Assignment3

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Question 1

Part 1

First we generate data for Part1.

```
#generate data
library(rpart)
library(tidyverse)
```

```
## ── Attaching packages ─────────────────────────────────────── tidyverse
1.3.0 ──
```

```
## ✓ ggplot2 3.3.0     ✓ purrr   0.3.3
## ✓ tibble  3.0.0     ✓ dplyr   0.8.5
## ✓ tidyr   1.0.2     ✓ stringr 1.4.0
## ✓ readr   1.3.1     ✓ forcats 0.5.0
```

```
## Warning: package 'tibble' was built under R version 3.6.2
```

```
## ── Conflicts ────────────────────────────────────────── tidyverse_confli
cts() ──
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library(splines)
n=300
set.seed(1)
u=sort(runif(n)*5*pi)
y = sin(u)+rnorm(n)/4
df = data.frame(x=u,y=y)
number_of_weak_learners = 100
number_of_knots_split = 6
polynomial_degree = 2
fit = rpart(y~x,data=df)
```

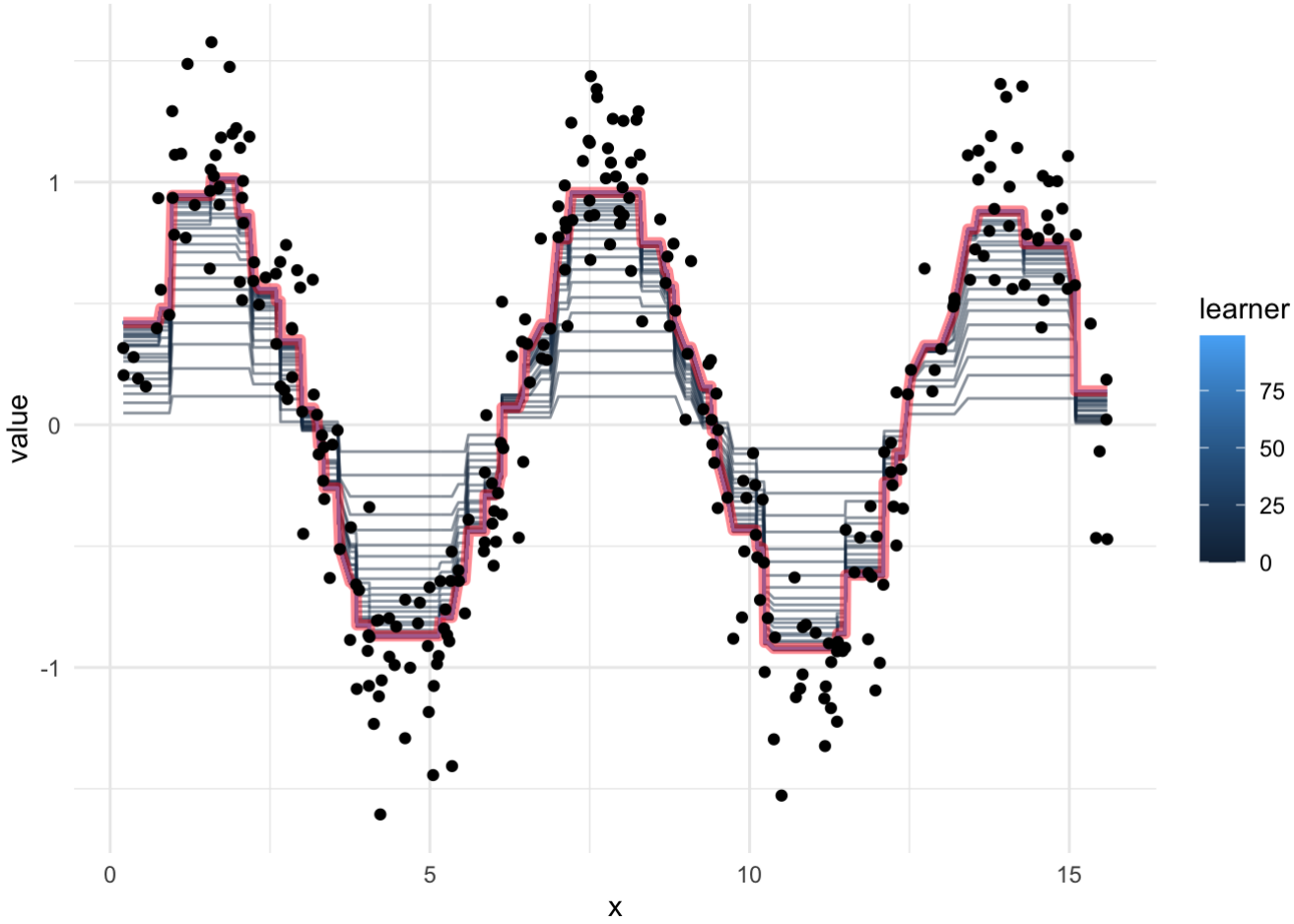
For Q0 and Q1, define a function called plot_fitted.

```
plot_fitted <- function(v,fit)
{number_of_weak_learners = 100
yp = predict(fit,newdata=df)
df$yr = df$y - v*yp
YP = v*yp
list_of_weak_learners = list(fit)
for(t in 2:number_of_weak_learners){
  # Fit linear spline
  fit = rpart(yr~x,data=df)

  # Generate new prediction
  yp=predict(fit,newdata=df)

  # Update residuals
  df$yr=df$yr - v*yp

  # Bind to new data point
  YP = cbind(YP,v*yp)

  # Store fitted model in list
  list_of_weak_learners[[t]] = fit
}


##############################################
##### Getting predictions for each boost #####
##############################################
for (i in 1:number_of_weak_learners){
  # Calculating performance of first i weak_learners

  # Summing weak learner residuals
  if(i==1){yp_i = YP[,1:i]
  }else{yp_i=apply(YP[,1:i],1,sum) #<- strong learner
  }

  # Binds new cols
  col_name = paste0('yp_',i)
  df = df %>% bind_cols(yp=yp_i)
}

df111 = df[,c(-2,-3)]
# Re-arrange sequences to get pseudo residuals
plot_wl = df111 %>%
  pivot_longer(cols = starts_with("yp")) %>%
  mutate(learner = str_match(name,"[0-9]+")) %>%
  mutate(learner = as.integer(ifelse(is.na(learner),0,learner)))

# Plot final learner
final_learner = plot_wl %>% filter(learner == (number_of_weak_learners-1))

# Plot progression of learner
ggplot() +
  # Visualizing all learners
```

```
  geom_line(aes(x = x, y = value, group = learner, color =learner),
            data = plot_wl,alpha=0.5) +
  # Final learner
  geom_line(aes(x = x, y = value, group = learner, color =learner),
            data = final_learner,alpha=0.5,color = 'firebrick1',size = 2)  +
  geom_point(aes(x = x, y= y),data = df)+ # true values
  theme_minimal()
}
```

