A Brief Review of Electromagnetic Scattering Properties and Interactions With Carbon Nanotubes

Antonius Torode

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Abstract

Electromagnetic radiation is a key concept in understanding modern technology and properties of interactions between various matter. The current models and developments of scattering effects against Carbon nanotubes are examined and discussed. A variety of the current modeling techniques are mentioned including method of moment analysis and recursive methods along with examples of the data and conclusions collected from multiple research teams. This leads to some possible implications and applications which are briefly mentioned and discussed such as electromagnetic shielding and possible manipulation of scattering wave patterns.

I. Introduction

Electromagnetic (EM) waves play a pivotal role in scientific and technological advancements. Due to the vast range of frequencies they can radiate at, they interact with various forms of matter in differing ways. It is of key importance that we understand how EM waves interact with matter in order to know how it will both effect and advance technological development. One important property of EM waves is that it can scatter when interacting with matter. One example of this is Raman spectroscopy in which an EM ray will interact with the phonon's within a material and either absorb or emit energy. The EM ray will then scatter off of the material after interacting with it allowing us to gather the scattered radiation and learn about various properties of the material based on the interaction.

EM radiation can be defined as "energy resulting from the acceleration of electric charge and the associated electric fields and magnetic fields [8]." This energy can be regarded as propagating waves that involve oscillating magnetic and electric waves that are orthogonal to eachother and the direction of propagation. These waves require no medium to travel through like that of sound waves. Using Maxwell's equations, an electric or magnetic field in a vacuum can be shown to obey the wave equations

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{1}$$

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}. \tag{2}$$

From these, the speed of both of these waves is similar and turns out to be the speed of EM radiation which is given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 299,792,458 \frac{m}{s}. \tag{3}$$

From this wave equation, we can deduce that EM radiation is in fact a wave and thus it must satisfy behavioral properties of waves.

Scattering is a process of waves in which radiation is forced to deviate from it's original trajectory due to coming in contact with matter or forces. This includes reflection, refraction and diffraction. Other notable forms of EM scattering include Mie, Rayleigh, Brillouin, Compton and others [1]. Rayleigh scattering, which will be mentioned later, is the scattering of EM radiation by particles that have a smaller size than that of the EM radiation wavelength. This is the phenomenon generally responsible for rainbows and various colors that appear in the skies [9].

Similarly, scattering can refer to the interaction with a single particle or through a medium.

To predict the scattering effects that EM radiation will exhibit when interacting with matter, different geometries must be considered. In one case, a cylindrical arrangement can be studied in which EM radiation is scattered from various angles. Different types of scattering must also be considered and explored to fully understand how the interaction will occur. H. L. Chen and L. Gao developed a ”full-wave electromagnetic theory” in order to study the radially emitted scattering effects off nanocylinders [5]. They showed that in certain situations, non-Rayleigh vanishing asymptotic behavior and non-Rayleigh diverging behavior are observed when dealing with nanocylinders instead of the typical Rayleigh scattering. They also found that in some scenarios, a surface electric plasmon resonance arises which causes electric and magnetic fields to be enhanced which effects the EM scattering efficiency.

II. Carbon Nanotubes

Carbon nanotubes (CNTs) are an allotrope of Carbon which are arranged in a cylindrical form. They have unique properties that make then potentially useful in electronics and a wide variety of other applications. Due to this, it is important that we understand the EM scattering properties that are exhibited when EM waves and CNTs interact. There are many various ways of studying CNTs, however due to the small size of the structures it can prove difficult. Because of this, the EM scattering properties of CNTs have not yet been extensively studied [3, 4]. Of the studies that have been done, computer simulations are primarily used to model the CNTs of varying shapes and how they interact with EM waves. A few of the popular approaches are to use large arrays of CNT composites while utilizing a Method of Moment (MoM) analysis [3, 4]. This is a statistical method that is based on large numbers [11] which makes it useful for studying CNTs in this manner. Another technique used in some cases is to approximate the CNTs as strings. It appears that with a large number of CNTs, this approach makes for a reasonably accurate approximation [4, 10].

