A Brief Review of Piezoelectric Crystal Oscillators

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April 27, 2017

Abstract

Piezoelectric crystals exhibit properties that allow them to be very useful in modern electronics. The oscillators created by use of these materials allow for precise timing and frequency ranges with very low variation in their operating ranges. In this review I will give some examples of circuits that use the crystal oscillators such as the often used Pierce and Colpitt cells. I will similarly discuss techniques for measuring and studying the circuits while performing some analysis on typical oscillator values and ranges. Some discussion then follows on topics such as locus diagrams, three-point oscillators and challenges with four-point variations.

I. Introduction and History

In order to understand a crystal oscillator, it is important to discuss the property of Piezoelectricity. This phenomenon was discovered by Jacques and Pierre Curie in 1881. When given a crystal that has no center of symmetry, an electric charge can be created by introducing a pressure at the ends of a polar axis of a crystal [1]. This electrical charge is induced by a movement of electrons which appears due to the sheering and stress applied to the crystal (figure 1). It can also be defined simply as an electrical potential that is generated from mechanical deformations of the crystal surfaces [2]. Similarly, a electrical charge can change the shape of the crystal to produce a mechanical charge [3].

A crystal oscillator is an oscillator that utilizes piezoelectric crystals to determine a fixed frequency. It is commonly created by sputtering two metallic films onto the parallel faces of a crystal and applying an alternating electric field. The electric field causes the crystal material to vibrate at its natural frequency which is determined by the cut (or shape) of the crystal being used. The crystal's resonant frequency is a direct function of the dimensions and shape [2]. The natural frequency of the crystals can range from in the kilohertz range to megahertz. This natural vibration of the material produce an alternating electric field which spreads across the crystal and does not suffer from frequency drift due to the crystalline structure [4]. Piezo-electric devices, such as a crystal oscillator can be referred to as transducers due to the ability of converting mechanical energy into electrical energy or vise versa [3].

The crystal oscillator is useful in that it can be used to replace circuits with tuned oscillators to provide a fixed resonant frequency. It can similarly be coupled to oscillator circuits that are tuned to frequencies that are approximately equal to the natural frequency of the crystals being used. This adds an additional prevention of frequency drifts that may not have previously been there [1]. A frequency drift is the tendency of a frequency to shift from the desired value by either increasing or decreasing. This can cause problems in circuits that require precise frequencies and timings to maintain a specific precision. This device is used in almost all timing systems such as clocks and watches in telecommunications equipment including personal computers [5] and primarily used in conjunction with quartz due to the material properties it exhibits and the high availability of the material [2].

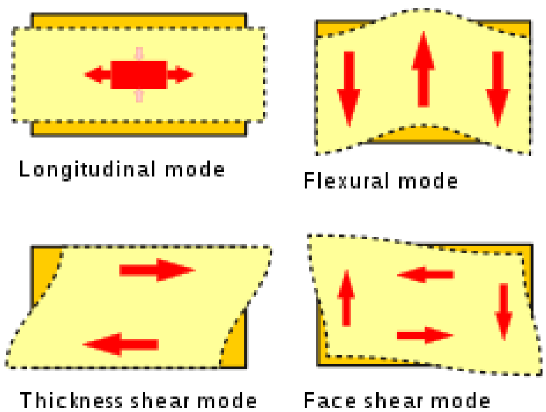

Figure 1. Variations of crystal oscillation modes due to sheer and stress [6].

Almost any object made of an elastic material can be used in this manner as all materials will have a resonant frequency. However, due to the large material dependence of this phenomenon, it can be greatly effected by temperature. Since some materials have their properties highly dependent on the temperature, there will be shifts of the natural frequency when in different conditions which make some materials better than others when used in electronic components. For this reason, quartz and other crystals are useful because they have a very low temperature dependence in regard to their natural frequencies. There are many crystal substances that can be used in these oscillators but another quartz is primarily used due to it's greater mechanical strength when compared to others [3]. Low cost ceramic materials can also be used in cases where high precision and relatively accurate timing are not needed. An example of a material that can be used in this manner is Rochelle Salt which produced a high voltage per unit of strain when it is effected by a deformation, however it is very sensitive to heat, moisture, aging, and mechanical shock [2], which makes it not suitable for many applications.

