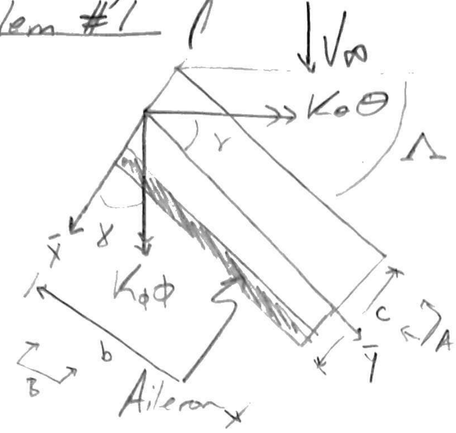
Victoria Nayorski | Hw #4 | Page 1

Problem #1



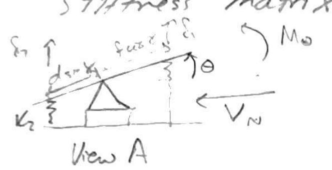
$\phi \rightarrow$ bendy
$\theta \rightarrow$ torsion

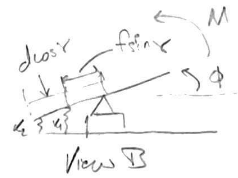
$Q = q_n S a_0$

$q_n = q \cos^2 \Lambda$

Lift @ c/4
$K_0 = 0$

a) Only the stiffness matrix changes... derive new stiffness matrix





$\delta_1 = +(f\cos\gamma)\theta - (f\sin\gamma)\phi$
$\delta_2 = -(d\sin\gamma)\theta - (d\cos\gamma)\phi$

$$\sum M_x = 0 = M_\phi - [K_2 d^2 - K_1 f^2](\sin\gamma\cos\gamma)\theta - [K_2 d^2 \cos^2\gamma + K_1 f^2 \sin^2\gamma]\phi$$

$$\sum M_y = 0 = M_\theta - [K_1 f^2 \cos^2\gamma + K_2 d^2 \sin^2\gamma]\theta - [K_2 d^2 - K_1 f^2](\sin\gamma\cos\gamma)\phi$$

$$K_2 d^2 = K_\phi$$

$$K_1 f^2 = K_\theta$$

With this can rewrite:

$$M_\phi = (K_\phi - K_\theta)\sin\gamma\cos\gamma\,\theta + (K_\phi\cos^2\gamma + K_\theta\sin^2\gamma)\phi$$

$$M_\theta = (K_\theta\cos^2\gamma + K_\phi\sin^2\gamma)\theta + (K_\phi - K_\theta)\sin\gamma\cos\gamma\,\phi$$

We end up with:

$$\begin{bmatrix} K_\phi\cos^2\gamma + K_\theta\sin^2\gamma & (K_\phi - K_\theta)\sin\gamma\cos\gamma \\ (K_\phi - K_\theta)\sin\gamma\cos\gamma & K_\phi\sin^2\gamma + K_\theta\cos^2\gamma \end{bmatrix} \begin{bmatrix} \phi \\ \theta \end{bmatrix} = \begin{bmatrix} M_\phi \\ M_\theta \end{bmatrix}$$

Need to derive moments due to lift:

$\phi \rightarrow \int_0^b l y \, dy$

$\theta \rightarrow \int_0^b l e \, dy$

$l = q_n a_0 c \alpha_s + q_n c_{l_\delta} c \delta$

$m = \int_0^b c_{m_\delta} q_n c^2 \delta \, dy \rightarrow m = c_{m_\delta} c^2 q_n b \delta$

$$\phi \rightarrow \int_0^b (a_0 q_n c \bar{y} + c_{l_\delta} q_n c \bar{y}) \, d\bar{y}$$

$$\tfrac{1}{2}(a_0 q_n c \bar{y}^2 + c_{l_\delta} q_n c \bar{y}^2) \Big|_0^b \rightarrow \tfrac{1}{2}(a_0 q_n c b^2 \alpha_s + c_{l_\delta} q_n c b^2 \delta)$$

$$\theta \rightarrow \int_0^b (a_0 q_n c e + c_{l_\delta} q_n c e) \, dy$$

$$a_0 q_n c e \bar{y} + c_{l_\delta} q_n c e \bar{y} \Big|_0^b \rightarrow a_0 q_n c e b \alpha_s + c_{l_\delta} q_n c e b \delta$$

$$\alpha_s = \frac{\cancel{\alpha}^{\,0}}{\cancel{\cos\Lambda}} + \theta - \phi \tan\Lambda \rightarrow \alpha_s = \theta - \phi \tan\Lambda$$

$$M_\phi = a_0 q_n \underbrace{cb}_{S} \left(-b/2 \tan\Lambda \, \phi + b/2 \, \theta\right) + e a_0 q_n c b \left(\frac{c_{l_\delta}}{a_0} b/2 \; 1/e\right) \delta_0$$

$$M_\theta = a_0 q_n \underbrace{cb}_{S} \left(-e \tan\Lambda \, \phi + e\theta\right) + e a_0 q_n \underbrace{cb}_{S} \left(\frac{c_{l_\delta}}{a_0} + \frac{c_{m_\delta}}{a_0} \frac{c}{e}\right) \delta_0$$

$$\begin{bmatrix} M_\phi \\ M_\theta \end{bmatrix} = Q \begin{bmatrix} -b/2 \tan\Lambda & b/2 \\ -e \tan\Lambda & e \end{bmatrix} \begin{bmatrix} \phi \\ \theta \end{bmatrix} + eQ \begin{bmatrix} \frac{c_{l_\delta}}{a_0}(b/2)(1/e) \\ c_{m_\delta}/a_0 (c/e) + c_{l_\delta}/a_0 \end{bmatrix} \delta_0$$

Start matching to problem statement...

Move aero matrix to the left side...

$$\begin{bmatrix} K_\phi \cos^2\gamma + K_\theta \sin^2\gamma + (bQ/2)\tan\Lambda & (K_\phi - K_\theta)\sin\gamma\cos\gamma - Qb/2 \\ (K_\phi - K_\theta)\sin\gamma\cos\gamma + (eQ)\tan\Lambda & K_\phi \sin^2\gamma + K_\theta \cos^2\gamma - Qe \end{bmatrix} \begin{bmatrix} \phi \\ \theta \end{bmatrix} =$$

$$eQ \underbrace{\begin{bmatrix} c_{l_\delta}/a_0 (b/2)(1/e) \\ c_{m_\delta}/a_0 (c/e) + c_{l_\delta}/a_0 \end{bmatrix}}_{C} \delta_0$$

→ It matches!

→ There shouldn't be a 2 under the K_{21} term on the right.

b)

$$\det\left(\begin{bmatrix}\bar{K}_{11} & \bar{K}_{12}\\ \bar{K}_{21} & \bar{K}_{22}\end{bmatrix}\right) = 0$$

$$\bar{K}_{11}\bar{K}_{22} - \bar{K}_{12}\bar{K}_{21} = 0$$

$$\left(K_{11}^* + (bQ/2)\tan\Lambda\right)\left(K_{22}^* - Qe\right) - \left(K_{12}^* - Qb/2\right)\left(K_{21}^* + Qe\tan\Lambda\right) = 0$$

$$K_{11}^*K_{22}^* - K_{11}^*Qe + K_{22}^*\left(\frac{bQ}{2}\right)\tan\Lambda - K_{12}^*K_{21}^* - K_{12}^*Qe\tan\Lambda - \cancel{Q^2be/2\tan\Lambda} + K_{21}^*(Qb/2) + \cancel{Q^2be/2\tan\Lambda} = 0$$

$$Q\left(\frac{K_{21}^*b}{2} + K_{11}^*e + K_{12}^*e\tan\Lambda \mp \frac{K_{22}^*b\tan\Lambda}{2}\right) = -K_{12}^*K_{21}^* + K_{11}^*K_{22}^*$$

Multiply each-side by (-)

$$\boxed{Q_D = \frac{K_{11}^*K_{22}^* - K_{12}^*K_{21}^*}{K_{11}^*e - \frac{K_{21}^*}{2}b + \left(K_{12}^*e - K_{22}^*\frac{b}{2}\right)\tan\Lambda} = \underbrace{q_D\cos^2\Lambda}_{q_n} S a_0}$$

c) Lift of aileron due to bending and twisting from aileron torsion deflection

$$L_{Flex} = \int_0^b q_n a_0 c(\theta - \phi\tan\Lambda)\,dy + \int_0^b q_n C_{l_\delta} c\,\delta\,dy$$

$$L_{flex} = q_n a_0 \underbrace{cb}_{S}(\theta - \phi\tan\Lambda) + q_n a_0 \underbrace{cb}_{S}\frac{C_{l\delta}}{a_0}\delta_0$$

Need to solve for θ and ϕ

$$\begin{bmatrix}\bar{K}_{11} & \bar{K}_{12}\\ \bar{K}_{21} & \bar{K}_{22}\end{bmatrix}\begin{bmatrix}\phi\\ \theta\end{bmatrix} = Qe\begin{bmatrix}c_1\\ c_2\end{bmatrix}\delta_0$$

$$\begin{bmatrix}\phi\\ \theta\end{bmatrix} = \frac{Qe\,\delta_0}{\bar{K}_{11}\bar{K}_{22} - \bar{K}_{12}\bar{K}_{21}}\begin{bmatrix}\bar{K}_{22} & -\bar{K}_{12}\\ -\bar{K}_{21} & \bar{K}_{11}\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix}$$

$$\begin{bmatrix}\phi\\ \theta\end{bmatrix} = \frac{Qe\,\delta_0}{\bar{K}_{11}\bar{K}_{22} - \bar{K}_{12}\bar{K}_{21}}\begin{bmatrix}\bar{K}_{22}c_1 - \bar{K}_{12}c_2\\ -\bar{K}_{21}c_1 + \bar{K}_{11}c_2\end{bmatrix}$$

$$\boxed{L_{Flex} = \left[\frac{Q^2e\left[(\bar{K}_{11}c_2 - \bar{K}_{21}c_1) - (\bar{K}_{22}c_1 - \bar{K}_{12}c_2)\tan\Lambda\right]}{\bar{K}_{11}\bar{K}_{22} - \bar{K}_{12}\bar{K}_{21}} + Q\frac{C_{l\delta}}{a_0}\right]\delta_0}$$

It matches!

d) We have L_{flex}. We need L_{Rigid}. $Q = q_n S a_0$

For rigid $\theta = \phi = 0 \Rightarrow L_R = \underbrace{q_n S \frac{C_{l\delta}}{a_0} a_0}_{Q} \delta_0$

$$L_R = Q \frac{C_{l\delta}}{a_0} \delta_0$$

Cancel Q and δ_0

$$\boxed{\frac{L_{flex}}{L_{rigid}} = \frac{Qe\left[(\bar{K}_{11} c_2 - \bar{K}_{21} c_1) - (\bar{K}_{22} c_1 - \bar{K}_{12} c_2)\tan\Lambda\right]}{\left(C_{l\delta}/a_0\right)\left(\bar{K}_{11}\bar{K}_{22} - \bar{K}_{12}\bar{K}_{21}\right)} + 1}$$

e) Get in terms of $\frac{C_{l\delta}}{a_0}, \frac{C_{m\delta}}{a_0}, \frac{b}{c}, \frac{K_\phi}{K_\theta}, \Lambda$

$$Q = q_n S a_0 \qquad q_n = q \cos^2\Lambda$$

Glauert's theory:

$$\frac{C_{l\delta}}{a_0} = \frac{1}{\pi}\left(\cos^{-1}(1-2E) + 2\sqrt{E(1-E)}\right)$$

$$\frac{C_{m\delta}}{a_0} = -\frac{1}{\pi}(1-E)\sqrt{(1-E)E}$$

Solved for in Matlab

$$c_1 \rightarrow \frac{C_{l\delta}}{a_0} \frac{b}{e} \frac{1}{2}$$

$$c_2 \rightarrow \frac{C_{m\delta}}{a_0} \frac{c}{e} + \frac{C_{l\delta}}{a_0} \rightarrow \frac{C_{l\delta}}{a_0}\left(\frac{C_{m\delta}}{a_0}\left(\frac{a_0}{C_{l\delta}}\right)\frac{c}{e} + \cancel{\frac{C_{l\delta}}{a_0}}^{1}\right)$$

↳ We can cancel out $C_{l\delta}/a_0$ on top & bottom...

