FunWork #3

The objective of this assignment is to apply Linear Matrix Inequalities (LMIs) to the design of linear controllers stabilizing a nonlinear system about a **nonzero** equilibrium state. Your controllers are to be tested on the nonlinear model.

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1. (**10 pts**) Start with the non-linear model of the double inverted pendulum on a cart (DIPC) from FunWork #1. Show that there does not exist u_e that would make

$$\boldsymbol{x}_e = \begin{bmatrix} 0.1 & 60^\circ & 45^\circ & 0 & 0 & 0 \end{bmatrix}^\top$$

an equilibrium state.

2. (**10 pts**) Add an extra input in the non-linear DIPC model, namely, the torque at the first joint. Thus the system's input is

$$\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^\top,$$

where u_1 is the force applied to the cart and u_2 is the torque applied at the first joint. In summary, we now have a three-output two-input system. Show that there is no $\boldsymbol{u}_e = \begin{bmatrix} u_{1e} & u_{2e} \end{bmatrix}^\top$ that would make $\boldsymbol{x}_e = \begin{bmatrix} 0.1 & 60^\circ & 45^\circ & 0 & 0 & 0 \end{bmatrix}^\top$ an equilibrium state.

3. (**10 pts**) Add the third extra input in the non-linear DIPC model, namely, the torque at the second joint. Thus the system's input is

$$\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^\top,$$

where u_1 is the force applied to the cart, u_2 is the torque applied at the first joint, and u_3 is the torque applied at the second joint. In summary, we have now a three-output three-input system. Find $\boldsymbol{u}_e = \begin{bmatrix} u_{1e} & u_{2e} & u_{3e} \end{bmatrix}^\top$ that makes

$$\boldsymbol{x}_e = \begin{bmatrix} 0.1 & 60^\circ & 45^\circ & 0 & 0 & 0 \end{bmatrix}^\top$$

an equilibrium state.

4. (**10 pts**) Perform linearization about $(\boldsymbol{x}_e, \boldsymbol{u}_e)$.

5. (**10 pts**) Design a state-feedback controller, $\boldsymbol{u} = -\boldsymbol{K}_x\boldsymbol{x}$, using LMIs and test it on the non-linear model. Generate plots of of the state variables versus time on the time interval $[0, 3]$ secs. When performing your simulations you have to use one of MATLAB's `ode` functions, for example, `ode23` or `ode45`. Compare their performance and see if you can notice any differences in your plots.

6. (**10 pts**) Implement and perform animation of the closed-loop system comprised of the state-feedback controller driving the nonlinear model of the DIPC;

7. (**10 pts**) Use LMIs to design an output-feedback controller, $\boldsymbol{u} = -\boldsymbol{K}_o\boldsymbol{x}$. Generate plots of state variables versus time on the time interval $[0, 3]$ secs. This can be one figure with subplots.

8. (**10 pts**) Simulate the performance of the combined state-feedback controller-observer compensator and compare its performance against the output feedback controller. Generate plots of state variables versus time, where $\tilde{x}_i$ is an estimate of x_i. Note that you can always set your observer's initial conditions to be zero, if you wish, because you have complete access to your design.

9. (**20 pts**) Implement and perform animation of the closed-loop system comprised of the combined optimal controller-observer compensator driving the nonlinear model of the DIPC. You may have to re-design your observer from the previous assignment.