Superfluidity for Liquid Helium and Experimental Sound Speed Results below the λ-Point

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Abstract

In this work, we cool liquid helium to a superfluid phase to measure its properties. We produced the correct conditions such that second sound waves are generated through liquid Helium-4. We record peak amplitudes and use them to calculate speeds of sound through the liquid helium in its superfluid phase. We used a method of determining the speeds through a medium, which prove to give consistent results at room temperatures. As cooled, systematic errors arose through a 120 Hz noise in low frequency ranges. Unknowingly, we collected a large amount of data on this noise which lead to odd results until eventually using those results to confirm which resonant peaks in a data set are actually due to second sound. We are able to reproduce similar results for second sound speeds at varying temperatures when comparing to published literature results.

I. Introduction & Background

The purpose of this work is to achieve superfluidity of Liquid Helium-4 and to re-create the sound speed versus temperature relationship from R.J Donnelly [1]. Superfluidity can be defined as "the property of flowing without friction or viscosity [2]." This property is historically shown to be exhibited by liquid Helium-4 below approximately 2.18 K under conditions of atmospheric pressure. This temperature is known as the λ-point due to the specific heat versus temperature behavior around this point correlating to a λ shape.

Superfluidity was originally discovered in liquid Helium-4 by Pyotr Kapitsa and John F. Allen [3]. Helium-4 is just one elemental isotope where superfluidity occurs at low temperatures. This phenomenon can occur in other isotopes such as Helium-3 which was the basis for a physics Nobel prize award in 1996 [4]. For the purposes of this paper, we will focus on liquid and gaseous Helium-4. Helium-4 has two different and easily distinguishable liquid phases, a warmer phase (Helium I) and a colder phase (Helium II). There is a wave-like excitation in Helium II which is called "second sound." It has been shown that there are clear visible phase transitions of helium at low temperatures from Helium I to Helium II (the superfluid phase). Helium I approximates the behavior of other known liquids with a boiling point of 4.2 K.

When liquid Helium-4 is cooled below the λ-point, the liquid will abruptly stop boiling. This implies Helium II has a large heat conductivity and the heat conductivity increases by a factor of 1 million during this cooling transition [5]. The liquid will visually go from rapidly boiling to an apparently calm fluid. As a superfluid, liquids can exhibit unusual and unique properties. Due to having zero viscosity, liquid flow can occur without the loss of any kinetic energy. If the helium is kept in an open vessel, a thin film of fluid will climb up the sides of the vessel and overflow which is known as a "Rollin film" [7]. This behavior cannot be understood through current classical models, but only as a quantum mechanical phenomenon.

Due to the strange behaviors of helium as a superfluid, some experiments have shown what appeared to be contradicting results where the liquid behaves as both having zero viscosity and non-zero viscosity. A solution to this contradiction is to think of liquid Helium-4 as containing two different components, a normal component which has viscosity and a superfluid component which does not. The superfluid component also has zero entropy and contains no friction. There is a high light sensitivity that must also be considered when working with liquid helium.

Light can supply mechanical energy to trigger motions such as "the fountain effect", where a fountain of liquid helium can be created using a heat source [5].

II. Theory and Calculations

For our apparatus, we will have a cylindrical cavity (which will be described later) in which we produce waves. The sound field generated in a cylindrical container is given by the wave equation for the pressure (p) as

$$\nabla^2 p = \frac{1}{v^2}\frac{\partial^2 p}{\partial t^2}, \tag{1}$$

where t is time, and v is the speed of sound through the containers medium. This can be solved by imposing the ansatz

$$p(r,\theta,z,t) = R(r)\Theta(\theta)Z(z)e^{i\omega t}, \tag{2}$$

where r, θ, z represent standard cylindrical coordinates. Solutions to $R(r)$,$\Theta(\theta)$ and $Z(z)$ are then of the form

$$\Theta(\theta) = e^{im\theta} \tag{3}$$
$$R(r) = c_1 J_m(kr) + c_2 Y_m(kr) \tag{4}$$
$$Z(z) = c_3 \sin(kz) + c_4 \cos(kz), \tag{5}$$