Look @ Bottom

$$(K_{11}^* + bQ/2 \tan\Lambda)(K_{22}^* - Qe) - (K_{12}^* - Qb/2)(K_{21}^* + eQ\tan\Lambda)$$

$$K_{11}^* K_{22}^* - K_{11}^* Qe + K_{22}^*(bQ/2)\tan\Lambda - \cancel{bQ/2 \tan\Lambda\, Qe}$$
$$- K_{12}^* K_{21}^* - K_{12}^* eQ\tan\Lambda + K_{21}^* Q\, b/2 + \cancel{eQ^2 (b/2)\tan\Lambda}$$

$$-Q(K_{11}^* e - K_{22}^*(b/2)\tan\Lambda + K_{12}^* e\tan\Lambda - K_{21}^* b/2) + (K_{11}^* K_{22}^* - K_{12}^* K_{21}^*)$$

Gets Q_D back:

$$Q_D = \frac{K_{11}^* K_{22}^* - K_{12}^* K_{21}^*}{K_{11}^* e - K_{21}^* b/2 + (K_{12}^* e - K_{22}^* b/2)\tan\Lambda}$$

$$K_{11}^* - K_{21}^* b/2 + (K_{12}^* e - K_{22}^* b/2)\tan\Lambda = \frac{K_{11}^* K_{22}^* - K_{12}^* K_{21}^*}{Q_D}$$

bottom:

$$-\frac{Q}{Q_D}(K_{11}^* K_{22}^* - K_{12}^* K_{21}^*) + (K_{11}^* K_{22}^* - K_{12}^* K_{21}^*)$$

$$(K_{11}^* K_{22}^* - \underbrace{K_{12}^* K_{21}^*}_{(K_{12}^*)^2})(1 - Q/Q_D)$$

$$\left(\frac{K_\phi}{K_\theta} - 1\right)\left(\frac{K_\phi}{K_\theta} - 1\right)$$

$$K_\theta^2\left(\frac{K_\phi}{K_\theta}\cos^2\gamma + \sin^2\gamma\right)\left(\frac{K_\phi}{K_\theta}\sin^2\gamma + \cos^2\gamma\right) - K_\theta^2\left(\frac{K_\phi}{K_\theta} - 1\right)^2\cos^2\gamma\sin^2\gamma$$

$$\cancel{\left(\frac{K_\phi}{K_\theta}\right)^2\cos^2\gamma\sin^2\gamma} + \frac{K_\phi}{K_\theta}\cos^4\gamma + \frac{K_\phi}{K_\theta}\sin^4\gamma + \cancel{\sin^2\gamma\cos^2\gamma}$$

$$-\cancel{\left(\frac{K_\phi}{K_\theta}\right)^2\cos^2\gamma\sin^2\gamma} + 2\frac{K_\phi}{K_\theta}\cos^2\gamma\sin^2\gamma - \cancel{\cos^2\gamma\sin^2\gamma}$$

$$\frac{K_\phi}{K_\theta}\left(\cancel{\frac{K_\phi}{K_\theta}}\cos^4\gamma + 2\cancel{\frac{K_\phi}{K_\theta}}\cos^2\gamma\sin^2\gamma + \cancel{\frac{K_\phi}{K_\theta}}\sin^4\gamma\right)$$

$$\frac{K_\phi}{K_\theta}(\cos^2\gamma + \sin^2\gamma)^2$$

bottom: $K_\theta^2\left(\frac{K_\phi}{K_\theta}\right)(\cos^2\gamma + \sin^2\gamma)^2(1 - Q/Q_D)$

Re-arrange the top:

$$Qe[(\bar{K}_{11} + \bar{K}_{12}\tan\Lambda)c_2 - (\bar{K}_{21} + \bar{K}_{22}\tan\Lambda)c_1]$$

$$\bar{K}_{11} + \bar{K}_{12}\tan\Lambda \rightarrow K_{11}^* + K_{12}^*\tan\Lambda + \cancel{(bQ/2)\tan\Lambda - Q_b/2\tan\Lambda}$$

$$\bar{K}_{21} + \bar{K}_{22}\tan\Lambda \rightarrow K_{21}^* + K_{22}^*\tan\Lambda + \cancel{(eQ)\tan\Lambda} - \cancel{Qe\tan\Lambda}$$

$$Qe\left[(K_{11}^* + K_{12}^*\tan\Lambda)\left(\frac{C_{m_\delta}}{a_0}\left(\frac{a_0}{C_{\ell_\delta}}\right)\left(\frac{c}{e}\right) + 1\right) - (K_{21}^* + K_{22}^*\tan\Lambda)\left(\frac{b}{e}\right)\left(\frac{1}{2}\right)\right]$$

$$Qe\cancel{K_\theta}\left[\left(\frac{K_\phi}{K_\theta}\cos^2\gamma + \sin^2\gamma + \left(\frac{K_\phi}{K_\theta} - 1\right)\sin\gamma\cos\gamma\tan\Lambda\right)\left(\frac{C_{m_\delta}}{a_0}\left(\frac{a_0}{C_{\ell_\delta}}\right)\frac{c}{e} + 1\right)\right.$$
$$\left. - \left(\left(\frac{K_\phi}{K_\theta} - 1\right)\sin\gamma\cos\gamma + \frac{K_\phi}{K_\theta}\sin^2\gamma\tan\Lambda + \cos^2\gamma\tan\Lambda\right)\left(\frac{b}{e}\right)\left(\frac{1}{2}\right)\right]$$

↪ cancel out K_θ from top & bottom

The end result:

$$\frac{L_F}{L_R} = \frac{Qe\left[\left(\frac{K_\phi}{K_\theta}\cos^2\gamma + \sin^2\gamma + \left(\frac{K_\phi}{K_\theta}-1\right)\sin\gamma\cos\gamma\tan\Lambda\right)\left(\frac{C_{m_\delta}}{a_0}\left(\frac{a_0}{C_{\ell_\delta}}\right)\left(\frac{c}{e}\right)+1\right) - \left(\left(\frac{K_\phi}{K_\theta}-1\right)\sin\gamma\cos\gamma + \frac{K_\phi}{K_\theta}\sin^2\gamma\tan\Lambda + \cos^2\gamma\tan\Lambda\right)\left(\frac{b}{e}\right)\left(\frac{1}{2}\right)\right]}{K_\theta\left(\frac{K_\phi}{K_\theta}\right)\left(\cos^2\gamma+\sin^2\gamma\right)^2\left(1-Q/Q_D\right)} + 1$$

↳ The top won't really simpify more

↳ Need to make up values for $Q, e, K_\theta, Q/Q_D$

$Q = \frac{1}{2}\rho V^2 c b a_0$ $(80)^2$

$V = 50\,m/s$

$\rho = 1.225\,Kg/m^3$

$e/c = 0.1$

$b/c = 6$

$c = 2m$ (problem #2) ← borrowing

$b = 12$

Choosing to use values for $Q/Q_D = [0.5, 0.75, 1, 1.25]$

$e = 0.2\,m$

Found online ⟶ $K_\theta = 1{,}600$ Kg m / rad

$\frac{b}{e} = \frac{b}{c}\left(\frac{c}{e}\right) = 6(10) = 60$

$\frac{K_\phi}{K_\theta} = 3$ $\Lambda = [-30°, 0°, 30°]$

Now we can graph on Matlab attachment

↳ Prof said values for Q/Q_D or Q/Q_R

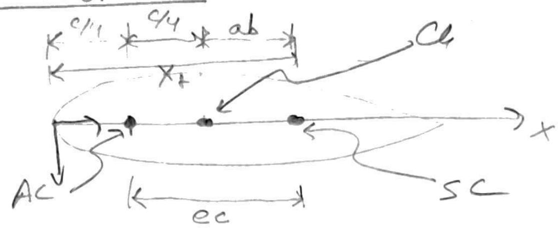
↳ Already turned in hw; not going to rederive to Q/Q_R.

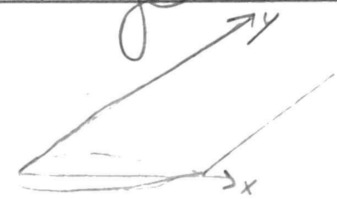
~ Set $M_{Roll} = 0$

Note: When $Q/Q_D = 1$, then the denominator goes to 0, and $\frac{L_F}{L_R} \longrightarrow \infty$

(hence the empty graph)

Problem No. 2





a. $z(x,y,t) = (y/s)^2 h_1(t) + (x - x_f)(y/s)\theta_1(t)$

1) $T = \int_{wing} \frac{1}{2} dm\, \dot{z}^2$

$(x-x_f)(x-x_f)$
$x^2 - 2xx_f + x_f^2$

$$T = \frac{m}{2}\int_0^s\int_0^c \left(\left(y/s\right)^2\dot{h}_1 + (x-x_f)(y/s)\dot{\theta}_1\right)^2 dx\,dy$$

$$= \frac{m}{2}\int_0^s\int_0^c \left[(y/s)^2\dot{h}_1\right]^2 + \left[(y/s)^2\dot{h}_1\right]\left[(x-x_f)(y/s)\dot{\theta}_1\right] + \left[(x-x_f)(y/s)\dot{\theta}_1\right]^2 dx\,dy$$

$$T = \frac{m}{2}\int_0^s \left[(y/s)^2\dot{h}_1\right]^2 x + \left[(y/s)^2\dot{h}_1\right]\left[\left(\tfrac{1}{2}x^2 - x_f x\right)(y/s)\dot{\theta}_1\right] + \left[(y/s)\dot{\theta}\right]^2\left[\tfrac{1}{3}x^3 - x^2x_f + x_f^2x\right]\Big|_0^c dx\,dy$$

$$T = \frac{m}{2}\int_0^s \left[(y/s)^2\dot{h}_1\right]^2 c + 2\left[(y/s)^2\dot{h}_1\right]\left[\left(\tfrac{1}{2}c^2 - x_f c\right)(y/s)\dot{\theta}_1\right] + \left[(y/s)\dot{\theta}_1\right]^2\left[\tfrac{1}{3}c^3 - x_f c^2 + x_f^2 c\right] dy$$

$$T = \frac{m}{2}\left(\tfrac{1}{5}y^5/s^4\,\dot{h}_1^2 c + \tfrac{2\cdot 1}{4}y^4/s^3\,\dot{h}_1\dot{\theta}_1\left(\tfrac{1}{2}c^2 - x_f c\right) + \tfrac{1}{3}y^3/s^2\,\dot{\theta}_1^2\left(\tfrac{1}{3}c^3 - x_f c^2 + x_f^2 c\right)\Big|_0^s\right)$$

$$T = \frac{m}{2}\left(\tfrac{1}{5}s\dot{h}_1^2 c + \tfrac{2\cdot 1}{4}s\dot{h}_1\dot{\theta}_1\left(\tfrac{1}{2}c^2 - x_f c\right) + \tfrac{1}{3}s\dot{\theta}_1^2\left(\tfrac{1}{3}c^3 - x_f c^2 + x_f^2 c\right)\right)$$

