Generations Of Nuclear Activity (G.I.N.A)
Manual & Programming Documentation
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(Document continually under development)

Developer: Antonius Torode
Michigan State University
Department of Physics & Astronomy
National Superconducting Cyclotron Laboratory

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The Original Maintainer of this work is: Antonius Torode.

The Current Maintainer of this work is: Antonius Torode.

This document is designed for the sole purpose of providing documentation for the Generations of Nuclear Activity (GINA) program. GINA is a program written for and created at the National Superconducting Cyclotron Laboratory (NSCL). Throughout this document, "GINA" will be used as a reference to the "Generations of Nuclear Activity" program. This acronym is solely designed for use in conjunction with the program and was originally created by the creator of this document. GINA is designed for specific purposes at the NSCL. It can be adapted to work in other situations and for alternate purposes. GINA is not be to used for gaining profit. GINA may be freely used and adapted to be used solely at Michigan State University and the NSCL.

This document is continually in development as GINA is. The version number of this document is to correspond with the version of GINA it covers.

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Contents

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(Indeed, this is a contradiction.)

Generations of Nuclear Activity (GINA)

GINA(Generations of Implanted Nuclear Activity) is a program designed to work with an experimental tape station. An experimental tape system is a system designed to have some form of tape in which a small portion of the tape acts as a target for radioactive isotopes. At some point, the tape should be able to move such that a different point of the tape becomes the target.

GINA performs multiple functions related to the radioactive isotopes at the target location. First, GINA calculates the number of radioactive isotopes at a target location based on some initial and constant rate of isotope implantation over some time. Next, GINA uses this information to determine the radioactive decay rates of each isotope at the target. Finally, GINA does a calculation of the contamination on the target relative to the primary isotope. Explanations of these calculations can be found later in this manual.

When a radioactive isotope is implanted on a target, it may naturally decay into some daughter isotope. The daughter isotope may then decay into a granddaughter isotope and so on. Because of this, GINA takes the half-lives of the primary isotope and each generation isotope up to four isotopes as initial input values. These are what is used to perform the calculations described in the above paragraph.

Original GINA Calculations

Preamble

This chapter was the original document designed to go along with the python code it was written for (the original GINA program). For this reason, relevant the code was attached in code blocks. It is not updated to include references to the GINA program which is the C++ implementation. The derivations in this chapter were originally done on scrap paper and had a LaTeXdocument to keep relevant solutions.

Introduction

The rate of decay of a material is given by

$$\frac{dN(t)}{dt} = -\lambda N(t) = -\frac{N(t)}{\tau}, \tag{2.1}$$

where N is the number of radioactive nuclei, λ is the decay constant and τ is the mean lifetime of the particles within the material. The decay relates to the half life $T_{1/2}$ by

$$\lambda T_{1/2} = \ln(2) \implies T_{1/2} = \tau \ln(2). \tag{2.2}$$

From Eq. 2.1, we can say

$$\frac{dN(t')}{N(t')} = -\frac{1}{\tau}dt' \implies \int_{N(t)}^{N(t+\Delta t)} \frac{dN(t')}{N(t')} = -\int_{t}^{t+\Delta t} \frac{1}{\tau}dt' \implies \ln\left[\frac{N(t+\Delta t)}{N(t)}\right] = -\frac{\Delta t}{\tau}. \tag{2.3}$$

We can re-write Eq. 2.3 in exponential form which gives

$$N(t+\Delta t) = N(t)e^{-\frac{\Delta t}{\tau}} = N(t)e^{-\frac{\Delta t}{T_{1/2}}\ln(2)} \implies N(\Delta t) = N(0)e^{-\frac{\Delta t}{T_{1/2}}\ln(2)}. \tag{2.4}$$

