TMR4160 Project Report

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Introduction

The task given was to numerically solve the Poisson equation under Dirichlet and Neumann boundary conditions. The Dirichlet case was pretty straightforward to implement and performed pretty well on first try. In the Neumann case interperting the boundary condition (that the derivative normally over the boundary is 0) proved very challenging and frustrating.

Another source of error was that arrays are 1-indexed, which matters when you're trying to determine the coordinates from the matrix index.

Background

The problem presented is to numerically solve the two-dimensional Poisson equation $\nabla^2\varphi := \frac{\delta^2\varphi}{\delta x^2} + \frac{\delta^2\varphi}{\delta y^2} = f(x,y)($ find $\varphi)$ under three different conditions

1. With $f(x,y) = 1$ generally and $\varphi(x,y) = \frac{1}{4}(x^2 + y^2)$ on the boundary(Dirichlet condition)
2. With $f(x,y) = 12 - 12x - 12y$ generally, $\varphi(0,0) = 0$ and $\frac{d\varphi}{dn} = 0$ on the boundary(Neumann condition), where n is a vector normal to the boundary.
3. With $f(x,y) = (6-12x)(3y^2 - 2y^3) + (3x^2 - 2x^3)(6-12y)$ generally, $\varphi(0,0) = 0$ and $\frac{d\varphi}{dn} = 0$ on the boundary(Neumann condition), where n is a vector normal to the boundary.

We were to use the discrete abstraction $\nabla^2\varphi := (\varphi(x+h,y) + \varphi(x-h,y) + \varphi(x,y+h) + \varphi(x,y-h) - 4\varphi(x,y))/h^2 = f(x,y)$. We were permitted to calculate over a quadratic field with points divided up uniformly in both directions.

Analysis and Design

Starting from the discrete equation we were allowed to use I decided to use relaxation, as it's relatively straightforward and performs well under most circumstances.

In order to use a relaxation method we need an update rule. Taking a hint as to the desired result from Wikipedia's article on relaxation we perform a transformation:

$$\begin{aligned}(\varphi(x+h,y)+\varphi(x-h,y)+\varphi(x,y+h)+\varphi(x,y-h)-4\varphi(x,y))/h^2 &= f(x,y) \\ \varphi(x+h,y)+\varphi(x-h,y)+\varphi(x,y+h)+\varphi(x,y-h)-4\varphi(x,y) &= h^2 f(x,y) \\ \varphi(x+h,y)+\varphi(x-h,y)+\varphi(x,y+h)+\varphi(x,y-h)-h^2 f(x,y) &= 4\varphi(x,y) \\ (\varphi(x+h,y)+\varphi(x-h,y)+\varphi(x,y+h)+\varphi(x,y-h)-h^2 f(x,y))/4 &= \varphi(x,y)\end{aligned} \tag{1}$$

Using the relaxation strategy, we repeatedly set $\varphi(x,y)$ to be $(\varphi(x+h,y)+\varphi(x-h,y)+\varphi(x,y+h)+\varphi(x,y-h)-h^2 f(x,y))/4$ until some convergence criterium is fulfilled.

So far so good. And for the Dirichlet boundary condition, we don't need any more analytics than this. The solution is straightforward. However, now remains the thorny issue on how to best enforce $\frac{d\varphi}{dn}=0$ for the Neumann boundary conditions.

Neumann Boundary Condition Enforcing

The first part ($\varphi(0,0)=0$) is straightforward, as we let that point be zero and never change it. There are some optimizations to be done near the origin, but they are just Enforcing $\frac{d\varphi}{dn}=0$, however, proved very challenging. There are two particular cases that needs consideration:

1. The sides of the square
2. The corners of the square

Let ut consider each in isolation.

Side options:

1. Approximate $\varphi(x,y)$ to be linear on the border, assuming that the outer points ($x=1$ etc.) are the border. Since $\frac{d\varphi}{dn}=0$ we get that the two neighbours in the n direction of a boundary point are equal to that point. This changes the math from (1) a bit. Assuming (x,y) is on the left border($x=0$):

$$\begin{aligned}(\varphi(x+h,y)+\varphi(x-h,y)+\varphi(x,y+h)+\varphi(x,y-h)-4\varphi(x,y))/h^2 &= f(x,y) \\ (\varphi(x,y)+\varphi(x,y)+\varphi(x,y+h)+\varphi(x,y-h)-4\varphi(x,y))/h^2 &= f(x,y) \\ (\varphi(x,y+h)+\varphi(x,y-h)-2\varphi(x,y))/h^2 &= f(x,y) \\ \varphi(x,y+h)+\varphi(x,y-h)-2\varphi(x,y) &= h^2 f(x,y) \\ \varphi(x,y+h)+\varphi(x,y-h)-h^2 f(x,y) &= 2\varphi(x,y) \\ (\varphi(x,y+h)+\varphi(x,y-h)-h^2 f(x,y))/2 &= \varphi(x,y)\end{aligned} \tag{2}$$

   - Optionally, we can say that, in order for this to be consistent with how the in-memory matrix actually looks, the boundary point's neighbor inside the matrix is updated along with the boundary point. This means that, for $x=0$, after updating $\varphi(x+h,y) == \varphi(x,y)$.
   - This means that the amount of points in any of the two directions is $\frac{1}{h}+1$

2. Approximate $\varphi(x,y)$ to be linear on the border, assuming that the border lies next to the outer points ($x=h/2$ etc.). This deviates from the approach hitherto taken in this report, as this idea came fairly late. Since $\frac{d\varphi}{dn}=0$ we get that the two neighbours in the n direction of a boundary point are equal to that point. This changes the result in (3). Assuming (x,y) is on the left border($x=h/2$):

$$(\varphi(x,y+h)+\varphi(x,y-h)+\varphi(x+h,y)-h^2 f(x,y))/3 = \varphi(x,y)$$

Let this be the OUTERBORDER strategy

- This means that the amount of points in any of the two directions is $length = \frac{1}{h}$. We can derive this from

$$\begin{aligned}
h &= \frac{1-2\frac{h}{2}}{length-1} \\
length-1 &= \frac{1-2\frac{h}{2}}{h} \\
length &= \frac{1-2\frac{h}{2}}{h}+1 \\
length &= \frac{1-h}{h}+\frac{h}{h} \\
length &= \frac{1}{h}
\end{aligned}$$

  Explanation: At the right-hand side we see

  - 1: This is the distance between the borders
  - $2\frac{h}{2}$: This is the distance lost because the two outer points each need $\frac{h}{2}$ units of distance between themselves and the border
  - $length-1$: "length" is the amount of points in a direction. We subtract one because if you part a bread in three, there will be only two closest-piece-to-closest-piece distances between them if you line them up.

Corner options:

1. Designate the point nearest to the corner that can still be determined by the relaxation rule as special. Set the corner and the two neighboring points equal to this one. Let (x, y) be the top right corner

$$\begin{aligned}
\varphi(x,y) &= \varphi(x-h,y-h) \\
\varphi(x-h,y) &= \varphi(x-h,y-h) \\
\varphi(x,y-h) &= \varphi(x-h,y-h)
\end{aligned}$$

2. Since $\frac{d\varphi}{dn} = 0$, we can assume that φ does not change much between the corner and its neighbors. We simply average (for top right corner: $(x, y) = (max, max)$):

$$\varphi(max,max) = (\varphi(max-1,max)+\varphi(max,max-1)\varphi(max-1,max-1))/3$$

   - In the same way, we can say that the points near the origin will have φ be pretty close to zero and we can set this at the start of each iteration:

$$\begin{aligned}
\varphi(0,h) &= 0 \\
\varphi(h,0) &= 0 \\
\varphi(h,h) &= 0
\end{aligned}$$

   Let this be the RESET strategy

3. In accordance with the OUTERBORDER strategy outlined above, we handle the corners similarly. For the upper right corner $(\varphi(1-h/2, 1-h/2))$:

$$\varphi(x,y) = (\varphi(x-h,y)+\varphi(x,y-h)-h^2 f(x,y))/2$$

Analytical Solutions

For the first task (the Dirichlet one) we need $\nabla^2\varphi = 1$in general and $\varphi(x, y) = \frac{1}{4}(x^2 + y^2)$ on the boundary. Turns out that $\varphi(x, y) = \frac{1}{4}(x^2 + y^2)$ satisfies both.