For the Lagrange Eq, we need...

$$\frac{\partial T}{\partial h_1} = 0, \quad \frac{\partial T}{\partial \theta_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{h}_1}\right) = \frac{d}{dt}\left(\tfrac{1}{5}sc\dot{h}_1 + \tfrac{1}{4}s\dot{\theta}_1\left(\tfrac{1}{2}c^2 - x_f c\right)\right)\cdot m$$
$$= \left[\tfrac{1}{5}sc\ddot{h}_1 + \tfrac{1}{4}s\ddot{\theta}_1\left(\tfrac{1}{2}c^2 - x_f c\right)\right]\cdot m$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = \frac{d}{dt}\left(\tfrac{1}{4}s\dot{h}_1\left(\tfrac{1}{2}c^2 - x_f c\right) + \tfrac{1}{3}s\dot{\theta}_1\left(\tfrac{1}{3}c^3 - x_f c^2 + x_f^2 c\right)\right)\cdot m$$
$$= \left[\tfrac{1}{4}s\ddot{h}_1\left(\tfrac{1}{2}c^2 - x_f c\right) + \tfrac{1}{3}s\ddot{\theta}_1\left(\tfrac{1}{3}c^3 - x_f c^2 + x_f^2 c\right)\right]\cdot m$$

Need to solve for potential energy:

$$U = \tfrac{1}{2}\int_0^s \left(\frac{\partial^2 z}{\partial y^2}\right)^2 EI + \tfrac{1}{2}\int_0^s \left(\frac{\partial \theta}{\partial y}\right)^2 GJ$$

$$U = \tfrac{1}{2}\int_0^s EI\left(\frac{2h_1}{s^2}\right)^2 dy + \tfrac{1}{2}\int_0^s GJ\left(\theta_1/s\right)^2 dy$$

$$U = \tfrac{1}{2} EI \left(\frac{2h}{s^2}\right)^2 y \Big|_0^s + \tfrac{1}{2} GJ \left(\frac{\theta_1}{s}\right)^2 y \Big|_0^s$$

$$U = \tfrac{1}{2} EI \left(\frac{2h_1}{s^2}\right)^2 s + \tfrac{1}{2} GJ \left(\frac{\theta_1}{s}\right)^2 s$$

$$U = 2EI h_1^2 / s^3 + \tfrac{1}{2} GJ \theta_1^2 / s$$

For Lagrange's Eq we need:

$$\frac{\partial U}{\partial h_1} = \frac{4EI h_1}{s^3} \qquad \frac{\partial U}{\partial \theta_1} = \frac{GJ\theta_1}{s}$$

Next we need to solve for δW:

$$\delta W = -\int_0^s dL\, \delta z + \int_0^s dM\, \delta\theta$$

$$\delta z = (y/s)^2 \delta h_1 \qquad \delta\theta = (y/s)\,\delta\theta_1$$

$$dL = q c a_w \left(y/s\, \theta_1 + (y/s)^2 \frac{\dot{h}_1}{V} \right) dy$$

$$dM = q c^2 \left[e a_w \left(y/s\, \theta_1 + (y/s)^2 \frac{\dot{h}_1}{V} \right) + M_{\dot\theta} \left(y/s \frac{c}{4V} \right) \dot{\theta}_1 \right] dy$$

$$Q_{h_1} = \frac{\partial(\delta W)}{\partial(h_1)} = -\int_0^s q c a_w \left(\frac{y^3}{s^3} \theta_1 + \frac{y^4}{s^4} \frac{\dot{h}_1}{V} \right) dy$$

$$Q_{h_1} = -q c a_w \left(\tfrac{1}{4} s \theta_1 + \tfrac{1}{5} s \dot{h}_1 / V \right)$$

$$Q_{\theta_1} = \frac{\partial(\delta W)}{\partial(\theta_1)} = \int_0^s \left[q c^2 \left(e a_w \left(y/s\, \theta_1 + \frac{y^2}{s^2} \frac{\dot{h}_1}{V} \right) + M_{\dot\theta} \left(y/s \frac{c}{4V} \right) \right) \right] \left(\frac{y}{s} \right) dy$$

$$Q_{\theta_1} = q c^2 \left[e a_w \left(\tfrac{1}{3} s \theta_1 + \tfrac{1}{4} s \dot{h}_1 / V \right) + M_{\dot\theta} \left(\frac{cs}{12V} \right) \dot{\theta}_1 \right]$$

We should have everything we need for Lagrange:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \cancelto{0}{\frac{\partial T}{\partial x}} + \frac{\partial U}{\partial x} = \frac{\partial(\delta W)}{\partial(\delta x)}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) + \frac{\partial U}{\partial x} = \frac{\partial(\delta W)}{\partial(\delta x)}$$

Combine:

$$\boxed{\begin{aligned} & m\begin{bmatrix} \frac{1}{5}sc & \frac{s}{4}\left(\frac{c^2}{2}-x_f c\right) \\ \frac{1}{4}s\left(\frac{c^2}{2}-x_f c\right) & \frac{s}{3}\left(\frac{c^3}{3}-x_f c^2+x_f^2 c\right)\end{bmatrix}\begin{bmatrix}\ddot{h}_1 \\ \ddot{\theta}_1\end{bmatrix} + \\ & \begin{bmatrix} (\frac{1}{5})qca_w(s/V) & 0 \\ (\frac{1}{4})qc^2ea_w(s/V) & -qc^3(s/12V)M_{\dot{\theta}}\end{bmatrix}\begin{bmatrix}\dot{h}_1 \\ \dot{\theta}_1\end{bmatrix} + \\ & \begin{bmatrix} 4EI/s^3 & qca_w(s/4) \\ 0 & GJ/s - qc^2ea_w(s/3)\end{bmatrix}\begin{bmatrix}h_1 \\ \theta_1\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix}\end{aligned}}$$

2) At this point, I will switch over to Matlab

$$q = \frac{1}{2}\rho V^2$$

Solve for e:

$c \rightarrow$ total length

$$ec = c/4 + ab$$

$$x_f = \underbrace{c/4 + c/4}_{c/2} + ab \qquad (x_f - c/2) = ab$$

$$e = \frac{(c/4 + ab)}{c}$$

Reduce order of equations

$$M\ddot{x} + D\dot{x} + Kx = 0$$

$$M\ddot{x} = -D\dot{x} - Kx \rightarrow \ddot{x} = -M^{-1}D\dot{x} - M^{-1}Kx$$

$$\ddot{x} = [[-M^{-1}D][-M^{-1}K]]\begin{bmatrix}\dot{x} \\ x\end{bmatrix}$$

$$\ddot{x} = [[-M^{-1}K][-M^{-1}D]]\underbrace{\begin{bmatrix}x \\ \dot{x}\end{bmatrix}}_{y} \qquad \dot{y} = \begin{bmatrix}\dot{x} \\ \ddot{x}\end{bmatrix}$$

$$\ddot{x} = [0 \quad I]\,\dot{y}$$

$$\dot{y} = \underbrace{\begin{bmatrix}[0] & [I] \\ -M^{-1}K & -M^{-1}D\end{bmatrix}}_{A} y \qquad (A - I\lambda)x = 0$$

$A \leftarrow$ plant matrix

$$-\zeta\omega \pm i\omega\sqrt{1-\zeta^2}$$

3) When no aerodynamic dampy terms are added, we look at where the frequency terms coalesce. In this case $V_F = 105\ m/s$

With aerodynamic dampy forces added, we look where the dampy ratio becomes negative (not possible). This happens at $V_F = 82\ m/s$.

4) Flutter usually occurs before divergence, but still needs to be checked.

Look at real part of eigenvalue

or

use $|K| = 0$ ← Pg. 188 (Section 10.8)

(calc done on Matlab)

$$\boxed{V = 173.5711\ m/s}$$

b. $z(x,y,t) = (y/s)^2 h_1(t) + (y/s)^3 h_2(t) + (x-x_f)(y/s)\theta_1(t) + (x-x_f)(y/s)^2\theta_2(t)$

1) $T = \int \frac{1}{2}\, dm\, \dot{z}^2$ $\qquad (x-x_f) \to x^2 - 2xx_f + x_f^2$

$$T = \frac{m}{2}\int_0^s\int_0^c \left[(y/s)^2\dot h_1 + (y/s)^3\dot h_2 + (x-x_f)(y/s)\dot\theta_1 + (x-x_f)(y/s)^2\dot\theta_2\right]^2 dx\,dy$$

$$\begin{aligned}T = \frac{m}{2}\int_0^s\int_0^c \Big(& (y/s)^4\dot h_1^2 + 2(y/s)^5\dot h_1\dot h_2 + 2(x-x_f)(y/s)^3\dot h_1\dot\theta_1 + 2(x-x_f)(y/s)^4\dot h_1\dot\theta_2\\
& + \cancel{(y/s)^5\dot h_1\dot h_2} + (y/s)^6\dot h_2^2 + 2(x-x_f)(y/s)^4\dot h_2\dot\theta_1 + 2(x-x_f)(y/s)^5\dot h_2\dot\theta_2\\
& + \cancel{(x-x_f)(y/s)^3\dot h_1\dot\theta_1} + \cancel{(x-x_f)(y/s)^4\dot h_2\dot\theta_1} + (x-x_f)^2(y/s)^2\dot\theta_1^2\\
& + 2(x-x_f)^2(y/s)^3\dot\theta_1\dot\theta_2 + \cancel{(x-x_f)(y/s)^4\dot h_1\dot\theta_2} + \cancel{(x-x_f)(y/s)^5\dot h_2\dot\theta_2}\\
& + \cancel{(x-x_f)^2(y/s)^3\dot\theta_1\dot\theta_2} + (x-x_f)^2(y/s)^4\dot\theta_2^2\Big)\,dx\,dy\end{aligned}$$

