ME597/AAE556 – Fall 2022
Purdue University
West Lafayette, IN

Homework Set No. 4

Assignment date: Friday, November 4
Due date: Tuesday, November 15

Please submit your completed homework assignment by

- Scanning and uploading assignment to ***Gradescope***.

Instructions:

- Please upload your HW with following this name code, where ** is the assignment number (e.g., HW**=HW1): HW**ME597/AAE556_Fall22_NameInitial_Lastname
- This assignment is strictly individual.
- Your work will be evaluated considering the shown procedure to obtain final answers.
- The procedure and results should be **clear** and **ordered**.
- Consider asking questions about the assignment in advance to avoid inconveniences caused by unexpected events close to the submission date.
- **Please ensure your work is submitted via *Gradescope***.

Name____

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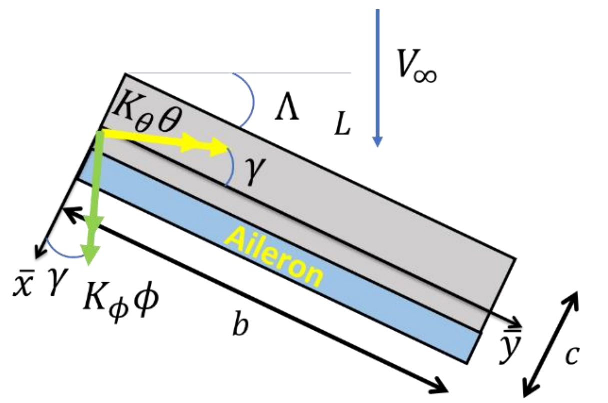
Homework Set No. 4
Problem No. 1 – 50 points

The idealized swept wing flying door model consists of the familiar semi-rigid surface restrained by bending and torsion springs to resist rotations ϕ and θ, respectively, but these are aligned at an angle γ with respect to the $\bar{x}$ and $\bar{y}$ axes, as shown in the figure. These springs have stiffnesses K_ϕ and K_θ. The wing planform dimensions are: swept semi-span b and chord c. The wing's planform area is S and the 2D lift curve slope is a_0. Assume that lift is applied at the aerodynamic axis at the $\frac{c}{4}$ location from the leading edge and consider the effect of sweep in the lift production. The root incidence angle is $\alpha_0 = 0$.



The purpose of this problem is to illustrate how the tailoring of the wing principal axes affect aileron effectiveness.

Find:

(a) Derive the equations of static equilibrium in matrix form in terms of the vector $[\phi \quad \theta]^T$.
Show that using $Q = q_n S a_0$ and $q_n = q\ \cos^2\Lambda$ these are:

$$\begin{pmatrix} \bar{K}_{11} & \bar{K}_{12} \\ \bar{K}_{21} & \bar{K}_{22} \end{pmatrix} \begin{bmatrix} \phi \\ \theta \end{bmatrix} = q_n S a_0 \delta_0 \begin{bmatrix} \dfrac{c_{l\delta}}{a_0}\dfrac{b}{2e} \\ \dfrac{c_{l\delta}}{a_0}\left(1 + \dfrac{c}{e}\dfrac{c_{m\delta}}{a_0}\right) \end{bmatrix} = Qe \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \delta_0$$

where: $\bar{K}_{11} = K_\theta(K_{11}^* + \frac{bQ}{K_\theta}\frac{\tan\Lambda}{2})$, $\bar{K}_{12} = K_\theta(K_{12}^* - \frac{Q}{K_\theta}\frac{b}{2})$, $\bar{K}_{21} = K_\theta(K_{12}^* + \frac{bQ}{K_\theta}\frac{e}{b}\frac{\tan\Lambda}{2})$, and $\bar{K}_{22} = K_\theta(K_{22}^* - \frac{bQ}{K_\theta}\frac{e}{b})$, and:
$K_{11}^* = K_\phi \cos^2\gamma + K_\theta \sin^2\gamma$, $K_{22}^* = K_\phi \sin^2\gamma + K_\theta \cos^2\gamma$, and
$K_{12}^* = K_{21}^* = (K_\phi - K_\theta)\cos\gamma \sin\gamma$

(b) Derive the equation for the divergence dynamic pressure parameter Q_D in terms of the K_{ij}^* stiffnesses, b, e, and $\tan\Lambda$

