1 Constants and units

$$m_e \text{ (mass of an electron)} = 9.10938291(40) \times 10^{-31} kg \tag{1}$$

$$= 510.9989 \text{ keV}/c^2 \tag{2}$$

$$m_p \text{ (mass of a proton)} = 0.938272 \text{ GeV}/c^2 \tag{3}$$

2 Mathematics

Trigonometric Identities

$$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \pm \cos(\theta) sin(\phi) \tag{4}$$
$$\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta) sin(\phi) \tag{5}$$

Series expansions.

$$\textbf{[Def]:} \quad f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(z-a)^2 + \frac{1}{3!}f'''(a)(x-a)^4 + \cdots \tag{6}$$
$$f(x) = e^x \implies f(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots \tag{7}$$
$$f(x) = \ln(1+x) \implies f(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots [|x| < 1] \tag{8}$$
$$f(x) = \sin(x) \implies f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \tag{9}$$
$$f(x) = \cos(x) \implies f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \tag{10}$$
$$f(x) = \sinh(x) \implies f(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \tag{11}$$
$$f(x) = \cosh(x) \implies f(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \tag{12}$$
$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots [|x| < 1] \tag{13}$$

3 Differential Equations

$$\ddot{x} + Ax = B \implies x(t) = \frac{B}{A} + C_1 \cos(\sqrt{A}t) + C_2 \sin(\sqrt{A}t) \tag{14}$$

$$\ddot{x} - Ax = B \implies x(t) = -\frac{B}{A} + C_1 e^{\sqrt{A}t} + C_2 e^{-\sqrt{A}t} \tag{15}$$

$$\text{or } \ldots \implies x(t) = -\frac{B}{A} + C_1 \sinh(\sqrt{A}t) + C_2 \cosh(\sqrt{A}t) \tag{16}$$

4 Statistics

The binomial coefficient

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} \tag{17}$$

A probability distribution

$$\mathcal{P}_n(m;p) = \binom{n}{m} p^m (1-p)^{n-m} \tag{18}$$

The Poisson Distribution

$$\mathcal{P}(m,\lambda) = \frac{\lambda^m}{m!} e^{-\lambda} \tag{19}$$

The mean number of events is

$$\langle m \rangle = \sum_{m=0}^{\infty} m \frac{\lambda^m}{m!} e^{-\lambda} = \lambda \tag{20}$$

And the standard deviation is

$$\sigma = \sqrt{\lambda} \tag{21}$$

The normal, or Gaussian distribution

$$\mathcal{P}(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(x-\mu)^2}{2\sigma^2} \right] \tag{22}$$

Given some function $f(x_1, x_2, \ldots, x_n)$, the error of a calculation can be determined by

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)^2 \sigma_{x_n}^2 \tag{23}$$

5 Classical Mechanics

Dot and cross products

$$\vec{r} \cdot \vec{s} = rs\cos(\theta) = r_x s_x + r_y s_y + r_z s_z \tag{24}$$

$$\vec{r} \times \vec{s} = (r_y s_z - r_z s_y, r_z s_x - r_x s_z, r_x s_y - r_y s_x) = \det \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{vmatrix} \tag{25}$$

Newtons Second Law in Cartesian coordinates

$$\vec{F} = m\vec{a} = m\ddot{\boldsymbol{r}} \iff \begin{cases} F_x = m\ddot{x} \\ F_y = m\ddot{y} \\ F_z = m\ddot{z} \end{cases} \tag{26}$$

Newtons Second Law in 2D polar coordinates

$$\vec{F} = m\vec{a} \iff \begin{cases} F_r = m(\ddot{r} - r\dot{\phi}^2) \\ F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \end{cases} \tag{27}$$

Newtons Second Law in cylindrical polar coordinates

$$\vec{F} = m\vec{a} \iff \begin{cases} F_r = m(\ddot{\rho} - \rho\dot{\phi}^2) \\ F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) \\ F_z = m\ddot{x} \end{cases} \tag{28}$$

The Lorentz Force on a charged particle.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{29}$$

Equation of motion for a rocket

$$m\dot{v} = -\dot{m}v_{ex} + F^{external} \tag{30}$$

The center of mass of several particles

$$\vec{R} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha = \frac{m_1\vec{r}_1 + \cdots m_N\vec{r}_N}{M} \tag{31}$$

$$\vec{R} = \frac{1}{M}\int \vec{r}dm = \frac{1}{M}\int \rho\vec{r}dV \tag{32}$$

6 Special Relativity

Relativistic time dilation and length contraction.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1-\beta^2}} = \gamma \Delta t_0 \tag{33}$$

$$\Delta l = \Delta l_0 \sqrt{1-\beta^2} = \frac{\Delta l_0}{\gamma} \tag{34}$$

$$\beta = \frac{v}{c} \tag{35}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{36}$$

