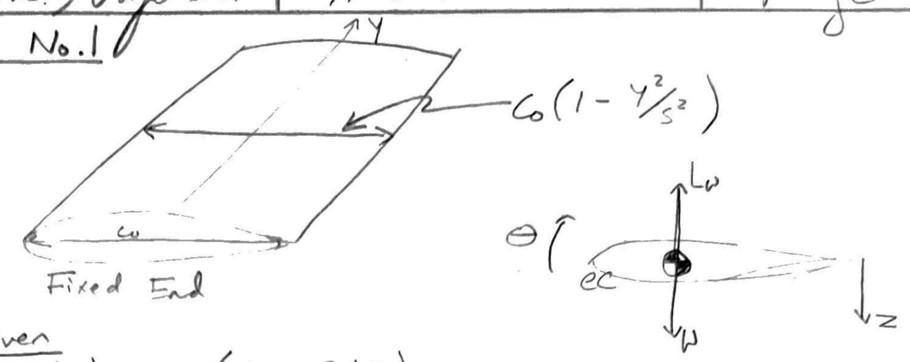
Problem No. 1



Given

$C(y) = c_o(1 - y^2/s^2)$

$GJ(y) = (GJ)_o(1 - y^2/s^2)$

$dL = q\,c(y)\,a_w\,(\alpha_o + \theta(y))$

All forces act at AC

AC & CG same location

AC distance e from SC

a) $\theta(y) = y/s\ \theta_T \Rightarrow$ Solve for lift distribution and divergence speed

Need to solve for trim θ using energy methods.

$$U = 2\left(\frac{1}{2}\int_0^s GJ(y)\left(\frac{\partial\theta}{\partial y}\right)^2\right)dy \qquad \theta = \alpha_o + \theta_T\, y/s$$

$$U = \int_0^s (GJ)_o(1 - y^2/s^2)(\theta_T/s)^2\,dy$$

$$\int_0^s (GJ)_o\left([\theta_T/s]^2 - [\theta_T/s]^2(y/s)^2\right)dy$$

$$U = (GJ)_o\left((\theta_T/s)^2 y - (\theta_T/s)^2\left(\tfrac{1}{3}\right)y^3/s^2\right)\Big|_0^s$$

$$U = (GJ)_o(\theta_T)^2\left(\tfrac{2}{3}\right)\left(\tfrac{1}{s}\right)$$

$$\delta W = W\delta z + 2\,dL\cdot(ec\,\delta\theta - \delta z)$$

$$= W\delta z + 2\int_0^s q\,c_o a_w(1 - y^2/s^2)(\alpha_o + \theta_T\, y/s)(ec\,\delta\theta - \delta z)\,dy$$

$$\delta W = W\delta z + 2\int_0^s q\,c_o a_w(1 - y^2/s^2)(\alpha_o + \theta_T\, y/s)\,e\,c\,(1 - y^2/s^2)\,y/s\,\delta\theta_T\,dy$$
$$- 2\int_0^s q\,c_o a_w(1 - y^2/s^2)(\alpha_o + \theta_T\, y/s)\,\delta z\,dy$$

$$\alpha_o + \theta_T\, y/s - y^2/s^2\,\alpha_o - \theta_T\, y^3/s^3 \rightarrow \alpha_o s + \tfrac{1}{2}\theta_T s - \tfrac{1}{3}\alpha_o s - \tfrac{1}{4}\theta_T s$$

$$(\quad)(y/s - y^3/s^3) \rightarrow y/s\,\alpha_o - y^3/s^3\,\alpha_o + y^2/s^2\,\theta_T - y^4/s^4\,\theta_T - y^3/s^3\,\alpha_o + y^5/s^5\,\alpha_o$$

$$\rightarrow \cancel{\tfrac{1}{2}\alpha_o s} - \cancel{\tfrac{1}{4}\alpha_o s} + \tfrac{5}{5\cdot 3}\theta_T s - \tfrac{3}{3\cdot 5}\theta_T s - \cancel{\tfrac{1}{4}\alpha_o s} + \tfrac{1}{6}\alpha_o s$$

$$\delta W = W\delta z + 2q\,c_o^2\,e\,a_w s\left(\tfrac{1}{6}\alpha_o + \tfrac{2}{15}\theta_T\right)\delta\theta_T$$
$$- 2q\,c_o a_w s\left(\tfrac{2}{3}\alpha_o + \tfrac{1}{4}\theta_T\right)\delta z$$

$$Q_z = \frac{\partial(\delta W)}{\partial(\delta z)} = W - 2qc_0 a_w s\left(\tfrac{2}{3}\kappa_0 + \tfrac{1}{4}\theta_T\right)$$

$$Q_{\theta_T} = \frac{\partial(\delta W)}{\partial(\delta \theta_T)} = 2qc_0^2 e a_w s\left(\tfrac{1}{6}\kappa_0 + \tfrac{2}{15}\theta_T\right)$$

