Entry College Level Problem Solving: 001

Please write all answers on this page. You may use scrap paper to do your work on but please attach any scrap paper used after completion. Do not use a calculator.

Question 1. Using the mass-energy equivalence and energy conservation formulas, derive a formula for a particles momentum (p) in terms of kinetic energy (K) and rest energy ($E_0 = mc^2$).

$$E = K + mc^2 \text{ ...conservation of energy} \tag{1}$$
$$E^2 = (pc)^2 + (mc^2)^2 \text{ ...mass-energy equivalence} \tag{2}$$

Question 2. The limit definition of the derivative of a function $f : \mathbb{R} \to \mathbb{R}$ is given by the formula

$$\frac{\partial}{\partial x} f(x) = f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{3}$$

Compute the derivative of the function $g : \mathbb{R} \to \mathbb{R}$ where $g(x) = x^2 + 2x + 1$ using the limit definition.

Question 3. Compute the derivative of the function $h : \mathbb{R} \to \mathbb{R}$ where $h(x) = x^3 + 5x$ using the limit definition.

Question 4. On planet X, the height of an object at some time t that is thrown upwards at some initial velocity v_0 is given by $h(t) = v_0 t - 30t^2$. In order to determine the velocity $v(t)$ and acceleration $a(t)$ we can use

$$v(t) = h'(t) \tag{4}$$
$$a(t) = h''(t) = v'(t). \tag{5}$$

Calculate the velocity and acceleration functions on planet X.

Question 5. The quadratic formula is used to determine the roots (zeros) of any polynomial expression of the form $0 = ax^2 + bx + c$ and is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6}$$

Find the roots of $4 = 2x^2 + 3x + 5$.

Question 6. Find an equation for the roots of $c - 2b = cx^2 + 4ab - 2cx$ in simplest form.

Question 7. In order to quickly calculate the sum (addition) of all values from 1 to n, we can use the formula

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \tag{7}$$

Using the above formula, calculate the sum of all natural numbers from 1 to 500.

Question 8. Using equation (7) from above, determine the sum of all natural numbers from 1 to 60 plus the sum of all natural numbers from 80 to 120.

Question 9. Similar to the previous summation equation, the sum of the square of all natural numbers from 1 to n can be determined by

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}. \tag{8}$$

And furthermore

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}. \tag{9}$$

Use equations (7)-(9) to calculate

$$\sum_{i=1}^{15} i^3 + i^2 + i \tag{10}$$

Question 10. Use equations (7)-(9) to calculate

$$\sum_{i=1}^{10} 4i^3 + 2i^2 + 2i \tag{11}$$

Question 11. Solve the following system of equations for x, y, z.

$$2x + 7y + 3z = 14 \tag{12}$$
$$4x + 5z + 2y = 5 \tag{13}$$
$$z + 9x + y = 2 \tag{14}$$

Question 12. The relativistic energy and momentum of a particle are given by the following equations respectively:

$$E = \gamma mc^2 \tag{15}$$
$$p = \gamma mv, \tag{16}$$

where m is the mass of the particle, c is the speed of light, v is the velocity of the particle and

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}. \tag{17}$$

Solve for the velocity of a particle in terms of E, p, and c. (*Hint: This is easy, though requires a very simple algebra trick.)