where J_m and Y_m are 1st and 2nd kind Bessel functions, respectively. If we use a source field that is uniformly symmetric about the z-axis, then there will be no θ dependence and thus $m = 0$. If we assume that the container has rigid boundaries at $Z = 0, L$ and $R = a$, then we can impose appropriate boundary conditions. With this assumption, the radial and axial particle velocities are zero which make the boundary conditions

$$\frac{\partial Z}{\partial z} = \frac{\partial R}{\partial r} = 0. \tag{6}$$

Applying these boundary conditions give us the radial and axial modes

$$k_{zn} = \frac{\pi}{L}n \tag{7}$$

$$k_{rjm} = \frac{j_{mj}}{a}, \tag{8}$$

where $n \in \mathbb{N}$. From $\omega = vk$ and $k^2 = k_{zn}^2 + k_{rjm}^2$, we can solve for the resonant frequencies which are given by

$$\omega_{nj}^2 = 4\pi^2 f_{nj}^2 = v^2\left[\left(\frac{j_{0j}}{a}\right)^2 + \left(\frac{\pi n}{L}\right)^2\right]. \tag{9}$$

At low frequencies one would only expect to see plane-wave-like axial modes[6] which allows us to solve Eqn. 9 for the speed of sound using $j_{0j} = 0$ which gives

$$f_{nj} \approx \frac{nv}{2L}. \tag{10}$$

By measuring the frequency at adjacent n values, one can calculate the speed of sound through the cavity. In practice, to do so would require at least two measurements. Suppose we measure a resonant frequency f_1 at some n. Then the next corresponding resonant frequency f_2 would occur at $n + 1$. Therefore by taking the difference between these frequencies using Eqn. 10 we would have

$$f_2 - f_1 = \frac{v}{2L} \implies v = 2L\Delta f. \tag{11}$$

Since we left the node index n arbitrary, we know this calculation will be good for any two nearby resonant frequencies. This can be used in practice by sweeping through a range of frequencies with a small step size. By collecting multiple peaks for the same sample, we can find the average change in frequency as

$$\Delta f_{ave} = \frac{1}{N-1}\sum_{i=0}^{N-1}(f_{i+1} - f_i), \tag{12}$$

where N is the number of peaks and each peak has an incrementing index i.

To apply Eqn. 11 we must consider our sources of error. Both L and Δf are measured quantities and thus the error of this calculation σ_v is given by

$$\sigma_v = \sqrt{\left(\frac{\partial v}{\partial L}\right)^2 \sigma_L^2 + \left(\frac{\partial v}{\partial \Delta f}\right)^2 \sigma_f^2} \tag{13}$$
$$= \sqrt{(2\Delta f \sigma_L)^2 + (2L\sigma_f)^2}, \tag{14}$$

where σ_L and σ_f are the uncertainties on L and Δf respectively.

For particles moving in a medium, we would expect the speed of sound waves to be related to the kinetic energy. Similarly, the kinetic energy can be related to temperature which thus gives us a relationship

$$k_B T \propto \frac{1}{2}mv^2 \implies v \propto \sqrt{\frac{T}{m}}, \tag{15}$$

where k_B is the Boltzmann constant. This allows us to naively check our results as we change gases or temperatures to confirm if the results match this relationship.

III. Experimental Setup

To measure sound speeds within a medium, we used a brass cylindrical cavity shown in Fig. 1. At 300 K, the cavity had the dimensions of $L = 4.0$ cm and $r = 0.5$ cm. When driven by a frequency such that $n\lambda = 2L$, a greatly enhanced acoustic amplitude is achieved due to the occurrence of a standing wave resonance. By sweeping the frequency over a large range, multiple resonances can be observed. Once done, we can then use Eqn. 11 to calculate the speed of sound.

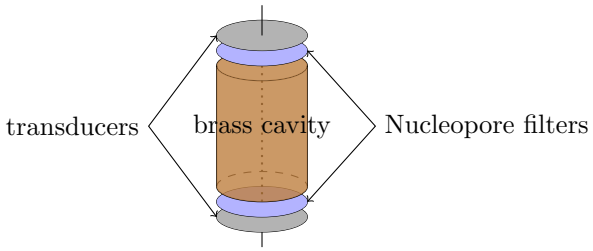

Figure 1: A brass cylindrical experimental cavity.