$T=0$

Lagrange Eq

$$\cancel{\frac{d}{dt}\frac{\partial T}{\partial \dot\theta_T}}^{\,0} - \cancel{\frac{\partial T}{\partial \theta_T}}^{\,0} + \cancel{\frac{\partial \mathcal{D}}{\partial \theta_T}}^{\,0} + \frac{\partial U}{\partial \theta_T} = \frac{\partial(\delta W)}{\partial(\delta \theta_T)}$$

$$\cancel{\frac{d}{dt}\frac{\partial T}{\partial \dot z}}^{\,0} - \cancel{\frac{\partial T}{\partial z}}^{\,0} + \cancel{\frac{\partial \mathcal{D}}{\partial z}}^{\,0} + \cancel{\frac{\partial U}{\partial z}}^{\,0} = \frac{\partial(\delta W)}{\partial(\delta z)}$$

$$\frac{\partial U}{\partial \theta_T} = \frac{4}{3}(GJ)_0 \frac{1}{s}\theta_T$$

$$\frac{4}{3}(GJ)_0 \frac{1}{s}\theta_T + 2qc_0^2 e a_w s\left(\tfrac{1}{6}\kappa_0 + \tfrac{2}{15}\theta_T\right) = 0$$

$$2qc_0 a_w s\left(\tfrac{2}{3}\kappa_0 + \tfrac{1}{4}\theta_T\right) = W$$

$$\begin{bmatrix} \tfrac{1}{3} q c_0^2 e a_w s & \tfrac{4}{3}(GJ)_0 \tfrac{1}{s} + \tfrac{4}{15} q c_0^2 e a_w s \\ \tfrac{4}{3} q c_0 a_w s & \tfrac{1}{2} q c_0 a_w s \end{bmatrix} \begin{bmatrix} \kappa_0 \\ \theta_T \end{bmatrix} = \begin{bmatrix} 0 \\ W \end{bmatrix}$$

$$\tfrac{1}{3} q c_0^2 e a_w s\left(\tfrac{1}{2} q c_0 a_w s\right) - \tfrac{4}{3} q c_0 a_w s\left(\tfrac{4}{3}(GJ)_0 \tfrac{1}{s} + \tfrac{4}{15} q c_0^2 e a_w s\right)$$

$$\det = \tfrac{1}{6} q^2 c_0^3 a_w^2 e s^2 - \tfrac{16}{9}(GJ)_0 q c_0 a_w + \tfrac{16}{45} q^2 c_0^3 a_w^2 e s^2$$

$$\begin{bmatrix} \kappa_0 \\ \theta_T \end{bmatrix} = \frac{1}{\det} \begin{bmatrix} \tfrac{1}{2} q c_0 a_w s & -\tfrac{4}{3}(GJ)_0 \tfrac{1}{s} - \tfrac{4}{15} q c_0^2 e a_w s \\ -\tfrac{4}{3} q c_0 a_w s & \tfrac{1}{3} q c_0^2 e a_w s \end{bmatrix} \begin{bmatrix} 0 \\ W \end{bmatrix}$$

$$\kappa_0 = \frac{\left[-\tfrac{4}{3}(GJ)_0 \tfrac{1}{s} - \tfrac{4}{15} q c_0^2 e a_w s\right] W}{\tfrac{1}{6} q^2 c_0^3 a_w^2 e s^2 - \tfrac{16}{9}(GJ)_0 q c_0 a_w + \tfrac{16}{45} q^2 c_0^3 a_w^2 e s^2}$$

$$\theta_T = \frac{\tfrac{1}{3} q c_0^2 e a_w s W}{\tfrac{1}{6} q^2 c_0^3 a_w^2 e s^2 - \tfrac{16}{9}(GJ)_0 q c_0 a_w + \tfrac{16}{45} q^2 c_0^3 a_w^2 e s^2}$$

