max planck institut informatik

SIC Saarland Informatics Campus

High Level Computer Vision

Image Classification, Linear Classifier & Losses @ April 24, 2024

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https://cms.sic.saarland/hlcvss24/

Max Planck Institute for Informatics & Saarland University,
Saarland Informatics Campus Saarbrücken

Overview Today's Lecture

- Image Classification
  - using data-driven approaches (machine learning)
  - K-nearest neighbor classifier
  - linear classification (parametric approach)
  - loss functions and regularization
  - softmax classifier
  - optimization via gradient descent

- What is Deep Learning
  - intuition why deep learning can help
  - integrated learning of features and classifier

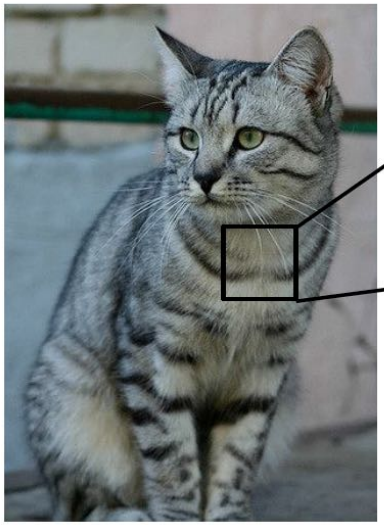
Image Classification: a core task in computer vision

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

Image Classification

The Problem: Semantic Gap




```
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 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
 [114 108  85  55  55  69  64  54  64  87 112 129  98  74  84  91]
 [133 137 147 103  65  81  80  65  52  54  74  84 102  93  85  82]
 [128 137 144 140 109  95  86  70  62  65  63  63  60  73  86 101]
 [125 133 148 137 119 121 117  94  65  79  80  65  54  64  72  90]
 [127 125 131 147 133 127 126 131 111  96  89  75  61  64  72  84]
 [115 114 109 123 150 148 131 118 113 109 100  92  74  65  72  78]
 [ 89  93  90  97 108 147 131 118 113 114 113 109 106  95  77  80]
 [ 63  77  86  81  77  79 102 123 117 115 117 125 125 130 115  87]
 [ 62  65  82  89  78  71  80 101 124 126 119 101 107 114 131 119]
 [ 63  65  75  88  89  71  62  81 120 138 135 105  81  98 110 118]
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 [123 107  96  86  83 112 153 149 122 109 104  75  80 107 112  99]
 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```

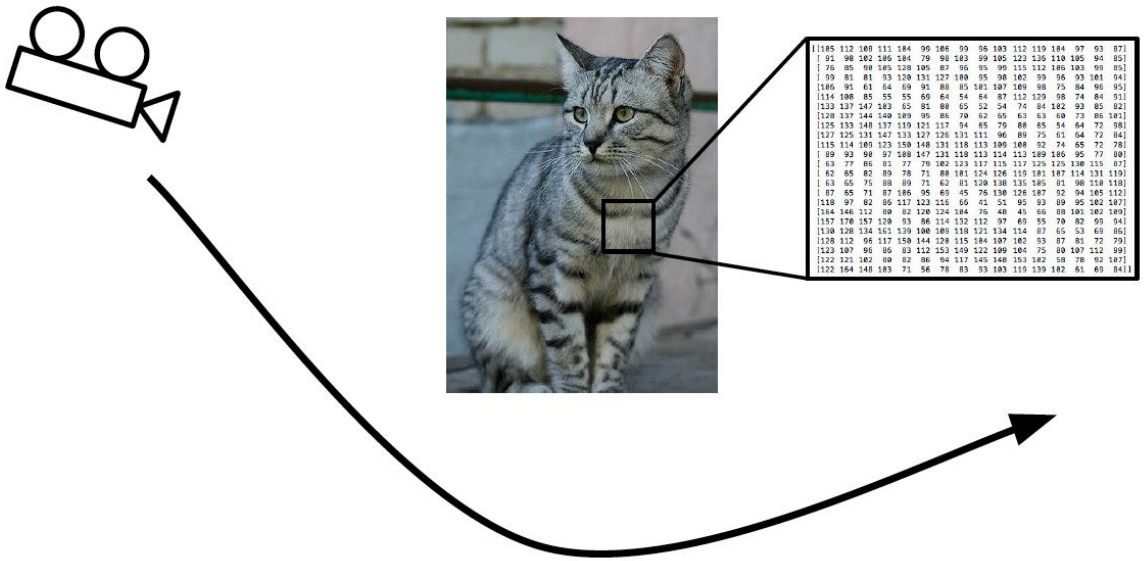
What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Image Classification

Challenges: Viewpoint variation



All pixels change when the camera moves!

Image Classification

Challenges: Illumination

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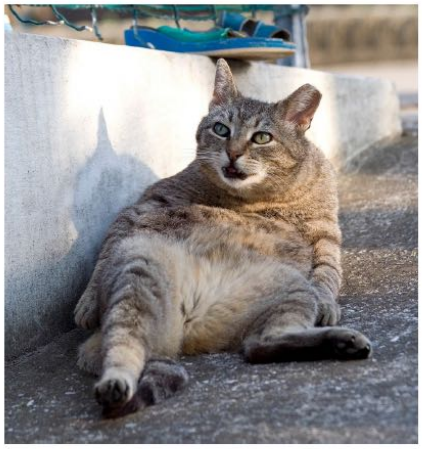
Image Classification

Challenges: Deformation



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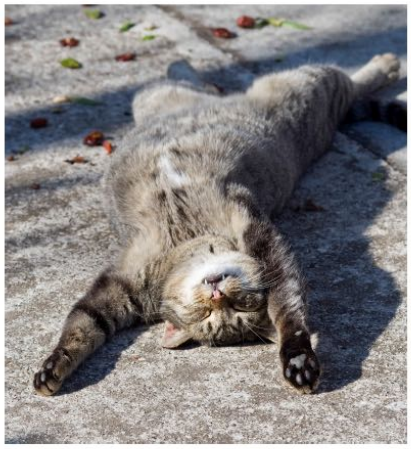

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This image by sare bear is licensed under CC-BY 2.0

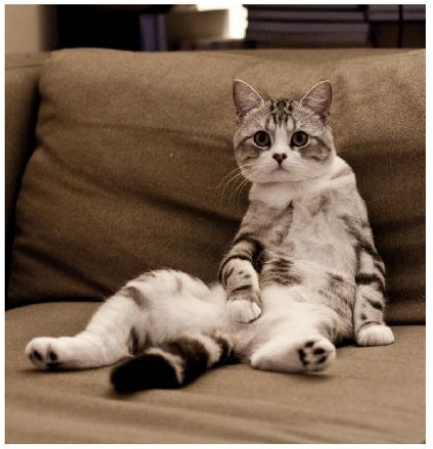

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Image Classification

Challenges: Occlusion

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Image Classification

Challenges: Background Clutter

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Image Classification

Challenges: Intraclass variation

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Image Classification

An image classifier

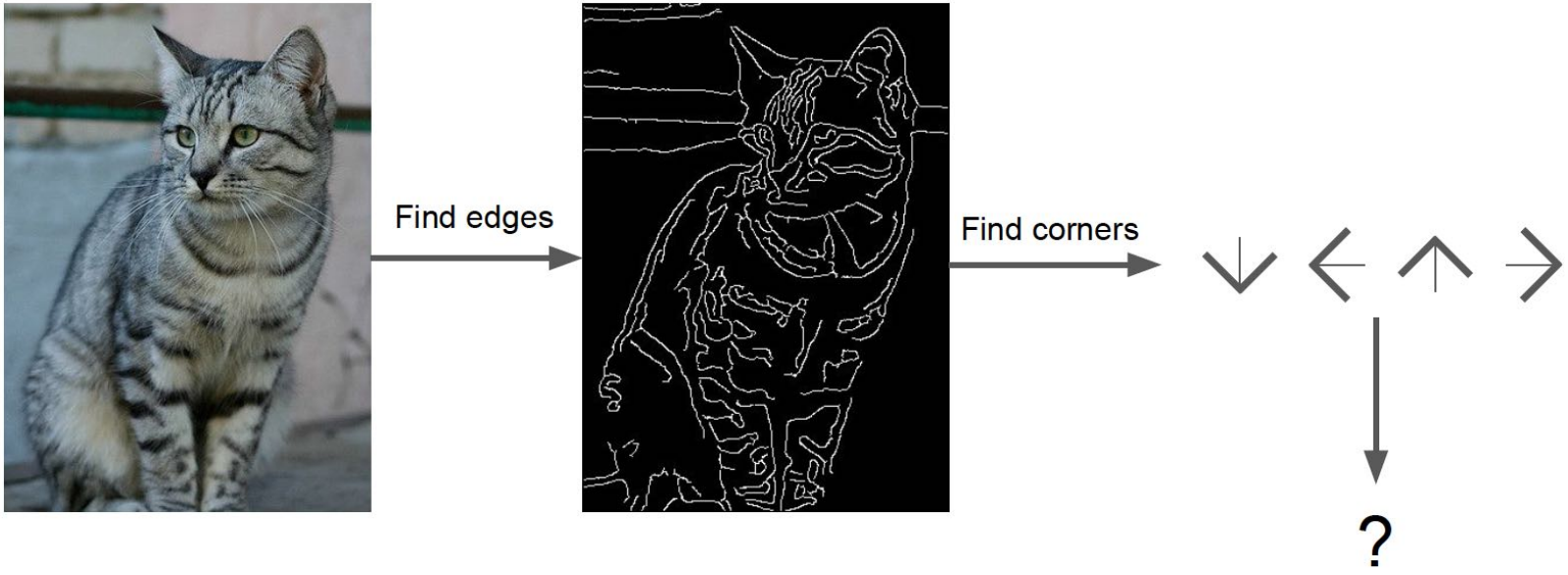
```
def classify_image(image):
  # Some magic here?
  return class_label
```

Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for recognizing a cat, or other classes.

Image Classification

Attempts have been made



John Canny, "A Computational Approach to Edge Detection", IEEE TPAMI 1986

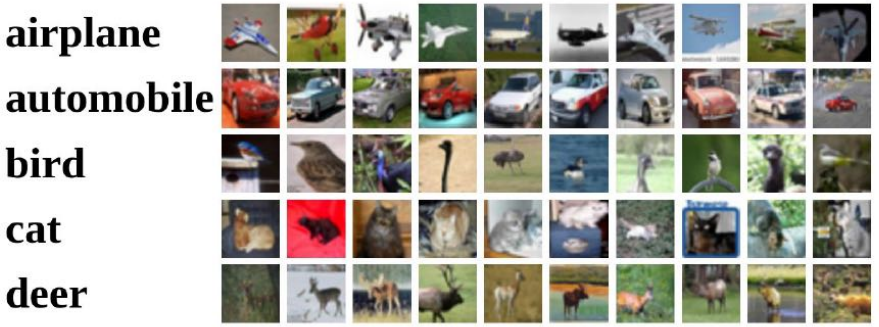
Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning to train a classifier
3. Evaluate the classifier on new images

```
def train(images, labels):
  # Machine learning!
  return model
```

```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

Example training set



First Classifier: Nearest Neighbor

```
def train(images, labels):
  # Machine learning!
  return model
```

