tornikeo_e5

February 14, 2024

[1]:
```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mpl
import pandas as pd
```

Download the book

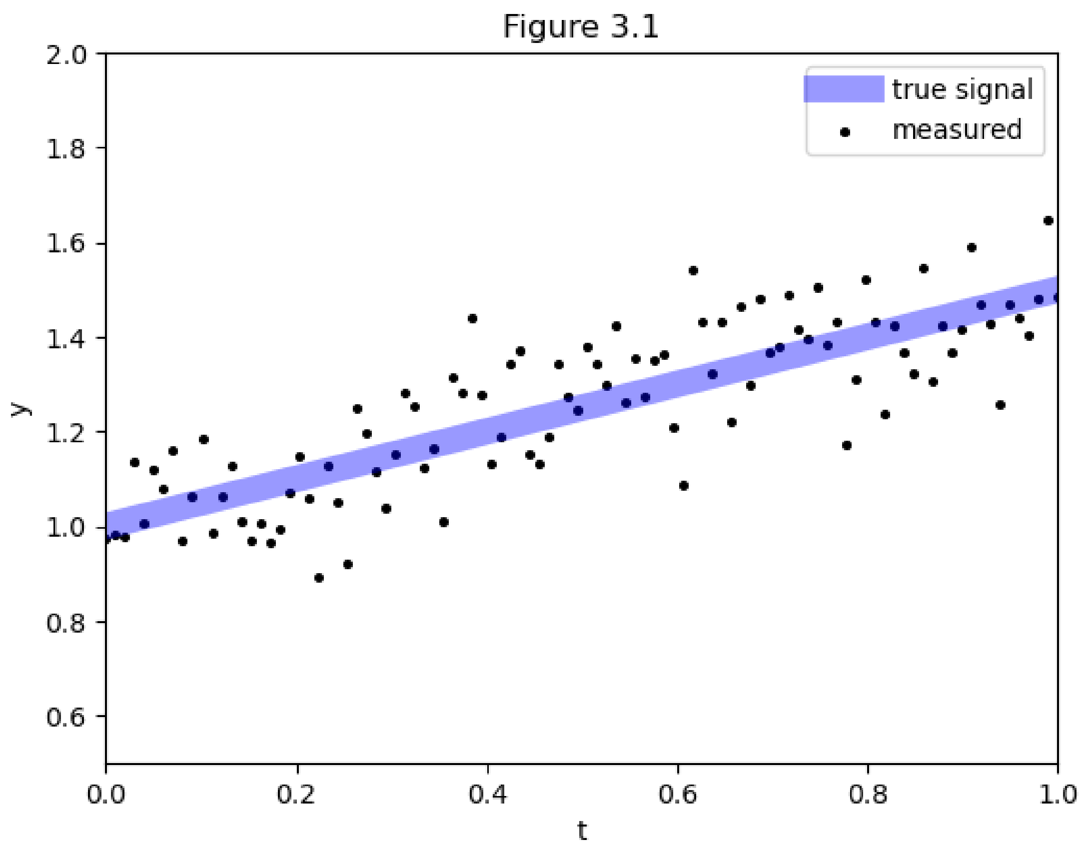
- Simo Särkkä and Lennart Svensson (2023). Bayesian Filtering and Smoothing, Second Edition. Cambridge University Press. https:// users.aalto.fi/~ssarkka/pub/bfs_book_2023_online.pdf

0.1 Task 1

1. Reproduce the example in Figure 3.1.

[55]:
```python
from scipy import stats
sigma = .1
theta = np.array([[1, .5]]).T

H_k = np.linspace([1, 0],[1, 1],100)
t_k = H_k[:,1]
y_signal = H_k @ theta
y_meas = stats.norm.rvs(loc=y_signal, scale=sigma)
plt.plot(t_k, y_signal, linewidth=10., c='b', label='true signal', alpha=.4)
plt.scatter(t_k, y_meas, c='k', s=7, label='measured')
plt.xlabel("t")
plt.ylabel("y")
plt.ylim(.5, 2)
plt.xlim(0, 1)
plt.legend()
plt.title("Figure 3.1")
plt.show()
```



