Chapter 9

Conclusion

9.1 Conclusions

The overall objective of this thesis was to develop a general and versatile solution to the identification of moving vehicle interaction forces on a bridge deck. This objective has been achieved both numerically and experimentally through the extension of the general inverse theory developed by Trujillo (1975). This project has followed a logical progression from the outset, from the detailed introduction to general inverse theory and its application to the problem of force identification, through to the extension of the first order regularisation, to that of identifying moving vehicle interaction forces on both 1 and 2d models of bridges. These objectives have been achieved as follows

In chapter 2, a detailed review of the method of force identification in structural dynamics as developed by Trujillo was presented. The mathematical theory behind inverse dynamics was discussed in detail, both separately and collectively in its application to the problem of force identification. The three areas of maths utilised in the force identification process are,

- Conversion of the equilibrium equation of motion to one in state space.
- A least squares minimisation scheme with regularisation known as Tikhonov regularisation.
- Dynamic programming to solve the recursive least squares formulation.

The inverse dynamics approach using the method of general inverse theory was validated using numerical experiments on a cantilevered beam. The example illustrates the ill-conditioning of the problem, and the need to have a regularisation term in the formulation of the problem. These examples show how the first order regularisation out performs the zeroth.

In chapter 4, the powerful mathematical tool of Tikhonov regularisation, described in chapter 2, is applied to the Bridge-Weigh-In-Motion (B-WIM) equations to reduce the errors inherent in this ill-conditioned system and to better illustrate the concept of regularisation. A new formulation of the B-WIM equations is developed which employs the method of Tikhonov regularisation coupled with the original equations developed by

Moses (1978). The new improved algorithm was tested numerically using both simple and complex vehicle bridge models; in all cases the new algorithm was found to improve the identified axle weights over that of the conventional Moses equations. In some cases the new improved algorithm outperforms that of the conventional by orders of magnitude, even so the errors in the regularised algorithm can still be as high as 30%, and are not really acceptable for the purposes of legal enforcement. However the worst cases encountered in both the conventional and regularised solutions were for poor road profiles, the regularised solutions for the smooth profiles performed very well.

What is clear from the analysis of the regularised B-WIM algorithm is that in all cases it performs better than the conventional algorithm. What is particularly advantageous of this approach is its relative ease of implementation and its compatibility with existing commercial B-WIM systems. Whereby no additional instrumentation is necessary and calibration factors would be identical to those originally identified in the field. It is therefore thought that the regularised solution would improve the current classification of in-service commercial B-WIM systems without any additional instrumentation or calibration making it relatively easy and inexpensive.

The method of general inverse theory was extended to the problem of identifying moving forces on simply supported beam. The theory follows a direct extension of the theory of inverse dynamics outlined in chapter 2. The method of moving force identification using the zeroth order regularisation, as outlined by Law & Fang (2001) is discussed, and was then extended to first order regularisation. The effectiveness of this extension and its improvement over that of the zeroth order regularisation was illustrated through the use of numerical examples. It was shown that the first order regularisation gave better results than that of the zeroth. It was therefore inferred that the first order regularisation is a more robust method of identifying moving forces.

The algorithm was tested using the simulated strains from a number of vehicle crossing events. A number of vehicle models were employed: constant forces, a series of sinusoidal forces and a 4 degree of freedom sprung mass model excited by a road profile. The algorithm was able to predict multiple forces on the bridge regardless of their magnitude, frequency or phase. The force history was also accurately predicted for the case of frequency matching between force and bridge.

In chapter 6, both the numerical and theoretical deficiencies of the first order regularisation theory developed in chapter 5 were addressed. The first order regularisation method of chapter 5, required the use of the full stiffness and mass matrices in the conversion of the equilibrium equation of motion to first order system suitable for dynamic programming. Although this is perfectly valid theoretically, numerical the computational time and the storage requirements for the dynamic programming routine increase proportionally with each dimensional increase in the system. The “curse of dimensionality” as coined by Bellman (1967 a & b), refers to the exponentially increasing storage requirements involved in the dynamic programming with each additional dimension of the mathematical space. The solution adopted to overcome these enormous storage requirements is the eigenvalue reduction technique (Busby & Trujillo 1986).