For the second task (first Neumann one) we need $\nabla^2\varphi = 12 - 12x - 12y$ generally, $\varphi(0, 0) = 0$ and $\frac{d\varphi}{dn} = 0$ on boundary. With some trial and error, it can be determined that $3x^2 + 3y^2 - 2x^3 - 2y^3$ satisfies those.

For the third task (second Neumann one) we need $\nabla^2\varphi = (6-12x)(3y^2-2y^3)+(3x^2-2x^3)(6-12y)$ generally, $\varphi(0, 0) = 0$ and $\frac{d\varphi}{dn} = 0$ on boundary. This seems like a very hard nut to crack until you realise that:

$$(6-12x)(3y^2-2y^3)+(3x^2-2x^3)(6-12y) = \frac{\delta^2 i}{\delta x^2}j + i\frac{\delta^2 j}{\delta y^2} \tag{3}$$

and that

$$\nabla^2(ij) = \frac{\delta^2 ij}{\delta x^2} + \frac{\delta^2 ij}{\delta y^2} \tag{4}$$

$$\nabla^2(ij) = \frac{\delta^2 i}{\delta x^2} j + i\frac{\delta^2 j}{\delta x^2} + \frac{\delta^2 i}{\delta y^2} j + i\frac{\delta^2 j}{\delta y^2} + 2\frac{\delta i}{\delta x}\frac{\delta j}{\delta x} + 2\frac{\delta i}{\delta y}\frac{\delta j}{\delta y} \tag{5}$$

Here all the terms reduce to 0 except $\frac{\delta^2 i}{\delta x^2} j + i\frac{\delta^2 j}{\delta y^2}$, which means, together with the other requirements, that $\varphi(x, y) = (3y^2 - 2y^3)(3x^2 - 2x^3)$.

Memory use

No memory is allocated dynamically and only the matrix holding the values itself has a size that depends on inter-point distance. In terms of big-O notation, memory use is $O(1/h^2)$, where h is the inter-point distance. An interesting note is that, as the matrix is the main source of memory consumption, setting real to be 8 bytes means memory consumption roughly doubles.

Convergence criterium

There were many possible ones to consider

1. Setting a minimum permitted ratio of current average change to previous average change
2. Setting a minimum permitted ratio of current maximum change to previous maximum change
3. For Neumann: Using the fact that we know points close to origin to be close to zero, we can say that we want (1,1) to be closer to zero than a certain difference.
4. Observing that for 1 and 2, the changes cannot be permitted to rise, they can be combined and measure both of these and let none of them rise
5. Modifying 4, the all time lows of average change and maximum change is tracked, and a minimum permitted ratio for each current of these two to the all time low developed. If both are breached at the same time, the algorithm is said to have converged.

Implementation

List of functions and description of each:

F	the right side of the poisson equation
ANALYTICAL	computes the analytical solution
SIMPLEESTIMATE	computes a new estimate for the value of (x*h,y*h). Not valid on the boundary
NEWESTIMATE	computes the value of ((x-1)*h,(y-h)*h). Valid everywhere.
HIGHESTCHANGEFUN	returns the highest change in (phi-)value given new old value, new assignment and previous highest chang
LOWCHANGE	returns the lesser of two variables. Intended to compare change magnitudes to determine the lowest one.
ANALYTICALERROR	computes the analytically determined average squared error

Here is the most interesting code, which is for the Neumann case

```
FUNCTION NEWESTIMATE(X,Y,H,PHIS,LENGTH)
  !computes the value of ((x-1)*h,(y-h)*h). Valid everywhere.
  INTEGER X
  INTEGER  Y
  REAL H
  INTEGER LENGTH
  REAL NEWVALUE
  REAL :: PHIS(LENGTH,LENGTH)
  REAL NEWESTIMATE
  REAL SIMPLEESTIMATE
  REAL F
  IF (X==1 .AND. Y==1) THEN
    NEWESTIMATE = 0.0
  ELSE IF (X==1 .AND. Y==LENGTH) THEN
    NEWESTIMATE = (1.0/2.0)*(PHIS(X+1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
  ELSE IF (X==LENGTH .AND. Y==1) THEN
    NEWESTIMATE = (1.0/2.0)*(PHIS(X-1,Y)+PHIS(X,Y+1)-H*H*F(X,Y,H))
  ELSE IF (X==LENGTH .AND. Y==LENGTH) THEN
    NEWESTIMATE = (1.0/2.0)*(PHIS(X-1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
  ELSE IF (X==1) THEN
    NEWESTIMATE = (1.0/3.0)*(PHIS(X,Y+1)+PHIS(X,Y-1)+PHIS(X+1,Y)-H*H*F(X,Y,H))
  ELSE IF (X==LENGTH) THEN
    NEWESTIMATE = (1.0/3.0)*(PHIS(X,Y+1)+PHIS(X,Y-1)+PHIS(X-1,Y)-H*H*F(X,Y,H))
  ELSE IF (Y==1) THEN
    NEWESTIMATE = (1.0/3.0)*(PHIS(X+1,Y)+PHIS(X-1,Y)+PHIS(X,Y+1)-H*H*F(X,Y,H))
  ELSE IF (Y==LENGTH) THEN
    NEWESTIMATE = (1.0/3.0)*(PHIS(X+1,Y)+PHIS(X-1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
  ELSE
    NEWESTIMATE = SIMPLEESTIMATE(X,Y,H,PHIS,LENGTH)
  END IF
  RETURN
END FUNCTION


FUNCTION SIMPLEESTIMATE(X,Y,H,PHIS,LENGTH)
  !computes a new estimate for the value of (x*h,y*h). Not valid on the boundary
  INTEGER X,Y
  REAL H
```

```
  REAL SIMPLEESTIMATE
  INTEGER LENGTH
  REAL F
  REAL :: PHIS(LENGTH,LENGTH)
  SIMPLEESTIMATE = (1.0/4.0)*(PHIS(X+1,Y)+PHIS(X,Y+1)+PHIS(X-1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
  RETURN
END FUNCTION
```

And the main loop:

```
NUMITERATIONS = 0
  !the actual computation is performed here
  DO WHILE ((LOWHIGHESTCHANGE/HIGHESTCHANGE) > 1.1 .OR. (LOWAVGCHANGE/AVGCHANGE) > 1.000008)
    NUMITERATIONS = NUMITERATIONS + 1
    LOWAVGCHANGE = LOWCHANGE(LOWAVGCHANGE, AVGCHANGE)
    LOWHIGHESTCHANGE = LOWCHANGE(LOWHIGHESTCHANGE, HIGHESTCHANGE)
    HIGHESTCHANGE = 0.0
    AVGCHANGE = 0.0
    DO J=1,LENGTH
      DO I=1,LENGTH
          NEWVALUE = NEWESTIMATE(I,J,H,PHIS,LENGTH)
          HIGHESTCHANGE = HIGHESTCHANGEFUN(PHIS(I,J),NEWVALUE,HIGHESTCHANGE)
          AVGCHANGE = AVGCHANGE + ((PHIS(I,J)-NEWVALUE)**2)/SIZE
          PHIS(I,J) = NEWVALUE
      END DO
    END DO
    !WRITE (*,*) "HIGHESTCHANGE IS ", HIGHESTCHANGE
    !WRITE (*,*) "AVGCHANGE IS ", AVGCHANGE
    !WRITE (*,*) "LOWAVGCHANGE IS ", LOWAVGCHANGE
    !WRITE (*,*) "LOWHIGHESTCHANGE IS ", LOWHIGHESTCHANGE
    !WRITE (*,*) ""
  END DO
```

The display code is almost entirely taken from the Mesa project's "array" example. What's changed is the function that gets the height of the points

```
static float
getinm(float x, float y, float minx, float maxx, float miny, float maxy)
{
  int i = ((x+(-minx))/(maxx-minx))*LENGTH;
  int j = ((y+(-miny))/(maxy-miny))*LENGTH;
  int index = i*LENGTH+j;
  if(index<0){index=0;}
  if(index>LENGTH*LENGTH){index=LENGTH*LENGTH-1;}
  //printf("x is %f, y is %f, i is %d, j is %d, index is %d\n",x,y,i,j,index);      //disabled
  return inm[index];
}
```

And the start of the main function, which reads in the data:

