

# Master Thesis

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# 1 Preface

## Contents

<b>1</b>	<b>Preface</b>	<b>2</b>
<b>2</b>	<b>Introduction</b>	<b>3</b>
<b>3</b>	<b>Theory</b>	<b>3</b>
3.1	Bridge Weigh-in-Motion . . . . .	3
3.1.1	Moses' Algorithm . . . . .	3
3.2	Influence lines . . . . .	4
3.2.1	Matrix method . . . . .	5
3.2.2	Optimization . . . . .	6
3.3	Finding the train's speed . . . . .	6
3.4	The axle distances . . . . .	6
<b>4</b>	<b>Method</b>	<b>6</b>
4.1	Programming a BWIM system . . . . .	6
4.1.1	Finding the speed of the train . . . . .	7
4.2	Finding influence lines . . . . .	7
4.2.1	Matrix method . . . . .	8
4.2.2	Optimization . . . . .	8
4.3	System setup . . . . .	8
4.4	Testing . . . . .	8
<b>5</b>	<b>Analysis</b>	<b>8</b>
5.1	The matrix method . . . . .	9
5.2	Optimized influence lines . . . . .	9
5.3	Differences between the methods . . . . .	10
5.4	Problems . . . . .	10
<b>6</b>	<b>Conclusion and summary</b>	<b>10</b>
<b>A</b>	<b>Dynamics</b>	<b>10</b>
A.1	Rocking and vertical dynamic forces . . . . .	10
<b>B</b>	<b>Bibliography</b>	<b>10</b>

## 2 Introduction

Random citation [1] embeddeed in text. Random citation [1] embeddeed in text.

## 3 Theory

### 3.1 Bridge Weigh-in-Motion

A Bridge Weigh-in-Motion system is based on measurements of a bridge's deformation. The BWIM system uses these measurements to calculate passing vehicles axle loads. There are different approaches to assembling such a system, but they typically consists of a strain gauge measuring the strain induced by passing vehicles, a axle detector used to find the vehicle speed and spacing of axles and a computer or data storage device. An algorithm then is able to use the data gathered from the axle detector and strain gauge to calculate axle loads [2].

#### 3.1.1 Moses' Algorithm

Moses' algorithm is based on the fact that a moving load along a bridge will set up stresses in proportion to the product of the value of the influence line and the axle load magnitude. The influence line being defined as the bending moment at the point of measurement due to a unit axle load crossing the bridge [2].

Moses' algorithm is built from the fact that a moving unit load on a bridge will induce stresses proportional to the product of the value of the influence line and the axle load magnitude.

Each individual girder's stress is related to moment:

$$\underbrace{\text{stress in i'th girder}}_{\sigma_i} = \frac{\underbrace{\text{bending moment i'th girder}}_{M_i}}{\underbrace{\text{section modulus}}_{W_i}} \quad (1)$$

Expressing the moment in terms of strain gives

$$M_i = W_i \sigma_i = \underbrace{\text{Modulus of elasticity}}_E \times W_i \times \underbrace{\text{strain in i'th girder}}_{\varepsilon_i} \quad (2)$$

The sum of the individual girder moments is therefore:

$$M = \sum_{i=1}^N M_i = \sum_{i=1}^N E W_i \varepsilon_i = E W \sum_{i=1}^N \varepsilon_i \quad (3)$$

The sum of the girder strains is proportional to the gross bending moment. The total bending moment and the measured strain is thus directly related by  $EW$ . These constants can be calculated through the bridge's dimensions and material properties. However through measuring the effects of a known vehicle passing the bridge these constants can be derived.

