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ANTONIUS' HANDBOOK

Useful Formulas, Constants, Units and Definitions Volume I - Mathematical Mansion Version 2.016

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Preface

This document is a compilation of useful formulas, definitions, constants, and general information used throughout my own schooling as a reference while furthering education. It's purpose is to provide a complete 'encyclopedia' per say of various mathematical and significant ideas used often. The idea and motivation behind it is to be a quick reference providing easily accessible access to necessary information for either double checking or recalling proper formula for use in various situations due to my own shortcomings in matters of memorization. All the material in this document was either directly copied from one of the references listed at the end or derived from scratch. On occasion typos may exist due to human error but will be corrected when discovered.

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Courses Covered In This Book

This document encompasses a large portion of formula used throughout specific courses at Michigan state University. The courses which have information pertaining to something in this book are more than just listed below; however, below is a list of classes that the author took whilst compiling the information in this book. All course numbers correspond to Michigan State University courses at the time of adding them.

- | | |
|--|---|
| • AST 207/208/304: Astrophysics I/II/III | • PHY 415: Methods Of Theoretical Physics |
| • PHY 215: Thermodynamics & Modern Physics | • PHY 440: Electronics |
| • MTH 310: Abstract Algebra/Number Theory | • PHY 471/472: Quantum Physics I/II |
| • PHY 321: Classical Mechanics I | • PHY 481/482: Electricity and Magnetism I/II |
| • PHY 410: Thermal & Statistical Physics | • PHY 492: Introduction to Nuclear Physics |

The information in this book is in no way limited to the material used within the courses above. They serve as a simple guideline to what you will find within this document. For more information about this book or details about how to obtain your own copy please visit:

<https://msu.edu/~torodean/AHandbook.html>

Disclaimer

This book contains formulas, definitions, and theorems that by nature are very precise. Due to this, some of the material in this book was taken directly from other sources such as but not limited to Wolfram Mathworld. This is only such in cases where a change in wording could cause ambiguities or loss of information quality. Following this, all sources used are listed in the references section.

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Constants and units

1.1: Physical Constants

Constant	Symbol	Value	Units
Speed of light in a vacuum	$c \equiv 1/\sqrt{\mu_0\epsilon_0}$	2.99792458×10^8	m/s
Elementary charge	e	$1.602176565(35) \times 10^{-19}$	C
Gravitational constant	G	$6.67384(80) \times 10^{-11}$	$\text{m}^3\text{kg}^{-1}\text{s}^{-2}$
Avagadro's number	N_a	$6.02214129(27) \times 10^{23}$	$\text{mol}\cdot\text{s}^{-1}$
Planck constant	h	$6.62606872(52) \times 10^{-34}$	J·s
		4.135668×10^{-15}	eV·s
	hc	1239.84	eV·nm
Reduced planck constant	$\hbar \equiv h/2\pi$	1.05×10^{-34}	J·s
Permittivity of the vacuum	ϵ_0	8.854×10^{-12}	$\text{C}^2\text{N}^{-1}\text{m}^{-2}$
Permeability of the vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
Permeability of the vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
Boltzmann constant	k_B	$1.38064852 \times 10^{-23}$	J/K
		8.61733×10^{-5}	eV/K
Stefan-Boltzmann constant	$\sigma_B \equiv \frac{\pi^2 k_B^4}{60\hbar^3 c^3}$	$5.670367(13) \times 10^{-8}$	$\text{W}\cdot\text{m}^{-2}\text{K}^{-4}$
Thomson cross-section	σ_e	6.652×10^{-29}	m^2
The Bohr Magneton	$\mu_B \equiv \frac{e\hbar}{2m}$	5.788×10^{-5}	eV/T
		9.274×10^{-24}	Am^2
Mass of an electron	m_e	$9.10938291(40) \times 10^{-31}$	kg
		510.9989	keV/c^2
Mass of a proton	m_p	$1.6726218 \times 10^{-27}$	kg
		938.27203	MeV/c^2
Mass of a neutron	m_n	$1.6749274 \times 10^{-27}$	kg
		939.56536	MeV/c^2
Unified amu	u	$1.660538782 \times 10^{-27}$	kg
		931.494028	MeV/c^2