Analysis - Decays Over Time

RADIOACTIVE PARTICLE NUMBERS

Let the superscripts m, d and gd denote a mother, daughter and granddaughter (the nuclei the daughter nuclei decays into) nuclei respectively and T represent the total time that has passed. Consider a steady beam that is depositing radioactive particles onto a material at a constant rate of $R(t)$ (particles/second). Initially assume there are no radioactive particles on the material. That is

$$N_0^m = 0 \qquad N_0^d = 0 \qquad N_0^{gd} = 0 \tag{2.5}$$

Over some small amount of time[1] Δt ($T = \Delta t$), the total particles placed on the material will be

$$N_1^m = R(t)\Delta t. \tag{2.6}$$

None of the radioactive particles will have decayed yet so $N_1^d = 0$ and $N_1^{gd} = 0$. The mother nuclei may decay into the daughter nuclei at this point but the daughter nuclei will not have yet had a chance to decay and thus $N_2^{gd} = 0$.

```
#[PYTHON CODE]#
#Sets the initial conditions.
N_m[0]=0
N_m[1]=R*del_t
N_d[0]=0
N_d[1]=0
N_gd[0]=0
N_gd[1]=0
```

After another small amount of time Δt ($T = 2\Delta t$), the total radioactive particles on the material will increase by $R(t)\Delta t$ and decrease by some amount

$$N_2^d = D_1^m = N_1^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right), \tag{2.7}$$

as the radioactive particles decay into the daughter nuclei. Thus, the total radioactive mother nuclei after $T = 2\Delta t$ is

$$N_2^m = 2N_1^m - N_1^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right). \tag{2.8}$$

After yet another small amount of time Δt ($T = 3\Delta t$), the total radioactive particles on the material will increase by $N_1^m = R(t)\Delta t$ and decrease by some amount

$$D_2^m = N_2^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right), \tag{2.9}$$

as the radioactive particles decay into the daughter nuclei. At this point, the total number of mother nuclei will be

$$\begin{aligned} N_3^m &= N_2^m + N_1^m - N_2^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right) && (2.10)\\ &= N_1^m + N_2^m e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}. && (2.11)\end{aligned}$$

At this point, the number of radioactive daughter nuclei will increase by ΔN_3^d and decrease by some amount based on the decay time. Thus at $T = 3\Delta t$ we will have

$$\begin{aligned} N_3^d &= N_2^d + \Delta N_3^d - N_2^d \left(1 - e^{-\frac{\Delta t}{T_{1/2}^d}\ln(2)}\right) && (2.12)\\ &= N_2^d + N_2^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right) - N_2^d \left(1 - e^{-\frac{\Delta t}{T_{1/2}^d}\ln(2)}\right) && (2.13)\\ &= N_2^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right) + N_2^d e^{-\frac{\Delta t}{T_{1/2}^d}\ln(2)}. && (2.14)\end{aligned}$$

[1]For sufficiently small Δt or $R(t)$, we can neglect the decays that happen between Δt time increments.

At this stage, there will also be daughter nuclei that decay and so

$$N_3^{gd} = N_2^d \left(1 - e^{-\frac{\Delta t}{T_{1/2}^d}\ln(2)}\right). \tag{2.15}$$

For sufficiently large $T_{1/2}^{gd}$, we can ignore any further decays and still maintain a good approximation for our system. At this point, any possible increase/decrease in particles and all possible decays are accounted for. Since we can represent the particle numbers at $T = 3\Delta t$ using terms for the previous time steps, we can generalize this to some $T = n\Delta t$ for $n \in \mathbb{N}$ giving

$$N_n^m = N_1^m + N_{n-1}^m e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)} \tag{2.16}$$

$$N_n^d = N_{n-1}^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right) + N_{n-1}^d e^{-\frac{\Delta t}{T_{1/2}^d}\ln(2)} \tag{2.17}$$

$$N_n^{gd} = N_{n-1}^{gd} + N_{n-1}^d \left(1 - e^{-\frac{\Delta t}{T_{1/2}^d}\ln(2)}\right). \tag{2.18}$$

These relations can be proven by trivial induction.

```
#[PYTHON CODE]#
#Calculation of particle number
for n in range(2,size):
    N_m[n]=N_m[1]+N_m[n-1]*np.exp(-del_t*np.log(2)/HL_m)
    N_d[n]=N_m[n-1]*(1-np.exp(-del_t*np.log(2)/HL_m))+N_d[n-1]*np.exp(-del_t*np.
        log(2)/HL_d)
    N_gd[n]=N_gd[n-1]+N_d[n-1]*(1-np.exp(-del_t*np.log(2)/HL_d))
```