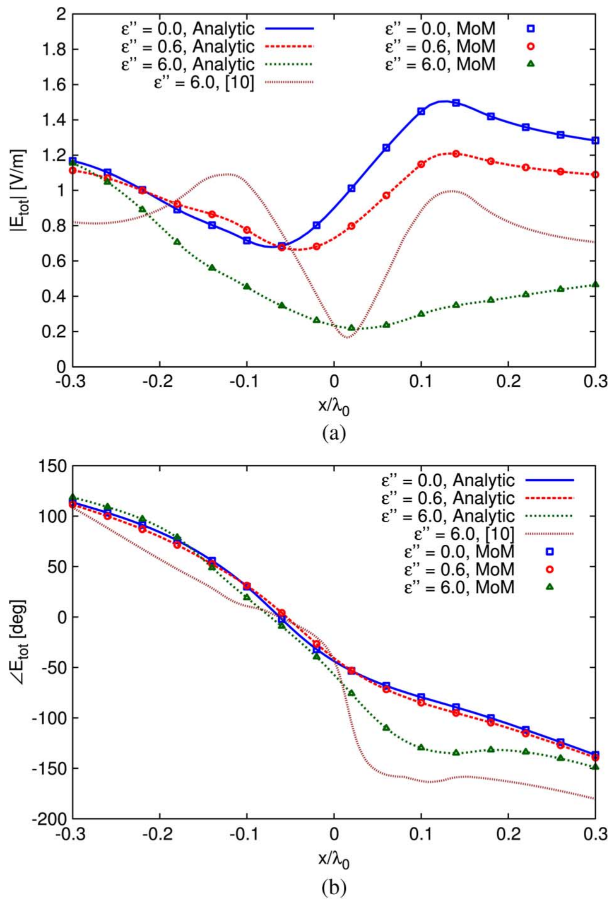

Figure 1: This demonstrates a comparison of the scattered electric field from an analytic method compared to MoM analysis. Figure taken from [6]

Analysis of EM scattering using MoM is a widely used method. However, as shown by Matteo Pastorino et al., there are similar recursive procedures that are also used when analyzing scattering properties [6]. Matteo uses Mathieu functions to represent the total electric field of different multi layered cylinders. They then determine the scattered electric field to be given by

$$E_s(u,v) = \sum_{m=0}^{\infty} e_m^{N+1} Mc_m^{(4)}(q_{N+1},u)ce_m(q_{N+1},v) + \sum_{m=0}^{\infty} o_m^{N+1} Ms_m^{(4)}(q_{N+1},u)se_m(q_{N+1},v).$$

with $Mc_m^{(4)}$ and $Ms_m^{(4)}$ denoting the even and odd

m^{th} order radial Mathieu functions of the fourth kind.

As defined by Wolfram Mathworld, "the Mathieu functions are the solutions to the Mathieu differential equation

$$\frac{d^2V}{dv^2} + [a - 2q\cos(2v)]V = 0." \tag{4}$$

These functions appear often in physical problems containing elliptical shapes. Unfortunately, some regard them as difficult to employ due to them being nearly impossible to represent in a simple analytic way [12]. Even though these functions are not simple to deal with, it gives a proper representation of the electric field scattered off of these cylinders. Due to this, we can get information that we would perhaps be unable to get otherwise.

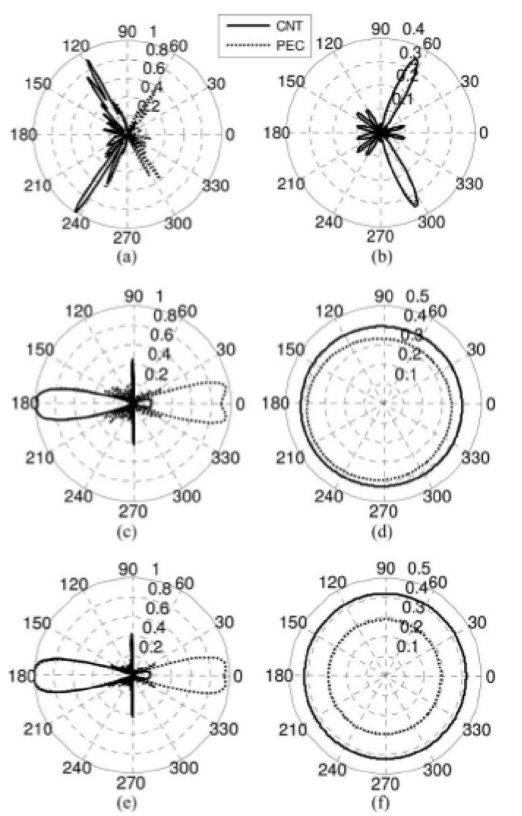

Figure 2: This demonstrates a comparison of a scattered field direction and pattern of PEC versus that of CNTs. Figure taken from [10]

Using this equation for the scattering, they then use a first-order Taylor formula to approximate complex arguments of the Mathieu functions [6]. This gives various results which can be compared to results similarly obtained using MoM analysis as shown in Figure 1. Thus, as we can see, the analytic results agree with the results using MoM. This shows there are multiple methods that can be used when comparing scattering effects from nanocylinders which could be adapted in the study of CNTs. It is important to note that this method as used by Mathieu was not explicitly used to study CNTs but instead the scattering of generic multi-layered cylindrical shapes which are comparable to CNTs due to their geometrical similarities.