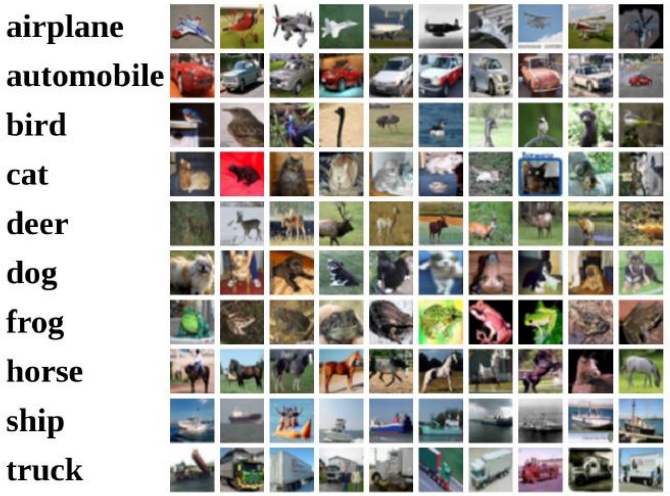
→ Memorize all data and labels

```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

→ Predict the label of the most similar training image

Example Dataset: CIFAR10

10 classes
50,000 training images
10,000 testing images



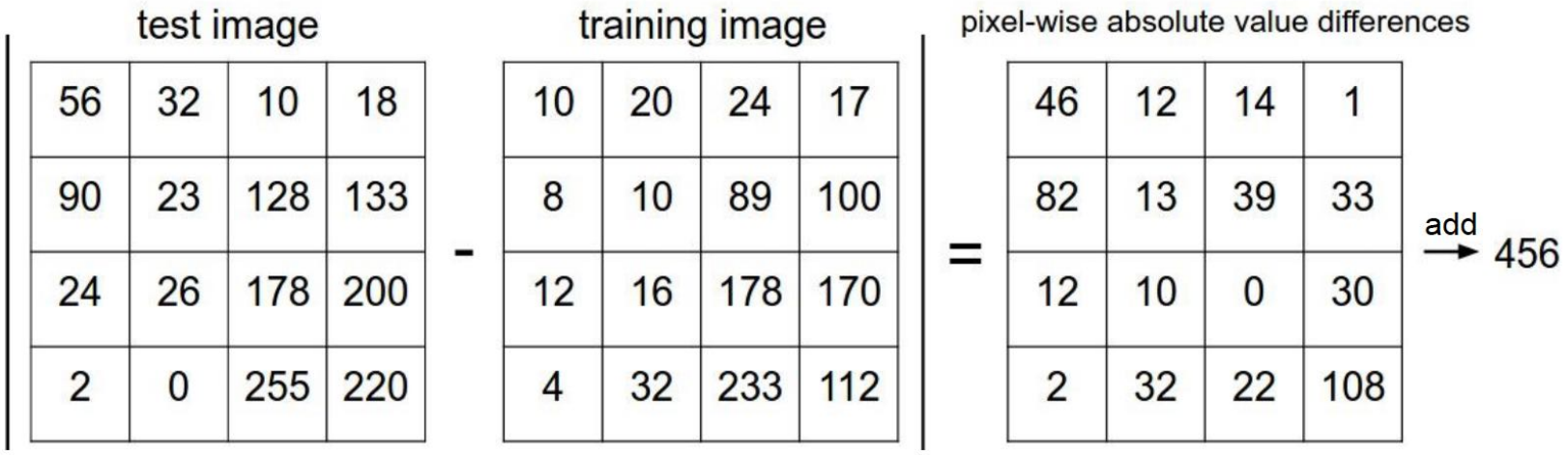
Test images and nearest neighbors

Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

First Classifier: Nearest Neighbor Classifier

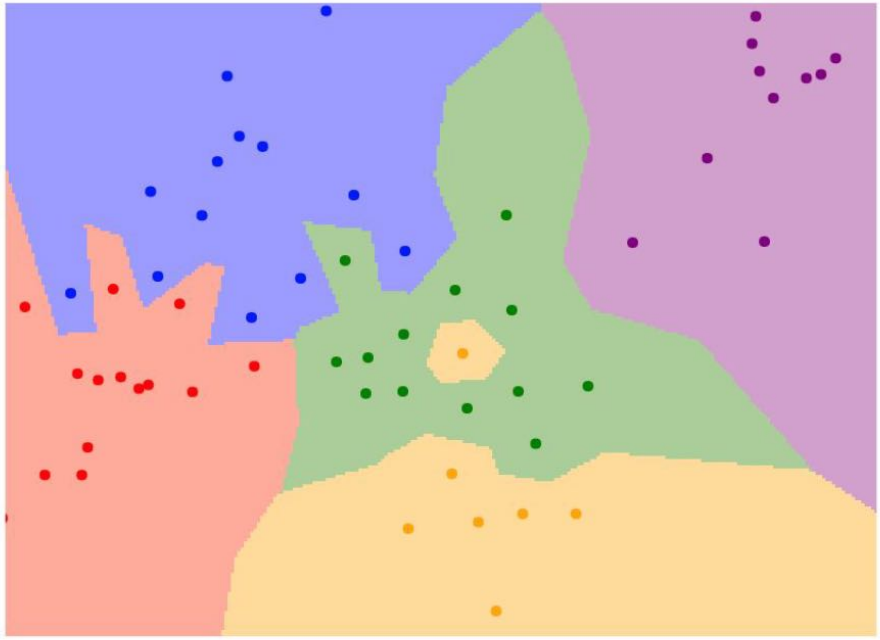
Distance Metric to compare images

L1 distance: $d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$



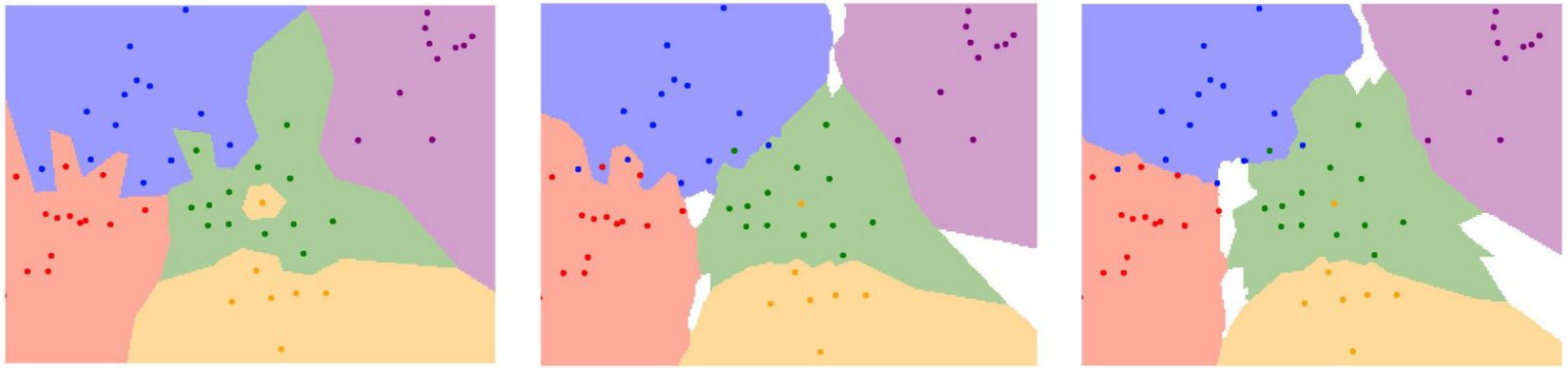
First Classifier: Nearest Neighbor Classifier

What does this look like?



K-Nearest Neighbor Classifier

Instead of copying label from nearest neighbor,
take **majority vote** from K closest points



K = 1 K = 3 K = 5

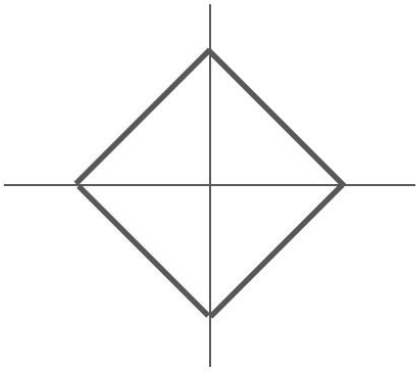
K-Nearest Neighbor Classifier

What does this look like?

K-Nearest Neighbor Classifier: Distance Metric

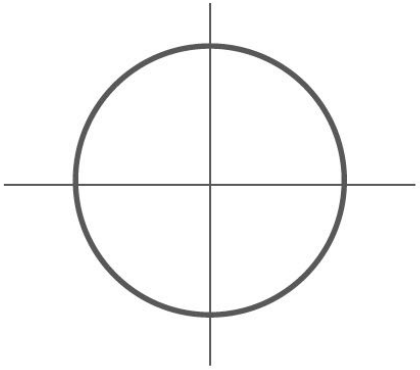
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



L2 (Euclidean) distance

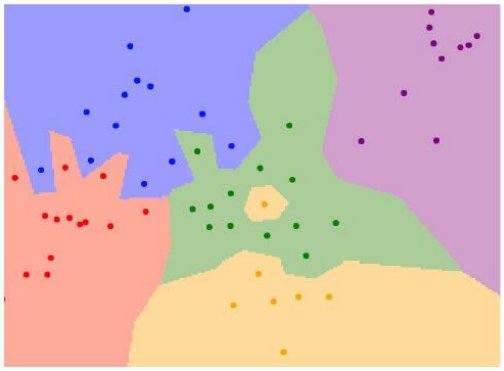
$$d_2(I_1, I_2) = \sqrt{\sum_p \left(I_1^p - I_2^p\right)^2}$$



K-Nearest Neighbor Classifier: Distance Metric

L1 (Manhattan) distance

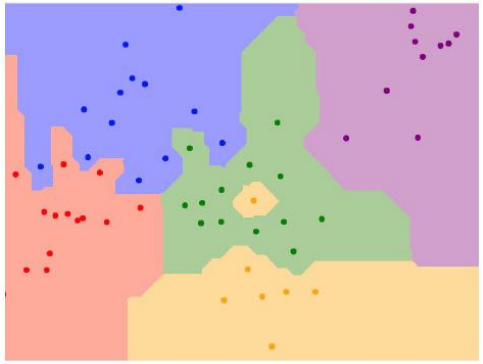
$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



K = 1

L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$



K = 1

Hyperparameters

What is the best value of **k** to use?
What is the best **distance** to use?

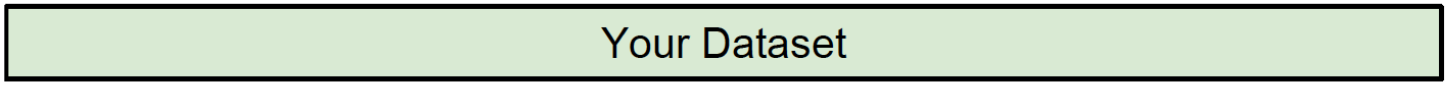
These are **hyperparameters**: choices about the algorithm that we set rather than learn

Very problem-dependent.
Must try them all out and see what works best.

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: K = 1 always works perfectly on training data



Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

BAD: No idea how algorithm will perform on new data

Idea #3: Split data into **train**, **val**, and **test**; choose hyperparameters on val and evaluate on test

Better!

Setting Hyperparameters

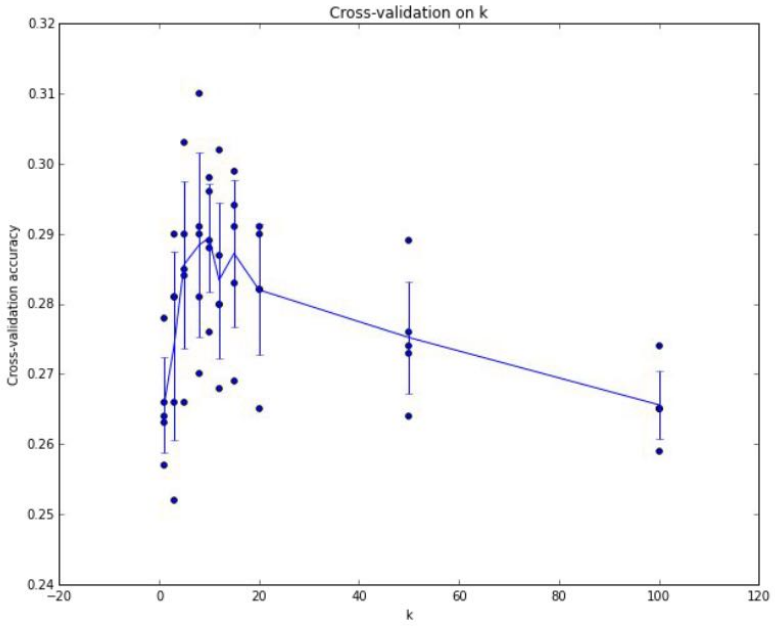
Your Dataset

Idea #4: **Cross-Validation**: Split data into **folds**,
try each fold as validation and average the results

fold 1	fold 2	fold 3	fold 4	fold 5	test