A thin polymer membrane (15μm) with a metalized surface is grounded and set up so that an electric field can be applied between it and the rigid end of the cylinder. This creates a force on the membrane which exerts a force on the fluid within the cavity. This force gives rise to a sound wave (or density oscillation) within the fluid. The membrane itself is composed of a porous polycarbonate polymer (Nuclepore) that contains a high density of 1μm pores. For normal gases and liquids, the viscosity is sufficiently high such that it cannot flow through the pores over an oscillation period. The membrane thus acts as an impenetrable shield. When this cavity is filled with liquid Helium II, the superfluid component has a low viscosity which allows the fluid to flow through the pores easily. However, the normal component is blocked similar to a typical fluid. The two components flow together in such a way that a constant total density is maintained. This is a behavior that is characteristic to second sound and thus the setup serves as an efficient second sound wave generator [8].

Our experimental setup can be seen in Fig 2. and consists of an experimental chamber (containing our cavity described above) which we can pump under vacuum to under 2 mmHg. Directly surrounding the experimental chamber is an inner vacuum chamber which serves as a buffer between the experiment chamber and the surrounding chamber. Around the inner vacuum chamber is an open chamber that we add liquid nitrogen to for cooling. Lastly, there is one more outer layer of vacuum which insulates the liquid nitrogen from the outer air. The main idea is such that we can add a small amount of gas to the inner vacuum chamber and the particles will carry energy away from the experimental chamber and towards the liquid nitrogen chamber, thus cooling the experimental chamber. In practice, this also works in reverse when we have a colder experimental chamber than the liquid nitrogen, and thus we require the ability to vacuum pump the gas out of it.

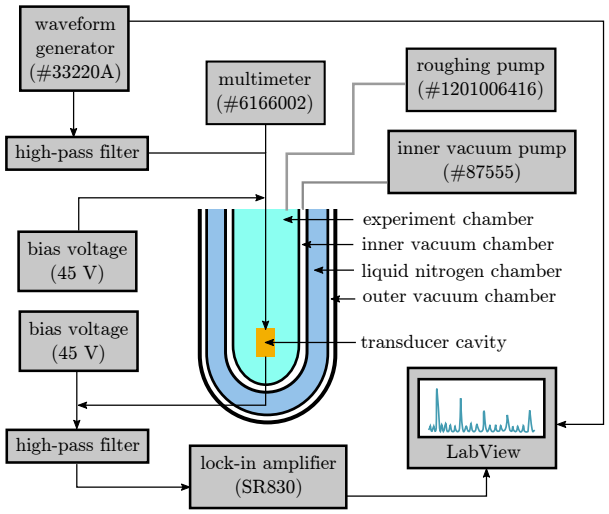

Figure 2: The experimental setup of our multi-layer dewar system.

Within our experiment chamber, we had a material which we monitored the resistance of. We used a program in Lab View that was correlated such that the resistance of the material will give us a temperature reading within the chamber. As our temperature decreased, the resistance increased. We also had a pressure gauge connected to the chamber that was correlated to read in units of liquid helium temperature. Thus, as we pumped our chamber containing liquid helium towards vacuum, we were able to see the temperature reading directly.

IV. Initial Results & Analysis

First, we began by filling our experimental chamber with nitrogen gas. While at atmospheric pressure and room temperature, we swept over a frequency range from 2000-20000 Hz and recorded the amplitude at each frequency step (Fig. 3). From this data, we can determine the frequency peaks to occur at $f_1 = 4400$ Hz, $f_2 = 8700$ Hz, $f_3 = 13000$ Hz and $f_4 = 17300$ Hz. The error on these measurements is proportional to the step size used to find them, which in this case is ± 100 Hz. From this, we can use Eqn. 12 to determine the average frequency change which for nitrogen gas gives $\Delta f_{ave} = 4300$ Hz. Thus we can calculate the speed of sound through nitrogen gas using Eqn. 11 which gives

$$v_N^{300K} = 344 \pm 8 \text{ m/s}. \tag{16}$$

When comparing this to literature (at 20°C), with a value of $v_N = 349$ m/s [9], we see there is a 1.4% error between the two values and the value from literature is well within our uncertainty. This confirms that our apparatus was working properly.