$$\begin{aligned}T = \frac{m}{2}\int_0^s \Big(& (y/s)^4\dot h_1^2 x + (y/s)^6\dot h_2^2 x + 2(y/s)^5\dot h_1\dot h_2 x + 2(\tfrac12 x^2 - xx_f)(y/s)^3\dot h_1\dot\theta_1\\
& + 2(\tfrac12 x^2 - xx_f)(y/s)^4\dot h_1\dot\theta_2 + 2(\tfrac12 x - xx_f)(y/s)^4\dot h_2\dot\theta_1\\
& + 2(\tfrac12 x^2 - xx_f)(y/s)^5\dot h_2\dot\theta_2 + (\tfrac13 x^3 - x^2x_f + xx_f^2)(y/s)^2\dot\theta_1^2\\
& + 2(\tfrac13 x^2 - x^2x_f + xx_f^2)(y/s)^3\dot\theta_1\dot\theta_2\\
& + (\tfrac13 x^3 - x^2x_f + xx_f^2)(y/s)^4\dot\theta_2^2\Big)\Big|_0^c dy\end{aligned}$$

$$\begin{aligned}T = m/2\Big(& \tfrac15 y^5/s^4\,\dot h_1^2 c + \tfrac17 y^7/s^6\,\dot h_2^2 c + \tfrac13 y^6/s^5\,\dot h_1\dot h_2 c + \tfrac25(\tfrac12 c^2 - cx_f)(y^5/s^4)\dot h_1\dot\theta_2\\
& + \tfrac12(\tfrac12 c^2 - cx_f)(y^4/s^3)\dot h_1\dot\theta_1 + \tfrac13(\tfrac12 c^2 - cx_f)(y^6/s^5)\dot h_2\dot\theta_2\\
& + \tfrac25(\tfrac12 c^2 - cx_f)(y^5/s^4)\dot h_2\dot\theta_1\\
& + \tfrac13(\tfrac13 c^3 - c^2x_f + cx_f^2)(y^3/s^2)\dot\theta_1^2\\
& + \tfrac12(\tfrac13 c^3 - c^2x_f + cx_f^2)(y^4/s^3)\dot\theta_1\dot\theta_2\\
& + \tfrac15(\tfrac13 c^3 - c^2x_f + cx_f^2)(y^5/s^4)\dot\theta_2^2\Big)\Big|_0^s\end{aligned}$$

$$\begin{aligned}T = sm/2\Big(& \tfrac15\dot h_1^2 c + \tfrac17\dot h_2^2 c + \tfrac13\dot h_1\dot h_2 c + \tfrac12(\tfrac12 c^2 - cx_f)\dot h_1\dot\theta_1\\
& + \tfrac25(\tfrac12 c^2 - cx_f)\dot h_1\dot\theta_2 + \tfrac25(\tfrac12 c^2 - cx_f)\dot h_2\dot\theta_1\\
& + \tfrac13(\tfrac12 c^2 - cx_f)\dot h_2\dot\theta_2\\
& + \tfrac13(\tfrac13 c^3 - c^2x_f + cx_f^2)\dot\theta_1^2\\
& + \tfrac12(\tfrac13 c^3 - c^2x_f + cx_f^2)\dot\theta_1\dot\theta_2\\
& + \tfrac15(\tfrac13 c^3 - c^2x_f + cx_f^2)\dot\theta_2^2\Big)\end{aligned}$$

For Lagrange's Eqs:

$$\frac{\partial T}{\partial h_1} = 0, \quad \frac{\partial T}{\partial h_2} = 0, \quad \frac{\partial T}{\partial \theta_1} = 0, \quad \frac{\partial T}{\partial \theta_2} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{h}_1}\right) = \frac{sm}{2}\frac{d}{dt}\left(\frac{2}{5}\dot{h}_1 c + \frac{1}{3}\dot{h}_2 c + \frac{1}{2}\left(\frac{1}{2}c^2 - cx_f\right)\dot{\theta}_1 + \frac{2}{5}\left(\frac{1}{2}c^2 - cx_f\right)\dot{\theta}_2\right)$$

$$= sm\left(1/5\,\ddot{h}_1 c + 1/6\,\ddot{h}_2 c + \frac{1}{4}\left(\frac{1}{2}c^2 - cx_f\right)\ddot{\theta}_1 + 1/5\left(1/2\,c^2 - cx_f\right)\ddot{\theta}_2\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{h}_2}\right) = \frac{sm}{2}\frac{d}{dt}\left(\frac{2}{7}\dot{h}_2 c + \frac{1}{3}\dot{h}_1 c + \frac{2}{5}\left(\frac{1}{2}c^2 - cx_f\right)\dot{\theta}_1 + \frac{1}{3}\left(\frac{1}{2}c^2 - cx_f\right)\dot{\theta}_2\right)$$

$$= sm\left(1/7\,\ddot{h}_2 c + 1/6\,\ddot{h}_1 c + 1/5\left(1/2\,c^2 - cx_f\right)\ddot{\theta}_1 + 1/6\left(1/2\,c^2 - cx_f\right)\ddot{\theta}_2\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = \frac{sm}{2}\frac{d}{dt}\left(\frac{1}{2}\left(\frac{1}{2}c^2 - cx_f\right)\dot{h}_1 + \frac{2}{5}\left(\frac{1}{2}c^2 - cx_f\right)\dot{h}_2 + \frac{2}{3}\left(\frac{1}{3}c^3 - c^2x_f + cx_f^2\right)\dot{\theta}_1 + \frac{1}{2}\left(\frac{1}{3}c^3 - c^2x_f + cx_f^2\right)\dot{\theta}_2\right)$$

$$= sm\left(\frac{1}{4}\left(\frac{1}{2}c^2 - cx_f\right)\ddot{h}_1 + 1/5\left(1/2\,c^2 - cx_f\right)\ddot{h}_2 + 1/3\left(1/3\,c^3 - c^2x_f + cx_f^2\right)\ddot{\theta}_1 + 1/4\left(1/3\,c^3 - c^2x_f + cx_f^2\right)\ddot{\theta}_2\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = \frac{sm}{2}\frac{d}{dt}\left(\frac{2}{5}\left(\frac{1}{2}c^2 - cx_f\right)\dot{h}_1 + \frac{1}{3}\left(\frac{1}{2}c^2 - cx_f\right)\dot{h}_2 + \frac{1}{2}\left(\frac{1}{3}c^3 - c^2x_f + cx_f^2\right)\dot{\theta}_1 + \frac{2}{5}\left(\frac{1}{3}c^3 - c^2x_f + cx_f^2\right)\dot{\theta}_2\right)$$

$$= sm\left(1/5\left(1/2\,c^2 - cx_f\right)\ddot{h}_1 + 1/6\left(1/2\,c^2 - cx_f\right)\ddot{h}_2 + 1/4\left(1/3\,c^3 - c^2x_f + cx_f^2\right)\ddot{\theta}_1 + 1/5\left(1/3\,c^3 - c^2x_f + cx_f^2\right)\ddot{\theta}_2\right)$$

Need to Solve for potential energy:

$$U = 1/2\int_0^s \left(\frac{\partial^2 z}{\partial y^2}\right)^2 EI\,dy + 1/2\int_0^s \left(\frac{\partial \theta}{\partial y}\right)^2 GJ\,dy$$

$$U = 1/2\int_0^s \left(2/s^2\,h_1 + 6y/s^3\,h_2\right)^2 EI\,dy + \frac{1}{2}\int_0^s \left(1/s\,\theta_1 + 2y/s^2\,\theta_2\right)^2 GJ\,dy$$

$$U = s/2\left(4/s^4\,h_1^2 + \frac{24s^2}{2}/s^5\,h_1h_2 + \frac{36s^3}{3}/s^6\,h_2^2\right)EI + \frac{1}{2}\int\left(1/s^2\,\theta_1^2 + 4y/s^3\,\theta_1\theta_2 + 4y^2/s^4\,\theta_2^2\right)GJ\,dy$$

$$U = \left(2/s^3\,h_1^2 + 6/s^3\,h_1h_2 + 6/s^3\,h_2^2\right)EI + 1/2\left(1/s^2\,\theta_1^2 + 2y^2/s^3\,\theta_1\theta_2 + 4/3\,y^3/s^4\,\theta_2^2\Big|_0^s\right)GJ$$

$$U = \left(2/s^3\,h_1^2 + 6/s^3\,h_1h_2 + 6/s^3\,h_2^2\right)EI + \left(1/2s\,\theta_1^2 + 1/s\,\theta_1\theta_2 + 2/3s\,\theta_2^2\right)GJ$$

$$\frac{\partial U}{\partial h_1} = \left(\frac{4}{s^3}h_1 + \frac{6}{s^3}h_2\right)EI$$

$$\frac{\partial U}{\partial h_2} = \left(\frac{6}{s^3}h_1 + \frac{12}{s^3}h_2\right)EI$$

$$\frac{\partial U}{\partial \theta_1} = \left(\frac{1}{s}\theta_1 + \frac{1}{s}\theta_2\right)GJ$$

$$\frac{\partial U}{\partial \theta_2} = \left(\frac{1}{s}\theta_1 + 4/3\,\frac{1}{s}\theta_2\right)GJ$$

Solve for δW:

$$\delta W = -\int_0^s dL\,\delta z + \int_0^s dM\,\delta\theta$$

$$\delta z = (y/s)^2 \delta h_1 + (y/s)^3 \delta h_2$$

$$\delta\theta = (y/s)\delta\theta_1 + (y/s)^2 \delta\theta_2$$

$$dL = q c a_w \left(\theta + \frac{\dot z}{V}\right) dy, \quad dM = q c^2 \left[e a_w \left(\theta + \frac{\dot z}{V}\right) + M_{\dot\theta} \frac{\dot\theta c}{4V}\right] dy$$

$$dL = q c a_w \left((y/s)\theta_1 + (y/s)^2 \theta_2 + \frac{(y/s)^2 \dot h_1 + (y/s)^3 \dot h_2}{V} \right) dy$$

$$dM = q c^2 \left[e a_w \left((y/s)\theta_1 + (y/s)^2 \theta_2 + \frac{(y/s)^2 \dot h_1 + (y/s)^3 \dot h_2}{V} \right) + M_{\dot\theta} \frac{\left((y/s)\dot\theta_1 + (y/s)^2 \dot\theta_2\right) c}{4V} \right] dy$$

$$Q_{h_1} = \frac{\partial(\delta W)}{\partial(\delta h_1)} = -\int_0^s q c a_w \left((y/s)\theta_1 + (y/s)^2 \theta_2 + \frac{(y/s)^2 \dot h_1 + (y/s)^3 \dot h_2}{V} \right) (y/s)^2 dy$$

$$\Big| \; Q_{h_1} = -q c a_w \left(\tfrac{1}{4} s\theta_1 + \tfrac{1}{5} s \theta_2 + \frac{\tfrac{1}{5} s \dot h_1 + \tfrac{1}{6} s \dot h_2}{V} \right)$$

