence line for sensor closest Trondheim, train 4

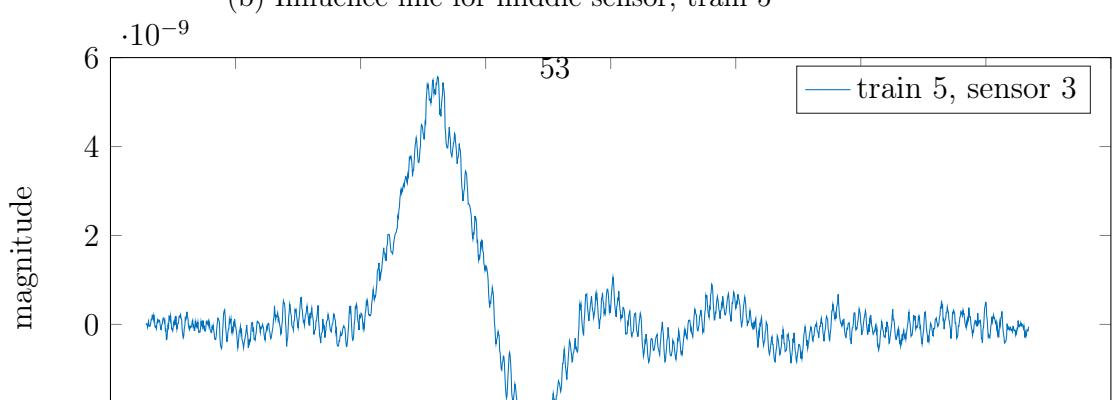
Figure 31: Influence lines train 4

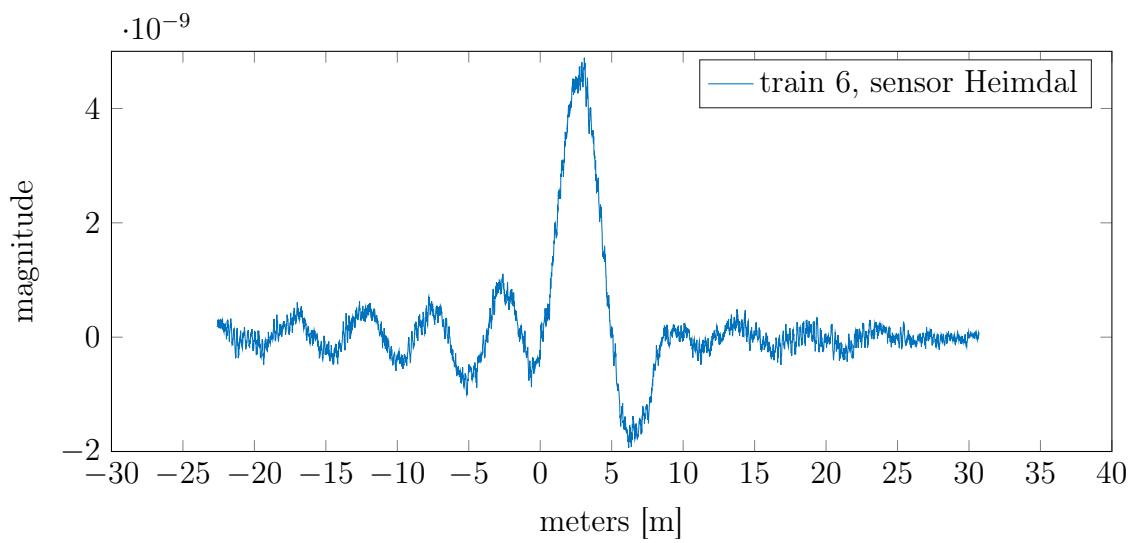


(a) Influence line for sensor towards Heimdal, train 5

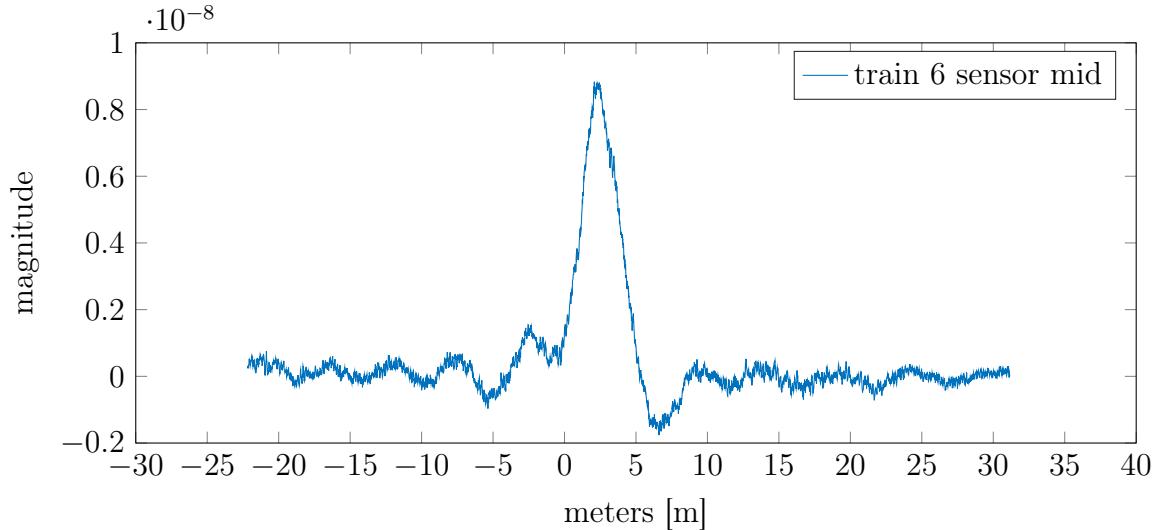


(b) Influence line for middle sensor, train 5

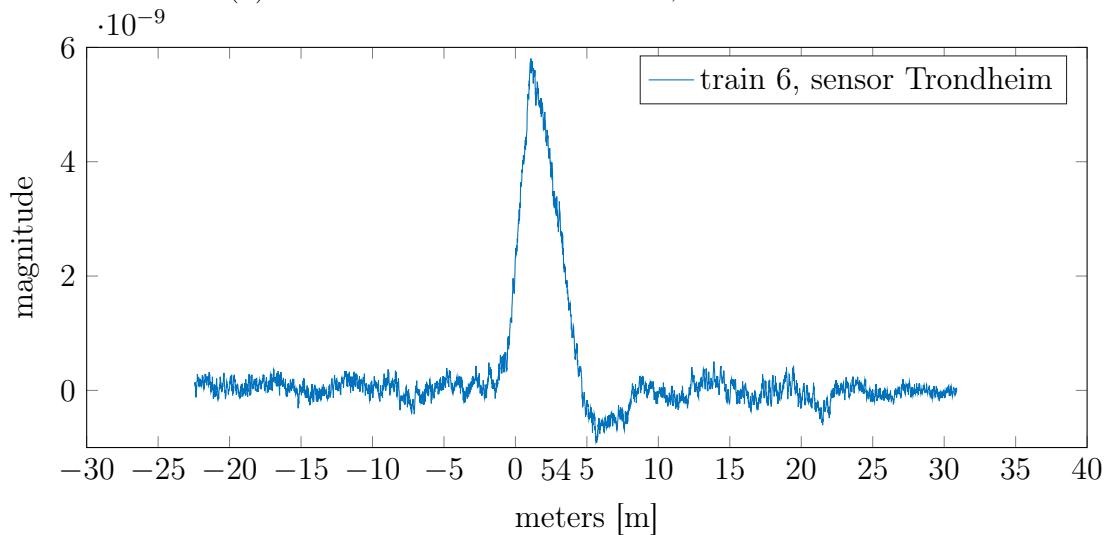




(a) Influence line for sensor towards Heimdal, train 6

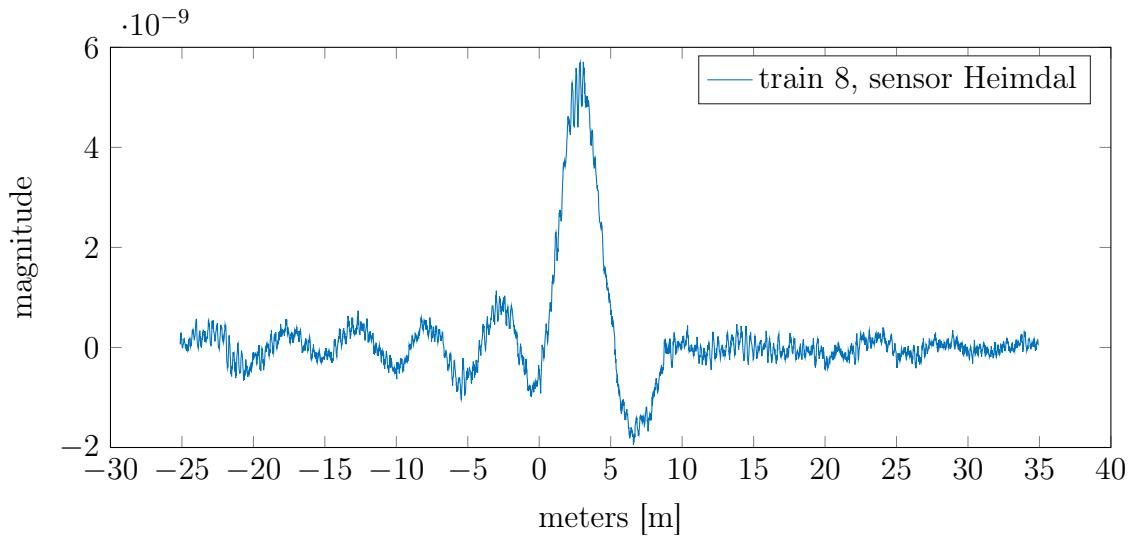


(b) Influence line for middle sensor, train 6

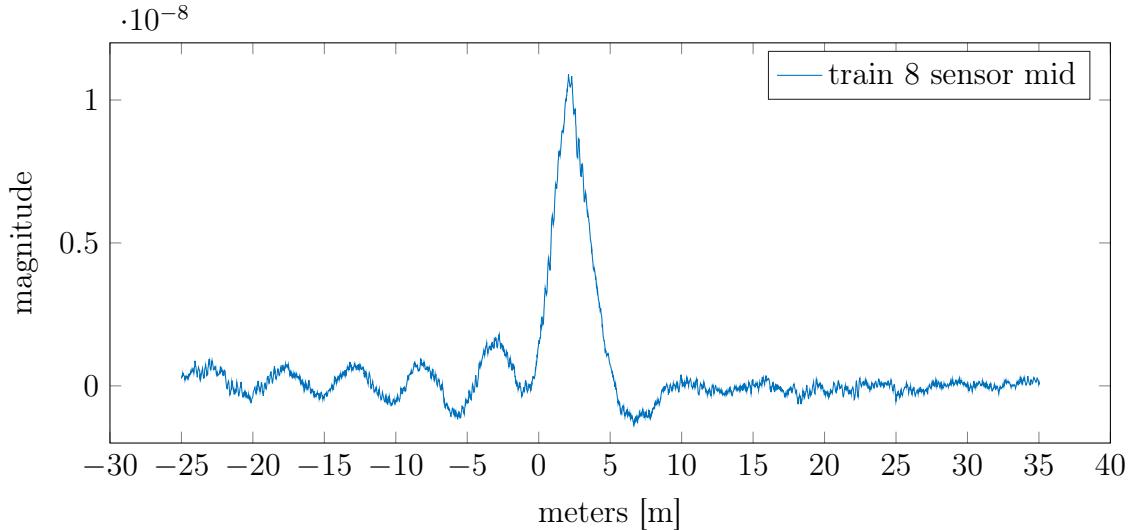


(c) Influence line for sensor closest Trondheim, train 6

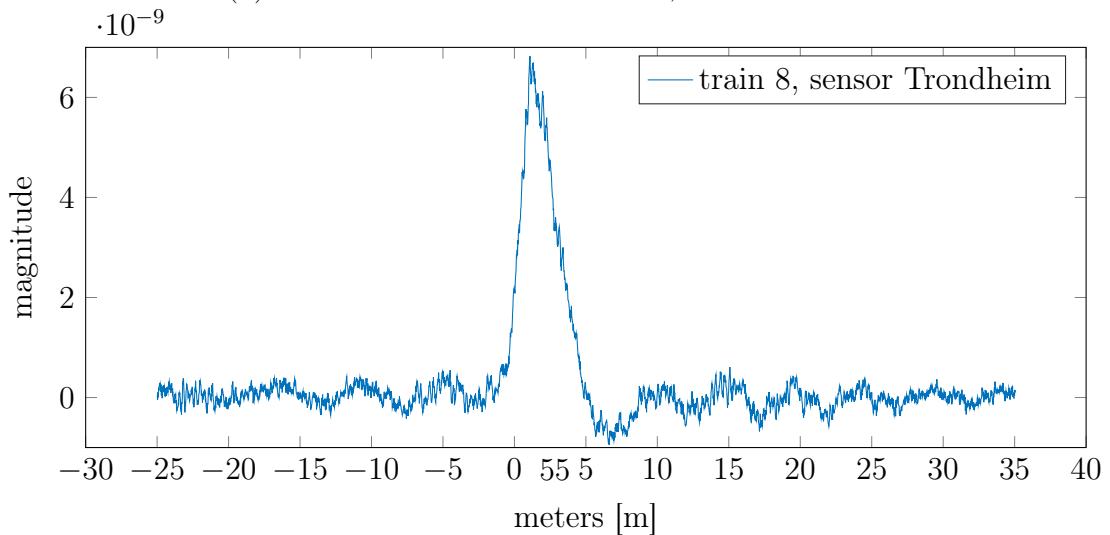
Figure 33: Influence lines train 6



(a) Influence line for sensor towards Heimdal, train 8