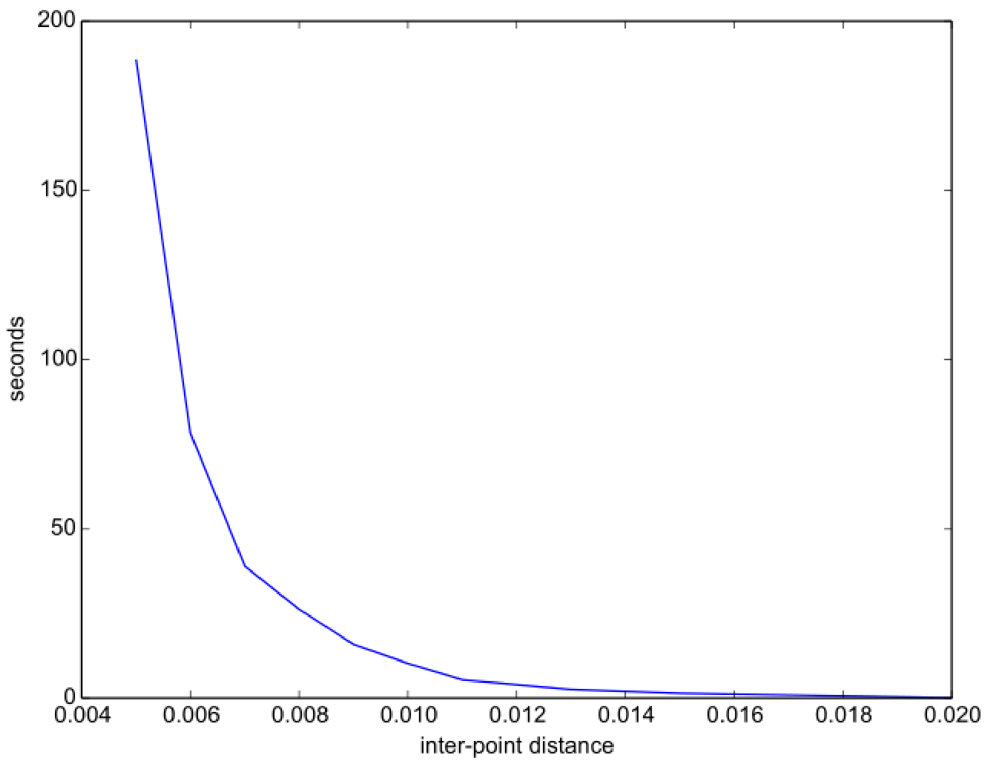
```
int
main(int argc, char *argv[])
{
  srand(time(NULL));
  float in = 0.0;
  int control = 0;



  FILE *f = fopen("results.txt", "r");
  fscanf(f,"%d",&LENGTH);
  fscanf(f,"%d",&control);
```

```
assert(LENGTH==control);
float arr[LENGTH*LENGTH];
inm = &arr;
for(int i=0; i<LENGTH*LENGTH; i++)
{
  fscanf(f,"%f",&in);
  inm[i] = in;
  //inm[i] = rand()%GLmaxdiff;
  if (in < lowest)
  {
    lowest = in;
  }
  else if (in > highest)
  {
    highest = in;
  }
  //printf("%f\n", in);
}
fclose(f);
```

Testing

In the end, the OUTERBORDER strategy is markedly superior to anything else by a solid margin. For a convergence criterium, we settled on option 5 from above.

For small inter-point distances, the neumann programs are really slow. A possible reason is that the constant point at the origin must have the consequences of its value propagated through the matrix and this requires more iterations, even as each iteration becomes more costly. In the graph below we set the average squared error to be 0.1 and see how long it needs to run to achieve this.



Conclusion, Evaluation and Further Work

A possible way to speed up the algorithm would be to first do a run with large inter-point distance, and then use the result to initialize the bigger matrix. Unfortunately this has not been tried during the work with this report. I am a bit unsure about the convergence criterium, but I have no better suggestions.

Appendix A

User manual

To compile and run the Neumann case, use "gfortran neumannFinal.f95 -fimplicit-none -O3 -o neumannFinal -fdefault-real-8 && time ./neumannFinal"

Remember to comment in or out the correct f and analytical functions!

To compile and run the Dirichlet case, use "gfortran dirichlet.f95 -fimplicit-none -O3 -o dirichlet -fdefault-real-8 && time ./dirichlet"

To compile the display program use "clang -o array array.c shaderutil.c -lGL -lglut -lm -lGLEW -O3". To run it, be in the same folder as the result.txt file and use "./array" or "./array/array" depending on where the executable is. Use of Clang is of course optional - GCC should work as well.