Addition. Instead of using m, g and gd for mother, daughter and granddaughter respectively, suppose we allow the superscript to denote the generation as a number, where $(0) \equiv m$, $(1) \equiv d$ and $(2) \equiv gd$ and so on such that (i) would represent the i-th generation. Using this scheme, this process can be generalized for i number of generations as

$$N_n^{(0)} = N_1^{(0)} + N_{n-1}^{(0)} e^{-\frac{\Delta t}{T_{1/2}^{(0)}}\ln(2)} \tag{2.19}$$

$$N_n^{(i)} = N_{n-1}^{(i-1)} \left(1 - e^{-\frac{\Delta t}{T_{1/2}^{(i-1)}}\ln(2)}\right) + N_{n-1}^{(i)} e^{-\frac{\Delta t}{T_{1/2}^{(i)}}\ln(2)}. \tag{2.20}$$

If we assume the lats stage has a near infinite half life, the limiting case will produce a final step to be equivalent to that of Eq. 2.18.

Important Relations

DECAY RATES - METHOD I

The values N_n^m, N_n^d and N_n^{gd} all represent a total number of particles of a specific nuclei. If we are interested in the decays D of the radioactive particles, we can represent them based on the number of particles are present. That is

$$D_n^m = N_n^m \left(1 - e^{-\frac{\Delta t}{T_{1/2}^m}\ln(2)}\right) \tag{2.21}$$

$$D_n^d = N_n^d \left(1 - e^{-\frac{\Delta t}{T_{1/2}^d}\ln(2)}\right). \tag{2.22}$$

These represent the today decays per unit Δt. The contamination is then defined as the number of decays the daughter is producing with respect to the number of decays the mother nuclei is producing.

$$C_n = \frac{D_n^d}{D_n^m} \times 100\% = \text{contamination } \%. \tag{2.23}$$

```
#[PYTHON CODE]#
#Calculation of decay rate (devided by del_t to make the units per second)
for n in range(0,size):
    D_m[n]=N_m[n]*(1-np.exp(-del_t*np.log(2)/HL_m))/del_t
    D_d[n]=N_d[n]*(1-np.exp(-del_t*np.log(2)/HL_d))/del_t
    if D_m[n] > 0:
        C[n]=D_d[n]/D_m[n]*100
    t[n]=n*del_t
```

DECAY RATES - METHOD II

An alternate, and nicer method of calculating the decay rate can be found from Eq. 2.1. By definition this is the decay rate, so using the total particle numbers we calculate in Eq. 2.16 - 2.18, we can determined the decay rate (note that we are taking the absolute value because we are interested in the decays per second) as

$$D_n^m = \frac{dN_n^m}{dt} = \frac{N_n^m \ln(2)}{T_{1/2}^m} \tag{2.24}$$

$$D_n^d = \frac{dN_n^d}{dt} = \frac{N_n^d \ln(2)}{T_{1/2}^d}. \tag{2.25}$$

These represent the today decays per second. Using this method, the contamination will still be calculated in the same way using Eq. 2.23.

```
#[PYTHON CODE]#
#Alternate calculation of decay rate
for n in range(0,size):
    D2_m[n]=N_m[n]*np.log(2)/HL_m
    D2_d[n]=N_d[n]*np.log(2)/HL_d
    if D2_m[n] > 0:
        C2[n]=D2_d[n]/D2_m[n]*100
```

Addition. Begin with the notation from the previous 'Addition'. The decay rate for any generation would simply be

$$D_n^{(i)} = \frac{dN_n^{(i)}}{dt} = \frac{N_n^{(i)} \ln(2)}{T_{1/2}^{(i)}}, \tag{2.26}$$

and the contamination of the original implant would be given by

$$C_n = \frac{\sum_{k=1}^{\infty} D_n^{(k)}}{D_n^{(0)}} \times 100\% = \text{contamination } \%. \tag{2.27}$$