1.2: Stellar Data

Spectral Type	T_{eff} (K)	M/M_{\odot}	L/L_{\odot}	R/R_{\odot}	V_{mag}
O5	44,500	60	7.9×10^5	12	-5.7
B5	15,400	5.9	830	3.9	-1.2
A5	8,200	2.0	14	1.7	1.9
F5	6,440	1.4	3.2	1.3	3.4
G5	5,770	0.92	0.79	0.92	4.9
K5	4,350	0.67	0.15	0.72	6.7
M5	3,170	0.21	0.011	0.27	12.3

General Mathematics

Definitions

$$\begin{array}{lcl} \sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix}) & (2.0.1) & \left| \begin{array}{lcl} \cos(x) = \frac{1}{2}(e^{ix} + e^{-ix}) & (2.0.4) \\ \sinh(x) = \frac{1}{2}(e^x - e^{-x}) & (2.0.2) & \left| \begin{array}{lcl} \cosh(x) = \frac{1}{2}(e^x + e^{-x}) & (2.0.5) \\ = -i \sin(ix) & (2.0.3) & \left| \begin{array}{lcl} = \cos(ix) & (2.0.6) \end{array} \right. \end{array} \right. \end{array} \right. \end{array}$$

Curl Theorem: A special case of Stokes' theorem in which \vec{F} is a vector field and M is an oriented, compact embedded 2-manifold with boundary in \mathbb{R}^3 , and a generalization of Green's theorem from the plane into three-dimensional space. The curl theorem states

$$\int_S (\nabla \times \vec{F}) \cdot d\vec{a} = \int_{\partial S} \vec{F} \cdot d\vec{s} \quad (2.0.7)$$

Green's theorem is a vector identity which is equivalent to the curl theorem

$$\iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_{\partial S} P(x, y) dx + Q(x, y) dy \quad (2.0.8)$$

The **divergence theorem** is also known as Gauss's theorem (e.g., Arfken 1985) or the Gauss-Ostrogradsky theorem. Let V be a region in space with boundary ∂V . Then the volume integral of the divergence $\nabla \cdot \vec{F}$ of \vec{F} over V and the surface integral of \vec{F} over the boundary ∂V of V are related by

$$\int_V (\nabla \cdot \vec{F}) dV = \int_{\partial V} \vec{F} \cdot d\vec{a} \quad (2.0.9)$$

The **gradient theorem** (where the integral is a line integral) is

$$\int_a^b (\nabla f) \cdot d\vec{s} = f(b) - f(a) \quad (2.0.10)$$

The **Gamma function** Γ and the **Riemann zeta function** ζ are given by

$$\Gamma(z) \equiv \int_0^\infty t^{z-1} e^{-t} dt \quad (2.0.11)$$

$$\zeta(z) = \sum_{k=1}^\infty \frac{1}{k^z} \implies \zeta'(z) = - \sum_{k=1}^\infty \frac{\ln(k)}{k^z} \quad (2.0.12)$$

$$\zeta(z)\Gamma(z) = \int_0^\infty \frac{u^{z-1}}{e^u - 1} du \quad (2.0.13)$$

The most general case of the **binomial theorem** is the binomial series identity

$$(x + y)^n = \sum_{i=1}^n \binom{n}{k} x^{n-k} y^k \quad (2.0.14)$$

Complex Analysis

Complex Numbers

The set of complex numbers is defined such that

$$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}. \quad (3.1.1)$$

A complex number can be defined by its real part and its imaginary part

$$i^2 = -1 \iff i = \sqrt{-1} \iff \frac{1}{i} = -i \quad (3.1.2)$$

$$z = x + iy \iff z^* = x - iy \quad (3.1.3)$$

We can express the real and imaginary parts of a complex number in terms of the number and its complex conjugate

$$\Re(z) = \frac{1}{2}(z + z^*) \quad (3.1.4)$$

$$\Im(z) = \frac{1}{2}i(z - z^*) \quad (3.1.5)$$

Just like a two-dimensional vector, a complex number has the magnitude $|z|$ as well as an angle θ with respect to the horizontal axis of the complex plane.

$$|z|^2 = z^*z = x^2 + y^2 = |z|e^{-i\theta}|z|e^{i\theta} \quad (3.1.6)$$

$$\tan(\theta) = \frac{\Im(z)}{\Re(z)} = \frac{y}{x} = \frac{i(z - z^*)}{(z + z^*)} \quad (3.1.7)$$