Appendix B

Code listing

Neumann(neumannFinal.f95):

```
!gfortran neumannFinal.f95 -fimplicit-none -O3 -o neumannFinal -fdefault-real-8 && time ./neumannFina

      FUNCTION F(X,Y,H)
        !the right side of the poisson equation
        INTEGER X,Y
        REAL H
        REAL F
        REAL :: XC
        REAL :: YC
        XC = (X-0.5)*H
        YC = (Y-0.5)*H
        !F=12-12*XC-12*YC                                              !Switch which of these
        F=(6-12*XC)*(3*YC**2-2*YC**3) + (3*XC**2-2*XC**3)*(6-12*YC)
        RETURN
      END FUNCTION

      FUNCTION ANALYTICAL(X,Y,H)
        !computes the analytical solution
        REAL ANALYTICAL
        INTEGER :: X
        INTEGER :: Y
        REAL :: H
        REAL :: XC
        REAL :: YC
        XC = (X-0.5)*H
        YC = (Y-0.5)*H
        !ANALYTICAL = 3*XC**2 + 3*YC**2 - 2*XC**3 - 2*YC**3            !Switch which of these a
        ANALYTICAL = (3*YC**2 - 2*YC**3)*(3*XC**2 - 2*XC**3)
        RETURN
      END FUNCTION

      FUNCTION NEWESTIMATE(X,Y,H,PHIS,LENGTH)
        !computes the value of ((x-1)*h,(y-h)*h). Valid everywhere.
        INTEGER X
        INTEGER  Y
        REAL H
        INTEGER LENGTH
        REAL NEWVALUE
        REAL :: PHIS(LENGTH,LENGTH)
        REAL NEWESTIMATE
        REAL SIMPLEESTIMATE
        REAL F
        IF (X==1 .AND. Y==1) THEN
```

```
      NEWESTIMATE = 0.0
    ELSE IF (X==1 .AND. Y==LENGTH) THEN
      NEWESTIMATE = (1.0/2.0)*(PHIS(X+1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
    ELSE IF (X==LENGTH .AND. Y==1) THEN
      NEWESTIMATE = (1.0/2.0)*(PHIS(X-1,Y)+PHIS(X,Y+1)-H*H*F(X,Y,H))
    ELSE IF (X==LENGTH .AND. Y==LENGTH) THEN
      NEWESTIMATE = (1.0/2.0)*(PHIS(X-1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
    ELSE IF (X==1) THEN
      NEWESTIMATE = (1.0/3.0)*(PHIS(X,Y+1)+PHIS(X,Y-1)+PHIS(X+1,Y)-H*H*F(X,Y,H))
    ELSE IF (X==LENGTH) THEN
      NEWESTIMATE = (1.0/3.0)*(PHIS(X,Y+1)+PHIS(X,Y-1)+PHIS(X-1,Y)-H*H*F(X,Y,H))
    ELSE IF (Y==1) THEN
      NEWESTIMATE = (1.0/3.0)*(PHIS(X+1,Y)+PHIS(X-1,Y)+PHIS(X,Y+1)-H*H*F(X,Y,H))
    ELSE IF (Y==LENGTH) THEN
      NEWESTIMATE = (1.0/3.0)*(PHIS(X+1,Y)+PHIS(X-1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
    ELSE
      NEWESTIMATE = SIMPLEESTIMATE(X,Y,H,PHIS,LENGTH)
    END IF
    RETURN
END FUNCTION


FUNCTION SIMPLEESTIMATE(X,Y,H,PHIS,LENGTH)
    !computes a new estimate for the value of (x*h,y*h). Not valid on the boundary
    INTEGER X,Y
    REAL H
    REAL SIMPLEESTIMATE
    INTEGER LENGTH
    REAL F
    REAL :: PHIS(LENGTH,LENGTH)
    SIMPLEESTIMATE = (1.0/4.0)*(PHIS(X+1,Y)+PHIS(X,Y+1)+PHIS(X-1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
    RETURN
END FUNCTION

FUNCTION HIGHESTCHANGEFUN(OLD,NEW,PREVHIGHEST)
    !returns the Highest change in (phi-)value given a new old value, new assignment and the prev
    REAL :: OLD
    REAL :: NEW
    REAL :: PREVHIGHEST
    REAL :: CHANGE
    REAL HIGHESTCHANGEFUN
    CHANGE = ABS(OLD-NEW)
    IF (CHANGE .GE. PREVHIGHEST) THEN
      HIGHESTCHANGEFUN = CHANGE
    ELSE
      HIGHESTCHANGEFUN = PREVHIGHEST
    END IF
    RETURN
END FUNCTION

FUNCTION LOWCHANGE(OLDLOWEST, LASTLOW)
    !returns the lesser of two variables. Intended to compare change magnitudes to determine the 
    REAL LOWCHANGE
    REAL OLDLOWEST
    REAL LASTLOW
    IF (OLDLOWEST .LT. LASTLOW) THEN
      LOWCHANGE = OLDLOWEST
    ELSE
      LOWCHANGE = LASTLOW
```

```
  END IF
END FUNCTION

FUNCTION ANALYTICALERROR(PHIS,LENGTH,H,SIZE)
  !computes the analytically determined average squared error
  INTEGER SIZE
  INTEGER LENGTH
  REAL H
  REAL :: PHIS(LENGTH,LENGTH)
  REAL ANALYTICAL
  REAL AVGERROR
  INTEGER I,J
  REAL ANALYTICALERROR
  AVGERROR = 0.0
  DO J=1,LENGTH
    DO I=1,LENGTH
    AVGERROR = AVGERROR + ((PHIS(I,J)-ANALYTICAL(I,J,H))**2)/SIZE
    END DO
  END DO
  ANALYTICALERROR = AVGERROR
  RETURN
END FUNCTION

PROGRAM SOLVER
  REAL, PARAMETER :: H = 0.01
  INTEGER, PARAMETER :: LENGTH = (1.0/H)
  INTEGER, PARAMETER :: SIZE = LENGTH*LENGTH
  REAL :: PHIS(LENGTH, LENGTH)
  INTEGER, PARAMETER :: out_unit=20
  REAL :: HIGHESTCHANGE
  REAL :: NEWVALUE
  REAL SIMPLEESTIMATE
  REAL NEWESTIMATE
  REAL ANALYTICAL
  INTEGER I,J
  REAL HIGHESTCHANGEFUN
  REAL LOWHIGHESTCHANGE
  REAL AVGCHANGE
  REAL LOWAVGCHANGE
  REAL LOWCHANGE
  REAL AVGERROR
  REAL NUMAVGERROR
  INTEGER NUMITERATIONS
  REAL ANALYTICALERROR
  REAL :: r(5,5)
  INTEGER :: SEED
  LOWAVGCHANGE = 20000.0
  AVGCHANGE = 10000.0
  LOWHIGHESTCHANGE = 20000.0
  HIGHESTCHANGE = 10000.0

  WRITE (*,*) "LENGTH IS ", LENGTH
  !a friendly reminder that the real coordinate is (x-0.5)*h, not x*h
  WRITE (*,*) "(LENGTH-0.5)*H IS ", ((LENGTH-0.5)*H)

  DO I=1,LENGTH
    DO J=1,LENGTH
      PHIS(I,J) = RAND(SEED)*10.0 !SEED is intentionally uninitialized in the hope of providing
    END DO
```

```
END DO

NUMITERATIONS = 0
!the actual computation is performed here
DO WHILE ((LOWHIGHESTCHANGE/HIGHESTCHANGE) > 1.0000126 .OR. (LOWAVGCHANGE/AVGCHANGE) > 1.000
  NUMITERATIONS = NUMITERATIONS + 1
  LOWAVGCHANGE = LOWCHANGE(LOWAVGCHANGE, AVGCHANGE)
  LOWHIGHESTCHANGE = LOWCHANGE(LOWHIGHESTCHANGE, HIGHESTCHANGE)
  HIGHESTCHANGE = 0.0
  AVGCHANGE = 0.0
  DO J=1,LENGTH
    DO I=1,LENGTH
        NEWVALUE = NEWESTIMATE(I,J,H,PHIS,LENGTH)
        HIGHESTCHANGE = HIGHESTCHANGEFUN(PHIS(I,J),NEWVALUE,HIGHESTCHANGE)
        AVGCHANGE = AVGCHANGE + ((PHIS(I,J)-NEWVALUE)**2)/SIZE
        PHIS(I,J) = NEWVALUE
    END DO
  END DO
  !WRITE (*,*) "HIGHESTCHANGE IS ", HIGHESTCHANGE
  !WRITE (*,*) "AVGCHANGE IS ", AVGCHANGE
  !WRITE (*,*) "LOWAVGCHANGE IS ", LOWAVGCHANGE
  !WRITE (*,*) "LOWHIGHESTCHANGE IS ", LOWHIGHESTCHANGE
  !WRITE (*,*) ""
END DO

WRITE (*,*) "ratio", (LOWHIGHESTCHANGE/HIGHESTCHANGE) > 1.00000000
WRITE (*,*) "ratio", LOWHIGHESTCHANGE/HIGHESTCHANGE
WRITE (*,*) "ratio", (LOWAVGCHANGE/AVGCHANGE) > 1.00000000
WRITE (*,*) "ratio", LOWAVGCHANGE/AVGCHANGE



WRITE (*,*) "NUMITERATIONS IS ", NUMITERATIONS
!finds and prints the numerically determined error
NUMAVGERROR = 0.0
DO I=1,LENGTH
  DO J=1,LENGTH
  NUMAVGERROR = NUMAVGERROR + ((PHIS(I,J)-NEWESTIMATE(I,J,H,PHIS,LENGTH))**2)/SIZE
  END DO
END DO
WRITE (*,*) "AVERAGE SQUARE NUMERICALLY ESTIMATED ERROR IS ", NUMAVGERROR

!performance metrics. Analytically determined error is printed
AVGERROR = 0.0
DO I=1,LENGTH
  DO J=1,LENGTH
  AVGERROR = AVGERROR + ((PHIS(I,J)-ANALYTICAL(I,J,H))**2)/SIZE
  END DO
END DO
WRITE (*,*) "AVERAGE SQUARE ANALYTICALLY DETERMINED ERROR IS ", AVGERROR

!writes results to file
open (unit=out_unit,file="results.txt",action="write",status="replace")
WRITE (OUT_UNIT,'(I4)') LENGTH
WRITE (OUT_UNIT,'(I4)') LENGTH
DO I=1,LENGTH
  DO J=1,LENGTH
      WRITE (OUT_UNIT,'(F0.5)') PHIS(I,J)
  END DO
```

```
        END DO
        close (out_unit)
      END
```

