Entry College Level Problem Solving: 001

Question 1. Using the mass-energy equivalence and energy conservation formulas, derive a formula for a particles momentum (p) in terms of kinetic energy (K) and rest energy ($E_0 = mc^2$).

$$E = K + mc^2 \ldots \text{conservation of energy} \tag{1}$$
$$E^2 = (pc)^2 + (mc^2)^2 \ldots \text{mass-energy equivalence} \tag{2}$$

Question 2. The force of gravitational attraction between two objects from Newtonian mechanics is given by

$$F = \frac{Gm_1m_2}{r^2}. \tag{3}$$

Find the force between the center of the moon and earth given that the distance between the earth to the moon is 3.84×10^8 m, the mass of the moon is 7.35×10^{22} kg, the mass of the earth is 5.97×10^{24} kg, and the gravitational constant G is 6.67×10^{-11} $\text{m}^3\text{kg}^{-1}\text{s}^{-2}$. Do not forget to include units in your answer and do not use a calculator.

Question 3. The relativistic energy and momentum of a particle are given by the following equations respectively:

$$E = \gamma mc^2 \tag{4}$$
$$p = \gamma mv, \tag{5}$$

where m is the mass of the particle, c is the speed of light, v is the velocity of the particle and

$$\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} \tag{6}$$

Solve for the velocity of a particle in terms of E, p, and c. (*Hint: This is easy, though requires a very simple algebra trick.)

Question 4. The frequency (f) of a wave that is moving towards an observer at speed v is given by

$$f = f_0 \sqrt{\frac{c+v}{c-v}}, \tag{7}$$

where f_0 is the frequency the wave would be observed at if it were at rest with respect to the observer. Solve for the speed that the wave is traveling at.

Question 5. Using the frequency equation from the previous question, is the observed frequency f greater than, less than, or equal to f_0 while a wave is moving towards an observer?

Question 6. Similar to the frequency equation from question 4, a relationship exists between the observed wavelength (λ) and the wavelength that would be observed if a wave would be at rest with respect to an observer (λ_0). This relationship is

$$\lambda = \lambda_0 \sqrt{\frac{c-v}{c+v}}. \tag{8}$$

Is the observed wavelength λ greater than, less than, or equal to λ_0 while a wave is moving towards an observer?

Question 7. Using your answers to the previous questions, what relationship can be deduced between frequency and wavelength (write an equation or relation between f and λ).

Question 8. You are given the following relations:

$$1 = A^2 + B^2 \tag{9}$$
$$A = CB \tag{10}$$
$$1 = DB \tag{11}$$
$$1 = EA \tag{12}$$
$$B = AF. \tag{13}$$

Prove (using the above) the following two relations:

$$1 = D^2 - C^2 \tag{14}$$
$$1 = E^2 - F^2 \tag{15}$$

The dot and cross products of two vectors $\vec{r} = (r_x, r_y, r_z)$ and $\vec{s} = (s_x, s_y, s_z)$ are defined as

$$\vec{r} \cdot \vec{s} = r_x s_x + r_y s_y + r_z s_z \tag{16}$$
$$\vec{r} \times \vec{s} = (r_y s_z - r_z s_y, r_z s_x - r_x s_z, r_x s_y - r_y s_x) \tag{17}$$

Question 9. Given the two vectors, $\vec{A} = (3, 7, 9)$ and $\vec{B} = (5, 6, 2)$, find $\vec{A} \cdot \vec{B}$.

Question 10. Given the two vectors from question 9, find $\vec{A} \times \vec{B}$.