

2.a

$$\frac{\partial L_n(\omega, x_n, y_n)}{\partial \omega_i} = [\sigma(\omega, x_n) - y_n] \cdot \frac{\partial \sigma(\omega, x_n)}{\partial \omega_i}$$

$$L_n = \frac{1}{2} [\sigma(\omega, x_n) - y_n]^2$$

$$\sigma(\omega, x_n) = \frac{1}{1 + e^{-\omega^T x_n}}$$

$$\begin{aligned} \frac{\partial \sigma(\omega, x_n)}{\partial \omega_i} &= \frac{\partial [(1 + e^{-\omega^T x_n})^{-1}]}{\partial \omega_i} = -(1 + e^{-\omega^T x_n})^{-2} \cdot \frac{\partial (e^{-\omega^T x_n})}{\partial \omega_i} \\ &= -(1 + e^{-\omega^T x_n})^{-2} \cdot \frac{\partial (e^{-\sum \omega_j x_{nj}})}{\partial \omega_i} = -(1 + e^{-\omega^T x_n})^{-2} \cdot (-x_i e^{-\sum \omega_j x_{nj}}) \end{aligned}$$

$$\cancel{\frac{x_i}{1 + e^{-\omega^T x_n}}} = \frac{x_i e^{-\sum \omega_j x_{nj}}}{1 + e^{-\omega^T x_n}} = x_i \cdot \sigma(\omega, x_n) \cdot \frac{e^{-\sum \omega_j x_{nj}}}{1 + e^{-\omega^T x_n}}$$

$$= x_i \cdot \sigma(\omega, x_n) \cdot \frac{e^{-\omega^T x_n}}{1 + e^{-\omega^T x_n}} = x_i \cdot \sigma(\omega, x_n) \cdot \frac{1 + e^{-\omega^T x_n} - 1}{1 + e^{-\omega^T x_n}}$$

$$= x_i \cdot \sigma(\omega, x_n) \cdot \left[1 - \frac{1}{1 + e^{-\omega^T x_n}} \right] = x_i \cdot \sigma(\omega, x_n) \cdot [1 - \sigma(\omega, x_n)]$$

So:

$$\frac{\partial L_n(\omega, x_n, y_n)}{\partial \omega_i} = [\sigma(\omega, x_n) - y_n] \cdot x_i \cdot \sigma(\omega, x_n) \cdot (1 - \sigma(\omega, x_n))$$