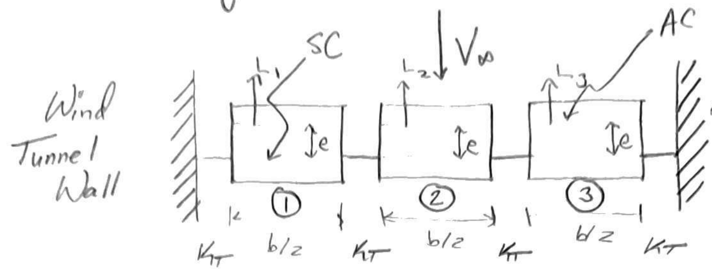
Victoria Nagorski | Hw #2 | Page 1

Problem No. 1



→ identical wings

a) Equations of static equilibrium

$$\sum M_1 = L_1 e - K_T \theta_1 + K_T(\theta_2 - \theta_1) = 0$$

$$L_1 e - K_T\theta_1 + K_T\theta_2 - K_T\theta_1 = 0$$

$$L_1 e - 2K_T\theta_1 + K_T\theta_2 = 0 \qquad (a)$$

$$\sum M_2 = L_2 e - K_T(\theta_2 - \theta_1) + K_T(\theta_3 - \theta_2) = 0$$

$$L_2 e - K_T\theta_2 + K_T\theta_1 + K_T\theta_3 - K_T\theta_2 = 0$$

$$L_2 e - 2K_T\theta_2 + K_T\theta_1 + K_T\theta_3 = 0 \qquad (b)$$

$$\sum M_3 = L_3 e - K_T(\theta_3 - \theta_2) - K_T\theta_3 = 0$$

$$L_3 e - K_T\theta_3 + K_T\theta_2 - K_T\theta_3 = 0$$

$$L_3 e - 2K_T\theta_3 + K_T\theta_2 = 0 \qquad (c)$$

Define Lift as $L = qSC_{L\alpha}(\alpha_0 + \theta)$ and plug into (a), (b), and (c).

$$(a.2) \quad qSC_{L\alpha}(\alpha_0 + \theta_1)e - 2K_T\theta_1 + K_T\theta_2 = 0$$

$$(b.2) \quad qSC_{L\alpha}(\alpha_0 + \theta_2)e - 2K_T\theta_2 + K_T\theta_1 + K_T\theta_3 = 0$$

$$(c.3) \quad qSC_{L\alpha}(\alpha_0 + \theta_3)e - 2K_T\theta_3 + K_T\theta_2 = 0$$

We can rewrite in matrix form (to compare later):

$$\begin{bmatrix} (qSeC_{L\alpha} - 2K_T) & K_T & 0 \\ K_T & (qSeC_{L\alpha} - 2K_T) & K_T \\ 0 & K_T & (qSeC_{L\alpha} - 2K_T) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} =$$

$$- qSeC_{L\alpha}\alpha_0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

We can rewrite to make cleaner... multiply the negative across

$$\begin{bmatrix} (2K_T - qSeC_{L\alpha}) & -K_T & 0 \\ -K_T & (2K_T - qSeC_{L\alpha}) & -K_T \\ 0 & -K_T & (2K_T - qSeC_{L\alpha}) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = qSeC_{L\alpha}\alpha_0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Rewrite one more time in terms of stiffness matrix:

$$\boxed{K_T \underbrace{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}}_{\text{Stiffness Matrix}} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + qSeC_{L\alpha} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = qSeC_{L\alpha}\alpha_0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

b) Strain energy as a function of the torsional displacements:

$$\boxed{U_1 = \tfrac{1}{2}K_T\theta_1^2 + \tfrac{1}{2}K_T(\theta_2 - \theta_1)^2}$$

$$\frac{\partial U_1}{\partial \theta_1} = K_T\theta_1 + K_T(\theta_2 - \theta_1)(-1)$$

$$\frac{\partial U_1}{\partial \theta_1} = 2K_T\theta_1 - K_T\theta_2$$

$$\boxed{U_2 = \tfrac{1}{2}K_T(\theta_2 - \theta_1)^2 + \tfrac{1}{2}K_T(\theta_3 - \theta_2)^2}$$