BEAM CUTTOFF & PARTICLE LOSS

We will introduce a new variable T_c which will represent a cutoff of the beam that is implanting the radioactive particles. After this time, the system will reset and the original system will be instantaneously[2] moved to a new location. So, when $T = iT_c$ for $i \in \mathbb{N}$, we will have our particle number values reset to zero, and thus

$$N_i^m = 0 \qquad N_i^d = 0 \qquad N_i^{gd} = 0. \tag{2.28}$$

This will of course occur at the n values such that $n = \frac{iT_c}{\Delta t}$. Let the total particles that have decayed be denoted by $\bar{N}$. The total decayed particles can be determined by

$$\bar{N}_n^m = N_n^d + N_n^{gd}. \tag{2.29}$$

The ratio of total particles implanted to the particles that have decayed is then simply

$$\frac{nN_1^m}{\bar{N}_n^m} = \frac{nN_1^m}{N_n^d + N_n^{gd}}. \tag{2.30}$$

The deposited implant loss B_n^ℓ (representing the total % of the implanted particles that have been decayed) is then the inverse

$$B_n^\ell = \frac{\bar{N}_n^m}{nN_1^m} \times 100\% = \frac{N_n^d + N_n^{gd}}{nN_1^m} \times 100\%. \tag{2.31}$$

The deposited implant health B^h is then the percentage of original implant that still exists and is given simply by

$$B_n^h = 100\% - B_n^\ell. \tag{2.32}$$

```
#[PYTHON CODE]#
#Determines the implant loss and Health
for n in range(1,size):
    B_l[n] = 100*(N_d[n]+N_gd[n])/(n*N_m[1])
    B_h[n]=100-P_l[n]
```

Addition. Considering an arbitrary number of generations, the decayed particles would be given by

$$\bar{N}_n^{(i)} = \sum_{k=i+1}^{\infty} N_n^{(k)}, \tag{2.33}$$

to which it follows that the implant loss be given by

$$B_n^\ell = \frac{\bar{N}_n^{(0)}}{nN_1^{(0)}} \times 100\% = \frac{1}{nN_1^{(0)}} \sum_{k=1}^{\infty} N_n^{(k)} \times 100\%. \tag{2.34}$$

The implant health would still be given by

$$B_n^h = 100\% - B_n^\ell. \tag{2.35}$$

[2]In reality this would not be instantaneous.

GINA Features

The current version of GINA is a graphical user interface (GUI) program that was designed in Qt Creator on a Linux computer. Upon an initial run of GINA, there are default values filled out in every required field so that the user can generate an analysis and get an example of what GINA will provide.

User Interface

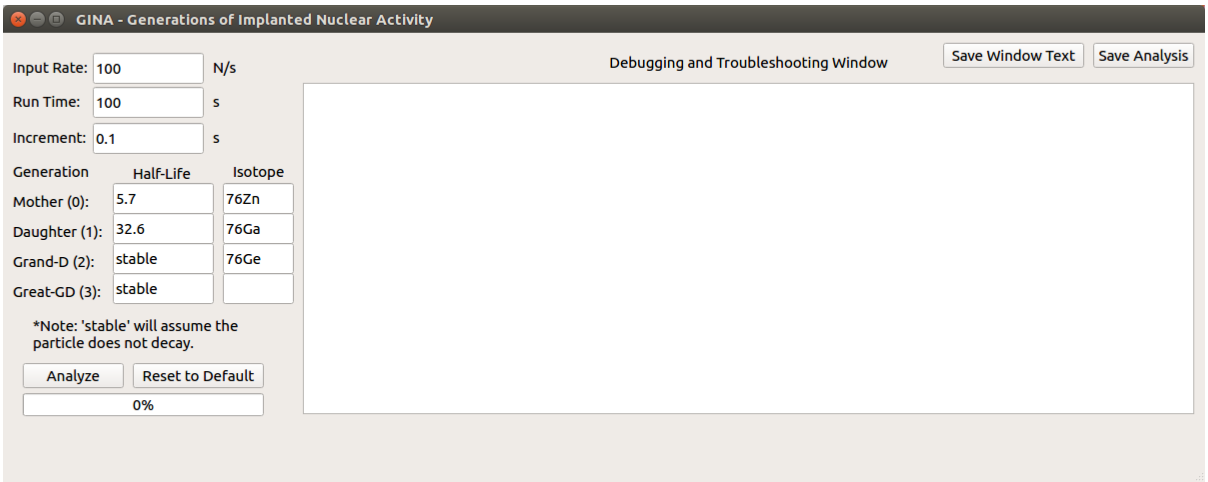

Figure 3.1: An example of what the GINA GUI looks like when first ran.