Dirichlet(dirichlet.f95):

```
!gfortran dirichlet.f95 -fimplicit-none -O3 -o dirichlet -fdefault-real-8 && time ./dirichlet

      FUNCTION F(X,Y,H)
        !the right side of the poisson equation
        INTEGER X,Y
        REAL H
        REAL F
        F=1
        RETURN
      END FUNCTION

      FUNCTION ANALYTICAL(X,Y,H)
        !computes the analytical solution
        REAL ANALYTICAL
        INTEGER :: X
        INTEGER :: Y
        REAL :: H
        REAL :: XC
        REAL :: YC
        XC = (X-1)*H
        YC = (Y-1)*H
        ANALYTICAL = 0.25*(XC**2+YC**2)
        RETURN
      END FUNCTION

      FUNCTION NEWESTIMATE(X,Y,H,PHIS,LENGTH)
        !computes the value of ((x-1)*h,(y-h)*h). Valid everywhere.
        INTEGER X
        INTEGER  Y
        REAL H
        INTEGER LENGTH
        REAL NEWVALUE
        REAL :: PHIS(LENGTH,LENGTH)
        REAL NEWESTIMATE
        REAL SIMPLEESTIMATE
        REAL F
        REAL :: ORIGINDEMAND
        REAL :: XC
        REAL :: YC
        XC = (X-1)*H
        YC = (Y-1)*H
        IF (X==1 .OR. Y==1 .OR. X==LENGTH .OR. Y==LENGTH) THEN
          NEWESTIMATE = 0.25*(XC**2+YC**2)
        ELSE
          NEWESTIMATE = SIMPLEESTIMATE(X,Y,H,PHIS,LENGTH)
        END IF
        RETURN
      END FUNCTION


      FUNCTION SIMPLEESTIMATE(X,Y,H,PHIS,LENGTH)
        !computes a new estimate for the value of (x*h,y*h). Not valid on the boundary
        INTEGER X,Y
        REAL H
```

```
    REAL SIMPLEESTIMATE
    INTEGER LENGTH
    REAL F
    REAL :: PHIS(LENGTH,LENGTH)
    SIMPLEESTIMATE = (1.0/4.0)*(PHIS(X+1,Y)+PHIS(X,Y+1)+PHIS(X-1,Y)+PHIS(X,Y-1)-H*H*F(X,Y,H))
    RETURN
  END FUNCTION

  FUNCTION HIGHESTCHANGEFUN(OLD,NEW,PREVHIGHEST)
    !returns the gighest change in (phi-)value given a new old value, new assignment and the prev
    REAL :: OLD
    REAL :: NEW
    REAL :: PREVHIGHEST
    REAL :: CHANGE
    REAL HIGHESTCHANGEFUN
    CHANGE = ABS(OLD-NEW)
    IF (CHANGE .GE. PREVHIGHEST) THEN
      HIGHESTCHANGEFUN = CHANGE
    ELSE
      HIGHESTCHANGEFUN = PREVHIGHEST
    END IF
    RETURN
  END FUNCTION

  FUNCTION LOWCHANGE(OLDLOWEST, LASTLOW)
    !returns the lesser of two variables. Intended to compare change magnitudes to determine the l
    REAL LOWCHANGE
    REAL OLDLOWEST
    REAL LASTLOW
    IF (OLDLOWEST .LT. LASTLOW) THEN
      LOWCHANGE = OLDLOWEST
    ELSE
      LOWCHANGE = LASTLOW
    END IF
  END FUNCTION

  FUNCTION ANALYTICALERROR(PHIS,LENGTH,H,SIZE)
    !computes the analytically determined average squared error
    INTEGER SIZE
    INTEGER LENGTH
    REAL H
    REAL :: PHIS(LENGTH,LENGTH)
    REAL ANALYTICAL
    REAL AVGERROR
    INTEGER I,J
    REAL ANALYTICALERROR
    AVGERROR = 0.0
    DO J=1,LENGTH
      DO I=1,LENGTH
      AVGERROR = AVGERROR + ((PHIS(I,J)-ANALYTICAL(I,J,H))**2)/SIZE
      END DO
    END DO
    ANALYTICALERROR = AVGERROR
    RETURN
  END FUNCTION

  PROGRAM SOLVER
    REAL, PARAMETER :: H = 0.01
    INTEGER, PARAMETER :: LENGTH = (1.0/H)+1
```

```
INTEGER, PARAMETER :: SIZE = LENGTH*LENGTH
REAL :: PHIS(LENGTH, LENGTH)
INTEGER, PARAMETER :: out_unit=20
REAL :: HIGHESTCHANGE
REAL :: NEWVALUE
REAL SIMPLEESTIMATE
REAL NEWESTIMATE
REAL ANALYTICAL
INTEGER I,J
REAL HIGHESTCHANGEFUN
REAL LOWHIGHESTCHANGE
REAL AVGCHANGE
REAL LOWAVGCHANGE
REAL LOWCHANGE
REAL AVGERROR
REAL NUMAVGERROR
INTEGER NUMITERATIONS
REAL ANALYTICALERROR
LOWAVGCHANGE = 20.0
AVGCHANGE = 1.0
LOWHIGHESTCHANGE = 20.0
HIGHESTCHANGE = 10.0

!a friendly reminder that what the real coordinate is (x-1)*h, not x*h
WRITE (*,*) "(LENGTH-1)*H IS ", ((LENGTH-1)*H)

DO I=1,LENGTH
  DO J=1,LENGTH
    PHIS(I,J) = RAND(0)*10
  END DO
END DO

NUMITERATIONS = 0
!the actual computation is performed here
DO WHILE  ((LOWHIGHESTCHANGE/HIGHESTCHANGE) > 1.00001 .OR. (LOWAVGCHANGE/AVGCHANGE) > 1.00001)
  NUMITERATIONS = NUMITERATIONS + 1
  LOWAVGCHANGE = LOWCHANGE(LOWAVGCHANGE, AVGCHANGE)
  LOWHIGHESTCHANGE = LOWCHANGE(LOWHIGHESTCHANGE, HIGHESTCHANGE)
  HIGHESTCHANGE = 0.0
  AVGCHANGE = 0.0
  DO J=1,LENGTH
    DO I=1,LENGTH
        NEWVALUE = NEWESTIMATE(I,J,H,PHIS,LENGTH)
        HIGHESTCHANGE = HIGHESTCHANGEFUN(PHIS(I,J),NEWVALUE,HIGHESTCHANGE)
        AVGCHANGE = AVGCHANGE + ((PHIS(I,J)-NEWVALUE)**2)/SIZE
        PHIS(I,J) = NEWVALUE
    END DO
  END DO
END DO



WRITE (*,*) "NUMITERATIONS IS ", NUMITERATIONS
!finds and prints the numerically determined error
NUMAVGERROR = 0.0
DO I=1,LENGTH
  DO J=1,LENGTH
  NUMAVGERROR = NUMAVGERROR + ((PHIS(I,J)-NEWESTIMATE(I,J,H,PHIS,LENGTH))**2)/SIZE
  END DO
```

```
    END DO
    WRITE (*,*) "AVERAGE SQUARE NUMERICALLY ESTIMATED ERROR IS ", NUMAVGERROR

    !performance metrics. Analytically determined error is printed
    AVGERROR = 0.0
    DO I=1,LENGTH
      DO J=1,LENGTH
      AVGERROR = AVGERROR + ((PHIS(I,J)-ANALYTICAL(I,J,H))**2)/SIZE
      END DO
    END DO
    WRITE (*,*) "AVERAGE SQUARE ANALYTICALLY DETERMINED ERROR IS ", AVGERROR

    !writes results to file
    open (unit=out_unit,file="results.txt",action="write",status="replace")
    WRITE (OUT_UNIT,'(I4)') LENGTH
    WRITE (OUT_UNIT,'(I4)') LENGTH
    DO I=1,LENGTH
      DO J=1,LENGTH
          WRITE (OUT_UNIT,'(F0.5)') PHIS(I,J)
      END DO
    END DO
    close (out_unit)
  END
```

