Ridge Regression: L2 Norm.

$$p(\beta | Y, X) \propto p(Y|\beta) \cdot p(\beta | b_i, S_i).$$

$$\propto \exp(-\tfrac{1}{2}(Y_i - X_i^T\beta)\cdot(Y_i - X_i^T\beta)) \cdot \exp(-\tfrac{1}{2}(\tfrac{|\beta_i|^2}{S_i}))$$, since $\sigma = 1$, $b_i = 0$.

$$\propto \exp(-\tfrac{1}{2}[(Y_i - X^T\beta)\cdot(Y_i - X_i^T\beta) + (\tfrac{|\beta_i|^2}{S_i})])$$

$$= \exp(-\tfrac{1}{2}[(Y - X^T\beta)^T(Y - X^T\beta) + \lambda|\beta_i|^2])$$, where $\lambda = 1/S_i$.

$$\log(p(\beta|Y,X)) \propto -\tfrac{1}{2}[(Y - X^T\beta)^T(Y - X^T\beta) + \lambda|\beta_i|^2]$$

$$= -\tfrac{1}{2}[\,||Y - X^T\beta||_2^2 + \lambda||\beta||_2^2\,].$$

since β_i is $p\times1$ vector.
We have $|\beta_i| = \sqrt{\sum_1^p \beta_i^2}$ and
we have $|\beta_i|^2 = \sum_1^p \beta_i^2 = ||\beta||_2^2$

Lasso Regression: L1 norm.

$$p(\beta | Y, X) \propto p(Y|\beta) \cdot p(\beta | b_i, S_i)$$

$$\propto \exp(-\tfrac{1}{2}(Y_i - X_i^T\beta)\cdot(Y_i - X_i^T\beta)) \cdot \exp(-\tfrac{|\beta_i|}{S_i})$$, since $\sigma = 1$, $b_i = 0$.

$$\propto \exp(-\tfrac{1}{2}[(Y_i - X^T\beta)\cdot(Y_i - X_i^T\beta) + \tfrac{1}{2}(\tfrac{|\beta_i|}{S_i})])$$

$$= \exp(-\tfrac{1}{2}[(Y - X^T\beta)^T(Y - X^T\beta) + \lambda|\beta_i|])$$, where $\lambda = 1/2S_i$

$$\log(p(\beta|Y,X)) \propto -\tfrac{1}{2}[(Y - X^T\beta)^T(Y - X^T\beta) + \lambda||\beta||\,].$$

since β_i is $p\times1$ vector,
$|\beta_i| = \sqrt{\sum \beta_i^2} = ||\beta||_2$