Q0

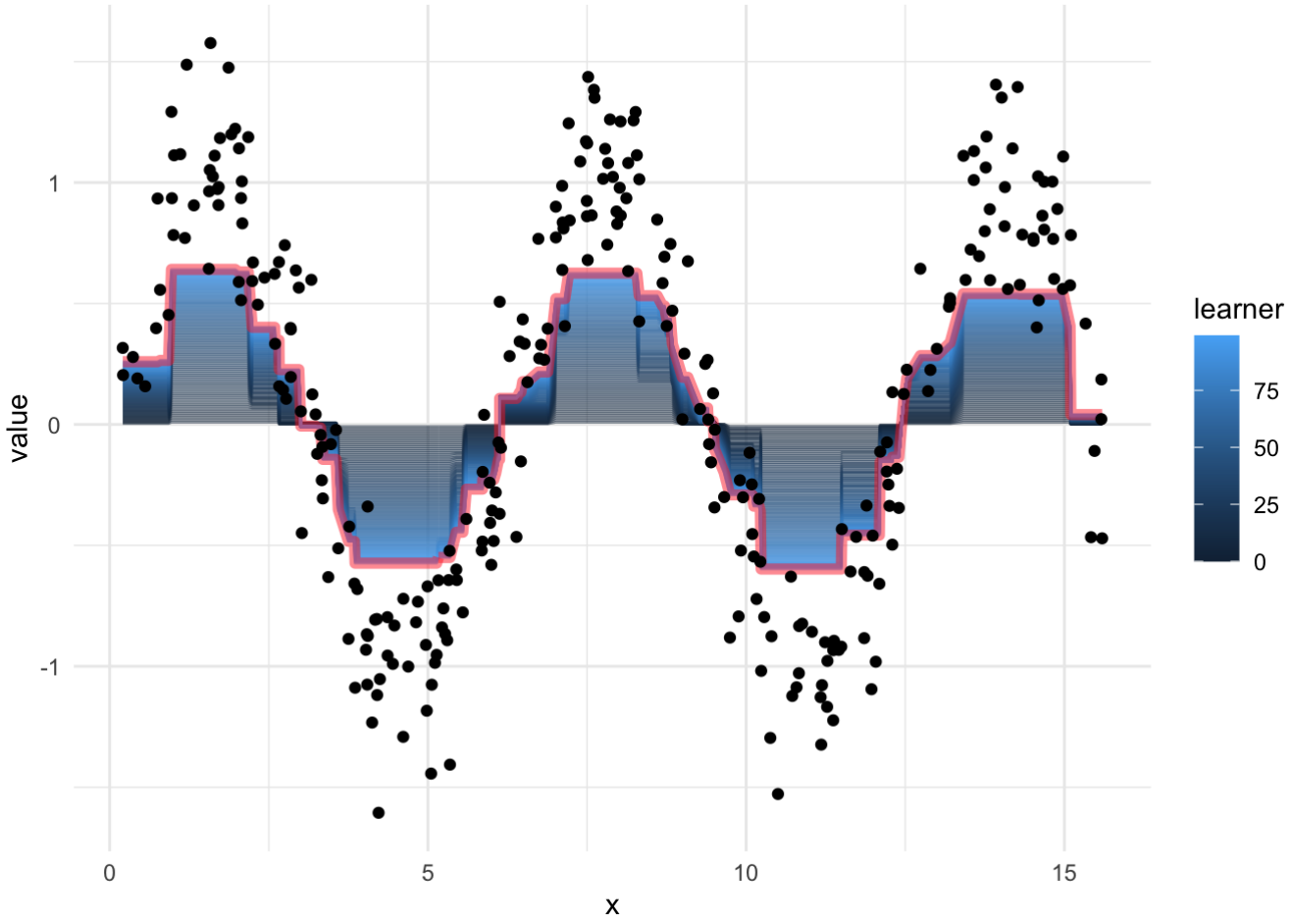
```
plot_fitted(0.125,fit)
```



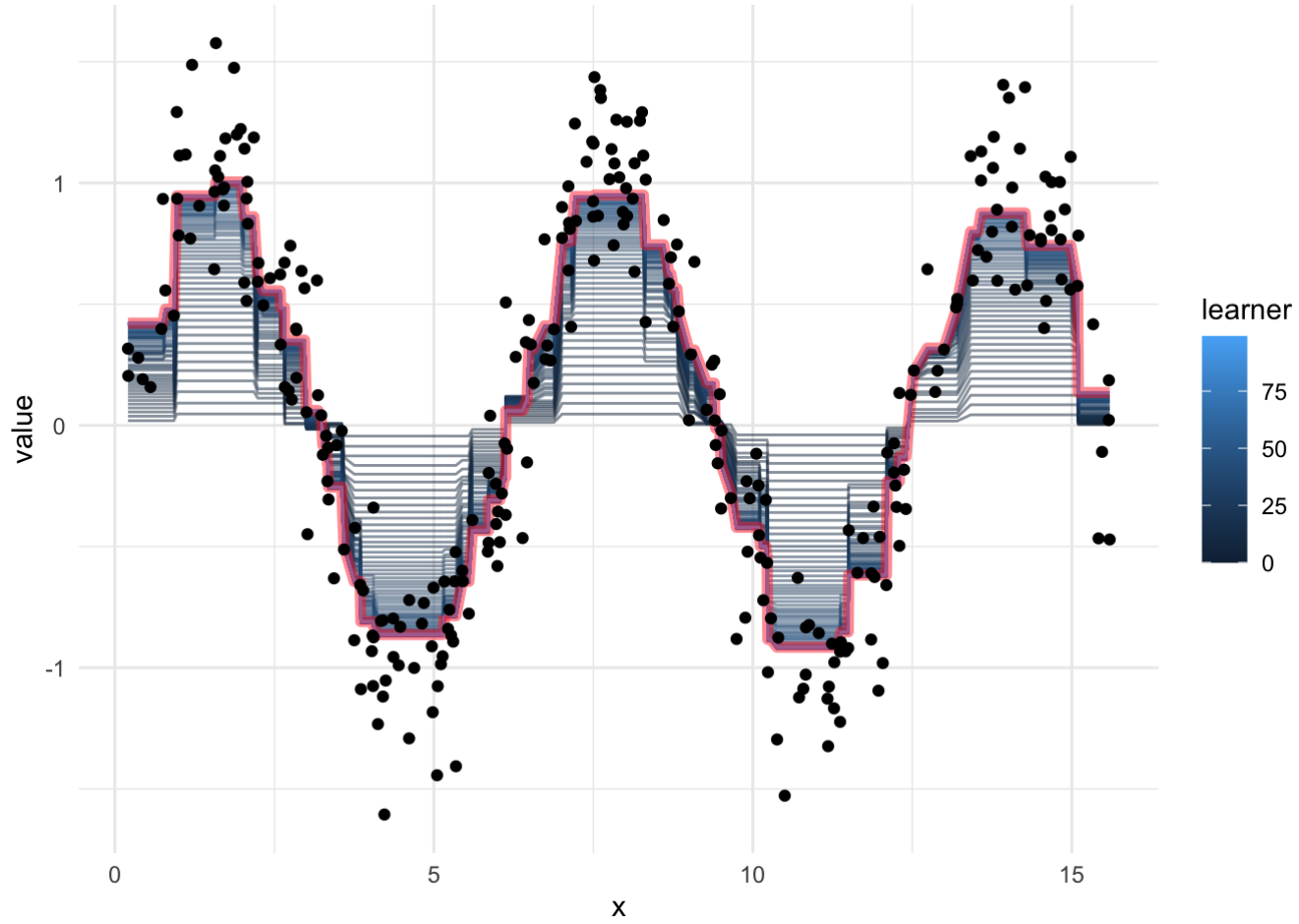
Q1

v=0.125 is plotted in Q0. Here we plot for v=0.01 and v=0.05.

```
plot_fitted(0.01,fit)
```




```
plot_fitted(0.05,fit)
```



