Discovering Symmetry

Stephan Tornier
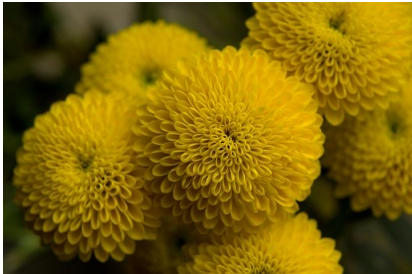
THE UNIVERSITY OF
NEWCASTLE
AUSTRALIA
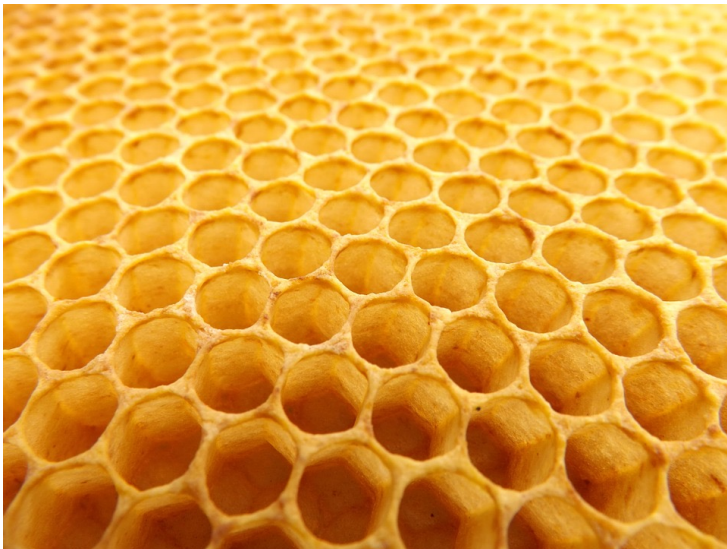
January 14, 2021

Symmetry
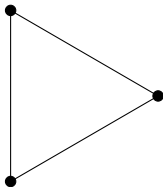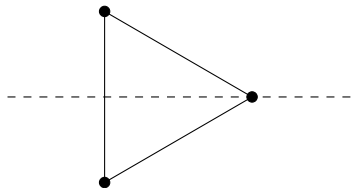
Beauty

Efficiency

Cost-effectiveness

Dynamical approach

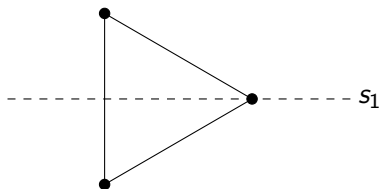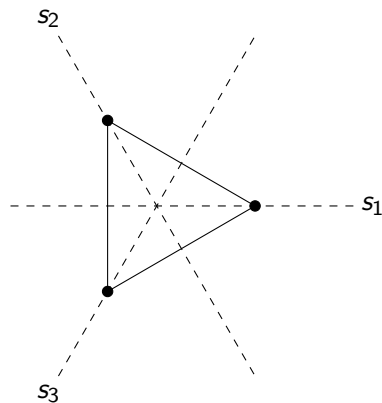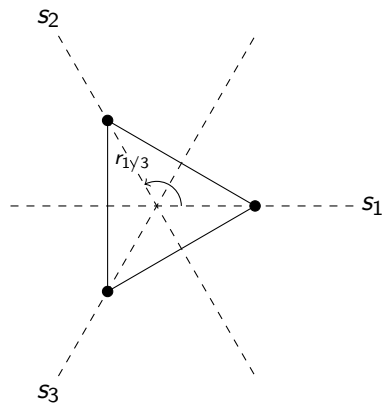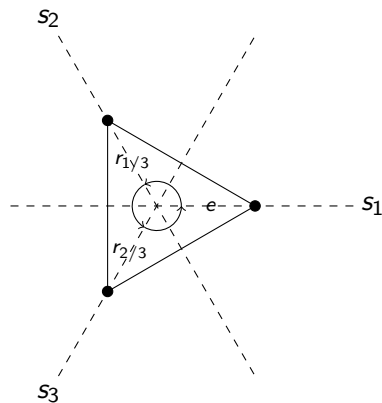# Dynamical approach

Dynamical approach

s_1

Dynamical approach

Dynamical approach

Dynamical approach

Dynamical approach
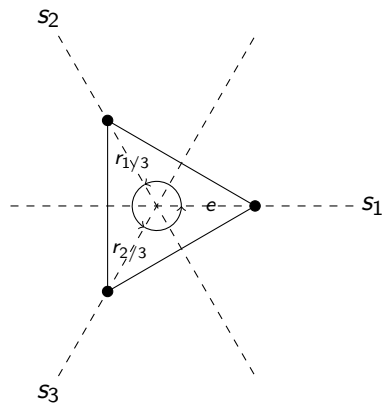
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e						
s_1						
s_2						
s_3						
$r_{1/3}$						
$r_{2/3}$						