Lorentz Transformations for space and time coordinates.

$$x' = \gamma(x - vt) \tag{37}$$

$$y' = y \tag{38}$$

$$z' = z \tag{39}$$

$$t' = \gamma(t - vx/c^2) \tag{40}$$

The relativistic velocity transformation is.

$$u' = \frac{u - v}{1 - vu/c^2} \tag{41}$$

$$u = \frac{u' + v}{a + vu'/c^2} \tag{42}$$

The rest energy of a particle

$$E_0 = mc^2 \tag{43}$$

the lorentz transformation for momentum and energy is.

$$p'_x = \gamma(p_x - vE/c^2) \tag{44}$$

$$p'_y = p_y \tag{45}$$

$$p'_z = p_z \tag{46}$$

$$E' = \gamma(E - vp_x) \tag{47}$$

Relativistic mass and momentum.

$$E = \gamma mc^2 \tag{48}$$

$$p = \gamma mv \tag{49}$$

Combining the above equations give

$$\frac{E}{p} = \frac{c^2}{v} \implies E = \frac{pc^2}{v} \tag{50}$$

Mass-energy equivalence

$$E^2 = (mc^2)^2 + (pc)^2 \tag{51}$$
$$E = K_E + E_0 \tag{52}$$
$$K_E = \text{ Kinetic Energy}$$

Combining the above equation gives

$$p = \frac{1}{c}\sqrt{K_E^2 + 2K_E E_0} \tag{53}$$

Invariant dot product in c=1 notation

$$A \cdot B = (E, \vec{p}) \cdot (U, \vec{q}) = EU - \vec{p} \cdot \vec{q} \tag{54}$$

Relativistic frequency and wavelength shifts

$$f = f_0\sqrt{\frac{c \pm v}{c \mp v}} \tag{55}$$
$$\lambda = \lambda_0\sqrt{\frac{c \mp v}{c \pm v}} \tag{56}$$

Space-time equivalence (same in all reference frames)

$$S \equiv (c\Delta t)^2 - (\Delta x)^2 \equiv E^2 - (pc)^2 \tag{57}$$

7 Thermodynamics

Useful constants:

$$\text{The specific heat of water is } c = 4186 \text{ J/(kg·K)}. \tag{58}$$
$$1 \text{ cal} = 4.186 \text{ J} \tag{59}$$

Temperature relationships.

$$^{\circ}F = \frac{9}{5}^{\circ}C + 32 \tag{60}$$
$$^{\circ}C = \frac{5}{9}(^{\circ}F - 32) \tag{61}$$
$$^{\circ}K =^{\circ} C + 273.15 \tag{62}$$

The heat required to raise the temperature of a mass m by ΔT is

$$Q = cm\Delta T \tag{63}$$

he temperature of an object determines the radiated power of the object, which is given by the **Stefan-Boltzmann equation**

$$P_{radiated} = \sigma\epsilon A T^4 \tag{64}$$
$$\sigma = 5.67 \times 10^{-8} \text{ W}/K^4 m^2 \tag{65}$$
$$\epsilon = \text{emissivity, and } 0 \leq \epsilon \leq 1 \tag{66}$$

The work done on a system in going from initial volume (V_i) to a final volume (V_f) is

$$W = \int dW = \int_{V_i}^{V_f} p dV. \tag{67}$$

The first law of thermodynamics

$$\Delta E_{internal} = Q - W \tag{68}$$

different processes include

(i) An adiabatic process is one in which $Q = 0$.

(ii) In a constant-volume process, $W = 0$.

(iii) In a closed-loop process, $Q = W$.

(iv) In an adiabatic free expansion, $Q = W = \Delta E_{internal} = 0$.

If heat is added to an object, its change in temperature is given by

$$\Delta T = \frac{Q}{C} \tag{69}$$
$$C = \text{ heat capacity of the object} \tag{70}$$

If heat is added to an object with mass m, its change in temperature is given by

$$\Delta T = \frac{Q}{cm} \tag{71}$$

$$c = \text{ specific heat of the object} \tag{72}$$

The ideal gas law

$$PV = nRT \tag{73}$$

$$R = 1.38106504(24) \times 10^{-23} \text{ J/K} \tag{74}$$

With a constant number of moles we get from the ideal gas law the following relation:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \tag{75}$$

Dalton's law - The total pressure exerted by a mixture of gases is equal to the sum of the partial pressures pf the gases in the mixture.