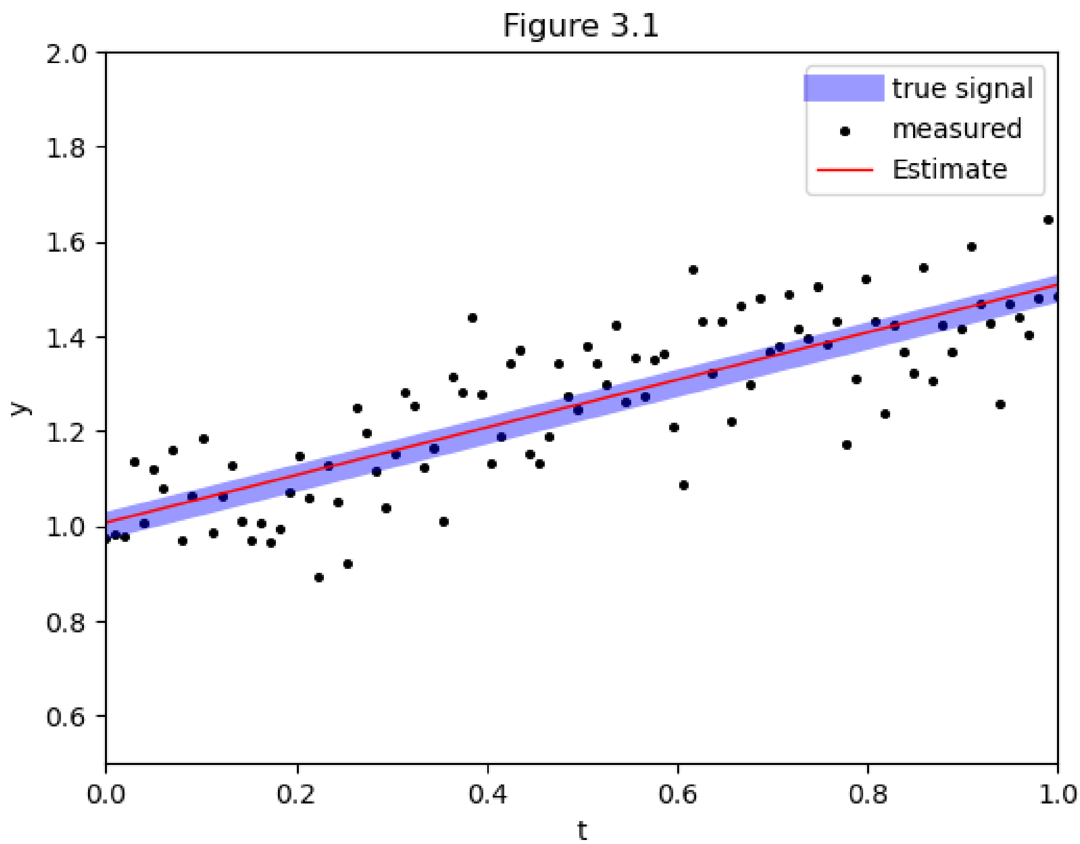
0.2 Task 2

2. Reproduce the example in Figure 3.2.

[56]:
```
P_0 = (np.eye(2) * sigma)
m_0 = np.ones_like(theta)
P_0_inv = np.linalg.inv(P_0)
a = ( P_0_inv + sigma**-2 * H_k.T @ H_k)
b = (sigma**-2 * H_k.T @ y_meas + P_0_inv @ m_0)
m_t = np.linalg.inv(a) @ b
P_t = np.linalg.inv(P_0_inv + sigma**-2 * H_k.T @ H_k)

y_pred = H_k @ m_t
plt.plot(t_k, y_signal, linewidth=10., c='b', label='true signal', alpha=.4)
plt.scatter(t_k, y_meas, c='k', s=7, label='measured')
plt.plot(t_k, y_pred, linewidth=1., c='r', label='Estimate')
plt.xlabel("t")
plt.ylabel("y")
plt.ylim(.5, 2)
plt.xlim(0, 1)
```

```
plt.legend()
plt.title("Figure 3.1")
plt.show()
```



