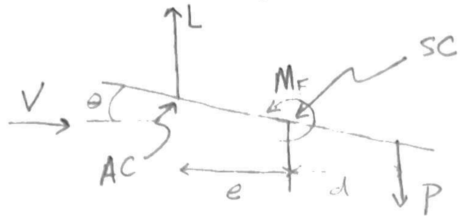
Problem #1




Positive clockwise

Wing area $\rightarrow S$
concentrated load $\rightarrow P$
$\Theta_0 = 0$
$\Theta$ [rads]
$M_E = K_1\Theta + K_2\Theta^3$
$K_1$ & $K_2$ [Nm/rad]
$L = qSa_1\Theta$
$e = X_{sc} - X_{AC}$

a) Torsional Equilibrium Eq:

$$\sum M = Pd + Le - K_1\Theta - K_2\Theta^3 = 0$$

$$Pd = -qSa_1\Theta e + K_1\Theta + K_2\Theta^3$$

$$\boxed{Pd = (K_1 - qSa_1e)\Theta + K_2\Theta^3}$$

b) Nondimensialize:

$$\frac{Pd}{K_1} = \Big(\cancel{\frac{K_1}{K_1}}^{1} - \underbrace{\frac{qSa_1e}{K_1}}_{\hat{q}}\Big)\Theta + \frac{K_2\Theta^3}{K_1}$$

$$\hat{q} = \frac{qSea_1}{K_1}$$

$$\boxed{\frac{Pd}{K_1} = (1-\hat{q})\Theta + \frac{K_2}{K_1}\Theta^3}$$

c) $K_2 = {}^8/_3\, qSea_1K_1$

(i) effective torsional stiffness $K_e$
$\frac{\partial\hat{P}}{\partial\Theta}$, where $\hat{P} = \frac{Pd}{K_1}$

$$\hat{P} = (1-\hat{q})\Theta + \underbrace{\frac{{}^8/_3\, qSea_1}{K_1}}_{\hat{q}}K_1\Theta^3$$

$$\hat{P} = (1-\hat{q})\Theta + {}^8/_3\,\hat{q}K_1\Theta^3$$

$$\boxed{K_e = \frac{\partial\hat{P}}{\partial\Theta} = (1-\hat{q}) + 8\hat{q}K_1\Theta^2}$$

$$\hat{K}_e = \frac{K_e}{K_1}$$

According to Weisshaar this is the non-dimensional effective torsional stiffness. But as defined in problem statement.

$$K_e = (1-\hat{q})K_1 + 8\hat{q}\Theta^2K_1$$ Using Weisshaar def

(ii)

$$K_e = (1-\hat{q}) + 8\hat{q}\,\theta^2$$

$\hat{q} = [0, 0.5, 1, 1.5]$

Look at attachment for plot

d) $\hat{P} = (1-\hat{q})\theta + \frac{8}{3}\hat{q}\,\theta^3$

$\theta \in (-0.5, 0.5)$

$\hat{q} = [0, 0.5, 1, 1.5]$

Look at attachment for plot.

1. Only one twist angle solution $\Rightarrow \bar{q}=0, \bar{q}=0.5$
2. The limit of linear divergence, i.e. for a system w/ $K_e=0 \Rightarrow \bar{q}=1$
3. The system shows multiple stable solutions $\Rightarrow \bar{q}=1, \bar{q}=1.5$

Both $\bar{q}=0$ and $\bar{q}=0.5$ are stable if we look the graphs in part (c). This means they are below the divergence speed, and therefore have only one twist angle solution. This is confirmed if we look at the graphs in part (d). We see the graph only crosses the x-axis once. The slope of the cross-point is also positive.

At $\bar{q}=1$, we see that the system is now neutrally stable. At $\theta=0$, the $K_e$ is 0. We want $K_e$ to be positive for stability. However, we see that $K_e$ also never becomes negative and becomes more positive with a positive twist. According to linear theory, a neutral stable system is going to have multiple stability points. If we look at graph (d), we see that the curve almost hugs the x-axis for some time.