$$dL = q\,c(y)\,a_w\,\alpha(y)\,dy$$

$$\frac{dL}{dy} = q\,c_0\,(1 - y^2/s^2)(\alpha_0 + 1/s\,\theta_T)\,a_w$$

$$\left(-4/3\,(GJ)_0\,1/s - 4/15\,q\,c_0^2\,e\,a_w\,s + 1/3\,q\,c_0^2\,e\,a_w\,y\right)W$$

$$\boxed{\frac{dL}{dy} = \frac{q\,c_0\,a_w\,(1 - y^2/s^2)\left(-4/3\,(GJ)_0\,1/s - 4/15\,q\,c_0^2\,e\,a_w\,s + 1/3\,q\,c_0^2\,e\,a_w\,y\right)W}{1/6\,q^2 c_0^3 a_w^2 e s^2 - 16/9\,(GJ)_0\,q\,c_0\,a_w + 16/45\,q^2 c_0^3 a_w^2 e s^2}}$$

Solve for divergent speed while trimmed:

Look at α_0 denominator.

$$0 = \overset{7.5}{\cancel{1/6}}\,q^2\,\cancel{c_0^3}\,\cancel{a_w^2}\,\cancel{e}\,\cancel{s^2} - \frac{\overset{80}{\cancel{16/9}}\,(GJ)_0\,q\,\cancel{c_0}\,\cancel{a_w}}{s^2\,e\,c_0^2\,a_w} + 16/\cancel{45}\,q^2\,\cancel{c_0^3}\,\cancel{a_w^2}\,\cancel{e}\,\cancel{s^2}$$

$$23.5\,q^{\cancel{2}} - \frac{80\,(GJ)_0}{s^2 e c_0^2 a_w}\,\cancel{q} = 0$$

$$q = 0$$

$$q_A = \frac{3.4\,(GJ)_0}{s^2 e c_0^2 a_w}$$

$$\cancel{\tfrac{1}{2}}\rho V_A^2 = \frac{3.4\,(GJ)_0}{s^2 e c_0^2 a_w\,\rho}(2)$$

$$\boxed{V_A = \sqrt{\frac{6.8\,(GJ)_0}{s^2 e c_0^2 a_w\,\rho}}}$$

b) $\theta(y) = y/s\,\theta_1 + y^2/s^2\,\theta_2$

Solve for U

$$U = \cancel{2}(1/\cancel{2})\int_0^s GJ(y)\left(\frac{\partial\theta}{\partial y}\right)^2 dy$$

$$\frac{\partial\theta}{\partial y} \to \frac{\theta_1}{s} + 2\frac{y}{s^2}\theta_2$$

$$U = \int_0^s (GJ)_0\left(1 - y^2/s^2\right)\left(\frac{\theta_1}{s} + 2\frac{y}{s^2}\theta_2\right)^2 dy$$

$$\left(\frac{\theta_1}{s} + 2\,y/s^2\,\theta_2\right)\left(\frac{\theta_1}{s} + 2\,y/s^2\,\theta_2\right)$$

$$\left(1 - y^2/s^2\right)\left(\frac{\theta_1^2}{s^2} + \cancel{4}\,y/s^3\,\theta_1\theta_2 + \cancel{2\frac{y}{s^3}\theta_1\theta_2} + 4\,y^2/s^4\,\theta_2^2\right)$$

$$\int_0^s\left(\frac{\theta_1^2}{s^2} + 4\,y/s^3\,\theta_1\theta_2 + 4\,y^2/s^4\,\theta_2^2 - \frac{y^2}{s^4}\theta_1^2 - 4\,y^3/s^5\,\theta_1\theta_2 - 4\,y^4/s^6\,\theta_2^2\right)dy$$

$$\frac{\theta_1^2}{s} + \frac{2}{s}\theta_1\theta_2 + {}^{5\cdot}\frac{4}{3}\frac{\theta_2^2}{s} - \frac{1}{3}\frac{\theta_1^2}{s} - \frac{1}{s}\theta_1\theta_2 - {}^{3\cdot}\frac{4}{5}\frac{\theta_2^2}{s}$$

$$U = (GJ)_0(1/s)\left(2/3\,\theta_1^2 + 8/15\,\theta_2^2 + \theta_1\theta_2\right)$$