In 1929, F. R. Lack presented research that outlines temperature dependencies and modes of vibrations that were observed in quartz crystal plates. He discusses that the importance of improving this technology was vital to keep it within the communication art standards. This is because after being originally developed, the usual manner of preparing the quartz plates was not sufficient for maintaining the high precision required for a good astronomical clock and to meet the standards needed at the time [7].

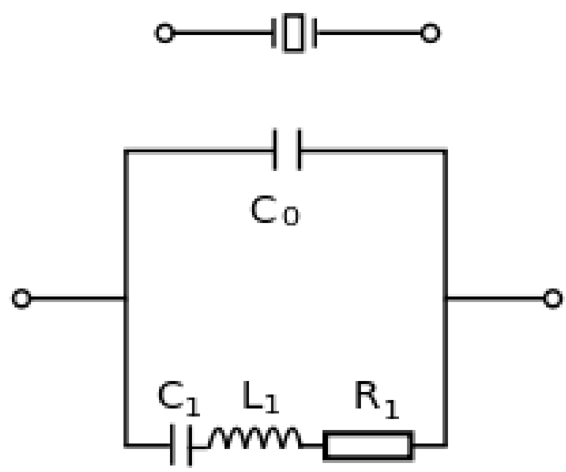

Figure 2. Schematic symbol and the Equivalent circuit of a piezoelectric crystal near its resonant frequency [6].

II. Electrical Model

A crystal oscillator component can be represented by an equivalent circuit with a resistor, capacitor, and inductor in series with each other and all in parallel with a capacitor [8, 9, 10] as shown in figure 2. The most direct way of determining the oscillating conditions in a circuit containing a crystal oscillator according to some are to analyze the differential equation for the current in a particular branch of a circuit [8]. Other methods include using impedances through complex phasor analysis.

If we consider the crystal oscillator setup in figure 2, we can determine the total complex impedance of the circuit. For simplicity, let $L_1 \equiv L$, $C_1 \equiv C$ and $R_1 \equiv R$. The complex impedance would then be given by

$$\widetilde{Z}_{tot} = \left(\frac{1}{\widetilde{Z}_{top}} + \frac{1}{\widetilde{Z}_{bot}} \right)^{-1} = \frac{\widetilde{Z}_{top} + \widetilde{Z}_{bot}}{\widetilde{Z}_{top}\widetilde{Z}_{bot}},$$

where $\widetilde{Z}_{top}$ and $\widetilde{Z}_{bot}$ are the impedances for the top and bottom paths respectively. Plugging in the correct impedances for each component then gives

$$\begin{aligned}\widetilde{Z}_{tot} &= \frac{\frac{1}{i\omega C_0} + \frac{1}{i\omega C} + i\omega L + R}{\left(\frac{1}{i\omega C_0}\right)\left(\frac{1}{i\omega C} + i\omega L + R\right)} \\ &= i\omega C_0 - \frac{i\omega C}{(C\omega^2 L - 1) - i\omega RC}\end{aligned}$$

If we then multiply the second term by one using the complex conjugate of the denominator we have a $\widetilde{Z}_{tot}$ of

$$i\omega C_0 - \frac{(iC^2\omega^3 L - i\omega C - \omega^2 RC^2)}{(C\omega^2 L - 1)^2 + \omega^2 R^2 C^2}, \qquad (1)$$

which allows us to take our the real and imaginary parts of $\widetilde{Z}_{tot}$ by inspection as

$$\begin{aligned}Re[\widetilde{Z}_{tot}] &= \frac{\omega^2 RC^2}{(C\omega^2 L - 1)^2 + \omega^2 R^2 C^2} \\ Im[\widetilde{Z}_{tot}] &= \omega C_0 - \frac{C^2\omega^3 L - \omega C_1}{(C\omega^2 L - 1)^2 + \omega^2 R^2 C^2}.\end{aligned}$$

