SR-71 Control Design

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Overview

- Problem Statement
- Open-Loop Characteristics
- Design Requirements
- Methodology
  - SDR
  - PI – SDR
  - NZSP
  - PI – NZSP
  - PIF – NZSP – CRW
- Closing Comments

Project Statement

Problem Statement

- The model based on Kelly Johnson's legendary Blackbird- the SR-71
- Determine natural flight characteristics, and then produce desired characteristics through direct digital design

Problem Statement

- MIMO System
- 5 States and 2 Controls
- Latitude dynamics chosen by recommendation of Dr V
  - Easier and more interesting to work with

Lateral Model SR-71

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\varphi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -0.0346 & 0.114 & -0.993 & 0.0101 & 0.0000174 \\ -1.393 & -0.261 & 0.0221 & 0.0000426 & -0.0000156 \\ 0.804 & 0.00225 & -0.0183 & -0.0000245 & 0 \\ -0.000149 & 1 & 0.113 & 0.0000171 & 0.0000503 \\ 0.0000399 & 0 & 1 & 0 & 0.0000625 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \varphi \\ \psi \end{bmatrix} + \begin{bmatrix} 0 & 0.00639 \\ 2.705 & 0.348 \\ -0.0446 & -0.938 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

$M_1 = 3.2$

$H_1 = 76,500$ feet

$\alpha_1 = 6.56^\circ$

$\bar{q} = 495.59$ psf

$\delta_e = -1.81^\circ$

$\delta_T = 71^\circ$ throttle lever angle

$\lambda_{1,2} = -0.0417 \pm 0.975j$

$\omega_{DR} = 0.976$ rad/sec

$\zeta_{DR} = 0.043$

$\lambda_3 = -0.2311$

$\tau_r = 4.327$ sec

$\lambda_4 = 0.0007$

$\tau_s = 1,428$ sec

Original Actuators

Actuators transfer function:

$$\frac{33}{s+33}$$

With time constant:

$$\tau = 0.03$$

Adjusting Actuators

- Adjusted actuators transfer function to:

$$\frac{10}{s + 10}$$

- Dynamics difficult to control with original actuators
- Changed design to use actuators more commonly used
- Provides better filtering on frequencies

Adding Actuators System

A_new =

-0.0346	0.1140	-0.9930	0.0101	0.0000	0	0.0064
-1.3930	-0.2610	0.0221	0.0000	0.0000	2.7050	0.3480
0.8040	0.0022	-0.0183	-0.0000	0	-0.0446	-0.9380
-0.0001	1.0000	0.1130	0.0000	0.0001	0	0
0.0000	0	1.0000	0	0.0001	0	0
0	0	0	0	0	-10.0000	0
0	0	0	0	0	0	-10.0000

B_new =

0	0
0	0
0	0
0	0
0	0
10	0
0	10

Adding Actuators System

C_new =

1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	-10	0
0	0	0	0	0	0	-10

D_new =

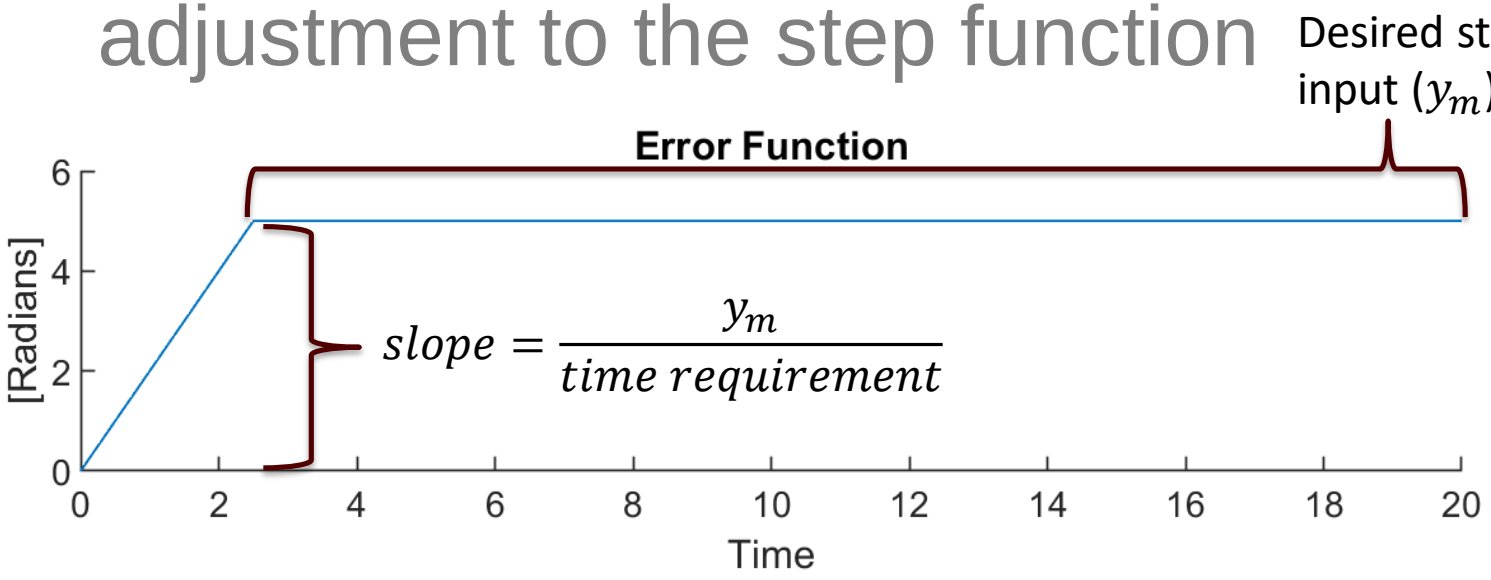
0	0
0	0
0	0
0	0
0	0
0	0
0	0
1	0
0	1
10	0
0	10

Step Input for NZSP

- Input was the constant being tracked
  - Track a 5° change in 4 seconds
  - Originally 10°, but SR-71 did not respond well to the change.

Step Input for NZSP

- Building step function for error
- Ramp exits in beginning to show natural adjustment to the step function



Initial Sinusoidal Input

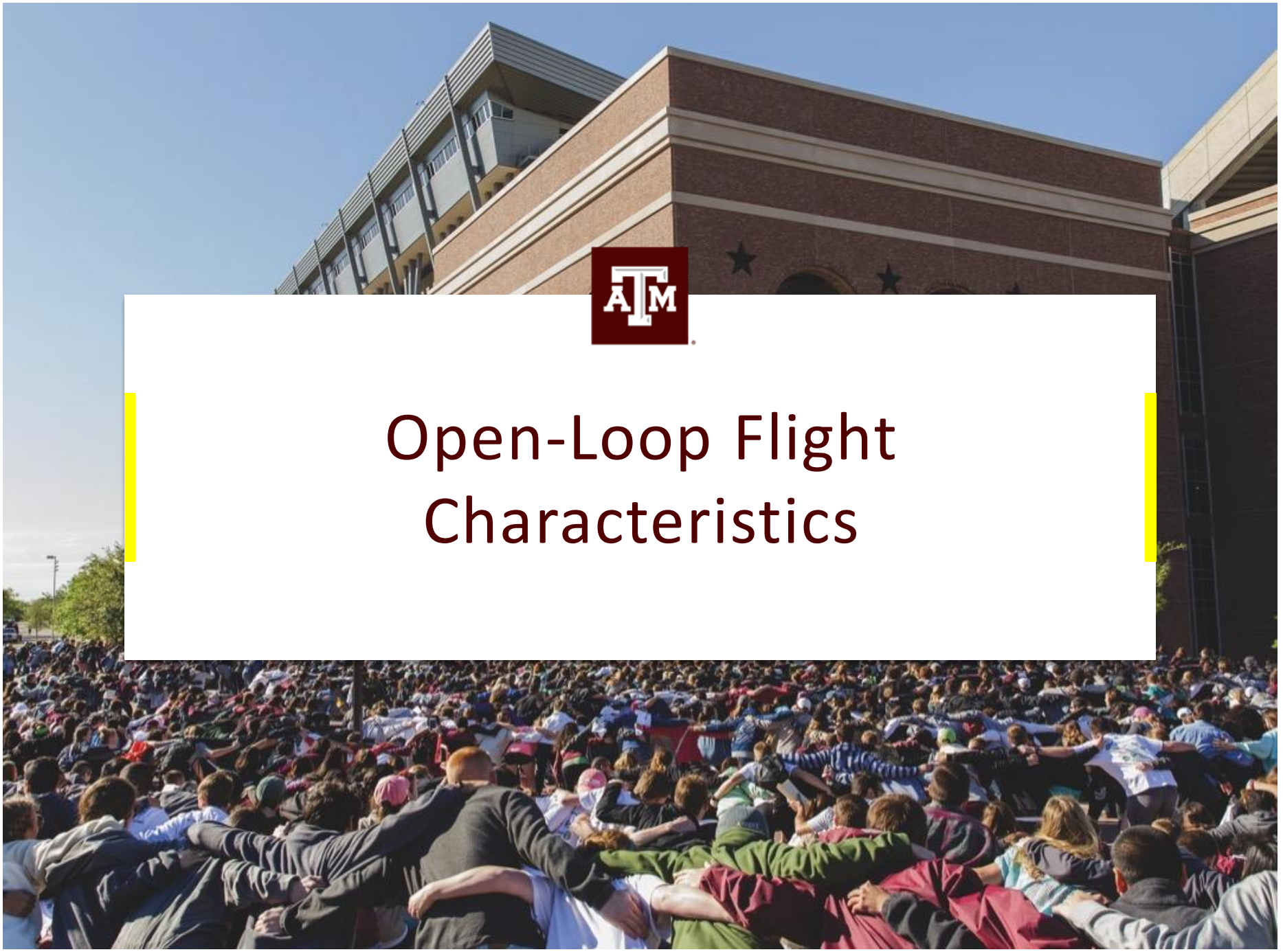
- Amplitude: $A = 0.786$
- Time: 2.83 sec
  - $\omega = \frac{2\pi}{T}$
- $y(t) = A\sin(\omega t)$

Adjusted Sinusoidal Input

- Amplitude: $A = 0.0873$
- Time: 16 sec
  - $\omega = \frac{2\pi}{T}$
- $y(t) = A\sin(\omega t)$
- Original design was too high of a frequency to be tracked without large error
  - PIF-NZSP-CRW did track $T = 4s$ well

Notes

- Weights in the matrices on the proceeding slides may show entries of 0
  - Values are not really 0
  - MATLAB outputs 0 for negligible values
- For NZSP controllers, only focused on the ψ state, control, and rates
- Constant disturbance applied to PI-SDR, PI-NZSP, and PIF-NZSP-CRW



Open-Loop Flight Characteristics

Open Loop Characteristics

- Eigenvalues (w/Actuators)

```
D =

  Columns 1 through 4

 -0.0417 + 0.9750i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i  -0.0417 - 0.9750i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i  -0.2311 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0015 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 7

  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
 -0.0007 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i -10.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i -10.0000 + 0.0000i
```

One eigenvalue is unstable- not a stable system.

Open Loop Characteristics

- Eigenvectors (w/Actuators)

```
V =

  Columns 1 through 4

  -0.0938 - 0.3932i  -0.0938 + 0.3932i   0.0046 + 0.0000i   0.0000 + 0.0000i
   0.5636 + 0.0000i   0.5636 + 0.0000i  -0.2231 + 0.0000i  -0.0000 + 0.0000i
  -0.3222 + 0.0838i  -0.3222 - 0.0838i  -0.0148 + 0.0000i   0.0014 + 0.0000i
  -0.0133 - 0.5401i  -0.0133 + 0.5401i   0.9726 + 0.0000i   0.1376 + 0.0000i
   0.0999 + 0.3261i   0.0999 - 0.3261i   0.0641 + 0.0000i   0.9905 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 7

  -0.0000 + 0.0000i   0.0034 + 0.0000i   0.0090 + 0.0000i
   0.0001 + 0.0000i  -0.2671 + 0.0000i  -0.0345 + 0.0000i
  -0.0007 + 0.0000i   0.0041 + 0.0000i   0.0928 + 0.0000i
  -0.0750 + 0.0000i   0.0267 + 0.0000i   0.0024 + 0.0000i
   0.9972 + 0.0000i  -0.0004 + 0.0000i  -0.0093 + 0.0000i
   0.0000 + 0.0000i   0.9633 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9950 + 0.0000i
```

Open-Loop Characteristics

- Damping Rations, Frequencies, and Rise Time Constants (w/Actuators)

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-6.75e-04	1.00e+00	6.75e-04	1.48e+03
1.49e-03	-1.00e+00	1.49e-03	-6.71e+02
-2.31e-01	1.00e+00	2.31e-01	4.33e+00
-4.17e-02 + 9.75e-01i	4.28e-02	9.76e-01	2.40e+01
-4.17e-02 - 9.75e-01i	4.28e-02	9.76e-01	2.40e+01
-1.00e+01	1.00e+00	1.00e+01	1.00e-01
-1.00e+01	1.00e+00	1.00e+01	1.00e-01

Open-Loop Characteristics

- Modal Analysis (w/Actuators)
  - $\dot{\xi} = A_m \xi + B_m U$

    $Y = C_m \xi$

Am =

-0.0417	0.9750	0.0000	0.0000	-0.0000	-0.0000	-0.0000
-0.9750	-0.0417	0.0000	0.0000	-0.0000	0.0000	0.0000
-0.0000	0.0000	-0.2311	-0.0000	0.0000	0.0000	-0.0000
0.0000	0.0000	-0.0000	0.0015	-0.0000	-0.0000	0.0000
-0.0000	-0.0000	0.0000	-0.0000	-0.0007	0.0000	-0.0000
0	0	0	0	0	-10.0000	0
0	0	0	0	0	0	-10.0000

Bm =

0.8491	2.4542
-0.2252	-0.3046
-10.2983	4.6543
45.6211	-22.1979
-44.6600	21.6968
10.3811	0
0	10.0502

Open-Loop Characteristics

- Modal Analysis (w/Actuators)
  - $\dot{\xi} = A_m\xi + B_mU$

    $Y = C_m\xi$

```
Cm =

   -0.0938   -0.3932    0.0046    0.0000   -0.0000    0.0034    0.0090
    0.5636         0   -0.2231   -0.0000    0.0001   -0.2671   -0.0345
   -0.3222    0.0838   -0.0148    0.0014   -0.0007    0.0041    0.0928
   -0.0133   -0.5401    0.9726    0.1376   -0.0750    0.0267    0.0024
    0.0999    0.3261    0.0641    0.9905    0.9972   -0.0004   -0.0093
         0         0         0         0         0    0.9633         0
         0         0         0         0         0         0    0.9950
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0   -9.6329         0
         0         0         0         0         0         0   -9.9500
```

Open-Loop Characteristics

- Modal Analysis (w/Actuators)

```
Modal_Matrix =

   -0.0938   -0.3932    0.0046    0.0000   -0.0000    0.0034    0.0090
    0.5636         0   -0.2231   -0.0000    0.0001   -0.2671   -0.0345
   -0.3222    0.0838   -0.0148    0.0014   -0.0007    0.0041    0.0928
   -0.0133   -0.5401    0.9726    0.1376   -0.0750    0.0267    0.0024
    0.0999    0.3261    0.0641    0.9905    0.9972   -0.0004   -0.0093
         0         0         0         0         0    0.9633         0
         0         0         0         0         0         0    0.9950

Z =

   -0.5634    0.2636   -2.4799    0.0253    0.0000    0.0849    0.2454
   -2.4256   -0.1049    0.5231   -0.0047   -0.0000   -0.0225   -0.0305
   -1.4227   -3.8226   -6.2751    0.0622    0.0004   -1.0298    0.4654
    0.6444   17.3645   30.1153    4.4168    0.3530    4.5621   -2.2198
    0.3011  -16.9943  -29.4323   -4.3922    0.6522   -4.4660    2.1697
         0         0         0         0         0    1.0381         0
         0         0         0         0         0         0    1.0050
```

Open-Loop Characteristics

- Modal Analysis (w/Actuators)
  - Dominant mode comes from the complex set of eigenvalues
    - Imaginary part is greater in magnitude than the real part
  - The last two modes will have more effective controlling
    - Magnitudes of the last two rows in B_m are larger

Open-Loop Characteristics

- Modal Analysis (w/Actuators)
  - Oscillatory modes do not produce good output
    - First two rows of C_m are smaller
  - State r has the greatest influence on the modes

Selection of Sample Period

- $\omega_s > 2\omega_c$
  - $\omega_s > 1.952 * 10^{-2}\ rad/s$
  - $T < 321.88\ s$
- $T = \frac{1}{3}\ to\ \frac{1}{2}$ rise time
  - $T = 1.442\ to\ 2.164\ s$
- $T = 0.6\ s$

Had unstable dynamics with anything greater

Selection of Sample Period

- Adding a PI controller made dynamics unstable while tracking and maintaining physical requirements.
- Changed sample period to something faster for a few controllers
  - $T = 0.05\ s$
  - PI-NZSP and PIF-NZSP-CRW

Controllability

```
Cont =

   1.0e+07 *

  Columns 1 through 7

         0         0         0    0.0000    0.0000    0.0000   -0.0000
         0         0    0.0000    0.0000   -0.0000   -0.0000    0.0003
         0         0   -0.0000   -0.0000    0.0000    0.0000   -0.0000
         0         0         0         0    0.0000    0.0000   -0.0000
         0         0         0         0   -0.0000   -0.0000    0.0000
    0.0000         0   -0.0000         0    0.0001         0   -0.0010
         0    0.0000         0   -0.0000         0    0.0001         0

  Columns 8 through 14

   -0.0000    0.0000    0.0001   -0.0004   -0.0009    0.0036    0.0090
    0.0000   -0.0028   -0.0003    0.0277    0.0035   -0.2772   -0.0347
   -0.0001    0.0000    0.0009   -0.0004   -0.0093    0.0042    0.0933
   -0.0000    0.0003    0.0000   -0.0028   -0.0002    0.0277    0.0024
    0.0000   -0.0000   -0.0001    0.0000    0.0009   -0.0004   -0.0093
         0    0.0100         0   -0.1000         0    1.0000         0
   -0.0010         0    0.0100         0   -0.1000         0    1.0000
```

```
Control =

    "True"
```

System is unstable but controllable.

Observability

Obs =

1.0e+07 *

0.0000	0	0	0	0	0	0
0	0.0000	0	0	0	0	0
0	0	0.0000	0	0	0	0
0	0	0	0.0000	0	0	0
0	0	0	0	0.0000	0	0
0	0	0	0	0	0.0000	0
0	0	0	0	0	0	0.0000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	-0.0000	0
0	0	0	0	0	0	-0.0000
-0.0000	0.0000	-0.0000	0.0000	0.0000	0	0.0000
-0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0000	0	-0.0000	-0.0000
-0.0000	0.0000	0.0000	0.0000	0.0000	0	0
0.0000	0	0.0000	0	0.0000	0	0
0	0	0	0	0	-0.0000	0
0	0	0	0	0	0	-0.0000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0.0000	0
0	0	0	0	0	0	0.0000
-0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0000	0.0000
0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000
-0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0000	0.0000	-0.0000	-0.0000
0	0	0	0	0	0.0000	0
0	0	0	0	0	0	0.0000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	-0.0001	0
0	0	0	0	0	0	-0.0001
0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.0000	0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000
-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000
0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0	-0.0001	0

...

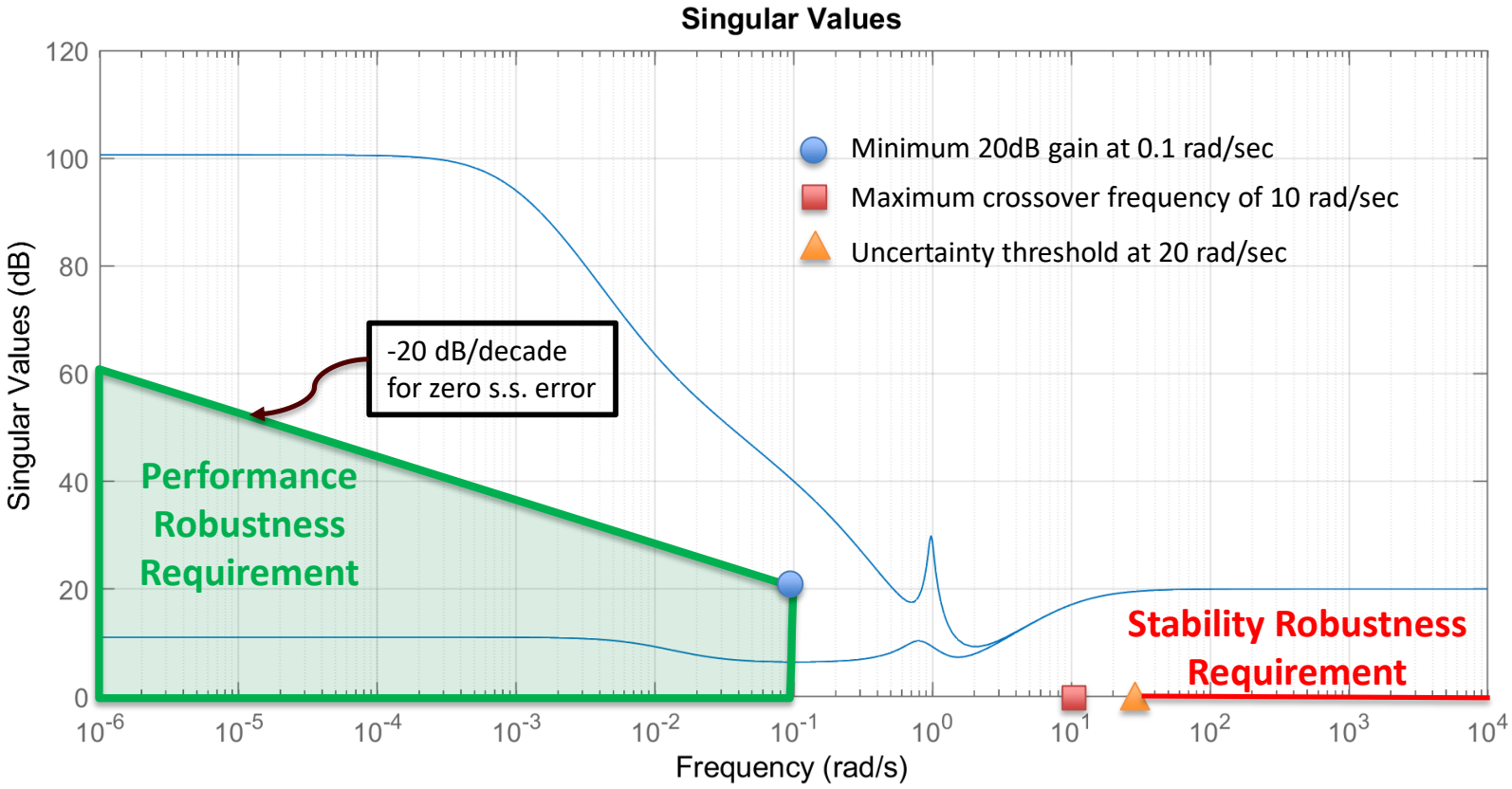
...

