

Tanner Orndorff

Homework 3

I certify that this is my work and no one else's.

Total Score: 100/100

Section 1.3 Exercises 1.a, 3.a: 20/20

Section 1.3 Computer Problems 1, 3 :20/20

Section 1.4 Exercise 1.b : 10/10

Section 1.4 Computer Problem 1.c : 10/10

Section 1.5 Exercise 1.1a use secant method : 10/10

Section 1.5 Exercise 1.1b use modified secant method with $x_0 = 1$ and $h = .0001$
(modified secant method covered in class): 10/10

Section 1.5 Computer Problem 1 : 20/20

Section 1.3 Exercise 1a 3a

$$f(r) = 0 \\ \text{back} \rightarrow |f(x_0)| \\ \text{for } |r - x_0|$$

- (1a) Forward Error of $f(x) = 4x - 3$

$$\begin{array}{c} f(0.74) - 3 \\ \hline f(0.04) \end{array} \quad \left| \begin{array}{l} 3/4 - 3 \\ 0.01 \end{array} \right|$$

Backward Error:

$$\begin{array}{c} 4(0.74) - 3 \\ \hline 0.04 \end{array}$$

- (3a) Multiplicity of root $r=0$ of $f(x) = 1 - \cos x$

$$f(0) = 1 - \cos(0)$$

$$f'(0) = 0$$

$$f''(0) = \sin(0)$$

$$f'''(0) = 0$$

$$f''''(0) = \cos(0)$$

$$f''''(0) = 1$$

Multiplicity of 2 @ $r=0$

Section 1.4 Exercise 1.b

- (1b) 2 steps of Newton's Method, $x_0 = 0$, $x^4 - x^2 + x - 1 = 0$

$$\cancel{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

$$f'(x) = 4x^3 - 2x + 1 = 0$$

$$x_1 = 0 - \frac{0^4 - 0^2 + 0 - 1}{4(0)^3 - 2(0) + 1}$$

$$x_1 = 0 - \frac{-1}{1}$$

$$x_1 = 0 + 1 \quad (x_1 = 1)$$

$$x_2 = 1 - \frac{1^4 - 1^2 + 1 - 1}{4(1)^3 - 2(1) + 1}$$

$$x_2 = 1 - \frac{1 - 1 + 1 - 1}{4 - 2 + 1} \quad x_2 = 1 - \frac{0}{3}$$

$$(x_2 = 1)$$

Computer Problem Exercise 1.3

① $\sin x - x$

$$f(0) = \sin(0) - 0$$

$$f'(0) = \cos(0) - 1$$

$$f''(0) = -\sin(0)$$

$m=3$

$$\text{Computer response: } -2.0735e^{-08}$$

$$FE = \left| -2.0735e^{-8} - -1 \right|$$

$$FE = \left| 0 - -2.073e^{-8} \right|$$

or $-2.073e^{-8}$

Section 1.5 Exercise 1.1a, 1.1b

(1a) two step secant method

$$x_0 = 1 \quad x_1 = 2$$

$$x^3 = 2x + 2$$

$$x_2 = 2 - \frac{(-8+6)(1)}{(-2)-(-1+4)} = (-1^3 + 2(1) + 2) \cancel{\neq}$$

$$x_2 = 2 - \frac{(-8+6)(1)}{(-2)-(-1+4)} \quad f(x_2) = -(8/5)^3 + 2(8/5) + 2$$

$$x_2 = 2 - \frac{-2}{-2-3} \quad f(x_2) = 1.104$$

$$x_2 = 2 - \frac{-2}{-5}$$

$$x_2 = \frac{10}{5} - \frac{2}{5} \quad \text{or} \quad \boxed{x_2 = \frac{8}{5}}$$

