Antonius' Compendium

Derivations, Formulas, Units and Definitions
Volume I
Version 2.035

Compiled by: Antonius W. Torode[1]
1. Applied Research Laboratories - University Of Texas: Austin
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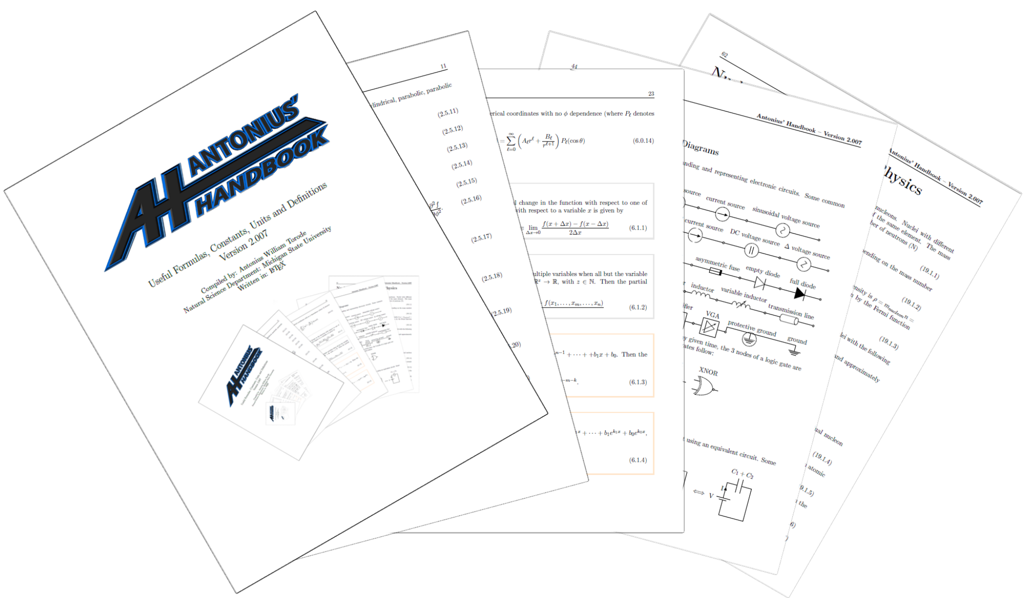

The original maintainer of this work is: Antonius Torode.

The current maintainer of this work is: Antonius Torode.

Chief Editor: Antonius Torode

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Preface

This document is a compilation of scratch work, derivations, useful formulas, definitions, constants, and general information used for my own studies as a reference while furthering self education. It's purpose is to provide a complete 'compendium' per say of various mathematical and significant ideas used often. The idea and motivation behind it is to be a quick reference providing easily accessible access to necessary information for either double checking or recalling proper formula for use in various situations due to my own shortcomings in matters of memorization. All the material in this document was either directly copied from one of the references listed at the end or derived from scratch. On occasion *typos may exist* due to human error but will be corrected when discovered.

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Topics Covered In This Book

- topic 1
- topic 2

The information in this book is in no way limited to the topics listed above. They serve as a simple guideline to what you will find within this document. For more information about this book or details about how to obtain your own copy please visit:

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Disclaimer

This book contains formulas, definitions, and theorems that by nature are very precise. Due to this, some of the material in this book was taken directly from other sources such as but not limited to Wolfram Mathworld. This is only such in cases where a change in wording could cause ambiguities or loss of information quality. Following this, all sources used are listed in the references section and cited when used.

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(Yes, this is a contradiction.)