0	0	0	0	0	0	-0.0100
-0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0000	-0.0001
-0.0000	-0.0000	0.0000	-0.0000	-0.0000	0.0028	0.0003
0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0000	-0.0009
-0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0003	-0.0000
0.0000	-0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0001
0	0	0	0	0	-0.0100	0
0	0	0	0	0	0	-0.0100
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0.1000	0
0	0	0	0	0	0	0.1000
-0.0000	-0.0000	0.0000	-0.0000	-0.0000	0.0004	0.0009
0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0277	-0.0035
-0.0000	0.0000	-0.0000	0.0000	0.0000	0.0004	0.0093
-0.0000	-0.0000	0.0000	-0.0000	-0.0000	0.0028	0.0002
0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0000	-0.0009
0	0	0	0	0	0.1000	0
0	0	0	0	0	0	0.1000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	-1.0000	0
0	0	0	0	0	0	-1.0000

```
Observable =

   "True"
```

System is unstable, but observable and controllable.

Frequency Domain Analysis



Frequency Domain Analysis

- Low Frequency
  - Top Line
    - Meets minimum singular value
    - Does not meet crossover requirement
    - Slope is greater than 20dB/dec
      - Zero steady-state error not met
  - Bottom Line
    - Does not meet any requirements

4.2.1 Low Frequency

- Minimum singular value large
- Attenuation of low frequency disturbances by a factor of 0.1
- Slope at least –20dB/decade
- Zero steady-state error
- Minimum crossover frequency of 0.1 rad/sec, maximum crossover frequency of 10 rad/sec.

Frequency Domain Analysis

- High Frequency
  - Requirements are not met
    - Never crosses over
    - Singular value flatlines at 20dB

4.2.2 High Frequency

- Maximum singular value small
- Linear model accurate to within 10% of actual plant for frequencies up to 2 rad/sec, where uncertainty grows without bound at 20 dB/decade thereafter:

$$m(\omega) = \frac{s+2}{20}$$

where $m(\omega)$ is the multiplicative modeling discrepancy bound.

Design Requirements

Design Requirements

- Position Limits
  - Ailerons: $\pm 14.0\ deg$
  - Rudder: $\pm 10.0\ deg$
- Rate Limits
  - Ailerons: $\pm 65.0\ deg/s$
  - Rudder: $\pm 30.0\ deg/s$
- Tracking
  - $\varphi = 10°$ in $1.9s$

Hard Requirements – Physically restricted

Soft Requirement

Physical requirements have no flexibility.

Changes in Requirements

- Tracking
  - It was found that the SR-71 does not turn well
    - Changed requirement to make project more manageable
  - $\varphi = 5°$ in $16s$

Sometimes requirements need to be adjusted through the design process.

Idealistic Design

- SDR
  - Meet physical requirements
  - Get best response from states as possible
    - Smooth time histories
    - Quick response
    - Quick settle time
    - Settle to 0

Preferred but not required specifications.

Idealistic Design

- NZSP
  - Minimize overshoot on ψ
  - Meet all physical requirements
  - Minimize error between desired and actual response

Preferred but not required specifications.

Closed-Loop || SDR Design

Characteristics

- Closed Loop A Matrix

```
A_cl =

   -0.0346    0.1140   -0.9930    0.0101    0.0000         0    0.0064
   -1.3930   -0.2610    0.0221    0.0000    0.0000    2.7050    0.3480
    0.8040    0.0022   -0.0183   -0.0000         0   -0.0446   -0.9380
   -0.0001    1.0000    0.1130    0.0000    0.0001         0         0
    0.0000         0    1.0000         0    0.0001         0         0
   -2.1702   -3.9051    2.1494   -2.2463   -0.0277  -11.0670   -0.3096
   -0.0144    0.0728    6.7004    0.3037    0.0087   -0.0151  -10.6307
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

-10.0500 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i -10.0084 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i  -0.4673 + 0.9448i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.4673 - 0.9448i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 7

  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
 -0.5092 + 0.5104i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i  -0.5092 - 0.5104i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0001 + 0.0000i
```

All eigenvalues in system are stable.

Characteristics

- Eigenvectors

```
V_cl =

  Columns 1 through 4

   0.0031 + 0.0000i  -0.0091 + 0.0000i  -0.2060 - 0.0795i  -0.2060 + 0.0795i
  -0.2646 + 0.0000i   0.0431 + 0.0000i   0.6632 + 0.0000i   0.6632 + 0.0000i
   0.0010 + 0.0000i  -0.0928 + 0.0000i  -0.0922 + 0.1562i  -0.0922 - 0.1562i
   0.0263 + 0.0000i  -0.0033 + 0.0000i  -0.2596 - 0.5626i  -0.2596 + 0.5626i
  -0.0001 + 0.0000i   0.0093 + 0.0000i   0.1716 + 0.0127i   0.1716 - 0.0127i
   0.9634 + 0.0000i  -0.0312 + 0.0000i  -0.1489 + 0.1777i  -0.1489 - 0.1777i
  -0.0334 + 0.0000i  -0.9942 + 0.0000i  -0.0547 + 0.0911i  -0.0547 - 0.0911i

  Columns 5 through 7

  -0.0043 - 0.0360i  -0.0043 + 0.0360i   0.0004 + 0.0000i
  -0.4009 + 0.4049i  -0.4009 - 0.4049i  -0.0000 + 0.0000i
  -0.0586 + 0.0317i  -0.0586 - 0.0317i  -0.0001 + 0.0000i
   0.8003 + 0.0000i   0.8003 + 0.0000i  -0.0136 + 0.0000i
   0.0884 + 0.0265i   0.0884 - 0.0265i   0.9999 + 0.0000i
  -0.0393 - 0.1348i  -0.0393 + 0.1348i   0.0002 + 0.0000i
  -0.0162 + 0.0250i  -0.0162 - 0.0250i   0.0003 + 0.0000i
```

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-7.41e-05	1.00e+00	7.41e-05	1.35e+04
-5.09e-01 + 5.10e-01i	7.06e-01	7.21e-01	1.96e+00
-5.09e-01 - 5.10e-01i	7.06e-01	7.21e-01	1.96e+00
-4.67e-01 + 9.45e-01i	4.43e-01	1.05e+00	2.14e+00
-4.67e-01 - 9.45e-01i	4.43e-01	1.05e+00	2.14e+00
-1.00e+01	1.00e+00	1.00e+01	9.99e-02
-1.00e+01	1.00e+00	1.00e+01	9.95e-02

Gains & Weighted Matrices

- Chosen Weighted Matrices

$$Q = \begin{bmatrix} 50.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$R = \begin{bmatrix} 8 & 0 \\ 0 & 65 \end{bmatrix}$$

Gains & Weighted Matrices

Q_hat =

26.4914	0.8502	-7.1272	0.0360	0.0001	0.1882	0.5026
0.8502	0.1031	-0.3303	0.1739	0.0000	0.0226	0.0268
-7.1272	-0.3303	5.9351	-0.0041	-0.0001	-0.0908	-0.4587
0.0360	0.1739	-0.0041	0.6002	0.0000	0.0343	0.0049
0.0001	0.0000	-0.0001	0.0000	0.0000	0.0000	0.0000
0.1882	0.0226	-0.0908	0.0343	0.0000	0.0551	0.0075
0.5026	0.0268	-0.4587	0.0049	0.0000	0.0075	0.0872

M1 =

0.3267	0.8042
0.0444	0.0509
-0.1874	-1.0390
0.0615	0.0094
0.0000	0.0000
0.0601	0.0154
0.0159	0.1392

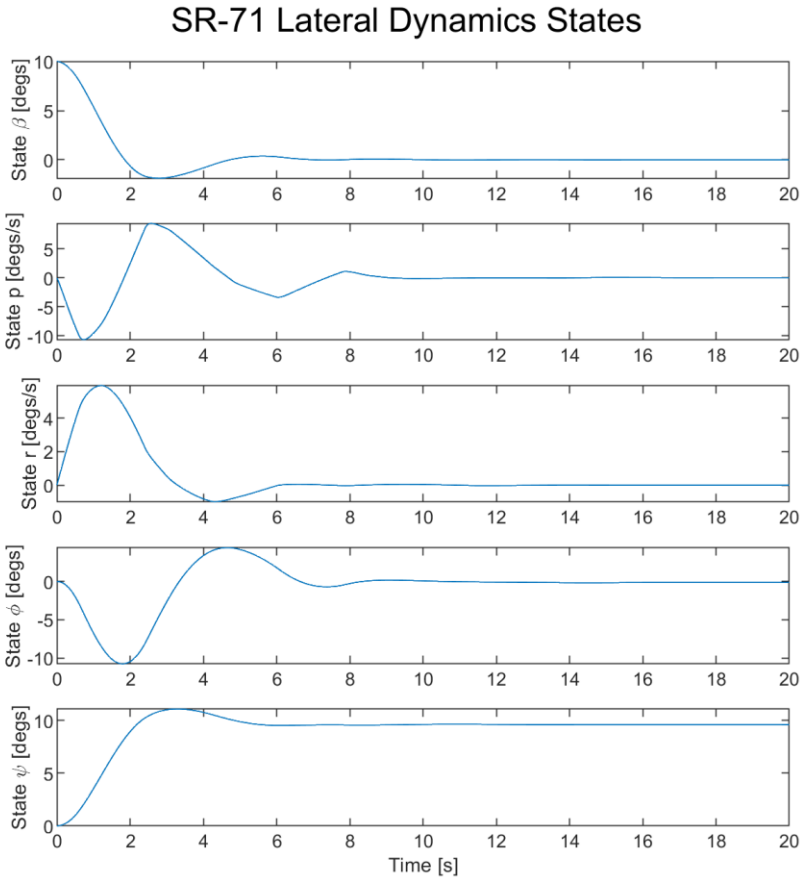
K =

0.2170	0.3905	-0.2149	0.2246	0.0028	0.1067	0.0310
0.0014	-0.0073	-0.6700	-0.0304	-0.0009	0.0015	0.0631

R_hat1 =

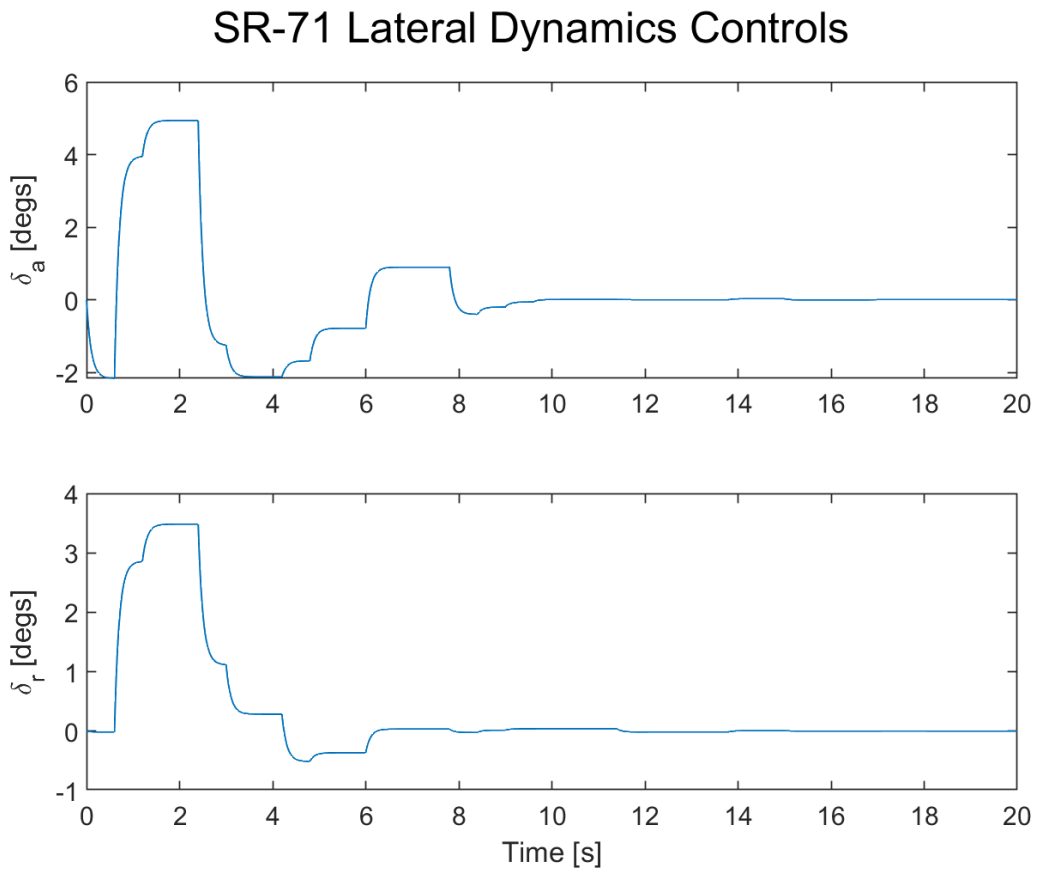
5.2727	0.0369
0.0369	39.7114

Plots - States



- States all settle at a value within $10\ s$
  - Only ψ does not settle at 0

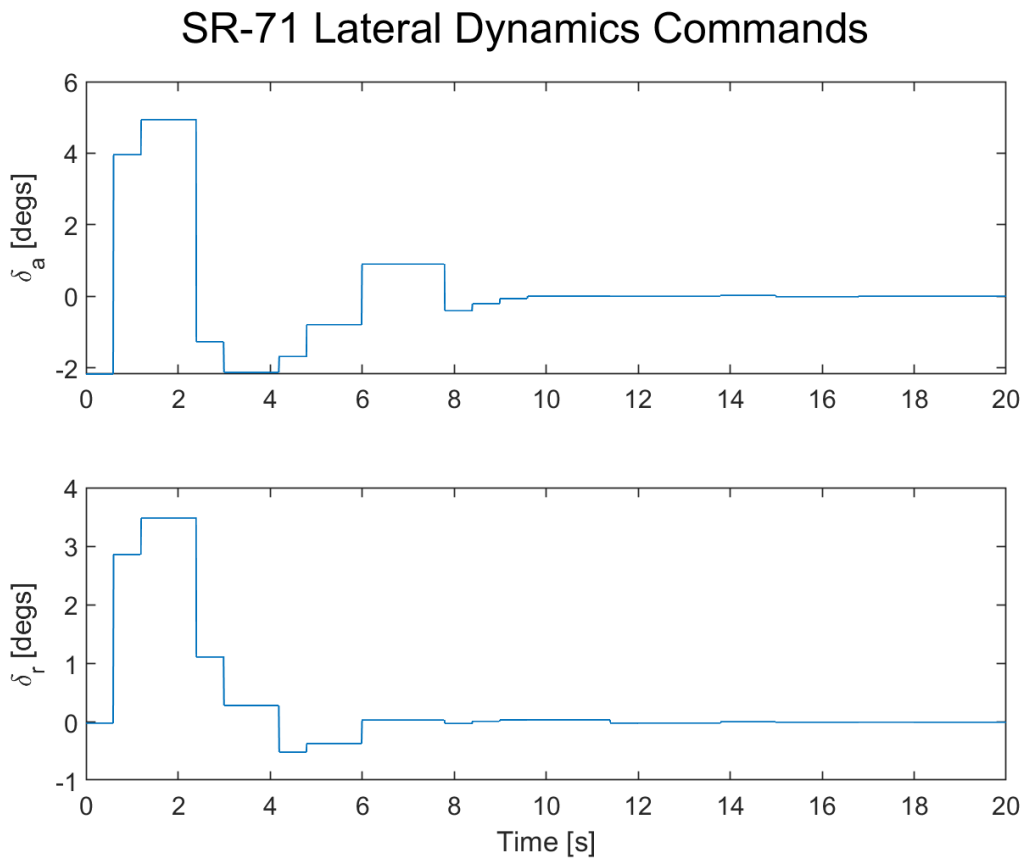
Plots - Controls



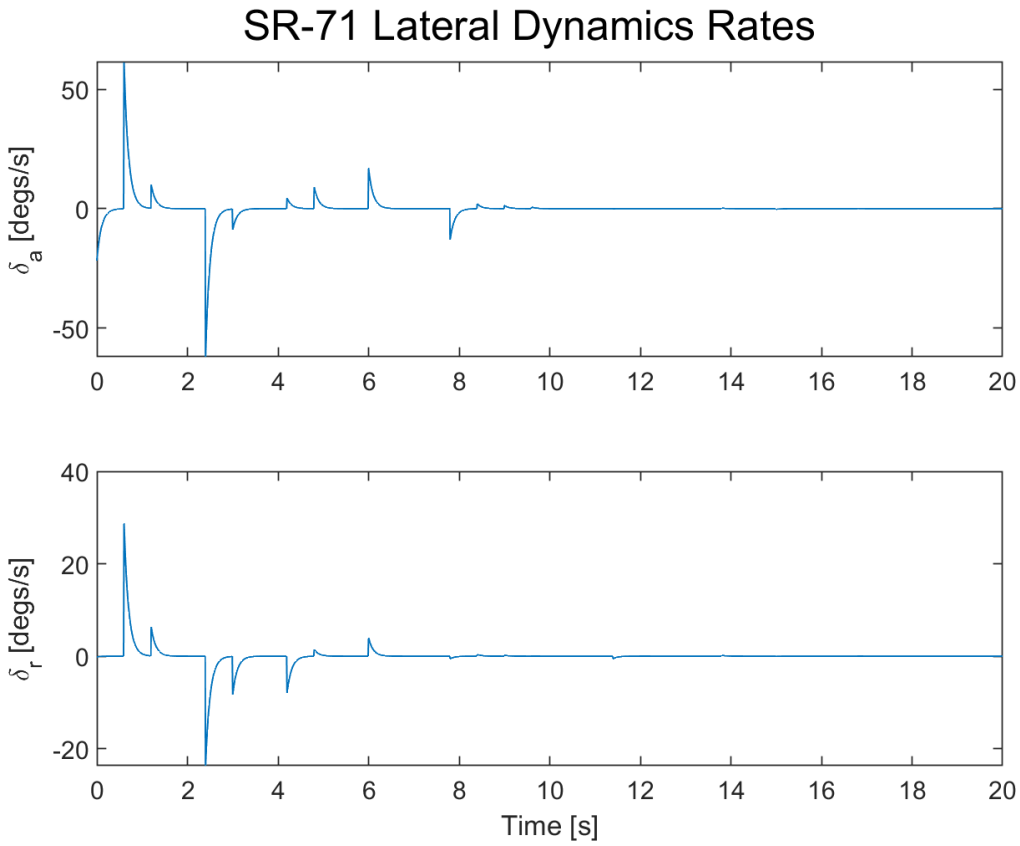
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands



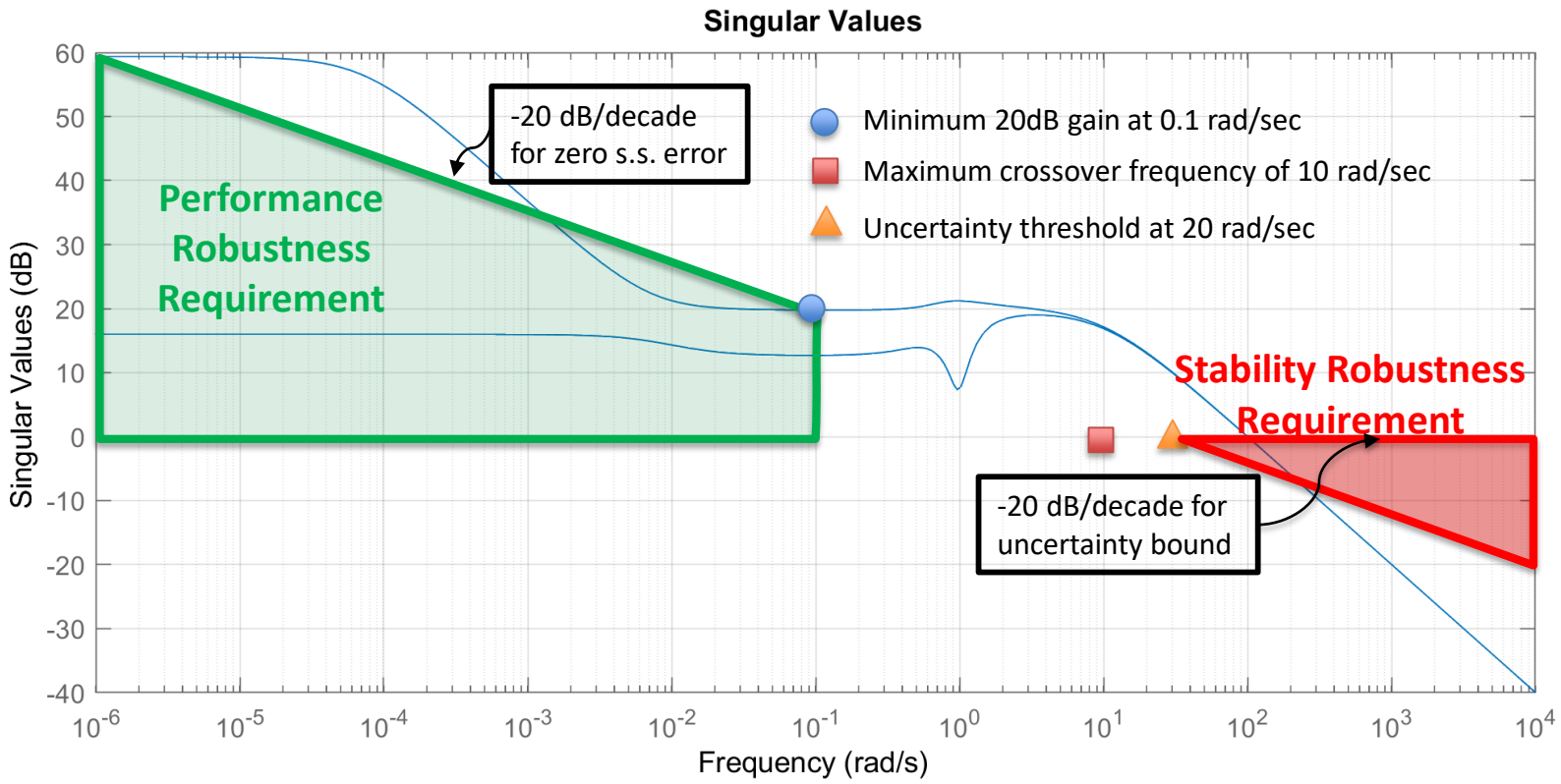
Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$

Frequency Domain Analysis



Frequency Domain Analysis

- Did not meet requirements for high or low frequencies

4.2.1 Low Frequency

- Minimum singular value large
- Attenuation of low frequency disturbances by a factor of 0.1
- Slope at least –20dB/decade
- Zero steady-state error
- Minimum crossover frequency of 0.1 rad/sec, maximum crossover frequency of 10 rad/sec.