Q2

A&B

My method is to measure the difference between previous RMSE and current RMSE, if the difference is less than 0.01, then make an early stop.

```
rm(list=ls())
n=500
set.seed(1)
u=sort(runif(n)*5*pi)
y = sin(u)+rnorm(n)/4
df = data.frame(x=u,y=y)
idx=sample(seq_len(n), size =50)
test <- df[idx, ]
remain <- df[-idx,]
df<-remain
row.names(df) <- NULL
valid_idx=sample(seq_len(450), size =50)
valid<-df[valid_idx,]
remain<-df[-valid_idx,]
df<-remain
row.names(df) <- NULL



fit_early_stop<-function(model){
diff = 100000000000
t=1
v=.05

yp = predict(model,newdata=df)
df$yr = df$y - v*yp
YP = v*yp
list_of_weak_learners = list(model)
y_valid_p=v*predict(model,newdata=valid)
y_rmse_valid=sqrt(mean(abs(valid$y-y_valid_p)**2))
while (diff>0.01)
{
  t=t+1
  fit = rpart(yr ~ x,data=df)
  yp=predict(fit,newdata=df)
  list_of_weak_learners[[t]] = fit
  df$yr=df$yr - v*yp
  YP = cbind(YP,v*yp)
  y_valid_p=y_valid_p+v*predict(fit,newdata=valid)
  y_rmse_valid_new=sqrt(mean(abs(valid$y-y_valid_p)**2))
  diff=abs(y_rmse_valid_new-y_rmse_valid)
  y_rmse_valid=y_rmse_valid_new

}

t=t-1

##################################################
##### Getting predictions for each boost #####
##################################################
for (i in 1:t){
  # Calculating performance of first i weak_learners
```

```
  # Summing weak learner residuals
  if(i==1){yp_i = YP[,1:i]
  }else{yp_i=apply(YP[,1:i],1,sum) #<- strong learner
  }
  # Binds new cols
  col_name = paste0('yp_',i)
  df = df %>% bind_cols(yp=yp_i)
}
df111 = df[,c(-2,-3)]
# Re-arrange sequences to get pseudo residuals
plot_wl = df111 %>%
  pivot_longer(cols = starts_with("yp")) %>%
  mutate(learner = str_match(name,"[0-9]+")) %>%
  mutate(learner = as.integer(ifelse(is.na(learner),0,learner)))

# Plot final learner
final_learner = plot_wl %>% filter(learner == (t-1))

# Plot progression of learner
ggplot() +
  # Visualizing all learners
  geom_line(aes(x = x, y = value, group = learner, color =learner),
            data = plot_wl,alpha=0.5) +
  # Final learner
  geom_line(aes(x = x, y = value, group = learner, color =learner),
            data = final_learner,alpha=0.5,color = 'firebrick1',size = 2)  +
  geom_point(aes(x = x, y= y),data = df)+ # true values
  ggtitle('Plot progression of learner with original learning paremater v=0.05')+
  theme_minimal()
cat('The number of trees is',t)
return(list_of_weak_learners)}

model=rpart(y~x,data=df)
list_of_weak_learners = fit_early_stop(model)
```

```
## The number of trees is 19
```

C

```
v=0.05
t=18
for (i in 1:18){
  weak_learner_i = list_of_weak_learners[[i]]

  if (i==1){pred = v*predict(weak_learner_i,test)}
  else{pred =pred + v*predict(weak_learner_i,test)}

  if(i==t){
    test = test %>% bind_cols(yp=pred)
  }
}
cat('The RMSE is',sqrt(sum((test$yp-test$y)**2))/50)
```

```
## The RMSE is 0.06358244
```

Q3

In this part, I will use grid-search to tune 3 parameters: minsplit,cp and maxdepth.

```
rm(list=ls())
n=500
set.seed(1)
u=sort(runif(n)*5*pi)
y = sin(u)+rnorm(n)/4
df = data.frame(x=u,y=y)
idx=sample(seq_len(n), size =50)
test <- df[idx, ]
remain <- df[-idx,]
df<-remain
row.names(df) <- NULL
valid_idx=sample(seq_len(450), size =50)
valid<-df[valid_idx,]
remain<-df[-valid_idx,]
df<-remain
row.names(df) <- NULL

fit_grid_search<-function(minsplit,cp,maxdepth){
model=rpart(y~x,data=df,control = rpart.control(minsplit = minsplit,cp = cp,maxdepth = m
axdepth))
list_of_weak_learners = list(model)
v=.05
number_of_weak_learners = 100
yp = predict(model,newdata=df)
df$yr = df$y - v*yp
YP = v*yp
list_of_weak_learners = list(model)
for(t in 2:number_of_weak_learners){
  # Fit linear spline
  fit = rpart(yr~x,data=df,control = rpart.control(minsplit = minsplit,cp = cp,maxdepth
 = maxdepth))

  # Generate new prediction
  yp=predict(fit,newdata=df,control = rpart.control(minsplit = minsplit,cp = cp,maxdepth
= maxdepth))

  # Update residuals
  df$yr=df$yr - v*yp

  # Bind to new data point
  YP = cbind(YP,v*yp)

  # Store fitted model in list
  list_of_weak_learners[[t]] = fit
}



for (i in 1:t){
  weak_learner_i = list_of_weak_learners[[i]]

  if (i==1){pred = v*predict(weak_learner_i,test)}
  else{pred =pred + v*predict(weak_learner_i,test)}
```

```
  if(i==t){
    test = test %>% bind_cols(yp=pred)
  }
}
return(sqrt(sum((test$yp-test$y)**2)))

}


res = data.frame(minsplit = rep(0,27),cp = rep(0,27),maxdepth = rep(0,27),RMSE = rep(0,2
7))

i=1
for (p1 in c(5,20,50)){
  for (p2 in c(0.1,0.01,0.001)){
    for (p3 in c(5,10,30)){
      res[i,] = c(p1,p2,p3,fit_grid_search(p1,p2,p3)/50)
      i=i+1
    }

  }
}
```

```
res
```

```
##    minsplit    cp maxdepth       RMSE
## 1         5 0.100        5 0.06683252
## 2         5 0.100       10 0.06683252
## 3         5 0.100       30 0.06683252
## 4         5 0.010        5 0.04643182
## 5         5 0.010       10 0.04591451
## 6         5 0.010       30 0.04591451
## 7         5 0.001        5 0.04536585
## 8         5 0.001       10 0.05088259
## 9         5 0.001       30 0.05310488
## 10       20 0.100        5 0.06683252
## 11       20 0.100       10 0.06683252
## 12       20 0.100       30 0.06683252
## 13       20 0.010        5 0.04628380
## 14       20 0.010       10 0.04560248
## 15       20 0.010       30 0.04560248
## 16       20 0.001        5 0.04420488
## 17       20 0.001       10 0.04573664
## 18       20 0.001       30 0.04586075
## 19       50 0.100        5 0.06683252
## 20       50 0.100       10 0.06683252
## 21       50 0.100       30 0.06683252
## 22       50 0.010        5 0.04648577
## 23       50 0.010       10 0.04631201
## 24       50 0.010       30 0.04631201
## 25       50 0.001        5 0.04555101
## 26       50 0.001       10 0.04545742
## 27       50 0.001       30 0.04545742
```

```
cat('Best Parameters are:','\n')
```

```
## Best Parameters are:
```

```
res[res$RMSE==min(res$RMSE),]
```