Display(array.c):

```
//clang -o array array.c shaderutil.c -lGL -lglut -lm -lGLEW -O3 && ./array

/**
 * Test variable array indexing in a vertex shader.
 * Brian Paul
 * 17 April 2009
 */

#include <assert.h>
#include <string.h>
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <GL/glew.h>
#include "glut_wrap.h"
#include "shaderutil.h"

int LENGTH;
float *inm;
#define GLmaxdiff 2.0
float highest = 0.0;
float lowest = 0.0;

/**
 * The vertex position.z is used as a (variable) index into an
 * array which returns a new Z value.
 */
static const char *VertShaderText =
   "uniform sampler2D tex1; \n"
   "uniform float HeightArray[20]; \n"
   "void main() \n"
   "{ \n"
   "   vec4 pos = gl_Vertex; \n"
   "   int i = int(pos.z * 9.5); \n"
```

```
   "   pos.z = HeightArray[i]; \n"
   "   gl_Position = gl_ModelViewProjectionMatrix * pos; \n"
   "   gl_FrontColor = pos; \n"
   "} \n";

static const char *FragShaderText =
   "void main() \n"
   "{ \n"
   "   gl_FragColor = gl_Color; \n"
   "} \n";


static GLuint fragShader;
static GLuint vertShader;
static GLuint program;

static GLint win = 0;
static GLboolean Anim = GL_TRUE;
static GLboolean WireFrame = GL_TRUE;
static GLfloat xRot = -70.0f, yRot = 0.0f, zRot = 0.0f;


static void
Idle(void)
{
   zRot = 90 + glutGet(GLUT_ELAPSED_TIME) * 0.05;
   glutPostRedisplay();
}


/** z=f(x,y) */
static float
fz(float x, float y)
{
   return fabs(cos(1.5*x) + cos(1.5*y));
}

static float
getinm(float x, float y, float minx, float maxx, float miny, float maxy)
{
  int i = ((x+(-minx))/(maxx-minx))*LENGTH;
  int j = ((y+(-miny))/(maxy-miny))*LENGTH;
  int index = i*LENGTH+j;
  if(index<0){index=0;}
  if(index>LENGTH*LENGTH){index=LENGTH*LENGTH-1;}
  //printf("x is %f, y is %f, i is %d, j is %d, index is %d\n",x,y,i,j,index);             //disabled f
  return inm[index];
}


static void
DrawMesh(void)
{
   GLfloat xmin = -2.0, xmax = 2.0;
   GLfloat ymin = -2.0, ymax = 2.0;
   GLuint xdivs = 60, ydivs = 60;
   GLfloat dx = (xmax - xmin) / xdivs;
   GLfloat dy = (ymax - ymin) / ydivs;
   GLfloat ds = 1.0 / xdivs, dt = 1.0 / ydivs;
```

```
    GLfloat x, y, s, t;
    GLuint i, j;

    float scale = GLmaxdiff/(highest-lowest);

    y = ymin;
    t = 0.0;
    for (i = 0; i < ydivs; i++) {
       x = xmin;
       s = 0.0;
       glBegin(GL_QUAD_STRIP);
       for (j = 0; j < xdivs; j++) {
          float z0 = scale*getinm(x, y, xmin,xmax,ymin,ymax), z1 = scale*getinm(x, y + dy, xmin,xmax,y

          glTexCoord2f(s, t);
          glVertex3f(x, y, z0);

          glTexCoord2f(s, t + dt);
          glVertex3f(x, y + dy, z1);
          x += dx;
          s += ds;
       }
       glEnd();
       y += dy;
       t += dt;
    }

    /*y = ymin;
    t = 0.0;
    for (i = 0; i < ydivs; i++) {
       x = xmin;
       s = 0.0;
       glBegin(GL_QUAD_STRIP);
       for (j = 0; j < xdivs; j++) {
          float z0 = fz(x, y), z1 = fz(x, y + dy);

          glTexCoord2f(s, t);
          glVertex3f(x, y, z0);

          glTexCoord2f(s, t + dt);
          glVertex3f(x, y + dy, z1);
          x += dx;
          s += ds;
       }
       glEnd();
       y += dy;
       t += dt;
    }*/
}


static void
Redisplay(void)
{
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);

    if (WireFrame)
       glPolygonMode(GL_FRONT_AND_BACK, GL_LINE);
    else
```

```
      glPolygonMode(GL_FRONT_AND_BACK, GL_FILL);

   glPushMatrix();
   glRotatef(xRot, 1.0f, 0.0f, 0.0f);
   glRotatef(yRot, 0.0f, 1.0f, 0.0f);
   glRotatef(zRot, 0.0f, 0.0f, 1.0f);

   glPushMatrix();
   DrawMesh();
   glPopMatrix();

   glPopMatrix();

   glPolygonMode(GL_FRONT_AND_BACK, GL_FILL);

   glutSwapBuffers();
}


static void
Reshape(int width, int height)
{
   glViewport(0, 0, width, height);
   glMatrixMode(GL_PROJECTION);
   glLoadIdentity();
   glFrustum(-1.0, 1.0, -1.0, 1.0, 5.0, 25.0);
   glMatrixMode(GL_MODELVIEW);
   glLoadIdentity();
   glTranslatef(0.0f, 0.0f, -15.0f);
}


static void
CleanUp(void)
{
   glDeleteShader(fragShader);
   glDeleteShader(vertShader);
   glDeleteProgram(program);
   glutDestroyWindow(win);
}


static void
Key(unsigned char key, int x, int y)
{
   const GLfloat step = 2.0;
   (void) x;
   (void) y;

   switch(key) {
   case 'a':
      Anim = !Anim;
      if (Anim)
         glutIdleFunc(Idle);
      else
         glutIdleFunc(NULL);
      break;
   case 'w':
      WireFrame = !WireFrame;
```

```
      break;
   case 'z':
      zRot += step;
      break;
   case 'Z':
      zRot -= step;
      break;
   case 27:
      CleanUp();
      exit(0);
      break;
   }
   glutPostRedisplay();
}


static void
SpecialKey(int key, int x, int y)
{
   const GLfloat step = 2.0;

   (void) x;
   (void) y;

   switch(key) {
   case GLUT_KEY_UP:
      xRot += step;
      break;
   case GLUT_KEY_DOWN:
      xRot -= step;
      break;
   case GLUT_KEY_LEFT:
      yRot -= step;
      break;
   case GLUT_KEY_RIGHT:
      yRot += step;
      break;
   }
   glutPostRedisplay();
}


static void
Init(void)
{
   GLfloat HeightArray[20];
   GLint u, i;

   if (!ShadersSupported())
      exit(1);

   vertShader = CompileShaderText(GL_VERTEX_SHADER, VertShaderText);
   fragShader = CompileShaderText(GL_FRAGMENT_SHADER, FragShaderText);
   program = LinkShaders(vertShader, fragShader);

   glUseProgram(program);

   /* Setup the HeightArray[] uniform */
   for (i = 0; i < 20; i++)
```

```
      HeightArray[i] = i / 20.0;
   u = glGetUniformLocation(program, "HeightArray");
   glUniform1fv(u, 20, HeightArray);

   assert(glGetError() == 0);

   glClearColor(0.4f, 0.4f, 0.8f, 0.0f);
   glEnable(GL_DEPTH_TEST);
   glColor3f(1, 1, 1);
}


int
main(int argc, char *argv[])
{
  srand(time(NULL));
  float in = 0.0;
  int control = 0;


  FILE *f = fopen("results.txt", "r");
  fscanf(f,"%d",&LENGTH);
  fscanf(f,"%d",&control);
  assert(LENGTH==control);
  float arr[LENGTH*LENGTH];
  inm = &arr;
  for(int i=0; i<LENGTH*LENGTH; i++)
  {
    fscanf(f,"%f",&in);
    inm[i] = in;
    //inm[i] = rand()%GLmaxdiff;
    if (in < lowest)
    {
      lowest = in;
    }
    else if (in > highest)
    {
      highest = in;
    }
    //printf("%f\n", in);
  }
  fclose(f);



    /**
    * Start OpenGL stuff
    */

   glutInit(&argc, argv);
   glutInitWindowSize(500, 500);
   glutInitDisplayMode(GLUT_RGB | GLUT_DOUBLE | GLUT_DEPTH);
   win = glutCreateWindow(argv[0]);
   glewInit();
   glutReshapeFunc(Reshape);
   glutKeyboardFunc(Key);
   glutSpecialFunc(SpecialKey);
   glutDisplayFunc(Redisplay);
   Init();
```

```
   if (Anim)
      glutIdleFunc(Idle);
   glutMainLoop();
   return 0;
}
```

Display support(glut_wrap.h)

```
#ifndef GLUT_WRAP_H
#define GLUT_WRAP_H

#ifdef HAVE_FREEGLUT
#  include <GL/freeglut.h>
#elif defined __APPLE__
#  include <GLUT/glut.h>
#else
#  include <GL/glut.h>
#endif

#ifndef GLAPIENTRY
#define GLAPIENTRY
#endif

#endif /* ! GLUT_WRAP_H */
```