$$\frac{\partial U_2}{\partial \theta_2} = K_T(\theta_2 - \theta_1)(1) + K_T(\theta_3 - \theta_2)(-1)$$

$$\frac{\partial U_2}{\partial \theta_2} = K_T\theta_2 - K_T\theta_1 - K_T\theta_3 + K_T\theta_2$$

$$\frac{\partial U_2}{\partial \theta_2} = -K_T\theta_1 + 2K_T\theta_2 - K_T\theta_3$$

$$\boxed{U_3 = \tfrac{1}{2}K_T(\theta_3 - \theta_2)^2 + \tfrac{1}{2}K_T\theta_3^2}$$

$$\frac{\partial U_3}{\partial \theta_3} = K_T(\theta_3 - \theta_2)(1) + K_T\theta_3$$

$$\frac{\partial U_3}{\partial \theta_3} = -K_T\theta_2 + 2K_T\theta_3$$

We now have the 3 different equations:

$$\frac{\partial U_1}{\partial \theta_1} = 2K_T\theta_1 - K_T\theta_2$$

$$\frac{\partial U_2}{\partial \theta_2} = -K_T\theta_1 + 2K_T\theta_2 - K_T\theta_3$$

$$\frac{\partial U_3}{\partial \theta_3} = -K_T\theta_2 + 2K_T\theta_3$$

Which can be re-written as:

$$\begin{bmatrix} \partial U_1/\partial\theta_1 \\ \partial U_2/\partial\theta_2 \\ \partial U_3/\partial\theta_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}}_{\text{Stiffness matrix}} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

↪ matches stiffness matrix derived in part a)

c) Divergent dynamic pressure:

Start w/ part a) ans

$$\frac{\cancel{K_T}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}}{\cancel{K_T}} + \frac{qSeC_{L\alpha}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}}{K_T} = \frac{qSeC_{L\alpha}\alpha_0}{K_T}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Define: $\bar{q} = \frac{qSeC_{L\alpha}}{K_T}$

$$\begin{bmatrix} 2-\bar{q} & -1 & 0 \\ -1 & 2-\bar{q} & -1 \\ 0 & -1 & 2-\bar{q} \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \bar{q}\alpha_0\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Solve determinant of matrix

$$(2-\bar{q})[(2-\bar{q})^2 - 1] + 1[(-1)(2-\bar{q}) - 0] + 0 = 0$$

$$(2-\bar{q})^3 - (2-\bar{q}) - (2-\bar{q}) = 0$$

$$(2-\bar{q})^3 - 2(2-\bar{q}) \longrightarrow (2-\bar{q})[(2-\bar{q})^2 - 2] = 0$$

$$(2-\bar{q})(2-\bar{q}) \rightarrow 4 - 4\bar{q} + \bar{q}^2 \qquad \bar{q}_1 = 2$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\bar{q}^2 - 4\bar{q} + 2 = 0$$

$$\bar{q}_{2,3} = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2} \rightarrow \frac{4 \pm \sqrt{8}}{2} \rightarrow \frac{\cancel{4}2 \pm \cancel{2}\sqrt{2}}{\cancel{2}}$$

$$\boxed{\bar{q}_1 = 2 \qquad \bar{q}_2 = 2 + \sqrt{2} \qquad \bar{q}_3 = 2 - \sqrt{2}}$$

d) Mode Shapes

$$\begin{bmatrix} 2-\bar{q} & -1 & 0 \\ -1 & 2-\bar{q} & -1 \\ 0 & -1 & 2-\bar{q} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

inverse of idenity is identity

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \bar{q} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \bar{q} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \Leftarrow$$ The eigenvalue problem $Ax = \lambda x$

$$\begin{aligned} 2\theta_1 - \theta_2 &= 2\theta_1 \qquad \theta_1 \to \text{arb},\ \theta_2 \to 0 \\ -\theta_1 + 2\theta_2 - \theta_3 &= 2\theta_2 \qquad \theta_1 = -\theta_3 \\ -\theta_2 + 2\theta_3 &= 2\theta_3 \end{aligned}$$