4.2.2 High Frequency

- Maximum singular value small
- Linear model accurate to within 10% of actual plant for frequencies up to 2 rad/sec, where uncertainty grows without bound at 20 dB/decade thereafter:

$$m(\omega) = \frac{s+2}{20}$$

where $m(\omega)$ is the multiplicative modeling discrepancy bound.

Closed-Loop || PI-SDR Design

Set-Up

- New State and Control Matrices

```
A_new =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0139    0.9974    0.0003   -0.0000    0.0000    0.0257    0.0033
    0.0080    0.0000    0.9998    0.0000    0.0000   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000    0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
         0         0         0         0         0    0.9048         0
         0         0         0         0         0         0    0.9048
         0         0    0.0100         0         0         0         0

  Column 8

         0
         0
         0
         0
         0
         0
         0
    1.0000
```

```
B_new =

    0.0000    0.0000
    0.0013    0.0002
   -0.0000   -0.0005
    0.0000    0.0000
   -0.0000   -0.0000
    0.0952         0
         0    0.0952
         0         0
```

Set-Up

- Observable: $rank(O) = 8$
- Controllable: $rank(C) = 8$

System is observable and controllable.

Characteristics

- Closed Loop A Matrix

```
A_cl =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0137    0.9969   -0.0001   -0.0003   -0.0003    0.0256    0.0033
    0.0079    0.0000    0.9995    0.0000   -0.0002   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000   -0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
    0.0143   -0.0336   -0.0321   -0.0225   -0.0310    0.8962    0.0016
    0.0319    0.0046    0.0492   -0.0001    0.0483    0.0010    0.9004
         0         0    0.0100         0         0         0         0

  Column 8

   -0.0000
    0.0002
    0.0001
    0.0000
    0.0000
    0.0171
   -0.0164
    1.0000
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

   0.9048 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9048 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9974 + 0.0093i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9974 - 0.0093i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 8

   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.9942 + 0.0053i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9942 - 0.0053i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9998 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
```

Unstable System

Characteristics

- Eigenvectors

```
V_cl =

  Columns 1 through 4

  -0.0069 + 0.0000i  -0.0015 + 0.0000i  -0.2301 - 0.3419i  -0.2301 + 0.3419i
   0.2559 + 0.0000i  -0.2148 + 0.0000i   0.4667 + 0.0000i   0.4667 + 0.0000i
  -0.0424 + 0.0000i  -0.0430 + 0.0000i  -0.3216 + 0.1356i  -0.3216 - 0.1356i
  -0.0251 + 0.0000i   0.0220 + 0.0000i  -0.1028 - 0.4324i  -0.1028 + 0.4324i
   0.0042 + 0.0000i   0.0043 + 0.0000i   0.2232 + 0.2827i   0.2232 - 0.2827i
  -0.8717 + 0.0000i   0.8377 + 0.0000i  -0.0975 - 0.0241i  -0.0975 + 0.0241i
  -0.4148 + 0.0000i  -0.4997 + 0.0000i  -0.1384 + 0.0631i  -0.1384 - 0.0631i
   0.0044 + 0.0000i   0.0045 + 0.0000i   0.2248 + 0.2821i   0.2248 - 0.2821i

  Columns 5 through 8

  -0.0072 - 0.0361i  -0.0072 + 0.0361i  -0.2861 + 0.0000i   0.0492 + 0.0000i
  -0.4373 + 0.4050i  -0.4373 - 0.4050i  -0.0123 + 0.0000i  -0.0000 + 0.0000i
  -0.0657 + 0.0308i  -0.0657 - 0.0308i   0.0084 + 0.0000i  -0.0000 + 0.0000i
   0.7707 + 0.0000i   0.7707 + 0.0000i   0.6370 + 0.0000i   0.1379 + 0.0000i
   0.0884 + 0.0277i   0.0884 - 0.0277i  -0.4683 + 0.0000i   0.3648 + 0.0000i
  -0.0279 - 0.1560i  -0.0279 + 0.1560i  -0.1177 + 0.0000i   0.0200 + 0.0000i
  -0.0276 + 0.0330i  -0.0276 - 0.0330i  -0.2397 + 0.0000i   0.0412 + 0.0000i
   0.0887 + 0.0276i   0.0887 - 0.0276i  -0.4706 + 0.0000i   0.9183 + 0.0000i
```

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
9.05e-01	-1.00e+00	9.05e-01	-1.11e+00
9.05e-01	-1.00e+00	9.05e-01	-1.11e+00
9.94e-01 + 5.27e-03i	-1.00e+00	9.94e-01	-1.01e+00
9.94e-01 - 5.27e-03i	-1.00e+00	9.94e-01	-1.01e+00
9.97e-01 + 9.32e-03i	-1.00e+00	9.97e-01	-1.00e+00
9.97e-01 - 9.32e-03i	-1.00e+00	9.97e-01	-1.00e+00
1.00e+00	-1.00e+00	1.00e+00	-1.00e+00
1.00e+00	-1.00e+00	1.00e+00	-1.00e+00

Gains & Weighted Matrices

- Chosen Weighted Matrices

```
Q =

     1     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0
     0     0     1     0     0     0     0     0
     0     0     0    10     0     0     0     0
     0     0     0     0    10     0     0     0
     0     0     0     0     0     1     0     0
     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     0    10

R =

    80     0
     0   100
```

Gains & Weighted Matrices

Q_hat =

Columns 1 through 7

0.7856	-0.3682	0.1570	-0.4370	0.2822	-0.0895	-0.0263
-0.3682	1.1415	0.1541	1.7016	0.0056	0.2458	0.0214
0.1570	0.1541	1.9803	0.2764	1.7513	0.0278	-0.1415
-0.4370	1.7016	0.2764	5.9987	0.0005	0.3337	0.0251
0.2822	0.0056	1.7513	0.0005	6.0002	-0.0048	-0.1190
-0.0895	0.2458	0.0278	0.3337	-0.0048	0.1058	0.0054
-0.0263	0.0214	-0.1415	0.0251	-0.1190	0.0054	0.0615
0.2821	0.0055	1.7512	0.0004	0.0000	-0.0048	-0.1190

Column 8

0.2821
0.0055
1.7512
0.0004
0.0000
-0.0048
-0.1190
6.0000

M1 =

-0.2544	-0.0826
0.5547	0.0539
0.0753	-0.3006
0.5978	0.0479
-0.0091	-0.2125
0.1844	0.0146
0.0125	0.0759
-0.0091	-0.2125

K =

Columns 1 through 7

-0.1502	0.3531	0.3371	0.2365	0.3261	0.0910	-0.0170
-0.3349	-0.0479	-0.5168	0.0013	-0.5074	-0.0100	0.0470

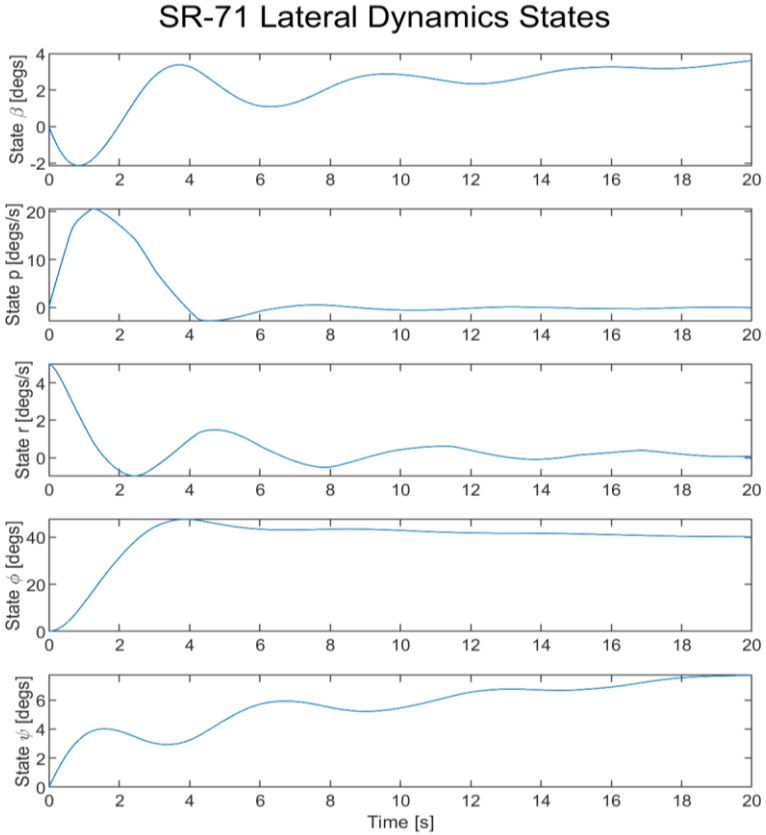
Column 8

-0.1800
0.1725

R_hat1 =

48.8474	0.0439
0.0439	60.5219

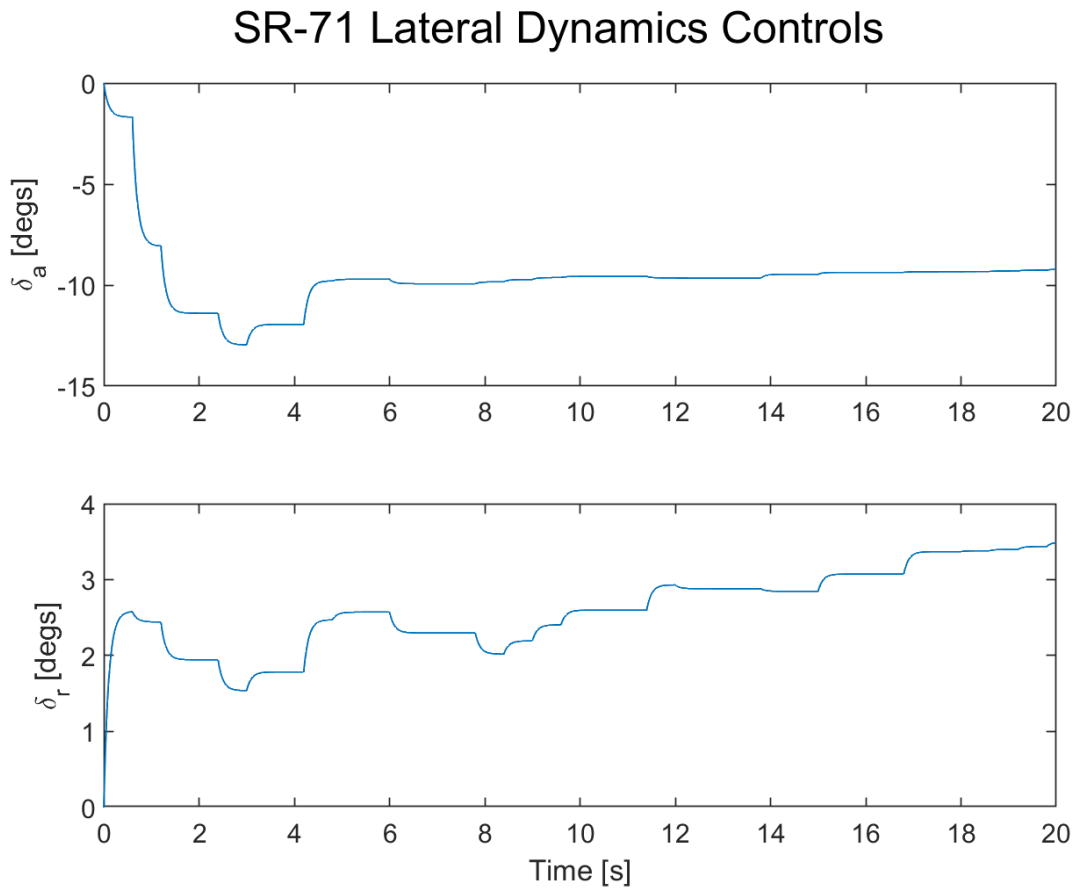
Plots - States



- Overall, state histories became worse with a PI added
  - ψ does not settle
  - ϕ no longer settles at 0
  - β does not settle
  - Settle times are larger

Overall, state histories were worse with PI added.

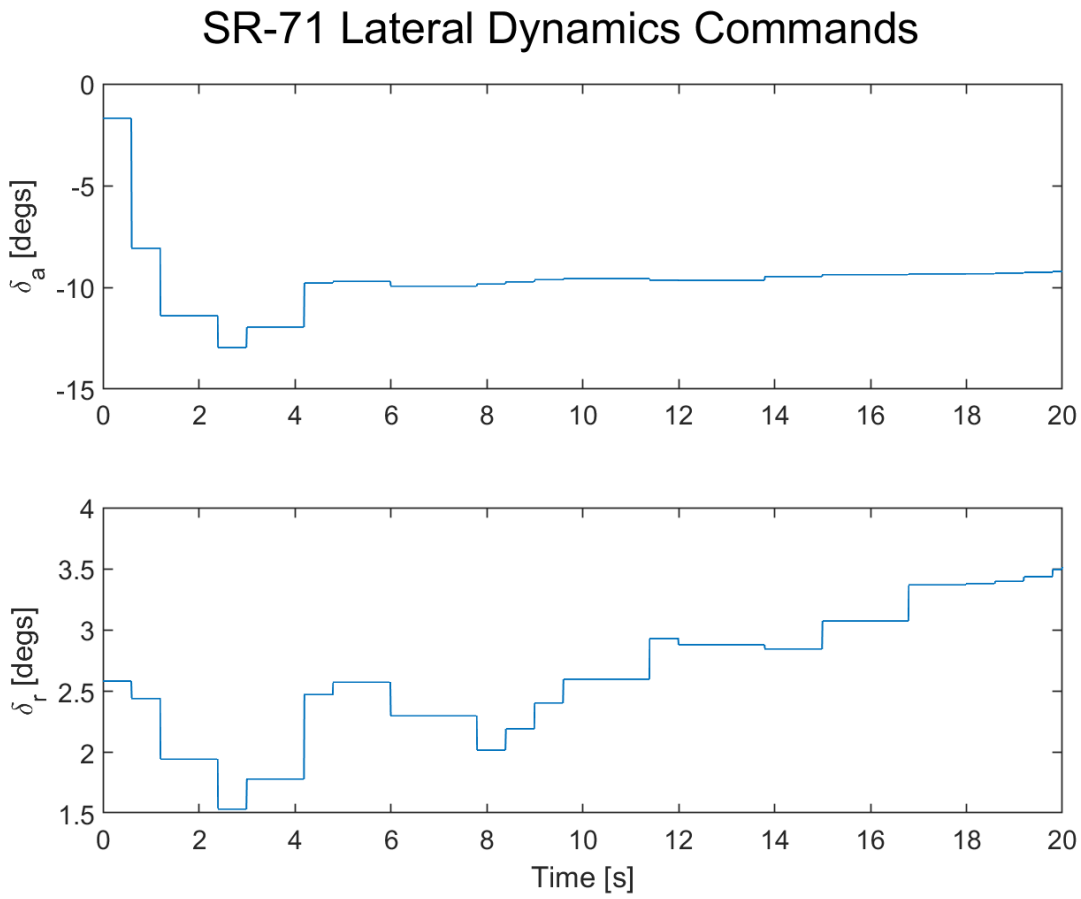
Plots - Controls



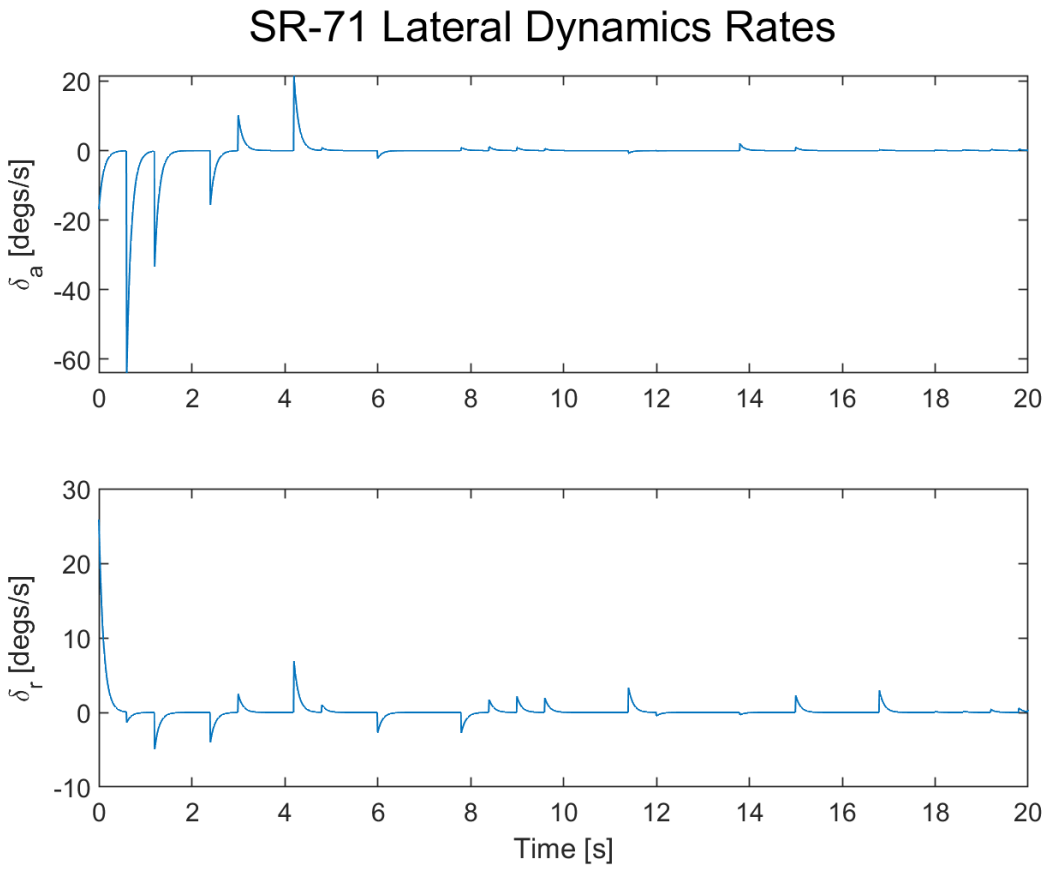
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands



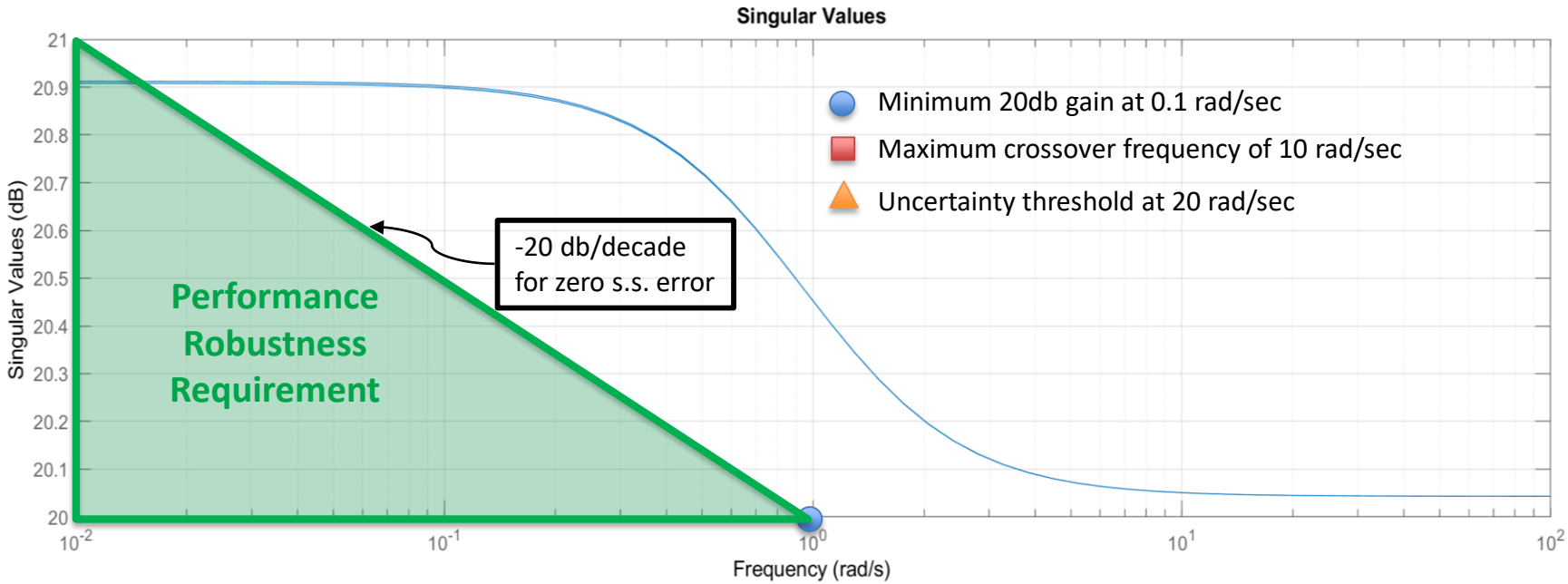
Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$

Frequency Domain Analysis



Frequency Domain Analysis

- Did not meet requirements for high or low frequencies

4.2.1 Low Frequency

- Minimum singular value large
- Attenuation of low frequency disturbances by a factor of 0.1
- Slope at least –20dB/decade
- Zero steady-state error
- Minimum crossover frequency of 0.1 rad/sec, maximum crossover frequency of 10 rad/sec.

4.2.2 High Frequency

- Maximum singular value small
- Linear model accurate to within 10% of actual plant for frequencies up to 2 rad/sec, where uncertainty grows without bound at 20 dB/decade thereafter:

$$m(\omega) = \frac{s+2}{20}$$

where $m(\omega)$ is the multiplicative modeling discrepancy bound.