$$\delta W = W\delta z + 2dL\cdot(e_c\,\delta\theta - \delta z)$$

$$\int_0^s qc(y)\,a_w\,\alpha(y)\,dy$$

$$\delta\theta \to y/s\,\delta\theta_1 + y^2/s^2\,\delta\theta_2$$

$$-2\int_0^s qc_0\left(1 - y^2/s^2\right)a_w\left(\alpha_0 + y/s\,\theta_1 + y^2/s^2\,\theta_2\right)\delta z$$

$$+2\int_0^s qc_0\left(1 - y^2/s^2\right)a_w\left(\alpha_0 + y/s\,\theta_1 + y^2/s^2\,\theta_2\right)(ec_0)\left(1 - y^2/s^2\right)\left(y/s\,\delta\theta_1 + y^2/s^2\,\delta\theta_2\right)$$

$$\left(1 - y^2/s^2\right)\left(\alpha_0 + y/s\,\theta_1 + y^2/s^2\,\theta_2\right)$$

(1) $\alpha_0 + y/s\,\theta_1 + y^2/s^2\,\theta_2 - y^2/s^2\,\alpha_0 - y^3/s^3\,\theta_1 - y^4/s^4\,\theta_2$

(2) $(y/s - y^3/s^3)(\ ) \to \alpha_0 y/s + y^2/s^2\,\theta_1 + y^3/s^3\,\theta_2 - y^3/s^3\,\alpha_0 - y^4/s^4\,\theta_1 - y^5/s^5\,\theta_2$
$- y^3/s^3\,\alpha_0 - y^4/s^4\,\theta_1 - y^5/s^5\,\theta_2 + y^5/s^5\,\alpha_0 + y^6/s^6\,\theta_1 + y^7/s^7\,\theta_2$

(3) $(y^2/s^2 - y^4/s^4)(\ ) \to \alpha_0 y^2/s^2 + y^3/s^3\,\theta_1 + y^4/s^4\,\theta_2 - y^4/s^4\,\alpha_0 - y^5/s^5\,\theta_1 - y^6/s^6\,\theta_2$
$- y^4/s^4\,\alpha_0 - y^5/s^5\,\theta_1 - y^6/s^6\,\theta_2 + y^6/s^6\,\alpha_0 + y^7/s^7\,\theta_1 + y^8/s^8\,\theta_2$

① $\rightarrow \int_0^s (\quad)\,dy$

$\cancel{K_0 s} + \cancel{\tfrac{1}{2}}\theta_1 s + \tfrac{5}{3}\theta_2 s - \cancel{\tfrac{1}{3}}K_0 s - \cancel{\tfrac{1}{4}}\theta_1 s - \tfrac{3}{5}\theta_2 s$

$s(\,^2/_3 K_0 + \tfrac{1}{4}\theta_1 + \,^2/_{15}\theta_2)$

② $\rightarrow \int_0^s (\quad)\,dy$ $\qquad \frac{5}{3} - \frac{6}{5} + \frac{1}{7} \rightarrow -\frac{1\cdot 3}{15\cdot 3} + \frac{1\cdot 5}{7\cdot 5} \rightarrow \frac{2}{35}$

$s(\tfrac{1}{2}\cancel{K_0} + \tfrac{1}{3}\theta_1 + \tfrac{1}{4}\theta_2 - \cancel{\tfrac{1}{4}K_0} - \cancel{\tfrac{1}{5}}\theta_1 - \tfrac{1}{6}\theta_2 - \cancel{\tfrac{1}{4}}K_0 - \cancel{\tfrac{1}{5}}\theta_1 - \tfrac{1}{6}\theta_2$
$+ \cancel{\tfrac{1}{6}K_0} + \cancel{\tfrac{1}{7}}\theta_1 + \cancel{\tfrac{1}{8}}\theta_2)$ $\qquad \frac{2\cdot 1}{2\cdot 4} - \frac{1}{3} + \frac{1}{8} \rightarrow \frac{3\cdot 3}{\cdot 8} - \frac{1\cdot 8}{3}$

$s(\,^1/_6 K_0 + \,^2/_{35}\theta_1 + \,^1/_{24}\theta_2)$

③ $\rightarrow \int_0^s (\quad)\,dy$ $\qquad \frac{1\cdot 7}{5\cdot 7} - \frac{2\cdot 5}{7\cdot 5} + \frac{1}{9} \rightarrow -\frac{3}{35} + \frac{1}{9} \rightarrow -\frac{27}{315} + \frac{35}{315}$