The magnitude of this complex impedance would then be given by $|\widetilde{Z}_{tot}| = \sqrt{Re[\widetilde{Z}_{tot}]^2 + Im[\widetilde{Z}_{tot}]^2}$. By examining (1) we can find a specific frequency such that the first term in the denominator will vanish. This is known as the series resonant frequency and is given by

$$\omega_s = \frac{1}{\sqrt{CL}} \qquad (2)$$

Within a crystal oscillator circuit, there exists two resonant frequencies [11]. It can be shown from the total impedance that what is known as the parallel resonant frequencies is given by

$$\omega_p = \sqrt{\frac{C + C_0}{LCC_0}}. \qquad (3)$$

If we would like to make a relationship between ω_s and ω_p, which is often useful, then (2) and (3) can be combined to relate the series and parallel frequencies as

$$\omega_p = \omega_s \sqrt{1 + \frac{C}{C_0}}, \qquad (4)$$

which can be expanded and expressed in the limit of $C_0 >> C$ as

$$\omega_p \approx \omega_s \left(1 + \frac{C}{2C_0}\right). \qquad (5)$$

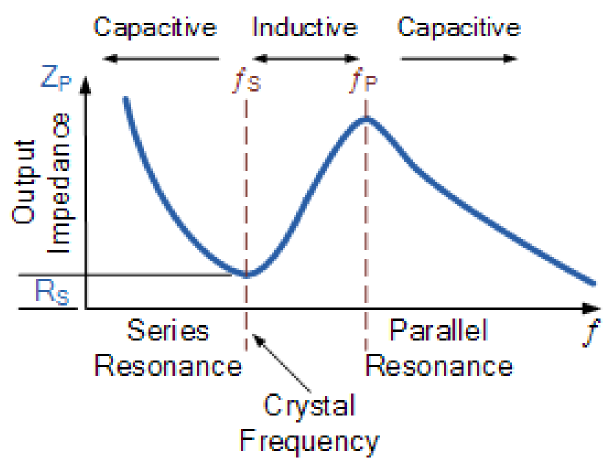

Figure 3. Output impedance of a crystal oscillator versus crystal frequency. In this plot, $Z_p \equiv |\widetilde{Z}_{tot}|$ and $R_s \equiv R$. Image from [3].

The series resonant frequency is the frequency at which the capacitor C resonates with the inductor L at the crystals operating frequency. This series resonance is the frequency that quartz crystal oscillators tend to operate at [3]. The parallel resonant frequency is established when L and C resonate with the parallel capacitor C_0. A plot of output impedance versus crystal frequency $f = \frac{\omega}{2\pi}$ is given in figure 3. The series resonant frequency is the point such that the output impedance is reduced t oa minimum and is equal to that of R. As the frequency increases, the oscillator behaves like an inductor until a maximum impedance is reached (the parallel resonance frequency). These frequencies cannot be used together when working with crystal oscillators. The frequency must be tuned to one or the other [3].

The typical values for crystals operating in the fundamental mode with a 5 MHz to 30 MHz frequency range, given by some sources [12] are

$$2 \text{ fF } \leq C \leq 20 \text{ fF} \tag{6}$$
$$10\Omega \leq R \leq 150\Omega \tag{7}$$
$$0.5 \text{ pF } \leq C_0 \leq 5 \text{ pF}. \tag{8}$$

The inductance L is then determined by the C values and ω. From (4), we can see that $\omega_s < \omega_p$. This means we can say our lowest frequency limit can correspond to a series resonance frequency. We can re-write (2) as

$$L = \frac{1}{C\omega_s^2}. \tag{9}$$

If we maintain a frequency minimum of 5 MHz we can use (9) to determine the upper limit of a typical inductance by finding the maximum value for our typical values, which give

$$L_{up} = \frac{1}{(2 \text{ fF})(5 \text{ MHz})^2} = 20 \text{ H},$$

Similarly, we can say the upper frequency limit corresponds to a parallel resonance, and so by (3), we have

$$L = \frac{C + C_0}{CC_0\omega_p^2}. \tag{10}$$

If we want a lower bound to our inductance we can minimize this using our typical values which gives

$$L_{low} = \frac{(20 \text{ fF} + 5 \text{ pF})}{(20 \text{ fF})(5 \text{ pF})(20 \text{ MHz})} \approx 125 \text{ mH}.$$

From these values we can conclude the typical crystal oscillator operating within the frequency range of 5 MHz to 30 MHz has an approximate typical inductance value of

$$125 \text{ mH} \leq L \leq 20 \text{ H}.$$

To put this in perspective, this value ranges from about one thousandth to about a fifth times the inductance of an industrial transformer respectively [13].