$$\bar{q} = 2 \Rightarrow \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = C_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$\bar{q} = 2+\sqrt{2}$

$$\begin{bmatrix} 2-(2+\sqrt{2}) & -1 & 0 \\ -1 & 2-(2+\sqrt{2}) & -1 \\ 0 & -1 & 2-(2+\sqrt{2}) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -\sqrt{2}\theta_1 - \theta_2 &= 0 \longrightarrow \theta_2 = -\sqrt{2}\theta_1 \\ -\theta_1 - \sqrt{2}\theta_2 - \theta_3 &= 0 \\ -\theta_2 - \sqrt{2}\theta_3 &= 0 \longrightarrow \theta_2 = -\sqrt{2}\theta_3 \end{aligned}\right\rangle \theta_1 = \theta_3$$

$$\bar{q} = 2+\sqrt{2} \longrightarrow \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = C_2 \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} \qquad -1+2-1=0$$

$\bar{q} = 2-\sqrt{2}$

$$\begin{bmatrix} 2-(2-\sqrt{2}) & -1 & 0 \\ -1 & 2-(2-\sqrt{2}) & -1 \\ 0 & -1 & 2-(2-\sqrt{2}) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} \sqrt{2}\theta_1 - \theta_2 &= 0 \qquad \theta_2 = \sqrt{2}\theta_1 \\ -\theta_1 + \sqrt{2}\theta_2 - \theta_3 &= 0 \\ -\theta_2 + \sqrt{2}\theta_3 &= 0 \qquad \theta_2 = \sqrt{2}\cdot\theta_3 \end{aligned}\right\rangle \theta_1 = \theta_3$$

$-1+2-1$

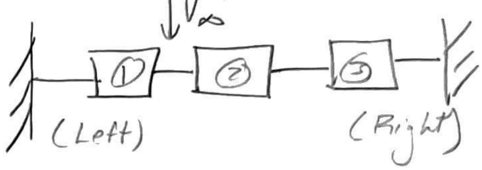
$$\bar{q} = 2-\sqrt{2} \longrightarrow \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = C_3 \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

$$\bar{q} = 2 \rightarrow \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = C_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\bar{q} = 2+\sqrt{2} \rightarrow \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = C_2 \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$$

$$\bar{q} = 2-\sqrt{2} \rightarrow \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = C_3 \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

→ eigenvectors are unique, and can be scaled arbitrarily



Note these are perturbed θs.

At $\bar{q} = 2$, We see that $\theta_1 = -\theta_2$ and $\theta_3 = 0$. What this means is that in order to stay in static equilibrium, for every degree wing on the left (1) twists, the wing on the far right needs to twist the same amount in the opposite direction. The middle wing, however, needs to maintain 0 twist.

At $\bar{q} = 2+\sqrt{2}$, we have a slightly different scenario to maintain static equilibrium. In this case, we want the far left (1) and far right wing (3) to twist the same amount in the same direction. The middle wing (2), however, need to twist in the opposite direction by $C_2\sqrt{2}$ amount.

At $\bar{q} = 2-\sqrt{2}$ is super similar to $\bar{q} = 2+\sqrt{2}$, but now the middle wing twists in the same direction as the left and right wing. The magnitude/amount of twist in said direction is $C_3\sqrt{2}$.