$s(\cancel{\tfrac{1}{3}}K_0 + \cancel{\tfrac{1}{4}\theta_1} + \,^1/_5\theta_2 - \cancel{\tfrac{1}{5}}K_0 - \cancel{\tfrac{1}{6}}\theta_1 - \,^1/_7\theta_2 - \cancel{\,^1/_5}K_0 - \cancel{\,^1/_6}\theta_1$
$- \,^1/_7\theta_2 + \cancel{\,^1/_7 K_0} + \cancel{\,^1/_8}\theta_1 + \,^1/_9\theta_2)$

$s(\,^2/_{35}K_0 + \,^1/_{24}\theta_1 + \,^8/_{315}\theta_2)$

Put it all together:

$$\delta W = \omega\,\delta z + qc_0^2 e a_w s(\,^1/_3 K_0 + \,^4/_{35}\theta_1 + \,^1/_{12}\theta_2)\delta\theta_1$$
$$+ qc_0^2 e a_w s(\,^4/_{35}K_0 + \,^1/_{12}\theta_1 + \,^{16}/_{315}\theta_2)\,\delta\theta_2$$
$$- qc_0 a_w s(\,^4/_3 K_0 + \,^1/_2\theta_1 + \,^4/_{15}\theta_2)\delta z$$

$$Q_z = \frac{\partial(\delta W)}{\partial(\delta z)} = \omega - qc_0 a_w s\left(\,^4/_3 K_0 + \,^1/_2\theta_1 + \,^4/_{15}\theta_2\right)$$
$$Q_{\theta_1} = \frac{\partial(\delta W)}{\partial(\delta\theta_1)} = qc_0^2 e a_w s\left(\,^1/_3 K_0 + \,^4/_{35}\theta_1 + \,^1/_{12}\theta_2\right)$$
$$Q_{\theta_2} = \frac{\partial(\delta W)}{\partial(\delta\theta_2)} = qc_0^2 e a_w s\left(\,^4/_{35}K_0 + \,^1/_{12}\theta_1 + \,^{16}/_{315}\theta_2\right)$$

Lagrange Eq: (generically)

$$\cancel{\frac{d}{dt}\frac{\partial T}{\partial \dot q}}^{\,0} - \cancel{\frac{\partial T}{\partial q}}^{\,0} + \cancel{\frac{\partial \mathcal{F}}{\partial q}}^{\,0} + \frac{\partial U}{\partial q} = \frac{\partial(\delta W)}{\partial(\delta q)}$$

$$\frac{\partial(\delta W)}{\partial(\delta q)} = \frac{\partial U}{\partial q}$$

$$\frac{\partial U}{\partial z} = 0$$
$$\frac{\partial U}{\partial \theta_1} = (GJ)_0\,^1/_s\left[(\,^4/_3)\theta_1 + \theta_2\right]$$
$$\frac{\partial U}{\partial \theta_2} = (GJ)_0\,^1/_s\left[(\,^{16}/_{15})\theta_2 + \theta_1\right]$$

Put together:

$$q c_o a_w s\left(\tfrac{4}{3}\chi_o + \tfrac{1}{2}\theta_1 + \tfrac{4}{15}\theta_2\right) = W$$

$$q c_o^2 e a_w s\left(\tfrac{1}{3}\chi_o + \tfrac{4}{35}\theta_1 + \tfrac{1}{12}\theta_2\right) = (GJ)_o \tfrac{1}{s}\left(\tfrac{4}{3}\theta_1 + \theta_2\right)$$

$$q c_o^2 e a_w s\left(\tfrac{4}{35}\chi_o + \tfrac{1}{12}\theta_1 + \tfrac{16}{315}\theta_2\right) = (GJ)_o \tfrac{1}{s}\left(\tfrac{16}{15}\theta_2 + \theta_1\right)$$

$$\begin{bmatrix} \tfrac{4}{3} q c_o a_w s & \tfrac{1}{2} q c_o a_w s & \tfrac{4}{15} q c_o a_w s \\ \tfrac{1}{3} q c_o^2 e a_w s & \tfrac{4}{35} q c_o^2 e a_w s - (GJ)_o \tfrac{4}{3s} & \tfrac{1}{12} q c_o^2 e a_w s - (GJ)_o \tfrac{1}{s} \\ \tfrac{4}{35} q c_o^2 e a_w s & \tfrac{1}{12} q c_o^2 e a_w s - (GJ)_o \tfrac{1}{s} & \tfrac{16}{315} q c_o^2 e a_w s - (GJ)_o \tfrac{16}{15s} \end{bmatrix} \begin{bmatrix} \chi_o \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} W \\ 0 \\ 0 \end{bmatrix}$$