fold 1	fold 2	fold 3	fold 4	fold 5	test
fold 1	fold 2	fold 3	fold 4	fold 5	test

Useful for small datasets, but not used too frequently in deep learning

Setting Hyperparameters



Example of
5-fold cross-validation
for the value of **k.**

Each point: single
outcome.

The line goes
through the mean, bars
indicated standard
deviation

(Seems that k ~= 7 works best
for this data)

Nearest Neighbor - not used for images :-)

- Very slow at test time
- Distance metrics on pixels are not informative

Original image is CC0 public domain

(all 3 images have same L2 distance to the one on the left)

K-Nearest Neighbors: Summary

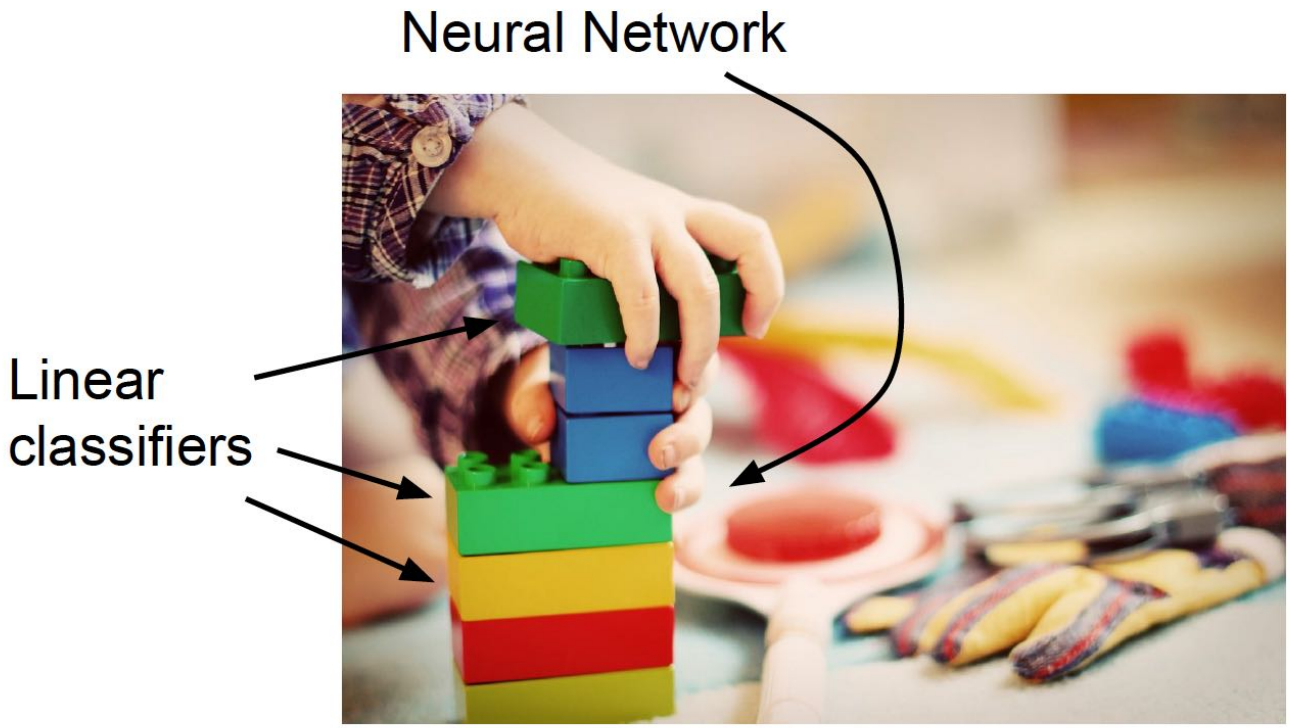
In **Image classification** we start with a **training set** of images and labels, and must predict labels on the **test set**

The **K-Nearest Neighbors** classifier predicts labels based on nearest training examples

Distance metric and K are **hyperparameters**

Choose hyperparameters using the **validation set**; only run on the test set once at the very end!

Linear Classification



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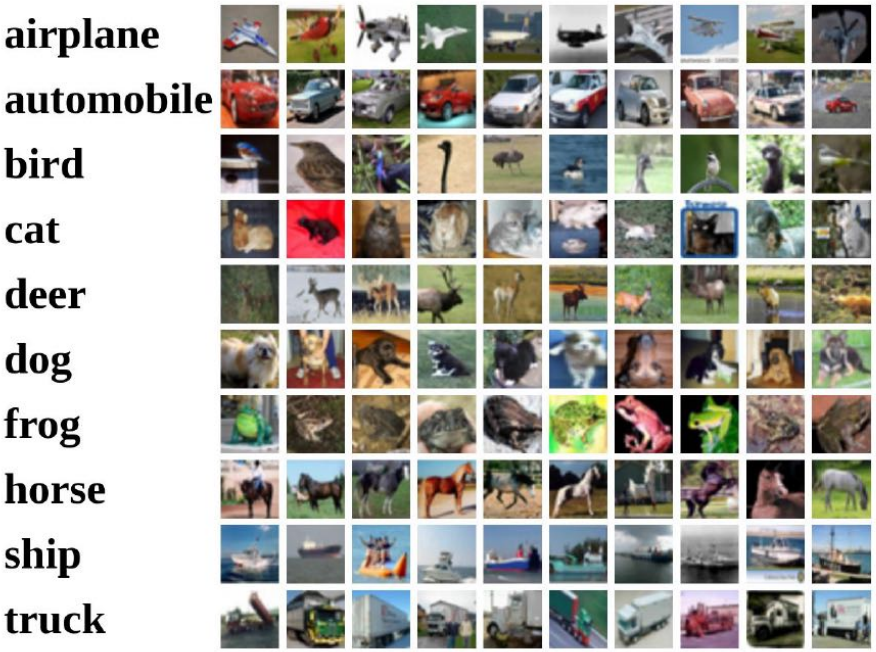
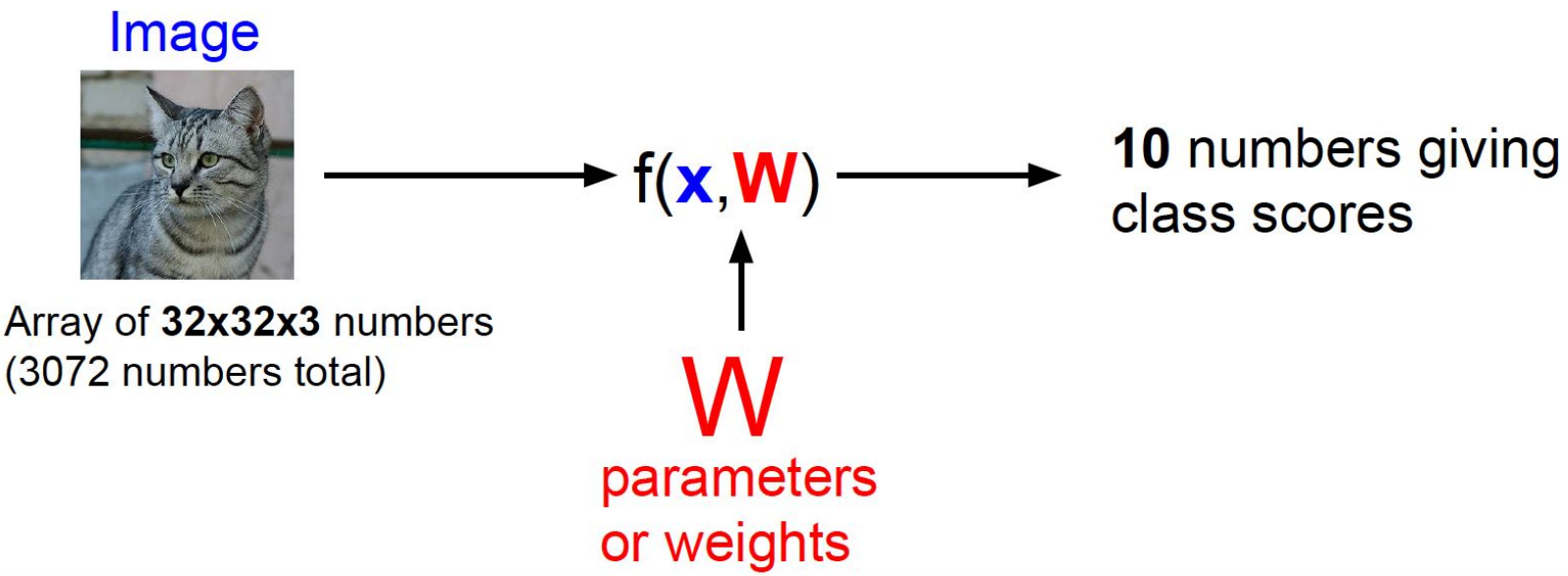
Recall CIFAR10



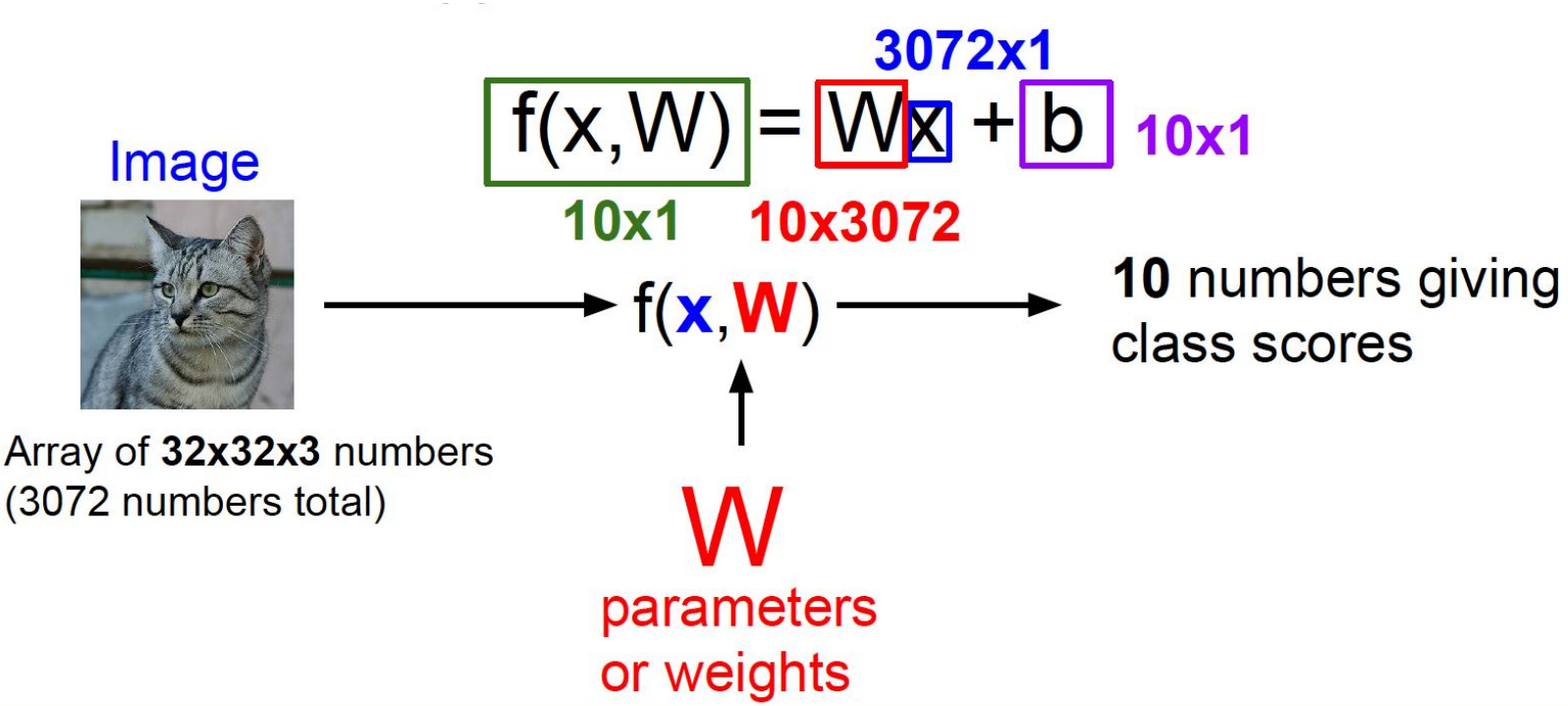
50,000 training images
each image is **32x32x3**

10,000 test images.

Parametric Approach

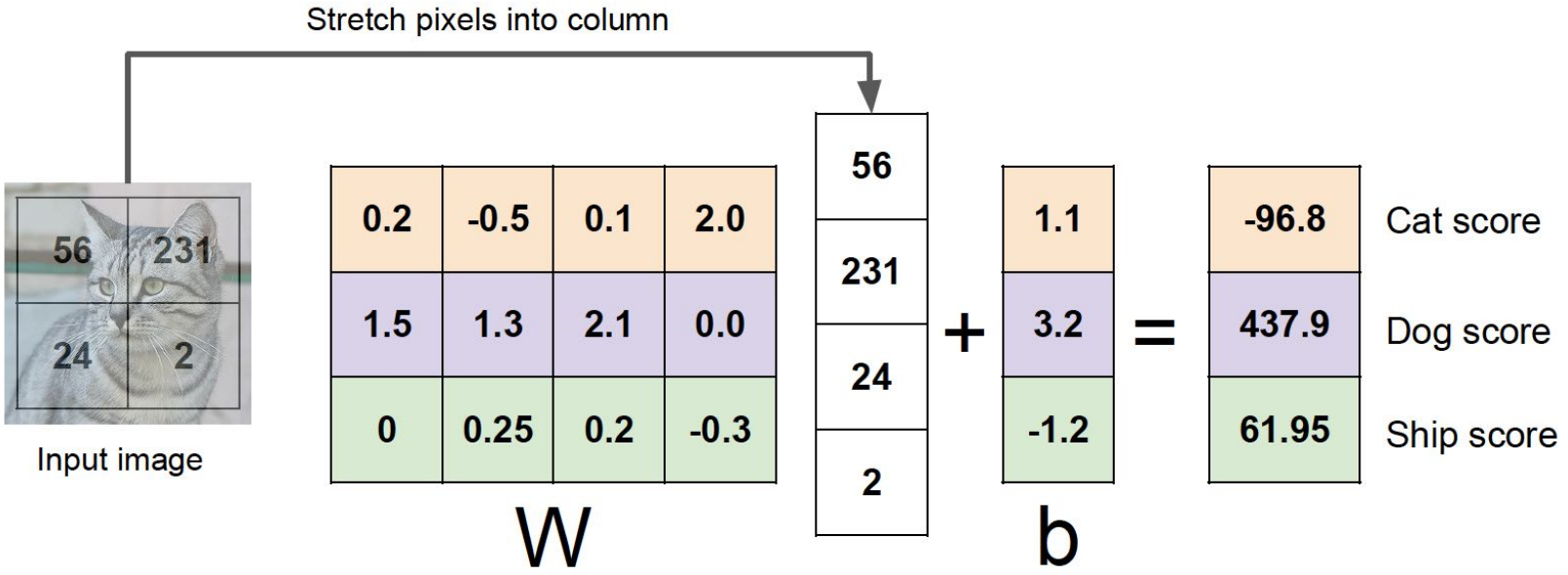


Parametric Approach: Linear Classifier



Parametric Approach: Linear Classifier

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

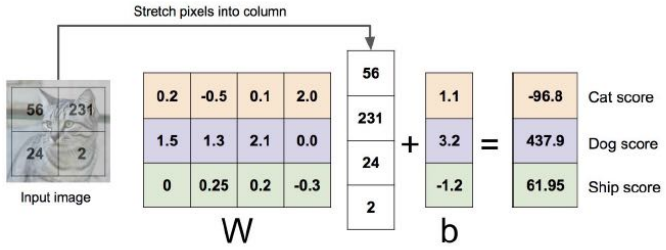


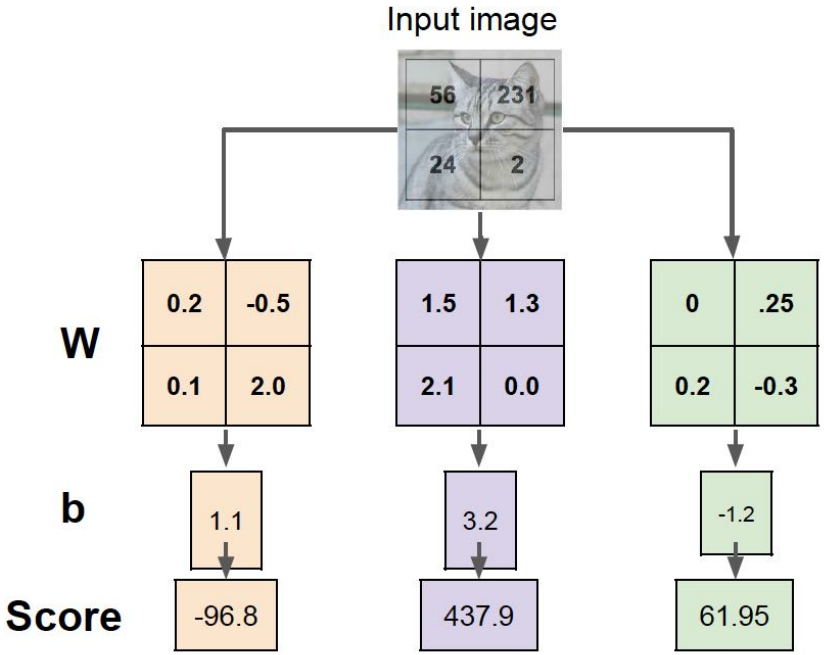
Parametric Approach: Linear Classifier

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

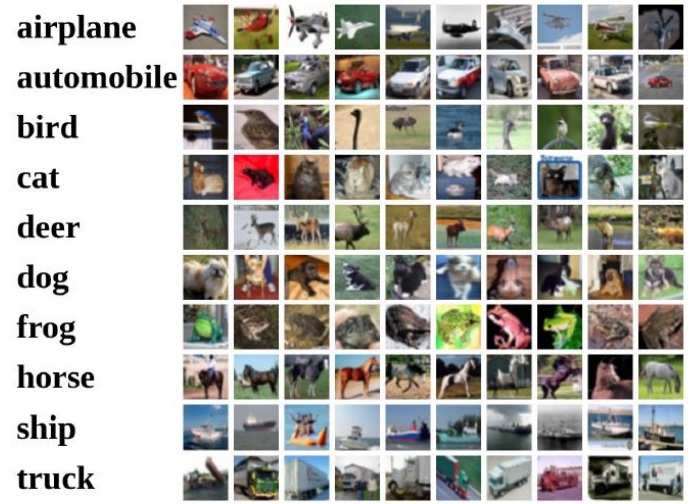
Algebraic Viewpoint

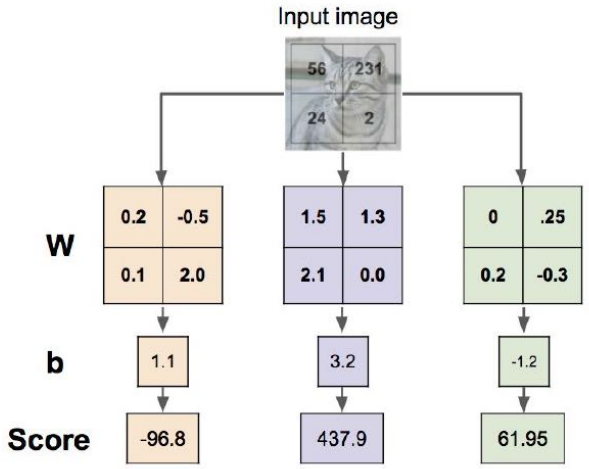
$f(x,W) = Wx$





Interpreting a Linear Classifier: Visual Viewpoint

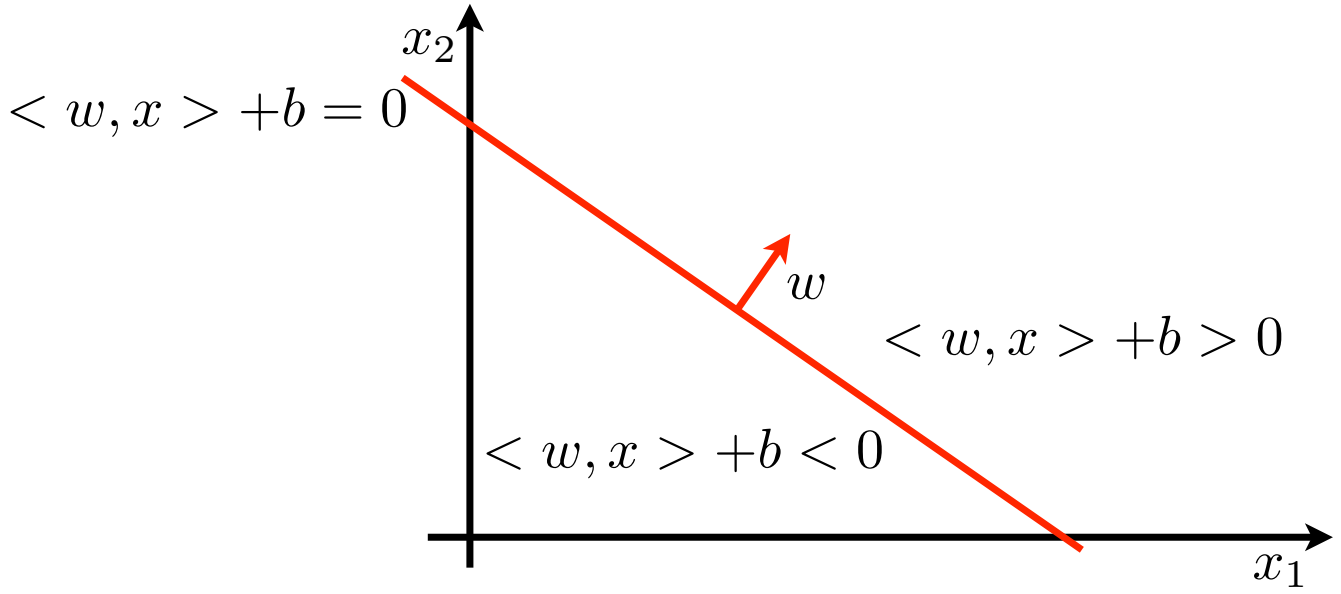




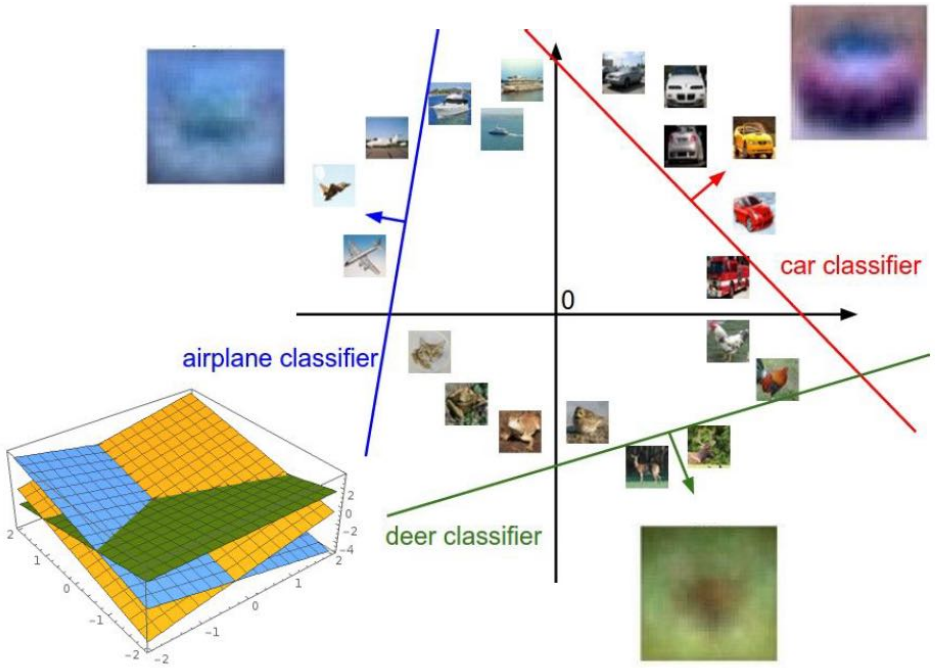
Interpreting a Linear Classifier: Geometric Viewpoint

- 2-class linear classifier:

$$f(x) = \text{sign}(< w, x > +b) = \begin{cases} 1 & \text{if } < w, x > +b > 0 \\ \text{-1} & \text{if } < w, x > +b \leq 0 \end{cases}$$



Interpreting a Linear Classifier: Geometric Viewpoint



Plot created using Wolfram Cloud

$$f(x,W) = Wx + b$$

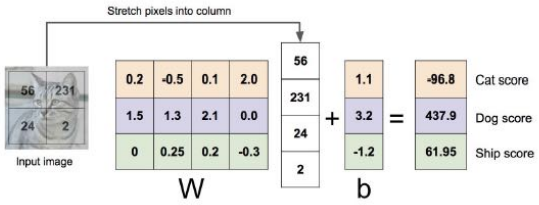
