Statistical Parameter Estimation 2024
Exercises 1
Deadline 17 January 2024, 23:59 Lappeenranta time

Let us consider a simple exponential decay

$$y(t) = a\exp(-bt) + \varepsilon(t). \tag{1}$$

Let the data be

$$y(t) = (0.3573, 0.3618, 0.1920, 0.1585, 0.1041, 0.1100, 0.0560, 0.0291, 0.0252, 0.0249, 0.04160)^T,$$
$$t = (0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0)^T,$$
$$\varepsilon(t) \sim \mathcal{N}(0, \sigma^2), \quad \text{with} \quad \sigma = 0.02.$$

Original simulation values are $a = 0.4$ and $b = 3$. We denote $\theta := (a, b)^T$.

1. Likelihood density
   - Write the likelihood density of θ given $y(t)$.
   - Derive the negative log-likelihood.
2. Optimisation: use the methods in the optimisation toolbox e.g. fminsearch in Matlab, to obtain a numerical optimizer for the maximum likelihood estimator of θ.
3. Integration
   - Use numerical integration to calculate the conditional mean estimator of θ. Any standard quadrature will do.
   - Similarly, calculate numerically marginal densities for a and b.
4. Priors
   - Choose a Gaussian prior for θ and define a posterior distribution. Calculate the negative log posterior. Calculate the MAP and CM estimators.
   - Do the same as above, but choose a prior distribution that is uniformly distributed.
5. Visualisation
   - Plot data and ground truth in the same plot.

- Take all the parameter estimates calculated above and plot the corresponding curves.
- Tune the prior parameters and study how the parameter values affect the estimators and curves.