Closed-Loop || NZSP Design Step Input

Characteristics

- Closed Loop A Matrix

```
A_cl =

   -0.0346    0.1140   -0.9930    0.0101    0.0000         0    0.0064
   -1.3930   -0.2610    0.0221    0.0000    0.0000    2.7050    0.3480
    0.8040    0.0022   -0.0183   -0.0000         0   -0.0446   -0.9380
   -0.0001    1.0000    0.1130    0.0000    0.0001         0         0
    0.0000         0    1.0000         0    0.0001         0         0
  -14.7257   -3.4672   -1.5007   -0.3640   -9.5390  -11.2911    0.0514
    5.3587    0.3465    6.6416    0.0321    5.8545    0.0423  -10.6641
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

-10.4304 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i -10.0664 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3439 + 1.0081i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3439 - 1.0081i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 7

  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
 -0.4977 + 0.3438i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i  -0.4977 - 0.3438i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0888 + 0.0000i
```

Stable System

Characteristics

- Eigenvectors

```
V_cl =

  Columns 1 through 4

   0.0033 + 0.0000i  -0.0087 + 0.0000i  -0.1463 - 0.1739i  -0.1463 + 0.1739i
  -0.2571 + 0.0000i   0.0238 + 0.0000i   0.6813 + 0.0000i   0.6813 + 0.0000i
   0.0056 + 0.0000i  -0.0920 + 0.0000i  -0.1464 + 0.0886i  -0.1464 - 0.0886i
   0.0246 + 0.0000i  -0.0013 + 0.0000i  -0.1926 - 0.5938i  -0.1926 + 0.5938i
  -0.0005 + 0.0000i   0.0091 + 0.0000i   0.1230 + 0.1032i   0.1230 - 0.1032i
   0.9659 + 0.0000i   0.0379 + 0.0000i  -0.0854 + 0.1596i  -0.0854 - 0.1596i
   0.0182 + 0.0000i  -0.9947 + 0.0000i  -0.0754 + 0.0314i  -0.0754 - 0.0314i

  Columns 5 through 7

   0.0334 - 0.0083i   0.0334 + 0.0083i  -0.0004 + 0.0000i
  -0.4208 + 0.2907i  -0.4208 - 0.2907i   0.0885 + 0.0000i
  -0.0268 + 0.0180i  -0.0268 - 0.0180i   0.0000 + 0.0000i
   0.8515 + 0.0000i   0.8515 + 0.0000i  -0.9961 + 0.0000i
   0.0534 + 0.0007i   0.0534 - 0.0007i  -0.0001 + 0.0000i
   0.0148 - 0.0855i   0.0148 + 0.0855i   0.0055 + 0.0000i
   0.0198 + 0.0166i   0.0198 - 0.0166i  -0.0003 + 0.0000i
```

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-8.88e-02	1.00e+00	8.88e-02	1.13e+01
-4.98e-01 + 3.44e-01i	8.23e-01	6.05e-01	2.01e+00
-4.98e-01 - 3.44e-01i	8.23e-01	6.05e-01	2.01e+00
-3.44e-01 + 1.01e+00i	3.23e-01	1.07e+00	2.91e+00
-3.44e-01 - 1.01e+00i	3.23e-01	1.07e+00	2.91e+00
-1.01e+01	1.00e+00	1.01e+01	9.93e-02
-1.04e+01	1.00e+00	1.04e+01	9.59e-02

Gains & Weighted Matrices

- Chosen Weighted Matrices

```
Q =

    0.0000         0         0         0         0         0         0
         0    0.0000         0         0         0         0         0
         0         0    1.0000         0         0         0         0
         0         0         0    0.0000         0         0         0
         0         0         0         0   30.0000         0         0
         0         0         0         0         0    3.0000         0
         0         0         0         0         0         0    5.0000


R =

     1     0
     0    40
```

Gains & Weighted Matrices

Q_hat =

0.1134	0.0027	0.4950	0.0002	0.8466	-0.0014	-0.0387
0.0027	0.0001	0.0108	0.0000	0.0166	-0.0000	-0.0009
0.4950	0.0108	2.5639	0.0008	5.2538	-0.0072	-0.1877
0.0002	0.0000	0.0008	0.0000	0.0013	-0.0000	-0.0001
0.8466	0.0166	5.2538	0.0013	18.0007	-0.0144	-0.3571
-0.0014	-0.0000	-0.0072	-0.0000	-0.0144	0.1500	0.0005
-0.0387	-0.0009	-0.1877	-0.0001	-0.3571	0.0005	0.2642

M1 =

-0.0039	-0.0949
-0.0001	-0.0023
-0.0166	-0.3945
-0.0000	-0.0002
-0.0274	-0.6374
0.1493	0.0011
0.0013	0.2801

K =

1.4726	0.3467	0.1501	0.0364	0.9539	0.1291	-0.0051
-0.5359	-0.0346	-0.6642	-0.0032	-0.5854	-0.0042	0.0664

R_hat1 =

1.9516	0.0033
0.0033	26.3329

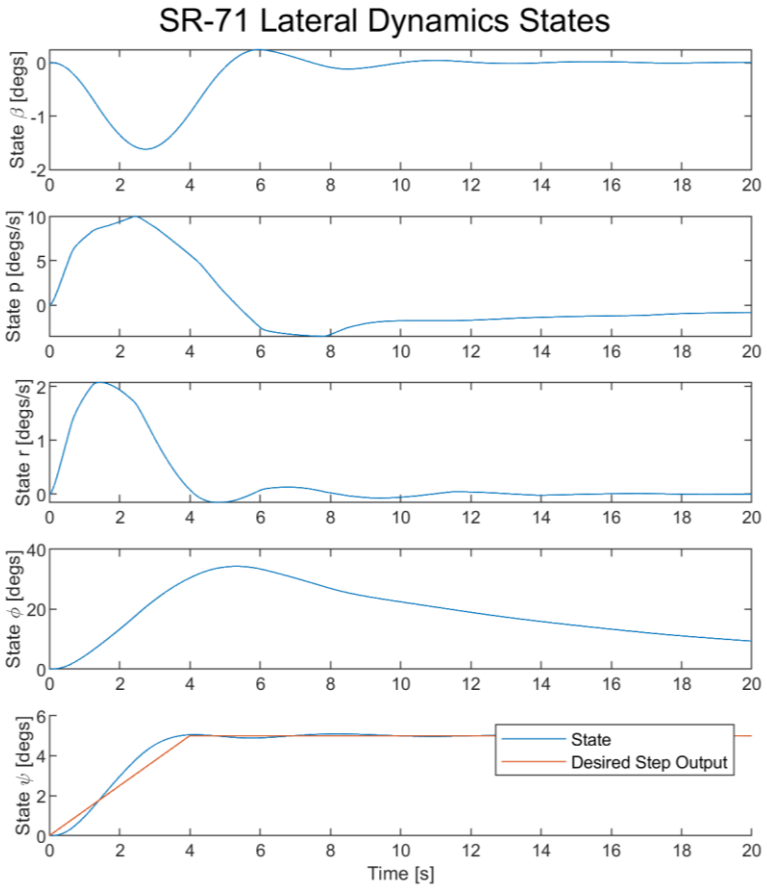
Characteristics

- Steady State Values

```
x_star =          u_star =

    0.0000            1.8642e-06
   -0.0000
   -0.0000
   -0.0006
    0.0873
         0
    0.0000
```

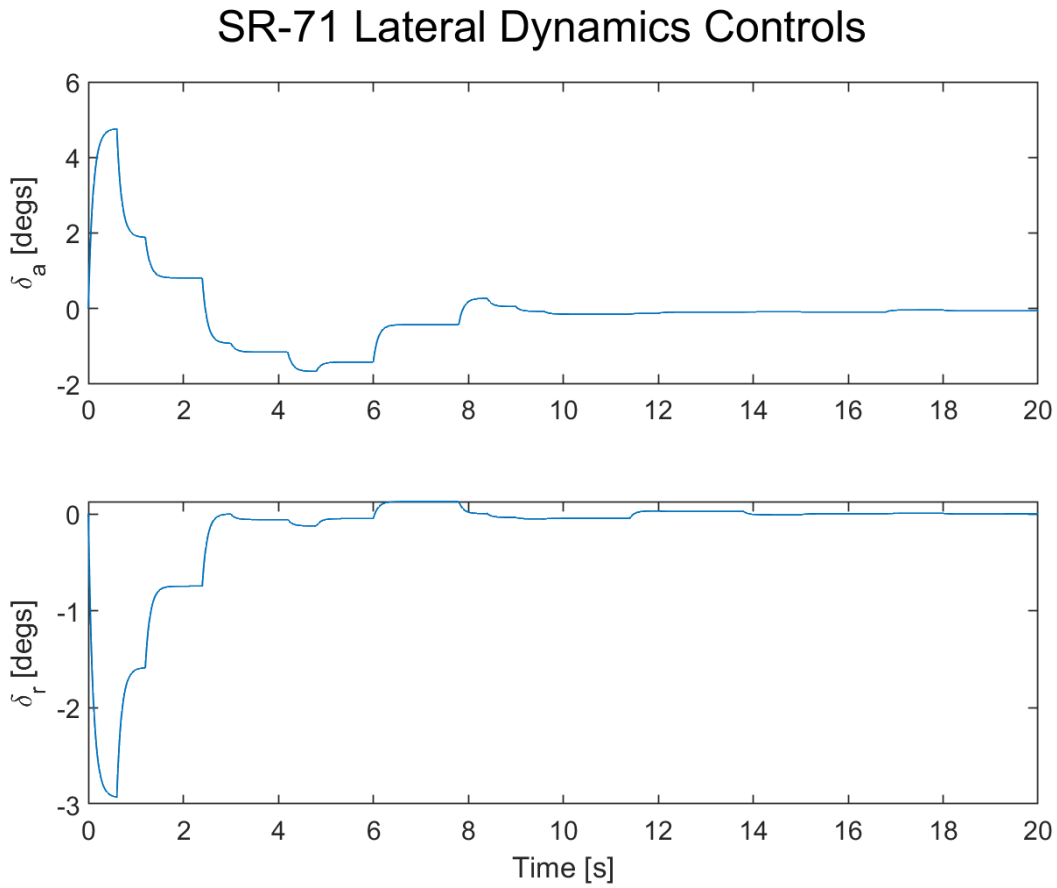
Units are in radians.

Plots - States



- The state followed the desired state output with minimal error
- Could not obtain 0 overshoot on state ψ

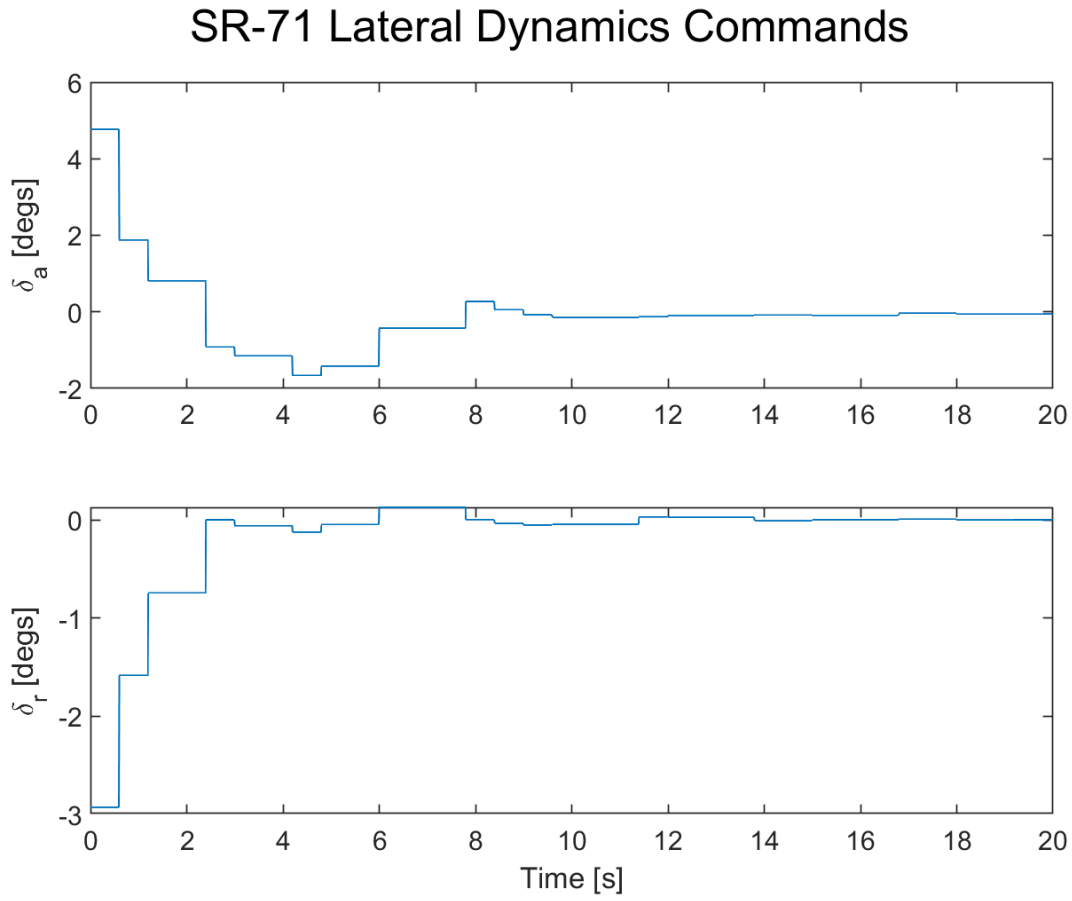
Plots - Controls



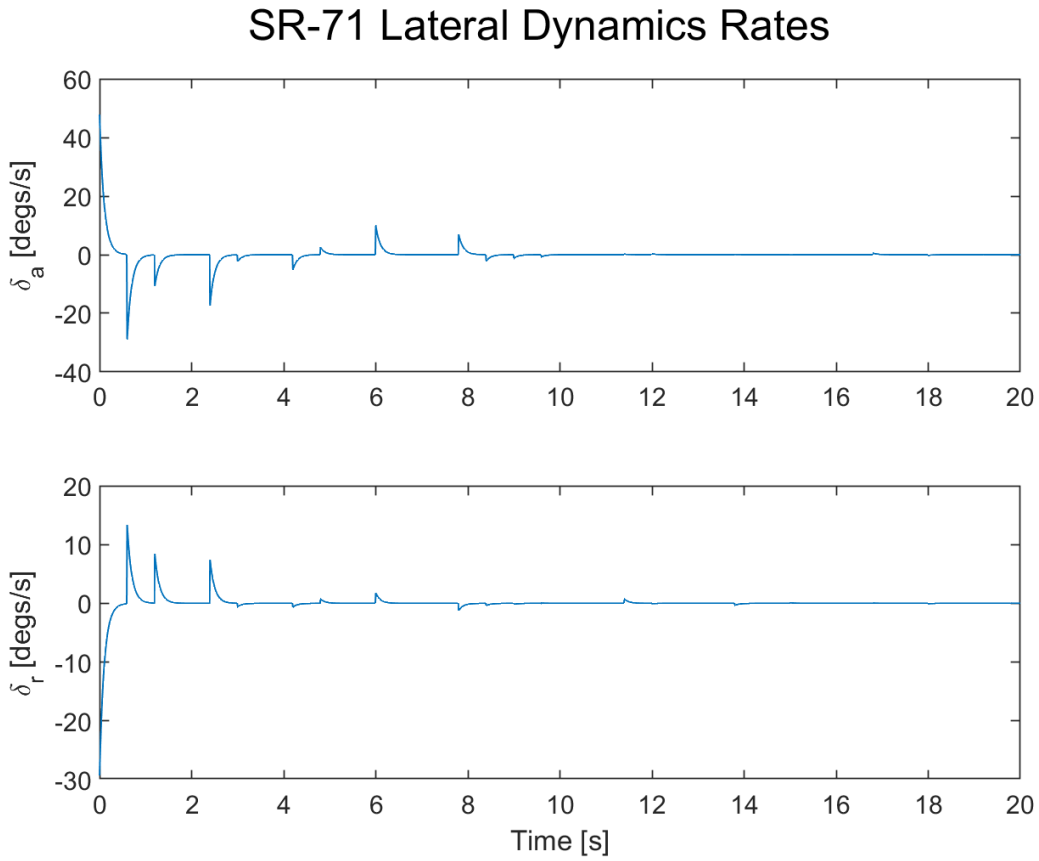
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands



Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$

Closed-Loop || NZSP Design
Sinusoidal Input

Characteristics

- Closed Loop A Matrix

```
A_cl =

   -0.0346    0.1140   -0.9930    0.0101    0.0000         0    0.0064
   -1.3930   -0.2610    0.0221    0.0000    0.0000    2.7050    0.3480
    0.8040    0.0022   -0.0183   -0.0000         0   -0.0446   -0.9380
   -0.0001    1.0000    0.1130    0.0000    0.0001         0         0
    0.0000         0    1.0000         0    0.0001         0         0
    2.2622   -6.3305   -3.6337   -6.0069   -1.7038  -11.5825    0.0965
    7.2532    0.3654   10.7208   -0.1604    9.6610    0.0385  -10.9781
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

-10.0175 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i -10.0003 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0271 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.5256 + 0.9331i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 7

  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
 -0.5256 - 0.9331i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i  -0.8891 + 0.8953i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i  -0.8891 - 0.8953i
```

Stable System

Characteristics

- Eigenvectors

```
V_cl =

  Columns 1 through 4

  -0.0092 + 0.0000i  -0.0034 + 0.0000i  -0.6868 + 0.0000i  -0.4048 - 0.0022i
   0.0522 + 0.0000i   0.2670 + 0.0000i  -0.0091 + 0.0000i   0.4262 + 0.0000i
  -0.0926 + 0.0000i  -0.0040 + 0.0000i   0.0037 + 0.0000i  -0.1551 + 0.3780i
  -0.0042 + 0.0000i  -0.0267 + 0.0000i   0.3170 + 0.0000i  -0.1526 - 0.3521i
   0.0092 + 0.0000i   0.0004 + 0.0000i  -0.1339 + 0.0000i   0.3786 - 0.0471i
  -0.0645 + 0.0000i  -0.9633 + 0.0000i  -0.2805 + 0.0000i  -0.2435 + 0.0975i
  -0.9921 + 0.0000i   0.0008 + 0.0000i  -0.5754 + 0.0000i  -0.0423 + 0.3522i

  Columns 5 through 7

  -0.4048 + 0.0022i  -0.0720 + 0.0002i  -0.0720 - 0.0002i
   0.4262 + 0.0000i   0.7362 + 0.0000i   0.7362 + 0.0000i
  -0.1551 - 0.3780i   0.0188 + 0.0611i   0.0188 - 0.0611i
  -0.1526 + 0.3521i  -0.4085 - 0.4191i  -0.4085 + 0.4191i
   0.3786 + 0.0471i   0.0239 - 0.0446i   0.0239 + 0.0446i
  -0.2435 - 0.0975i  -0.2115 + 0.2398i  -0.2115 - 0.2398i
  -0.0423 - 0.3522i   0.0258 + 0.0276i   0.0258 - 0.0276i
```

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-2.71e-02	1.00e+00	2.71e-02	3.69e+01
-5.26e-01 + 9.33e-01i	4.91e-01	1.07e+00	1.90e+00
-5.26e-01 - 9.33e-01i	4.91e-01	1.07e+00	1.90e+00
-8.89e-01 + 8.95e-01i	7.05e-01	1.26e+00	1.12e+00
-8.89e-01 - 8.95e-01i	7.05e-01	1.26e+00	1.12e+00
-1.00e+01	1.00e+00	1.00e+01	1.00e-01
-1.00e+01	1.00e+00	1.00e+01	9.98e-02

Gains & Weighted Matrices

- Chosen Weighted Matrices

Q_hat =

0.2176	-0.1800	0.7067	-0.4388	1.4110	-0.0413	-0.0595
-0.1800	0.6348	0.1247	1.7020	0.0277	0.1317	0.0084
0.7067	0.1247	3.9319	0.2778	8.7563	0.0119	-0.2830
-0.4388	1.7020	0.2778	5.9987	0.0022	0.3338	0.0250
1.4110	0.0277	8.7563	0.0022	30.0011	-0.0240	-0.5951
-0.0413	0.1317	0.0119	0.3338	-0.0240	0.0277	0.0028
-0.0595	0.0084	-0.2830	0.0250	-0.5951	0.0028	0.0714

K =

-0.2262	0.6331	0.3634	0.6007	0.1704	0.1582	-0.0097
-0.7253	-0.0365	-1.0721	0.0160	-0.9661	-0.0038	0.0978

R_hat1 =

4.3192	0.0139
0.0139	15.5618

M1 =

-0.0874	-0.1390
0.2591	0.0175
0.0220	-0.5810
0.5982	0.0477
-0.0456	-1.0623
0.0558	0.0060
0.0059	0.0955

Gains & Weighted Matrices

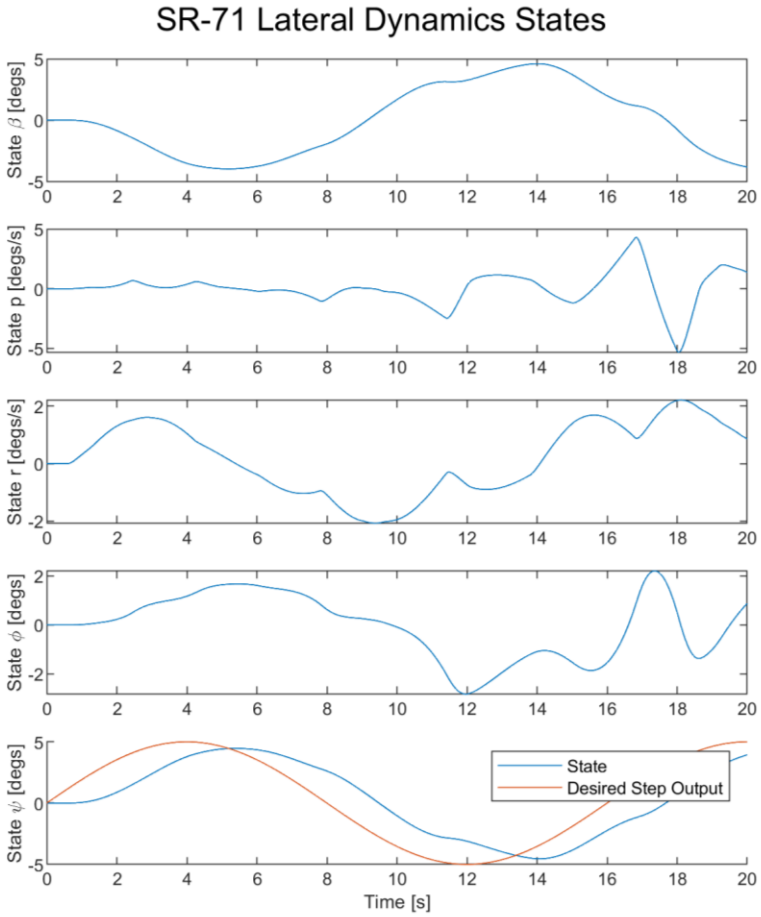
Q =

0.0000	0	0	0	0	0	0
0	0.0000	0	0	0	0	0
0	0	1.0000	0	0	0	0
0	0	0	10.0000	0	0	0
0	0	0	0	50.0000	0	0
0	0	0	0	0	0.0000	0
0	0	0	0	0	0	1.0000