Using (5) and the typical values for a crystal oscillator given in (6) - (8), we can compare ω_s to ω_p. First, suppose we have a crystal operating with $\omega_s = 5$ MHz. For our typical values above would have a lower limit of $C = 2$ fF. By Plugging in the possible values for C_0 found in (8), we get that $1.0002 \leq \frac{\omega_p}{\omega_s} \leq 1.002$. Similarly, using the upper limit of $C = 20$ fF, we get $1.002 \leq \frac{\omega_p}{\omega_s} \leq 1.02$. Combining these two results will give $1.0002 \leq \frac{\omega_p}{\omega_s} \leq 1.02$, which implies

$$5.001 \text{ MHz} \leq \omega_p \leq 5.1 \text{ MHz}, \tag{11}$$

for $\omega_s = 5MHz$. Now, consider $\omega_p = 20$ MHz. By a method analogous to that used above, we arrive at

$$19.6078 \text{ MHz} \leq \omega_s \leq 19.996 \text{ MHz}, \tag{12}$$

for $\omega_p = 20MHz$. By using (11) and (12), we can see that the parallel frequency and resonant frequencies will stay within 2% of each other using typical crystal oscillator values. This tells us that we cannot get two largely differing frequencies from using a single crystal oscillator

A crystal oscillator circuit can sustain oscillations by taking voltage signal from the resonating quartz and amplifying it. The signal is then fed back through the resonator until the output frequency energies are equal to the losses in the circuit. At this point, an oscillation can be sustained and the component becomes stable. When the system is active, positive feedback causes the noise to be amplified which will amplify the oscillations. The oscillator can be seen as a very sensitive frequency selection system that attenuates all of the frequencies save that around the resonant frequency. This is due to the narrow resonance band of the quartz crystal. The multiple frequency modes of a crystal can be set as the dominant resonant frequency as well as other overtone modes and any harmonic frequency which are simple integer multiple of any fundamental frequency. Overtone modes can be excited using specific oscillator circuits. The Pierce-Colpitts type circuits like that shown in figure 4 can easily be modified to oscillate at other overtone modes simply by thicker (and more robust) crystal plates [14].

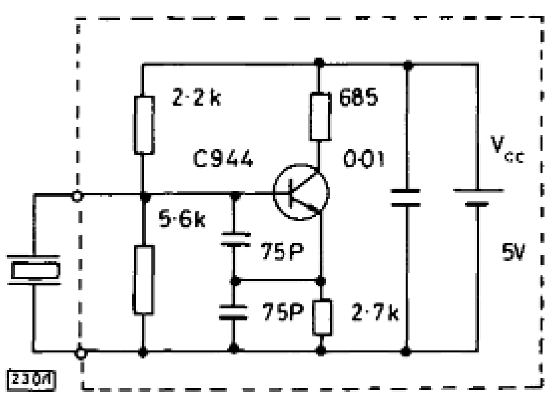

Figure 4. Pierce-Colpitts Crystal Oscillator Circuit [15].

III. Root Locus Diagrams

The performance of quartz have often been depicted using what is known as admittance circles as well as by measurements and calculations of impedances. These are represented on what is known as a locus diagram. These diagrams and form of analysis have proven effective in analyzing various crystal parameters. The diagrams are also useful in determining the most active circuits and which components should be used. This is done by changing the components and then post analyzing the diagrams until the desired results are achieved. They are also very useful in providing information necessary to determine frequency standards of a circuit [14].

Locus diagrams (more formally known as root locus diagrams) come about from root locus analysis which is a graphical method arising from control theory. It is used to determine how the roots of a system change while changing system parameters. This method was developed by Walter R. Evans and can be used to determine the stability of a system. The diagrams are a plots of a closed loop transfer function with respect to gain and are plotted in the complex plane. They can similarly be used to determine damping ratios within a design as well as a natural frequency of a system, such as that containing a crystal oscillator.

IV. Applications

There are many different circuits that utilize the crystal oscillators. One such circuit is known as the three-point oscillator shown in figure 6. In work done by Eric A. Vittoz, he describes this circuit and others which are designed for use in miniaturized resonators that run at high performance oscillations in the range of 2 MHz. The circuit provides a stable frequency between -10°C to 50°C. As mentioned earlier, these oscillator circuits are often used in watches, and this one is designed to be used in a watch which provides a minimum voltage supply of 1.1 V and a nominal current drain on the order of 1 μA. The complete circuit using this design is much more complicated than the general and simple cases. In figure 7, the complete three-point circuit that would be needed for a watch is shown. The image in the left of figure 6 is also an example of a locus diagram (mentioned previously) which helps determine where the maximum output impedances occur.