(b) Influence line for middle sensor, train 8



(c) Influence line for sensor closest Trondheim, train 8

Figure 34: Influence lines train 8

	sensor 1				sensor 2				sensor 3
axle	trains and their axle weights for sensors								
1	8397	10603	10782	10387	6034	7941	8024	7908	6612
2	9071	9965	10119	10125	6643	7207	6980	7149	5636
3	8562	9926	10896	9882	6669	8132	8664	8192	6889
4	8584	9599	10296	10140	6393	6600	6967	7025	4607
sum car	34614	40093	42093	40534	25739	29880	30635	30274	23744
5	13214	15027	15320	14171	10261	12783	12370	11916	10085
6	14186	14964	15796	16437	10926	11493	11402	12234	9863
7	11133	13979	15106	13520	8953	11922	12555	11743	9675
8	13822	13599	13540	14672	10612	10199	9617	10900	8904
sum loc	52355	57569	59762	58800	40752	46397	45944	46793	38527
sum tot	86969	97662	101855	99334	66491	76277	76579	77067	62271

Table 11: Table of axle weights for standard length influence lines, but with filtered strains for producing influence lines

	sensor 1				sensor 2				sensor 3							
	trains and their axle weights for sensors															
axle	train 3	train 4	train 5	train 6	train 8	train 3	train 4	train 5	train 6	train 8	train 3	train 4	train 5	train 6	train 8	
1	8563	10689	8578	11156	10617	9006	11278	9570	11195	11233	8341	8763	8402	9111	8532	
2	9343	10379	9170	10237	10284	7814	8628	7491	8176	8295	9581	9820	9440	10945	10043	
3	8709	10294	8817	11353	10353	9521	11446	9983	11940	11668	8837	8563	9203	8752	8320	
4	9057	9868	8451	10400	10285	8214	8586	7351	8336	8697	9073	9626	8547	10479	10262	
sum car	35672	41230	35016	43146	41539	34555	39938	34395	39647	39893	35832	36772	35592	39287	37157	
5	13392	15615	13546	15879	14865	14904	18402	15379	17434	17489	14064	14462	14660	13515	13475	
6	14581	14893	13859	15985	16313	13059	13336	11674	13391	14079	16116	15509	15121	18038	17548	
7	11303	15097	11479	15656	14380	13238	17679	13678	17332	17278	12374	13561	12595	13615	12636	
8	14184	12962	13616	13549	14350	12496	10913	11196	10910	12026	14788	13792	13933	16003	15501	
sum loc	53460	58567	52500	61069	59908	53697	60330	51927	59067	60872	57342	57324	56309	61171	59160	
sum tot	89132	99797	87516	10421	510144	788252	10026886322	98714	10076593174	94096	91901	10045896317				

Table 12: Table of axle weights for averaged influence lines, all trains