At $\bar{q}=1.5$, we now see an unstable system at small angle twists. Looking at (c), we see it would take a large angle twist for $K_e > 0$. This twist is unlikely. Looking at part (d), we see that 3 stability points exist: $\theta=0$, $\theta \approx 0.35$, $\theta \approx -0.35$. $\theta=0$ is the trivial solution, and is unstable due to the negative slope at that point. The other two points are stable because they have positive slopes. Those angle are considerable big twist angles.

It is observed that as $\bar{q}$ grows, the system begins to tend towards instability. As $\bar{q}$ crosses the divergence speed, more poles begin to arrise out of the system and move further away from $\theta = 0$. This means more and more twist is required to stabalize the system. At the $\bar{q} = 1$ value, we see all stability points near $\theta = 0$. This makes them plausible points to reach. However, as soon as we go to $\bar{q} = 1.5$, the points are much further away from $\theta = 0$.

Problem #2

$$n = \frac{\text{Total Lift}}{\text{Total Weight}} \qquad L_T = 2L + \cancel{L_f}^{\,0} = 2qSa_1(\alpha_0 + \theta)$$

$$W = 2w + W_f$$

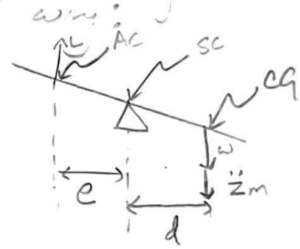
$$n = \frac{2qSa_1(\alpha_0 + \theta)}{2w + W_f} \qquad L = qSa_1(\theta + \alpha_0)$$

- angle of attack aircraft → $\alpha_0$ → ignore the tail
- wing twist → $\theta$
- wing area → $S$
- distance of CG from SC → $d$

a) $\alpha_0$ as a function of $n$

Looking at diagram... restoring force from spring
$M_s = K_T\theta$

Sum moments about shear center to get static equilibrium eq about each wing:




$$\sum M_s = K\theta - Le - mgd - m\ddot{z}d$$
$$= K_T\theta - Le - \underbrace{mg}_{w}\underbrace{(1 + \ddot{z}/g)}_{n}d$$

$(1 + \ddot{z}/g) = n$ ← applied acceleration

$\frac{m\ddot{z}}{mg} = n - 1$ ← real acceleration

$$\sum M_s = K_T\theta - Le - wnd = 0$$
$$L = qSa_1(\theta + \alpha_0) \qquad n = \frac{2L}{W}$$

$$\sum M_s = K_T\theta - Le - w\left(\frac{2L}{W}\right)d = 0$$

$$\sum M_s = K_T\theta - L\left(e + 2\frac{w}{W}d\right) = 0$$

$$K_T\theta = L\left(e + \frac{2w}{W}d\right)$$

$$K_T\theta = qSa_1(\alpha_0 + \theta)\left(e + 2\frac{w}{W}d\right)$$
$$K_T\theta - qSa_1\theta\left(e + 2\frac{w}{W}d\right) = qSa_1\alpha_0\left(e + 2\frac{w}{W}d\right)$$

$$K_T\theta - qSa_1\theta\left(e + 2\frac{\omega}{W}d\right) = qSa_1\alpha_0\left(e + 2\frac{\omega}{W}d\right)$$

$$\cancel{\left[K_T - qSa_1\left(e + 2\frac{\omega}{W}d\right)\right]}\theta = \frac{qSa_1\alpha_0\left(e + 2\frac{\omega}{W}d\right)}{K_T - qSa_1\left(e + 2\frac{\omega}{W}d\right)}$$

$$\theta = \frac{qSa_1 e\,\alpha_0\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}{K_T\left(1 - \frac{qSa_1 e}{K_T}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}$$

$$\bar{q} = \frac{qSa_1 e}{K_T}$$

differs from book, but I like $\bar{q}$ more than $\bar{q} = q/q_{D\,fixed}$ Easier for plotting purposes.

$$\theta = \frac{\bar{q}\,\alpha_0\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}{\left(1 - \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}$$

$$\theta = \frac{\bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}{\left(1 - \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}\alpha_0 \qquad (a)$$