A complex number can thus be expressed in terms of magnitude and the phase angle

$$z = |z|(\cos(\theta) + i \sin(\theta)) \quad (3.1.8)$$

Euler's Identity/relation

$$e^{i\theta} = \cos(\theta) + i \sin(\theta) \quad (3.1.9)$$

With the aid of Euler's identity, we can write any complex number as

$$z = |z|e^{i\theta} \quad (3.1.10)$$

$$z^n = |z|^n e^{in\theta} \quad (3.1.11)$$

A useful property of conjugates is

$$a^* + b^* = (a + b)^* \quad (3.1.12)$$

Powers and roots of a complex number can be determined from the exponential form of a complex number

$$z^n = (re^{i\theta})^n = r^n e^{in\theta} \quad (3.1.13)$$

$$(e^{i\theta})^n = e^{in\theta} = (\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta) \quad (3.1.14)$$

$$z^{1/n} = (re^{i\theta})^{1/n} = r^{1/n} e^{i\theta/n} = \sqrt[n]{r} \left(\cos \frac{\theta}{n} + i \sin \frac{\theta}{n} \right) \quad (3.1.15)$$

Much like in trigonometry, we can define complex numbers using trigonometric identities:

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}, \quad \sinh z = \frac{e^z - e^{-z}}{2}, \quad \cosh z = \frac{e^z + e^{-z}}{2} \quad (3.1.16)$$

The logarithm of a complex number can be manipulated as a normal log with

$$\ln(z) = \ln(re^{i\theta}) = \ln(r) + \ln(e^{i\theta}) = \ln(r) + i\theta \quad (3.1.17)$$

A few Trigonometric identities follow as:

$$\arcsin z = -i \ln(iz \pm \sqrt{1 - z^2}) \quad (3.1.18)$$

$$\arccos z = i \ln(z \pm \sqrt{z^2 - 1}) \quad (3.1.19)$$

$$\arctan z = \frac{1}{2i} \ln \left(\frac{1 + iz}{1 - iz} \right) \quad (3.1.20)$$

Abstract Algebra and Number Theory

Definition 5.1: Ring

A **ring** is a triple (R, \oplus, \odot) such that

- (i) R is a set.
- (ii) \oplus is a function (called ring addition) and $R \times R$ is a subset of the domain of \oplus . For $(a, b) \in R \times R$, $a \oplus b$ denotes the image of (a, b) under \oplus .
- (iii) \odot is a function (called ring multiplication) and $R \times R$ is a subset of the domain of \odot . For $(a, b) \in R \times R$, $a \odot b$ (and also ab) denotes the image of (a, b) under \odot .

and such that the following eight statements (axioms) hold:

- (1) [Closure of addition]: $a + b \in R$ for all $a, b \in R$.
- (2) [Associative addition]: $a + (b + c) = (a + b) + c$ for all $a, b, c \in R$.
- (3) [Commutative addition]: $a + b = b + a$ for all $a, b \in R$.
- (4) [Additive identity]: There exists an element in R , denoted by 0_R and called 'zero R ', such that $a = a + 0_R = a$ and $a = 0_R + a$ for all $a \in R$.
- (5) [Additive inverses]: For each $a \in R$ there exists an element in R , denoted by $-a$ and called 'negative a ', such that $a + (-a) = 0_R$.
- (6) [Closure for multiplication]: $ab \in R$ for all $a, b \in R$.
- (7) [Associative multiplication]: $a(bc) = (ab)c$ for all $a, b, c \in R$.
- (8) [Distributive laws]: $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a, b, c \in R$.

Definition 5.2: Commutative Ring

Let R be a ring. Then R is called commutative if

- (9) [Commutative multiplication]: $ab = ba$ for all $a, b \in R$.

Definition 5.3: Ring With Identity

Let R be a ring. We say that R is a ring with identity if there exists an element, denoted by 1_R and called 'one R ', such that

- (10) [Multiplicative identity]: $a = 1_R \cdot a$ and $a = a \cdot 1_R$ for all $a \in R$.

