Statistical Parameter Estimation 2024
Exercises 3
Deadline 31 January 2024, 23:59 Lappeenranta time

Problem 1: Consider the SIR model, consider modelling of synthetic data, and then make parameter estimation for β and γ.

- Get either an analytical solver, or a numerical solver, which outputs $S(t), I(t), R(t)$.
- Make synthetic test case with noise-perturbed observations, e.g.

$$S_{\text{observed}}(\mathbf{t}) = S_{\text{truth}}(\mathbf{t}) + \mathbf{e} \tag{1}$$

  where $\mathbf{e} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$

- Write posterior distribution for $(\beta, \gamma)^\intercal$.
- Use MCMC to get posterior estimates and uncertainty quantification, and evaluate MCMC chains.
- Plot predictive intervals.

Problem 2: Take one realisation of white noise, form posterior distribution, and obtain variance estimate with MCMC. Make the standard plots.

Problem 3:

1. Given one realisation of the Ornstein-Uhlenbeck process with fixed σ, use MCMC to obtain CM-estimate of λ. That is,
   - Draw one realisation of the OU process.
   - Formulate the posterior of λ.
   - Use MCMC to estimate λ.
   - Plot chains, ACFs, densities and compute ESS and OES.
   - Visualise posterior parameter estimates and Monte Carlo errors.
2. In order to see different behaviour of priors, do the same as above, but with three different priors
   - Use a Gaussian prior for $\lambda \sim \mathcal{N}(0, \sigma_{\text{pr}}^2)$.
   - Use a logarithmic transformation, that is $\log(\lambda) \sim \mathcal{N}(0, \sigma_{\text{pr}}^2)$.

- Use a uniform prior $\lambda \sim \text{Unif}(a, b)$, where $a < b$ are suitable constants.

Choose the prior parameters in such a way that you can see different effects of the priors.

3. Can you estimate all the parameters and are they identifiable?

   - Run the same analysis as above, but for σ with fixed λ,
   - Run the same analysis as above, but for parameter vector $\theta := (\lambda, \sigma)^\intercal$. Plot joint density and marginal densities.