

$$L_{\text{simple}}(w) = [\sigma(w, (1)) - 1]^2 + [\sigma(w, (0))]^2 + [\sigma(w, (1)) - 1]^2$$

$$\sigma(w, x) = \frac{1}{1 + e^{-w^T x}}, \quad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\sigma(w, (1)) = \frac{1}{1 + e^{-[w_1, w_2] \begin{bmatrix} 1 \\ 0 \end{bmatrix}}} = \frac{1}{1 + e^{-w_1}}$$

$$\sigma(w, (0)) = \frac{1}{1 + e^{-[w_1, w_2] \begin{bmatrix} 0 \\ 1 \end{bmatrix}}} = \frac{1}{1 + e^{-w_2}}$$

$$\sigma(w, (1)) = \frac{1}{1 + e^{-[w_1, w_2] \begin{bmatrix} 1 \\ 1 \end{bmatrix}}} = \frac{1}{1 + e^{-w_1 - w_2}}$$

$$\Rightarrow L_{\text{simple}}(w) = \left(\frac{1}{1 + e^{-w_1}} - 1 \right)^2 + \left(\frac{1}{1 + e^{-w_2}} \right)^2 + \left(\frac{1}{1 + e^{-w_1 - w_2}} - 1 \right)^2$$

$$\nabla_w L_{\text{simple}}(w) = \left[\frac{\partial L_{\text{simple}}(w)}{\partial w_1}, \frac{\partial L_{\text{simple}}(w)}{\partial w_2} \right]$$

Using Wolfram Alpha:

$$\frac{\partial L_{\text{simple}}(w)}{\partial w_1} = -2e^{w_1} (6e^{w_1 + w_2} + 3e^{2(w_1 + w_2)} + 3e^{3(w_1 + w_2)} + 3e^{2w_1 + w_2} + 3e^{w_2} + 1)$$

$$\frac{\partial L_{\text{simple}}(w)}{\partial w_1} = \frac{-2e^{w_1} (6e^{w_1 + w_2} + 3e^{2(w_1 + w_2)} + 3e^{3(w_1 + w_2)} + 3e^{2w_1 + w_2} + 3e^{w_2} + 1)}{(e^{w_1} + 1)^3 (e^{w_1 + w_2} + 1)^3}$$

$$\frac{\partial L_{\text{simple}}(w)}{\partial w_2} = \frac{2e^{2w_2}}{(e^{w_2} + 1)^3} - \frac{2e^{w_1 + w_2}}{(e^{w_1 + w_2} + 1)^3}$$

So:

$$\nabla_{\omega} L_{\text{sample}}(\omega) = \left[\frac{-2e^{\omega_1}(6e^{\omega_1+\omega_2} + 3e^{2(\omega_1+\omega_2)} + e^{3(\omega_1+\omega_2)} + 3e^{2\omega_1+\omega_2} + e^{3\omega_1+\omega_2} + e^{\omega_2} + 1)}{(e^{\omega_1}+1)^3(e^{\omega_1+\omega_2}+1)^3}, \frac{2e^{2\omega_2}}{(e^{\omega_2}+1)^3} - \frac{2e^{\omega_1+\omega_2}}{(e^{\omega_1+\omega_2}+1)^3} \right]$$