Electricity and Magnetism

Maxwell's Equations: The system of partial differential equations describing classical electromagnetism. \vec{P} is the polarization field, \vec{D} is the electric displacement field, ρ is the charge density, \vec{E} is the electric field, \vec{B} is the magnetic field, and \vec{J} is the current density. In the so-called cgs system of units, the Maxwell equations are given by

$$\begin{array}{l|l} \nabla \cdot \vec{E} = 4\pi\rho & (12.0.1) \\ \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} & (12.0.2) \end{array} \quad \left| \quad \begin{array}{l} \nabla \cdot \vec{B} = 0 & (12.0.3) \\ \nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} & (12.0.4) \end{array} \right.$$

In the MKS system of units (where ϵ_0 is the permittivity of free space and μ_0 is the permeability of free space), the equations are written

$$\begin{array}{l|l} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} & (12.0.5) \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} & (12.0.6) \end{array} \quad \left| \quad \begin{array}{l} \nabla \cdot \vec{B} = 0 & (12.0.7) \\ \nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} & (12.0.8) \end{array} \right.$$

From the field tensor and dual tensors, Maxwell's equations (where J^μ is the current density 4-vector) are given by

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu, \quad \frac{\partial G^{\mu\nu}}{\partial x^\nu} = 0 \quad \text{with} \quad J^\mu = (c\rho, J_x, J_y, J_z). \quad (12.0.9)$$

From Maxwell's equations, electric and magnetic fields can be shown to satisfy the wave equation in a vacuum allowing us to derive a speed for both fields which is equivalent to the speed of light (electromagnetic waves) in a vacuum.

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{and} \quad \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \implies c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}. \quad (12.0.10)$$

In the special case of a steady state, known as **electrostatics**, with stationary charges and currents,

$$\nabla \times \vec{E} = 0 \implies \oint \vec{E} \cdot d\vec{\ell} = 0 \quad (12.0.11)$$

The dipole moment is defined by

$$\vec{p} \equiv \sum_i q_i \vec{r}_i \quad (12.0.12)$$

$$\vec{p} \equiv \int_V \rho(\vec{r}') \vec{r}' d\tau' \quad (12.0.13)$$

If we consider both bound and free charges (where the free charges are the charges we place within a system), we have

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_{\text{bound}} + \rho_{\text{free}}}{\epsilon_0} \quad (12.0.14)$$

$$= \frac{-\nabla \cdot \vec{P}}{\epsilon_0} + \frac{\rho_{\text{free}}}{\epsilon_0} \quad (12.0.15)$$

$$\implies \nabla \cdot \vec{D} = \rho_{\text{free}} \quad (12.0.16)$$

$$\implies \oint_S \vec{D} \cdot d\vec{a} = Q_{\text{free}} = \int \rho_{\text{free}} d\tau'. \quad (12.0.17)$$

The **polarization field** of a linearly polarized dielectric is characterized by its dipole moment per unit volume and can be defined by the susceptibility constant χ_e and the dielectric constant ϵ_R ,

$$\vec{P} = \lim \frac{\Delta \vec{p}}{\Delta v} = \frac{1}{\Delta v} \sum_i \vec{p}_i \equiv \epsilon_0 \chi_e \vec{E} = \frac{\chi_e}{1 + \chi_e} \vec{D} = \frac{\chi_e}{\epsilon_R} \vec{D} \longrightarrow \begin{cases} \chi_e \rightarrow 0 & \implies \vec{P} \text{ for a vacuum} \\ \chi_e \rightarrow \infty & \implies \vec{P} \text{ for a metal} \end{cases} \quad (12.0.18)$$

Special Relativity

Relativistic time dilation and length contraction (where Δt_0 and Δl_0 are the proper time and length)

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \beta^2}} = \gamma \Delta t_0 \quad (14.0.1)$$

$$\Delta l = \Delta l_0 \sqrt{1 - \beta^2} = \frac{\Delta l_0}{\gamma} \quad (14.0.2)$$

$$\beta = \frac{v}{c} \quad (14.0.3)$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad (14.0.4)$$

Lorentz Transformations for space and time coordinates for a frame moving at a constant velocity v in the \hat{x} direction.

$$\bar{x} = \gamma(x - vt) \quad (14.0.5)$$

$$\bar{y} = y \quad \text{and} \quad \bar{z} = z \quad (14.0.6)$$

$$\bar{t} = \gamma(t - vx/c^2) \quad (14.0.7)$$

The relativistic velocity transformation is.

$$\bar{u}_x = \frac{u_x - v}{1 - vu_x/c^2} \iff u_x = \frac{\bar{u}_x + v}{1 + v\bar{u}_x/c^2} \quad (14.0.8)$$