R =

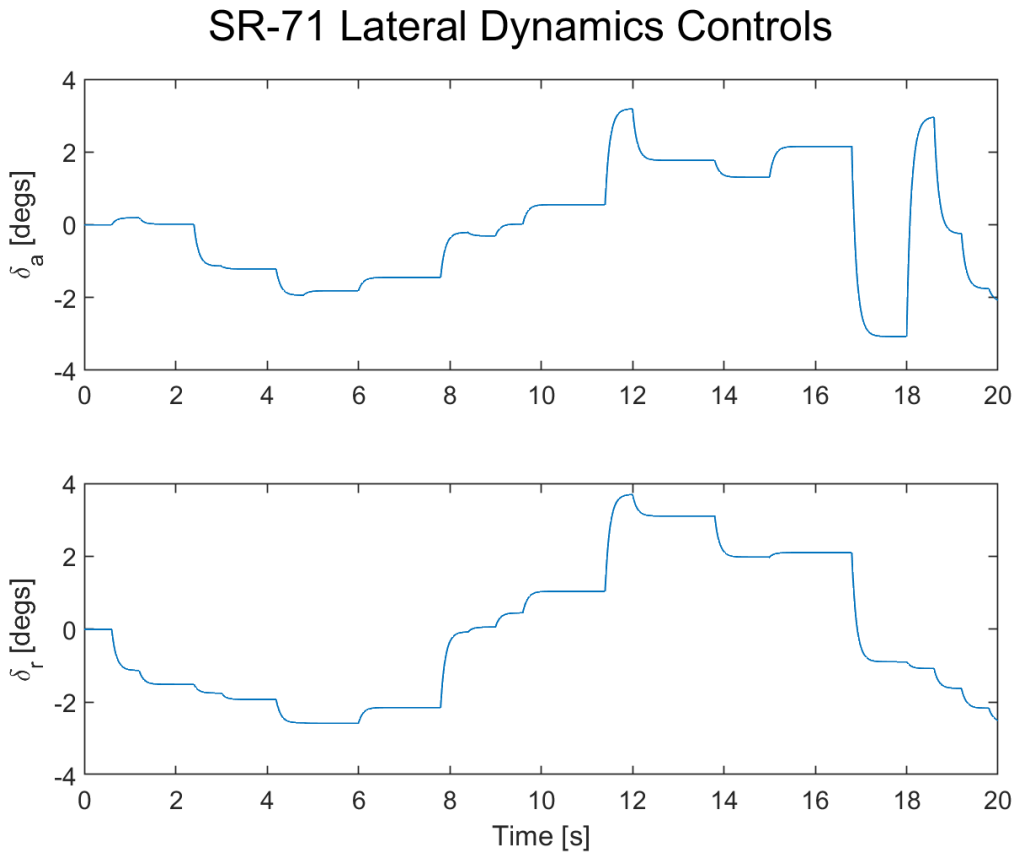
7	0
0	25

Plots - States



- The state followed the desired state output with minimal error
- Experienced a small phase shift between desired and actual state
  - Realistic to have some sort of lag in a system

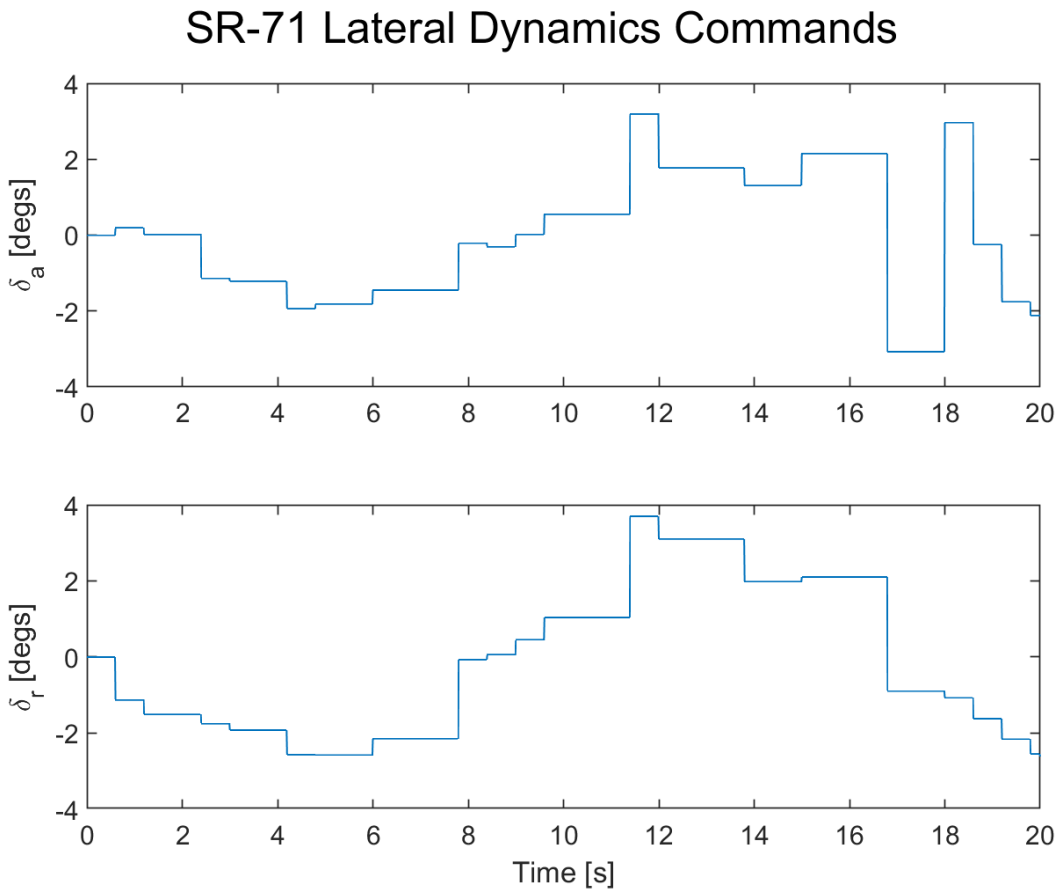
Plots - Controls



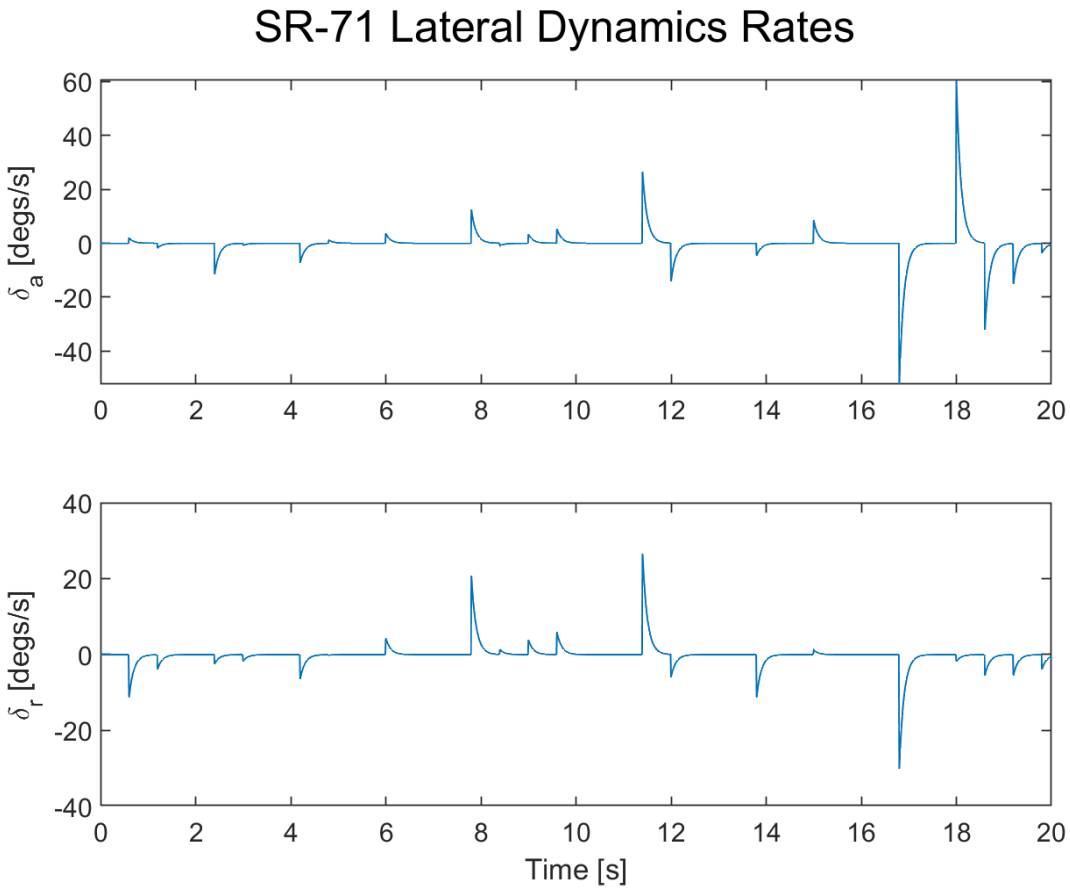
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands



Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$

Closed-Loop || PI-NZSP Design Step Input

Set-Up

- New State and Control Matrices

```
A_new =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0139    0.9974    0.0003   -0.0000    0.0000    0.0257    0.0033
    0.0080    0.0000    0.9998    0.0000    0.0000   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000    0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
         0         0         0         0         0    0.9048         0
         0         0         0         0         0         0    0.9048
         0         0         0         0    0.0100         0         0

  Column 8

         0
         0
         0
         0
         0
         0
         0
    1.0000
```

```
B_new =

    0.0000    0.0000
    0.0013    0.0002
   -0.0000   -0.0005
    0.0000    0.0000
   -0.0000   -0.0000
    0.0952         0
         0    0.0952
         0         0
```

Set-Up

- Observable: $rank(O) = 8$
- Controllable: $rank(C) = 8$

System is observable and controllable.

Characteristics

- Closed Loop A Matrix

```
A_cl =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0153    0.9967   -0.0005   -0.0003   -0.0018    0.0255    0.0033
    0.0078    0.0000    0.9996    0.0000   -0.0002   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000   -0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
   -0.1096   -0.0515   -0.0591   -0.0205   -0.1363    0.8886    0.0035
    0.0477    0.0027    0.0384   -0.0017    0.0543    0.0005    0.9009
         0         0         0         0    0.0100         0         0

  Column 8

    0.0000
   -0.0002
   -0.0000
   -0.0000
   -0.0000
   -0.0136
    0.0100
    1.0000
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

   0.9012 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9043 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9983 + 0.0093i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9983 - 0.0093i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 8

   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.9928 + 0.0050i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9928 - 0.0050i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9993 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9984 + 0.0000i
```

Unstable system

Characteristics

- Eigenvectors

```
V_cl =

  Columns 1 through 4

   0.0032 + 0.0000i   0.0089 + 0.0000i   0.4245 + 0.0000i   0.4245 + 0.0000i
  -0.2573 + 0.0000i  -0.0370 + 0.0000i  -0.3289 + 0.2487i  -0.3289 - 0.2487i
   0.0040 + 0.0000i   0.0923 + 0.0000i   0.0231 - 0.3685i   0.0231 + 0.3685i
   0.0247 + 0.0000i   0.0026 + 0.0000i   0.2760 + 0.2988i   0.2760 - 0.2988i
  -0.0004 + 0.0000i  -0.0092 + 0.0000i  -0.3859 + 0.0457i  -0.3859 - 0.0457i
   0.9660 + 0.0000i   0.0098 + 0.0000i   0.1223 - 0.0924i   0.1223 + 0.0924i
  -0.0001 + 0.0000i   0.9949 + 0.0000i  -0.0063 - 0.0779i  -0.0063 + 0.0779i
   0.0000 + 0.0000i   0.0010 + 0.0000i   0.1221 + 0.3906i   0.1221 - 0.3906i

  Columns 5 through 8

   0.0368 - 0.0349i   0.0368 + 0.0349i  -0.0975 + 0.0000i   0.0851 + 0.0000i
  -0.5262 + 0.3703i  -0.5262 - 0.3703i  -0.0629 + 0.0000i   0.1118 + 0.0000i
  -0.0452 - 0.0003i  -0.0452 + 0.0003i  -0.0010 + 0.0000i   0.0165 + 0.0000i
   0.7358 + 0.0000i   0.7358 + 0.0000i   0.9595 + 0.0000i  -0.7208 + 0.0000i
   0.0419 + 0.0296i   0.0419 - 0.0296i   0.0157 + 0.0000i  -0.1048 + 0.0000i
   0.0408 - 0.1794i   0.0408 + 0.1794i  -0.0442 + 0.0000i   0.0385 + 0.0000i
  -0.0057 + 0.0035i  -0.0057 - 0.0035i  -0.0817 + 0.0000i   0.0739 + 0.0000i
  -0.0201 - 0.0550i  -0.0201 + 0.0550i  -0.2388 + 0.0000i   0.6652 + 0.0000i
```

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
9.01e-01	-1.00e+00	9.01e-01	-1.11e+00
9.04e-01	-1.00e+00	9.04e-01	-1.11e+00
9.93e-01 + 5.00e-03i	-1.00e+00	9.93e-01	-1.01e+00
9.93e-01 - 5.00e-03i	-1.00e+00	9.93e-01	-1.01e+00
9.98e-01 + 9.33e-03i	-1.00e+00	9.98e-01	-1.00e+00
9.98e-01 - 9.33e-03i	-1.00e+00	9.98e-01	-1.00e+00
9.98e-01	-1.00e+00	9.98e-01	-1.00e+00
9.99e-01	-1.00e+00	9.99e-01	-1.00e+00

Gains & Weighted Matrices

- Chosen Weighted Matrices

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 9 & 0 \\ 0 & 70 \end{bmatrix}$$

Best response came by leaving state weights alone, and weighting controls.

Gains & Weighted Matrices

```
Q_hat =

  Columns 1 through 7

    0.0500   -0.0016   -0.0002   -0.0000    0.0000   -0.0001   -0.0000
   -0.0016    0.0494    0.0001    0.0012    0.0000    0.0028    0.0004
   -0.0002    0.0001    0.0500    0.0001    0.0013   -0.0000   -0.0010
   -0.0000    0.0012    0.0001    0.0500    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0013    0.0000    0.0500   -0.0000   -0.0000
   -0.0001    0.0028   -0.0000    0.0000   -0.0000    0.0318    0.0000
   -0.0000    0.0004   -0.0010    0.0000   -0.0000    0.0000    0.0316
    0.0000    0.0000    0.0000    0.0000    0.0013   -0.0000   -0.0000

  Column 8

    0.0000
    0.0000
    0.0000
    0.0000
    0.0013
   -0.0000
   -0.0000
    0.0500
```

```
K =

  Columns 1 through 7

    1.1517    0.5411    0.6209    0.2159    1.4319    0.1709   -0.0369
   -0.5014   -0.0284   -0.4039    0.0177   -0.5709   -0.0049    0.0412

  Column 8

    0.1432
   -0.1053
```

```
M1 =

   -0.0000   -0.0000
    0.0005    0.0001
   -0.0000   -0.0002
    0.0000    0.0000
   -0.0000   -0.0000
    0.0078    0.0000
    0.0000    0.0077
   -0.0000   -0.0000
```

```
R_hat1 =

    0.4529    0.0000
    0.0000    3.5029
```

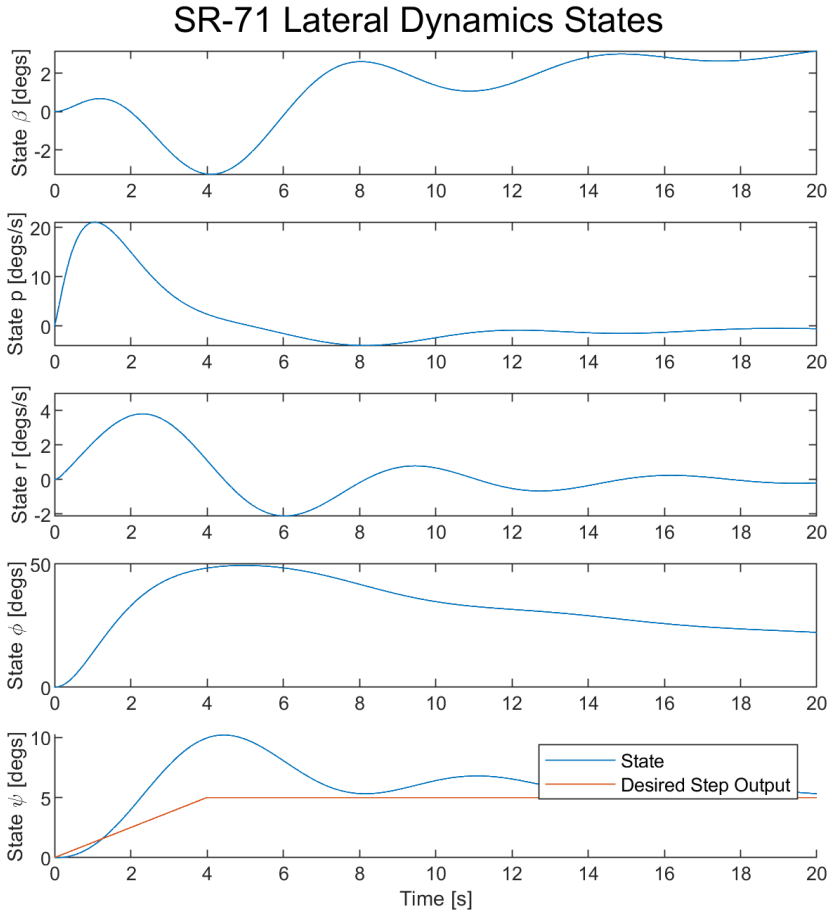
Characteristics

- Steady State Values

```
x_star =

    0.0000
   -0.0000
   -0.0000
   -0.0006
    0.0873
         0
    0.0000

u_star =

   1.8642e-06
```

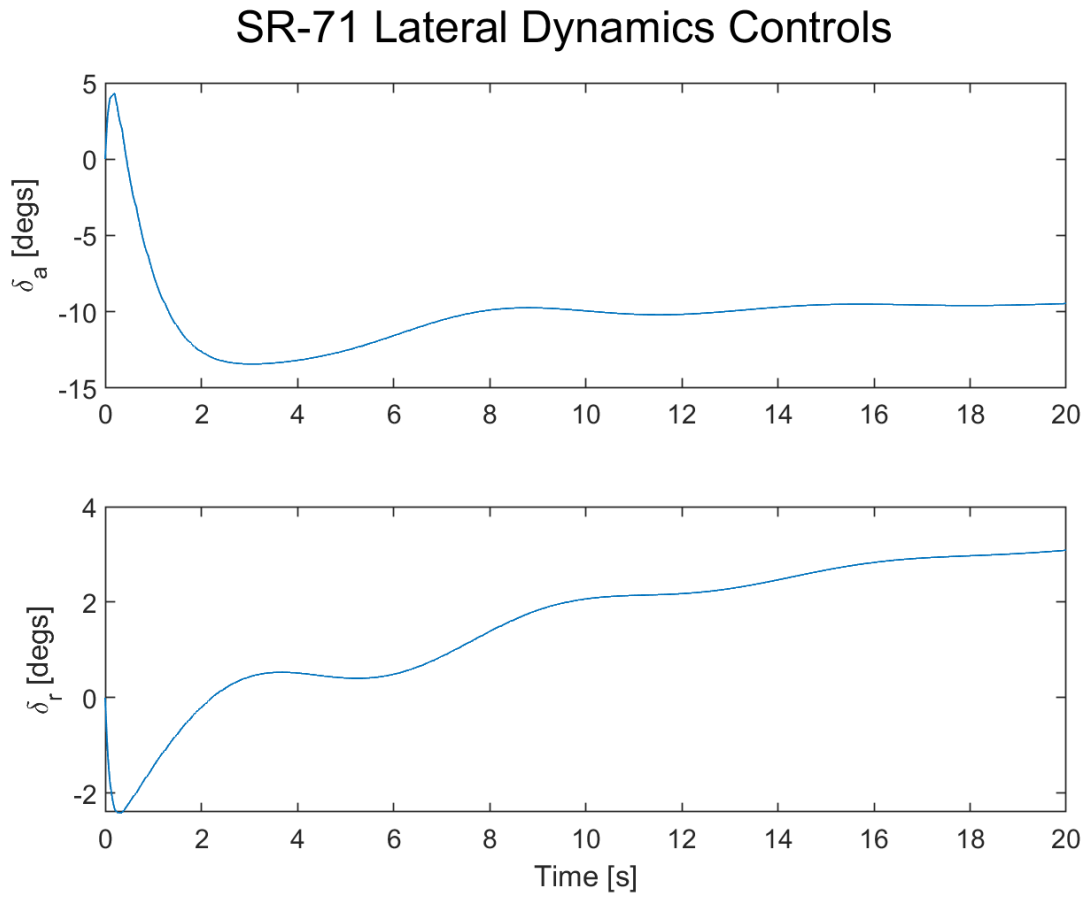
Units in radians.