$$Q_{h_2} = \frac{\partial(\delta W)}{\partial(\delta h_2)} = -\int_0^s q c a_w \left((y/s)\theta_1 + (y/s)^2 \theta_2 + \frac{(y/s)^2 \dot h_1 + (y/s)^3 \dot h_2}{V} \right) (y/s)^3 dy$$

$$\Big| \; Q_{h_2} = -q c a_w \left(\tfrac{1}{5} s\theta_1 + \tfrac{1}{6} s \theta_2 + \frac{\tfrac{1}{6} s \dot h_1 + \tfrac{1}{7} s \dot h_2}{V} \right)$$

$$Q_{\theta_1} = \frac{\partial(\delta W)}{\partial(\delta\theta_1)} = \int_0^s q c^2 \left[e a_w \left((y/s)\theta_1 + (y/s)^2 \theta_2 + \frac{(y/s)^2 \dot h_1 + (y/s)^3 \dot h_2}{V} \right) + M_{\dot\theta} \frac{c\left((y/s)\dot\theta_1 + (y/s)^2 \dot\theta_2\right)}{4V} \right] (y/s)$$

$$\Big| \; Q_{\theta_1} = q c^2 \left[e a_w \left(s/3\, \theta_1 + s/4\, \theta_2 + \frac{s/4\, \dot h_1 + s/5\, \dot h_2}{V} \right) + M_{\dot\theta} c \frac{s/3\, \dot\theta_1 + s/4\, \dot\theta_2}{4V} \right]$$

$$Q_{\theta_2} = \frac{\partial(\delta W)}{\partial(\delta\theta_2)} = \int_0^s q c^2 \left[e a_w \left((y/s)\theta_1 + (y/s)^2 \theta_2 + \frac{(y/s)^2 \dot h_1 + (y/s)^3 \dot h_2}{V} \right) + M_{\dot\theta} c \frac{(y/s)\dot\theta_1 + (y/s)^2 \dot\theta_2}{4V} \right] (y/s)^2$$

$$\Big| \; Q_{\theta_2} = q c^2 \left[e a_w \left(s/4\, \theta_1 + s/5\, \theta_2 + \frac{s/5\, \dot h_1 + s/6\, \dot h_2}{V} \right) + M_{\dot\theta} c \frac{s/4\, \dot\theta_1 + s/5\, \dot\theta_2}{4V} \right]$$

Put together:
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) + \frac{\partial U}{\partial x} = \frac{\partial(\delta W)}{\partial(\delta x)}$$

$$m\begin{bmatrix} s/5\,c & sc/6 & s/4\left(\tfrac{1}{2}c^2 - cx_f\right) & s/5\left(\tfrac{1}{2}c^2 - cx_f\right) \\ s/6\,c & sc/7 & s/5\left(\tfrac{1}{2}c^2 - cx_f\right) & s/6\left(\tfrac{1}{2}c^2 - cx_f\right) \\ s/4\left(\tfrac{1}{2}c^2 - cx_f\right) & s/5\left(\tfrac{1}{2}c^2 - cx_f\right) & s/3\left(\tfrac{1}{3}c^3 - c^2x_f + cx_f^2\right) & s/4\left(\tfrac{1}{3}c^3 - c^2x_f + cx_f^2\right) \\ s/5\left(\tfrac{1}{2}c^2 - cx_f\right) & s/6\left(\tfrac{1}{2}c^2 - cx_f\right) & s/4\left(\tfrac{1}{3}c^3 - c^2x_f + cx_f^2\right) & s/5\left(\tfrac{1}{3}c^3 - c^2x_f + cx_f^2\right) \end{bmatrix}\begin{bmatrix} \ddot{h}_1 \\ \ddot{h}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$+\begin{bmatrix} 4/s^3\,EI & 6/s^3\,EI & (s/4)qca_w & (s/5)qca_w \\ 6/s^3\,EI & 12/s^3\,EI & (s/5)qca_w & (s/6)qca_w \\ 0 & 0 & -(s/3)qc^2ea_w + (1/s)GJ & -(s/4)qc^2ea_w + (1/s)GJ \\ 0 & 0 & -(s/4)qc^2ea_w + (1/s)GJ & -(s/5)qc^2ea_w + (4/3)(1/s)GJ \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

$$+\begin{bmatrix} (s/5V)(qca_w) & (s/6V)(qca_w) & 0 & 0 \\ (s/6V)(qca_w) & (s/7V)(qca_w) & 0 & 0 \\ -(s/4V)(qc^2ea_w) & -(s/5V)(qc^2ea_w) & -M_{\dot{\theta}}c^3q(s/12V) & -M_{\dot{\theta}}c^3q(s/16V) \\ -(s/5V)(qc^2ea_w) & -(s/6V)(qc^2ea_w) & -M_{\dot{\theta}}c^3q(s/16V) & -M_{\dot{\theta}}c^3q(s/20V) \end{bmatrix}\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2) Look at Matlab $\quad M\ddot{x} + D\dot{x} + Kx = 0$

3. <u>Without</u> Aerodynamic damping

$V_F = 107$ m/s

↳ where $\zeta \leq 0$

<u>With</u> aerodynamic damping

$$\boxed{V_F = 81.45 \text{ m/s}}$$

↳ Yet again, where ζ crosses to negative since a negative damping ratio cannot exist.