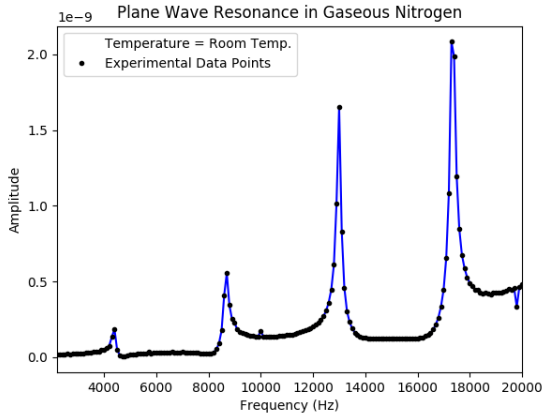

Figure 3: Plane wave resonance peaks generated at room temperature. The data was sampled at a rate of 100 Hz.

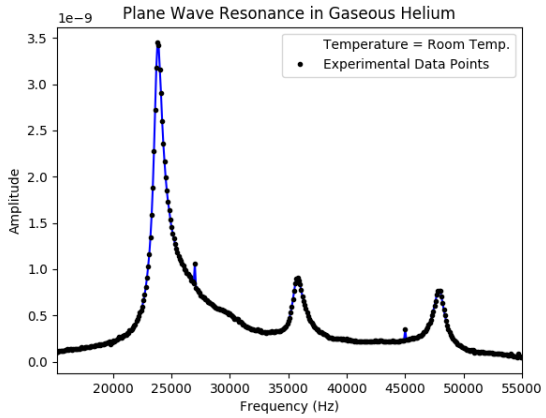

Figure 4: Plane wave resonance peaks generated at room temperature. The data was sampled at a rate of 100 Hz.

Following this, we did the same process for helium gas (Fig. 3). Using this, we can determine 3 clear peaks which give

$$v_{He}^{300K} = 978 \pm 8 \text{ m/s}. \tag{17}$$

When comparing this to a similar literature value (at 20°C) of $v_{He} = 1007$ m/s [9], we see there is a 4.7% error between the two values and our measured value only matches to within 4σ of the literature result. There are multiple reasons why this value is farther off (specifically lower) than what would be recorded in literature. First, the roughing pump we are using is very dated and thus unable to create as good of a vacuum as newer systems. This would have caused residual air molecules to remain in our experimental chamber while collecting data. Since we have already shown that the speed of sound through nitrogen gas (which is the primary component of air) is significantly slower than that of the expected helium gas result, then if there was some residual nitrogen or other air molecules, our measured results would be shifted towards that value as they are.

After confirming that our results are what we expect for room temperature, we began to cool our experiment chamber using liquid nitrogen. The experiment chamber was filled with nitrogen gas during this process. The resistance was monitored to ensure that it exhibits the expected increasing behavior as we cooled the experiment chamber. At first, there was a slight decrease (a dip) in the resistance. This is not expected as the resistance should increase as temperature decreases. We believe it was due to the wires connected to the resistor themselves cooling that cause this apparent dip in resistance. Immediately after, the results begin a steady increase as expected.

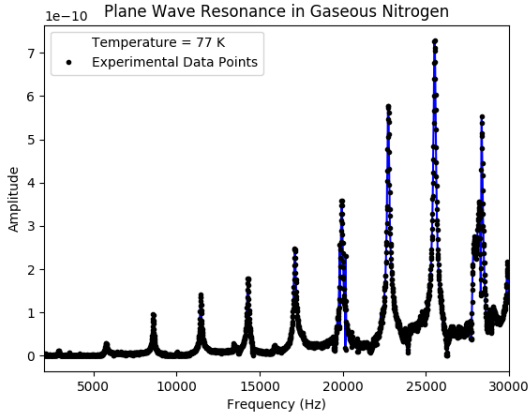

Figure 5: Plane wave resonance noise peaks generated at 77 K. The data was sampled at a rate of 10 Hz.

