# A Brief Review of Nuclear Pasta Models and Theoretical Predictions

Antonius Torode

April 26, 2017

**Abstract**

**Background:** Nuclear Pasta, which is named after it's structure, is a model that describes various structures of matter in very high density situations such as that of a neutron star. **Method:** Through simulations using algorithms such as minimum spanning trees, which determine how nuclei interact with those around it, we can model the structure that matter may take in neutron stars. Through these simulations, we can gain an understanding of the behavior that nuclear matter exhibits under specific conditions. These simulations take massive amounts of computational power and time to run but the results allow us to see the changes that density and other parameters have on matter formation. **Results:** These simulated structures (see figures in text) are referred to as nuclear pasta due to the pasta like structures that matter forms which resemble that of lasagna, spaghetti, gnocchi and others. **Conclusion:** Through the nuclear pasta simulations, we can determine that density has a high affect on the nuclear structure and that both coulomb attraction and short range nuclear interactions play a pivotal role in high density nuclear structures.

## I. Introduction

At the end of a massive stars life, a supernova can occur in which a massive change takes place in the structure of matter within the star. Shortly after the discovery of the neutron star in 1932, it was considered that stars over $\sim 10$ solar masses will supernova and become a compact star of neutrons which is the explanation of a neutron star [1]. Neutron stars have largely varying magnetic fields and spin periods which both span six to eight orders of magnitude [1]. The core becomes a proto-neutron star which is essentially a single gigantic nucleus compared to the separate $\sim 10^{55}$ nuclei that it was previously [2]. The structure of neutron stars is still very theoretical but it is believed that at very high densities, the approximately spherical nuclei begin to re-organize into exotic shapes in order to balance the short-range nuclear attraction and long range coulomb repulsions [1].

The structure of this nucleus within the neutron star is what is referred to as nuclear pasta. It is called nuclear pasta because the structures of the matter can vary in a plethora of variations which are simulated to look something resembling that of different pastas. These pasta structures are exotic in formation and all occur below the nuclear saturation density of $n_0 \approx 0.16\mathbf{fm}^{-3}$ [2]. In order to understand the properties and physics within neutron stars, it is important to understand the formation and structure of matter within these stars. The structure of various nuclear pastas is generally studied by simulating a varying density of the matter in the star from $n \lesssim 0.1n_0$ to $n \sim n_0$ [2]. Studying the structure of the neutron star between these densities gives us key insights on how neutron stars really work.

The discovery of nuclear pasta came about from work involving a liquid-drop model for nuclei. It was shown that matter in a high density star favors a structure of large nuclei with $Z > 100$ [3]. Lamb then determined that when the volume of nuclear matter reaches about $\sim 1/2$ that of the container it is within (or its region of the star in this case), the matter would "turn inside out" [2, 4]. This means that there would be regions of the star where the matter formed low density bubbles as well as high density clusters together. The combination of both of these low and high density regions is what creates the structures of nuclear pasta.

## II. Simulation Methods

For the purpose of this paper I will refer to the simulations primarily done in work by Schneider, et al [2], in which they isothermally expand nuclear matter. The analysis of nuclear pasta starts with algorithms that are used in determining the proton and neutron numbers and structures within a neutron star. One such method is described in Schneider's (et al.) work, in which two separate algorithms are used. These are referred to as "minimum spanning tree" (MST) and "minimum spanning tree in two-particle energy space" (MSTE) [2, 5]. Within this process, space is separated into clusters C and then correlations between the positions of protons and neutrons are searched for within coordinate space. The algorithms consists of logic that determines the positions of nuclei. An example of this, which is used by Schneider for the MST algorithm can be compressed as

$$i, j \in p \implies i \in C \text{ iff } \exists j \in C : |\vec{r}_i - \vec{r}_j| \leq r_{pp}$$
$$i \in n, j \in p \implies i \in C \text{ iff } \exists j \in C : |\vec{r}_i - \vec{r}_j| \leq r_{np}.$$

In the above notation, $i$ and $j$ represent two nucleons, $p$ and $n$ represent protons and neutrons respectively, and $|\vec{r}_i - \vec{r}_j|$ is defined as the distance between nucleon $i$ and the closest periodic image of nucleon $j$ [2], which accounts for the periodicity of the two-nucleon system. In this method, $r_{pp}$ and $r_{np}$ are both variable maximums which can be changed when running the simulations. In the case of Schneider, these values are determined by a two-nucleon correlation function and it's first non-zero term [2].