↳ This would a hard matrix to inverse so... call the matrix, P.

$$\begin{bmatrix} \chi_o \\ \theta_1 \\ \theta_2 \end{bmatrix} = P^{-1} \begin{bmatrix} W \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{dL}{dy} = q c_o a_w \left(1 - \tfrac{y^2}{s^2}\right) \begin{bmatrix} 1 & \tfrac{y}{s} & \tfrac{y^2}{s^2} \end{bmatrix} \begin{bmatrix} \chi_o \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

I used Matlab to solve $\frac{dL}{dy}$ and V_A.

There would be no way for me to do these calculations by hand without mistakes.

$R = C_o a_w s$ ↜ To simplify matrix

$M = (GJ)_o$

See attachment for calculations...

$$\frac{dL}{dy} = \frac{-(15W(s^2-y^2)(-223440M^2s + 12152MRcqs^2 - 127680MRcqsy + 95760MRcqy^2 + 603R^2c^2q^2s^3 + 3920R^2c^2q^2s^2y - 7788R^2c^2q^2sy^2)}{4s^4(1117200M^2 + 82880MRcqs - 2577R^2c^2q^2s^2}$$

$$V_A = \sqrt{\frac{280M(\sqrt{234505} + 296)}{2577Rc\rho s}}$$

$$R = C_0 a_w S$$

$$M = (GJ)_0$$

Hw #2 - Aeroelasticity - ME597/AAE556

Victoria Nagorski - 9/19/22

Contents

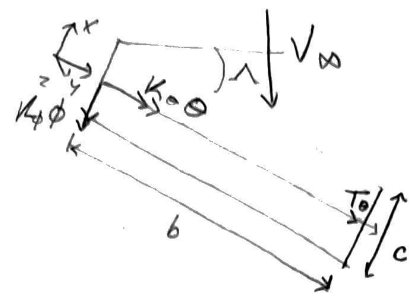
- Problem 1b

Problem 1b

```
syms q R c M s W y rho

% Solve for lift
P = [4/3*q*R    1/2*q*R 4/15*q*R;
        1/3*R*c*q  (4/35*q*c*R-4/3*M/s) (1/12*q*c*R-M/s);
        4/35*q*c*R (1/12*q*c*R-M/s) (16/315*q*c*R-16/15*M/s)];
thetas = P^-1 * [W;0;0]

dL_dy = q * R/s * (1 - y^2/s^2) * [1 y/s y^2/s^2]*thetas


% Solve for V_d
alpha0 = thetas(1);
[~,D] = numden(alpha0)
q = simplify(solve(D,q))

q_A = q(2)
V_A = sqrt(2/rho * q_A)
```

```
thetas =

-(15*W*(- 223440*M^2 + 12152*M*R*c*q*s + 603*R^2*c^2*q^2*s^2))/(4*(1117200*M^2*R*q + 82880*M*R^2*c*q^2*s - 2577*R^3*c^2*q^3*s^2))
                                      (2100*W*s*(- 7*R*q*s*c^2 + 228*M*c))/(1117200*M^2 + 82880*M*R*c*q*s - 2577*R^2*c^2*q^2*s^2)
                                    -(45*W*s*(- 649*R*q*s*c^2 + 7980*M*c))/(1117200*M^2 + 82880*M*R*c*q*s - 2577*R^2*c^2*q^2*s^2)


dL_dy =

(15*R*W*q*(y^2/s^2 - 1)*(- 223440*M^2 + 12152*M*R*c*q*s + 603*R^2*c^2*q^2*s^2))/(4*s*(1117200*M^2*R*q + 82880*M*R^2*c*q^2*s - 2577*R^3*c^2*q^3*s^2)) - (2100*R*W*q*y


D =

4468800*M^2*R*q + 331520*M*R^2*c*q^2*s - 10308*R^3*c^2*q^3*s^2


q =

                                         0
 (140*M*(234505^(1/2) + 296))/(2577*R*c*s)
-(140*M*(234505^(1/2) - 296))/(2577*R*c*s)


q_A =

(140*M*(234505^(1/2) + 296))/(2577*R*c*s)


V_A =

((280*M*(234505^(1/2) + 296))/(2577*R*c*rho*s))^(1/2)
```