We need to get (a) in terms of $\theta$ or $\alpha_0$.
Will require some further work...

$$L = qSa_1(\alpha_0 + \theta) \quad \text{Plug in (a)}$$

$$L = qSa_1\left(\alpha_0 + \frac{\bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}{\left(1 - \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}\alpha_0\right)$$

$$L = qSa_1\alpha_0\left(1 + \frac{\bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}{\left(1 - \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}\right)$$

$$L = qSa_1\alpha_0\,\frac{1 \cancel{- \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right) + \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}^{\to 0}}{1 - \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}$$

$$L = qSa_1\alpha_0\left(\frac{1}{1 - \bar{q}\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}\right)$$

$$L = q S a_1 \alpha_0 \left( \frac{1}{1 - \bar{q}\left(1 + 2\frac{w}{W}\frac{d}{e}\right)} \right) \quad (b)$$

We have the relationship given to us earlier that relates Lift, n, and Weight

$$n = \frac{2L}{W} \longrightarrow L = \frac{nW}{2} \quad (c)$$

Set equation (b) and (c) equal to eachother

$$q S a_1 \alpha_0 \left( \frac{1}{1 - \bar{q}\left(1 + 2\frac{w}{W}\frac{d}{e}\right)} \right) = \frac{nW}{2}$$

$$\alpha_0 = \frac{nW}{2qSa_1}\left(1 - \bar{q}\left(1 + 2\frac{w}{W}\frac{d}{e}\right)\right)$$ $\mu$

$W = 70{,}000\, g \cdot Kg$
$C_{L\alpha} = 6$
$S = 125\ m^2$
$V = 120\ m/s$
$\rho = 1.112\ Kg/m^3$
$\frac{\mu}{e} = \frac{1}{2}$

$q = \frac{1}{2}\rho V^2$

$$\boxed{\alpha_0 = \frac{nW}{2SC_{L\alpha}q}\left(1 - \frac{q}{q_{D\,div}}\left(1 + \frac{\mu}{e}\right)\right)} \quad (d)$$

$\bar{q} = q/q_{D\,diverg}$

b)

We can go back to equation (a):

$$\theta = \frac{\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}{\left(1 - \bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}\alpha_0$$

Make $\theta$ the independent variable now:

$$\alpha_0 = \frac{\left(1 - \bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}{\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}\theta$$

Plug into Lift like part a)

$$L = q S a_1 (\alpha_0 + \theta)$$

$$L = q S a_1 \theta\left(1 + \frac{\left(1 - \bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)}{\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}\right)$$

$$\cancel{\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)} + 1 - \cancel{\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}$$

$$L = q S a_1 \theta\left(\frac{1}{\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}\right) \qquad (e)$$

Use the $n = \frac{2L}{W}$ relationship:

$$L = \frac{nW}{2} = q S a_1 \theta\left(\frac{1}{\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)}\right)$$

Solve for $\theta$:

$$\theta = \frac{nW}{2 q S a_1}\left(\bar{q}_L\left(1 + 2\frac{\omega}{W}\frac{d}{e}\right)\right)$$

$\mu$

$$\boxed{\theta = \frac{nW}{2 q S C_{L\alpha}}\left(q/q_{L_{Div}}\left(1 + \mu/e\right)\right)} \qquad (f)$$

c)

$$\sum M_c = K_T\theta - Le - wnd = 0$$

$$\cancel{K_T}\theta - \frac{qSa_1e}{K_T}(\alpha_0 + \theta) = \frac{wnd}{K_T}$$

$$\frac{qSa_1e}{K_T} \rightarrow \frac{1}{q_D}$$

$$\cancel{\theta} + \frac{q}{q_D}(\alpha_0 + \theta) = -\frac{wnd}{K_T} + \theta$$

$$\frac{q}{\cancel{q_D}}(\alpha_0 + \theta) = \left(\theta - \frac{wnd}{K_T}\right) q_D$$

$$\boxed{q_D = \frac{q(\alpha_0 + \theta)}{\theta - \frac{wnd}{K_T}}}$$

We can plug in $\theta + \alpha_0$ other problems to simplify a little more...