Weigh in motion is an inverse type problem, the strain is measured and the cause of the strain is to be calculated. The theoretical bending moment corresponding to axle loads on the bridge at one strain sample, is given by:

$$M_k^T = \sum_{i=1}^N A_i I_{(k-C_i)} \quad (4)$$

$$C_i = (L_i \times f)/v \quad (5)$$

Where:

$N$  = the number of vehicle axles

$A_i$  = the weight of axle  $i$

$I_{k-C_i}$  = the influence line ordinate for axle  $i$  at sample  $k$

$L_i$  = the distance between axle  $i$  and the first axle in meters

$C_i$  = The number of strain samples corresponding to the axle distance  $L_i$

$f$  = the strain gauge's sampling frequency, in Hz

### 3.2 Influence lines

For a B-WIM system the influence line is defined as "the bending moment at the point of measurement due to a unit axle load moving along the bridge [3]". The influence line could be found through assembling a model of the bridge in any CAD or frame-program, this would however take a lot of time especially for more advanced bridge's. Depending on the support of the bridge the influence lines takes different theoretical forms, as seen in Figure 1. The true influence line for a bridge lie somewhere in between the simply supported and fixed version [3, p. 146]. Influence lines is a big source of error in a B-WIM system.

Znidaric and Baumgärter [3], did a study on the effect of choice of influence line. This study shows errors up to 10% for a short 2m bridge span and errors of several hundred percent for a 32m bridge span. This underlines the importance of using correct influence lines for a B-WIM system.

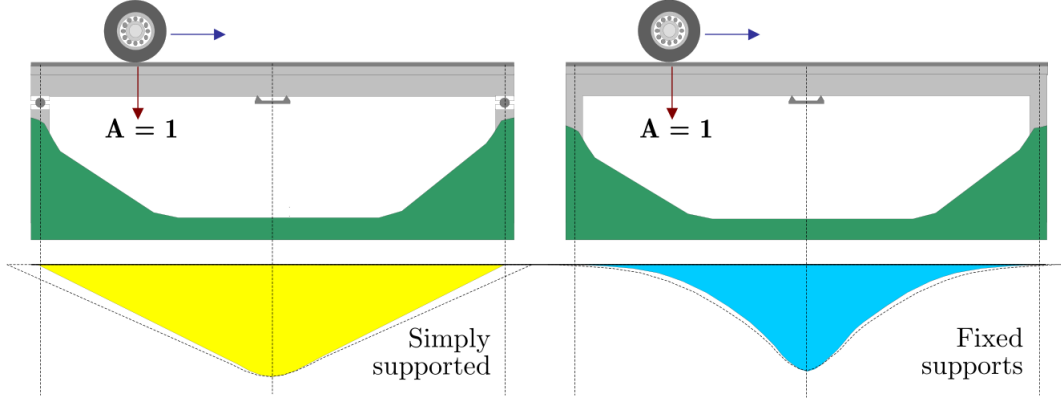


Figure 1: Influence lines for simply and fixed supported bridges, figure from [2]

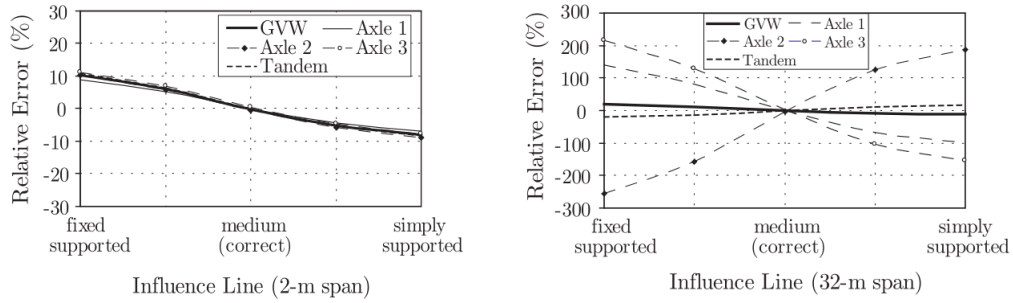


Figure 2: Errors of axle loads due to wrongly selected influence lines, figure from [2]

### 3.2.1 Matrix method

Quilligan [2] developed a 'matrix method' to calculate the influence line of a bridge through the measured strain induced by a vehicle. This method is derived from Moses', equation 6.

$$Error = \sum_{k=1}^K [\varepsilon_k^{measured} - \varepsilon_k^{theoretical}]^2 \quad (6)$$

Equation 6 were originally used to filter out the dynamic response of the bridge. The theoretical strain in this equation can be expressed as a product of axle loads and influence ordinates at sampling points, see equation 4, thus we can expand equation 6:

$$Error = \sum_{k=1}^K \left[ \varepsilon_k^{measured} - \left( \sum_{i=1}^N A_i I_{(k-C_i)} \right) \right]^2 \quad (7)$$