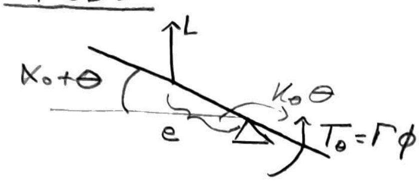
Problem No. 2

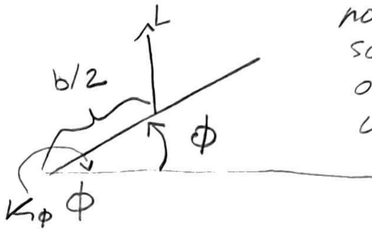


$\phi \to$ bending slope
$T_\theta = \Gamma \phi$
Has a α_0

★ **Note**: Drawing on problem statement is wrong... prof no respond on intent so I solved based on problem statement wording

FBDs

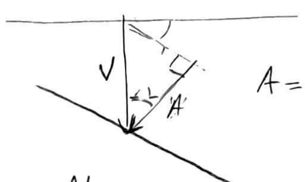




a) Derive static equilibrium eq using energy.

$$U = \frac{1}{2} K_\phi \phi^2 + \frac{1}{2} K_\theta \theta^2$$

Need to derive angles for lift:



$A = \cos\Lambda \, V$

$$z = h(\bar{y}) - \bar{x}\,\theta(\bar{y})$$

$$\begin{bmatrix} \bar{y} \\ \bar{x} \end{bmatrix} = \begin{bmatrix} C_\Lambda & S_\Lambda \\ -S_\Lambda & C_\Lambda \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix}$$

$$\phi = \frac{dh}{dy}$$

$$\alpha_s = -\left.\frac{dz}{dx}\right|_E = \frac{V_n}{V} \longrightarrow V_n = -V\left[\frac{dz}{d\bar{x}}\frac{d\bar{x}}{dx} + \frac{dz}{d\bar{y}}\frac{d\bar{y}}{dx}\right]$$

$$\bar{y} = C_\Lambda y + S_\Lambda x$$
$$\bar{x} = -S_\Lambda y + C_\Lambda x$$

$d\bar{y}/dx \to \sin\Lambda$ $\quad dz/d\bar{x} \to -\theta(\bar{y})$

$d\bar{x}/dx \to \cos\Lambda$ $\quad dz/d\bar{y} \to h'(\bar{y})$

$$\alpha_s = \frac{V_n}{V} = \theta(\bar{y})\cos\Lambda - \underbrace{h'(\bar{y})}_{\phi}\sin\Lambda$$

$$\alpha_s = \alpha_0 + \theta\cos\Lambda - \phi\sin\Lambda$$

$$\frac{V_n}{V\cos\Lambda} \Rightarrow \frac{\alpha_0}{\cos\Lambda} + \theta - \phi\tan\Lambda$$

$$\alpha_c = \frac{\alpha_0}{\cos\Lambda} + \theta - \phi\tan\Lambda$$

Two different sources produce work
→ Lift
→ T_0

We have the following equation:

$$\frac{\partial U}{\partial \phi}\delta\phi + \frac{\partial U}{\partial \theta}\delta\theta = \delta W$$

Solve for δW:

$$L = q_n S C_{L\alpha}\alpha_c$$

$$\delta W = (L \cdot b/4)\,\delta\phi + (L\cdot e + \Gamma\phi)\,\delta\theta$$

$$\frac{\partial U}{\partial \phi} \rightarrow K_\phi \phi$$
$$\frac{\partial U}{\partial \theta} \rightarrow K_\theta \theta$$

We now have 2 equations if we break up by $\delta\phi$ and $\delta\theta$

$$(L \cdot b/2) = K_\phi \phi$$
$$(L\cdot e + \Gamma\phi) = K_\theta\theta$$

$$q_n S C_{L\alpha}(b/2)\left(\frac{\alpha_0}{\cos\Lambda} + \theta - \phi\tan\Lambda\right) = K_\phi \phi$$