$$\left(\frac{nW}{2qSa_1}\right)\left[1 - \cancel{\bar{q}\left(1 + 2\frac{w}{W}\frac{d}{e}\right)} + \bar{q}\left(1 + 2\frac{w}{W}\frac{d}{e}\right)\right]$$

$$\alpha_0 + \theta \rightarrow \left(\frac{nW}{2qSa_1}\right)$$

$$q_D = \frac{\cancel{q}\left(\frac{nW}{2\cancel{q}Sa_1}\right)}{\theta - \frac{wnd}{K_T}} \rightarrow \frac{\frac{nW}{2Sa_1}}{\frac{K_T\theta}{K_T} - \frac{wnd}{K_T}} \rightarrow \frac{K_T\theta - wnd}{K_T}$$

$$q_D = \frac{nW}{2Sa_1} \cdot \frac{K_T}{K_T\theta - wnd}\,\frac{e}{e}$$

$$q_D = \frac{nWq_De}{2(K_T\theta - wnd)}$$ ← cancel out $q_D$ !!

If you plug in for both $\theta$s, all the "n"s cancel out. So we will leave the dependencies on $\alpha_0$ and $\theta$. I've tried a few variations.

# Hw #1 - Aeroelasticity - ME597/AAE556

Victoria Nagorski - 9/2/22

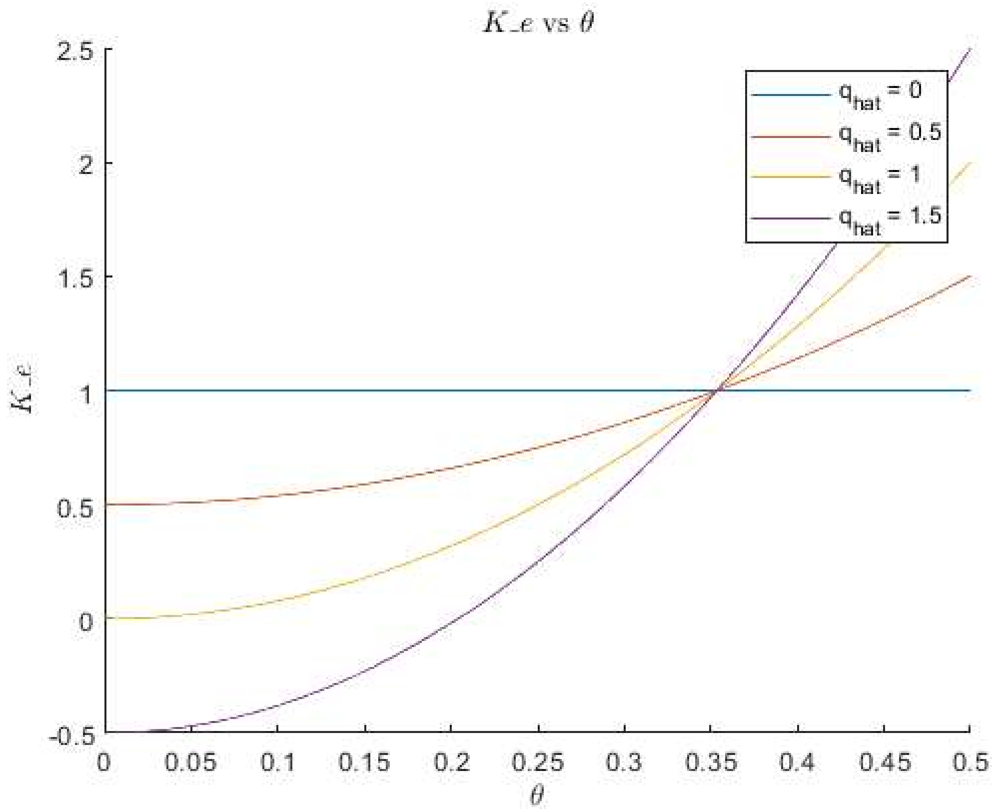
## Contents



## Problem No. 1c

```
clear;clc;close all;
% Initialize Variables
theta = 0:0.001:0.5;
K_e = @(qHat) (1-qHat) + 8*qHat.*theta.^2;
q_hat = [0,0.5,1,1.5];

% Graphing
figure
hold on
for i = 1:length(q_hat)
    qHat = q_hat(1,i);          % Pull out the q_hat value being used