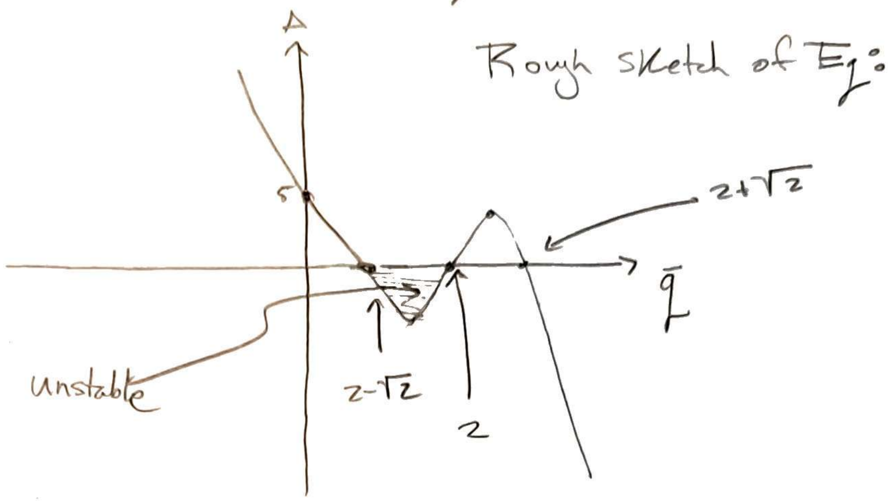
$\bar{q} = 2\pm\sqrt{2} \Rightarrow \left| \dfrac{\theta_1}{\theta_2} \right| = \dfrac{1}{\sqrt{2}}$ ← The ratio comparing twist ∡s between θ_1/θ_3 and θ_2.

They are equal

e) divergence dynamic pressure

$\bar{q}_1 = 2 \quad \bar{q}_2 = 2 + \sqrt{2} \quad \bar{q}_3 = 2 - \sqrt{2}$

Rough sketch of E_q:



$$(2-\bar{q})(\bar{q}^2 - 4\bar{q} + 2) = \Delta$$

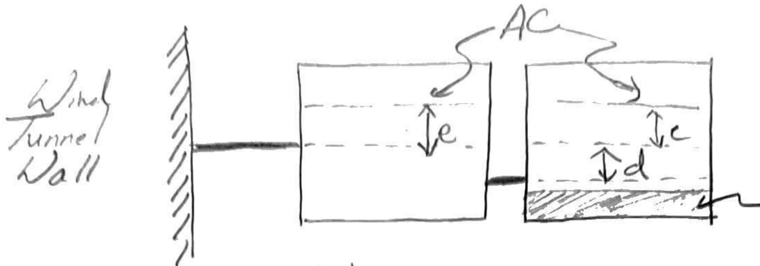
$$\cancel{2\bar{q}^2} - 8\bar{q} + 4 - \bar{q}^3 + \cancel{4\bar{q}^2} - 2\bar{q} = \Delta$$

$$-\bar{q}^3 + 6\bar{q}^2 - 10\bar{q} + 4 = \Delta$$

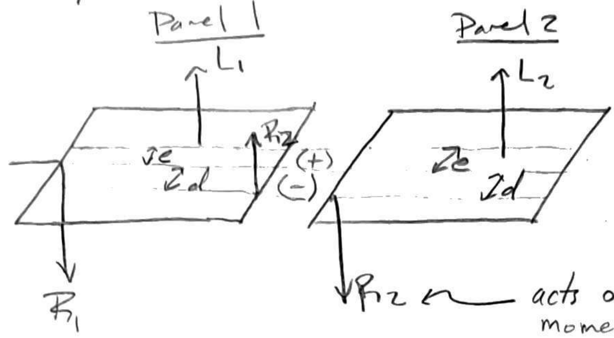
$\bar{q} = 2 - \sqrt{2}$ is the divergent dynamic pressure since static equilibrium is impossible since the system can no longer store energy caused by the pertubations.

At $\bar{q} = 2$, the system becomes stable again, but that doesn't really matter because the system already went unstable at an earlier dynamic pressure.

Problem No 2



→ Both lift from wing and aileron act at AC



$R_2 = L_2$

R_{12} acts on SC of Panel 2, so no extra moments on panel 2

$$L_1 = qSC_{L\alpha}(\alpha_0 + \theta_1)$$
$$L_2 = qSC_{L\alpha}(\alpha_0 + \theta_2) + qSC_{L_\delta}\delta_0$$

a) Static Equilibrium Eq: $\alpha_0 = 0$, $\delta = \delta_0$

$$\sum M_1 = \underbrace{L_1 e + K_T(\theta_2 - \theta_1) - K_T\theta_1}_{\text{Without aileron or additional forces caused by an offset.}} - \underbrace{L_2 d}_{\text{Caused by the SCs no longer being aligned and } L_2 \text{ not acting } SC_1.} = 0$$

