

Moving Force Identification of Axle Forces on Bridges

by

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A thesis submitted in partial fulfilment for the degree of
Doctor of Philosophy

April 2007

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Declaration

The author hereby declares that this thesis, in whole or part, has not been used to obtain any degree in this, or any other university. Except where reference has been given in the text, it is entirely the author's own work.

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April 2007

Abstract

Bridge Weigh-In-Motion (BWIM) systems are based on the measurement of strain on a bridge and the use of the measurements to estimate the static weights of passing traffic loads. Despite many advantages, BWIM algorithms have often failed to predict axle weights accurately due to random noise, vehicle and bridge dynamics and other factors such as variation in transverse position of the vehicle. This thesis presents three new algorithms to predict both the static axle weights and the time history of the interaction forces as a vehicle crosses a bridge.

WIM first emerged in the 1970's and there are currently commercial and prototype Bridge Weigh-in-Motion systems in use worldwide. Commercial Bridge Weigh-in-Motion systems are generally based on the algorithm developed by Moses. The BWIM algorithm of Moses (1979) solves a set of simultaneous equations to calculate individual axle loads from the measured bridge response. Studies have shown, in general, good accuracy for estimating gross vehicle weights; however for individual axle weights, the results can be inaccurate. It has been found that the equations are ill-conditioned, particularly for closely spaced axles. The method of Tikhonov regularisation is applied to the original Moses' equations to reduce some of the inaccuracies inherent within the algorithm. The optimal regularisation parameter is calculated using the L-curve criterion. The new regularised solution to the Bridge Weigh-in-Motion equations is numerically tested using simulations of moving vehicles on a bridge. Results show that the regularised solution performs significantly better than the original approach of Moses and is insensitive to road surface roughness.

The second algorithm developed in the thesis is an extension of the method proposed by Law and Fang (2000). In this approach, instead of predicting the static axle forces, the time histories of the forces, or the applied vehicle bridge interaction forces, are calculated. The bridge is modelled using the finite element method and is represented as a beam. The equilibrium equation of motion is converted to a discrete time-integration scheme using the exponential matrix. The inverse problem of finding applied forces from measured responses is then formulated as a least squares problem with Tikhonov regularisation to reduce the ill conditioning. It is solved with dynamic programming

using Bellman's principle of optimality. Finally the optimal regularisation parameter is solved using Hansen's L-curve method. The algorithm is tested using simulations from a finite element model, and dynamic simulations from a four-degree of freedom vehicle bridge interaction model with a random road profile.

The second algorithm requires the use of the full stiffness and mass matrices in the formulation of the bridge dynamic model. Whilst this is acceptable for small finite element models, the method is not really feasible for large models with thousands of degrees of freedom. To overcome this constraint, an eigenvalue reduction technique has been employed to reduce the dimensionality of the system, thus improving the computational time of the moving force identification algorithm whilst maintaining the accuracy of the unmodified algorithm. The moving force identification algorithm using an eigenvalue reduction technique is tested using simulations from a two span continuous bridge, and the simulations from the Fryba vehicle bridge interaction model. Finally an error analysis is carried out for the completed 1-dimensional moving force identification algorithm.

The third and final algorithm is a generalisation of the 1-dimensional moving force identification algorithm into 2 dimensions. The bridge is again modelled using the finite element method, with an orthotropic plate-bending element being used to model the simply supported bridge. The eigenvalue reduction technique is again employed to reduce the dimension of the system. Strain measurements are simulated using an elaborate 3-dimensional vehicle-road-bridge interaction system developed by Gonzalez (2001). The strain is contaminated with noise and input into the moving force identification algorithm. An error analysis is carried out on the results.

Experimental testing was carried out on the Vransko Bridge in Slovenia, a simply supported bridge of beam and slab construction. The bridge was instrumented with strain sensors and axle detectors. A 3-axle calibration truck was used during the testing. A finite element model of the bridge was developed in Matlab. The model was validated using frequency analysis of the measured free vibration, and modal analysis of the finite element model. The algorithm is tested using the measured strain from the calibration truck.