4. Use $|K| = 0$ ← pg. 188 (Section 10.8)

$$\boxed{V_D = 158.0025 \text{ m/s}}$$

Hw #4 - Aeroelasticity - ME597/AAE556

Victoria Nagorski - 11/13/22

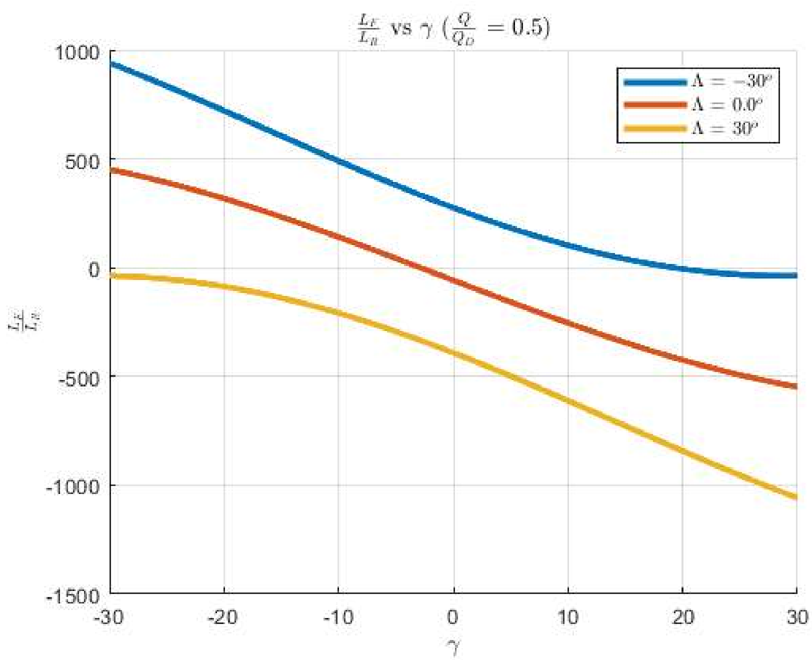
Contents

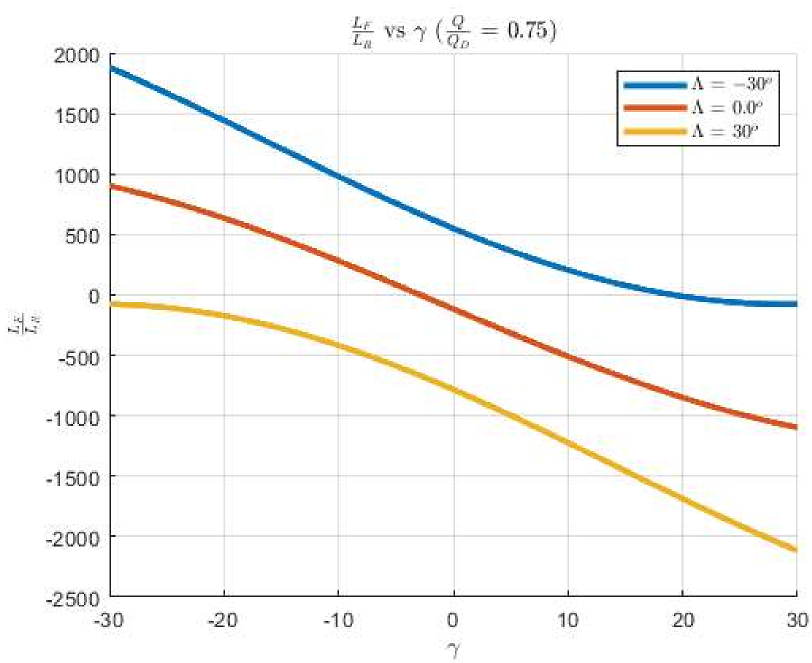
Problem No.1

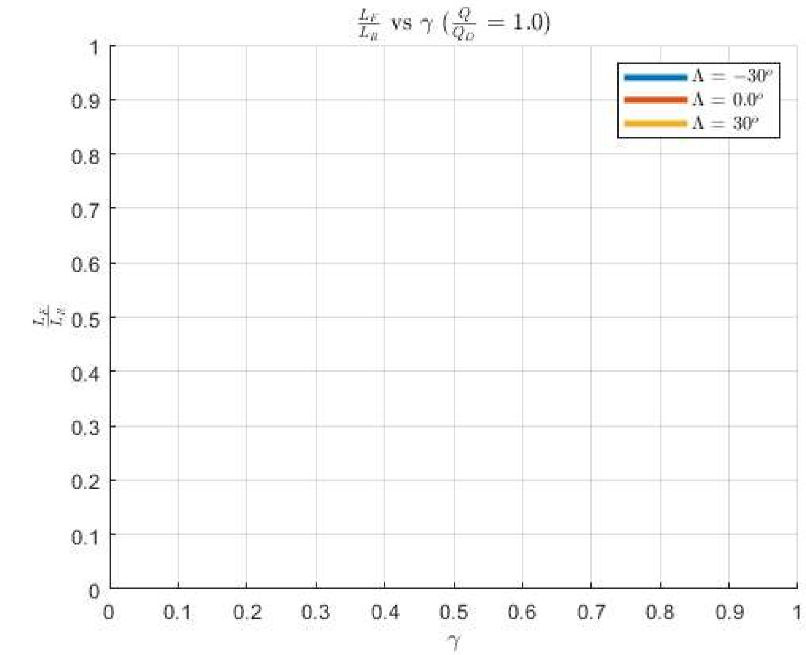
```
clear;clc;close all

% Define the variables
b_c = 6;
e_c = 0.1;
c_e = 1/e_c;
b_e = b_c/e_c;
c = 2;  % Barroed from problem 2
e = e_c*c;
b = b_c*c;
E = 0.15;
K_phi_theta = 3;
V = 50;
rho = 1.225;
a0 = 2*pi;
Q = (1/2)*rho*V^2*c*b*a0;
K_theta = 1600;     % Found this value online
Q_ratio = 0.5;

% Solve for coeffiecient ratios with Glauert's theory
c_l_a = (1/pi) * (acos(1-(2*E)) + 2*sqrt(E*(1-E)));
c_m_a = (-1/pi) * (1-E) * sqrt((1-E)*E);

% Variables to varry in graphs
Lambda = [-30 0 30].*pi/180;
gamma = (-30:0.01:30).*pi/180;

% Define the equation
bottom = K_theta.*K_phi_theta.*(cos(gamma).^2 + sin(gamma).^2).^2.*(1-Q_ratio);
L_F_R = @(L) Q*e*[(K_phi_theta*cos(gamma).^2+sin(gamma).^2+...
    (K_phi_theta-1).*sin(gamma).*cos(gamma).*tan(L)).*(1 + ...
    c_m_a/c_l_a*c_e) - ((K_phi_theta-1).*sin(gamma).*cos(gamma)+...
    K_phi_theta.*sin(gamma).^2.*tan(L)+...
    cos(gamma).^2*tan(L)).*(b_e/2)]./bottom + 1;

% Graphing
figure
hold on
for i = 1:length(Lambda)
    lambda = Lambda(1,i);          % Pull out the q_hat value being used
    temp_ratio = L_F_R(lambda);        % Solve for the K_e values
    plot(gamma.*180/pi,temp_ratio,'LineWidth',3)         % Graph everything
end
title('$\frac{L_F}{L_R}$ vs $\gamma$ ($\frac{Q}{Q_D}$ = 0.5)','Interpreter','latex')
xlabel('$\gamma$','Interpreter','latex')
ylabel('$\frac{L_F}{L_R}$','Interpreter','latex')
legend('$\Lambda$ = $-30^o$','$\Lambda$ = $0.0^o$','$\Lambda$ = $30^o$','Interpreter','latex')
grid('on')
hold off

Q_ratio = 0.75;

% Solve for coeffiecient ratios with Glauert's theory
c_l_a = (1/pi) * (acos(1-(2*E)) + 2*sqrt(E*(1-E)));
c_m_a = (-1/pi) * (1-E) * sqrt((1-E)*E);

% Variables to varry in graphs
Lambda = [-30 0 30].*pi/180;
gamma = (-30:0.01:30).*pi/180;

% Define the equation
```

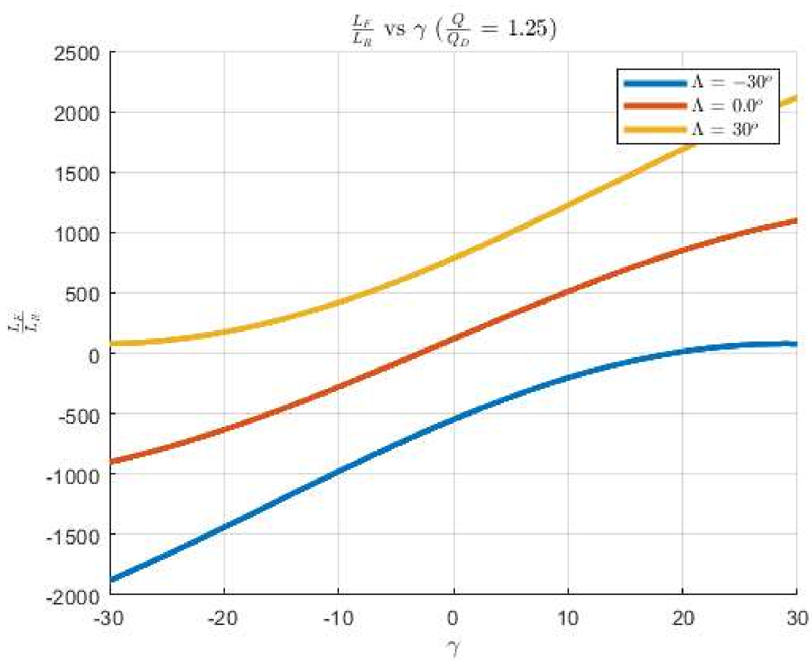
```
bottom = K_theta.*K_phi_theta.*(cos(gamma).^2 + sin(gamma).^2).^2.*(1-Q_ratio);
L_F_R = @(L) Q*e*[(K_phi_theta*cos(gamma).^2+sin(gamma).^2+...
    (K_phi_theta-1).*sin(gamma).*cos(gamma).*tan(L)).*(1 + ...
    c_m_a/c_l_a*c_e) - ((K_phi_theta-1).*sin(gamma).*cos(gamma)+...
    K_phi_theta.*sin(gamma).^2.*tan(L)+...
    cos(gamma).^2*tan(L)).*(b_e/2)]./bottom + 1;

% Graphing
figure
hold on
for i = 1:length(Lambda)
    lambda = Lambda(1,i);           % Pull out the q_hat value being used
    temp_ratio = L_F_R(lambda);        % Solve for the K_e values
    plot(gamma.*180/pi,temp_ratio,'LineWidth',3)         % Graph everything
end
title('$\frac{L_F}{L_R}$ vs $\gamma$ ($\frac{Q}{Q_D}$ = 0.75)','Interpreter','latex')
xlabel('$\gamma$','Interpreter','latex')
ylabel('$\frac{L_F}{L_R}$','Interpreter','latex')
legend('$\Lambda$ = $-30^o$','$\Lambda$ = $0.0^o$','$\Lambda$ = $30^o$','Interpreter','latex')
grid('on')
hold off

Q_ratio = 1.0;

% Solve for coeffiecient ratios with Glauert's theory
c_l_a = (1/pi) * (acos(1-(2*E)) + 2*sqrt(E*(1-E)));
c_m_a = (-1/pi) * (1-E) * sqrt((1-E)*E);

% Variables to varry in graphs
Lambda = [-30 0 30].*pi/180;
gamma = (-30:0.01:30).*pi/180;

% Define the equation
bottom = K_theta.*K_phi_theta.*(cos(gamma).^2 + sin(gamma).^2).^2.*(1-Q_ratio);
L_F_R = @(L) Q*e*[(K_phi_theta*cos(gamma).^2+sin(gamma).^2+...
    (K_phi_theta-1).*sin(gamma).*cos(gamma).*tan(L)).*(1 + ...
    c_m_a/c_l_a*c_e) - ((K_phi_theta-1).*sin(gamma).*cos(gamma)+...
    K_phi_theta.*sin(gamma).^2.*tan(L)+...
    cos(gamma).^2*tan(L)).*(b_e/2)]./bottom + 1;

% Graphing
figure
hold on
for i = 1:length(Lambda)
    lambda = Lambda(1,i);           % Pull out the q_hat value being used
    temp_ratio = L_F_R(lambda);        % Solve for the K_e values
    plot(gamma.*180/pi,temp_ratio,'LineWidth',3)         % Graph everything
end
title('$\frac{L_F}{L_R}$ vs $\gamma$ ($\frac{Q}{Q_D}$ = 1.0)','Interpreter','latex')
xlabel('$\gamma$','Interpreter','latex')
ylabel('$\frac{L_F}{L_R}$','Interpreter','latex')
legend('$\Lambda$ = $-30^o$','$\Lambda$ = $0.0^o$','$\Lambda$ = $30^o$','Interpreter','latex')
grid('on')
hold off

Q_ratio = 1.25;

% Solve for coeffiecient ratios with Glauert's theory
c_l_a = (1/pi) * (acos(1-(2*E)) + 2*sqrt(E*(1-E)));
c_m_a = (-1/pi) * (1-E) * sqrt((1-E)*E);

% Variables to varry in graphs
Lambda = [-30 0 30].*pi/180;
gamma = (-30:0.01:30).*pi/180;

% Define the equation
bottom = K_theta.*K_phi_theta.*(cos(gamma).^2 + sin(gamma).^2).^2.*(1-Q_ratio);
L_F_R = @(L) Q*e*[(K_phi_theta*cos(gamma).^2+sin(gamma).^2+...
    (K_phi_theta-1).*sin(gamma).*cos(gamma).*tan(L)).*(1 + ...
    c_m_a/c_l_a*c_e) - ((K_phi_theta-1).*sin(gamma).*cos(gamma)+...
    K_phi_theta.*sin(gamma).^2.*tan(L)+...
    cos(gamma).^2*tan(L)).*(b_e/2)]./bottom + 1;

% Graphing
figure
hold on
for i = 1:length(Lambda)
```

```
    lambda = Lambda(1,i);           % Pull out the q_hat value being used
    temp_ratio = L_F_R(lambda);         % Solve for the K_e values
    plot(gamma.*180/pi,temp_ratio,'LineWidth',3)         % Graph everything
end
title('$\frac{L_F}{L_R}$ vs $\gamma$ ($\frac{Q}{Q_D}$ = 1.25)','Interpreter','latex')
xlabel('$\gamma$','Interpreter','latex')
ylabel('$\frac{L_F}{L_R}$','Interpreter','latex')
legend('$\Lambda$ = $-30^o$','$\Lambda$ = $0.0^o$','$\Lambda$ = $30^o$','Interpreter','latex')
grid('on')
hold off
```