The set of influence ordinates  $I$  that minimizes *Error*, forms the wanted influence line.

$$\frac{\partial Error}{\partial I_R} = \frac{\partial \sum_{k=1}^K \left[ \varepsilon_k^{measured} - \left( \sum_{i=1}^N A_i I_{(k-C_i)} \right) \right]^2}{\partial I_R} \quad (8)$$

For a given number of known axle loads this equation comes down to a set of  $(K - C_n)$  number of linear equations. Rearranging the equations and writing them in matrix form leads to:

$$[AxleMatrix]_{K-C_N, K-C_N} \{I\}_{K-C_N, 1} = \{M\}_{K-C_N, 1} \quad (9)$$

Where:

$\{M\}$  = a vector depending on axle weights and measured strain,  $M_{i,1} = \left( \sum_{j=1}^N A_j \varepsilon_{(i+C_j)} \right)$

$[AxleMatrix]$  is a matrix depending only on the axle loads, defined as in 10.

$$[AxleMatrix] = \sum_{i=1}^N \sum_{j=i}^N [AxleMatrix] + (A_i A_j [Diagonal]_{C_j-C_i}) \quad (10)$$

Which produces the upper triangle of the symmetric AxleMatrix. Where:

$[Diagonal]_{C_j-C_i}$  = a diagonal matrix of ones, where diagonal is placed with an offset,  $C_j - C_i$ , from the center matrix diagonal.

Solving equation 9 for the influence ordinate vector gives the influence line for the strain history. This can be done through inversion of the AxleMatrix (equation 10) or other numerical solutions like a Cholesky factorization.

### 3.2.2 Optimization

testing [4].

## 3.3 Finding the train's speed

## 3.4 The axle distances

# 4 Method

## 4.1 Programming a BWIM system

Describe shortly how the BWIM system have been programmed. Keywords:

- Beam bridge model

- Producing a strain history through influence lines
- Finding the speed of the train
- Finding Axle distances
- Solving system for axle weights

This master project began by learning how a BWIM-system works, and to then create a working model performing BWIM. To not make this a too big project this meant building a simple beam model of a bridge in Matlab, and simulate moving loads crossing it. Through the theoretical influence lines of the beam a strain history was then produced for the given set of moving axle loads. This strain history could then be used as a base to build the code for a BWIM system.

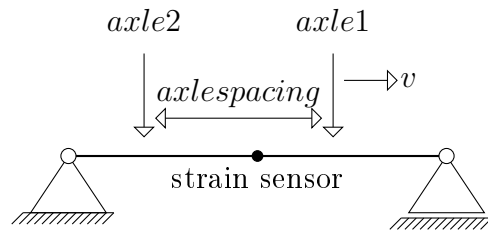


Figure 3: Beam model for initial BWIM

#### 4.1.1 Finding the speed of the train

Two working methods of finding the speed of a passing train were developed:

- By identifying peaks in the strain history for two different sensors, representing the same axle. The distance between the two sensors and the time difference between the found peaks should theoretically give a good estimate of the trains velocity.
- Through doing cross correlation between two sensors strain history. This involves finding the phase difference, or lag, between the signals. The known distance between the strain gauges should then along with a constant, based on distance between sensors, give a reliable estimate of train velocity. INSERT PLOT OF CORRELATION AND SHOW MATHEMATICAL EQUATION DESCRIBING CROSS CORRELATION.

## 4.2 Finding influence lines

Describe how influence lines have been found from the given strain history from Lerelva Bridge. Keywords:

- Matrix method
- Optimization
- Speed

#### 4.2.1 Matrix method

Describe the matrix method.

#### 4.2.2 Optimization

Describe how optimization can be used to find optimal influence lines for the bridge.