(a)
$$K_\phi\phi + q_n S C_{L\alpha}(b/2)\tan\Lambda\,\phi - q_n S C_{L\alpha}(b/2)\,\theta = q_n S C_{L\alpha}(b/2)\frac{\alpha_0}{\cos\Lambda}$$

$$q_n S C_{L\alpha} e\left(\frac{\alpha_0}{\cos\Lambda} + \theta - \phi\tan\Lambda\right) + \Gamma\phi = K_\theta\theta$$

(b)
$$K_\theta\theta - q_n S C_{L\alpha} e\,\theta + q_n S C_{L\alpha} e\tan\Lambda\,\phi - \Gamma\phi = q_n S C_{L\alpha} e\frac{\alpha_0}{\cos\Lambda}$$

Write into Matrix form: $[\theta \;\; \phi]^T$

$$\boxed{\begin{bmatrix} -q_n S C_{L\alpha}(b/2) & K_\phi + q_n S C_{L\alpha}(b/2)\tan\Lambda \\ K_\theta - q_n S C_{L\alpha} e & q_n S C_{L\alpha} e\tan\Lambda - \Gamma \end{bmatrix}\begin{bmatrix}\theta \\ \phi\end{bmatrix} = q_n S C_{L\alpha}\frac{\alpha_0}{\cos\Lambda}\begin{bmatrix} b/2 \\ e\end{bmatrix}}$$

b) Derive characteristic eq. Solve for divergent dynamic pressure

$$\begin{bmatrix} -q_n S C_{L\alpha}(b/2) & K_\phi + q_n S C_{L\alpha}(b/2)\tan\Lambda \\ K_\theta - q_n S C_{L\alpha} e & q_n S C_{L\alpha} e \tan\Lambda - \Gamma \end{bmatrix}$$

$$-q_n S C_{L\alpha}(b/2)(q_n S C_{L\alpha} e \tan\Lambda - \Gamma)$$
$$- (K_\theta - q_n S C_{L\alpha} e)(K_\phi + q_n S C_{L\alpha}(b/2)\tan\Lambda) = 0$$

$$-\cancel{(q_n S C_{L\alpha})^2(b/2)(e)\tan\Lambda} + q_n S C_{L\alpha}(b/2)\Gamma - K_\theta K_\phi$$
$$- K_\theta q_n S C_{L\alpha}(b/2)\tan\Lambda + \cancel{(q_n S C_{L\alpha})^2(b/2)(e)\tan\Lambda}$$
$$+ K_\phi q_n S C_{L\alpha}(e) = 0$$

$$\boxed{q_n S C_{L\alpha}(b/2)\Gamma - K_\theta K_\phi - K_\theta q_n S C_{L\alpha}(b/2)\tan\Lambda + K_\phi q_n S C_{L\alpha}(e) = 0}$$

Solve for q_n:

$$q_n S C_{L\alpha} e\left(\Gamma \frac{b}{2e} - K_\theta \frac{b}{2e}\tan\Lambda + K_\phi\right) = K_\theta K_\phi$$

$$q_n = \frac{K_\theta \cancel{K_\phi}}{\cancel{K_\phi}\, S C_{L\alpha} e}\left(\frac{\Gamma}{K_\phi}\frac{b}{2e} - \frac{K_\theta}{K_\phi}\frac{b}{2e}\tan\Lambda + 1\right)$$

$$\boxed{q_n = \frac{K_\theta}{S C_{L\alpha} e}\left(\frac{\Gamma}{K_\phi}\frac{b}{2e} - \frac{K_\theta}{K_\phi}\frac{b}{2e}\tan\Lambda + 1\right)}$$

c) Critical value of gain Γ above which wing divergence cannot occur

When $q_D < 0$, then divergence won't occur because then airspeed is an imaginary number

$$\frac{\cancel{K_\theta}}{\cancel{S C_{L\alpha} e}}\left(\frac{\Gamma}{K_\phi}\frac{b}{2e} - \frac{K_\theta}{K_\phi}\frac{b}{2e}\tan\Lambda + 1\right) < 0 - 1$$

$$+\frac{K_\theta}{K_\phi}\frac{b}{2e}\tan\Lambda$$

$$\frac{\Gamma}{\cancel{K_\phi}}\frac{\cancel{b}}{\cancel{2e}} < \left(\frac{K_\theta}{\cancel{K_\phi}}\frac{\cancel{b}}{\cancel{2e}}\tan\Lambda - 1\right)\frac{2e}{b}K_\phi$$

$$\boxed{\Gamma < K_\theta \tan\Lambda - \frac{2e}{b}K_\phi}$$