When the cooling process began, the resistance was around 153.5Ω. After several hours of cooling the chamber using liquid nitrogen, the resistance increased to a value of 335.9Ω. At this point, the chamber is in equilibrium with the liquid nitrogen which implies it is approximately 77 K. We collected data at this temperature and found the resonant frequencies in the cooled gaseous nitrogen (Fig. 5). Again we calculated the average frequency change between peaks and then the speed of sound which we determined was $v_N^{77K} = 225.9 \pm 0.8$ m/s. This result is expected. As the temperature decreases, the kinetic energy of the particles would decrease and thus the speed that a sound wave can propagate through the medium would decrease.

V. A Learning Experience Through False Measurements

After cooling our experiment chamber fully with liquid nitrogen, we transferred helium gas into the chamber in exchange for the nitrogen gas. We then placed liquid helium into the experiment chamber. At this stage, our resistance readings through Lab View are correlated such that we get a numerical temperature instead of just a resistance. Once placed into our experiment chamber, the liquid Helium was around 4.14 K. For our low level data, we started by using a small frequency step size at a low frequency range. Once again, what we believed to be the resonant peaks were measured (Fig. 6). In this case, the resolution was clearly not good compared to previous measurements. We believed this was likely due to the rapid boiling of the liquid helium at this temperature. The calculated speed from this data is then $v_{He}^{4.14K} = 9.6 \pm 0.3$ m/s.

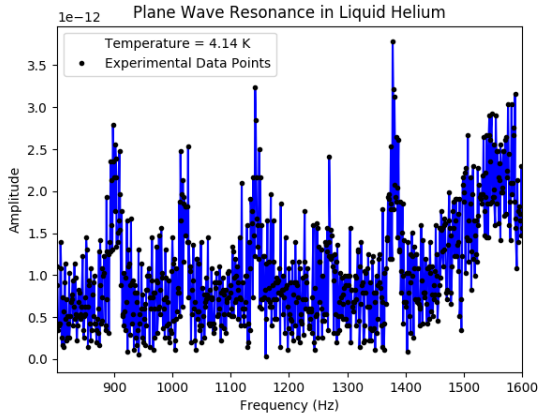

Figure 6: Plane wave resonance peaks generated at 4.14 K. The data was sampled at a rate of 1 Hz.

Somehow, while taking these measurements in the first trial, we completely glossed over the fact that there should be no second sound peaks for Helium-I. If we refer to results from literature, we see that the speed of sound through Helium-I at 4.22 K is about 180 m/s [10]. This corresponds to a frequency difference almost twenty times higher than what we measured.

The next step shortly after transferring liquid helium into our experiment chamber was to cool it below the λ-point and collect data. By pumping our experiment chamber under vacuum, we are able to cool our liquid helium under the λ-point. The temperature reading was very sensitive to how much we are pumping on our chamber. At first, we managed to get the Lab View temperature to read a steady value of around 2.125 K dropping at a steady rate that allowed us to capture multiple 'resonant' peaks before a change of 0.01 K (Fig. 7). We managed to collect a large amount of data points before our temperature reached about 1.675 K. The majority of these data points are represented by the blue points in Fig. 8. At this point, we re-adjusted the setup so that we were at a temperature closer to the λ-point. We adjusted the vacuum so that it was steadily dropping allowing us to collect multiple 'resonant' peaks within a 0.002 K temperature change. These temperature changes is what was used as the temperature error in Fig. 6. The green data points correspond to this second set of data.

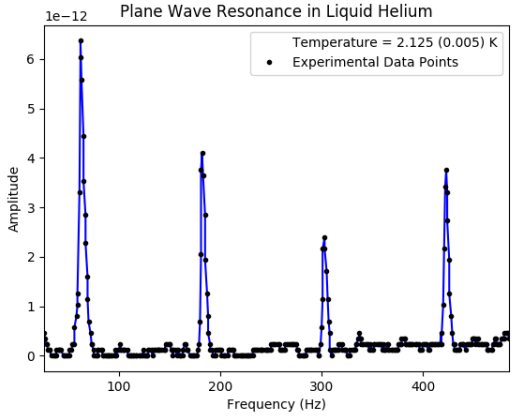

Figure 7: Plane wave resonance peaks generated at low temperature (near the λ-point). The data was sampled at a rate of 1 Hz.