Display support(shaderutil.c)

```
/**
 * Utilities for OpenGL shading language
 *
 * Brian Paul
 * 9 April 2008
 */


#include <assert.h>
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <GL/glew.h>
#include "glut_wrap.h"
#include "shaderutil.h"

/** time to compile previous shader */
static GLdouble CompileTime = 0.0;

/** time to linke previous program */
static GLdouble LinkTime = 0.0;

PFNGLCREATESHADERPROC CreateShader = NULL;
PFNGLDELETESHADERPROC DeleteShader = NULL;
PFNGLSHADERSOURCEPROC ShaderSource = NULL;
PFNGLGETSHADERIVPROC GetShaderiv = NULL;
PFNGLGETSHADERINFOLOGPROC GetShaderInfoLog = NULL;
PFNGLCREATEPROGRAMPROC CreateProgram = NULL;
PFNGLDELETEPROGRAMPROC DeleteProgram = NULL;
PFNGLATTACHSHADERPROC AttachShader = NULL;
PFNGLLINKPROGRAMPROC LinkProgram = NULL;
PFNGLUSEPROGRAMPROC UseProgram = NULL;
PFNGLGETPROGRAMIVPROC GetProgramiv = NULL;
PFNGLGETPROGRAMINFOLOGPROC GetProgramInfoLog = NULL;
```

```
PFNGLVALIDATEPROGRAMARBPROC ValidateProgramARB = NULL;
PFNGLUNIFORM1IPROC Uniform1i = NULL;
PFNGLUNIFORM1FVPROC Uniform1fv = NULL;
PFNGLUNIFORM2FVPROC Uniform2fv = NULL;
PFNGLUNIFORM3FVPROC Uniform3fv = NULL;
PFNGLUNIFORM4FVPROC Uniform4fv = NULL;
PFNGLUNIFORMMATRIX4FVPROC UniformMatrix4fv = NULL;
PFNGLGETACTIVEATTRIBPROC GetActiveAttrib = NULL;
PFNGLGETATTRIBLOCATIONPROC GetAttribLocation = NULL;

static void GLAPIENTRY
fake_ValidateProgram(GLuint prog)
{
   (void) prog;
}

GLboolean
ShadersSupported(void)
{
   if (GLEW_VERSION_2_0) {
      CreateShader = glCreateShader;
      DeleteShader = glDeleteShader;
      ShaderSource = glShaderSource;
      GetShaderiv = glGetShaderiv;
      GetShaderInfoLog = glGetShaderInfoLog;
      CreateProgram = glCreateProgram;
      DeleteProgram = glDeleteProgram;
      AttachShader = glAttachShader;
      LinkProgram = glLinkProgram;
      UseProgram = glUseProgram;
      GetProgramiv = glGetProgramiv;
      GetProgramInfoLog = glGetProgramInfoLog;
      ValidateProgramARB = (GLEW_ARB_shader_objects)
         ? glValidateProgramARB : fake_ValidateProgram;
      Uniform1i = glUniform1i;
      Uniform1fv = glUniform1fv;
      Uniform2fv = glUniform2fv;
      Uniform3fv = glUniform3fv;
      Uniform4fv = glUniform4fv;
      UniformMatrix4fv = glUniformMatrix4fv;
      GetActiveAttrib = glGetActiveAttrib;
      GetAttribLocation = glGetAttribLocation;
      return GL_TRUE;
   }
   else if (GLEW_ARB_vertex_shader && GLEW_ARB_fragment_shader
            && GLEW_ARB_shader_objects) {
      fprintf(stderr, "Warning: Trying ARB GLSL instead of OpenGL 2.x.  This may not work.\n");
      CreateShader = glCreateShaderObjectARB;
      DeleteShader = glDeleteObjectARB;
      ShaderSource = glShaderSourceARB;
      GetShaderiv = glGetObjectParameterivARB;
      GetShaderInfoLog = glGetInfoLogARB;
      CreateProgram = glCreateProgramObjectARB;
      DeleteProgram = glDeleteObjectARB;
      AttachShader = glAttachObjectARB;
      LinkProgram = glLinkProgramARB;
      UseProgram = glUseProgramObjectARB;
      GetProgramiv = glGetObjectParameterivARB;
      GetProgramInfoLog = glGetInfoLogARB;
```

```
      ValidateProgramARB = glValidateProgramARB;
      Uniform1i = glUniform1iARB;
      Uniform1fv = glUniform1fvARB;
      Uniform2fv = glUniform2fvARB;
      Uniform3fv = glUniform3fvARB;
      Uniform4fv = glUniform4fvARB;
      UniformMatrix4fv = glUniformMatrix4fvARB;
      GetActiveAttrib = glGetActiveAttribARB;
      GetAttribLocation = glGetAttribLocationARB;
      return GL_TRUE;
   }
   fprintf(stderr, "Sorry, GLSL not supported with this OpenGL.\n");
   return GL_FALSE;
}


GLuint
CompileShaderText(GLenum shaderType, const char *text)
{
   GLuint shader;
   GLint stat;
   GLdouble t0, t1;

   shader = CreateShader(shaderType);
   ShaderSource(shader, 1, (const GLchar **) &text, NULL);

   t0 = glutGet(GLUT_ELAPSED_TIME) * 0.001;
   glCompileShader(shader);
   t1 = glutGet(GLUT_ELAPSED_TIME) * 0.001;

   CompileTime = t1 - t0;

   GetShaderiv(shader, GL_COMPILE_STATUS, &stat);
   if (!stat) {
      GLchar log[1000];
      GLsizei len;
      GetShaderInfoLog(shader, 1000, &len, log);
      fprintf(stderr, "Error: problem compiling shader: %s\n", log);
      exit(1);
   }
   else {
      /*printf("Shader compiled OK\n");*/
   }
   return shader;
}


/**
 * Read a shader from a file.
 */
GLuint
CompileShaderFile(GLenum shaderType, const char *filename)
{
   const int max = 100*1000;
   int n;
   char *buffer = (char*) malloc(max);
   GLuint shader;
   FILE *f;
```

```
   f = fopen(filename, "r");
   if (!f) {
      fprintf(stderr, "Unable to open shader file %s\n", filename);
      free(buffer);
      return 0;
   }

   n = fread(buffer, 1, max, f);
   /*printf("read %d bytes from shader file %s\n", n, filename);*/
   if (n > 0) {
      buffer[n] = 0;
      shader = CompileShaderText(shaderType, buffer);
   }
   else {
      fclose(f);
      free(buffer);
      return 0;
   }

   fclose(f);
   free(buffer);

   return shader;
}


GLuint
LinkShaders(GLuint vertShader, GLuint fragShader)
{
   return LinkShaders3(vertShader, 0, fragShader);
}


GLuint
LinkShaders3(GLuint vertShader, GLuint geomShader, GLuint fragShader)
{
   GLuint program = CreateProgram();
   GLdouble t0, t1;

   assert(vertShader || fragShader);

   if (vertShader)
      AttachShader(program, vertShader);
   if (geomShader)
      AttachShader(program, geomShader);
   if (fragShader)
      AttachShader(program, fragShader);

   t0 = glutGet(GLUT_ELAPSED_TIME) * 0.001;
   LinkProgram(program);
   t1 = glutGet(GLUT_ELAPSED_TIME) * 0.001;

   LinkTime = t1 - t0;

   /* check link */
   {
      GLint stat;
      GetProgramiv(program, GL_LINK_STATUS, &stat);
      if (!stat) {
```

```
         GLchar log[1000];
         GLsizei len;
         GetProgramInfoLog(program, 1000, &len, log);
         fprintf(stderr, "Shader link error:\n%s\n", log);
         return 0;
      }
   }

   return program;
}


GLuint
LinkShaders3WithGeometryInfo(GLuint vertShader, GLuint geomShader, GLuint fragShader,
                             GLint verticesOut, GLenum inputType, GLenum outputType)
{
  GLuint program = CreateProgram();
  GLdouble t0, t1;

  assert(vertShader || fragShader);

  if (vertShader)
    AttachShader(program, vertShader);
  if (geomShader) {
    AttachShader(program, geomShader);
    glProgramParameteriARB(program, GL_GEOMETRY_VERTICES_OUT_ARB, verticesOut);
    glProgramParameteriARB(program, GL_GEOMETRY_INPUT_TYPE_ARB, inputType);
    glProgramParameteriARB(program, GL_GEOMETRY_OUTPUT_TYPE_ARB, outputType);
  }
  if (fragShader)
    AttachShader(program, fragShader);

  t0 = glutGet(GLUT_ELAPSED_TIME) * 0.001;
  LinkProgram(program);
  t1 = glutGet(GLUT_ELAPSED_TIME) * 0.001;

  LinkTime = t1 - t0;

  /* check link */
  {
    GLint stat;
    GetProgramiv(program, GL_LINK_STATUS, &stat);
    if (!stat) {
      GLchar log[1000];
      GLsizei len;
      GetProgramInfoLog(program, 1000, &len, log);
      fprintf(stderr, "Shader link error:\n%s\n", log);
      return 0;
    }
  }

  return program;
}


GLboolean
ValidateShaderProgram(GLuint program)
{
   GLint stat;
```

```
   ValidateProgramARB(program);
   GetProgramiv(program, GL_VALIDATE_STATUS, &stat);

   if (!stat) {
      GLchar log[1000];
      GLsizei len;
      GetProgramInfoLog(program, 1000, &len, log);
      fprintf(stderr, "Program validation error:\n%s\n", log);
      return 0;
   }

   return (GLboolean) stat;
}


GLdouble
GetShaderCompileTime(void)
{
   return CompileTime;
}


GLdouble
GetShaderLinkTime(void)
{
   return LinkTime;
}


void
SetUniformValues(GLuint program, struct uniform_info uniforms[])
{
   GLuint i;

   for (i = 0; uniforms[i].name; i++) {
      uniforms[i].location
         = glGetUniformLocation(program, uniforms[i].name);

      switch (uniforms[i].type) {
      case GL_INT:
      case GL_SAMPLER_1D:
      case GL_SAMPLER_2D:
      case GL_SAMPLER_3D:
      case GL_SAMPLER_CUBE:
      case GL_SAMPLER_2D_RECT_ARB:
      case GL_SAMPLER_1D_SHADOW:
      case GL_SAMPLER_2D_SHADOW:
      case GL_SAMPLER_1D_ARRAY:
      case GL_SAMPLER_2D_ARRAY:
      case GL_SAMPLER_1D_ARRAY_SHADOW:
      case GL_SAMPLER_2D_ARRAY_SHADOW:
         assert(uniforms[i].value[0] >= 0.0F);
         Uniform1i(uniforms[i].location,
                   (GLint) uniforms[i].value[0]);
         break;
      case GL_FLOAT:
         Uniform1fv(uniforms[i].location, 1, uniforms[i].value);
         break;
      case GL_FLOAT_VEC2:
```

```
         Uniform2fv(uniforms[i].location, 1, uniforms[i].value);
         break;
      case GL_FLOAT_VEC3:
         Uniform3fv(uniforms[i].location, 1, uniforms[i].value);
         break;
      case GL_FLOAT_VEC4:
         Uniform4fv(uniforms[i].location, 1, uniforms[i].value);
         break;
      case GL_FLOAT_MAT4:
         UniformMatrix4fv(uniforms[i].location, 1, GL_FALSE,
                          uniforms[i].value);
         break;
      default:
         if (strncmp(uniforms[i].name, "gl_", 3) == 0) {
            /* built-in uniform: ignore */
         }
         else {
            fprintf(stderr,
                    "Unexpected uniform data type in SetUniformValues\n");
            abort();
         }
      }
   }
}


/** Get list of uniforms used in the program */
GLuint
GetUniforms(GLuint program, struct uniform_info uniforms[])
{
   GLint n, max, i;

   GetProgramiv(program, GL_ACTIVE_UNIFORMS, &n);
   GetProgramiv(program, GL_ACTIVE_UNIFORM_MAX_LENGTH, &max);

   for (i = 0; i < n; i++) {
      GLint size, len;
      GLenum type;
      char name[100];

      glGetActiveUniform(program, i, 100, &len, &size, &type, name);

      uniforms[i].name = strdup(name);
      uniforms[i].size = size;
      uniforms[i].type = type;
      uniforms[i].location = glGetUniformLocation(program, name);
   }

   uniforms[i].name = NULL; /* end of list */

   return n;
}


void
PrintUniforms(const struct uniform_info uniforms[])
{
   GLint i;
```

```
   printf("Uniforms:\n");

   for (i = 0; uniforms[i].name; i++) {
      printf("  %d: %s size=%d type=0x%x loc=%d value=%g, %g, %g, %g\n",
             i,
             uniforms[i].name,
             uniforms[i].size,
             uniforms[i].type,
             uniforms[i].location,
             uniforms[i].value[0],
             uniforms[i].value[1],
             uniforms[i].value[2],
             uniforms[i].value[3]);
   }
}


/** Get list of attribs used in the program */
GLuint
GetAttribs(GLuint program, struct attrib_info attribs[])
{
   GLint n, max, i;

   GetProgramiv(program, GL_ACTIVE_ATTRIBUTES, &n);
   GetProgramiv(program, GL_ACTIVE_ATTRIBUTE_MAX_LENGTH, &max);

   for (i = 0; i < n; i++) {
      GLint size, len;
      GLenum type;
      char name[100];

      GetActiveAttrib(program, i, 100, &len, &size, &type, name);

      attribs[i].name = strdup(name);
      attribs[i].size = size;
      attribs[i].type = type;
      attribs[i].location = GetAttribLocation(program, name);
   }

   attribs[i].name = NULL; /* end of list */

   return n;
}


void
PrintAttribs(const struct attrib_info attribs[])
{
   GLint i;

   printf("Attribs:\n");

   for (i = 0; attribs[i].name; i++) {
      printf("  %d: %s size=%d type=0x%x loc=%d\n",
             i,
             attribs[i].name,
             attribs[i].size,
             attribs[i].type,
             attribs[i].location);
```

```
   }
}
```