$$0 = L_1 e + K_T\theta_2 - 2K_T\theta_1 - L_2 d$$

$$\sum M_2 = 0 = \underbrace{L_2(e+d) - K_T(\theta_2 - \theta_1)}_{\text{Without aileron}} + \underbrace{qScC_{MAC_\delta}\delta_0}_{\text{Added by aileron}}$$

$$0 = L_2 e + L_2 d - K_T\theta_2 + K_T\theta_1 + qScC_{MAC_\delta}\delta_0$$

Substitute lift eqs in:

$$eqSC_{L\alpha}\theta_1 - dqSC_{L\alpha}\theta_2 - dqSC_{L_\delta}\delta_0 + K_T\theta_2 - 2K_T\theta_1 = 0$$

$$eqSC_{L\alpha}\theta_2 + eqSC_{L_\delta}\delta_0 + dqSC_{L\alpha}\theta_2 + dqSC_{L_\delta}\delta_0 - K_T\theta_2 + K_T\theta_1 + qScC_{MAC_\delta}\delta_0 = 0$$

Rewrite multiply a negative through

$$\frac{2K_T\theta_1}{K_T} - \frac{K_T\theta_2}{K_T} - \frac{qSC_{L\alpha}}{K_T}(e\theta_1 - d\theta_2) = \frac{-dqSC_{L\delta}\delta_0}{K_T}$$

$$\frac{-K_T\theta_1}{K_T} + \frac{K_T\theta_2}{K_T} - \frac{qSC_{L\alpha}}{K_T}((e+d)\theta_2) = \frac{qSC_{L\delta}}{K_T}(e+d)\delta_0 + \frac{qScC_{MAC\delta}}{K_T}\delta_0$$

Write in Matrix form

$$\boxed{\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} - \frac{qSC_{L\alpha}e}{K_T}\begin{bmatrix} 1 & -\frac{d}{e} \\ 0 & 1+\frac{d}{e} \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \delta_0\frac{qSC_{L\alpha}e}{K_T}\begin{bmatrix} -d/e \; C_{L\delta}/C_{L\alpha} \\ (1+d/e)C_{L\delta}/C_{L\alpha} + c/e\,\frac{C_{MAC\delta}}{C_{L\alpha}} \end{bmatrix}}$$

b) We are given the following values:

$E = 0.15 \quad d/e = 1 \quad e/c = 0.10 \rightarrow c/e = 10$

With E, we can solve for $C_{L\delta}/C_{L\alpha}$ and $C_{MAC\delta}/C_{L\alpha}$ ...

$$C_{L\delta}/C_{L\alpha} = 1/\pi(\underbrace{\cos^{-1}(1-2E)}_{0.795} + \underbrace{2\sqrt{E(1-E)}}_{0.714}) = 0.4805$$

$$C_{MAC\delta}/C_{L\alpha} = -1/\pi\underbrace{(1-E)}_{0.85}\underbrace{\sqrt{(1-E)E}}_{0.357} = -0.0966$$

Plug in values: $\bar{q} = \frac{qSC_{L\alpha}e}{K_T}$

$$\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} - \bar{q}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \bar{q}\delta_0\begin{bmatrix} -0.4805 \\ 0.961 - 0.966 \end{bmatrix}$$

$$\left(\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} -\bar{q} & \bar{q} \\ 0 & -2\bar{q} \end{bmatrix}\right)\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \bar{q}\delta_0\begin{bmatrix} -0.4805 \\ -0.005 \end{bmatrix}$$

$$\begin{bmatrix} 2-\bar{q} & -1+\bar{q} \\ -1 & 1-2\bar{q} \end{bmatrix} \xrightarrow{\text{inv}} \frac{1}{(2-\bar{q})(1-2\bar{q}) + (-1+\bar{q})}\begin{bmatrix} 1-2\bar{q} & 1-\bar{q} \\ 1 & 2-\bar{q} \end{bmatrix}$$

$$2 - 4\bar{q} - \cancel{\bar{q}} + 2\bar{q}^2 - 1 + \cancel{\bar{q}}$$

$$2\bar{q}^2 - 4\bar{q} + 1$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \frac{\bar{q}\delta_0}{2\bar{q}^2 - 4\bar{q} + 1}\begin{bmatrix} 1-2\bar{q} & 1-\bar{q} \\ 1 & 2-\bar{q} \end{bmatrix}\begin{bmatrix} -0.4805 \\ -0.005 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \frac{\bar{q}\delta_0}{2\bar{q}^2 - 4\bar{q} + 1}\begin{bmatrix} -0.4805(1-2\bar{q}) - 0.005(1-\bar{q}) \\ -0.4805 - 0.005(2-\bar{q}) \end{bmatrix}$$