0.3 Task 3

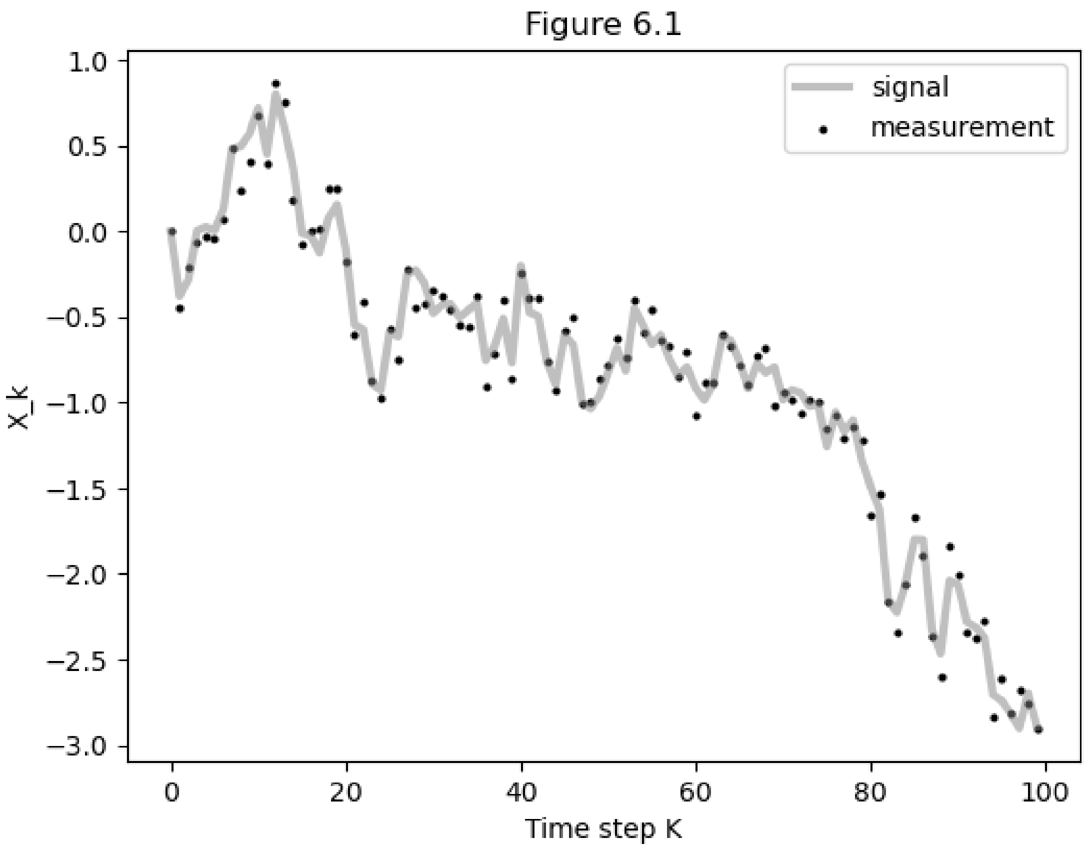
3. Reproduce the example in Figure 6.1.

[79]:
```
# Straight from the recurrent definition

steps = np.arange(0,100)
x_k = np.zeros_like(steps, dtype='float32')
y_k = np.zeros_like(x_k)
Q = .2
R = .1
for k in steps[1:]:
    x_k[k] = stats.norm.rvs(loc=x_k[k - 1], scale=Q) # Random walk
    y_k[k] = stats.norm.rvs(loc=x_k[k], scale=R) # Our sensors messing up

plt.plot(steps, x_k, linewidth=3., c='gray', alpha=.5, label='signal')
```

```python
plt.scatter(steps, y_k, s=4, c='k', marker='o', label='measurement')
plt.legend()
plt.xlim([0, 100])
plt.axis('tight')
plt.xlabel("Time step K")
plt.ylabel("X_k")
plt.title("Figure 6.1")
plt.show()
```



0.4 Task 4

4. Complete Exercise 6.1 (in page 106).

Derive the Kalman filter equations for the following linear-Gaussian filtering model with non-zero-mean noises:

$$x_k = Ax_{k-1} + q_{k-1}y_k = Hx_k + r_k$$

Where

$q_{k-1} \sim N(m_q, Q)$ and $r_k \sim N(m_r, R)$.

Well, a simple solution to this would be to just incorporate a bias term (known or unknown), to the state vector, i.e., to rewite the model like so:

$$x_k = Ax_{k-1} + \hat{q}_{k-1} - m_q$$
$$y_k = Hx_k + \hat{r}_k - m_r$$

In this case the $\hat{q}$ and $\hat{r}$ are now distributed as zero-centered normal gaussians.

In case the values m_q and m_r are known beforehand, we can move them to the right hand side, and solve these equations with regular Kalman filter:

$$\hat{x}_k = Ax_{k-1} + \hat{q}_{k-1}$$
$$\hat{y}_k = Hx_k + \hat{r}_k$$

Where $\hat{x}_k = x_k + m_q$ and $\hat{y}_k = y_k + m_r$

Unfortunately, I couldn't find the solution on the case where m_r and m_q aren't known initially.