ME597/AAE556 – Fall 2022
Purdue University
West Lafayette, IN

Final Exam

Assignment date: Monday, December 12
Due date: Friday, December 16

Please submit your completed homework assignment by

- Scanning and uploading assignment to ***Gradescope***.

Instructions:

- Please upload your HW with following this name code, where ** is the assignment number (e.g., HW**=HW1): HW**ME597/AAE556_Fall22_NameInitial_Lastname
- This assignment is strictly individual.
- Your work will be evaluated considering the shown procedure to obtain final answers.
- The procedure and results should be **clear** and **ordered**.
- Consider asking questions about the assignment in advance to avoid inconveniences caused by unexpected events close to the submission date.
- **Please ensure your work is submitted via *Gradescope***.

Name____

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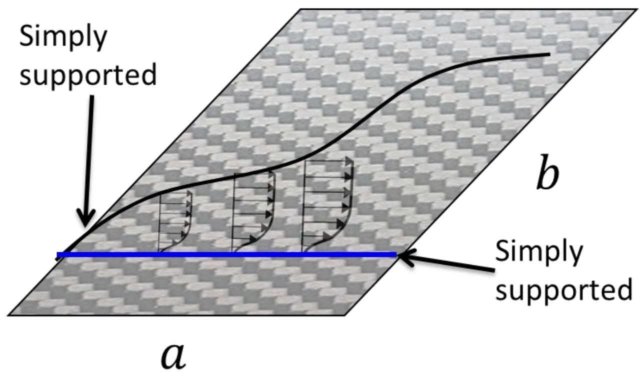
Problem No. 1 – 50 points

Consider the problem of an infinitely long panel $b \gg a$ subjected to supersonic flow. The differential equation describing this problem is:

$$\rho\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} + N_x\frac{\partial^2 w}{\partial x^2} + \frac{2q}{\sqrt{M^2-1}}\frac{\partial w}{\partial x} = 0 \qquad (1)$$



The last term represents the forcing from the aerodynamics, where q is the dynamics pressure and M is the Mach number. Consider the transverse displacement synchronous solution as:

$$w(x,t) = \phi(x)e^{i\omega t}$$

where $\phi(x)$ Is the spatial solution. This PDE has real coefficients which means that flutter is marked by the merging of the natural frequencies.

<u>Find:</u>

a) Substitute the synchronous solution $w(x,t) = \phi(x)e^{i\omega t}$ into equation (1).
b) Approximate the solution for Eq. (1) using the Galerkin method and a two-mode expansion with the eigenfunctions of the simply supported beam problem: $\phi(x) = \sum_{n=1}^{N} c_n \sin(\frac{\pi n x}{a})$ $, where\ n =$ $1,2$. This implies minimizing the error. See sections of Weisshaar and Ch 17 of Rao's Vibrations of Continuous Systems (added in the Brightspace).
c) Determine the flutter speed for this problem with a two-mode expansion, where $n = 1,2$.
d) Determine the flutter speed using a three-mode expansion, where $n = 1,2,3$, and compare the results

<u>All results require derivations and accompanying supporting code for grading.</u>

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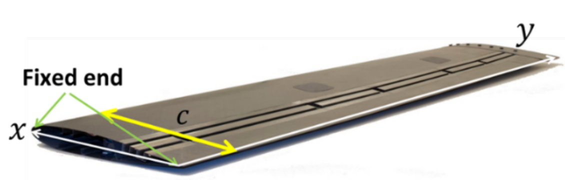
Problem No. 2 – 50 points

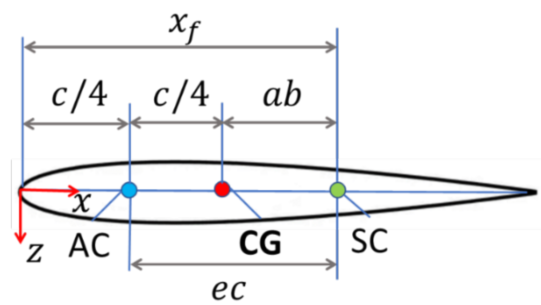
A fixed root wing has semi-span s, chord c, bending stiffness EI, and torsional rigidity GJ. The lift is calculated using strip theory with:

$$dL = \frac{1}{2}\rho V^2 c a_w \left(\theta(y,t) + \frac{\dot{h}(y,t)}{V}\right) dy$$

$$dM = \frac{1}{2}\rho V^2 c^2 \left(e a_w \left(\theta(y,t) + \frac{\dot{h}(y,t)}{V}\right) + M_{\dot{\theta}} \frac{\dot{\theta}(y,t)c}{4V}\right) dy$$

The wing's displacement is $z(x,y,t) = h(y,t) + x\theta(y,t) + p(t)$, where $h(y,t)$ and $\theta(y,t)$ are the continuous plunge and torsional displacements, and $p(t)$ is the rigid body plunge motion. Assume the wing's mass to be homogeneously distributed. The location of the aerodynamic, shear, and mass centers are at the shown locations below. The reference axis is located the leading edge.





The wing's properties are:

Semi-span (s)	7.5 m		Bending Stiffness: EI		$2\text{x}10^7$ Nm^2
Chord (c)	2 m		Bending Rigidity: GJ		$2\text{x}10^6$ Nm^2
Elastic axis (x_f)	0.48c		Lift curve slope, a_w		2π
Mass axis	0.5c		Air density ρ		1.225 kg/m^3
Mass per unit area, ρ	200 kg/m^2		M mass of fuselage		

<u>**Determine:**</u>

(a) Using a three-mode expansion for the displacement field $z(x,y,t) = \left(\frac{y}{s}\right)^2 h_1(t) + \left(x - x_f\right)\frac{y}{s}\theta_1(t) + p(t)$ obtain:

1. Discretized equations of motion in terms of the modal coordinates: $[h_1(t), \theta_1(t), p(t)]$.
2. Frequency vs. flow speed and Damping vs. flow speed graphs for:
    i. $M = \rho s c$
    ii. $M = 10\,\rho s c$
    iii. Determine the flutter speed V_F for both $M = \rho s c$ and $M = 10\,\rho s c$

(b) Obtain the same results as in point (a) using the five-mode expansion the displacement field for the displacement fields:
$z(x,y,t) = \left(\frac{y}{s}\right)^2 h_1(t) + \left(\frac{y}{s}\right)^3 h_2(t) + \left(x - x_f\right)\frac{y}{s}\theta_1(t) + \left(x - x_f\right)\left(\frac{y}{s}\right)^2 \theta_2(t) + p(t)$.

(c) Discuss the difference between the three- and five-mode expansions and the fuselage mass increase. On the flutter speed.

Attach the code you used to obtain them as appendix to your HW.
<u>All results require derivations and accompanying supporting code for grading.</u>