Acknowledgements

There are a number of people without whom this research could not have been completed. I would like to take this opportunity to extend my gratitude to these individuals.

I would like to express my deepest appreciation to my supervisors, Dr Arturo Gonzalez and Professor Eugene O'Brien. To Arturo, for his insight, guidance, friendship and continuous encouragement, who operated an open door policy, always making time to discuss whatever the problem was. To Eugene, for continually demanding more and for testing the limits of my capabilities. For his insight, guidance and excellent suggestions throughout this research. Without them, this task would not have been possible; it has been a pleasure and honour to work with them.

To Ales Žnidaric and Igor Lavric from the Slovenian National Building and Civil Engineering Institute for the collaborative work in this thesis.

The Lord Edward Fitzgerald memorial fund, for providing funding.

To my fellow researchers in the department of civil engineering, to Gavin O'Connell friendship & coffee, to Noel Boylan friendship, coffee & random nights out, to Padraic Rattigan and Niall Harris for sharing an office with me, friendship & tea breaks, to Fionnula McInerney friendship, coffee & cigarettes, to the tea room gang, Paul, Fiachria, Kieran McNally (music/making coffee), Kenny G, Brian, Sophie, Shane, Colin, Carl etc.

Finally I would like to thank my family, to my parents Mary and Padraic for their continuous support and encouragement, to my sister Aifric and my brother Padraic, your support throughout this period is greatly appreciated.

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Chapter 1

Introduction

1.1 Introduction

Since the latter half of the twentieth century, the growth and development of road transport in Europe has been significant. This growth has resulted in obvious increases in the volume of traffic, and an overall trend towards heavier vehicles. Today, heavy goods vehicles are responsible for the transportation of a large proportion of Europe's freight, consequently the size and weight of these vehicles has become an issue in bridge design.

Traditionally, static scales were used to obtain information on vehicle weights. However using this method it is necessary to stop truck haulers and weigh them individually, as well as the inconvenience to both parties, this method introduces a bias into the collected data as drivers tend to avoid these weigh stations. For these reasons interest in weigh-in-motion (WIM) technologies has grown considerably over time. WIM systems can potentially give unbiased information on the axle and gross vehicle weights of trucks while in motion, avoiding many of the problems associated with the traditional methods.

WIM systems can be loosely divided into two main categories; those that use sensors placed in the road surface and infer axle weights from the instantaneous measured response, and those that use the bridge as the instrument to infer the axle and gross vehicle weights. The latter of these two is known as bridge-weigh-in-motion (B-WIM).

B-WIM technologies first emerged in the late 1970's (Moses 1978) in the United States, as a possible alternative to pavement based systems. In general, the basic principle of B-WIM systems is that a load, in this case an axle or a group of axles, induces a continuous strain record in proportion to the influence ordinate and the load magnitude, and from this strain signal the axle weights can be inferred. Most of the current B-WIM systems use algorithms based on static equations of equilibrium combined with measurements at one single longitudinal section. Many alternative approaches have been discussed in the literature (Leming and Stalford 2003, Gonzalez et al. 2002, Gonzalez 2001), and extensive reviews of these methods can be found in Dempsey (1997) and Gonzalez (2001), but Moses' algorithm and variations thereon still appear to be the basis for commercial B-WIM systems. In chapter 4 of this thesis, another

variation on the Moses algorithm is presented for the first time. This method utilises the method of Tikhonov regularisation to significantly reduce the errors inherent in the ill-conditioned system generally formulated in B-WIM systems. However it should be noted that almost all of the WIM and B-WIM systems measure only the equivalent static weights of the vehicles, and this is not the focus of this thesis.