Dynamical approach

	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e					
s_1						
s_2						
s_3						
$r_{1/3}$						
$r_{2/3}$						

Dynamical approach

	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1				
s_1						
s_2						
s_3						
$r_{1/3}$						
$r_{2/3}$						

Dynamical approach
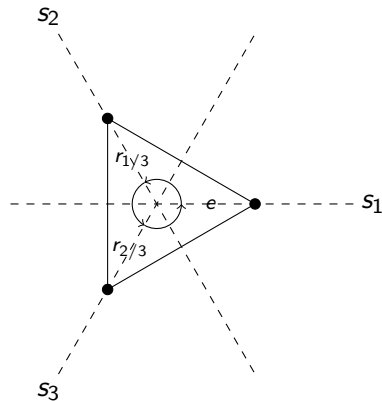
	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
s_1						
s_2						
s_3						
$r_{1/3}$						
$r_{2/3}$						

Dynamical approach
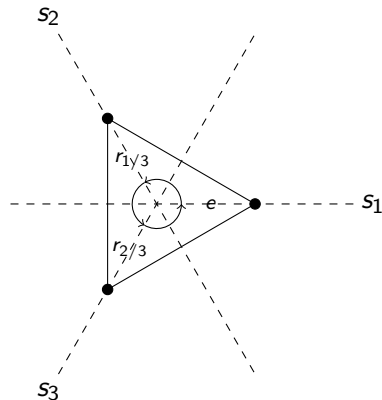
	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
s_1	s_1					
s_2	s_2					
s_3	s_3					
$r_{1/3}$	$r_{1/3}$					
$r_{2/3}$	$r_{2/3}$					

Dynamical approach
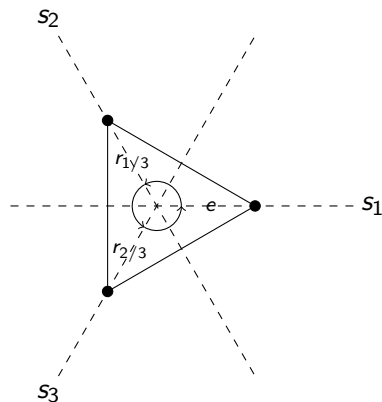
	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
s_1	s_1	e				
s_2	s_2					
s_3	s_3					
$r_{1/3}$	$r_{1/3}$					
$r_{2/3}$	$r_{2/3}$					

Dynamical approach

	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
s_1	s_1	e				
s_2	s_2		e			
s_3	s_3			e		
$r_{1/3}$	$r_{1/3}$					
$r_{2/3}$	$r_{2/3}$					

Dynamical approach

	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
s_1	s_1	e				
s_2	s_2		e			
s_3	s_3			e		
$r_{1/3}$	$r_{1/3}$				$r_{2/3}$	
$r_{2/3}$	$r_{2/3}$					

Dynamical approach
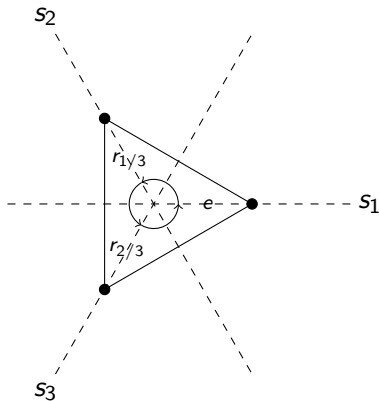
	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
s_1	s_1	e				
s_2	s_2		e			
s_3	s_3			e		
$r_{1/3}$	$r_{1/3}$				$r_{2/3}$	e
$r_{2/3}$	$r_{2/3}$				e	

Dynamical approach
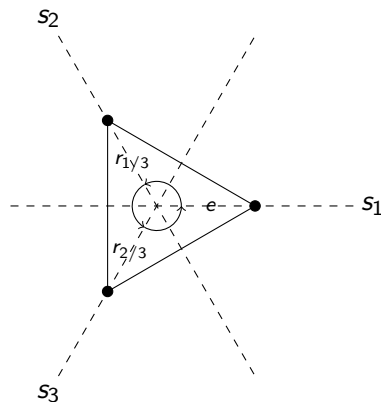
	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
e	e	s_1	s_2	s_3	$r_{1/3}$	$r_{2/3}$
s_1	s_1	e				
s_2	s_2		e			
s_3	s_3			e		
$r_{1/3}$	$r_{1/3}$				$r_{2/3}$	e
$r_{2/3}$	$r_{2/3}$				e	$r_{1/3}$