Plots - States



- Could not obtain 0 overshoot on state ψ
  - Large overshoot from desired output
- Overall, large error for system

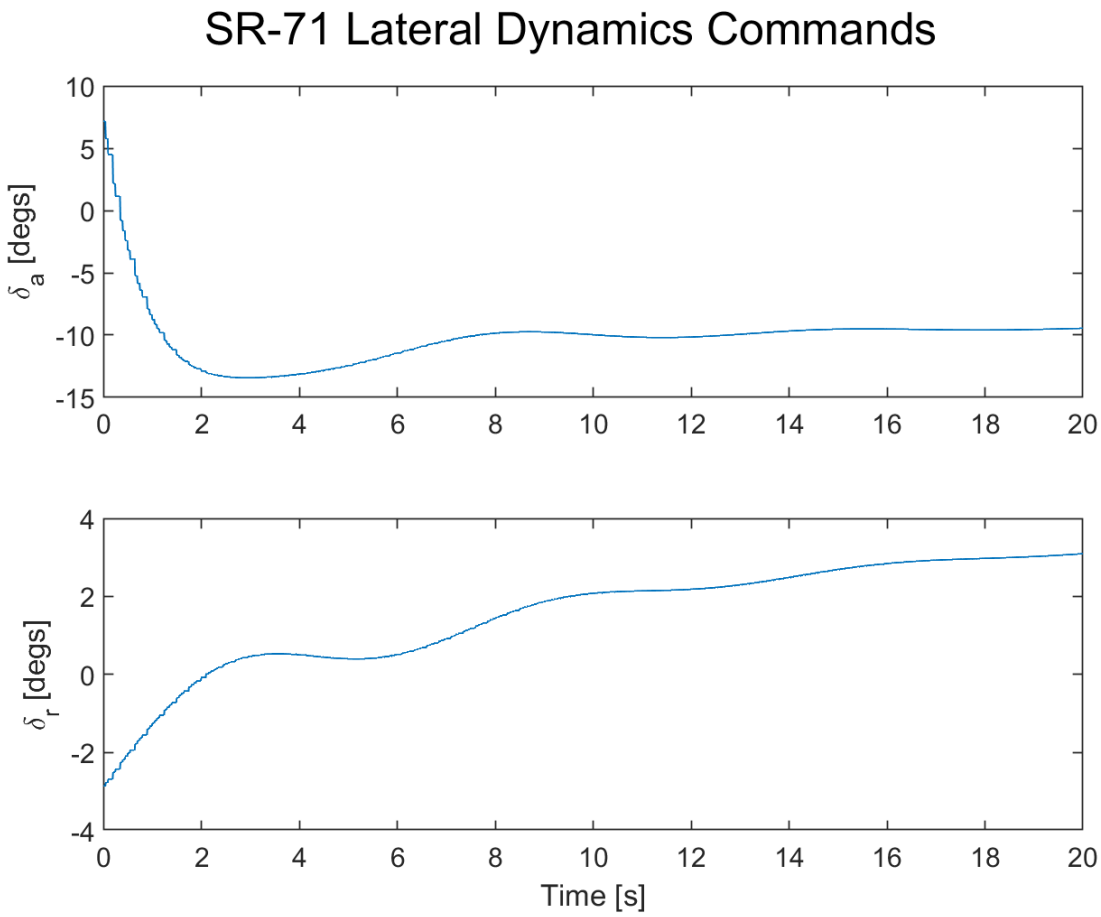
Plots - Controls



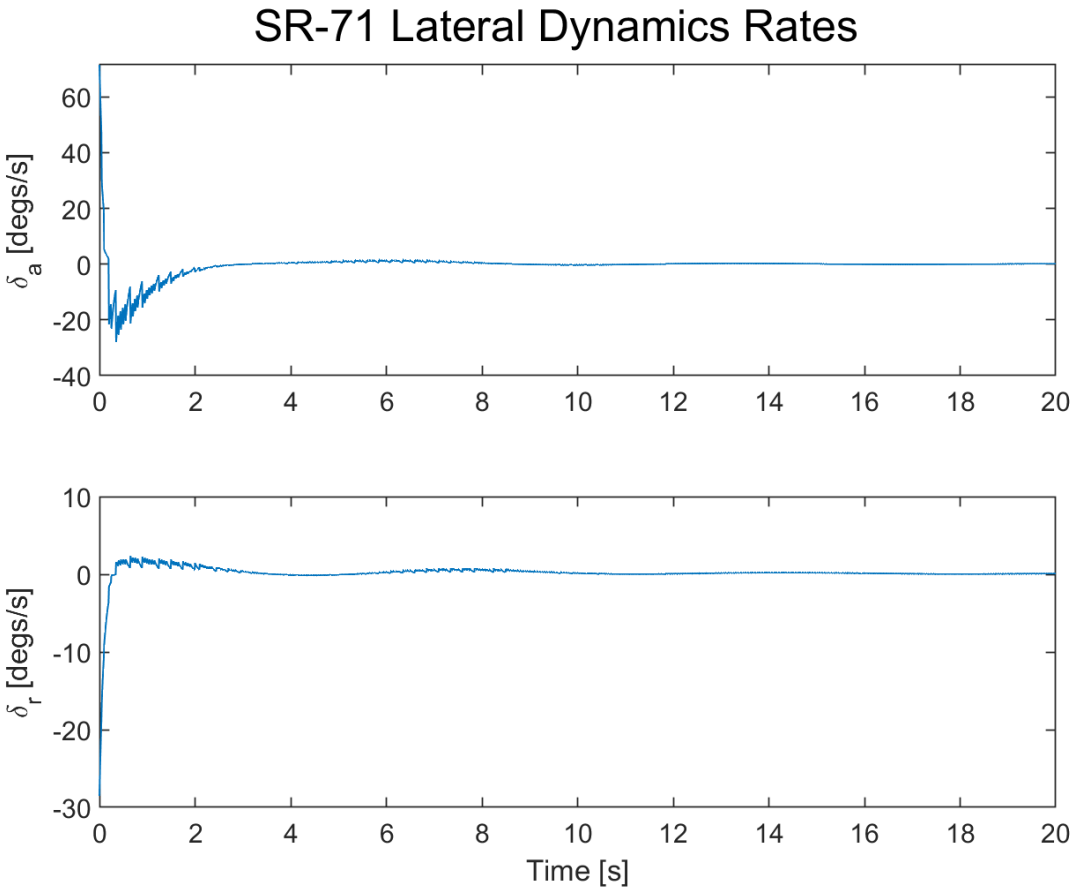
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands



Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$

Closed-Loop || PI-NZSP Design Sinusoidal Input

Set-Up

- New State and Control Matrices

```
A_new =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0139    0.9974    0.0003   -0.0000    0.0000    0.0257    0.0033
    0.0080    0.0000    0.9998    0.0000    0.0000   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000    0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
         0         0         0         0         0    0.9048         0
         0         0         0         0         0         0    0.9048
         0         0         0         0    0.0100         0         0

  Column 8

         0
         0
         0
         0
         0
         0
         0
    1.0000
```

```
B_new =

    0.0000    0.0000
    0.0013    0.0002
   -0.0000   -0.0005
    0.0000    0.0000
   -0.0000   -0.0000
    0.0952         0
         0    0.0952
         0         0
```

Set-Up

- Observable: $rank(O) = 8$
- Controllable: $rank(C) = 8$

System is observable and controllable.

Characteristics

- Closed Loop A Matrix

```
A_cl =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0150    0.9923   -0.0001   -0.0007   -0.0021    0.0244    0.0032
    0.0077    0.0001    0.9991    0.0000   -0.0007   -0.0004   -0.0088
   -0.0001    0.0100    0.0011    1.0000   -0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
   -0.0918   -0.3714   -0.0471   -0.0490   -0.1746    0.8079   -0.0058
    0.0821    0.0003    0.1431   -0.0005    0.1466   -0.0003    0.8853
         0         0         0         0    0.0100         0         0

  Column 8

    0.0000
   -0.0001
   -0.0001
   -0.0000
   -0.0000
   -0.0088
    0.0162
    1.0000
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

   0.9011 + 0.0198i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9011 - 0.0198i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.8968 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9942 + 0.0099i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 8

   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.9942 - 0.0099i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9994 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9986 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9988 + 0.0000i
```

Unstable System

Characteristics

- Eigenvectors

```
V_cl =

  Columns 1 through 4

   0.0029 + 0.0012i   0.0029 - 0.0012i   0.0047 + 0.0000i   0.0307 + 0.4183i
  -0.2469 - 0.0528i  -0.2469 + 0.0528i   0.1113 + 0.0000i  -0.0157 - 0.0428i
   0.0053 - 0.0004i   0.0053 + 0.0004i   0.0690 + 0.0000i   0.4368 + 0.1929i
   0.0217 + 0.0097i   0.0217 - 0.0097i  -0.0109 + 0.0000i  -0.0305 - 0.0161i
  -0.0005 - 0.0001i  -0.0005 + 0.0001i  -0.0063 + 0.0000i  -0.0457 - 0.4126i
   0.9671 + 0.0000i   0.9671 + 0.0000i  -0.5422 + 0.0000i  -0.0336 + 0.2131i
   0.0132 - 0.0158i   0.0132 + 0.0158i   0.8299 + 0.0000i   0.4928 + 0.0000i
   0.0000 + 0.0000i   0.0000 - 0.0000i   0.0006 + 0.0000i  -0.2901 + 0.2165i

  Columns 5 through 8

   0.0307 - 0.4183i  -0.1498 + 0.0000i   0.0056 + 0.0000i   0.0517 + 0.0000i
  -0.0157 + 0.0428i  -0.0542 + 0.0000i  -0.1361 + 0.0000i   0.0997 + 0.0000i
   0.4368 - 0.1929i  -0.0006 + 0.0000i  -0.0053 + 0.0000i   0.0077 + 0.0000i
  -0.0305 + 0.0161i   0.9585 + 0.0000i   0.9558 + 0.0000i  -0.8354 + 0.0000i
  -0.0457 + 0.4126i   0.0107 + 0.0000i   0.0368 + 0.0000i  -0.0639 + 0.0000i
  -0.0336 - 0.2131i  -0.0651 + 0.0000i  -0.0035 + 0.0000i   0.0261 + 0.0000i
   0.4928 + 0.0000i  -0.1255 + 0.0000i   0.0039 + 0.0000i   0.0442 + 0.0000i
  -0.2901 - 0.2165i  -0.1894 + 0.0000i  -0.2577 + 0.0000i   0.5317 + 0.0000i
```

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
8.97e-01	-1.00e+00	8.97e-01	-1.12e+00
9.01e-01 + 1.98e-02i	-1.00e+00	9.01e-01	-1.11e+00
9.01e-01 - 1.98e-02i	-1.00e+00	9.01e-01	-1.11e+00
9.94e-01 + 9.89e-03i	-1.00e+00	9.94e-01	-1.01e+00
9.94e-01 - 9.89e-03i	-1.00e+00	9.94e-01	-1.01e+00
9.99e-01	-1.00e+00	9.99e-01	-1.00e+00
9.99e-01	-1.00e+00	9.99e-01	-1.00e+00
9.99e-01	-1.00e+00	9.99e-01	-1.00e+00

Gains & Weighted Matrices

- Chosen Weighted Matrices

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 49 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.7000 & 0 \\ 0 & 30.0000 \end{bmatrix}$$

Gains & Weighted Matrices

Q_hat =

Columns 1 through 7

0.0541	-0.0857	0.0087	-0.0000	0.0008	-0.0065	-0.0011
-0.0857	2.4674	0.0028	0.0012	0.0000	0.1422	0.0183
0.0087	0.0028	0.5013	0.0001	0.0612	-0.0003	-0.0100
-0.0000	0.0012	0.0001	0.0500	0.0000	0.0000	0.0000
0.0008	0.0000	0.0612	0.0000	2.4500	-0.0000	-0.0008
-0.0065	0.1422	-0.0003	0.0000	-0.0000	0.1054	0.0014
-0.0011	0.0183	-0.0100	0.0000	-0.0008	0.0014	0.2217
0.0000	0.0000	0.0000	0.0000	0.0013	-0.0000	-0.0000

Column 8

0.0000
0.0000
0.0000
0.0000
0.0013
-0.0000
-0.0000
0.0500

M1 =

-0.0013	-0.0002
0.0247	0.0032
-0.0000	-0.0017
0.0000	0.0000
-0.0000	-0.0001
0.0253	0.0003
0.0003	0.0543
-0.0000	-0.0000

K =

Columns 1 through 7

0.9645	3.9030	0.4946	0.5145	1.8352	1.0186	0.0612
-0.8623	-0.0026	-1.5041	0.0049	-1.5407	0.0036	0.2051

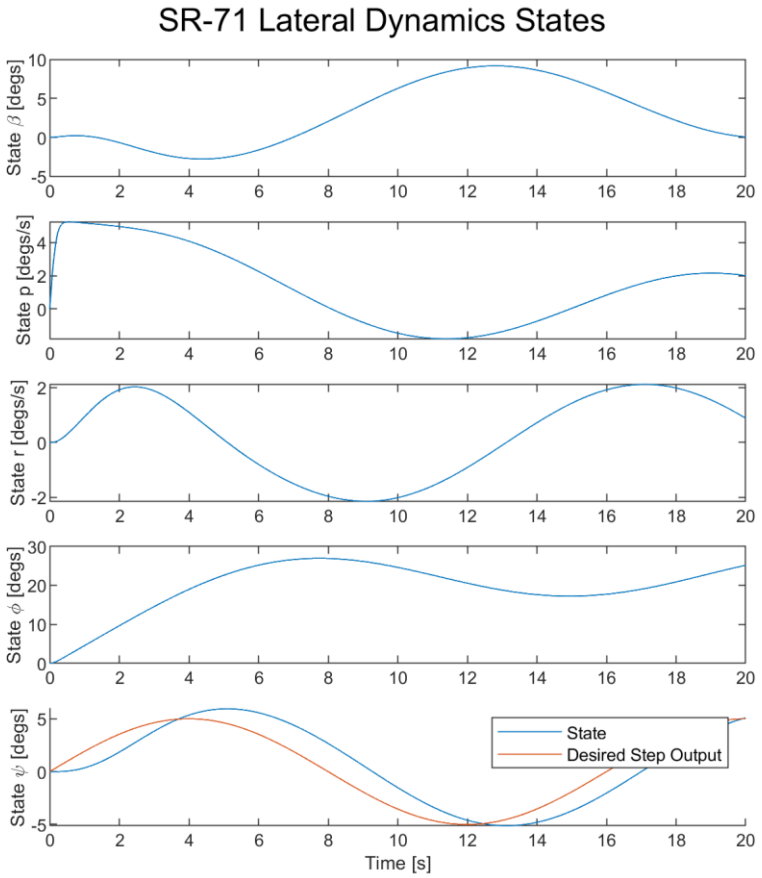
Column 8

0.0930
-0.1707

R_hat1 =

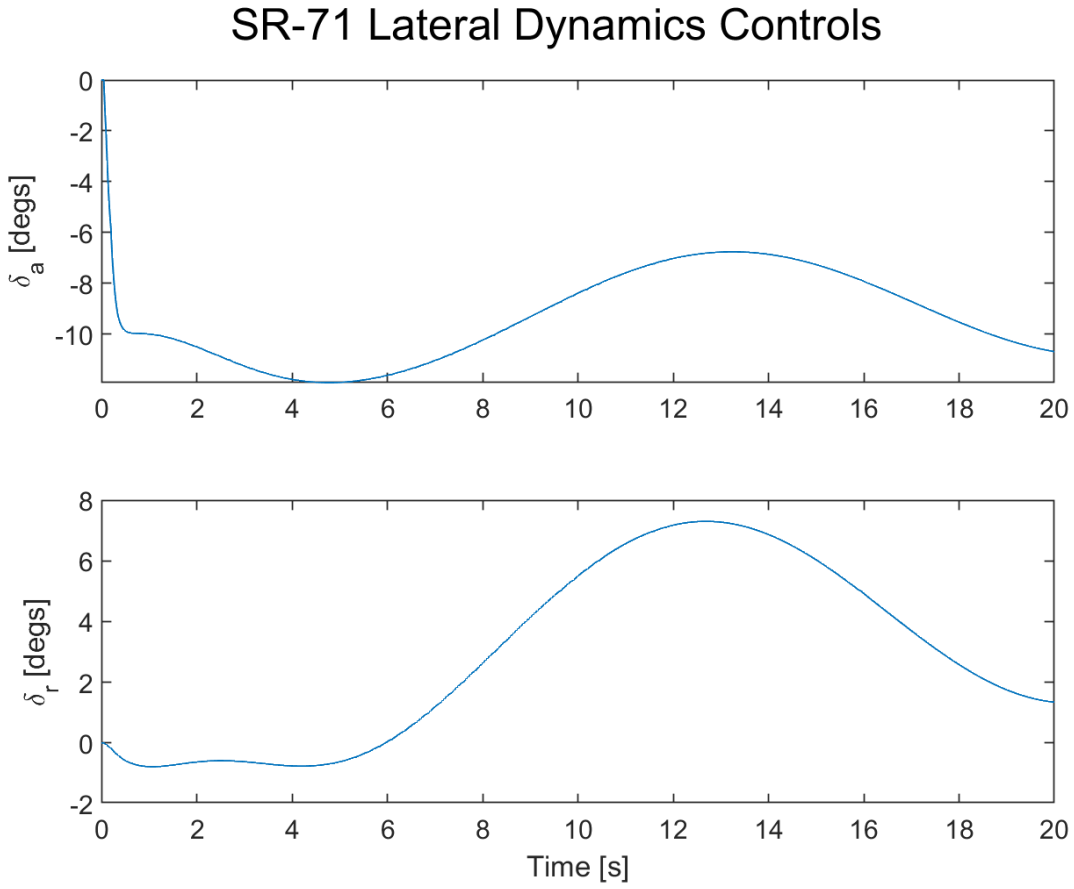
0.0942	0.0001
0.0001	1.5204

Plots - States



- The simulated state followed the desired state output with minimal error
  - System performed better without PI
    - Required faster sampling time

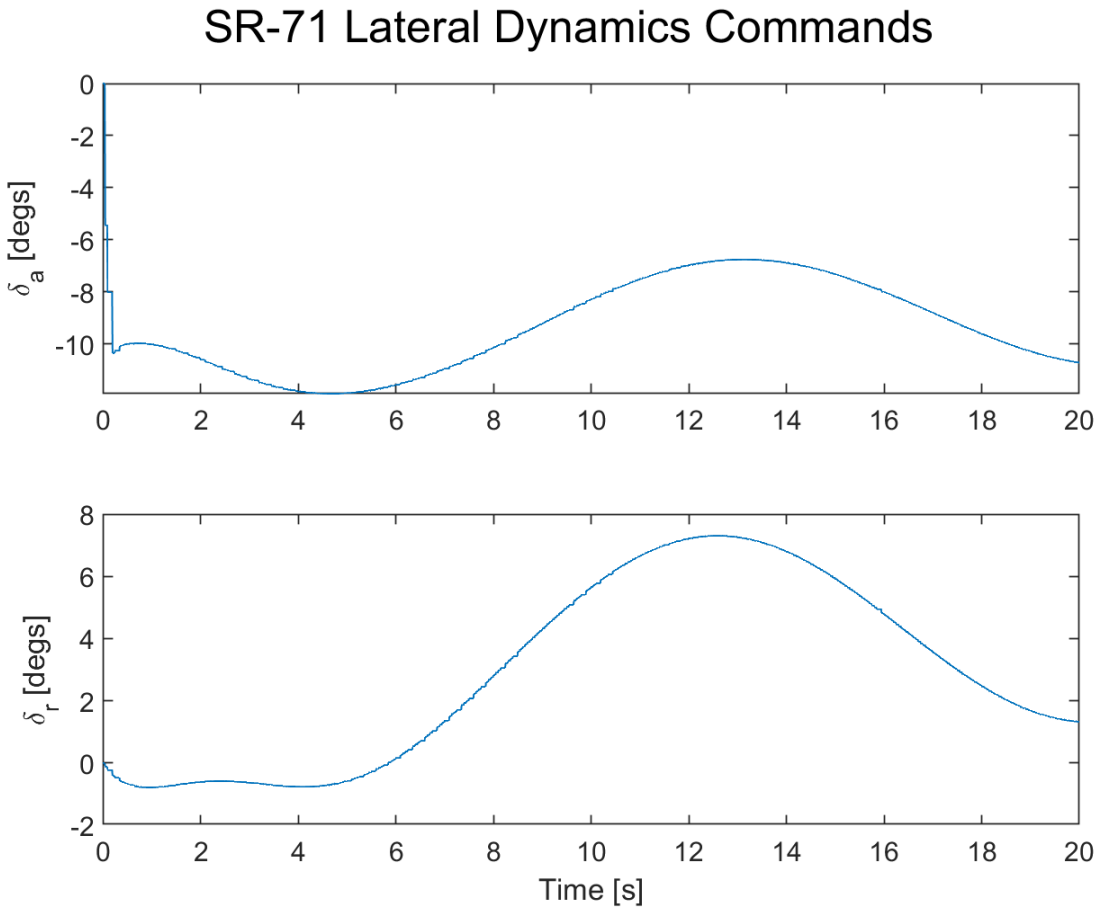
Plots - Controls



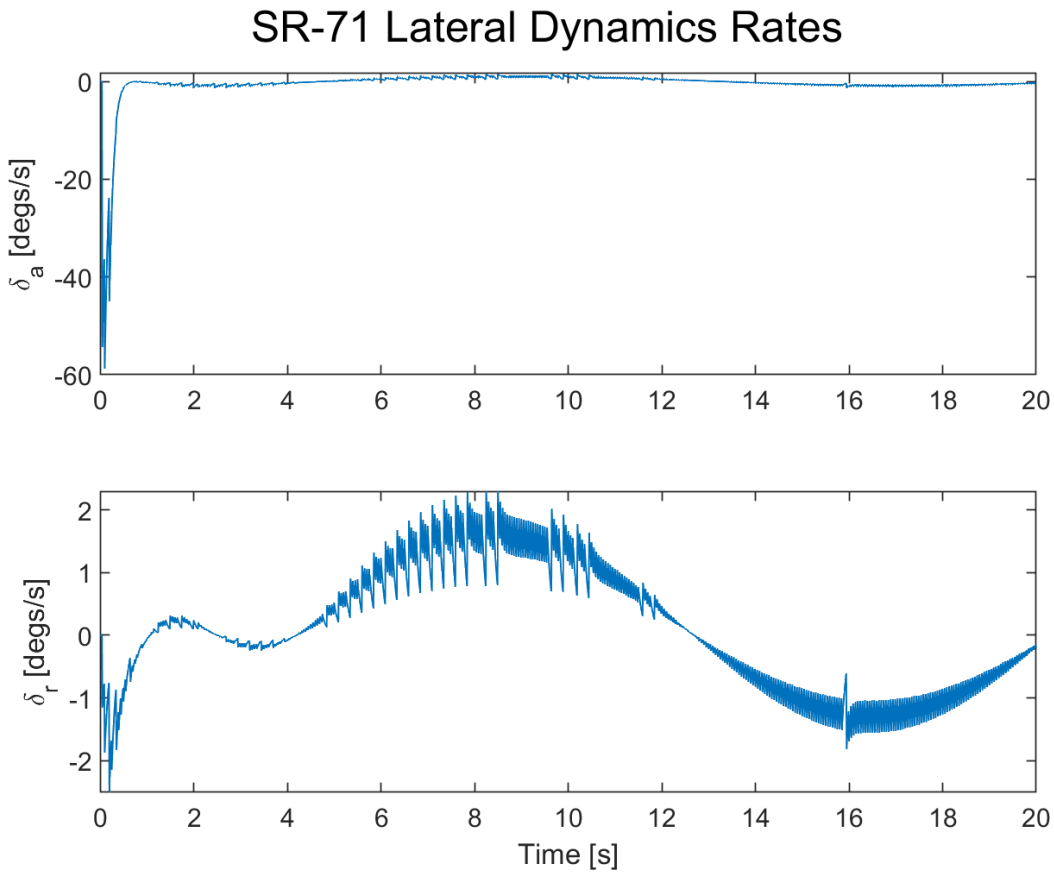
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands



Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$

Closed-Loop || PIF-NZSP-CRW Design
Step Input

Set-Up

- New State and Control Matrices

```
A_new =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0139    0.9974    0.0003   -0.0000    0.0000    0.0257    0.0033
    0.0080    0.0000    0.9998    0.0000    0.0000   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000    0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
         0         0         0         0         0    0.9048         0
         0         0         0         0         0         0    0.9048
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0    0.0100         0         0

  Columns 8 through 10

    0.0000    0.0000         0
    0.0013    0.0002         0
   -0.0000   -0.0005         0
    0.0000    0.0000         0
   -0.0000   -0.0000         0
    0.0952         0         0
         0    0.0952         0
    1.0000         0         0
         0    1.0000         0
         0         0    1.0000
```

```
B_new =

         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    0.0100         0
         0    0.0100
         0         0
```

Set-Up

- Observable: $rank(O) = 10$
- Controllable: $rank(C) = 10$

System is observable and controllable.