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \frac{\bar{q}\,\delta_0}{2\bar{q}^2-4\bar{q}+1}\begin{bmatrix}-0.4855 + 0.966\bar{q}\\ -0.4905 + 0.005\bar{q}\end{bmatrix}$$

$$L_1 = qSC_{L\alpha}\theta_1$$

$$L_2 = qSC_{L\alpha}\theta_2 + qSC_{L\delta}\delta_0$$

Plug in θ values

$$L_1 = qSC_{L\alpha}\left(\frac{\bar{q}\,\delta_0}{2\bar{q}^2-4\bar{q}+1}\right)(-0.4855 + 0.966\bar{q})$$

$$L_2 = qSC_{L\alpha}\left(\frac{\bar{q}\,\delta_0}{2\bar{q}^2-4\bar{q}+1}\right)(-0.4905 + 0.005\bar{q}) + qSC_{L\delta}\delta_0$$

$$= qSC_{L\alpha}\delta_0\left(\left(\frac{\bar{q}}{2\bar{q}^2-4\bar{q}+1}\right)(-0.4905 + 0.005\bar{q}) + \overbrace{\frac{C_{L\delta}}{C_{L\alpha}}}^{0.4805}\right)$$

$$L_2 = qSC_{L\alpha}\delta_0\left(\left(\frac{\bar{q}}{2\bar{q}^2-4\bar{q}+1}\right)(-0.4905 + 0.005\bar{q}) + 0.4805\right)$$

$$M_{roll} = \begin{bmatrix}\frac{b}{4} & \frac{3b}{4}\end{bmatrix}\begin{bmatrix}L_1\\ L_2\end{bmatrix} \longrightarrow M_{roll} = \frac{b}{4}L_1 + \frac{3b}{4}L_2$$

$$M_{roll} = b\,qSC_{L\alpha}\delta_0\left(\frac{\bar{q}}{2\bar{q}^2-4\bar{q}+1}\right)\Bigg(\frac{1}{4}(-0.4855 + 0.966\bar{q}) +$$

$$\frac{3}{4}(-0.4905 + 0.005\bar{q}) + \frac{2\bar{q}^2-4\bar{q}+1}{\bar{q}}\left(\frac{3}{4}\right)(0.4805)\Bigg)$$

$$M_{roll} = b\,qSC_{L\alpha}\delta_0\left[\left(\frac{\bar{q}}{2\bar{q}^2-4\bar{q}+1}\right)(-0.4893 + 0.2453\bar{q}) + 0.3604\right]$$

$$0.3604(2\bar{q}^2-4\bar{q}+1)$$

$$-0.4893\bar{q} + 0.2453\bar{q}^2 + 0.7208\bar{q}^2 - 1.4416\bar{q} + 0.3604$$

$$\boxed{M_{roll} = b\,qSC_{L\alpha}\delta_0\left(\frac{0.9661\bar{q}^2 - 1.9309\bar{q} + 0.3604}{2\bar{q}^2-4\bar{q}+1}\right) \qquad \bar{q} = \frac{qSC_{L\alpha}e}{K_T}}$$

↳ could probably plug in $C_{L\alpha} = 2\pi$, but not given

↳ would become messy if I plugged in $\bar{q}$

c) Solve for reversal speed

We have:

$$M_{roll} = bqSC_{L\alpha}\delta_0\left(\frac{0.9661\bar{q}^2 - 1.9309\bar{q} + 0.3604}{2\bar{q}^2 - 4\bar{q} + 1}\right)$$

Set $M_{roll} = 0$ (limit condition)