Exercise 1

Consider the figure below. Find and give names to all its symmetries, and record their compositions in the table.
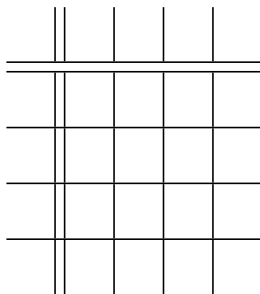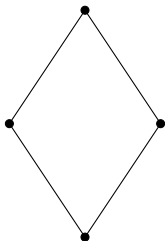
Exercise 2

Complete the following table of addition of integers modulo 4.

$+_4$	0	1	2	3
0				
1				
2				
3		0		

Exercise 3

Consider the mathematical expression below.

$$a + b + c \times d$$

For every choice of a, b, c and d, it assumes
a value. For example:

$(-1, 3, 2, 4) \mapsto -1 + 3 + 2 \times 4 = 10,$
$(-1, 4, 2, 3) \mapsto -1 + 4 + 2 \times 3 = 9.$
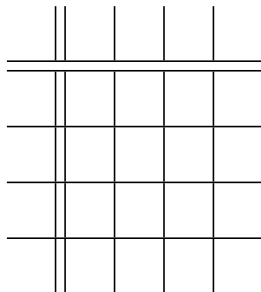
Find and give names to all rearrangements of the variables a, b, c and d
that leave the value of the expression unchanged for *every* choice, and
record their compositions in the table.
Note: By the above examples, swapping b and d is no such rearrangement.

Exercise 4

Consider the image below. Find and give names to all its symmetries, and record their compositions in the table.
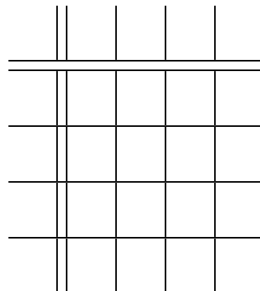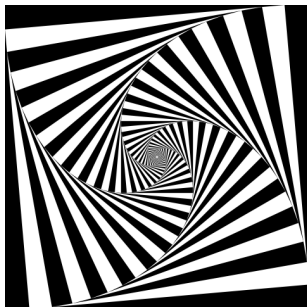
Results & comparison
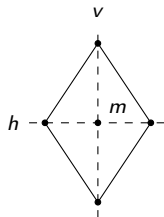
Results & comparison
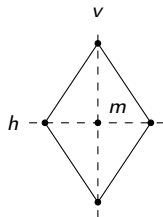
	e	h	v	m
e	e	h	v	m
h	h	e	m	v
v	v	m	e	h
m	m	v	h	e

Results & comparison
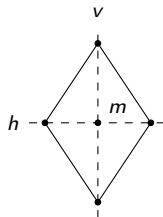
		e	h	v	m
---		---	---	---	---
e		e	h	v	m
h		h	e	m	v
v		v	m	e	h
m		m	v	h	e