Array of **32x32x3** numbers
(3072 numbers total)

Cat image by Nikita is licensed under CC-BY 2.0

Linear Classifier: Three Viewpoints

Algebraic Viewpoint

$$f(x,W) = Wx$$

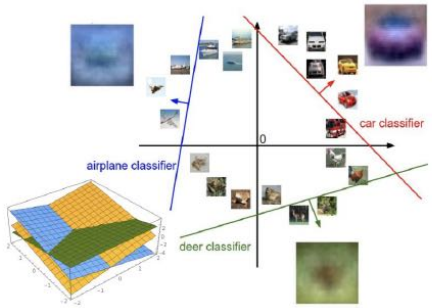


Visual Viewpoint

One template per class

Geometric Viewpoint

Hyperplanes cutting up space



Linear Classifier…

So far: Defined a (linear) score function $f(x,W) = Wx + b$

Example class scores for 3 images for some W:

How can we tell whether this W is good or bad?



airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

Linear Classifier…

airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.

2. Come up with a way of efficiently finding the parameters that minimize the loss function. **(optimization)**

Example

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

A **loss function** tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and
y_i is (integer) label

Loss over the dataset is a sum of loss over examples:

$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

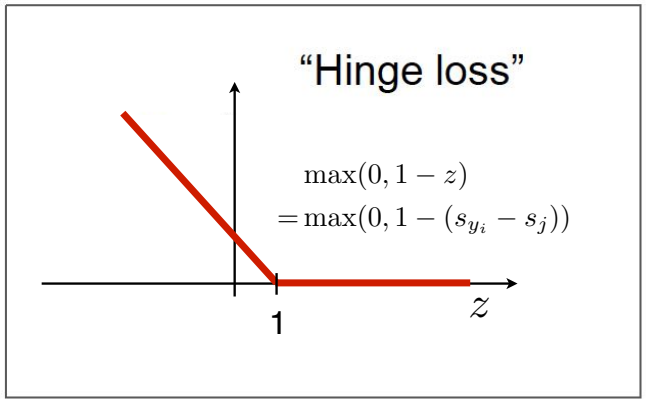
Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Multiclass SVM loss:



$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9		

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 6.3) + max(0, 6.6)
= 6.3 + 6.6
= 12.9

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Loss over full dataset is average:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 12.9)/3
= **5.27**

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: What happens to loss if car scores change a bit?

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q2: what is the min/max possible loss?

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q3: At initialization W is small so all s ≈ 0. What is the loss?

Multiclass Support Vector Machine (SVM) Loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q4: What if the sum was over all classes? (including j = y_i)

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

E.g. Suppose that we found a W such that L = 0.
Is this W unique?

No! 2W is also has L = 0!

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Before:

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

With W twice as large:

= max(0, 2.6 - 9.8 + 1)
+max(0, 4.0 - 9.8 + 1)
= max(0, -6.2) + max(0, -4.8)
= 0 + 0
= 0

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