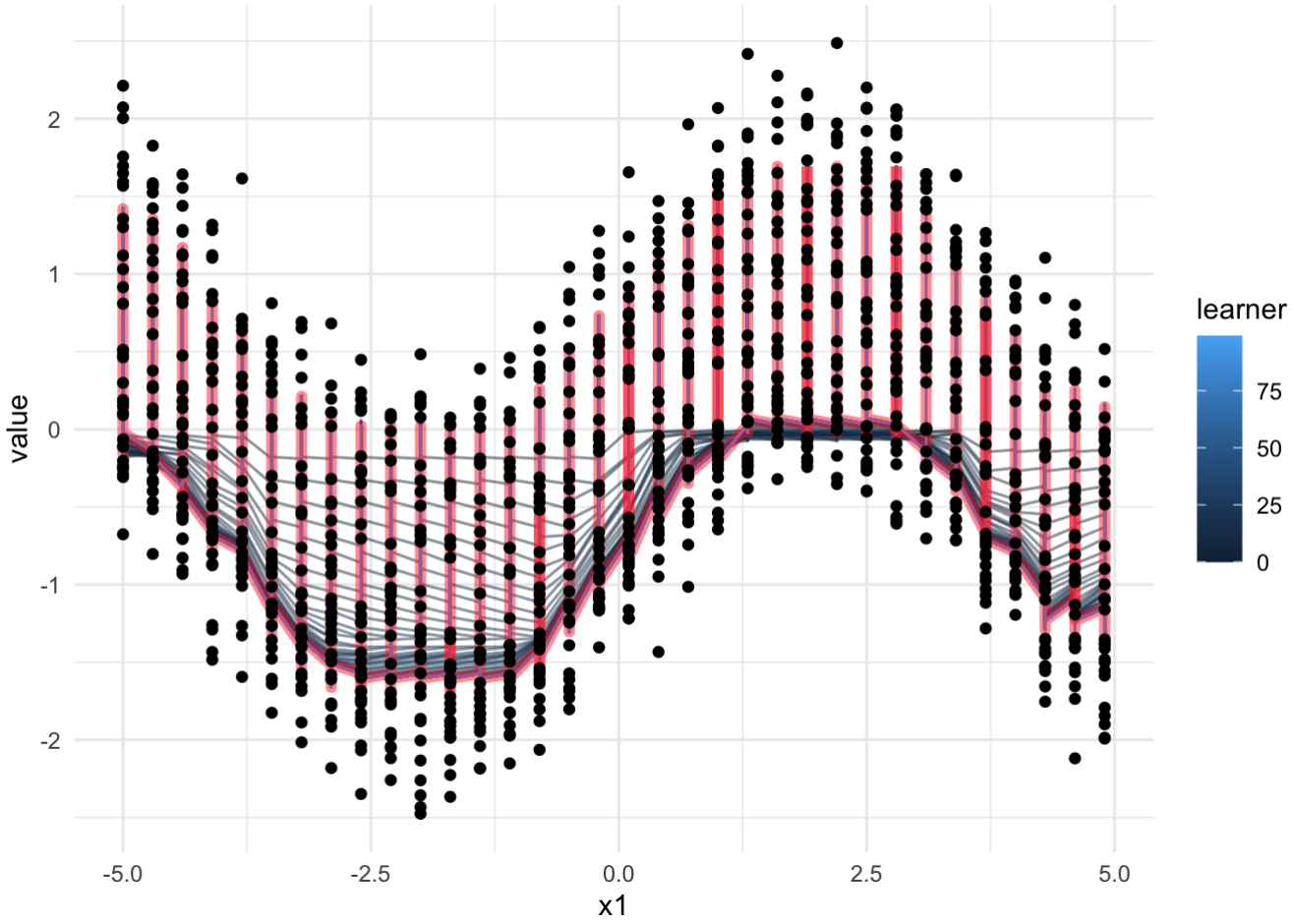
```
##    minsplit    cp maxdepth       RMSE
## 16       20 0.001        5 0.04420488
```

