ECE 680 Purdue University

FunWork #5

The objective of this assignment is to analyze necessary and sufficient conditions for the existence of unknown input observers for linear plants with unknown inputs and when there is a nonzero feed-through (or feed-forward) matrix and test an unknown input observer on the double inverted pendulum on a cart (DIPC).

Consider the following plant model,

$$\left.\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{u}_1 + \boldsymbol{B}_2\boldsymbol{u}_2 \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}_1\boldsymbol{u}_1 + \boldsymbol{D}_2\boldsymbol{u}_3, \end{aligned}\right\}$$

where $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is the state matrix, $\boldsymbol{B}_1 \in \mathbb{R}^{n\times m_1}$ is the control input matrix, $\boldsymbol{B}_2 \in \mathbb{R}^{n\times m_2}$ is the unknon input matrix, $\boldsymbol{C} \in \mathbb{R}^{p\times n}$ is the output matrix, $\boldsymbol{D}_1 \in \mathbb{R}^{p\times m_1}$ is the feed-through matrix, and $\boldsymbol{D}_2 \in \mathbb{R}^{p\times m_3}$ is the feed-through unknown input matrix.

1. (**25 pts**) Derive necessary and sufficient conditions for the existence of unknown input observer (UIO) for the above plant model.

   Express these conditions in terms of linear matrix inequalities.

2. (**25 pts**) Consider the non-linear continuous-time (CT) model of the double inverted pendulum on a cart (DIPC) from previous FunWork assignments. Use the linearized model and assume that the actuators and sensors are subjected to the adversarial attacks modeled by the unknown input. Construct a UIO for the such a system and simulate its behvaior.

3. (**25 pts**) Design a discrete-time (DT) UIO for the a discrete-time (DT) model. You as a designer select the sampling period. Design a DT state-feedback controller, $u[k] = -\boldsymbol{K}_x\boldsymbol{x}[k]$ using LMIs and test it on the non-linear CT model. Your input to the CT model is to be a piece-wise constant signal that you would receive using a zero-order hold (ZOH) element.

4. (**25 pts**) Animate the performance of the combined DT controller-UIO compensator driving the CT non-linear model. Your inputs to the CT model are to be piece-wise constant signals that you would receive using a zero-order hold (ZOH) element.