ME597/AAE556 – Fall 2022
Purdue University
West Lafayette, IN

Homework Set No. 1

Assignment date: Tuesday, August 30
Due date: Tuesday, September 13 at 11:59 PM (Eastern Time)

Please submit your completed homework assignment by

- Scanning and uploading assignment to ***Gradescope***.

Instructions:

- Please upload your HW with following this name code, where ** is the assignment number (e.g., HW**=HW1): HW**ME597/AAE556_Fall22_NameInitial_Lastname
- This assignment is strictly individual.
- Your work will be evaluated considering the **shown procedure to obtain final answers**.
- The procedure and results should be **clear** and **ordered**.
- Consider asking questions about the assignment in advance to avoid inconveniences caused by unexpected events close to the submission date.
- **Please ensure your work is submitted via *Gradescope***.

Name____

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Homework Set No. 1
Problem No. 1 – 50 points

Consider a 1D uncambered wing section with wing area S. A concentrated load P is applied aft of the shear center (SC) to create an applied moment $P\,d$. The wing has no initial incidence angle, $\theta_0 = 0$. The twist angle, θ (measured in radians), is resisted by a nonlinear torsional spring producing a restoring moment $M_E = K_1\theta + K_2\theta^3$, where K_1 and K_2 are measured in Nm/rad. The produced aerodynamic lift is calculated using $L = q\,S\,a_1\theta$ and is located at the aerodynamic center at a distance e from the SC.

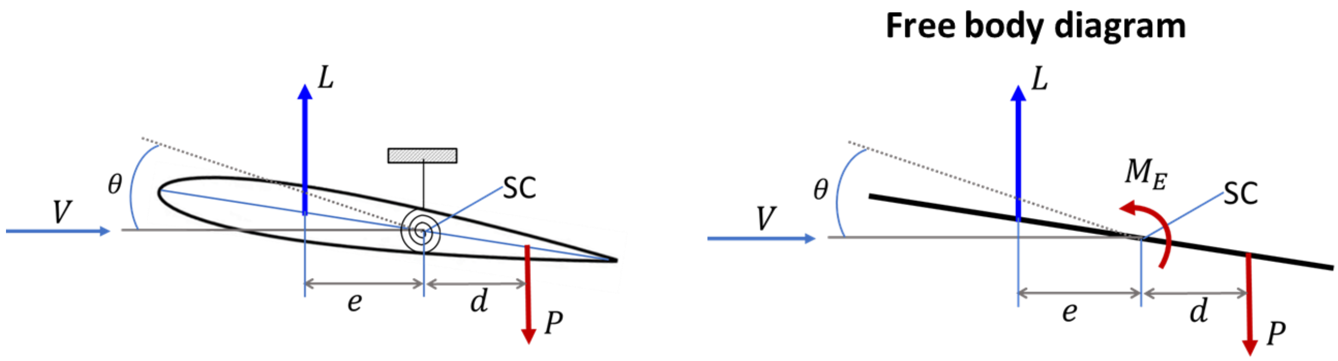

Figure 1: Model schematic and free body diagram

(a) Derive the torsional equilibrium equation with all terms in θ and its powers on one side and the static loading term $P\,d$ on the other.

(b) Derive a nondimensional form of this equation normalizing by K_1. Write the coefficients of θ in terms of the nondimensional quantity $\hat{q} = q\,S\,e\,a_1/K_1$

(c) Using the nondimensional equation from (b) setting $K_2 = \frac{8}{3} q\,S\,e\,a_1$, obtain:
   1. The effective torsional stiffness K_e by finding: $\frac{\partial \hat{P}}{\partial \theta}$, where $\hat{P} = \frac{Pd}{K_1}$
   2. Plot K_e as a function of θ for $\hat{q}$ taking values of $\hat{q} = [0, 0.5, 1, 1.5]$

(d) Plot the nondimensionalized applied torque $\hat{P}$ for the range $\theta \in (-0.5, 0.5)$ for $\hat{q} = [0, 0.5, 1, 1.5]$ and discuss the effect of increasing $\hat{q}$ on the stability of the system by identifying for which values we have:
   1. Only one twist angle solution.
   2. The limit of linear divergence, i.e. for a system with $K_2 = 0$.
   3. The system shows multiple stable solutions.

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Problem No. 2 – 50 points

An idealized aircraft is shown in the Figure 2 flying at an angle of attack α_0 with its two, uncambered, typical section "wings" twisted an amount θ. The wing lift for each of the two segments is $L = q\ S\ a_1\theta$ where θ is the wing elastic twist and S is the wing area of each wing, $S = c * 1$. During a banked, constant altitude turn this airplane develops a load factor of n, defined as

$$n = \frac{Total\ lift}{Total\ weight}$$

The total aircraft weight is $W\ =\ 2w\ + W_f$ where w is the weight of each wing and W_f is the weight of everything else. The total lift is (ignoring the tail) $L_{total}\ =\ 2L_w\ +\ L_T = 2qSa_1(\alpha_0 + \theta)$. The wing center of gravity is located a distance d from the wing shear center.

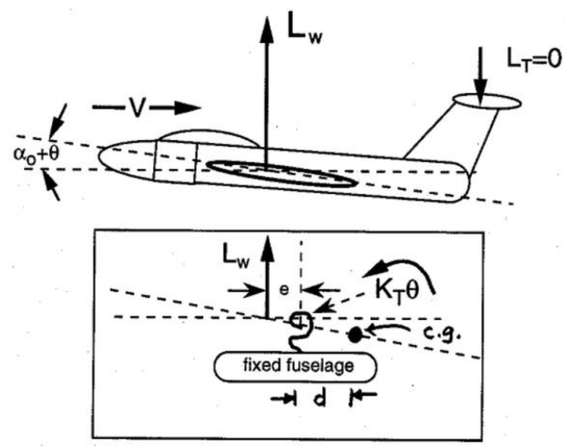

Figure 2: Aircraft and wing free body diagram

(a) Solve for the aircraft angle of attack α_0 as a function of n. Plot α_0 vs. n.
(b) Solve for the wing twist θ as a function of n.
(c) Plot θ vs. n.
(d) Solve for the divergence dynamic pressure q_D as a function of n.