Part 2

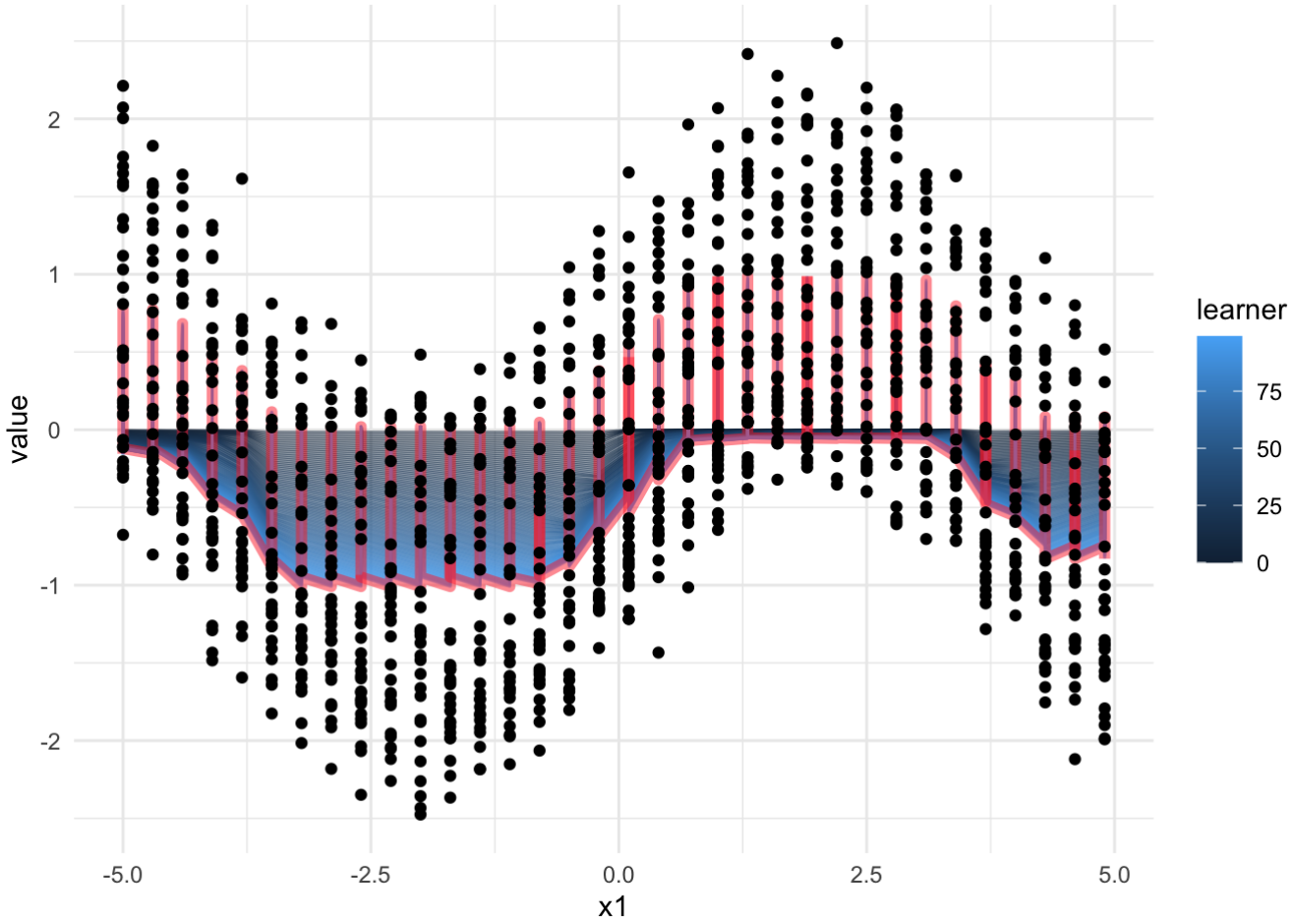
Q0&Q1

```
df = read.csv('/Users/zmt/Desktop/kernel_regression_2.csv')
colnames(df) = c('x1','x2','y')
plot_fitted <- function(v,fit)
{number_of_weak_learners = 100
yp = predict(fit,newdata=df)
df$yr = df$y - v*yp
YP = v*yp
list_of_weak_learners = list(fit)
for(t in 2:number_of_weak_learners){
  # Fit linear spline
  fit = rpart(yr~x1+x2,data=df)

  # Generate new prediction
  yp=predict(fit,newdata=df)

  # Update residuals
  df$yr=df$yr - v*yp

  # Bind to new data point
  YP = cbind(YP,v*yp)

  # Store fitted model in list
  list_of_weak_learners[[t]] = fit
}


##################################################
##### Getting predictions for each boost #####
##################################################
for (i in 1:number_of_weak_learners){
  # Calculating performance of first i weak_learners

  # Summing weak learner residuals
  if(i==1){yp_i = YP[,1:i]
  }else{yp_i=apply(YP[,1:i],1,sum) #<- strong learner
  }

  # Binds new cols
  col_name = paste0('yp_',i)
  df = df %>% bind_cols(yp=yp_i)
}

df111 = df[,c(-2,-3)]
# Re-arrange sequences to get pseudo residuals
plot_wl = df111 %>%
  pivot_longer(cols = starts_with("yp")) %>%
  mutate(learner = str_match(name,"[0-9]+")) %>%
  mutate(learner = as.integer(ifelse(is.na(learner),0,learner)))

# Plot final learner
final_learner = plot_wl %>% filter(learner == (number_of_weak_learners-1))

# Plot progression of learner
```