The MSTE algorithm correlates two nucleons in two particle energy space. The logic for this algorithm is given for any two nucleons $i$ and $j$ that

$$i \in C \text{ iff } \exists j \in C : e_{ij} < 0,$$

where $e_{ij}$ is the two-particle energy which is defined as

$$e_{ij} \equiv v_{ij} + \frac{|\vec{p}_i - \vec{p}_j|}{4\mu}.$$

In this algorithm, $\mu$ is the reduced mass of the two nuclei. This method has an advantage in that it takes into account the relative momenta of nucleons where as the MST algorithm does not. However, Schneider shows that both methods give similar results when determining the nuclear pasta formations.

## III. Nuclear Pasta Models

It is common to use a relatively small volume size and particle number in order to run simulations using nuclear matter with the required computational power and time. In the simulations used by Schneider, they use 51,200 particles in all of their simulations and a volume of 80 fm. In this work, they ran all of their simulations on the BigRed supercomputer at Indiana University. They typically used 128 cores for a time ranging from a few days to a week. Another version of their simulation was ran on the Kraken supercomputer which used approximately 1152 cores for about 150 hours. This shows it is clear that these simulations require a vast amount of computing power and by performing even small changes to the parameters or increased particle numbers can increase this computation time even greater.

They begin by settings the initial conditions for the nuclei, which include random initial velocities that are randomly selected from a Boltzmann distribution at a fundamental temperature of 1 MeV. They then choose a time to run each simulation for, which in their case is $t = 10,000 fm/c$ with time steps of $\Delta t = 1 fm/c$. They choose these times because they determined that it does not change the outcome by a noticeable amount if they use a time such as $t = 500,000 fm/c$ due to being able to determine when the system is in equilibrium clearly. Each model, after reaching equilibrium, is evolved using a stretching rate $\dot{\xi}$, which is varied throughout the simulations to produce different variations on the matter formations.

The simulation results from Schneider can be seen in figures 1 to 4. Each figure shows the model of nuclear matter at varying densities, which in each case are 0.090 $fm^{-3}$ (top left), 0.075 $fm^{-3}$ (bottom left), 0.050 $fm^{-3}$ (top right), and 0.025 $fm^{-3}$ (bottom right).

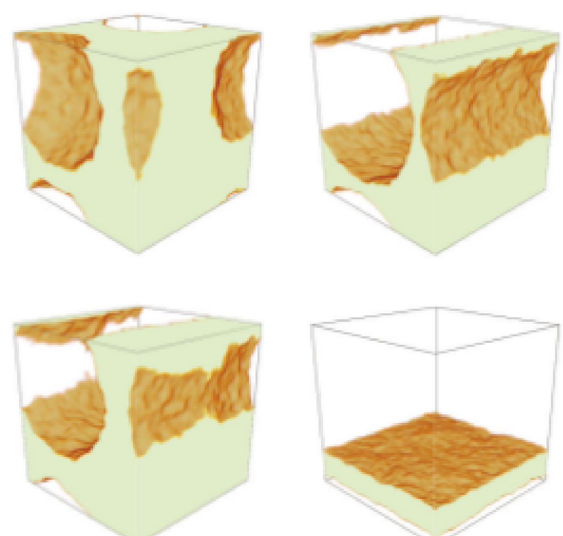

**Figure 1.** A run using a stretch factor $\dot{\xi} = 1.0 \times 10^{-6} c/fm$. Images reformatted from original publication [2].

These simulation models give us an idea of how matter organizes itself under certain conditions. As we can see from the figures, the different structures vary greatly but they all have a pasta like structure, which is where the term nuclear pasta comes from. For a specific example, the top right simulation in figure 4 resembles that of lasagna. Another example is the bottom right structure in figure 3 which almost resembles spaghetti. In this case, the matter orients itself in string like structures where as in lasagna it has a planar structure. Another enlarged example of this can be seen in figure 5, where a density of $0.010 fm^{-3}$ and a stretch rate of $\dot{\xi} = 1.0 \times 10^{-7} c/fm$. In this case, we can see that at a low density there is a lattice structure of the matter but it also tends to separate into smaller chunks of nuclei. This example resembles that of gnocchi (a type of pasta). All of these models are produced using a program known as ParaView [6].