The novel technique of using the bridge as an instrument to infer the dynamic wheel forces was first investigated by O'Connor & Chan (1990). Since then the field of moving force identification (MFI) has progressed rapidly. The advantageous of MFI over that of traditional static B-WIM methods are that it can potentially give a complete time history of the wheel forces applied to the bridge, the errors in B-WIM systems due to bridge dynamics no longer exist as the bridge dynamics are accounted for in the inverse model. The basic theory of the current MFI algorithms, which use a dynamic bridge model to predict a complete time history of the vehicle interaction forces, are detailed in Chapter 3.

1.2 Research objective

The focus of this thesis is to develop a general solution to the identification of vehicle interaction forces; as to date much of the attention of MFI has focused on the use of exact solution methods with the main exceptions being Chan 1990, Law & Fang 2001, Law et al 2004 and Pinkaew 2006. The principle of the solution to generalising the MFI problem lies in the concept that, if an accurate finite element model of the bridge has been developed, the new moving force identification algorithm can employ this model in the identification of the vehicle interaction forces imparted to the surface of the bridge.

1.3 Layout of thesis

Chapter 2 is an introduction to the mathematics behind general inverse theory (Trujillo 1975). The theory is reviewed and then implemented numerically. The theory outlined in this chapter forms the basis of almost all the subsequent algorithmic development in this thesis.

In chapter 3, a literature review is conducted on the current methods of moving force identification, which employ a bridge as the instrument of identification. As the focus of this thesis is predominately centred on the development of a more accurate general solution to the identification of vehicle interaction forces, MFI systems are reviewed in detail in order to obtain an understanding of the algorithms and techniques involved.

In chapter 4, the method of regularisation is applied to the original Moses B-WIM equations. The conventional and regularised B-WIM algorithms are tested using the simulated strain from a finite element model and a vehicle bridge interaction model provided by Green (1995).

In chapter 5, the method of moving force identification developed by Law and Fang (2001) is discussed and implemented; the method follows a direct extension of the theory outlined in chapter 2. Their algorithm employs the zeroth order regularisation for the prediction of moving forces. The method is extended here to allow for the first order regularisation of moving forces. Both algorithms are tested using the simulated strain from a finite element model and a vehicle bridge interaction model developed by Gonzalez (2001).

Chapter 6 highlights the difficulties of implementing the MFI algorithm of chapter 5 for large-scale finite element models where the number of degrees of freedom in the model is significant. If the algorithm of chapter 5 were to be executed for a moderately large finite element model, the storage requirements and the computational time would be enormous. A method to reduce the computational time and the storage requirements of the moving force identification algorithm without reducing the accuracy of the algorithm of chapter 5 is developed. The improved algorithm is tested using numerical experiments on both continuous two-span and simply supported beams.

In chapter 7, the 1-D moving force identification algorithm of chapters 5 and 6 is extended to 2-D. Again the numerical and theoretical deficiencies of chapters 5 and 6 are addressed and improved upon, resulting in an algorithm that can potentially identify both the imparted axle forces and the individual wheel forces imparted to the road surface on a bridge by a truck. The final 2-D MFI algorithm is tested using the simulated strain from a 3-D vehicle-bridge-road profile interaction MSc/NASTRAN

model provided by Gonzalez (2001). The new 2-D MFI algorithm requires an accurate FE model of the bridge, for this purpose a C_1 conforming plate element is developed in Matlab, the element is a variation of that developed by Bogner et al (1965).

Chapter 8 details the experimental programme carried out in Slovenia in collaboration with the ZAG research institute. Experimental testing was carried out on a simply supported beam and slab bridge, to test the 2-D MFI algorithm of chapter 7. The programme involves a pre-weighed three-axle truck, an instrumented bridge and recorded strain data due to the passage of the truck. A finite element model of the bridge is developed in Matlab, using the plate element of chapter 7 and grillage members to model the additional stiffness provided by the longitudinal and diaphragm beam members. Finally the moving forces of the truck are identified in the field using the algorithm of chapter 7.