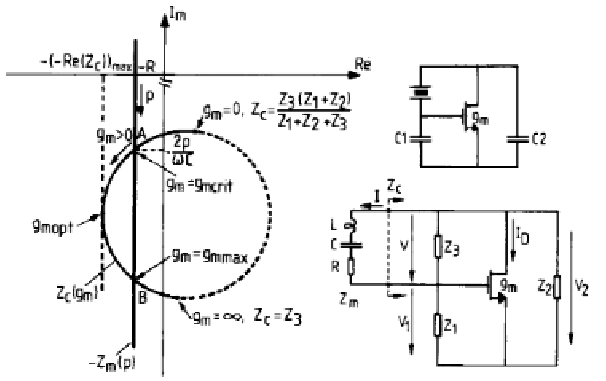

Figure 5. A Basic three-point oscillator (top right), the general form of the three point oscillator (bottom right), and the general complex plane representation (bottom left) [16].

It has been attempted to create a Variable-Range Four-Segment Quartz Crystal Oscillator but the attempts fail due to a needed large negative resistance in order to reach the parallel resonance frequency. Typical oscillator circuits use standard Colpitt and Pierce which is somewhat of a limit to the design schemes using this technology [5]. These circuits are not just used in timing devices and watches. Applications extend to communications receivers, photodynes, high-resolution NMR Spectrometer, microprocessors, and more [17, 18, 19].

Some common applications also include wide use in military applications such as navigation and guidance systems [10], as well as in celestial measurements and space tracking purposes. It is useful in these situations because a stable frequency can be used to track something at a rate set by the oscillator frequency and if there was little consistent, the tracking would drift away from a precise measurement.

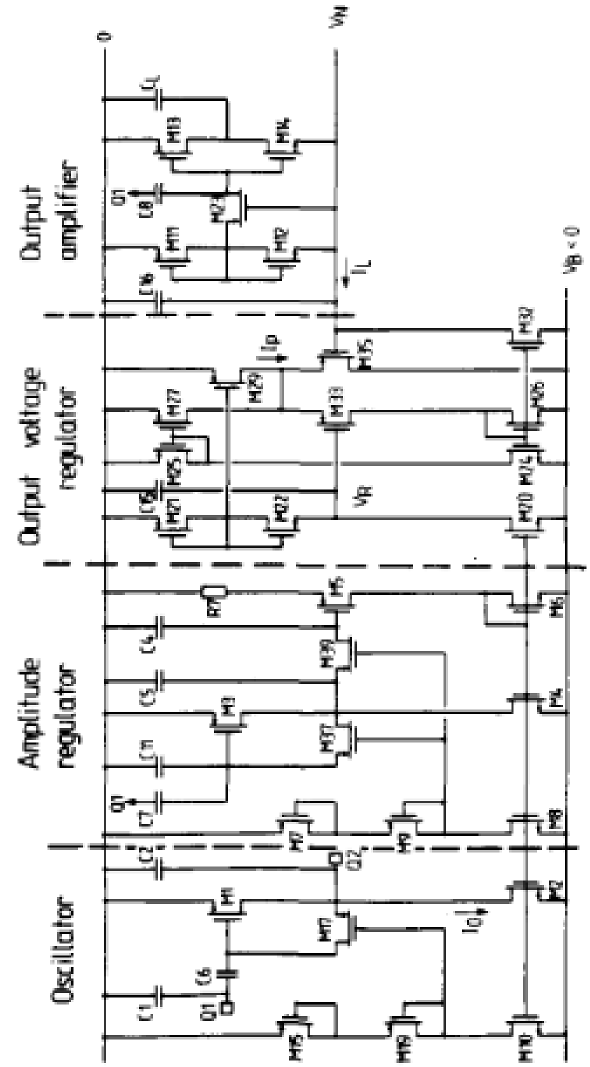

Figure 6. A complete three-point oscillator circuit that would be needed for a watch [16].

V. Conclusion

Crystal Oscillators are a key component in precise timing and frequency applications. They can be used to produce an accurate frequency output with a very small variation to the signal. They provide accurate timing circuits for use in watches and other electronics while being made of stable and temperature stable materials; primarily quartz and other crystals. They provide for multiple harmonic frequency outputs and can be designed to function at ranges in the kHz to MHz.

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