To this end an eigenvalue reduction technique was implemented to reduce the order of the system and allow for more complex bridge modelling without increasing the state vector by significant orders of magnitude. The effectiveness of the eigenvalue reduction technique was illustrated through the application of the first order identification of moving forces on a two span continuous beam.

The first order regularisation of moving forces was again improved by removing the assumption made in chapter 5, that if a force is located between nodes, one can regularise as if the force is located at the closet node. This assumption was replaced with the exact solution that forces located between nodes are distributed to the degrees of freedom of the particular element that they are acting on as product of the numerical value of the shape function of the element. The effectiveness of these improvements was illustrated by comparing the identified vehicle interaction forces, from both algorithms, using the simulated strain from the Fryba model of chapter 5. It should also be noted that the physical criterion employed to obtain the optimal regularisation parameter in chapter 5, was no longer required, the L-curve and the L-curve alone was sufficient to obtain the optimal regularisation parameter. Finally an error analysis was performed to assess the sensitivity of the modified moving force identification algorithm to various parameters.

In chapter 7, the theory developed in chapters 5 and 6, is extended to a 2d finite element model. As the moving force identification algorithm requires a finite element model of the bridge, a C_1 conforming orthtropic plate element was developed in Matlab. The element is a slight variation on the plate element developed by Bogner et al (1965), the significant differences being the orthotropic properties of the stiffness matrix, and the inclusion of rotary inertia in the generation of the mass matrix. The element is validated with the commercial finite element software package MSc/NASTRAN.

The algorithm of chapters 5 and 6 is again improved, by applying optimality conditions on the initial state of the system, and imposing known boundary conditions on the state vector during the forward sweep of the dynamic programming routine, for the distinct cases of the vehicle exiting and entering the bridge. The imposition of these boundary conditions results in a significant reduction in the errors particularly at the beginning of the identified force time histories. The final modification made to the moving force identification algorithm is the use of an operational definition of the corner of the L-curve (Hansen 1992, 1997). This was achieved by calculating the curvature of the L-curve with respect to the regularisation parameter λ, and plotting the curvature against the regularisation parameter on a semi log x-axis, the optimal regularisation parameter is then defined as the point of maximum curvature.

The algorithm was then tested using the simulated strain from an independently built 3-D vehicle-bridge-road profile interaction MSc/NASTRAN finite element model, provided by Gonzalez (2001), that was further contaminated with 2% Gaussian noise. Analysis was carried out on the accuracy of the algorithm for various velocities and road profiles. In generally the final MFI algorithm developed exhibited excellent results; in all cases the algorithm was able to identify both the magnitude and frequency of the applied interaction forces. Finally an error analysis was carried out to assess the effects of various properties on the accuracy of the solution. It was concluded from this error analysis that the number of measurement locations need not be very high; in fact it was illustrated how 3 sensors could be used to accurately predict the vehicle axle forces. However the use of three sensors did require upwards of 50 modes of vibration for acceptable results to be obtained. It should also be noted that from the analysis carried out on the transverse location of the forces, it was inferred that these 3 sensors were required to be in close proximity to the moving forces on the deck. This was achievable in numerical testing where the exact location of the vehicle is known. There was also no exact correlation ascertained between, the number of modes, the number of sensors and the transverse location of the truck, there are merely correlations for the particular events analysed.

Finally the 2d moving force identification algorithm of chapter 7 has been experimentally validated in the field in collaboration with ZAG. The experimental programme carried out in Slovenia was explained. The installation of the strain transducers and the axle detectors was detailed along with details of the data acquired. A finite element model of the bridge was developed in Matlab and a good match was found between the measured and theoretical frequencies of the bridge. However the match between the strain sensors and the physical behaviour of the model was poor in many cases, resulting in only two sensors being used in the inverse analysis.

Notwithstanding this the viability of using only two sensors was justified numerically first, and it was shown that valuable information of the truck dynamics can be identified using only two sensors. Finally the moving forces imparted by the truck to the road surface on the bridge have been identified in the field over a range of velocities, and in all cases the oscillation of the truck forces about the static axle weight is distinct.

From the identified forces of all runs of the test truck, it was shown that valuable information can be obtained on the nature of the imparted truck forces to the surface of the bridge. It was shown that not only can the force time histories be identified, but from these histories, the static loads, the impact factors and the frequencies of the applied forces can be inferred with clear repetition in these inferred values.