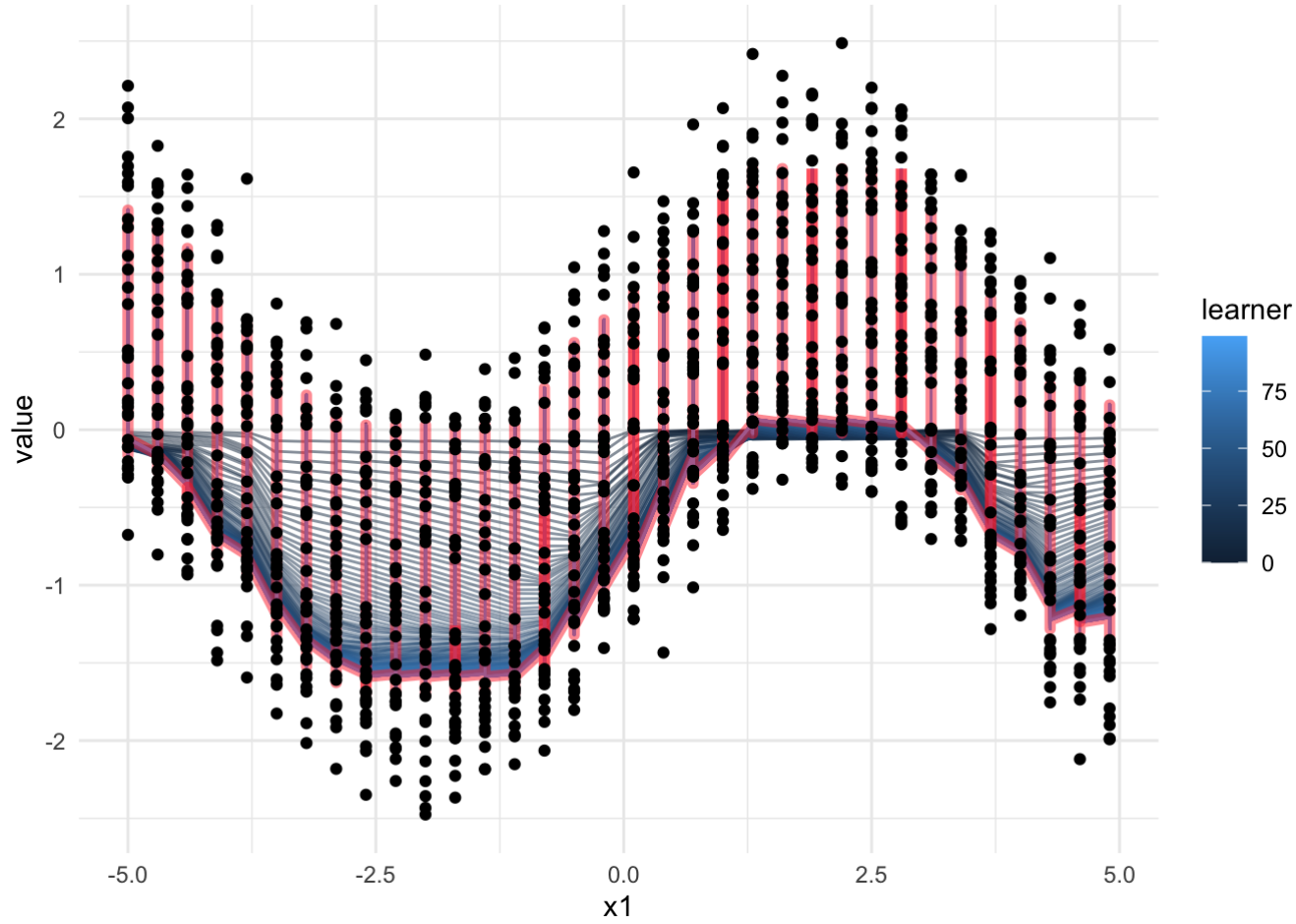
```
ggplot() +
  # Visualizing all learners
  geom_line(aes(x = x1, y = value, group = learner, color =learner),
            data = plot_wl,alpha=0.5) +
  # Final learner
  geom_line(aes(x = x1, y = value, group = learner, color =learner),
            data = final_learner,alpha=0.5,color = 'firebrick1',size = 2)  +
  geom_point(aes(x = x1, y= y),data = df)+ # true values
  theme_minimal()
}

model = rpart(y~x1+x2,data=df)
plot_fitted(0.125,fit=model)
```




```
plot_fitted(0.01,fit=model)
```




```
plot_fitted(0.05,fit=model)
```