E.g. Suppose that we found a W such that L = 0.
Is this W unique?

No! 2W is also has L = 0!
How do we choose between W and 2W?

Regularization

λ = regularization strength (hyperparameter)

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)}_{\text{Data loss}} + \underbrace{\lambda R(W)}_{\text{Regularization}}$$

Data loss: Model predictions should match training data

Regularization: Prevent the model from doing *too* well on training data

Simple examples

L2 regularization: $R(W) = \sum_k \sum_l W_{k,l}^2$

L1 regularization: $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2): $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$

More complex:

Dropout

Batch normalization

Stochastic depth, fractional pooling, etc

Softmax Classifier

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

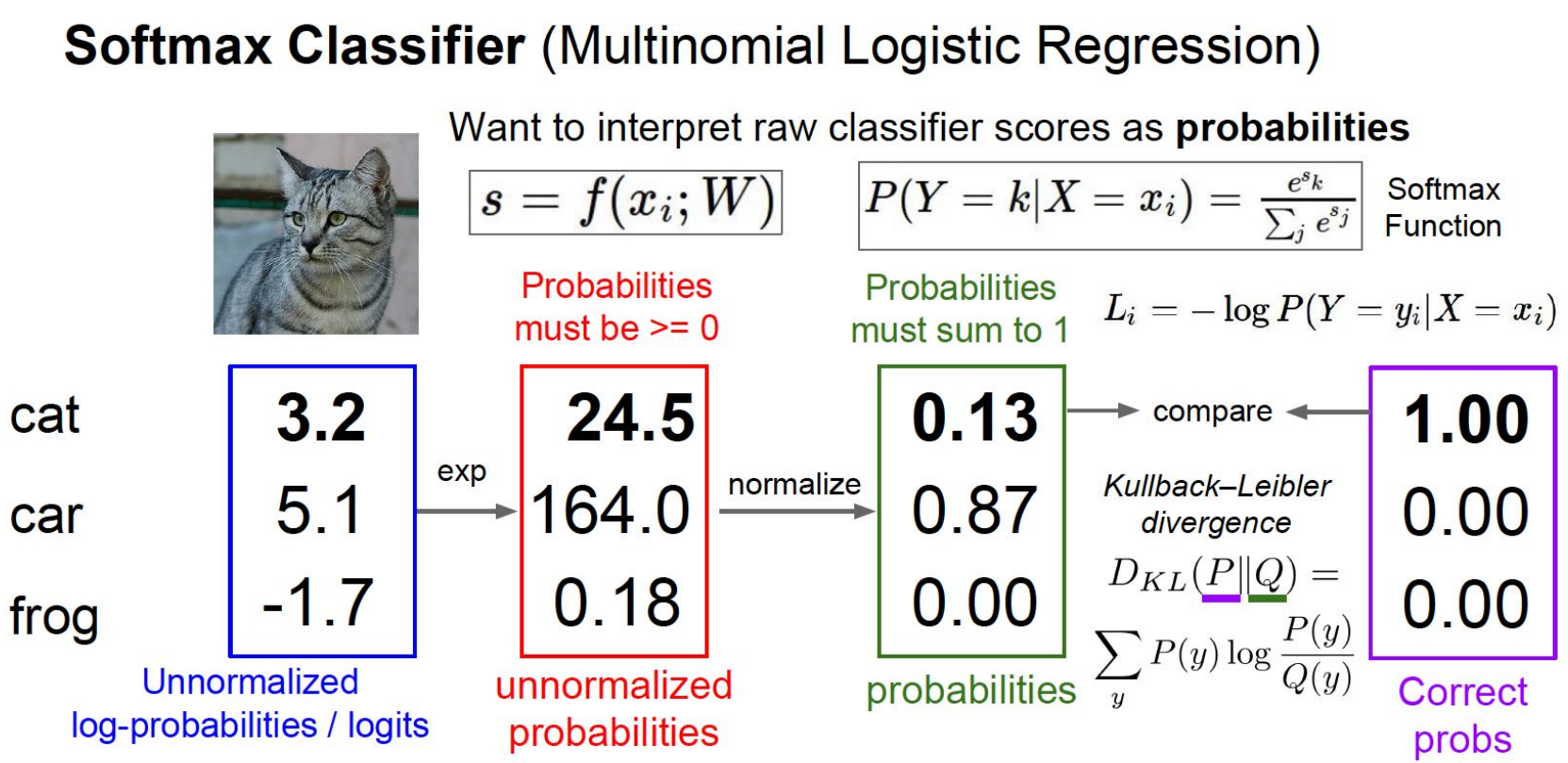
$$L_i = -\log P(Y = y_i|X = x_i)$$

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities
cat	3.2		24.5		0.13
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

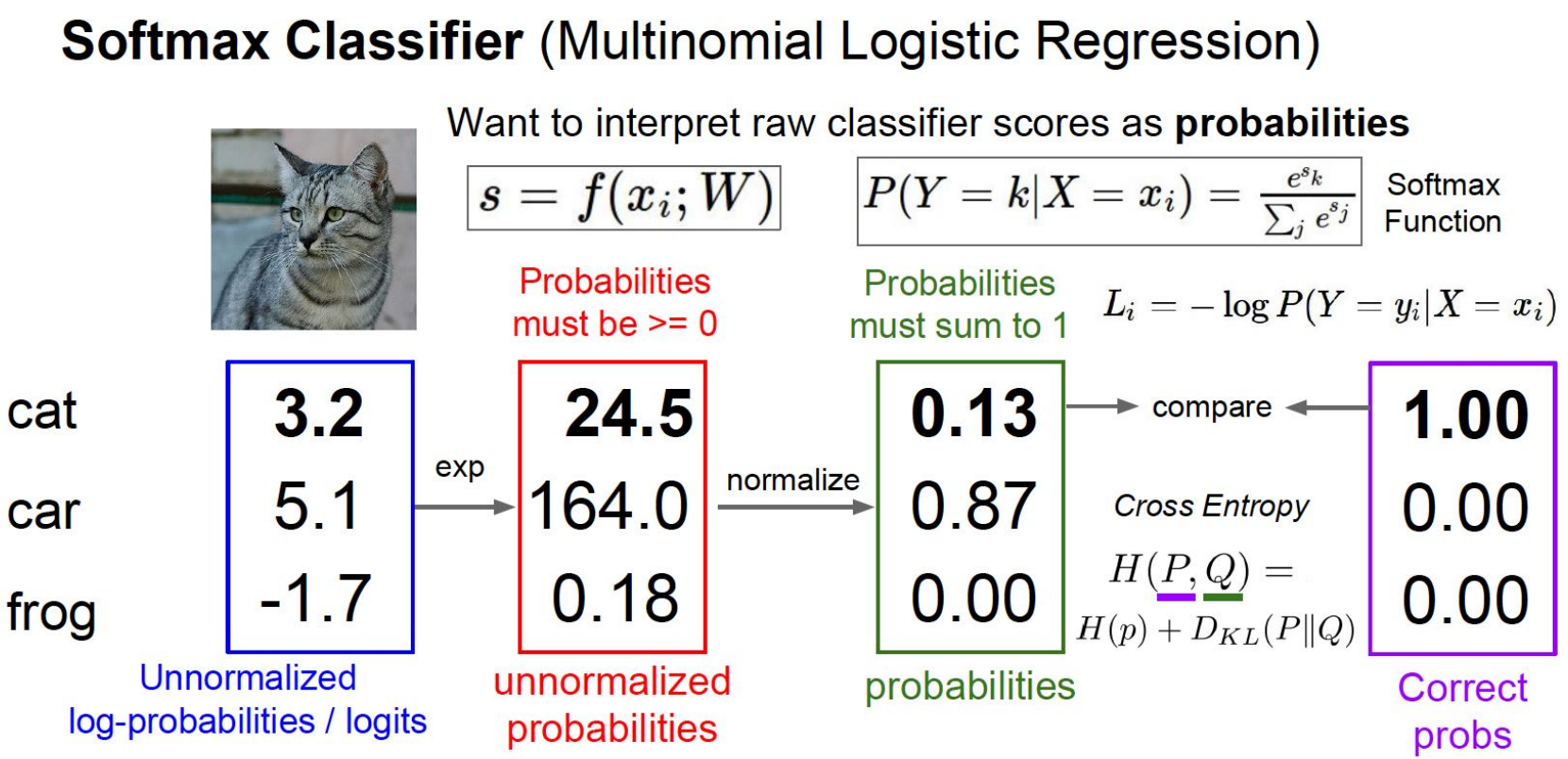
→ L_i = -log(0.13) = **2.04**

Maximum Likelihood Estimation
Choose probabilities to maximize the likelihood of the observed data

Cross Entropy Loss — Two Components



Cross Entropy Loss — Two Components



Cross Entropy Loss: Let's double check…

- Entropy & KL-divergence:

$$H(P^t) = -\sum_y P^t(y) \log P^t(y)$$

$$D_{KL}(P^t||Q) = \sum_y P^t(y) \log \frac{P^t(y)}{Q(y)}$$

- Cross Entropy the sum of both:

$$\begin{aligned} H(P^t, Q) &= H(P^t) + D_{KL}(P^t||Q) \\ &= \sum_y P^t(y) \left(\log \frac{P^t(y)}{Q(y)} - \log P^t(y) \right) \\ &= -\sum_y P^t(y) \log Q(y) \end{aligned}$$

Let's double check...

- Cross Entropy in our classification case:
  - Target "distribution" / output:

$$P^t(y) = \begin{cases} 1 & y = y_i \\ 0 & y \neq y_i \end{cases}$$

  - Output of the network:

$$Q(y|x_i) = P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

- Then Cross Entropy Loss for image x_i

$$\begin{aligned} L_i = L(x_i) &= -\sum_y P^t(y) \log Q(y|x_i) \\ &= -1 \cdot \log Q(y_i|x_i) \\ &= -\log P(Y = y_i|X = x_i) \quad = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right) \end{aligned}$$

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

$$L_i = -\log P(Y = y_i|X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

cat	**3.2**