$$P_{total} = P_1 + P_2 + P_3 + \cdots + P_n \tag{76}$$

The work done by an ideal gas at constant temperature is

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) \tag{77}$$

The average kinetic energy of an ideal gas

$$K_{ave} = \frac{1}{N}\sum_{i=1}^{N} K_i = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{2}mv_i^2 = \frac{1}{2}mv_{rms}^2 \tag{78}$$

The root-mean-square speed of gas molecules is

$$v_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{3RT}{m}} \tag{79}$$

For an adiabatic process, we have

$$dE_{internal} = -PdV = nC_V dT \tag{80}$$

$$C_V = \text{specific heat at constant volume} \tag{81}$$

$$C_P = \text{specific heat at constant pressure} \tag{82}$$

$$PV^\gamma = \text{constant} \tag{83}$$

$$\gamma = \frac{C_P}{C_V} \tag{84}$$

$$P_f V_f^\gamma = P_i V_i^\gamma \tag{85}$$

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \tag{86}$$

8 Quantum Physics

Electromagnetic wave frequency and wavelength

$$c = \nu\lambda \implies \nu = \frac{c}{\lambda} \implies \lambda = \frac{c}{\nu} \tag{87}$$

$$\nu = \text{frequency} \tag{88}$$

The energy in a photon (packet of light)

$$E = h\nu = \frac{hc}{\lambda} \tag{89}$$

The proportionality factor (Planck's constant) is

$$h = 6.6261 \times 10^{-34} J * s \tag{90}$$

$$= 4.1357 \times 10^{-15} eV * s$$

$$hc = 1240 eV * nm \tag{91}$$

$$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-}34 J * s \tag{92}$$

Wien's Displacement Law

$$\lambda_{MAX} T = 2.898 \times 10^{-3} m * K \tag{93}$$

Total Power Stefan-Boltzmann Law

$$R(T) = \int_0^\infty I(\lambda, T) d\lambda = \epsilon\sigma T^4 \tag{94}$$

$$\epsilon = \text{emmisivity (unitless)} \tag{95}$$

$$\sigma = 5.67 \times 10^{-8} \frac{w}{m^2 k^4} \tag{96}$$

Max Planck's Radiation Law:

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{97}$$

The kinetic energy of an emitted photoelectron is

$$KE = hv - \phi \tag{98}$$

$$E_{photon} = KE_{electron} + \phi \tag{99}$$

$$KE_{electrons} = 0 \text{ (at threshold)} \tag{100}$$

Where ϕ = binding energy of electron to metal surface (the work function).
Ruthford Scattering Formula

Any particle hitting an area σ around the nucleus will be scattered through an angle of θ or greater.

$$b = (r_{min}/2)\cot(\theta/2) \tag{101}$$

$$r_{min} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 K} \tag{102}$$

$$\sigma = \pi b^2 = \text{ cross sectional area} \tag{103}$$

$$\frac{e^2}{4\pi\epsilon_0} = 1.44 \times 10^{-9}\text{eV}\cdot\text{m} \tag{104}$$

A common unit of σ is one barn.

$$\text{barn (unit)} = 10^{-28}m^2 = 100fm^2 \tag{105}$$

Number of atoms per area = (atoms/volume)*thickness

$$n = \left(N_A \frac{atoms}{mole}\right)\left(\frac{1}{A}\frac{mole}{gm}\right)\left(\rho\frac{gm}{cm^3}\right) = \frac{\rho N_A}{A} \tag{106}$$

The Compton effect describes the photon wavelength λ' after a photon of wavelength λ scatters off an electron.

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - cos(\theta)) \tag{107}$$

The Compton wavelength of an electron is

$$\lambda_e = \frac{h}{m_e c} = 2.426 \times 10^{-12}m \tag{108}$$

Heisenberg Uncertainty relation

$$\Delta x \cdot \Delta p_x \geq \frac{1}{2}\hbar \tag{109}$$

Uncertainty relation of energy and the measurement of time.

$$\Delta E \cdot \Delta t \geq \frac{1}{2}\hbar \tag{110}$$

The de Broglie wavelength is defined as

$$\lambda = \frac{h}{p} = \frac{h}{mv\gamma} = \frac{h}{mv}\sqrt{1 - \frac{v^2}{c^2}} = \frac{hc}{\sqrt{K_E^2 + 2K_E E_0}} \tag{111}$$

Rutherford Scattering.

$$K = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{R_{min}} \Longleftrightarrow R_{min} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{K} \tag{112}$$

Z's are the atomic masses of the particles within the interaction and R_{min} is the minimum distance they reach (from center to center), and e is

$$e = 1.602177 \times 10^{-19} C \tag{113}$$

$$\epsilon \approx 8.854 \times 10^{-12} F/m \tag{114}$$

The Rutherford Scattering Formula

$$N(\theta) = \frac{N_i nt}{16r^2}(R_{min})^2 \frac{1}{\sin^4(\theta/2)} \tag{115}$$