$$x_3 = \frac{8}{5} - \frac{(1.104)(\frac{8}{5} - 2)}{(1.104) - (-2)}$$

$$x_3 = \frac{8}{5} - \frac{(1.104)(-.4)}{3.104}$$

$$x_3 = \frac{8}{5} - \frac{-0.4416}{3.104}$$

$$\boxed{x_3 = 1.742268041}$$

Section 1.5 Exercise 1.1b

$$x_0 = 1 \quad h = .0001 \quad e^x + x - 7$$

$$x_{i+1} = x_i - \frac{f(x_i) - f(x_i + 8x_i)}{f'(x_i + 8x_i) - f'(x_i)}$$

$$x_1 = 1 - \frac{.0001(1)(e^1 + 1 - 7)}{(e^{2.0001} + 2.0001 - 7) - (e^1 + 1 - 7)}$$

$$x_0 + \delta x_0 = 1 + 1(.0001)$$

$$x_2 = 1 - \frac{(1.0001)(-3.281718172)}{2.389895041} = (-3.281718172)$$

$$x_2 = 1 - \frac{-3.282046344}{3.71842277}$$

$$\delta x_1 = 1.578679508 \times 10^{-4}$$

$$f(x_1) = -.5727713801$$

$$x_2 = \boxed{1.578679508}$$

$$f(x_1 + \delta x_1) = -.571848021$$

$$x_3 = 1.578679508 - \frac{1.570... \times 10^{-4} (-.571848021)}{(-.571848021) - (-.5727713801)}$$

$$x_3 = \boxed{1.676607003}$$

Matlab code:

```
clc
%Computer Problem 1.3, 1
Xa1 = fzero('sin(x)-x',.1)
FE1 = abs(0 - fzero('sin(x)-x',.1))
BE1 = sin(Xa) - Xa
%Computer Problem 1.3, 3a
rootA = fzero('(2*x)*cos(x)-(2*x)+sin(x.^3)',[-.1,.2])
FE3 = abs(0 - fzero('(2*x)*cos(x)-(2*x)+sin(x.^3)',[-.1,.2]))
BE3 = (2*rootA)*cos(rootA)-(2*rootA)+sin(rootA.^3)
%%Computer Problem 1.3, 3b
BM3 = bisect(@(x) (2*x)*cos(x)-(2*x)+sin(x.^3), -.1,.2,.00001)
%Computer Problem 1.4, 1a
NEW = newtons(@(x) exp(x) + sin(x) - 4, @(x) exp(x) - cos(x), 1,8)
%computer Problem 1.5, 1
SecA = secantm(@(x) -x^3 + (2*x) + 2,1,2)
SecB = secantm(@(x) exp(x) + x - 7, 1, 2)
SecC = secantm(@(x) exp(x) + sin(x) - 4, 1, 2)
```

Outputs:

Xa1 =	rootA =
-2.0735e-08	1.6881e-04
FE1 =	FE3 =
2.0735e-08	1.6881e-04
BE1 =	BE3 =
0	0
NEW =	BM3 =
1.129986080000000	-6.1035e-05
SecA =	
1.769292354248405	
SecB =	
1.672821698628907	
SecC =	
1.129980498650832	

Function Matlab code:

```
%Program 1.1 Bisection Method
%Computes approximate solution of f(x) = 0
%input: function handle f; a,b such that f(a) * f(b) < 0
% and tolerance tol
%output: approximate solution xc
function xc=bisect(f,a,b,tol)
if sign(f(a))*sign(f(b)) >= 0
    error('f(a)f(b)<0 not satisfied!')
end
fa=f(a);
fb=f(b);
while(b-a)/2>tol
    c=(a+b)/2;
    fc=f(c);
    if fc == 0
        break
    end
    if sign(fc)*sign(fa)<0
        b=c;fb=fc;
    else
        a=c;fa=fc;
    end
end
xc = (a+b)/2;

%Program 1.4 Newton Method
function x=newtons(f,fp,xi,iter)
for i=0:iter
    xi=xi-(f(xi)/fp(xi));
end
format long
x = xi;
x = round(x,8);

%Program 1.5 Secant method
function x=secantm(f,xa,xi)
for i = 0:5
    temp = xi;
    xi=xi-((f(xi)*(xi-xa))/(f(xi)-f(xa)));
    xa = temp;
end
format long
x = xi;
```