Realistic CNTs can be generated by taking a large number of composites which together average to a distribution that would be found commercially [3]. From this, the scattering effects can be studied by comparing the EM reflections to that of a perfect electric conductor (PEC) [10]. The main differences between the scattering effects of a PEC compared to that of CNTs comes from the boundary conditions [10] which are governed by the equation

$$E_{qz}^{(i)}(z) + \sum_{q=1}^{Q} E_{qz}^{(s)}(z) = \frac{I^{(q)}(z)}{2\pi a_q \sigma_q}$$
$$\cong \frac{1}{2\pi a_q \sigma_q} \sum_{n=1}^{N} I_n^{(q)}(z). \tag{5}$$

The boundary conditions of materials and their composites are of vital importance when studying scattering effects. Even though EM radiation does not require a medium to transmit, it will interact with one it comes in contact with. In some cases such as water, air, or even the cosmic background radiation the interaction is small and the effect is minimal such as reducing the speed slightly or causing minor refraction. When interacting with larger groups of particles or solids such as CNTs the matter does not necessarily propagate directly through the matter as if it were a medium. Instead, depending on the boundary conditions of the material in question, the EM radiation can interact in a variety of ways. For example, shining visible light (which is a form of EM radiation) at a solid concrete wall will not yield visible light on the opposing side of the wall. However, if the wall had properties such as that of clear glass, the light would transmit directly through the wall.

For this reason, the properties and boundary conditions of the material it is interacting with has an utmost importance to understanding the scattering effects. Therefore, given equations governing the boundary conditions of a substance, we can determine how the EM radiation will interact with it upon contact and thus deduce the scattering effects. Figure 2 shows an example of this, where the EM radiation interacts with CNTs to scatter in various ways. These patterns are governed by the boundary conditions since the boundary is where the EM waves interact with the CNTs.

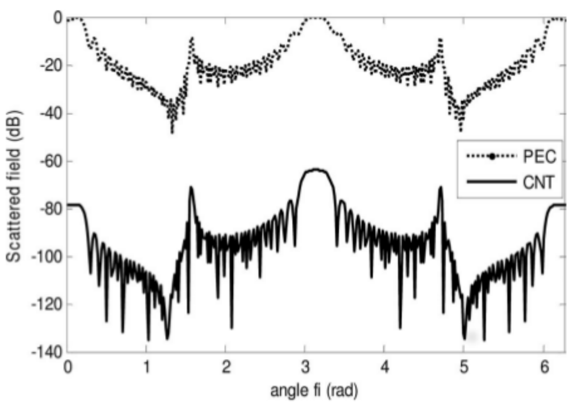

Figure 3: This demonstrates a comparison of a scattered field intensity of PEC versus that of CNTs. Figure taken from [10]

The scattering effects of EM wave can be measured and compared to PEC scattering cases. It has been found that many PEC arrays are reduced in their CNT cases (Figure 3). Similarly, you can compare the pattern of the scattering EM field to that of a PEC (Figure 2). The results in each case show a change in the behavior of EM scattering from interactions with CNTs than that of the PEC. This is of course important in studying this phenomenon because it shows that there will be major differences when using this material and various composites of it.

With the scattering effects observed, it is also prevalent that the geometry of CNT arrays have a major role in the produced scattering field. This gives the implication that this phenomenon can be used alongside CNTs to produce varying scattering fields. Similarly, a generally much lower scattered EM field in CNTs than the PEC counterpart are observed as well as a much smaller (in amplitude) induced current on the CNTs comparatively. Together, these allow for possibilities for CNTs to be used in various electronic applications as well as possible EM shielding or deflection devices. Due to the versatility of the CNT composites, EM scattering does not only play a major role in understanding the material, but it may prove to be a valuable thing in manipulating EM scattering patterns to desired outputs.

III. Conclusion

Although the scattering effects of EM radiation has not yet been extensively studied against CNTs, the data gathered so far shows multiple working methods for studying the phenomenon along with some properties that may serve well in application. Due to the versatility and increasing advancements of CNTs in electronics, the scattering properties may allow for unique usage. It appears MoM analysis is a popular method for studying the phenomenon which gives results that agree with other recursive methods in various situations. The scattering properties show CNTs exhibit different behaviors than other PEC materials which may have possible applications in controlled EM scattering or shielding.

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