Centripetal force due to coulomb attraction

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = ma_c = m\frac{v^2}{r} \implies v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mr} \tag{116}$$

$$\implies r = 4\pi\epsilon_0 \frac{n^2\hbar^2}{me^2} \tag{117}$$

Energy levels

$$E = KE + PE = -\frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r} \implies E = \frac{-E_0}{n^2}, \tag{118}$$

$$\text{where } E_0 = \alpha^2 mc^2/2 = 13.6 \text{ eV}. \tag{119}$$

Energy of emitted radiation

$$E = E_n - E_m = E_0\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{120}$$

Note. Using the Planck formula in the above equation leads to the Rydberg formula.

The Rydberg formula: Wavelength of the spectral lines in Hydrogen:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{121}$$

$$n \in \mathbb{N} = 1, 2, 3, 4, 5, \ldots \tag{122}$$

$$R_H = \frac{E_0}{hc} = \frac{13.6eV}{1240eV \cdot nm} \tag{123}$$

$$= 10,967,760m^{-1} \tag{124}$$

$$= 1.096776 \times 10^7 m^{-1} \text{ (Rydberg's constant)} \tag{125}$$

Note. ZnS (Zinc Sulfide) emits a faint flash of light when struck by an α−ray.

L quantized

$$L = mvr = n\hbar \tag{126}$$

Stationary state orbits

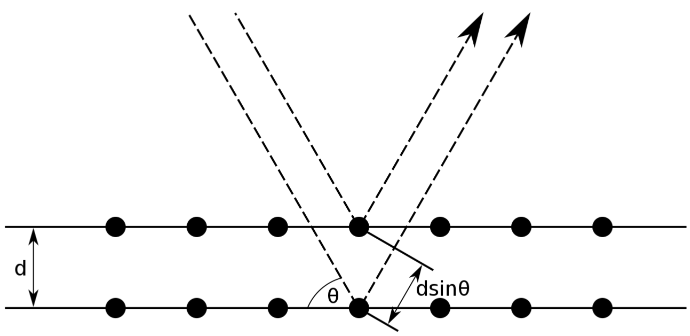
$$r = a_0 n^2 \tag{127}$$
$$a_0 = \text{Bohr Radius} \tag{128}$$

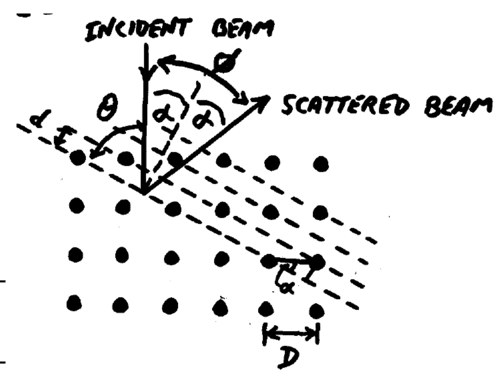
Stationary state energies

$$E_n = -Z^2 \frac{E_0}{n^2} \tag{129}$$
$$\Delta E = \frac{hc}{\lambda} \tag{130}$$

Bragg's Law: When scattering off of crystal structures, the wavelengths will peak at specific angles determined by the diagrams below

$$n\lambda = 2d\sin(\theta) = 2d\cos(\alpha) = 2D\sin(\alpha)\cos(\alpha) = D\sin(2\alpha) = D\sin(\phi) \tag{131}$$
$$d = Dsin(\alpha) \tag{132}$$
$$\phi = 2\alpha \tag{133}$$
$$\theta = 90^\circ - \alpha \tag{134}$$





9 Quantum Mechanics

A plane wave

$$\psi(x,t) = A\cos[2\pi(x-ct)/\lambda] \tag{135}$$
$$f = c/\lambda \tag{136}$$
$$T = 1/f \tag{137}$$
$$\psi(x,t) = A\cos(kx-\omega t) \tag{138}$$
$$k = 2\pi/\lambda \tag{139}$$
$$\omega = 2\pi f = 2\pi T \tag{140}$$

A periodic wave can be constructed from a sum of plane waves

$$\psi(x,t) = \sum_{i=1}^{n} A_i \cos(k_i x_i - \omega_i t) \tag{141}$$

Fourier Transform: A wave packet can be constructed as a continuous sum of plane waves

$$\psi(x,t) = \int A(k)\cos(kx-\omega t)dk \tag{142}$$

The wave equation

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial x^2} \tag{143}$$

The Schrödingr Equation

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi \tag{144}$$