The rest energy of a particle

$$E_0 = mc^2 \quad (14.0.9)$$

The lorentz transformation for momentum and energy is.

$$\bar{p}_x = \gamma(p_x - vE/c^2) \quad (14.0.10)$$

$$\bar{p}_y = p_y \quad \text{and} \quad \bar{p}_z = p_z \quad (14.0.11)$$

$$\bar{E} = \gamma(E - vp_x) \quad (14.0.12)$$

Relativistic mass and momentum (where m is the rest mass of an object measured in its rest frame).

$$E = \gamma mc^2 = cp^0 \quad (14.0.13)$$

$$p = \gamma mv \quad (14.0.14)$$

Combining the above equations give

$$\frac{E}{p} = \frac{c^2}{v} \implies E = \frac{pc^2}{v} \quad (14.0.15)$$

Mass-energy equivalence and kinetic energy (K_E).

$$E^2 = (mc^2)^2 + (pc)^2 \quad (14.0.16)$$

$$E = K_E + E_0 \quad (14.0.17)$$

$$K_E = (\gamma - 1)mc^2 \quad (14.0.18)$$

Combining the above equations gives

$$p = \frac{1}{c} \sqrt{K_E^2 + 2K_E E_0} \quad (14.0.19)$$

Invariant dot product in c=1 notation

$$A \cdot B = (E, \vec{p}) \cdot (U, \vec{q}) = EU - \vec{p} \cdot \vec{q} \quad (14.0.20)$$

Relativistic frequency and wavelength shifts

$$f = f_0 \sqrt{\frac{c \pm v}{c \mp v}} \iff \pm v = \frac{f^2 - f_0^2}{f^2 + f_0^2} \quad (14.0.21)$$

$$\lambda = \lambda_0 \sqrt{\frac{c \mp v}{c \pm v}} \iff \mp v = \frac{\lambda^2 - \lambda_0^2}{\lambda^2 + \lambda_0^2} \quad (14.0.22)$$

Space-time equivalence (same in all reference frames)

$$S \equiv (c\Delta t)^2 - (\Delta x)^2 \equiv E^2 - (pc)^2 \quad (14.0.23)$$

Assuming a frame moving with a constant velocity to another, we can relate the accelerations observed between two frames by

$$\bar{a}_x = a_x \left(1 - \frac{u_x}{c}\beta\right)^{-3} (1 - \beta^2)^{3/2} \quad (14.0.24)$$

$$\implies \bar{a}_x \approx a_x \left(1 + 3\beta \frac{u_x}{c} - \frac{3}{2}\beta^2\right) \quad (14.0.25)$$

Einstein Notation

The Lorentz components can be defined by $X^0 \equiv ct$, $X^1 \equiv x$, $X^2 \equiv y$, and $X^3 \equiv z$, from which the Lorentz transformations follow as

$$\bar{X}^0 = \gamma(X^0 - \beta X^1) \quad (14.0.26)$$

$$\bar{X}^1 = \gamma(X^1 - \beta X^0) \quad (14.0.27)$$

$$\bar{X}^2 = X^2 \quad (14.0.28)$$

$$\bar{X}^3 = X^3 \quad (14.0.29)$$

Thermal & Statistical Physics

States of a Model System

The multiplicity function for a system of N magnets with a spin excess $2s = N_{\uparrow} - N_{\downarrow}$ is

$$g(N, s) = \frac{N!}{(\frac{N}{2} + s)!(\frac{N}{2} - s)!} = \frac{N!}{N_{\uparrow}!N_{\downarrow}!}. \quad (17.1.1)$$

It is often useful to evaluate $g(N, s)$ within a logarithm in which the **Stirling approximation** becomes useful.

$$N! \approx N^N \sqrt{2\pi N} \exp\left(-N + \frac{1}{12N} + \dots\right). \quad (17.1.2)$$

It is often useful to take the logarithm of this which gives

$$\log N! \cong \frac{\log 2\pi}{2} + \left(N + \frac{1}{2}\right) \log N - N. \quad (17.1.3)$$