**Figure 2.** A run using a constant density. Images reformatted from original publication [2].

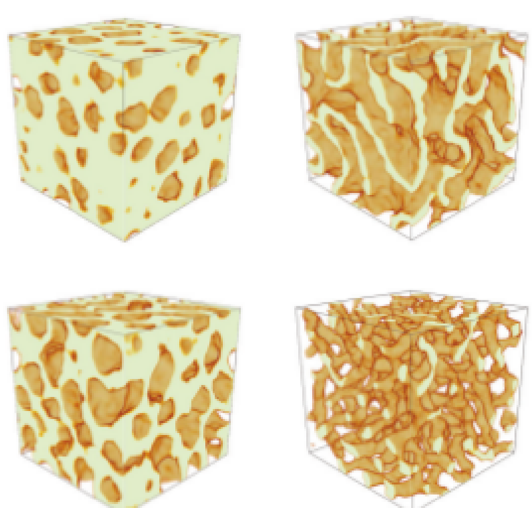

**Figure 3.** A run using a stretch factor $\dot{\xi} = 1.0 \times 10^{-5} c/fm$. Images reformatted from original publication [2].

Within the supplemental material of Schneider's work, there are also video simulations that show the whole cycle of the nuclear pasta model shown in figures 1-5. From the simulations we can see that as density decreases, the nuclear pasta formations tend to travel from lasagna, to spaghetti and finally to gnocchi. In other words, the structures that are formed from the nuclear matter appear to be combined into larger masses (per unit volume) for higher densities and smaller masses (smaller pieces but more of them) for lower densities.

**Figure 4.** A run using a stretch factor $\dot{\xi} = 1.0 \times 10^{-7} c/fm$. Images reformatted from original publication [2].

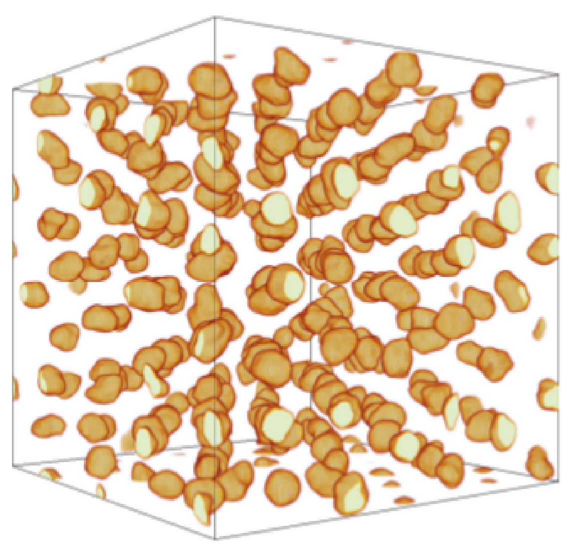

**Figure 4.** A run using a density of $0.010 fm^{-3}$ and a stretch factor $\dot{\xi} = 1.0 \times 10^{-7} c/fm$. Image from [2].

IV. Simulation Implications

It is determined by comparing different simulation results that there is a clear impact of the Coulomb force when it is added versus when it is ignored. This force is what gives the richness to the different nuclear pasta formations and without it the matter arrangements are very limited. The most prominent major phase transitions occur at around $0.040 fm^{-3}$ which is the transition from a lasagna formation, and $0.015 fm^{-3}$ which is the transition from spaghetti to gnocchi. These results can be used to determine lower limits on the densities in which these pasta shapes may occur in real situations such as that of neutron stars. This gives us theories to how that of high density stars behave on a subatomic level which can be used in conjunction with astronomical measurements to better refine our understanding of nuclei behavior. Within recent years, temperature effects, electron transport, and other affects are being considered within nuclear pasta structures in order to increase our understandings of nuclear and astronomical physics [7–12].

V. Conclusion

Nuclear pasta is a relatively new idea of study in which matter at high densities is studied. Coulomb repulsion combined with short-range nuclear attraction causes matter to structure itself in pasta-like configurations. Since this phenomenon occurs in high density situations such as that of neutron stars, the best way to study it is by simulation. Through various methods, simulations can be generated which show pasta like structures clearly and with a phase transitions from large pasta structures (e.g. lasagna) to smaller pasta structures (e.g. gnocchi). With different initial and varying conditions, different results can be determined. This work is important in understanding the structure of high density objects and how matter behaves in non-standard situations and enables us to refine our understandings of physics at a nuclear level.

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