Contents

Constants and units

1.0.1: Physical Constants

Constant	Symbol	Value	Units
Speed of light in a vacuum	$c \equiv 1/\sqrt{\mu_0\epsilon_0}$	2.99792458×10^8	m/s
Elementary charge	e	$1.602176565(35) \times 10^{-19}$	C
Gravitational constant	G	$6.67384(80) \times 10^{-11}$	$\mathrm{m^3kg^{-1}s^{-2}}$
Avagadro's number	N_a	$6.02214129(27) \times 10^{23}$	mol$\cdot s^{-1}$
Planck constant	h	$6.62606872(52) \times 10^{-34}$	J·s
		4.135668×10^{-15}	eV·s
	hc	1239.84	eV·nm
Reduced planck constant	$\hbar \equiv h/2\pi$	1.05×10^-34	J·s
Permittivity of the vacuum	ϵ_0	8.854×10^{-12}	$\mathrm{C^2N^{-1}m^{-2}}$
Permeability of the vacuum	μ_0	$4\pi \times 10^{-7}$	$\mathrm{N/A^2}$
Permeability of the vacuum	μ_0	$4\pi \times 10^{-7}$	$\mathrm{N/A^2}$
Boltzmann constant	k_B	$1.38064852 \times 10^{-23}$	J/K
		8.61733×10^{-5}	eV/K
Stefan-Boltzmann constant	$\sigma_\mathrm{B} \equiv \frac{\pi^2 k_B^4}{60\hbar^3 c^3}$	$5.670367(13) \times 10^{-8}$	$\mathrm{W{\cdot}m^{-2}K^{-4}}$
Thomson cross-section	σ_e	6.652×10^{-29}	m^2
The Bohr Magneton	$\mu_B \equiv \frac{e\hbar}{2m}$	5.788×10^{-5}	eV/T
		9.274×10^{-24}	$\mathrm{Am^2}$
Mass of an electron	m_e	$9.10938291(40) \times 10^{-31}$	kg
		510.9989	keV/c^2
Mass of a proton	m_p	$1.6726218 \times 10^{-27}$	kg
		938.27203	MeV/c^2
Mass of a neutron	m_n	$1.6749274 \times 10^{-27}$	kg
		939.56536	MeV/c^2
Unified amu	u	$1.660538782 \times 10^{-27}$	kg
		931.494028	$\mathrm{MeV/c^2}$

1.0.2: Stellar Data

Spectral Type	T_{eff} (K)	$M/M_\odot$	$L/L_\odot$	$R/R_\odot$	V_{mag}
O5	44,500	60	7.9×10^5	12	-5.7
B5	15,400	5.9	830	3.9	-1.2
A5	8,200	2.0	14	1.7	1.9
F5	6,440	1.4	3.2	1.3	3.4
G5	5,770	0.92	0.79	0.92	4.9
K5	4,350	0.67	0.15	0.72	6.7
M5	3,170	0.21	0.011	0.27	12.3

1.0.3: Astronomical Constants

Constant	Symbol	Value	Units
Mass of Earth	$M_\oplus$	5.974×10^{24}	kg
Mass of Sun	$M_\odot$	1.989×10^{30}	kg
Mass of Moon	$M_☾$	7.36×10^{22}	kg
Equatorial radius of Earth	$R_\oplus$	6.378×10^{6}	m
Equatorial radius of Sun	$R_\odot$	6.6955×10^{8}	m
Equatorial radius of Moon	$R_☾$	1.737×10^{6}	m
Mean density of Earth		5515	$\mathrm{kg{\cdot}m^{-3}}$
Mean density of Sun		1408	$\mathrm{kg{\cdot}m^{-3}}$
Mean density of Moon		3346	$\mathrm{kg{\cdot}m^{-3}}$
Earth-Moon distance		3.84×10^{8}	m
Earth-Sun distance		1.496×10^{11}	m
Luminosity of Sun	$L_\odot$	3.839×10^{26}	W
Effective temp. of Sun		5778	K
Hubble constant	H_0	70 ± 5	$\mathrm{km{\cdot}s^{-1}Mpc^{-1}}$
Parsec	pc	206264.81	AU
		3.0856776×10^{16}	m
		3.2615638	ly
Astronomical Unit	AU	1.496×10^{11}	m
Light year	ly	9.461×10^{15}	m
1 year on Earth	yr	365.25	days
		3.15576×10^{7}	s

1.0.4: Solar System

Planet	Symbol	Mass (kg)	Radius (m)	Sun-Distance (km)
Mercury	☿	3.285×10^{23}	2.44×10^{6}	5.791×10^{10}
Venus	♀	4.867×10^{24}	6.052×10^{6}	1.082×10^{11}
Mars	♂	6.39×10^{23}	3.390×10^{6}	2.279×10^{11}
Jupiter	♃	1.898×10^{27}	3.83×10^{11}	7.785×10^{11}
Saturn	♄	5.683×10^{26}	5.8232×10^{7}	1.429×10^{12}
Uranus	⛢	8.681×10^{25}	2.5362×10^{7}	2.871×10^{12}
Neptune	♅	1.024×10^{26}	2.4622×10^{7}	4.498×10^{12}
Pluto	♇	1.309×10^{22}	1.187×10^{6}	5.906×10^{12}

1.0.5: Unit conversions

The International System of Units (SI) defines seven units of measure as a basic set from which all other SI units can be derived. These are [length](m), [time](s), [mass](kg), [electric current] $\equiv$ [Ampere](A), [temperature](K), [luminous intensity](cd), [amount of substance](mol).