The GINA GUI contains numerous small text fields for user input. These are all labeled appropriately. These include input rate, run time, increment, half-lives, and isotope labels and can be seen on the left side in figure 3.1.

- Input rate - This is the rate at which radioactive isotopes are implanted onto a portion of the tape. This will not change the final results other than in magnitude.
- Run time - This determines the time frame that the program will run it's calculations for. It also determines the maximum x-axis values for the plots that can be generated.
- Increment - This determines the time increment between each value that is calculated by the program. For more accurate results (especially when dealing with isotopes with very short half lives), a short increment time is needed. The shorter the increment time, the more calculations are needed to be done by the program and thus the longer it will take to run it. Due to the process by which GINA performs all of the calculations, the accuracy of the data is also increased with a smaller increment size, though it does become negligible at sufficiently small increment sizes.

- Half-lives - These are the half lives of an isotope.
- Isotope label - These are the labels of which isotope corresponds to which half-life.

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Analysis and Analysis Text

This section is still under construction...

User Inputs

This section is still under construction...

Graphs and Graph Settings

This section is still under construction...

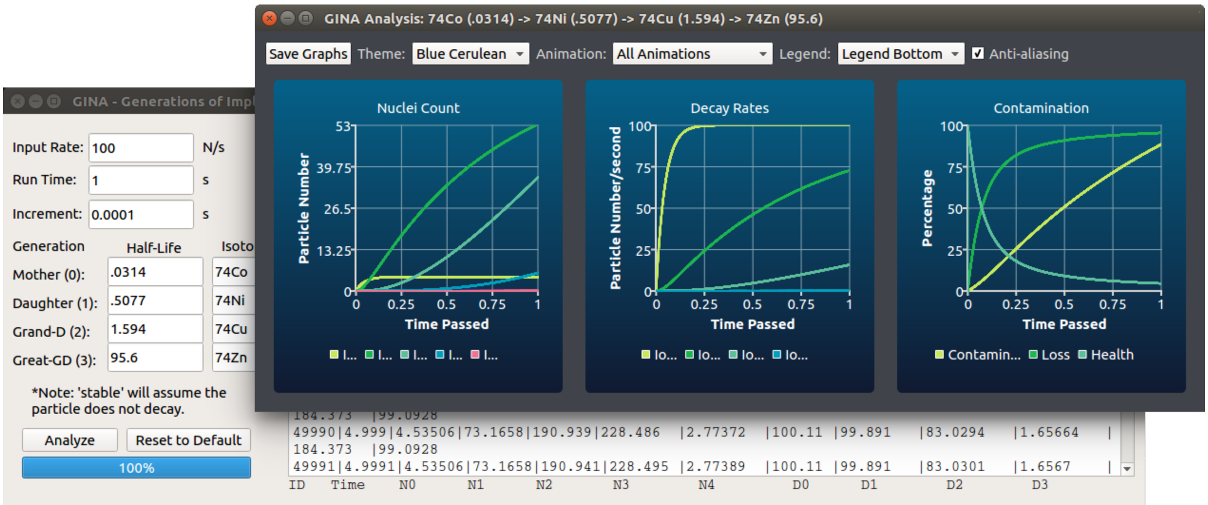

Figure 3.2: An example of what the GINA graphs looks like when analysis is first ran.

Advanced Options

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Appendix I: Useful Coding Features

This section is still under construction...

References

[1] Definition of Beta Decay: http://www2.lbl.gov/abc/wallchart/chapters/03/2.html

[2] Half life values: http://www.nndc.bnl.gov/chart/