Characteristics

- Closed Loop A Matrix

```
A_cl =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0139    0.9974    0.0003   -0.0000    0.0000    0.0257    0.0033
    0.0080    0.0000    0.9998    0.0000    0.0000   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000    0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
         0         0         0         0         0    0.9048         0
         0         0         0         0         0         0    0.9048
    0.0033   -0.0153   -0.0103   -0.0074   -0.0068   -0.0060    0.0003
    0.0339    0.0025    0.0616    0.0003    0.0597    0.0003   -0.0062
         0         0         0         0    0.0100         0         0

  Columns 8 through 10

    0.0000    0.0000         0
    0.0013    0.0002         0
   -0.0000   -0.0005         0
    0.0000    0.0000         0
   -0.0000   -0.0000         0
    0.0952         0         0
         0    0.0952         0
    0.9471    0.0013    0.0000
    0.0004    0.9641    0.0051
         0         0    1.0000
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

   0.9060 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9094 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9505 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9816 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 8

   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.9912 + 0.0140i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9912 - 0.0140i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9946 + 0.0049i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9946 - 0.0049i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 9 through 10

   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.9996 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9991 + 0.0000i
```

Characteristics

- Eigenvectors

V_cl =

Columns 1 through 4

0.0092 + 0.0000i	0.0038 + 0.0000i	0.0111 + 0.0000i	-0.1598 + 0.0000i
-0.0347 + 0.0000i	-0.2802 + 0.0000i	-0.4477 + 0.0000i	0.0142 + 0.0000i
0.0939 + 0.0000i	0.0044 + 0.0000i	0.0058 + 0.0000i	-0.2965 + 0.0000i
0.0024 + 0.0000i	0.0295 + 0.0000i	0.0881 + 0.0000i	0.0104 + 0.0000i
-0.0095 + 0.0000i	-0.0005 + 0.0000i	-0.0011 + 0.0000i	0.1595 + 0.0000i
-0.0008 + 0.0000i	0.9583 + 0.0000i	0.8021 + 0.0000i	0.0042 + 0.0000i
0.9948 + 0.0000i	0.0016 + 0.0000i	0.0016 + 0.0000i	-0.7191 + 0.0000i
-0.0000 + 0.0000i	0.0462 + 0.0000i	0.3851 + 0.0000i	0.0034 + 0.0000i
0.0119 + 0.0000i	0.0001 + 0.0000i	0.0008 + 0.0000i	-0.5799 + 0.0000i
0.0010 + 0.0000i	0.0001 + 0.0000i	0.0002 + 0.0000i	-0.0866 + 0.0000i

Columns 5 through 8

-0.0357 + 0.2354i	-0.0357 - 0.2354i	0.0738 + 0.0098i	0.0738 - 0.0098i
-0.2879 - 0.1375i	-0.2879 + 0.1375i	-0.4297 + 0.3875i	-0.4297 - 0.3875i
0.2764 + 0.2363i	0.2764 - 0.2363i	0.0019 + 0.0130i	0.0019 - 0.0130i
0.0245 + 0.1661i	0.0245 - 0.1661i	0.7891 + 0.0000i	0.7891 + 0.0000i
0.0334 - 0.2158i	0.0334 + 0.2158i	0.0100 - 0.0149i	0.0100 + 0.0149i
0.0426 - 0.0002i	0.0426 + 0.0002i	0.0034 - 0.1167i	0.0034 + 0.1167i
0.5762 + 0.0000i	0.5762 + 0.0000i	0.0700 + 0.0212i	0.0700 - 0.0212i
0.0387 + 0.0061i	0.0387 - 0.0061i	0.0092 - 0.1098i	0.0092 + 0.1098i
0.5228 + 0.0850i	0.5228 - 0.0850i	0.0649 + 0.0236i	0.0649 - 0.0236i
-0.1209 + 0.0522i	-0.1209 - 0.0522i	-0.0237 + 0.0060i	-0.0237 - 0.0060i

Columns 9 through 10

-0.2957 + 0.0000i	0.1235 + 0.0000i
-0.0385 + 0.0000i	0.0281 + 0.0000i
-0.0003 + 0.0000i	0.0080 + 0.0000i
0.8599 + 0.0000i	-0.3112 + 0.0000i
0.0062 + 0.0000i	-0.0861 + 0.0000i
-0.1235 + 0.0000i	0.0519 + 0.0000i
-0.2477 + 0.0000i	0.1041 + 0.0000i
-0.1229 + 0.0000i	0.0514 + 0.0000i
-0.2465 + 0.0000i	0.1031 + 0.0000i
-0.1384 + 0.0000i	0.9235 + 0.0000i

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
9.06e-01	-1.00e+00	9.06e-01	-1.10e+00
9.09e-01	-1.00e+00	9.09e-01	-1.10e+00
9.51e-01	-1.00e+00	9.51e-01	-1.05e+00
9.82e-01	-1.00e+00	9.82e-01	-1.02e+00
9.91e-01 + 1.40e-02i	-1.00e+00	9.91e-01	-1.01e+00
9.91e-01 - 1.40e-02i	-1.00e+00	9.91e-01	-1.01e+00
9.95e-01 + 4.90e-03i	-1.00e+00	9.95e-01	-1.01e+00
9.95e-01 - 4.90e-03i	-1.00e+00	9.95e-01	-1.01e+00
9.99e-01	-1.00e+00	9.99e-01	-1.00e+00
1.00e+00	-1.00e+00	1.00e+00	-1.00e+00

Gains & Weighted Matrices

- Chosen Weighted Matrices

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.0000 & 0 \\ 0 & 3.2000 \end{bmatrix}$$

Gains & Weighted Matrices

Q_hat =

Columns 1 through 7

0.0504	-0.0016	0.0139	-0.0000	0.0017	-0.0001	-0.0004
-0.0016	0.0494	0.0001	0.0012	0.0000	0.0028	0.0004
0.0139	0.0001	0.7530	0.0001	0.1249	-0.0007	-0.0150
-0.0000	0.0012	0.0001	0.0500	0.0000	0.0000	0.0000
0.0017	0.0000	0.1249	0.0000	5.0001	-0.0001	-0.0017
-0.0001	0.0028	-0.0007	0.0000	-0.0001	0.3163	0.0000
-0.0004	0.0004	-0.0150	0.0000	-0.0017	0.0000	0.3164
-0.0000	0.0005	-0.0001	0.0000	-0.0000	0.0775	0.0000
-0.0001	0.0001	-0.0026	0.0000	-0.0002	0.0000	0.0775
0.0000	0.0000	0.0000	0.0000	0.0013	-0.0000	-0.0000

Columns 8 through 10

-0.0000	-0.0001	0.0000
0.0005	0.0001	0.0000
-0.0001	-0.0026	0.0000
0.0000	0.0000	0.0000
-0.0000	-0.0002	0.0013
0.0775	0.0000	-0.0000
0.0000	0.0775	-0.0000
1.0291	0.0000	-0.0000
0.0000	0.0791	-0.0000
-0.0000	-0.0000	0.0500

M1 =

-0.0000	-0.0000
0.0000	0.0000
-0.0000	-0.0000
0.0000	0.0000
-0.0000	-0.0000
0.0013	0.0000
0.0000	0.0013
0.0256	0.0000
0.0000	0.0018
-0.0000	-0.0000

R_hat1 =

0.0508	0.0000
0.0000	0.1601

K =

Columns 1 through 7

-0.3343	1.5257	1.0302	0.7384	0.6844	0.6019	-0.0343
-3.3899	-0.2521	-6.1625	-0.0349	-5.9683	-0.0321	0.6175

Columns 8 through 10

5.2945	-0.1281	-0.0031
-0.0448	3.5885	-0.5088

Characteristics

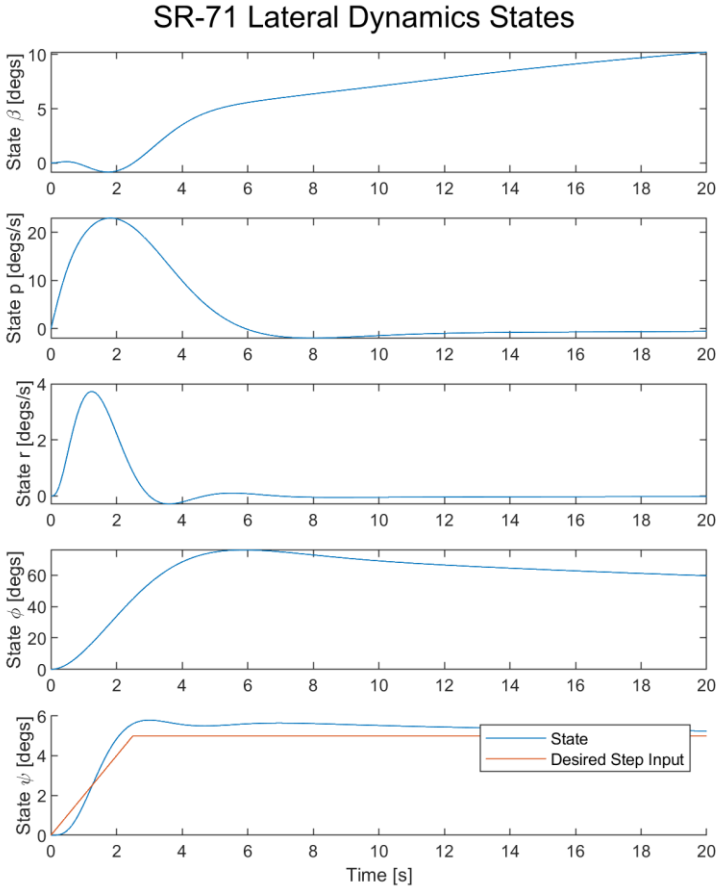
- Steady State Values

```
x_star =

    0.0000
   -0.0000
   -0.0000
   -0.0006
    0.0873
         0
    0.0000
```

```
u_star =

   1.8642e-06
```

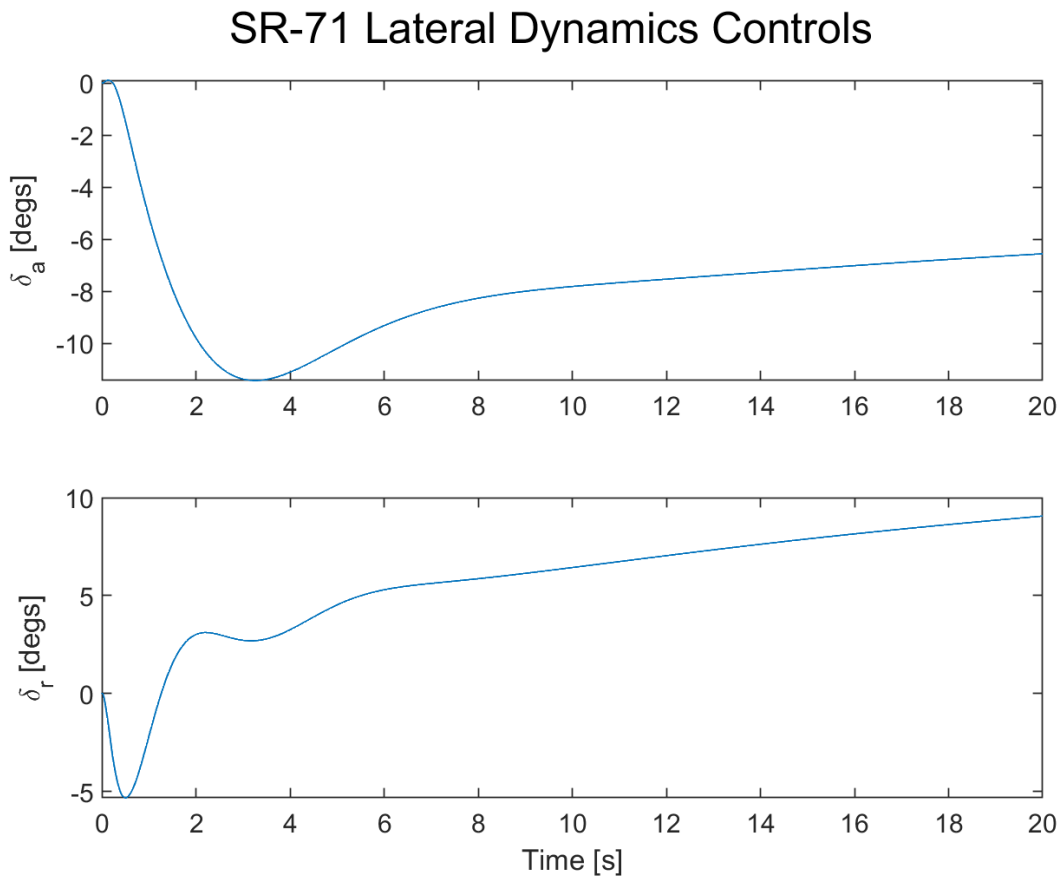
Units are in radians.

Plots - States



- Less error than PI-NZSP
- Does not quite settle at correct value
- Less overshoot in ψ

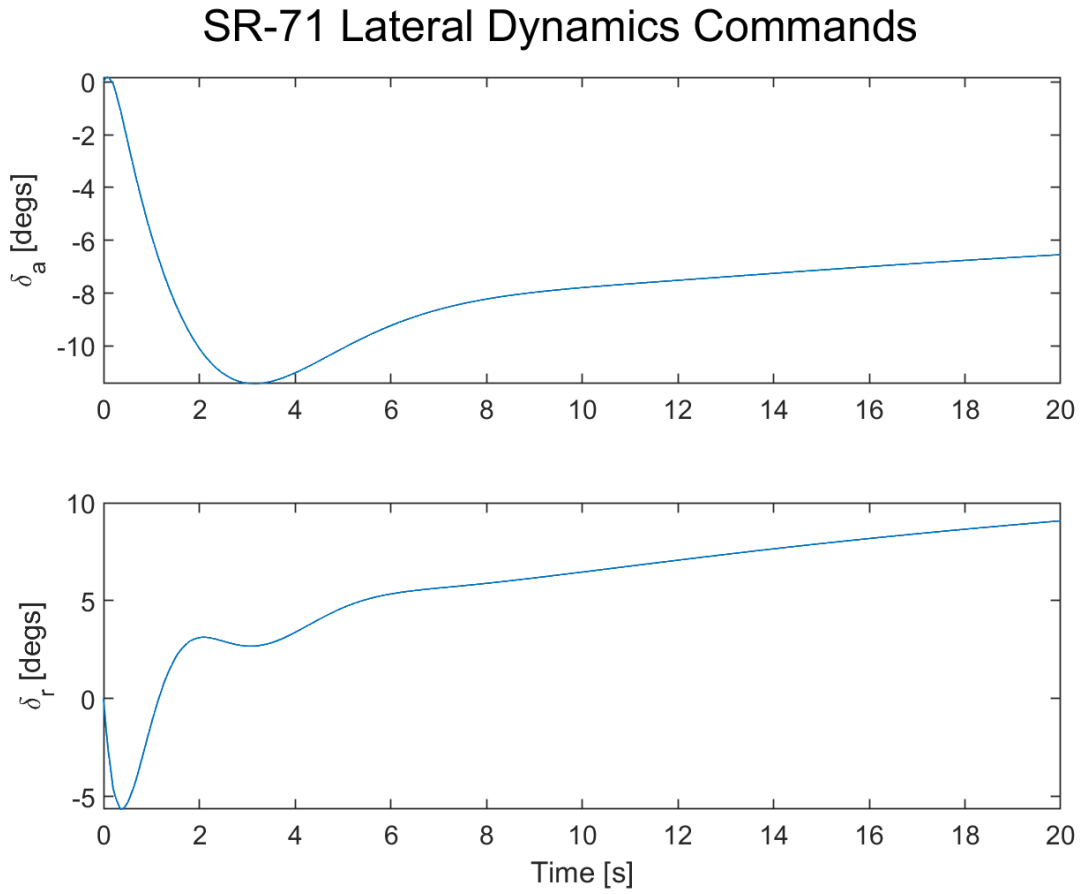
Plots - Controls



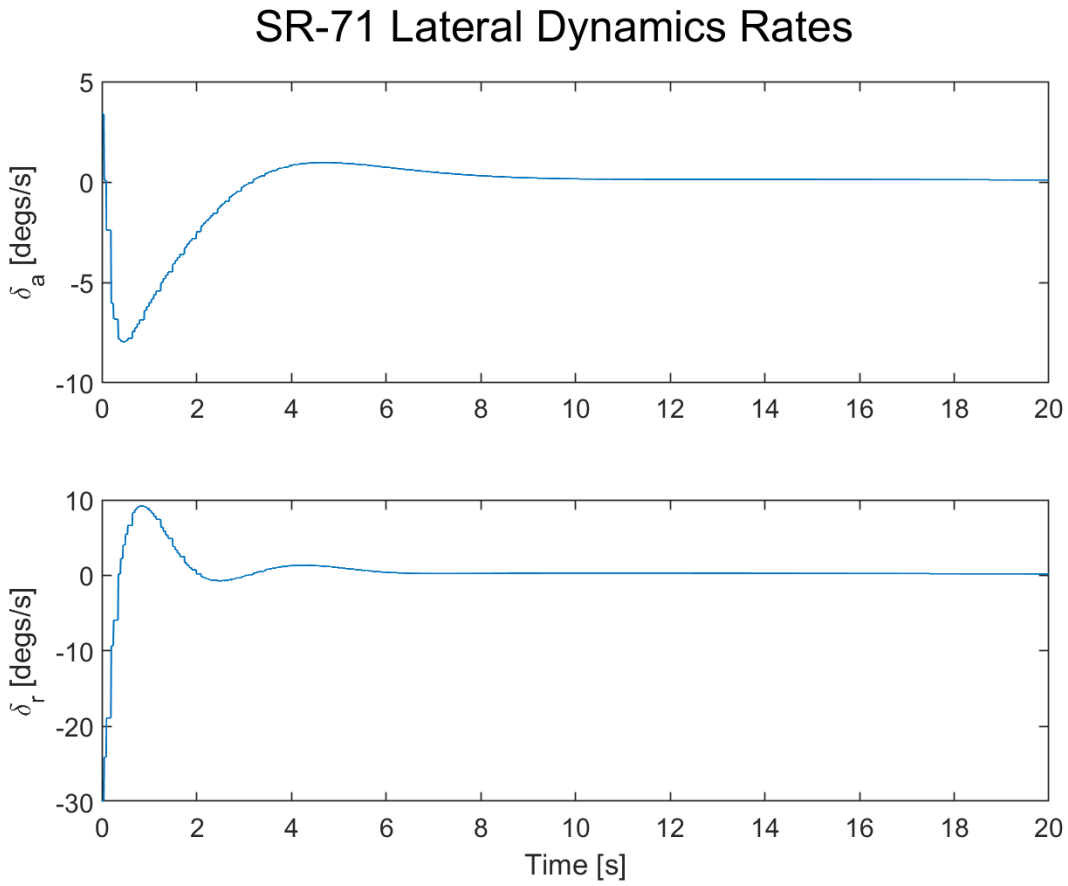
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands

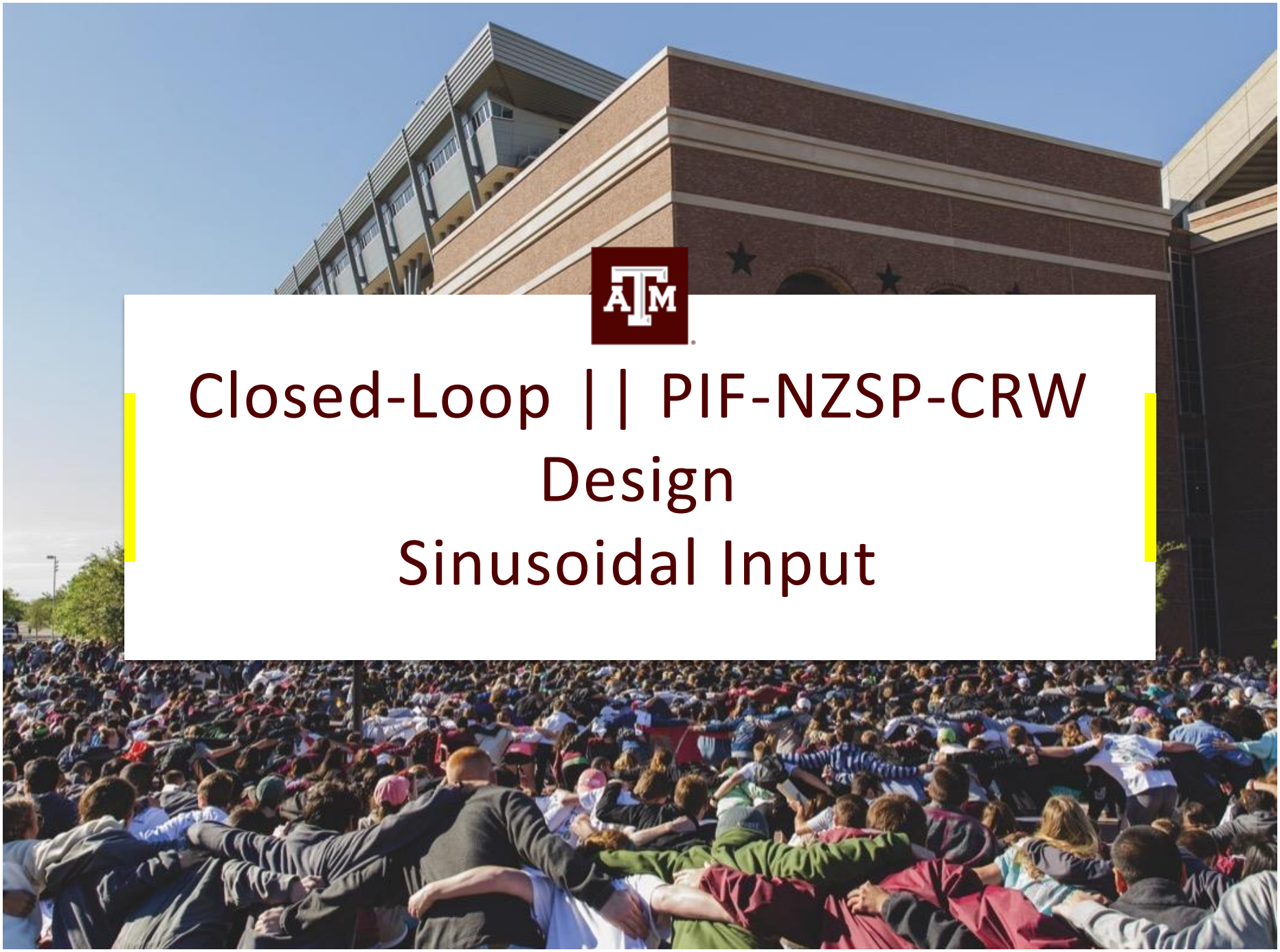


Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$



Closed-Loop || PIF-NZSP-CRW Design
Sinusoidal Input

Set-Up

- New State and Control Matrices

```
A_new =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0139    0.9974    0.0003   -0.0000    0.0000    0.0257    0.0033
    0.0080    0.0000    0.9998    0.0000    0.0000   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000    0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
         0         0         0         0         0    0.9048         0
         0         0         0         0         0         0    0.9048
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0    0.0100         0         0

  Columns 8 through 10

    0.0000    0.0000         0
    0.0013    0.0002         0
   -0.0000   -0.0005         0
    0.0000    0.0000         0
   -0.0000   -0.0000         0
    0.0952         0         0
         0    0.0952         0
    1.0000         0         0
         0    1.0000         0
         0         0    1.0000
```

```
B_new =

         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    0.0100         0
         0    0.0100
         0         0
```

Set-Up

- Observable: $rank(O) = 10$
- Controllable: $rank(C) = 10$

System is observable and controllable.