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Softmax Function

Maximize probability of correct class

$$L_i = -\log P(Y = y_i|X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

Q: What is the min/max possible loss L_i?
A: min 0, max infinity

Softmax Classifier (Multinomial Logistic Regression)

cat **3.2**

car 5.1

frog -1.7

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

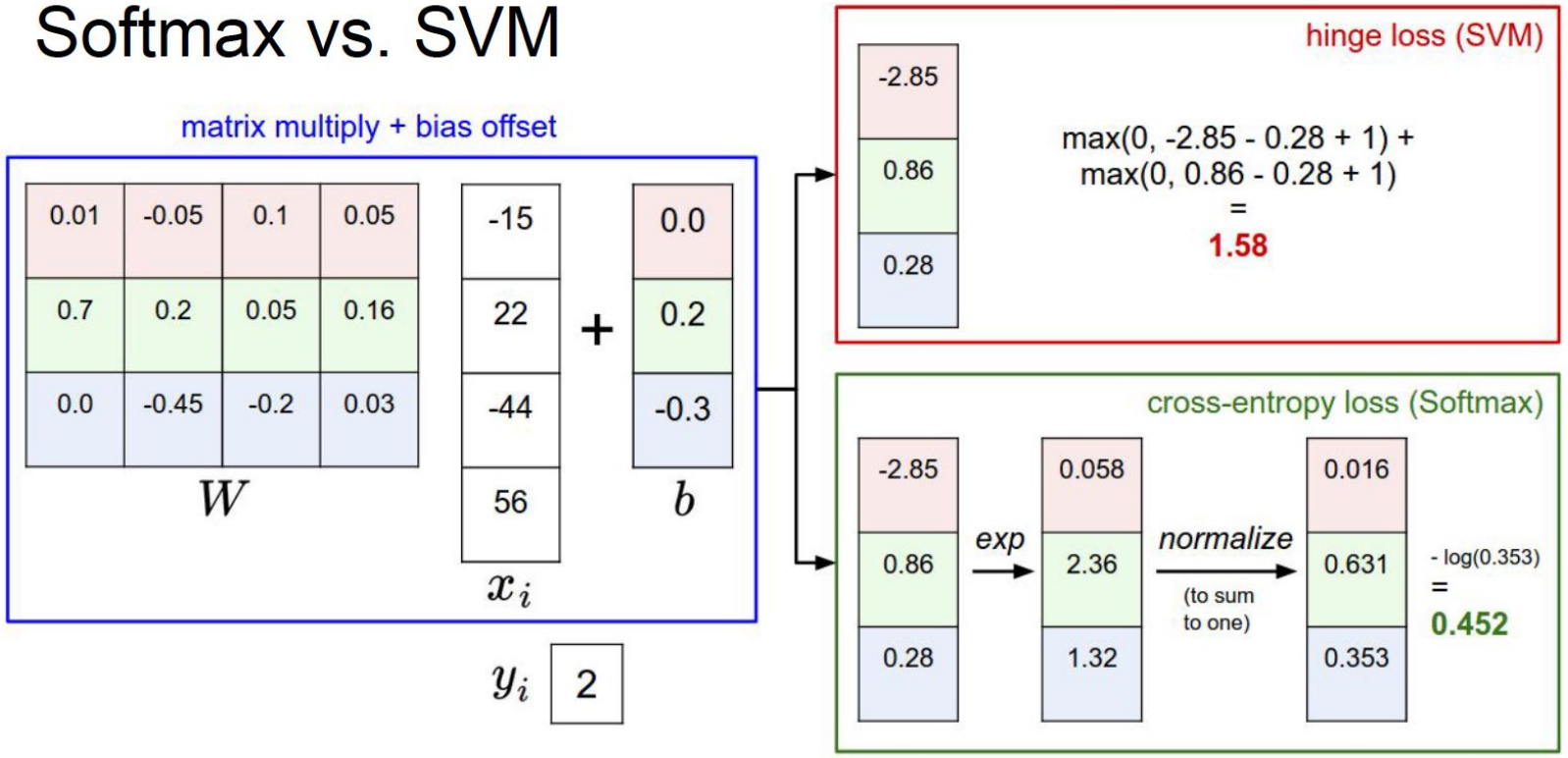
$$L_i = -\log P(Y = y_i|X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

Q2: At initialization all s will be approximately equal; what is the loss?
A: log(C), eg log(10) ≈ 2.3

Softmax vs. SVM



Recap… (as computational graph)

How do we find the best W?

- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

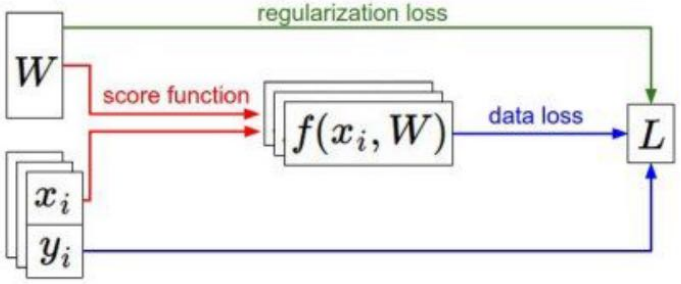
Softmax

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

SVM

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$ Full loss



Optimization — Strategies

- #1 Random Search
  - can work…
  - but very expensive
- #2 “Follow the Slope”
  - aka: gradient descent…

Overview Today's Lecture

- Image Classification
  - using data-driven approaches (machine learning)
  - K-nearest neighbor classifier
  - linear classification (parametric approach)
  - loss functions and regularization
  - softmax classifier
  - optimization via gradient descent

- **What is Deep Learning**
  - intuition why deep learning can help
  - integrated learning of features and classifier

Simple Neural Networks

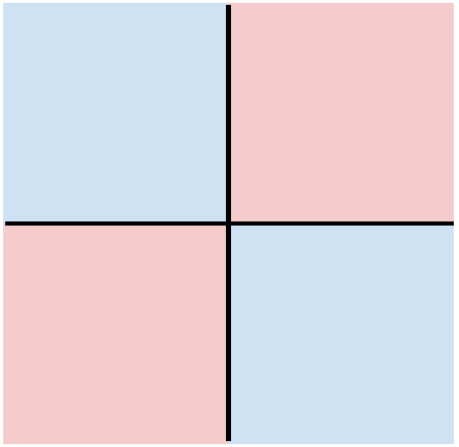
(**Before**) Linear score function: $f = Wx$

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Hard Cases for a Linear Classifier

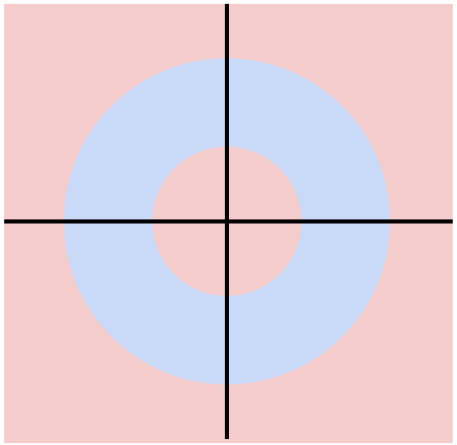
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



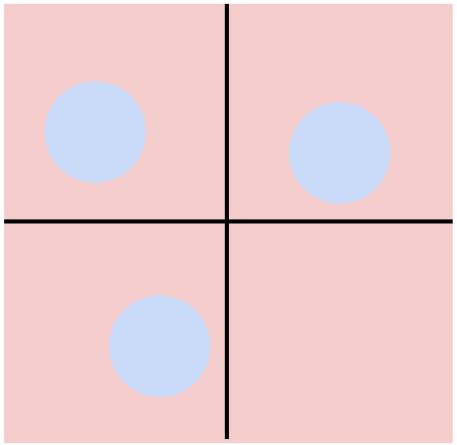
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



Class 1:
Three modes

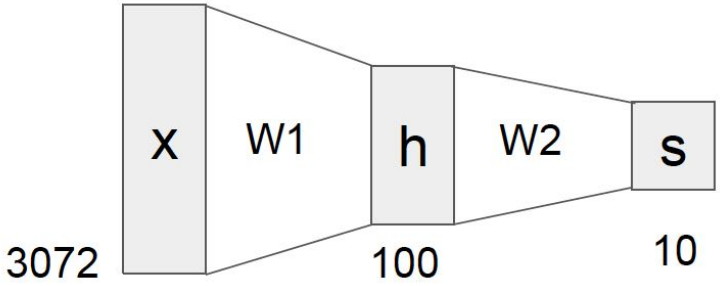
Class 2:
Everything else



Simple Neural Networks

(**Before**) Linear score function: $f = Wx$

(**Now**) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$



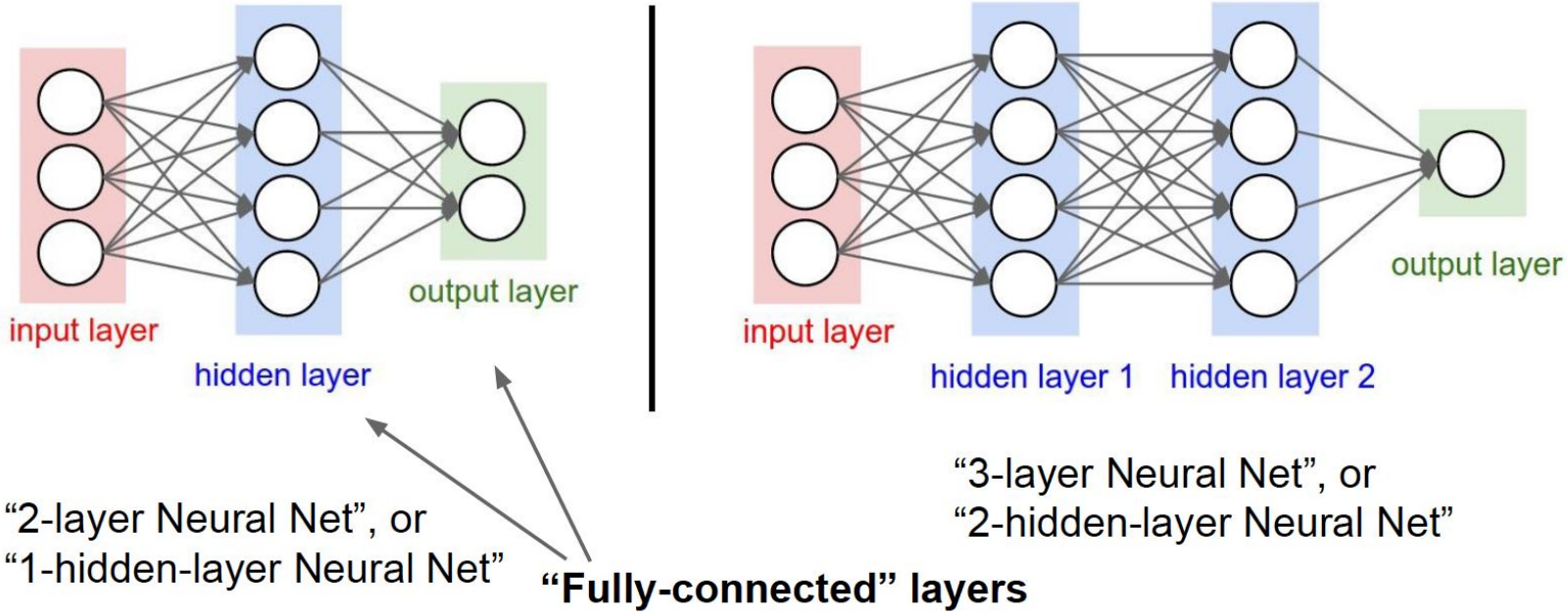
or 3-layer Neural Network

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Neural Networks - Classic Architectures

- (Multi-Layer) Perceptron (also called Fully Connected Network/Layers)