    temp_Ke = K_e(qHat);        % Solve for the K_e values
    plot(theta,temp_Ke)         % Graph everything
end
title('$K\_{e}$ vs $\theta$','Interpreter','latex')
xlabel('$\theta$','Interpreter','latex')
ylabel('$K\_{e}$','Interpreter','latex')
legend('q_{hat} = 0','q_{hat} = 0.5','q_{hat} = 1','q_{hat} = 1.5')
hold off
```

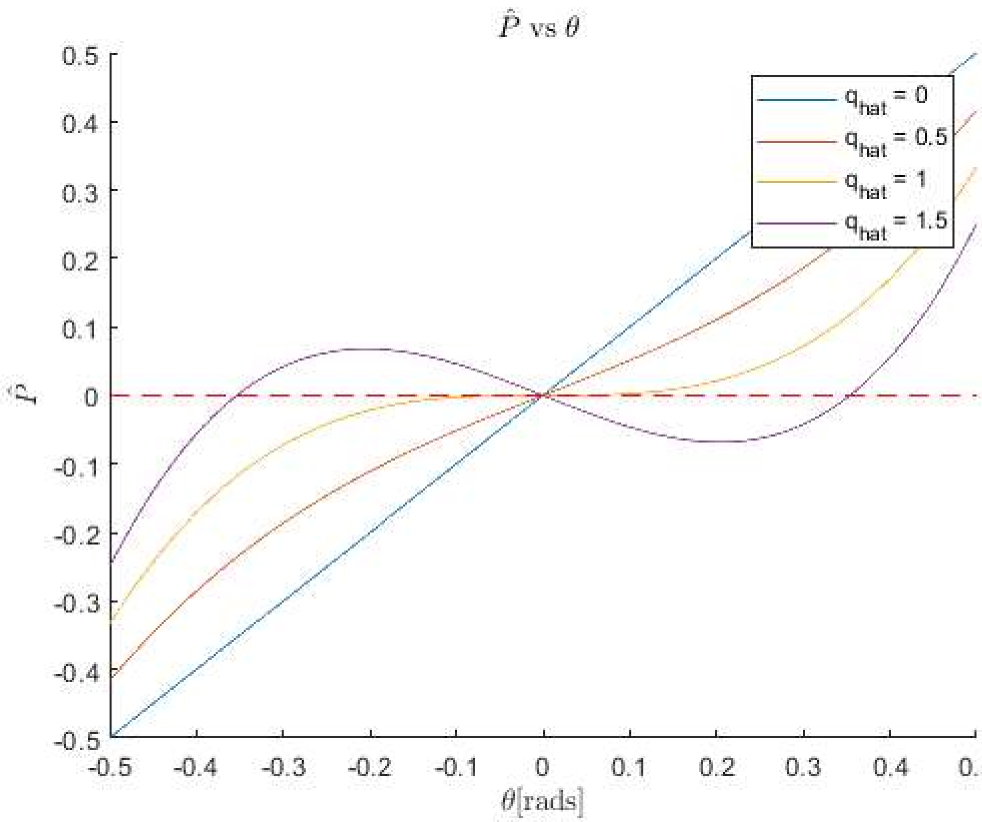



## Problem No. 1d

```
clear;clc;close all;

% Initialize Variables
theta = -0.5:0.0001:0.5;
P_hat = @(qHat) (1-qHat).*theta + 8/3*qHat.*theta.^3;
q_hat = [0,0.5,1,1.5];

% Graphing
figure
hold on
for i = 1:length(q_hat)
    qHat = q_hat(1,i);              % Pull out the q_hat value being used
    temp_Phat = P_hat(qHat);        % Solve for the P_hat values
    plot(theta,temp_Phat)           % Graph everything
end
x = [-0.5 0.5];
y = [0 0];
line(x,y,'Color','red','LineStyle','--')
title('$\hat{P}$ vs $\theta$ ','Interpreter','latex')
xlabel('$\theta$[rads]','Interpreter','latex')
ylabel('$\hat{P}$','Interpreter','latex')
legend('q_{hat} = 0','q_{hat} = 0.5','q_{hat} = 1','q_{hat} = 1.5')
hold off
```