Get rid of consts S, $C_{L\alpha}$, δ_0, and b

The bottom will multiply away...

$$0 = (0.9661\bar{q}^2 - 1.9309\bar{q} + 0.3604)\,q \quad q_1 = 0 \quad \bar{q}_1 = 0$$

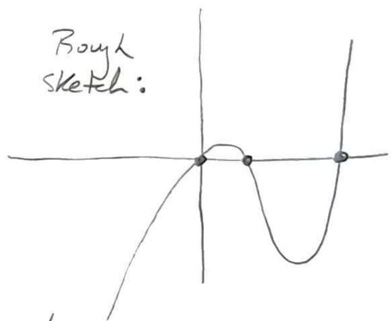
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \frac{1.9309 \pm \sqrt{(1.9309)^2 - 4(0.9661)(0.3604)}}{2(0.9661)}$$

$$\frac{1.9309 \pm 1.5283}{1.9322}$$

$\bar{q}_3 = 1.7903$ $\boxed{\bar{q}_2 = 0.2084}$

(non-dimensional)

Rough sketch:



Dynamic pressure $\bar{q}_2 = 0.2084$ is where reversal begins to happen. This suggests that K_T is not stiff enough for the airspeed being traveled before the wing begins to twist enough to cause a reversal effect.

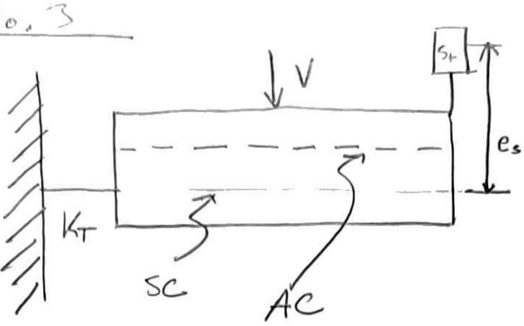
$$\bar{q} = q\frac{SC_{L\alpha}e}{K_T} \qquad q = \rho\frac{V^2}{2}$$

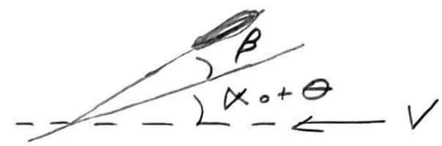
$$q = \frac{\bar{q}K_T}{SC_{L\alpha}e} \qquad \sqrt{\frac{\cancel{\rho}V^2}{\cancel{2}}} = \sqrt{\frac{\bar{q}K_T 2}{SC_{L\alpha}e\rho}}$$

$$\boxed{V_R = \sqrt{\frac{0.4168K_T}{SC_{L\alpha}e\rho}}}$$

Reversal Speed

Problem No. 3





$$L_s = q S_{tip} C_{L_{\alpha s}} \alpha_s$$

$$L = q S C_{L_\alpha} \alpha \quad w/ \alpha = \alpha_o + \theta$$

K [in-lb/rad]

$K * \beta$ restory torque

a) $\alpha_o \neq 0$

In terms of $\bar{q} = \frac{q S e C_{L_\alpha}}{K_T}$; $S_R = \frac{S_{Tip} C_{L_{\alpha s}} e_s}{S C_{L_\alpha} e}$; $K_R = \frac{K}{K_T}$

Equations of Eq ; derive characteristic eq for q_D.

→ Rigid Wing!

$$\sum M_{sc} = 0 = Le - K_T \theta + L_2 e_s$$

$$0 = q S C_{L_\alpha} (\alpha_o + \theta) e + K\beta - K_T \theta,$$

$$\boxed{\left(\frac{q S e C_{L_\alpha}}{K_T} - \frac{K_T}{K_T} \right) \theta + \frac{K\beta}{K_T} = -\frac{q S C_{L_\alpha} e \alpha_o}{K_T} \quad (a)}$$

We also have the following relationship

$$e_s q S_{Tip} C_{L_{\alpha s}} (\alpha_o + \theta + \beta) = K\beta$$

$$\boxed{\frac{q S_{Tip} e_s C_{L_{\alpha s}}}{K_T} \theta + \frac{(q S_{Tip} e_s C_{L_{\alpha s}} - K)}{K_T} \beta = -\frac{q S_{Tip} e_s C_{L_{\alpha s}} \alpha_o}{K_T}} \quad (b)$$