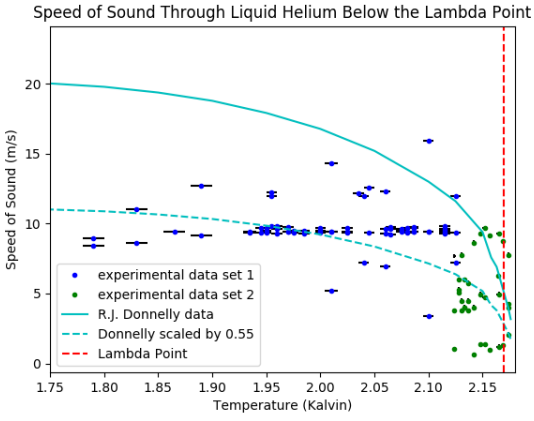

Figure 8: A comparison of our initial data and the data from R.J. Donnelly. The errors correspond to the temperature reading fluctuation and our range for speed calculated from the step size of frequency.

One of the main goals of the experiment was to reproduce the R. J. Donnelly data. In Fig. 8, the light blue line represents the R. J. Donnelly data,

which did not correspond to the data that was collected during this part of the experiment. The main feature of dropping in speed close to the λ-point is apparent by the second data set that was collected (green), however there does not appear to be a nice curve like that of the Donnelly data. A key feature of the data set that we had obtained is that the majority of our data points follow a somewhat flat line. We were measuring our 'resonant' peaks at different temperatures, however the values seemed to consistently give us a $\Delta f \approx 120$ Hz which caused all of our data to be about a factor of two from what we expected and follow an incorrect trend.

VI. Corrected Experimental Results

After performing our initial analysis from the data above, we had to step back and think about what went wrong with our experiment. The main difference between our room temperature data, which gave expected results, and our low temperature data was the frequency range that we used. At one point we also realized that the frequency of the power outlets our equipment are attached to operate at 60 Hz. By combining these two ideas, we had an idea about what was happening to our data and how we could test the results.

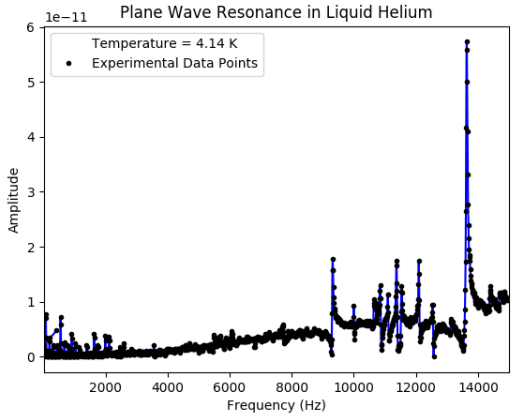

Figure 9: 120 Hz noise resonant peaks (low frequencies) and plane wave resonance peaks generated at low temperature (4.14 K). The data was sampled at a rate of 1 Hz.

After a few weeks of performing another experiment, we were presented with the opportunity to revisit this and decided to collect one last set of data with the new knowledge we had from our analysis. The first thing we did after obtaining an experiment chamber filled with liquid Helium-I was to measure the resonant peaks over a large range of frequency (Fig 9). We knew from literature that we should see resonant peaks that correspond to a spacing of approximately 2250 Hz using liquid Helium-I around 4.14 K [10]. We noticed right away that the 120 Hz peaks we saw were present, however they died off as the frequency range increased. At around 9000 Hz, we started seeing larger peaks that could be representative of our sound resonant peaks. After taking this data, we cooled our apparatus as low as we could get it and took one more data set to compare.