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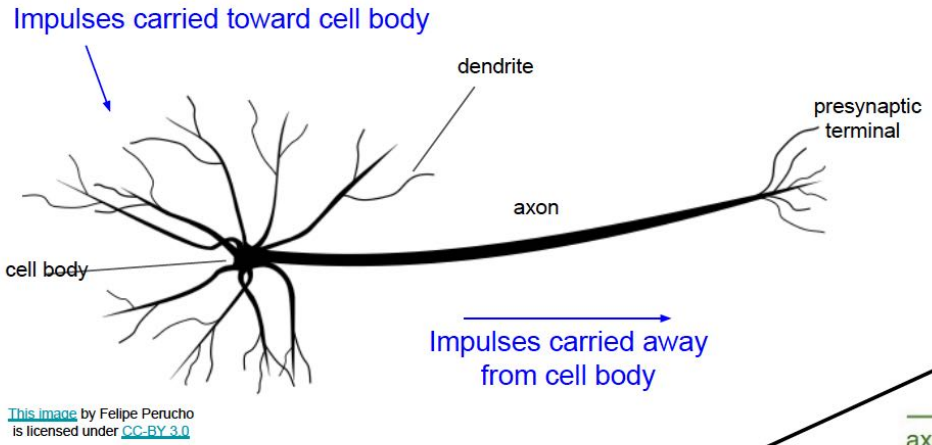
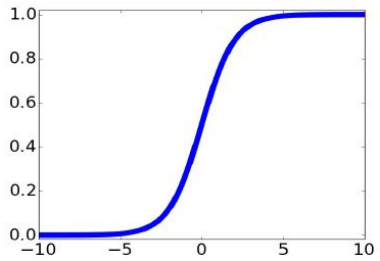
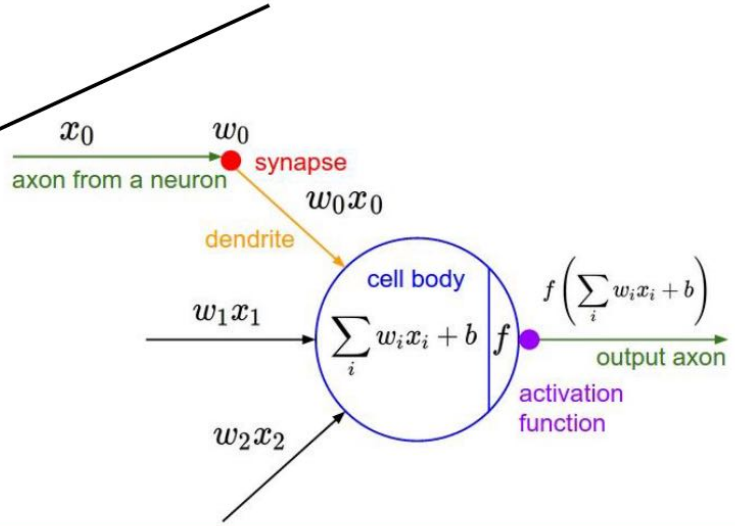
Some Inspiration from the (Human) Brain





sigmoid activation function

$$\frac{1}{1+e^{-x}}$$



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Some Inspiration from the (Human) Brain

Be very careful with your brain analogies!

Biological Neurons:

- Many different types
- Dendrites can perform complex non-linear computations
- Synapses are not a single weight but a complex non-linear dynamical system
- Rate code may not be adequate

[Dendritic Computation. London and Hausser]

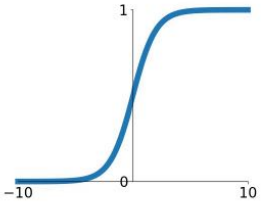
slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Activation Functions

Activation functions

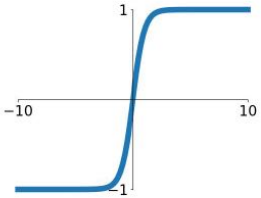
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



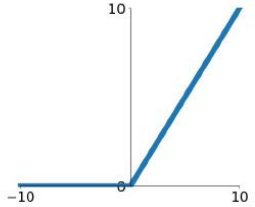
tanh

$$\tanh(x)$$



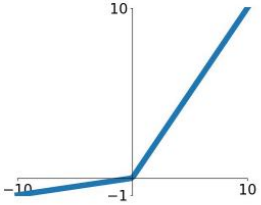
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

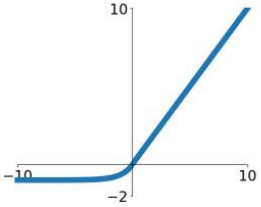


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

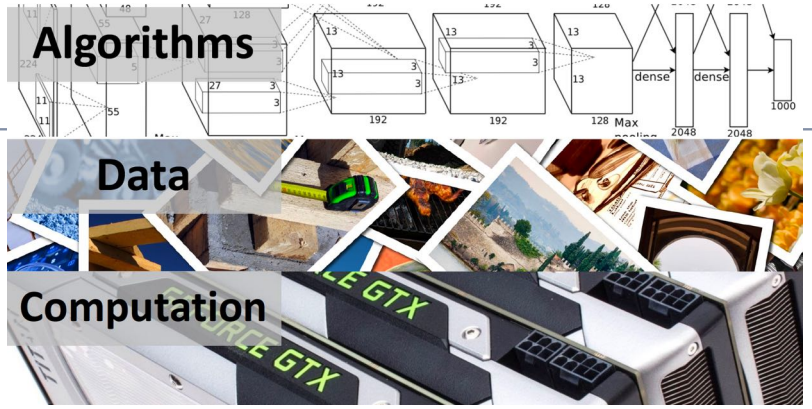
Summary

- Neural Networks

  - We arrange neurons into fully-connected layers
  - The abstraction of a **layer** has the nice property that it allows us to use efficient vectorized code (e.g. matrix multiplies)
  - Neural networks are not really *neural*

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

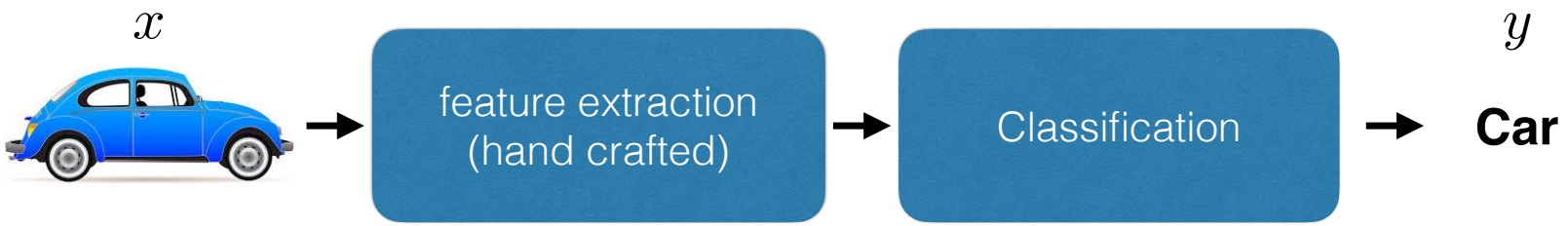
Deep Learning Ingredients



- Deep Learning is based on
  - Availability of large datasets
  - Massive parallel compute power
  - Advances in machine learning over the years

- Strong improvements due to
  - Internet (availability of large-scale data)
  - GPUs (availability of parallel compute power)
  - Deep / hierarchical models with end-to-end learning

Traditional Approach



- Feature extraction
  - often hand crafted and fixed
  - might be too general (not task-specific enough)
  - might be too specific (does not generalize to other tasks)
- How to achieve best classification performance
  - more complex classifier (e.g. multi-feature, non-linear)?
  - how specialized for the task?

Features Alone Can Explain (Almost) 10 Years of Progress

[Benenson,Omran,Hosang,Schiele@ECCV workshop'14]