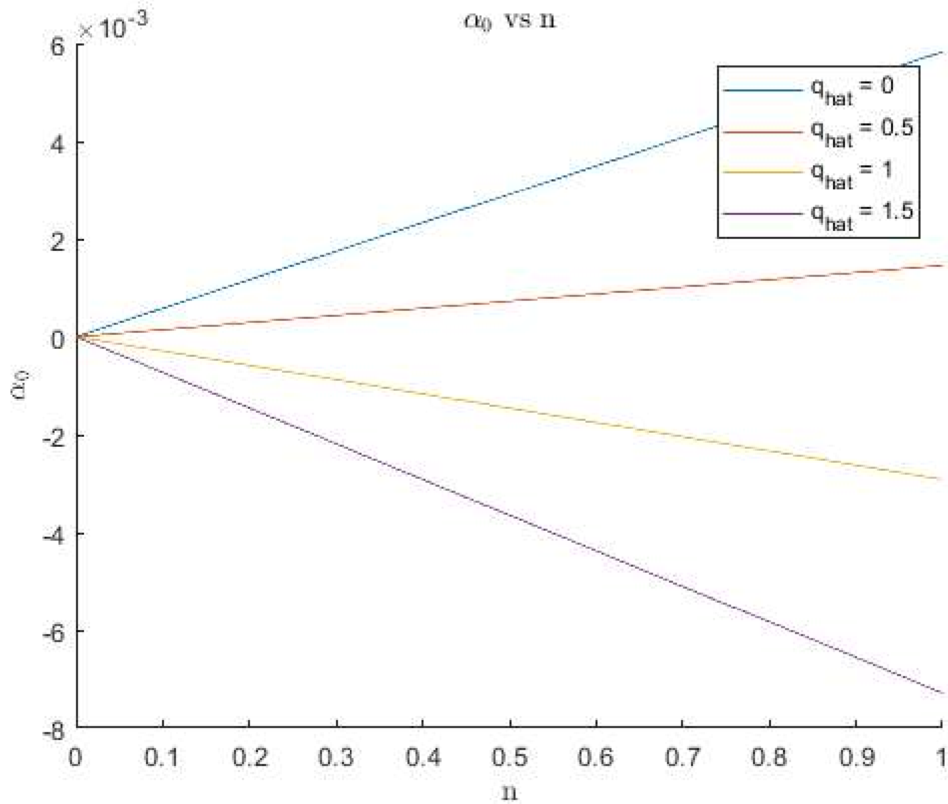



## Problem No. 2a

```
clear;clc;close all;
% Initialize Variables
V = 120;            % Velocity [m/s]
rho = 1.112;        % Density [kg/m^3]
W = 70000;          % Total weight [N]
muRatio = 1/2;      % Mu/e
q = (1/2)*rho*V^2;  % Dynamic pressure (typical) [N/m^2]
S = 125;            % Wing  area [m^2]
a1 = 6;             % Alpha vs lift coefficient slope
n = 0:0.0001:1;

% Define the Equations
alpha_0 = @(qHat) (n.*W./(2*q*S*a1)) .* (1-qHat*(1 + muRatio));
q_hat = [0,0.5,1,1.5];

% Graphing
figure
hold on
for i = 1:length(q_hat)
    qHat = q_hat(1,i);              % Pull out the q_hat value being used
    temp_alpha0 = alpha_0(qHat);    % Solve for the P_hat values
    plot(n,temp_alpha0)             % Graph everything
end
title('$\alpha_0$ vs n','Interpreter','latex')
xlabel('n','Interpreter','latex')
ylabel('$\alpha_0$','Interpreter','latex')
legend('q_{hat} = 0','q_{hat} = 0.5','q_{hat} = 1','q_{hat} = 1.5')
hold off
```