While well below the λ-point, we again measured our resonant peaks. This time we only used a frequency range from 0 Hz to 5000 Hz since the 120 Hz peaks appeared to die off well before 5000 Hz (Fig. 10). This time we clearly can see both the low frequency 120 Hz peaks that we were seeing before but also another set of peaks that have a spacing of about a factor of two larger than the low level noise. Eureka! These are peaks that do not appear to be the same as the low level noise, and also do not appear in our data above the λ-point and thus we concluded that these must be our second sound peaks. After confirming that we see second sound resonant peaks, we continued to collect a few runs of data at varying temperatures to see if we could reproduce the data from R. J. Donnelly.

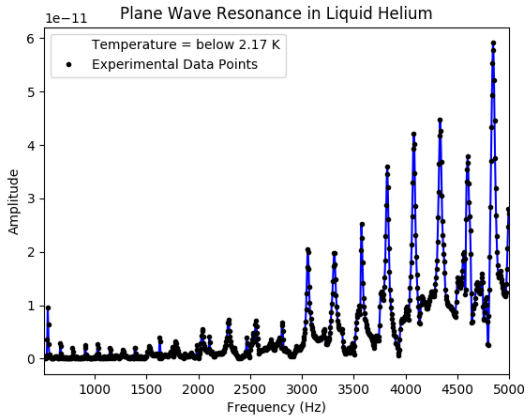

Figure 10: 120 Hz noise resonant peaks (low frequencies) and second sound resonance peaks generated at low temperature (below 2.17 K). The data was sampled at a rate of 1 Hz.

Our final data points are plotted along with the Donnelly data in Fig. 11. Two of our data points fit the Donnelly data exactly, and the other three points are within a few σ uncertainty. The error on these values are likely slightly larger than reported due to unknown sources of error due to the nature of the environment that this experiment was performed in. Possible sources of error include degradation on the equipment from previous use, inaccurate temperature dependence of cavity properties and possibly

an issue with the resistance versus temperature correlation. In some sources, it is discussed that there are corrections having to do with pressures that can cause systematic errors. These were not looked into in this experiment. As for our resonant peaks that appear 120 Hz apart at low frequencies, we believe this is from the wall power sources and is due to all odd or even modes being picked up by our amplifier.

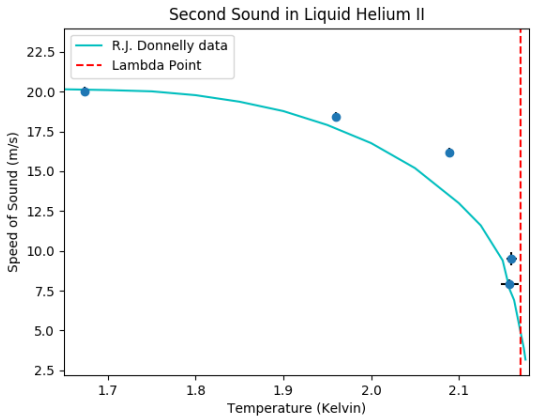

Figure 11: A comparison of our final data and the data from R.J Donnelly. The errors correspond to the temperature reading fluctuation and our range for speed calculated from the step size of frequency.

Medium	Temp. (K)	Speed (m/s)
Nitrogen gas	300	344
Helium gas	300	978
Nitrogen gas	77	225.9

Table 1: A chart of measured sound speeds through a medium at various temperatures.

VII. Conclusion

We are able to measure the speed of a sound wave through a medium using the resonant frequency peak values. Accurate results can be determined at room temperature and pressures which for our environment show $v_{He}^{300K} = 978 \pm 8$ m/s and $v_{N}^{300K} = 344 \pm 8$ m/s. It was shown that as temperature decreases, there is a decrease in sound speed through a medium. At first, we were unknowingly measuring resonant peaks that came about from electrical noise. After analyzing the data, we were able to correct for our mistake and properly determine second sound resonant peaks. For liquid helium, we were able to achieve a superfluid phase which exhibits a characteristic temperature versus second sound speed curve similar to previously published results.

VIII. Acknowledgements

I would like to acknowledge Michigan State University for providing the equipment and opportunity to take the advanced lab class where I performed this superfluidity experiment. I would also like to thank both professors teaching the Advanced lab course at Michigan State University for providing insight and assistance on various problems and challenges faced during this experiment.

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