Addition
modulo 4

Results & comparison

	e	h	v	m
e	e	h	v	m
h	h	e	m	v
v	v	m	e	h
m	m	v	h	e

Addition
modulo 4

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Results & comparison

	e	h	v	m
e	e	h	v	m
h	h	e	m	v
v	v	m	e	h
m	m	v	h	e

$$a \underset{s}{\overset{b}{+}} b + c \underset{t}{\overset{b}{\times}} d$$

Addition modulo 4

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Results & comparison

	e	h	v	m
e	e	h	v	m
h	h	e	m	v
v	v	m	e	h
m	m	v	h	e

$$a \underset{s}{\overset{b}{+}} b + c \underset{t}{\overset{b}{\times}} d$$

	e	s	t	b
e	e	s	t	b
s	s	e	b	t
t	t	b	e	s
b	b	t	s	e

Addition modulo 4

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Results & comparison

	e	h	v	m
e	e	h	v	m
h	h	e	m	v
v	v	m	e	h
m	m	v	h	e

$$a + b + c \times d$$

	e	s	t	b
e	e	s	t	b
s	s	e	b	t
t	t	b	e	s
b	b	t	s	e

Addition modulo 4

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Results & comparison

	e	h	v	m
e	e	h	v	m
h	h	e	m	v
v	v	m	e	h
m	m	v	h	e

$$a \underset{s}{\overset{b}{+}} b + c \underset{t}{\overset{b}{\times}} d$$

	e	s	t	b
e	e	s	t	b
s	s	e	b	t
t	t	b	e	s
b	b	t	s	e

Addition modulo 4

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

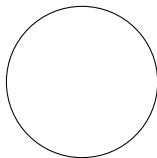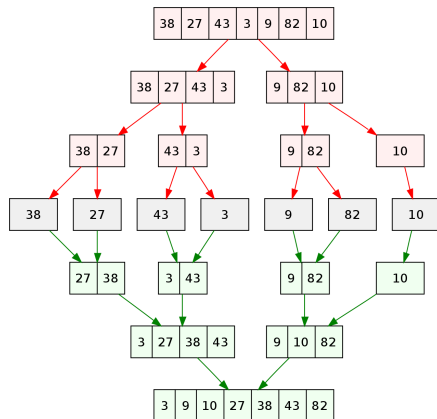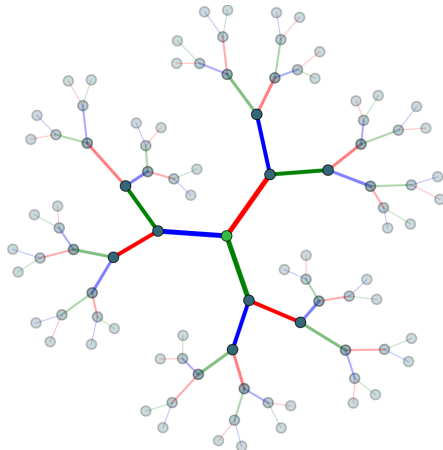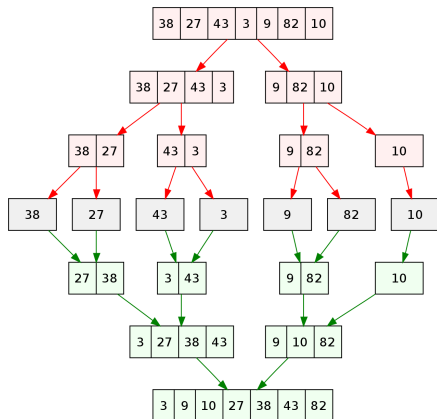|	e	$r_{1/4}$	$r_{2/4}$	$r_{3/4}$
e	e	$r_{1/4}$	$r_{2/4}$	$r_{3/4}$
$r_{1/4}$	$r_{1/4}$	$r_{2/4}$	$r_{3/4}$	e
$r_{2/4}$	$r_{2/4}$	$r_{3/4}$	e	$r_{1/4}$
$r_{3/4}$	$r_{3/4}$	e	$r_{1/4}$	$r_{2/4}$

More symmetry
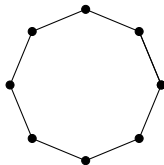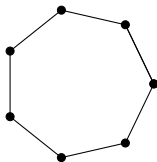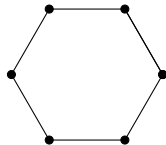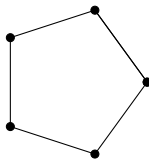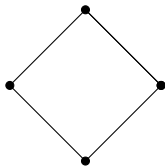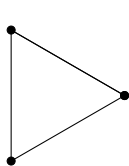
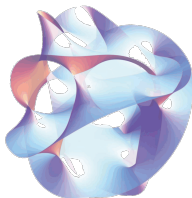# More symmetry
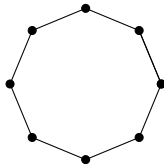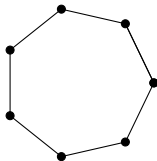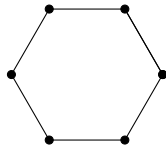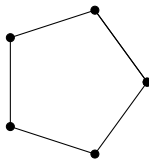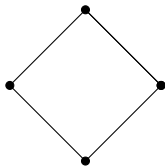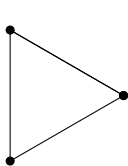
Biology

Biology
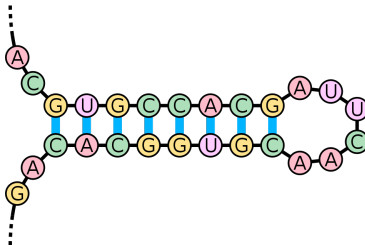
Computer Science

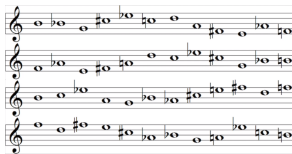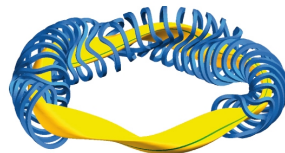# Computer Science

And everywhere else

Physics

Music

Engineering

Evolution

Information Technology

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