### 4.3 System setup

To test the BWIM-program on actual data, we Gunnstein, Daniel set up a BWIM-system to gather strain data from actual train passings. The subject bridge were Lerelva-Bridge in Trondheim, a typical Norwegian steel railway bridge. Three strain gauges, 3mm 120ohms from HBM, were placed by the support towards Trondheim on the first section of the longitudinal stringer. The sensors were placed with 1m spacing around the middle of the stringer section. These strain gauges were connected to a National Instruments compactDAQ with module NI 9235 which produced an continuous data flow to a standard laptop. A Kipor generator was brought for power.

### 4.4 Testing

Keywords:

- Comparing calculated strain with measured strain

## 5 Analysis

This chapter will describe how the BWIM system performs. What works? Why? How? etc. The main focus should perhaps be placed on identifying the pros and cons of the matrix method and optimization method.

Should include:

- Compare theoretical and calculated influence lines. Also include influence lines found through Abaqus.



- Check how influence lines found through matrix method and optimization reproduces the strain history
- Test obtained influence line by running the bwim routine on the hitherto unused freight train. (Depends on getting info about the train). Also Do this test on the other trains.

## 5.1 The matrix method

The matrix method creates an influence line for a specific strain history given a known train with known axle weights and velocity. Thus if the strain signal were recreated given with given parameters, the signal would be a almost exact replica of the measured strain signal, where the differences should originate from the effect of sensor noise. The found influence line would however be for this specific train and the passing's dynamic effects on the bridge, which is likely to vary from train to train. Therefore an averaging of a sufficient number of calculated influence lines should reduce or eliminate the dynamic effects from the influence line.

The analysis of the matrix method is based on 4 different train passings, and 3 sensor readings on each passing. The trains in these measurements is of the same type (not entirely shure!!) but the exact weight is not known. The weight of each axle were approximated by distributing carriage and locomotive gross weight. Passengers in the passing trains were not accounted for, and may lead to some deviation from ideal results.

TODO:

- Show the found influence lines for some sensors
- discuss the plots
- reproduce strain signal, and compare with measured signal
- show averaged influence line, and perform the same tests
- show interpolation of this averaged influence line
- perform the same test with this interpolated influence line
- the alternative should also be done, interpolate each found influence line and average them, then reproduce the strain signal, and find difference through comparison.

## 5.2 Optimized influence lines

Perform the same procedures as for the matrix method

## 5.3 Differences between the methods

Compare the optimized influence lines and the matrix method influence lines. This should be done in a thorough manner.

## 5.4 Problems

- Big problem with identifying exactly when train enters and leaves the bridge. This results in guesswork when placing influence line in a coordinate system. Where does the bridge begin and end in the influence line.. The only definite certainty seems to be placing the index of the maximum magnitude of the influence line in the correct position according to the measuring sensor's location.
- This could be problematic when using the found influence lines

# 6 Conclusion and summary

## A Dynamics

A train traversing a railway bridge creates actions in longitudinal, lateral, and vertical directions. Braking and traction from a passing train causes longitudinal forces. Rocking, or rotations around an axis parallel to the longitudinal axis of the bridge, and vertical dynamic forces are created by structure-track-vehicle conditions and interactions.

### A.1 Rocking and vertical dynamic forces

Lateral rocking of moving vehicles provide amplification of vertical wheel loads. This amplification increases the stresses in the members supporting the track. Superstructure-vehicle interaction creates a vertical dynamic amplification of moving loads, which will result in vibrations causing additional stresses in members supporting the track.

The unloaded simply supported beam frequency  $\omega_1 = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{m}}$ , provides a basic indicator of superstructure vertical dynamic response.

## B Bibliography

[1] J. Doe, *The Book without Title*. Dummy Publisher, 2100.

- [2] M. Quilligan, *Bridge Weigh-in Motion : Development of a 2-D multi-vehicle algorithm*. PhD thesis, KTH, Civil and Architectural Engineering, 2003. NR 20140805.
- [3] J. B. . C. . O'Brien, E., "Second european conference on weigh-in-motion of road vehicles : Lisbon, 14th - 16th september, 1998," pp. 139, 152, 1988.
- [4] A. Liljencrantz, R. Karoumi, and P. Olofsson, "Implementing bridge weigh-in-motion for railway traffic," *Computers and Structures*, vol. 85, no. 1-2, pp. 80–88, 2007.