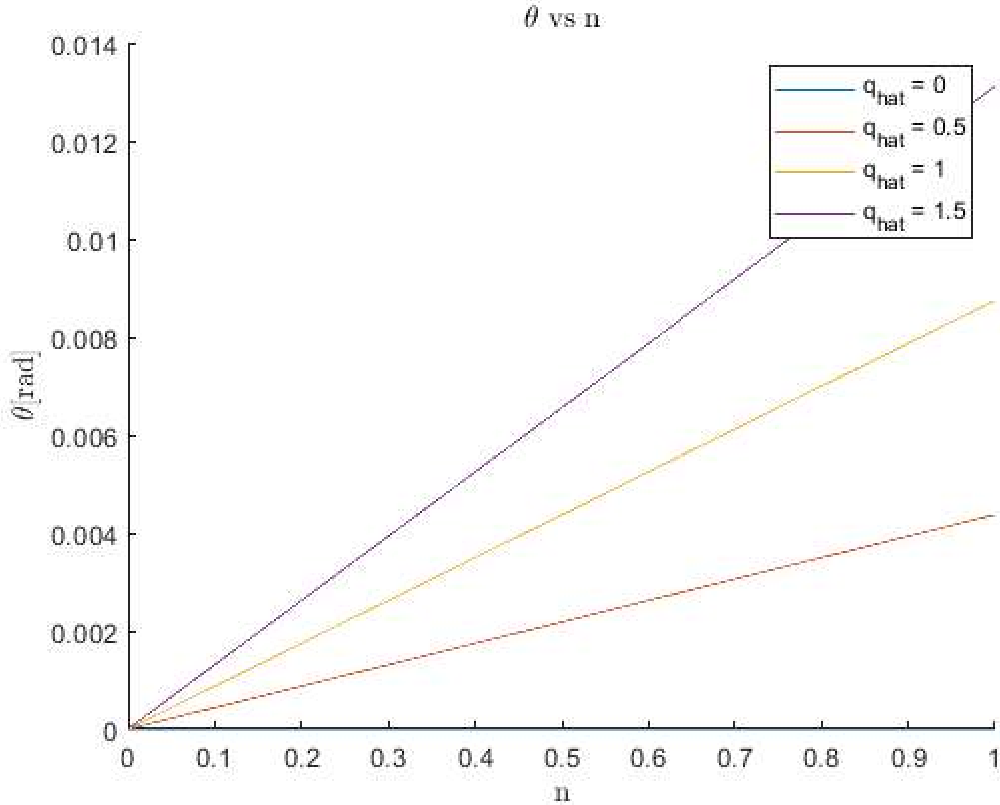



## Problem No. 2c

```
clear;clc;close all;
% Initialize Variables
V = 120;             % Velocity [m/s]
rho = 1.112;         % Density [kg/m^3]
W = 70000;           % Total weight [N]
muRatio = 1/2;       % Mu/e
q = (1/2)*rho*V^2;   % Dynamic pressure (typical) [N/m^2]
S = 125;             % Wing  area [m^2]
a1 = 6;              % Alpha vs lift coefficient slope
n = 0:0.0001:1;

% Define the Equations
theta = @(qHat) (n.*W./(2*q*S*a1)) .* (qHat*(1 + muRatio));
q_hat = [0,0.5,1,1.5];

% Graphing
figure
hold on
for i = 1:length(q_hat)
    qHat = q_hat(1,i);              % Pull out the q_hat value being used
    temp_theta = theta(qHat);       % Solve for the P_hat values
    plot(n,temp_theta)              % Graph everything
end
title('$\theta$ vs n','Interpreter','latex')
xlabel('n','Interpreter','latex')
ylabel('$\theta$[rad]','Interpreter','latex')
legend('q_{hat} = 0','q_{hat} = 0.5','q_{hat} = 1','q_{hat} = 1.5')
hold off
```

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