ECE 680 Purdue University

FunWork #2

- You can prepare your assignment using LaTeX (preferable) or MS Word.
- You can submit html or pdf file of your MATLAB m-file prepared using the cell mode. Use the publish button in the toolbar to obtain an html file, or go to the workspace and type

  ```
  publish('your m-file name without extension','pdf')
  ```

  to obtain a pdf file.
- Produce a video of your animation, using, for example, the `avi` format, post it on the YouTube, and link the posted video to your assignment document so that the grader can easily access your animation.
- Submit either html or pdf file of your assignment.

The objective of this assignment is to use your linear system methods and apply them to constructing controllers for a nonlinear system.

Start with the non-linear model of the double inverted pendulum on a cart (DIPC) from FunWork #1.

1. (**10 pts**) Solve the Lyapunov matrix equation for the linearized open-loop system. Take $\boldsymbol{Q} = \boldsymbol{I}_6$. Is the open-loop system asymptotically stable, that is, is the equilibrium state of interest asymptotically stable in the sense of Lyapunov? Explain.
2. (**10 pts**) Design a linear state-feedback controller using the linearized model.

3. (**10 pts**) Find the transfer function of the closed-loop system comprised of the linearized model driven by the linear state-feedback controller.

4. (**10 pts**) Construct a Lyapunov function for the closed-loop system comprised of the linearized model driven by the state-feedback controller, solve the Lyapunov matrix equation, and check if the equilibrium state of interest of the closed-loop system is asymptotically stable in the sense of Lyapunov.

5. (**10 pts**) Add an extra input in the double inverted pendulum on a cart (DIPC) nonlinear model from the previous FunWork, namely, the torque at the first joint. Thus the system's overall input is

$$\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^\top,$$

   where u_1 is the force applied to the cart and u_2 is the torque applied at the first joint. In summary, we have now a three-output two-input system.

   Design a stabilizing state-feedback controller using the linearized model. Write a MATLAB script that animates the behavior of the closed-loop system comprised of the linear state-feedback controller driving the DIPC nonlinear model for different initial conditions.

6. (**10 pts**) Construct a Lyapunov function for the closed-loop system comprised of the linearized model driven by the combined controller-observer compensator, solve the Lyapunov matrix equation, and check if the equilibrium state of interest of the closed-loop system is asymptotically stable in the sense of Lyapunov.

7. (**10 pts**) Use the linearized model to design a Luenberger observer.

8. (**10 pts**) Find the transfer function of the closed-loop system comprised of the linearized model driven by the combined controller-observer compensator.

9. (**20 pts**) Implement and perform animation of the closed-loop system comprised of the combined controller-observer compensator driving the DIPC nonlinear model.