(c) Show that the lift produced by the aileron is:

$$L_{flex} = \left(\frac{(Q^2 e(\,(c_2\bar{K}_{11} - c_1\bar{K}_{21}) - (c_1\bar{K}_{22} - c_2\bar{K}_{12})\tan\Lambda)}{\bar{K}_{11}\bar{K}_{22} - \bar{K}_{21}\bar{K}_{12}} + Q\frac{c_{l_\delta}}{a_0} \right)\delta_0$$

(d) Derive the aileron effectiveness:

$$\frac{L_{flex}}{L_{rigid}}$$

(e) For the parameters: $\frac{b}{c} = 6$, $E = 0.15$, and $\frac{K_\phi}{K_\theta} = 3$, plot the aileron effectiveness vs. γ for $\Gamma = [-30^o, 0^o, 30^o]$. Use Glauert's theory to calculate the derivatives $\frac{c_{l\delta}}{a_0}$ and $\frac{c_{m\delta}}{a_0}$ as a function of E.

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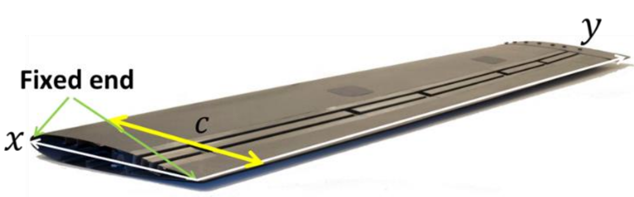
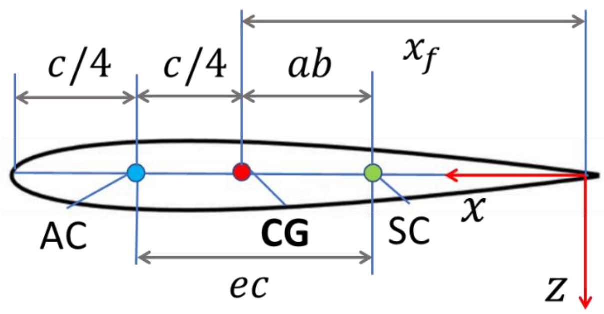
Problem No. 2 – 50 points

A fixed root wing has semi-span s, chord c, bending stiffness EI, and torsional rigidity GJ. The lift is calculated using strip theory where $dL = qca_w\theta(t)dy$ and that lift acts at the aerodynamic center. The wing's displacement is $z(x, y, t) = h(y, t) + x\theta(y, t)$. Assume the wing's mass to be homogeneously distributed. The location of the aerodynamic, shear, and mass centers are at the shown locations below. The reference axis is located the trailing edge.





The wing's properties are:

Semi-span (s)	7.5 m		Bending Stiffness: EI		$2\text{x}10^7$ Nm^2
Chord (c)	2 m		Bending Rigidity: GJ		$2\text{x}10^6$ Nm^2
Elastic axis (x_f)	0.48c		Lift curve slope, a_w		2π
Mass axis	0.5c		Air density ρ		1.225 kg/m^3
Mass per unit area	200 kg/m^2				

Determine:

(a) Using a two-mode expansion for the displacement field $z(x, y, t) = \left(\frac{y}{s}\right)^2 h_1(t) + (x - x_f)\frac{y}{s}\theta_1(t)$ obtain:
1. Discretized equations of motion in terms of the modal coordinates: $[h_1(t), \theta_1(t)]$.
2. Frequency vs. flow speed and Damping vs. flow speed graphs.
3. Determine the flutter speed V_F
4. Determine the divergence speed V_D

(b) Using a four-mode expansion for the bending and twist fields: $z(x, y, t) = \left(\frac{y}{s}\right)^2 h_1(t) + \left(\frac{y}{s}\right)^3 h_2(t) + (x - x_f)\frac{y}{s}\theta_1(t) + (x - x_f)\left(\frac{y}{s}\right)^2 \theta_2$ obtain:
1. Discretized equations of motion in terms of the modal coordinates: $[h_1(t), h_2(t), \theta_1(t), \theta_2(t)]$.
2. Frequency vs. flow speed and Damping vs. flow speed graphs.
3. Determine the flutter speed V_F
4. Determine the divergence speed V_D

Attach the code you used to obtain them as appendix to your HW.