Put (a) and (b) into a matrix, but also sub in for $\bar{q}$ and K_R:

$$\begin{bmatrix} \bar{q} - 1 & K_R \\ \frac{q S_{Tip} e_s C_{L_{\alpha s}}}{K_T} & \frac{q S_{Tip} e_s C_{L_{\alpha s}}}{K_T} - K_R \end{bmatrix} \begin{bmatrix} \theta \\ \beta \end{bmatrix} = -\alpha_o \begin{bmatrix} \bar{q} \\ \frac{q S_{Tip} e_s C_{L_{\alpha s}}}{K_T} \end{bmatrix}$$

Characteristic eq is the determinant of

$$\begin{bmatrix} \bar{q}-1 & K_R \\ \frac{q S_{Tip} e_s C_{L\alpha_s}}{K_T} & \frac{q S_{Tip} e_s C_{L\alpha}}{K_T} - K_R \end{bmatrix}$$

$$\bar{q}-1\left(\frac{q S_{tip} e_s C_{L\alpha s}}{K_T} - K_R\right) - K_R\left(\frac{q S_{Tip} e_s C_{L\alpha s}}{K_T}\right) = 0$$

$$\frac{q S_{Tip} e_s C_{L\alpha s}}{K_T} \cdot \frac{C_{L\alpha}}{C_{L\alpha}} \cdot \frac{e}{e} \cdot \frac{S}{S} \longrightarrow \bar{q}\ \frac{S_{Tip} e_s C_{L\alpha s}}{C_{L\alpha} e S} \rightarrow S_R$$

$$\bar{q}-1(\bar{q} S_R - K_R) - K_R(\bar{q} S_R) = 0$$

$$S_R \bar{q}^2 - K_R \bar{q} - S_R \bar{q} + K_R - K_R S_R \bar{q} = 0$$

$$\frac{\cancel{S_R}\bar{q}^2}{\cancel{S_R}} - \left(\frac{K_R}{S_R} + \frac{\cancel{S_R}}{\cancel{S_R}} + \frac{K_R \cancel{S_R}}{\cancel{S_R}}\right)\bar{q} + \frac{K_R}{S_R} = 0$$

$$\bar{q}^2 - \left(\frac{K_R}{S_R} + 1 + K_R\right)\bar{q} + \frac{K_R}{S_R} = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ae}}{2a}$$

$$\boxed{\bar{q} = \frac{\left(\frac{K_R}{S_R} + 1 + K_R\right) \pm \sqrt{\left(\frac{K_R}{S_R} + 1 + K_R\right)^2 - 4\left(\frac{K_R}{S_R}\right)}}{2}}$$

b) $K_R = 10$ and $K_R = 1 \implies$ Plot these

See Attached

We notice that we must graph minus version of above eq or $\bar{q}$ becomes too large.

We also notice that higher values of K_R shift the values of $\bar{q}$ higher as the value of S_R grows larger. $\bar{q}$ also decreases as S_R increases.

Hw #2 - Aeroelasticity - ME597/AAE556

Victoria Nagorski - 9/19/22

Contents

Problem No. 3b

```
clear;clc;close all;
% Initialize Variables
Sr = 0:0.001:1;
q_hat = @(K_r) (((K_r./Sr)+1+K_r)-sqrt(((K_r./Sr)+1+K_r).^2-(4.*K_r./Sr)))./2;
Kr = [1,10];

% Graphing
figure
hold on
for i = 1:length(Kr)
    K_r = Kr(1,i);          % Pull out the q_hat value being used
    temp_q = q_hat(K_r);        % Solve for the K_e values
    plot(Sr,temp_q)         % Graph everything
end
title('$\hat{q}$ vs $S_R$','Interpreter','latex')
xlabel('$S_R$','Interpreter','latex')
ylabel('$\hat{q}$','Interpreter','latex')
legend('k_{R} = 1','k_{R} = 10')
hold off
```