Display support(shaderutil.h)

```
#ifndef SHADER_UTIL_H
#define SHADER_UTIL_H


#ifdef __cplusplus
extern "C" {
#endif


struct uniform_info
{
   const char *name;
   GLuint size;  /**< number of value[] elements: 1, 2, 3 or 4 */
   GLenum type;  /**< GL_FLOAT, GL_FLOAT_VEC4, GL_INT, GL_FLOAT_MAT4, etc */
   GLfloat value[16];
   GLint location;  /**< filled in by InitUniforms() */
};

#define END_OF_UNIFORMS   { NULL, 0, GL_NONE, { 0, 0, 0, 0 }, -1 }


struct attrib_info
{
   const char *name;
   GLuint size;  /**< number of value[] elements: 1, 2, 3 or 4 */
   GLenum type;  /**< GL_FLOAT, GL_FLOAT_VEC4, GL_INT, etc */
   GLint location;
};


extern GLboolean
ShadersSupported(void);

extern GLuint
CompileShaderText(GLenum shaderType, const char *text);

extern GLuint
CompileShaderFile(GLenum shaderType, const char *filename);

extern GLuint
LinkShaders(GLuint vertShader, GLuint fragShader);

extern GLuint
LinkShaders3(GLuint vertShader, GLuint geomShader, GLuint fragShader);

extern GLuint
LinkShaders3WithGeometryInfo(GLuint vertShader, GLuint geomShader, GLuint fragShader,
                             GLint verticesOut, GLenum inputType, GLenum outputType);

extern GLboolean
ValidateShaderProgram(GLuint program);

extern GLdouble
GetShaderCompileTime(void);
```

```
extern GLdouble
GetShaderLinkTime(void);

extern void
SetUniformValues(GLuint program, struct uniform_info uniforms[]);

extern GLuint
GetUniforms(GLuint program, struct uniform_info uniforms[]);

extern void
PrintUniforms(const struct uniform_info uniforms[]);

extern GLuint
GetAttribs(GLuint program, struct attrib_info attribs[]);

extern void
PrintAttribs(const struct attrib_info attribs[]);

/* These pointers are only valid after calling ShadersSupported.
 */
extern PFNGLCREATESHADERPROC CreateShader;
extern PFNGLDELETESHADERPROC DeleteShader;
extern PFNGLSHADERSOURCEPROC ShaderSource;
extern PFNGLGETSHADERIVPROC GetShaderiv;
extern PFNGLGETSHADERINFOLOGPROC GetShaderInfoLog;
extern PFNGLCREATEPROGRAMPROC CreateProgram;
extern PFNGLDELETEPROGRAMPROC DeleteProgram;
extern PFNGLATTACHSHADERPROC AttachShader;
extern PFNGLLINKPROGRAMPROC LinkProgram;
extern PFNGLUSEPROGRAMPROC UseProgram;
extern PFNGLGETPROGRAMIVPROC GetProgramiv;
extern PFNGLGETPROGRAMINFOLOGPROC GetProgramInfoLog;
extern PFNGLVALIDATEPROGRAMARBPROC ValidateProgramARB;
extern PFNGLUNIFORM1IPROC Uniform1i;
extern PFNGLUNIFORM1FVPROC Uniform1fv;
extern PFNGLUNIFORM2FVPROC Uniform2fv;
extern PFNGLUNIFORM3FVPROC Uniform3fv;
extern PFNGLUNIFORM4FVPROC Uniform4fv;
extern PFNGLGETACTIVEATTRIBPROC GetActiveAttrib;
extern PFNGLGETATTRIBLOCATIONPROC GetAttribLocation;

#ifdef __cplusplus
}
#endif

#endif /* SHADER_UTIL_H */
```