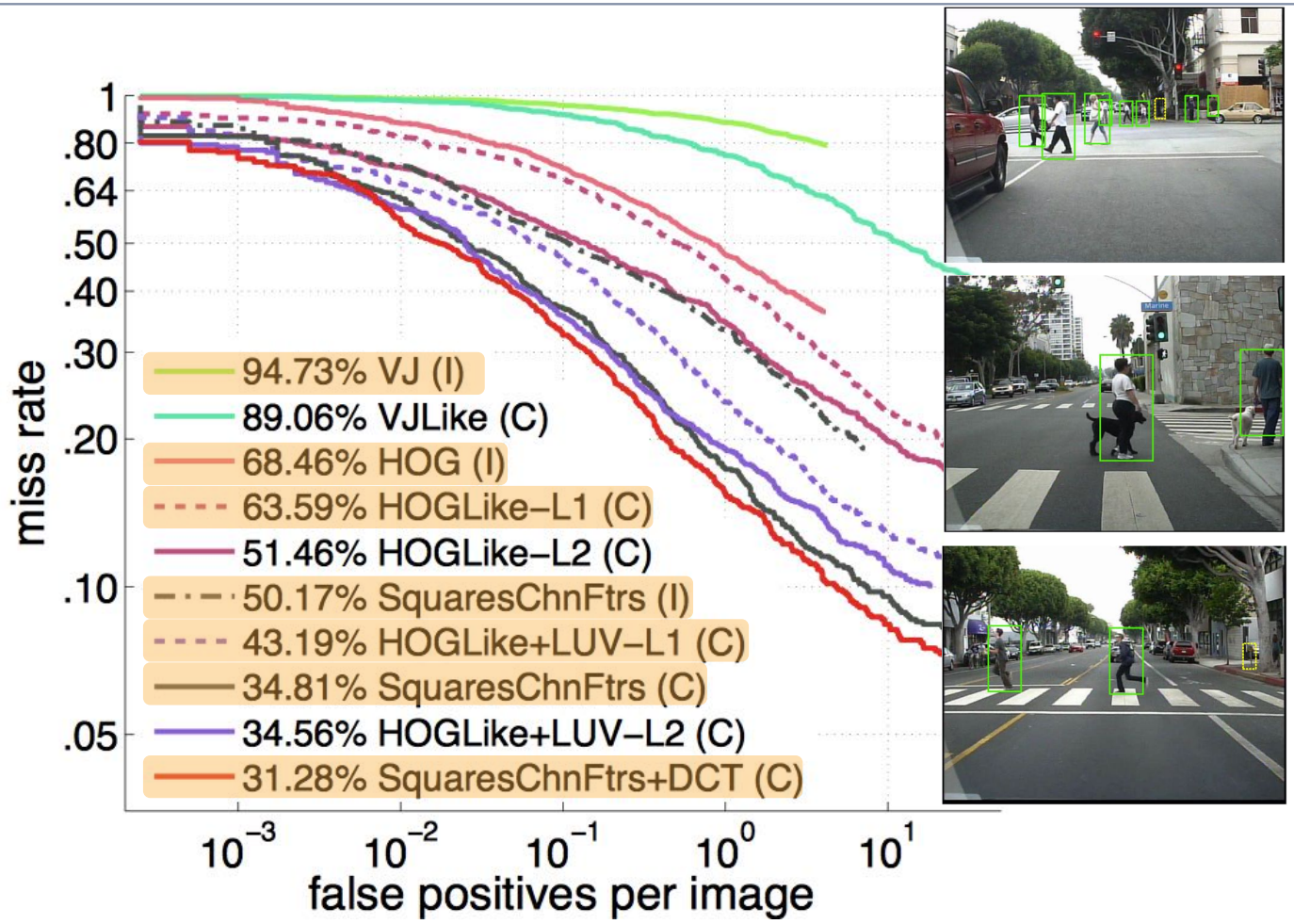
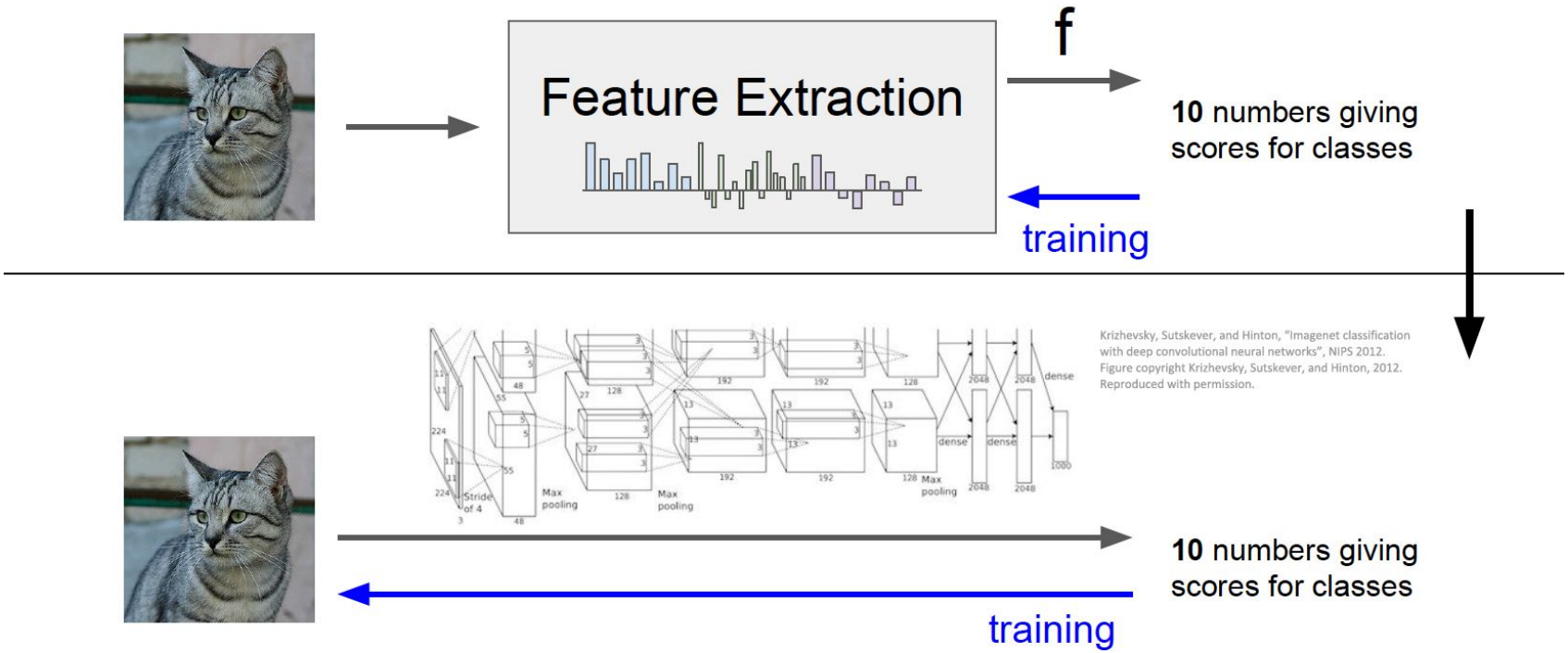
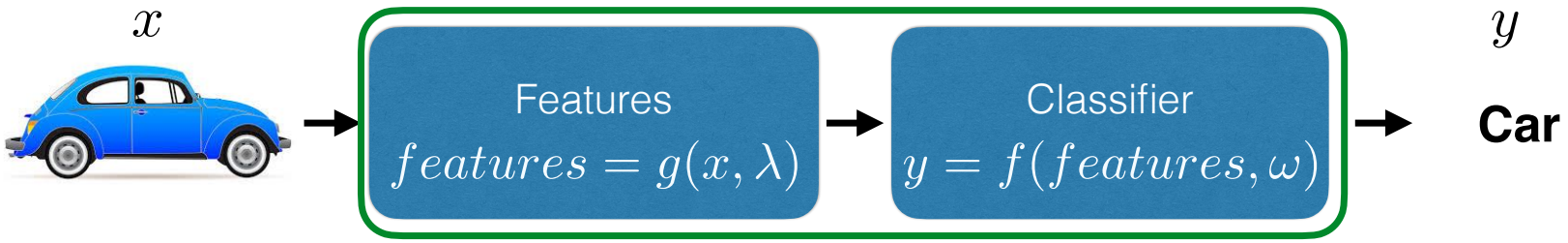


Image Features vs. Deep Learning



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

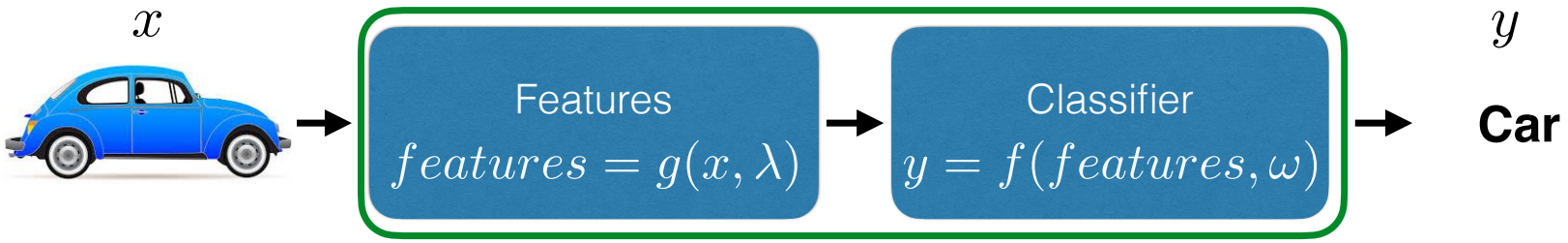
Deep Learning: Trainable Features



"End-to-End" System

- Parameterized feature extraction
- Features should be
  - efficient to compute
  - efficient to train (differentiable)
- **Joint** training of **feature** extraction and **classification**
  - Feature extraction and classification merge into one pipeline

Deep Learning: Joint Training of all Parameters

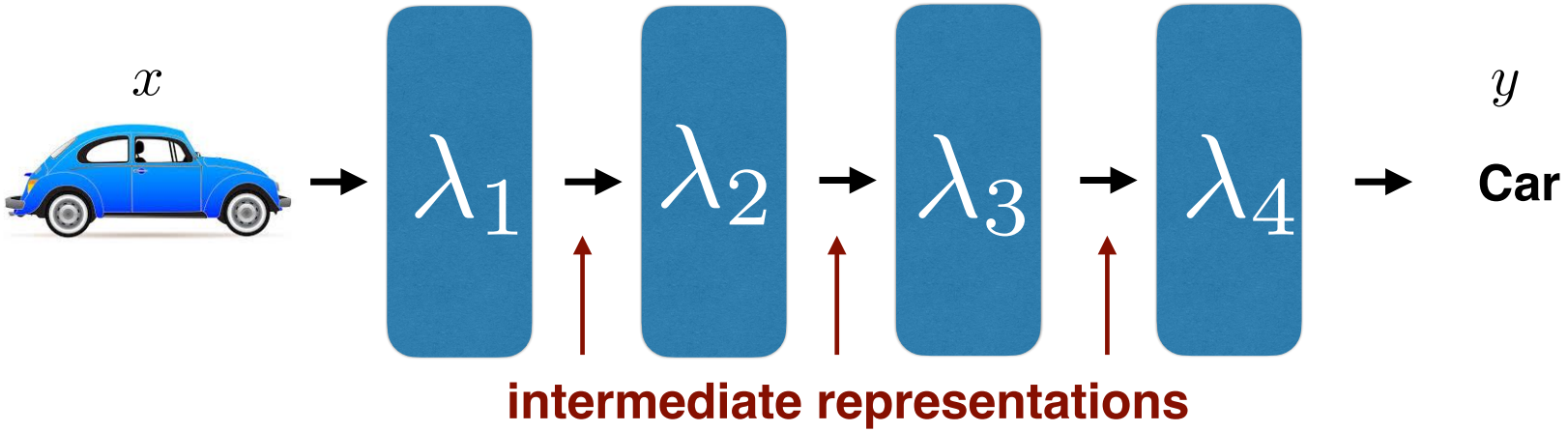


"End-to-End" System

- All parts are adaptive
  - No differentiation between feature extraction and classification
  - Non linear transformation from input to desired output

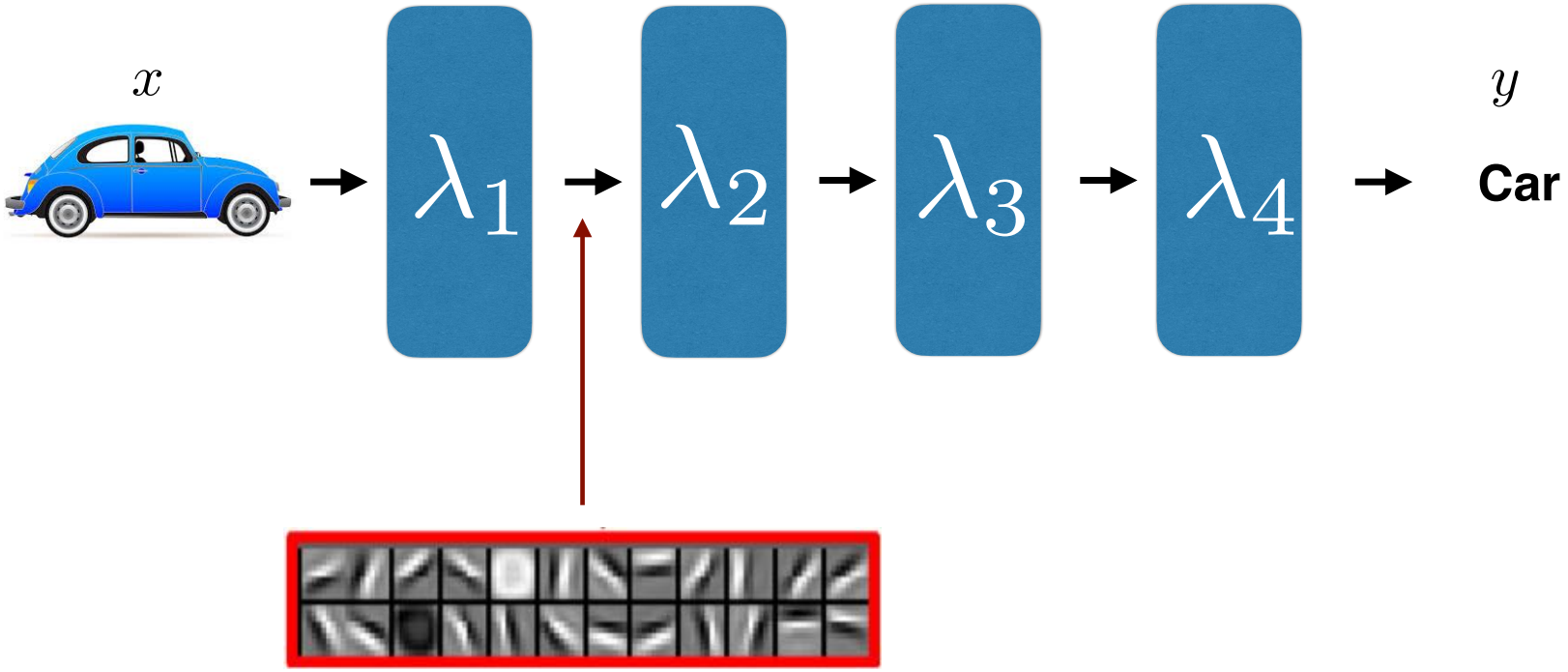
- How can we build such systems?
  - Composition of simple building blocks can lead to complex systems (e.g. neurons - brain)

Deep Learning: Complex Functions by Composition



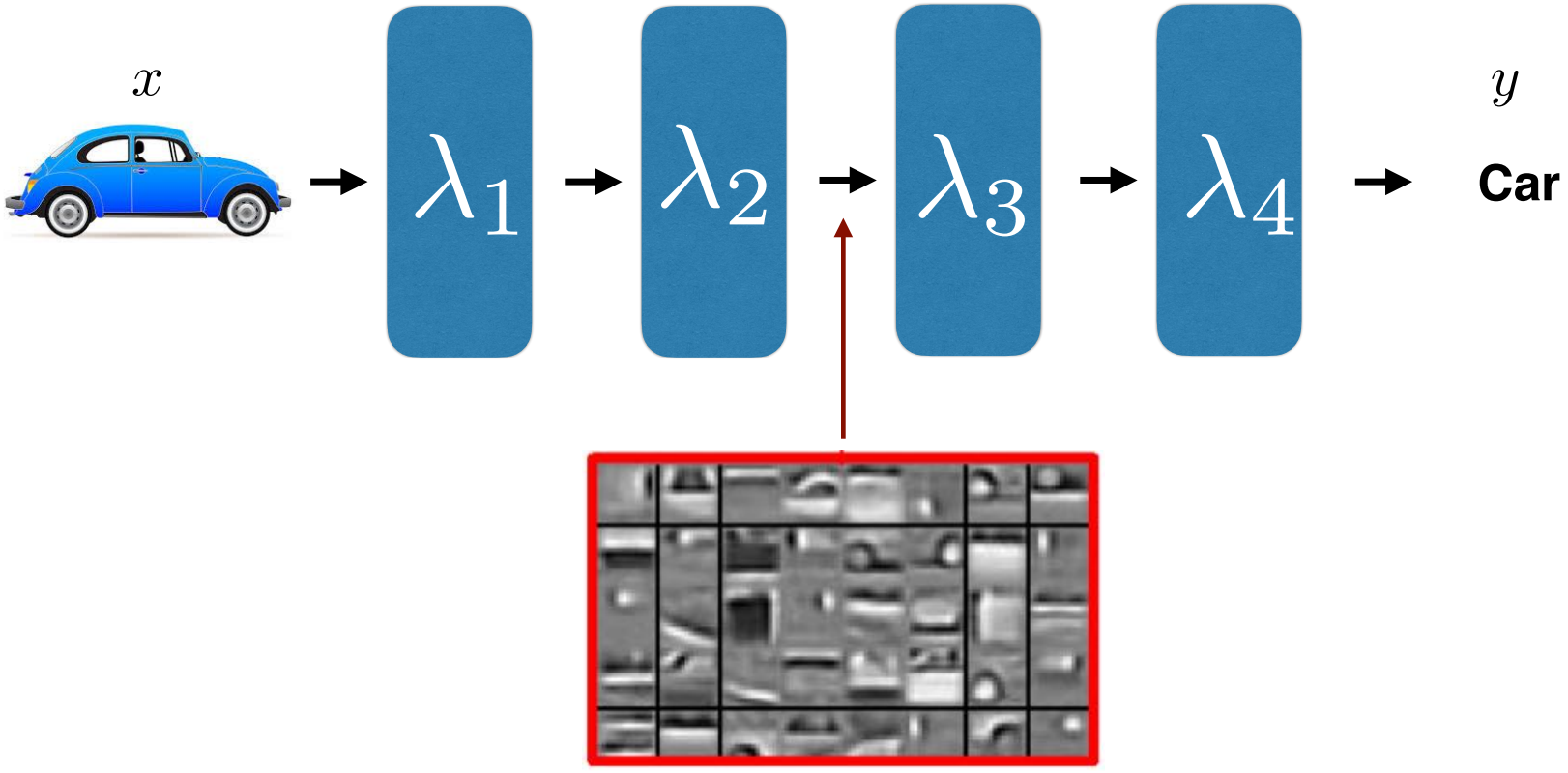
- How can we build such systems?
  - Composition of simple building blocks can lead to complex systems (e.g. neurons - brain) each block has trainable parameters
  - Each block has trainable parameters λ_i

Deep Learning: Complex Functions by Composition



Lee et al. “Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations”

Deep Learning: Complex Functions by Composition



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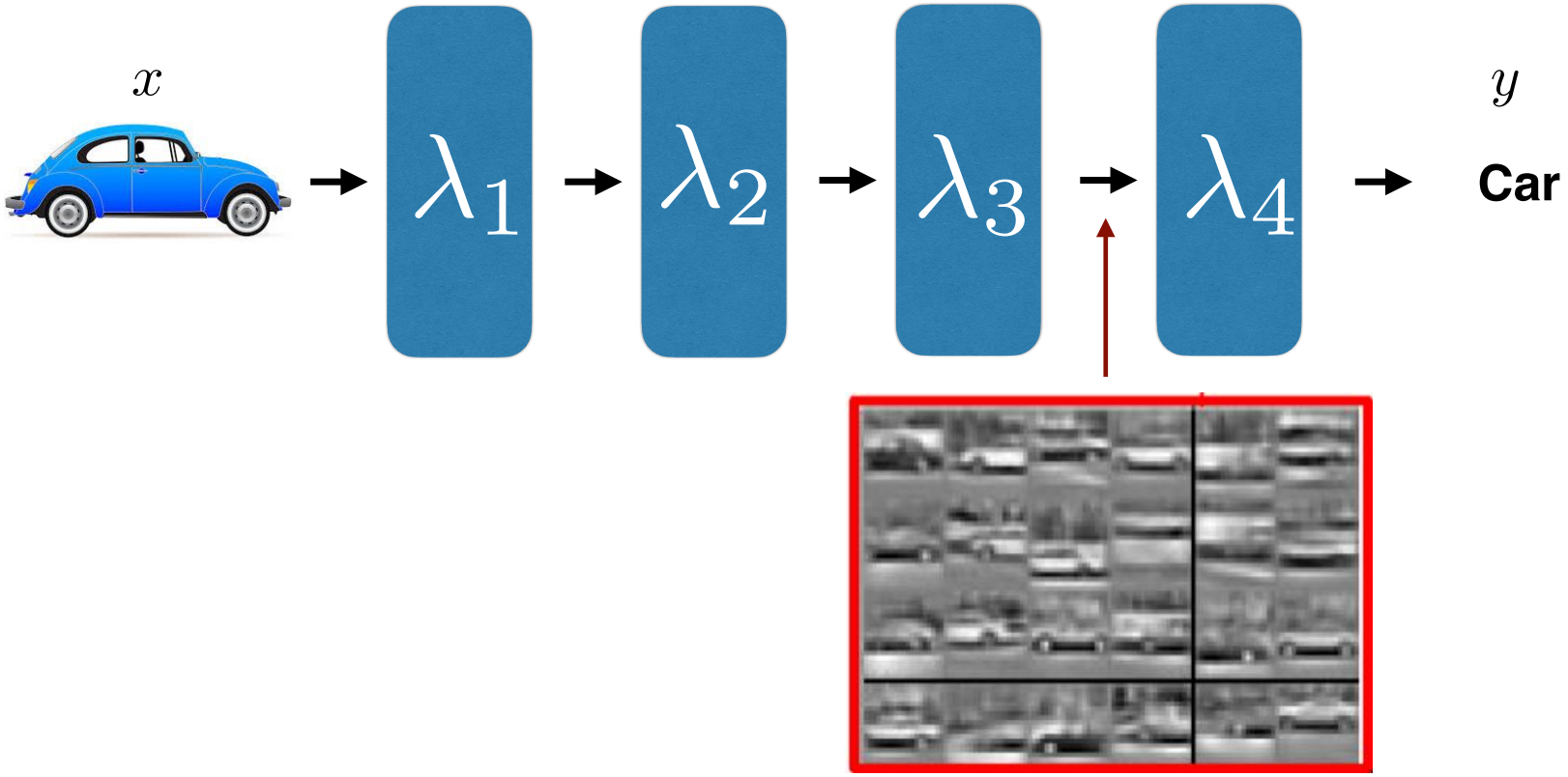
Deep Learning: Complex Functions by Composition



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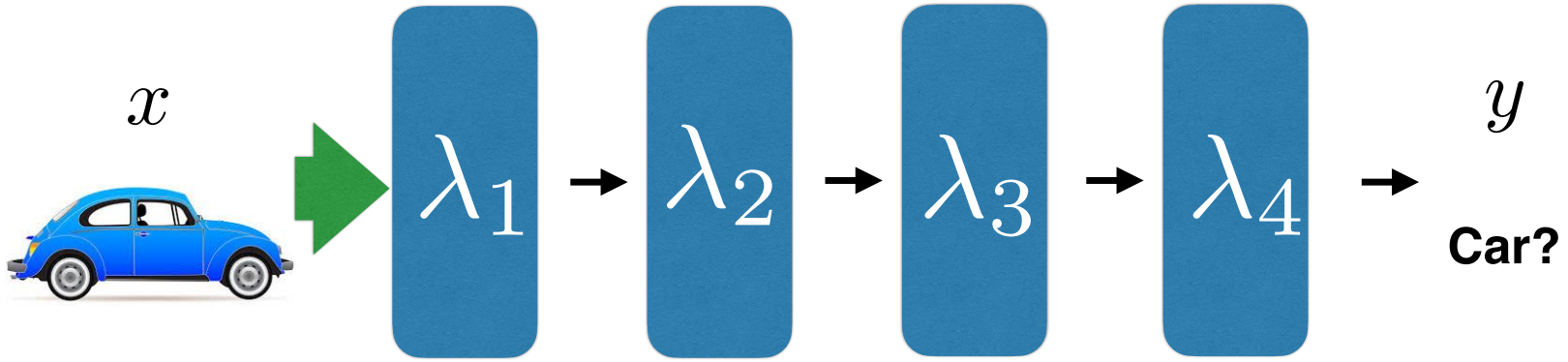
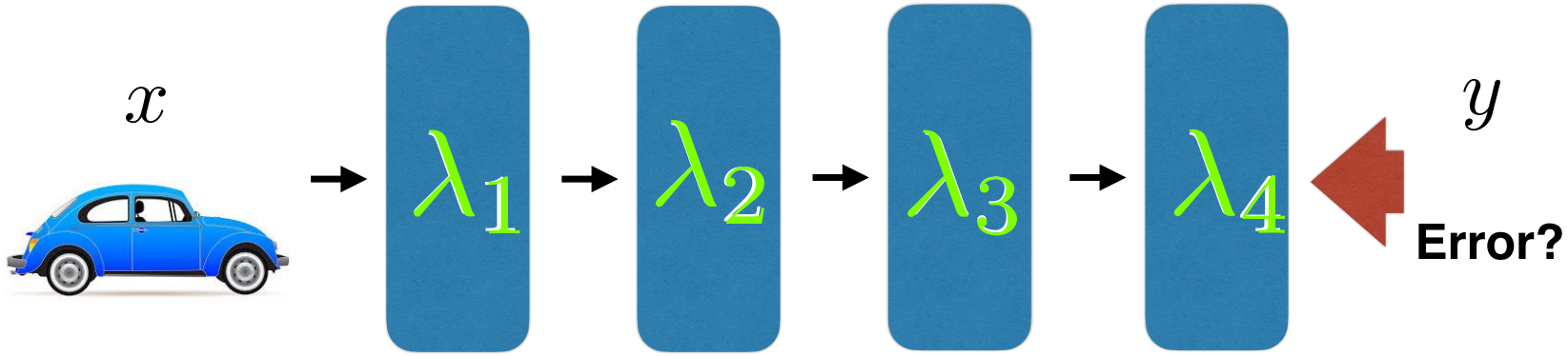
Training: Overview



- Setting
  - generate output y for input x (forward pass)

Training: Overview



- Setting
  - generate output y for input x (forward pass)
  - when there is an error, propagate error backwards to update weights (error back propagation)

Summary of Main Ideas in Deep Learning

1. **Learning of feature extraction** (across many layers)
2. **Efficient** and **trainable** systems by **differentiable** building blocks
3. Composition of deep architectures via **non-linear modules**
4. "**End-to-End**" training: no differentiation between feature extraction and classification