Characteristics

- Closed Loop A Matrix

```
A_cl =

  Columns 1 through 7

    0.9996    0.0011   -0.0099    0.0001    0.0000    0.0000    0.0001
   -0.0139    0.9974    0.0003   -0.0000    0.0000    0.0257    0.0033
    0.0080    0.0000    0.9998    0.0000    0.0000   -0.0004   -0.0089
   -0.0001    0.0100    0.0011    1.0000    0.0000    0.0001    0.0000
    0.0000    0.0000    0.0100    0.0000    1.0000   -0.0000   -0.0000
         0         0         0         0         0    0.9048         0
         0         0         0         0         0         0    0.9048
    0.0257   -0.0865   -0.0211   -0.0826   -0.0133   -0.0213   -0.0010
    0.0486   -0.0029    0.0928   -0.0046    0.0959   -0.0011   -0.0126
         0         0         0         0    0.0100         0         0

  Columns 8 through 10

    0.0000    0.0000         0
    0.0013    0.0002         0
   -0.0000   -0.0005         0
    0.0000    0.0000         0
   -0.0000   -0.0000         0
    0.0952         0         0
         0    0.0952         0
    0.9350   -0.0018   -0.0008
   -0.0018    0.9470    0.0086
         0         0    1.0000
```

Characteristics

- Eigenvalues

```
D_cl =

  Columns 1 through 4

   0.9041 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9149 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9538 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9734 + 0.0302i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 5 through 8

   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.9734 - 0.0302i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9920 + 0.0121i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9920 - 0.0121i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.9991 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Columns 9 through 10

   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.9997 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.9860 + 0.0000i
```

Unstable System

Characteristics

- Eigenvectors

```
V_cl =

  Columns 1 through 4                                                                                  Columns 9 through 10

   0.0035 + 0.0000i   0.0112 + 0.0000i  -0.0350 + 0.0000i  -0.0044 + 0.0177i      0.4973 + 0.0000i   0.0608 + 0.0000i
  -0.2654 + 0.0000i  -0.0295 + 0.0000i   0.0311 + 0.0000i  -0.2931 - 0.3763i      0.0024 + 0.0000i  -0.7291 + 0.0000i
   0.0056 + 0.0000i   0.1033 + 0.0000i  -0.1675 + 0.0000i   0.0102 + 0.0183i      0.0005 + 0.0000i   0.0058 + 0.0000i
   0.0263 + 0.0000i   0.0020 + 0.0000i  -0.0026 + 0.0000i  -0.0235 + 0.1140i     -0.0747 + 0.0000i   0.5177 + 0.0000i
  -0.0006 + 0.0000i  -0.0116 + 0.0000i   0.0354 + 0.0000i   0.0018 - 0.0048i     -0.0177 + 0.0000i  -0.0041 + 0.0000i
   0.9636 + 0.0000i  -0.0279 + 0.0000i   0.0443 + 0.0000i   0.6813 + 0.0000i      0.2027 + 0.0000i   0.3346 + 0.0000i
   0.0164 + 0.0000i   0.9882 + 0.0000i  -0.8740 + 0.0000i   0.0523 + 0.0315i      0.4166 + 0.0000i   0.0431 + 0.0000i
  -0.0079 + 0.0000i  -0.0030 + 0.0000i   0.0228 + 0.0000i   0.4911 + 0.2164i      0.2020 + 0.0000i   0.2855 + 0.0000i
  -0.0001 + 0.0000i   0.1044 + 0.0000i  -0.4495 + 0.0000i   0.0277 + 0.0394i      0.4152 + 0.0000i   0.0367 + 0.0000i
   0.0001 + 0.0000i   0.0014 + 0.0000i  -0.0077 + 0.0000i  -0.0012 + 0.0005i      0.5648 + 0.0000i   0.0030 + 0.0000i

  Columns 5 through 8

  -0.0044 - 0.0177i  -0.0223 + 0.2777i  -0.0223 - 0.2777i   0.1297 + 0.0000i
  -0.2931 + 0.3763i  -0.0032 - 0.0121i  -0.0032 + 0.0121i   0.0005 + 0.0000i
   0.0102 - 0.0183i   0.3281 + 0.2379i   0.3281 - 0.2379i   0.0077 + 0.0000i
  -0.0235 - 0.1140i  -0.0042 - 0.0250i  -0.0042 + 0.0250i  -0.0148 + 0.0000i
   0.0018 + 0.0048i   0.0145 - 0.2777i   0.0145 + 0.2777i  -0.0868 + 0.0000i
   0.6813 + 0.0000i  -0.0808 + 0.1420i  -0.0808 - 0.1420i   0.0527 + 0.0000i
   0.0523 - 0.0315i   0.5657 + 0.0000i   0.5657 + 0.0000i   0.1093 + 0.0000i
   0.4911 - 0.2164i  -0.0922 + 0.1198i  -0.0922 - 0.1198i   0.0522 + 0.0000i
   0.0277 - 0.0394i   0.5183 + 0.0721i   0.5183 - 0.0721i   0.1083 + 0.0000i
  -0.0012 - 0.0005i  -0.1654 + 0.0967i  -0.1654 - 0.0967i   0.9727 + 0.0000i
```

Characteristics

- Damping Rations, Frequencies, and Rise Time Constants

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
9.04e-01	-1.00e+00	9.04e-01	-1.11e+00
9.15e-01	-1.00e+00	9.15e-01	-1.09e+00
9.54e-01	-1.00e+00	9.54e-01	-1.05e+00
9.73e-01 + 3.02e-02i	-1.00e+00	9.74e-01	-1.03e+00
9.73e-01 - 3.02e-02i	-1.00e+00	9.74e-01	-1.03e+00
9.86e-01	-1.00e+00	9.86e-01	-1.01e+00
9.92e-01 + 1.21e-02i	-1.00e+00	9.92e-01	-1.01e+00
9.92e-01 - 1.21e-02i	-1.00e+00	9.92e-01	-1.01e+00
9.99e-01	-1.00e+00	9.99e-01	-1.00e+00
1.00e+00	-1.00e+00	1.00e+00	-1.00e+00

Gains & Weighted Matrices

- Chosen Weighted Matrices

$$Q = \begin{bmatrix} 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Gains & Weighted Matrices

Q_hat =

Columns 1 through 7

2.4977	-0.0789	-0.0610	-0.0021	0.0008	-0.0062	0.0004
-0.0789	2.4715	0.0030	0.1244	0.0000	0.1424	0.0183
-0.0610	0.0030	0.0541	0.0142	0.0625	0.0002	-0.0010
-0.0021	0.1244	0.0142	5.0000	0.0000	0.0050	0.0004
0.0008	0.0000	0.0625	0.0000	2.5000	-0.0000	-0.0009
-0.0062	0.1424	0.0002	0.0050	-0.0000	0.0422	0.0014
0.0004	0.0183	-0.0010	0.0004	-0.0009	0.0014	0.6955
-0.0012	0.0247	0.0000	0.0006	-0.0000	0.0098	0.0003
0.0000	0.0032	-0.0002	0.0001	-0.0001	0.0003	0.1703
0.0000	0.0000	0.0000	0.0000	0.0013	-0.0000	-0.0000

Columns 8 through 10

-0.0012	0.0000	0.0000
0.0247	0.0032	0.0000
0.0000	-0.0002	0.0000
0.0006	0.0001	0.0000
-0.0000	-0.0001	0.0013
0.0098	0.0003	-0.0000
0.0003	0.1703	-0.0000
0.0533	0.0001	-0.0000
0.0001	0.1141	-0.0000
-0.0000	-0.0000	0.0500

M1 =

-0.0000	-0.0000
0.0003	0.0000
0.0000	-0.0000
0.0000	0.0000
-0.0000	-0.0000
0.0002	0.0000
0.0000	0.0028
0.0013	0.0000
0.0000	0.0025
-0.0000	-0.0000

R_hat1 =

0.0500	0.0000
0.0000	0.0501

K =

Columns 1 through 7

-2.5700	8.6500	2.1085	8.2588	1.3338	2.1284	0.1016
-4.8555	0.2893	-9.2766	0.4564	-9.5892	0.1126	1.2644

Columns 8 through 10

6.5008	0.1773	0.0756
0.1799	5.2951	-0.8645

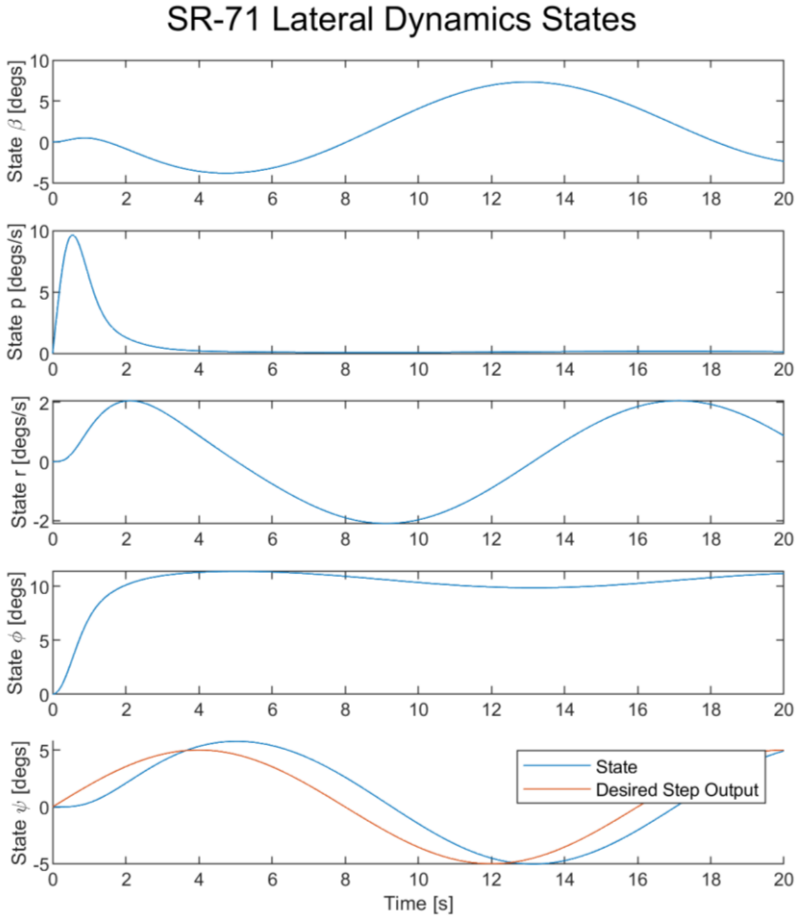
Characteristics

- Steady State Values

```
x_star =

    0.0000
   -0.0000
   -0.0000
   -0.0006
    0.0873
         0
    0.0000

u_star =

    1.8642e-06
```

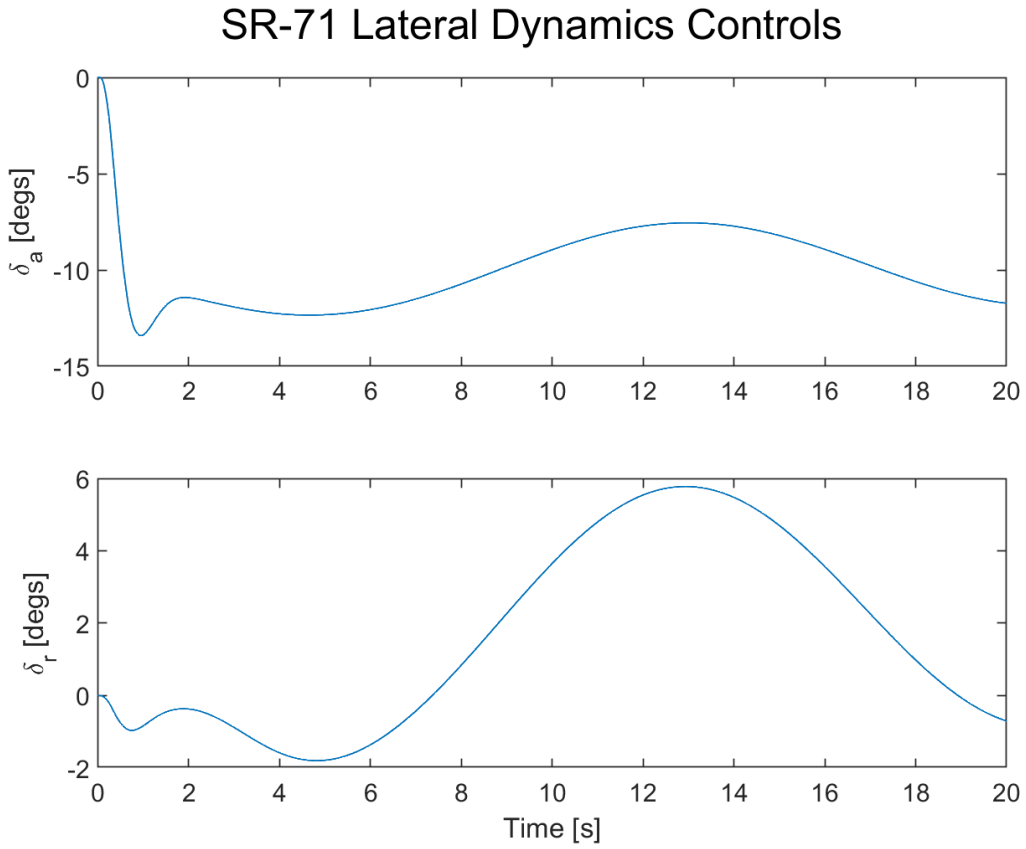
Units are in radians.

Plots - States



- The simulated state followed the desired state output with minimal error
  - Comparable to PI at low frequencies
    - Performs better than PI at higher frequencies
    - Easier on controls than PI

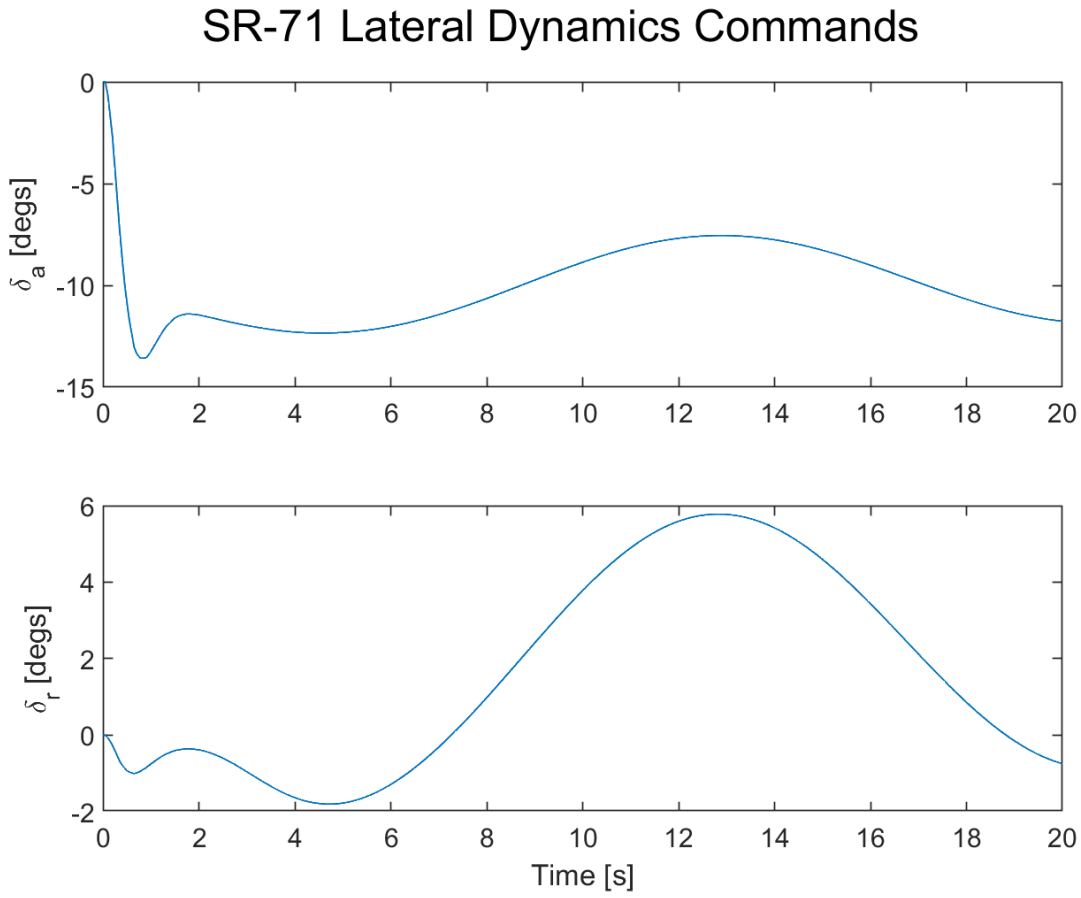
Plots - Controls



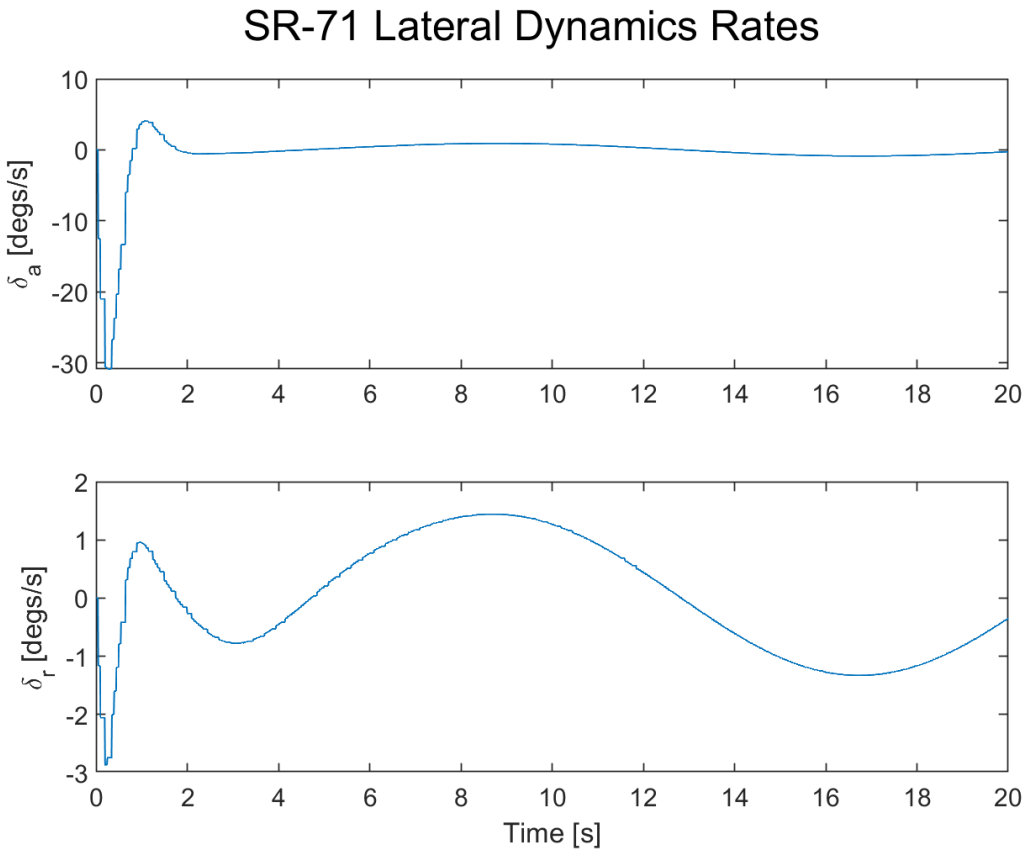
Requirement:
Aileron positions shall stay under $\pm 14.0\ degs$

Requirement:
Rudder positions shall stay under $\pm 10.0\ degs$

Plots - Commands



Plots - Rates



Requirement:
Aileron rates shall stay under
$\pm 65.0\ degs/sec$

Requirement:
Rudder rates shall stay under
$\pm 30.0\ degs/sec$

Closing Comments

Closing Comments

- SDR
  - Hard to get a good disturbance response
    - Could not bring all states to 0
  - Was able to meet physical requirements on control surfaces

Closing Comments

- PI-SDR
  - Much harder to meet physical requirements of control surfaces compared to SDR
  - Easy to saturate R matrix trying to meet physical requirements

Closing Comments

- NZSP
  - Focused on ψ since a tracking problem
  - Was able to meet physical requirements on control surfaces
  - Tried to minimize overshoot
    - Consequently, slowed convergence time
  - Sinusoidal tracking had visibly reduced amplitude

Closing Comments

- PI-NZSP
  - Much greater overshoot on ψ state
    - Makes sense since PI controllers are supposed to increase rise time and decrease settling time
  - Sinusoidal tracking was about the same result as without PI
    - Was using a faster sampling rate in this design

Closing Comments

- PIF-NZSP-CRW
  - Step track
    - Much better results in comparison to just PI-NZSP
    - Preference to the NZSP controller
  - Sinusoidal track
    - Preferred method to track sinusoidal waves at higher frequencies
    - As shown, not a large difference at low frequencies

Next Iteration

- Iterate different sampling rates into design
  - Cut sampling time to be significantly smaller after noticing an inability to control system
  - See if there is an in-between value that produces acceptable results

Next Iteration

- Iterate different frequencies for sinusoid tracking
- “Bake in” frequency analysis as controllers are designed
  - Design more robust controllers
  - Not reserved for post design analysis