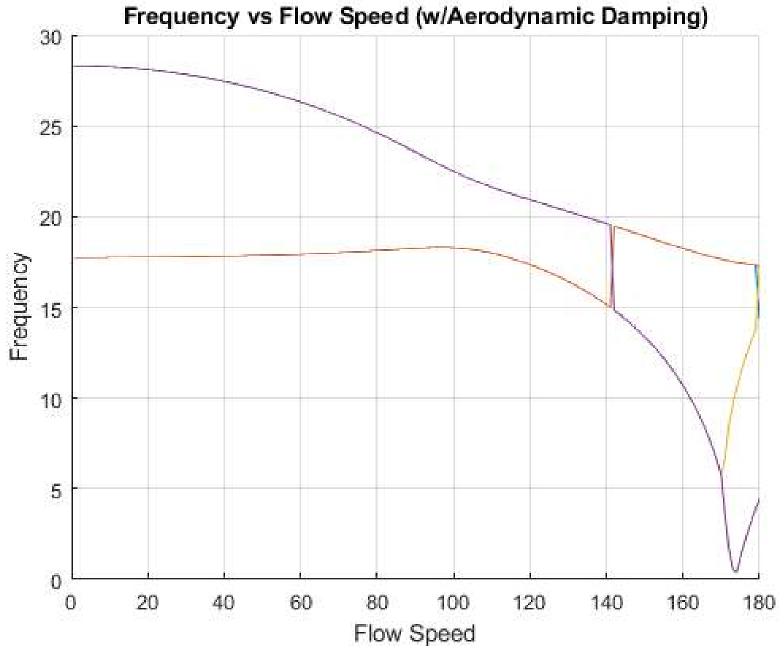
Problem No.2 Part a

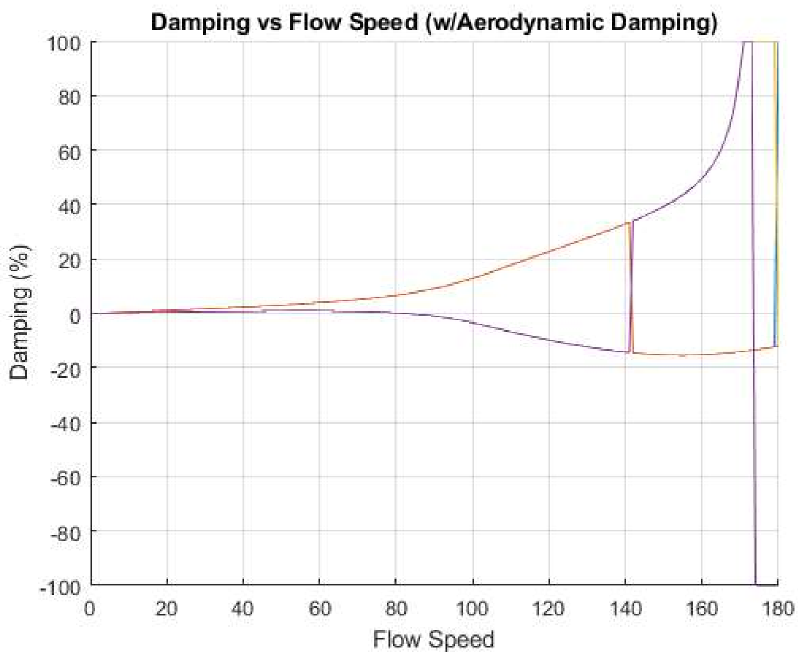
```
clear;clc;close all

% Define the variables
s = 7.5;                  % [m]
c = 2;                    % [m]
xf = 0.48*c;              % [m]
m = 200;                  % [kg/m^2]
EI = 2e7;                 % [Nm^2]
GJ = 2e6;                 % [Nm^2]
aw = 2*pi;                % Lift curve
rho = 1.225;              % [kg/m^3]
ab = xf - c/2;
e = (c/4 + ab)/c;
M_thetad = -1.2;           % Non-dimensional pitch damping derivative

syms V lamda
q = (1/2)*rho*V^2;        % Dynamic pressure

% Set up matrices (Mx_ddot + Dx_dot + Kx = 0)
```

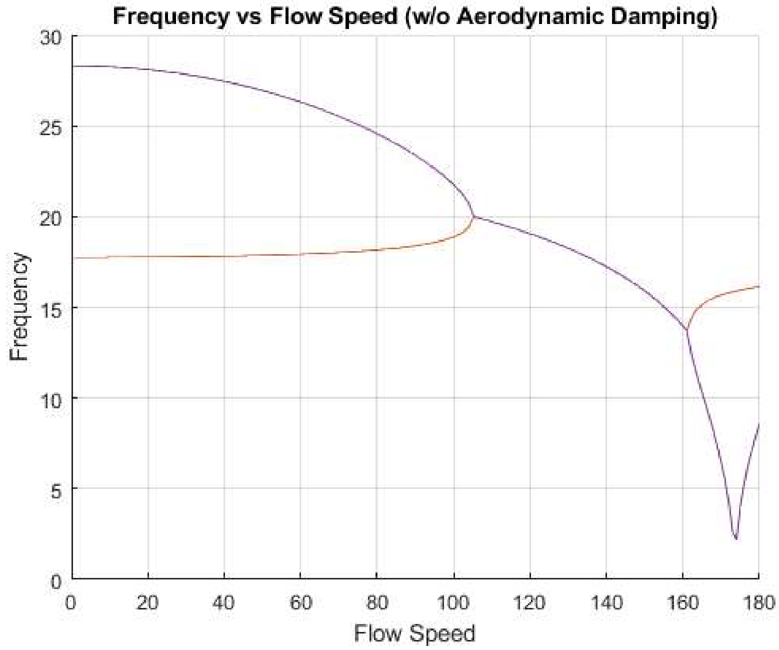
```
M = m.*[(s/5)*c                     (s/4)*(((c^2)/2) - xf*c);
        (s/4)*(((c^2)/2) - xf*c)    (s/3)*(((c^3)/3) - xf*(c^2) + (xf^2)*c)];

D = [(1/5)*q*c*aw*s/V                       0;
     (-1/4)*q*c^2*e*aw*s/V      (-s/12)*q*c^3*M_thetad/V];

K = [4*EI/s^3       q*c*aw*(s/4);
        0           GJ/s - q*c^2*e*aw*(s/3)];

% Reduce order of equations
A = [zeros(2,2)   eye(2);
     -inv(M)*K  -inv(M)*D];

% Plug in velocities
Velocities = 0:1:180;
Frequencies = zeros(4,length(Velocities));
Damping = zeros(4,length(Velocities));
for i = 1:length(Velocities)
    v = Velocities(i);
    Temp = double(subs(A,V,v));
    [wn,zeta] = damp(Temp);
    Frequencies(:,i) = wn;
    Damping(:,i) = zeta;
end

% Plot everything for w/Aerodynamic Damping
figure
hold on
plot(Velocities,Frequencies(1,:))
plot(Velocities,Frequencies(2,:))
plot(Velocities,Frequencies(3,:))
plot(Velocities,Frequencies(4,:))
title('Frequency vs Flow Speed (w/Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Frequency')
grid('on')
hold off

figure
hold on
plot(Velocities,100.*Damping(1,:))
plot(Velocities,100.*Damping(2,:))
plot(Velocities,100.*Damping(3,:))
plot(Velocities,100.*Damping(4,:))
title('Damping vs Flow Speed (w/Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Damping (%)')
grid('on')
hold off

% Reduce order of equations
A = [zeros(2,2) eye(2);
     -inv(M)*K  zeros(2,2)];

% Plug in velocities
Velocities = 0:1:180;
Frequencies = zeros(4,length(Velocities));
Damping = zeros(4,length(Velocities));
for i = 1:length(Velocities)
    v = Velocities(i);
    Temp = double(subs(A,V,v));
    [wn,zeta] = damp(Temp);
    Frequencies(:,i) = wn;
    Damping(:,i) = zeta;
end

% Plot everything for w/o Aerodynamic Damping
figure
hold on
plot(Velocities,Frequencies(1,:))
plot(Velocities,Frequencies(2,:))
plot(Velocities,Frequencies(3,:))
plot(Velocities,Frequencies(4,:))
title('Frequency vs Flow Speed (w/o Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Frequency')
grid('on')
```

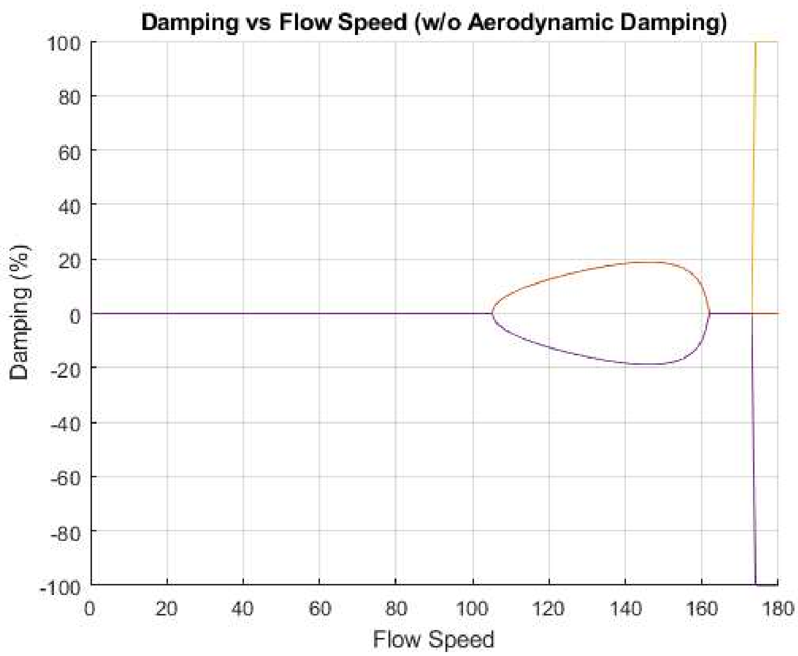
```
hold off

figure
hold on
plot(Velocities,100.*Damping(1,:))
plot(Velocities,100.*Damping(2,:))
plot(Velocities,100.*Damping(3,:))
plot(Velocities,100.*Damping(4,:))
title('Damping vs Flow Speed (w/o Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Damping (%)')
grid('on')
hold off

% Solve for V_D
V_D = double(solve(det(K)==0,V))
```

```
V_D =

 -173.5711
  173.5711
```


Frequency vs Flow Speed (w/Aerodynamic Damping)
Frequency
Flow Speed
0
5
10
15
20
25
30
0
20
40
60
80
100
120
140
160
180


Damping vs Flow Speed (w/Aerodynamic Damping)
Damping (%)
Flow Speed
-100
-80
-60
-40
-20
0
20
40
60
80
100
0
20
40
60
80
100
120
140
160
180