Unit Symbol	Unit	Equivalence
C	[Coulomb]	[Ampere][time]
N	[Newton]	[mass][length][time]$^{-2}$
P	[Pascal]	[mass][length]$^{-1}$[time]$^{-2}$
J	[Joule]	[mass][length]^{2}[time]$^{-2}$
W	[Watt]	[mass][length]^{2}[time]$^{-3}$
		[Ohm][Ampere]^{2}
		[Volt]^{2}[Ohm]$^{-1}$
V	[Volt]	[mass][length]^{2}[time]$^{-3}$[Ampere]$^{-1}$
Wb	[Weber]	[mass][length]^{2}[time]$^{-2}$[Ampere]$^{-1}$
T	[Tesla]	[mass][time]$^{-2}$[Ampere]$^{-1}$
H	[henry]	[mass][length]^{2}[time]$^{-2}$[Ampere]$^{-2}$
Ω	[Ohm]	[mass][length]^{2}[time]$^{-3}$[Ampere]$^{-2}$
F	[Farad]	[mass]$^{-1}$[length]$^{-2}$[time]^{4}[Ampere]^{2}
Hz	[Hertz]	[time]$^{-1}$

1.0.6: Number Sets ($i \equiv \sqrt{-1}$)

Symbol	Set	Symbol	Set
$\mathbb{R}$	Real numbers	$\emptyset$	{}
$\mathbb{N} \equiv \mathbb{N}_1$	{1,2,3,4,...}	$\mathbb{Z}$	{...,-2,1,0,1,2,...}
$\mathbb{Z}^+ \equiv \mathbb{N}_0$	{0,1,2,3,...}	$\mathbb{Z}^-$	{0,-1,-2,-3,-4,...}
$\mathbb{C}$	$\{x+iy \mid x,y \in \mathbb{R}\}$	$\mathbb{Q}$	$\{\frac{x}{y} \mid x,y \in \mathbb{Z}\}$
$\mathbb{I}$	$\{ix \mid x \in \mathbb{R}\}$	$\mathbb{U}$	Universal Set [a]
$\mathbb{A}$	Algebraic Numbers[b]	$\mathbb{T}$	Transcendental Numbers [c]

[a] Definition: The set containing all objects or elements and of which all other sets are subsets.
[b] Any number that is a solution to a polynomial equation with rational coefficients.
[c] Any number that is not an Algebraic Number.

1.0.7: Mathematical Notation

$\forall$	For all	$\exists$	There exists	$\because$	Because
$\in$	Is an element of	$\notin$	Is not an element of	$\therefore$	Therefore
$\Longrightarrow$	Implies	$\Longleftrightarrow$	Bi conditional	$\approx$	Approximately
$\longrightarrow$	Mapped to	$\not\subseteq$	Is not a subset of	$\ll$	Much smaller than
$\subset$	Is a subset of	$\subseteq$	Is a subset or equal to	$\gg$	Much greater than
$\propto$	Is proportional to	$\equiv$	Is equivalent to	$\cup / \cap$	Union/Intersection
$\perp$	Is perpendicular to	$\parallel$	Is parallel to	$:$ or $\mid$	Such that

Statistics and Probability

A **state** (or outcome) is particular condition that something is in at a specific time.

Definition 2.0.1: System

A **system** is an activity, experiment, process, or model with states or outcomes that are typically subject to uncertainty.

Definition 2.0.2: Sample Space

A **sample space** of an system is the set of all possible states of a system.

Definition 2.0.3: Event

An **event** (also may be referred to as a trial or measurement) is any subset or collection of states contained in the sample space of a system. An event is a **simple event** if it consists of exactly one state and a **compound event** if it consistst of more than one state.

Definition 2.0.4: Probability

An **probability** p can be defined as the asymptotic frequency of a system in the state s ($s \in \Omega$, where Ω is the sample space of the system) by the total number of occurances of that state N_s in the limit of an infinite number of events N.

$$p(s) = \lim_{N\to\infty} \frac{N_s}{N} \tag{2.0.1}$$

$$p(s) \in [0,1] \forall s \in \Omega \tag{2.0.2}$$

For a system with n states, the total probabilities of all states must normalize to one.

$$\sum_{i=0}^{n} p(i) = \sum_{s\in\Omega} p(i) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{n} N_i = 1 \tag{2.0.3}$$

A **Bayesian probability** is defined as a person's knowledge of the outcome of a trial, based on the evidence at their disposal - often encompanied by an associated error. A **model probability** is an assumption or guess for the probability given the possibility of an infinite nuber of trials.

References