In the limit $s/N \ll 1$, with $N \gg 1$, we have the Gaussian approximation

$$g(N, s) \cong g(N, 0) \exp\left(\frac{-2s^2}{N}\right) \quad (17.1.4)$$

$$g(N, 0) \simeq 2^N \sqrt{\frac{2}{\pi N}}. \quad (17.1.5)$$

The exact value of $g(N, 0)$ is given by

$$g(N, 0) = \frac{N!}{(N/2)!(N/2)!}. \quad (17.1.6)$$

The average value, or mean value, of a function $f(s)$ taken over a probability distribution $P(s)$ is defined as

$$\langle f \rangle = \sum_s f(s)P(s), \quad (17.1.7)$$

$$1 = \sum_s P(s). \quad (17.1.8)$$

The binomial distribution has the property

$$\sum_s g(N, s) = 2^N. \quad (17.1.9)$$

If all states of the model spin system are equally likely, the average value of s^2 is

$$\langle s^2 \rangle = \frac{\int_{-\infty}^{\infty} s^2 g(N, s) ds}{\int_{-\infty}^{\infty} g(N, s) ds} = \frac{N}{4} \quad (17.1.10)$$

The energy interaction of a single magnetic moment \vec{m} with a fixed external magnetic field \vec{B} is

$$U = -\vec{m} \cdot \vec{B}. \quad (17.1.11)$$

For a model system of N elementary magnets, each with two allowed orientations in a uniform magnetic field \vec{B} , the total potential energy U is

$$U = \sum_{i=0}^N U_i = -\vec{B} \cdot \sum_{i=0}^N m_i \quad (17.1.12)$$

$$= -2smB = -MB. \quad (17.1.13)$$

Entropy And Temperature

If $P(s)$ is the probability that a system is in the state X , the average value of a quantity X is

$$\langle X \rangle = \sum_s X(s)P(s). \quad (17.2.1)$$

The number of combined systems 1 and 2 (with $s = s_1 + s_2$) is

$$g(s) = \sum_s g_1(s_1)g_2(s - s_1). \quad (17.2.2)$$

The relation $s = k_B \sigma$ connects the conventional entropy S with the fundamental entropy σ . The **entropy** $\sigma(N, U)$ is given by

$$\sigma(N, U) = \log g(N, U). \quad (17.2.3)$$

The fundamental temperature τ is defined by the relation

$$\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U} \right)_{N, V}. \quad (17.2.4)$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1A	2A	3A	4A	5A	6A	7A	8	9	10	11	12	13	14	15	16	17	18
1 H Hydrogen 1.008	2 He Helium 4.003	3 Li Lithium 6.941	4 Be Beryllium 9.012	5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180	11 Na Sodium 22.990	12 Mg Magnesium 24.305	13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.867	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.631	33 As Arsenic 74.922	34 Se Selenium 78.971	35 Br Bromine 79.904	36 Kr Krypton 84.958
37 Rb Rubidium 84.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.95	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.711	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.904	54 Xe Xenon 131.294
55 Cs Cesium 132.905	56 Ba Barium 137.328	57-71 Lanthanide Series	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.217	78 Pt Platinum 195.085	79 Au Gold 196.967	80 Hg Mercury 200.592	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium [208.982]	85 At Astatine 209.987	86 Rn Radon 222.018
87 Fr Francium 223.020	88 Ra Radium 226.025	89-103 Actinide Series	104 Rf Rutherfordium [261]	105 Db Dubnium [262]	106 Sg Seaborgium [266]	107 Bh Bohrium [264]	108 Hs Hassium [269]	109 Mt Meitnerium [268]	110 Ds Darmstadtium [269]	111 Rg Roentgenium [272]	112 Cn Copernicium [277]	113 Uut Ununtrium unknown	114 Fl Flerovium [289]	115 Uup Ununpentium unknown	116 Lv Livermorium [298]	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown

Periodic Table of the Elements

Atomic Number	Symbol	Name	Atomic Mass
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57 La Lanthanum 138.905	58 Ce Cerium 140.116	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.243	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.500	67 Ho Holmium 164.930	68 Er Erbium 167.259	69 Tm Thulium 168.934	70 Yb Ytterbium 173.055	71 Lu Lutetium 174.967
89 Ac Actinium 227.028	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.080	99 Es Einsteinium [254]	100 Fm Fermium 257.095	101 Md Mendelevium 258.1	102 No Nobelium 259.101	103 Lr Lawrencium [262]

Alkali Metal	Alkaline Earth	Transition Metal	Basic Metal	Semimetal	Nonmetal	Halogen	Noble Gas	Lanthanide	Actinide
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