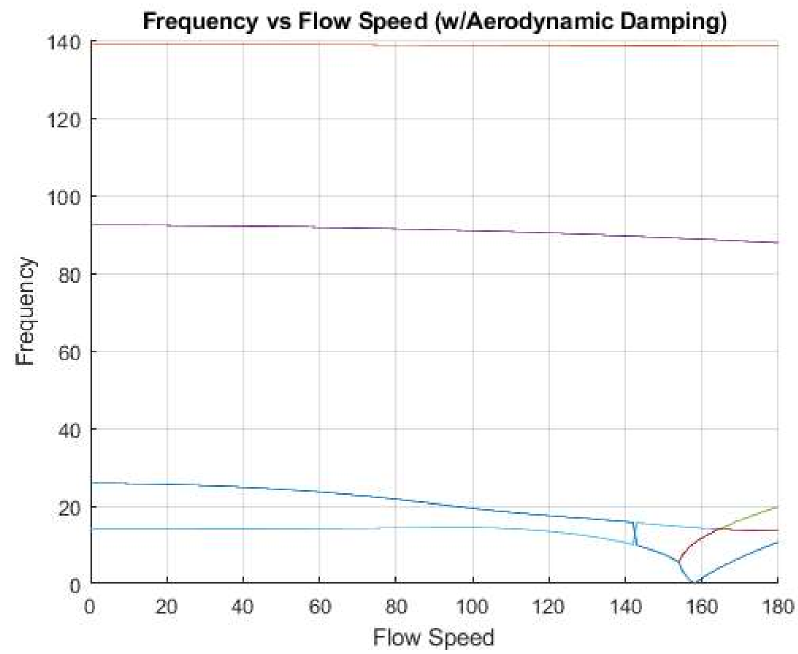
Problem No.2 Part b

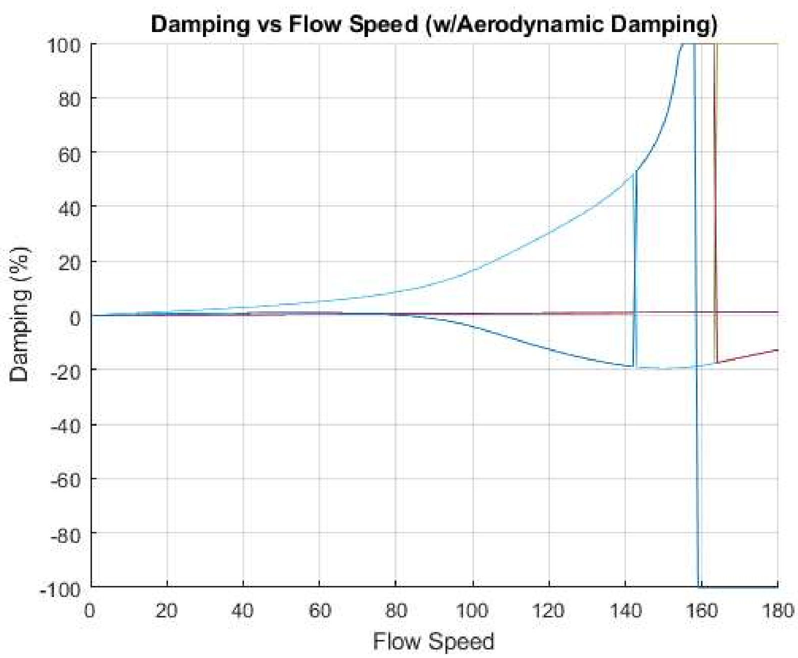
```
clear;clc;close all

% Define the variables
s = 7.5;                    % [m]
c = 2;                      % [m]
xf = 0.48*c;                % [m]
m = 200;                    % [kg/m^2]
EI = 2e7;                   % [Nm^2]
GJ = 2e6;                   % [Nm^2]
aw = 2*pi;                  % Lift curve
rho = 1.225;                % [kg/m^3]
ab = xf - c/2;
e = (c/4 + ab)/c;
M_thetad = -1.2;            % Non-dimensional pitch damping derivative

syms V lamda
q = (1/2)*rho*V^2;          % Dynamic pressure

% Set up matrices (Mx_ddot + Dx_dot + Kx = 0)
```

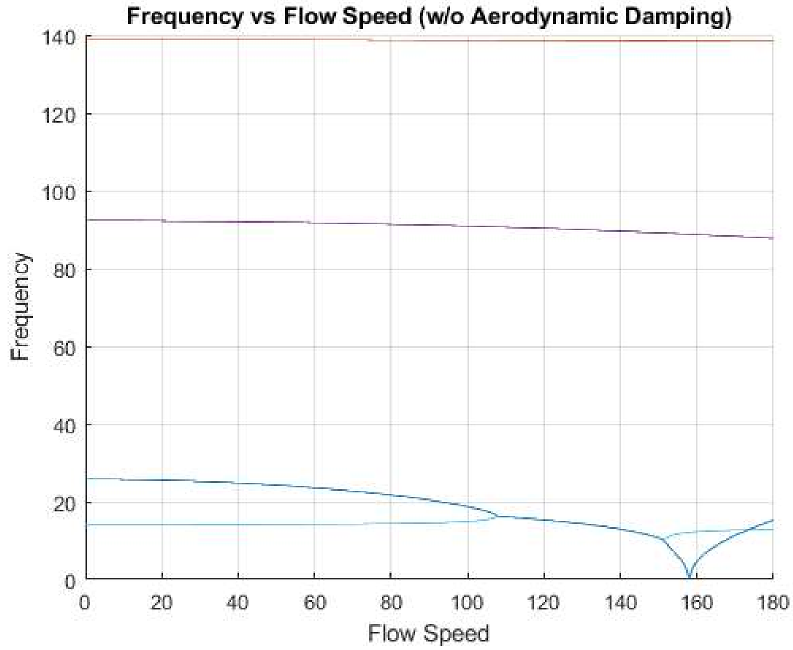
```matlab
M = m.*[(s/5)*c   (s/6)*c   (s/4)*((1/2)*c^2 - c*xf)   (s/5)*((1/2)*c^2 - c*xf);
        (s/6)*c   (s/7)*c   (s/5)*((1/2)*c^2 - c*xf)   (s/6)*((1/2)*c^2 - c*xf);
        (s/4)*((1/2)*c^2 - c*xf)   (s/5)*((1/2)*c^2 - c*xf)   (s/3)*((1/3)*c^3 - c^2*xf + c*xf^2)   (s/4)*((1/3)*c^3 - c^2*xf + c*xf^2);
        (s/5)*((1/2)*c^2 - c*xf)   (s/6)*((1/2)*c^2 - c*xf)   (s/4)*((1/3)*c^3 - c^2*xf + c*xf^2)   (s/5)*((1/3)*c^3 - c^2*xf + c*xf^2)];

K = [(4/s^3)*EI   (6/s^3)*EI   (s/4)*q*c*aw   (s/5)*q*c*aw;
     (6/s^3)*EI  (12/s^3)*EI   (s/5)*q*c*aw   (s/6)*q*c*aw;
            0           0       -(s/3)*q*c^2*aw*e + (1/s)*GJ   -(s/4)*q*c^2*aw*e + (1/s)*GJ;
            0           0       -(s/4)*q*c^2*aw*e + (1/s)*GJ   -(s/5)*q*c^2*aw*e + (4/(3*s))*GJ];

D = [(s/(5*V))*(q*c*aw)         (s/(6*V))*(q*c*aw)       0                            0;
     (s/(6*V))*(q*c*aw)         (s/(7*V))*(q*c*aw)       0                            0;
     -(s/(4*V))*(q*c^2*e*aw)   -(s/(5*V))*(q*c^2*e*aw)   -M_thetad*c^3*q*(s/(12*V))  -M_thetad*c^3*q*(s/(16*V));
     -(s/(5*V))*(q*c^2*e*aw)   -(s/(6*V))*(q*c^2*e*aw)   -M_thetad*c^3*q*(s/(16*V))  -M_thetad*c^3*q*(s/(20*V))];

% Reduce order of equations
A = [zeros(4,4) eye(4);
     -inv(M)*K  -inv(M)*D];

% Plug in velocities
Velocities = 0:1:180;
Frequencies = zeros(8,length(Velocities));
Damping = zeros(8,length(Velocities));
for i = 1:length(Velocities)
    v = Velocities(i);
    Temp = double(subs(A,V,v));
    [wn,zeta] = damp(Temp);
    Frequencies(:,i) = wn;
    Damping(:,i) = zeta;
end

% Plot everything for w/Aerodynamic Damping
figure
hold on
plot(Velocities,Frequencies(1,:))
plot(Velocities,Frequencies(2,:))
plot(Velocities,Frequencies(3,:))
plot(Velocities,Frequencies(4,:))
plot(Velocities,Frequencies(5,:))
plot(Velocities,Frequencies(6,:))
plot(Velocities,Frequencies(7,:))
plot(Velocities,Frequencies(8,:))
grid('on')
title('Frequency vs Flow Speed (w/Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Frequency')
hold off

figure
hold on
plot(Velocities,100.*Damping(1,:))
plot(Velocities,100.*Damping(2,:))
plot(Velocities,100.*Damping(3,:))
plot(Velocities,100.*Damping(4,:))
plot(Velocities,100.*Damping(5,:))
plot(Velocities,100.*Damping(6,:))
plot(Velocities,100.*Damping(7,:))
plot(Velocities,100.*Damping(8,:))
title('Damping vs Flow Speed (w/Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Damping (%)')
grid('on')
hold off

% Reduce order of equations
A = [zeros(4,4) eye(4);
     -inv(M)*K  zeros(4,4)];

% Plug in velocities
Velocities = 0:1:180;
Frequencies = zeros(8,length(Velocities));
Damping = zeros(8,length(Velocities));
for i = 1:length(Velocities)
    v = Velocities(i);
    Temp = double(subs(A,V,v));
    [wn,zeta] = damp(Temp);
    Frequencies(:,i) = wn;
```

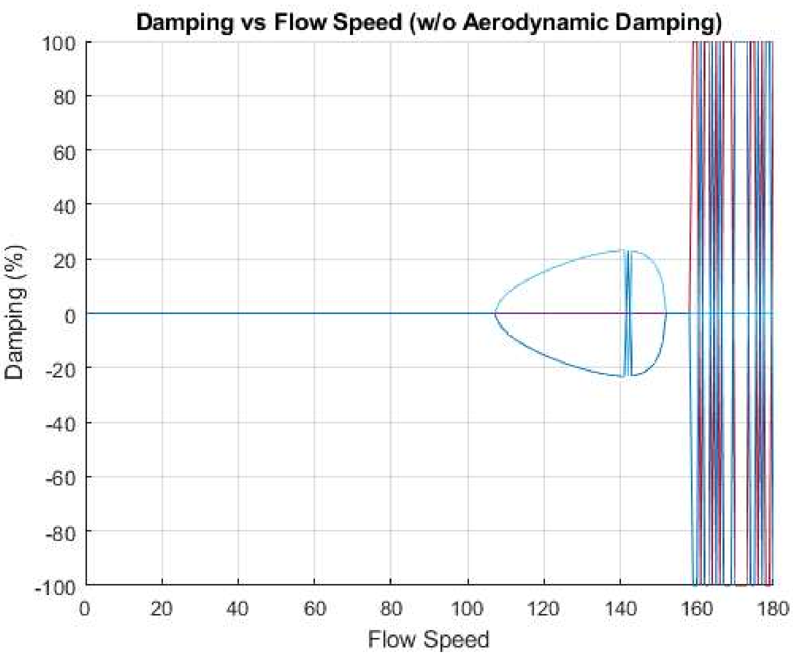
```
    Damping(:,i) = zeta;
end

% Plot everything for w/o Aerodynamic Damping
figure
hold on
plot(Velocities,Frequencies(1,:))
plot(Velocities,Frequencies(2,:))
plot(Velocities,Frequencies(3,:))
plot(Velocities,Frequencies(4,:))
plot(Velocities,Frequencies(5,:))
plot(Velocities,Frequencies(6,:))
plot(Velocities,Frequencies(7,:))
plot(Velocities,Frequencies(8,:))
title('Frequency vs Flow Speed (w/o Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Frequency')
grid('on')
hold off

figure
hold on
plot(Velocities,100.*Damping(1,:))
plot(Velocities,100.*Damping(2,:))
plot(Velocities,100.*Damping(3,:))
plot(Velocities,100.*Damping(4,:))
plot(Velocities,100.*Damping(5,:))
plot(Velocities,100.*Damping(6,:))
plot(Velocities,100.*Damping(7,:))
plot(Velocities,100.*Damping(8,:))
title('Damping vs Flow Speed (w/o Aerodynamic Damping)')
xlabel('Flow Speed')
ylabel('Damping (%)')
grid('on')
hold off

% Solve for V_D
V_D = double(solve(det(K)==0,V))
```

```
V_D =

  158.0025
  568.4792
 -158.0025
 -568.4792
```


Frequency vs Flow Speed (w/Aerodynamic Damping)
Frequency
Flow Speed
0
20
40
60
80
100
120
140
160
180


Damping vs Flow Speed (w/Aerodynamic Damping)
Damping (%)
Flow Speed
100
80
60
40
20
0
-20
-40
-60
-80
-100





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