Q2

A&B

```
rm(list=ls())
df = read.csv('/Users/zmt/Desktop/kernel_regression_2.csv')
n=length(df[,1])
colnames(df) = c('x1','x2','y')
idx=sample(seq_len(n), size =50)
test <- df[idx, ]
remain <- df[-idx,]
df<-remain
row.names(df) <- NULL
valid_idx=sample(seq_len(n-50), size =50)
valid<-df[valid_idx,]
remain<-df[-valid_idx,]
df<-remain
row.names(df) <- NULL
fit_early_stop<-function(model){
diff = 100000000000
t=1
v=.05

yp = predict(model,newdata=df)
df$yr = df$y - v*yp
YP = v*yp
list_of_weak_learners = list(model)
y_valid_p=v*predict(model,newdata=valid)
y_rmse_valid=sqrt(mean(abs(valid$y-y_valid_p)**2))
while (diff>0.01)
{
  t=t+1
  fit = rpart(yr ~ x1+x2,data=df)
  yp=predict(fit,newdata=df)
  list_of_weak_learners[[t]] = fit
  df$yr=df$yr - v*yp
  YP = cbind(YP,v*yp)
  y_valid_p=y_valid_p+v*predict(fit,newdata=valid)
  y_rmse_valid_new=sqrt(mean(abs(valid$y-y_valid_p)**2))
  diff=abs(y_rmse_valid_new-y_rmse_valid)
  y_rmse_valid=y_rmse_valid_new

}

t=t-1

##############################################
##### Getting predictions for each boost #####
##############################################
for (i in 1:t){
  # Calculating performance of first i weak_learners

  # Summing weak learner residuals
  if(i==1){yp_i = YP[,1:i]
  }else{yp_i=apply(YP[,1:i],1,sum) #<- strong learner
  }
  # Binds new cols
```

```
  col_name = paste0('yp_',i)
  df = df %>% bind_cols(yp=yp_i)
}
df111 = df[,c(-2,-3)]
# Re-arrange sequences to get pseudo residuals
plot_wl = df111 %>%
  pivot_longer(cols = starts_with("yp")) %>%
  mutate(learner = str_match(name,"[0-9]+")) %>%
  mutate(learner = as.integer(ifelse(is.na(learner),0,learner)))

# Plot final learner
final_learner = plot_wl %>% filter(learner == (t-1))

# Plot progression of learner
ggplot() +
  # Visualizing all learners
  geom_line(aes(x = x, y = value, group = learner, color =learner),
            data = plot_wl,alpha=0.5) +
  # Final learner
  geom_line(aes(x = x, y = value, group = learner, color =learner),
            data = final_learner,alpha=0.5,color = 'firebrick1',size = 2)  +
  geom_point(aes(x = x, y= y),data = df)+ # true values
  ggtitle('Plot progression of learner with original learning paremater v=0.05')+
  theme_minimal()
cat('The number of trees is',t)
return(list_of_weak_learners)}

model=rpart(y~x1+x2,data=df)
list_of_weak_learners = fit_early_stop(model)
```

```
## The number of trees is 22
```

C

```
v=0.05
t=18
for (i in 1:18){
  weak_learner_i = list_of_weak_learners[[i]]

  if (i==1){pred = v*predict(weak_learner_i,test)}
  else{pred =pred + v*predict(weak_learner_i,test)}

  if(i==t){
    test = test %>% bind_cols(yp=pred)
  }
}
cat('The RMSE is',sqrt(sum((test$yp-test$y)**2))/100)
```

```
## The RMSE is 0.03939153
```

Q3

```
fit_grid_search<-function(minsplit,cp,maxdepth){
model=rpart(y~x1+x2,data=df,control = rpart.control(minsplit = minsplit,cp = cp,maxdepth
= maxdepth))
list_of_weak_learners = list(model)
v=.05
number_of_weak_learners = 100
yp = predict(model,newdata=df)
df$yr = df$y - v*yp
YP = v*yp
list_of_weak_learners = list(model)
for(t in 2:number_of_weak_learners){
  # Fit linear spline
  fit = rpart(yr~x1+x2,data=df,control = rpart.control(minsplit = minsplit,cp = cp,maxde
pth = maxdepth))

  # Generate new prediction
  yp=predict(fit,newdata=df,control = rpart.control(minsplit = minsplit,cp = cp,maxdepth
= maxdepth))

  # Update residuals
  df$yr=df$yr - v*yp

  # Bind to new data point
  YP = cbind(YP,v*yp)

  # Store fitted model in list
  list_of_weak_learners[[t]] = fit
}



for (i in 1:t){
  weak_learner_i = list_of_weak_learners[[i]]

  if (i==1){pred = v*predict(weak_learner_i,test)}
  else{pred =pred + v*predict(weak_learner_i,test)}

  if(i==t){
    test = test %>% bind_cols(yp=pred)
  }
}
return(sqrt(sum((test$yp-test$y)**2))/100)

}


res = data.frame(minsplit = rep(0,27),cp = rep(0,27),maxdepth = rep(0,27),RMSE = rep(0,2
7))

i=1
for (p1 in c(2,5,10)){
  for (p2 in c(0.01,0.1,0.001)){
    for (p3 in c(5,10,30)){
```

```
      res[i,] = c(p1,p2,p3,fit_grid_search(p1,p2,p3))
      i=i+1
    }

  }
}
```

```
cat('Best Parameters are:','\n')
```

```
## Best Parameters are:
```

```
res[res$RMSE==min(res$RMSE),][1,]
```

```
##   minsplit   cp maxdepth       RMSE
## 1        2 0.01        5 0.03939153
```

The best set is minsplit=2,cp=0.01,maxdepth=5

Question 2

Part 1

a. Yes. Since t-SNE will maintain the local structure of data, if two points are close after mapping by t-SNE, then it indicates the short distance between those two points in original dimension.
b. Perplexity is a guess about the number of close neighbors each point has and it is used to balance attention between local and global aspects of your data.
c. If the number of steps is too small, then t-SNE will end before it converges. If you see a t-SNE plot with strange “pinched” shapes, chances are the process was stopped too early. Unfortunately, there’s no fixed number of steps that yields a stable result. Different data sets can require different numbers of iterations to converge.
d. Sometimes you can read topological information off a t-SNE plot, but that typically requires views at multiple perplexities (multiple plots). There are two examples listed in the article. One of the simplest topological properties is containment. t-SNE with low perplexity value greatly exaggerates the size of the smaller group of points. Another is trefoil knot, low perplexity values failed to show global connectivity.

Part 2

```
library(tidyverse)
library(Rtsne)
library(RColorBrewer)
```

```
# Get MNIST data
mnist_raw <- read_csv("https://pjreddie.com/media/files/mnist_train.csv", col_names = FA
LSE)
```

```
## Parsed with column specification:
## cols(
##   .default = col_double()
## )
```

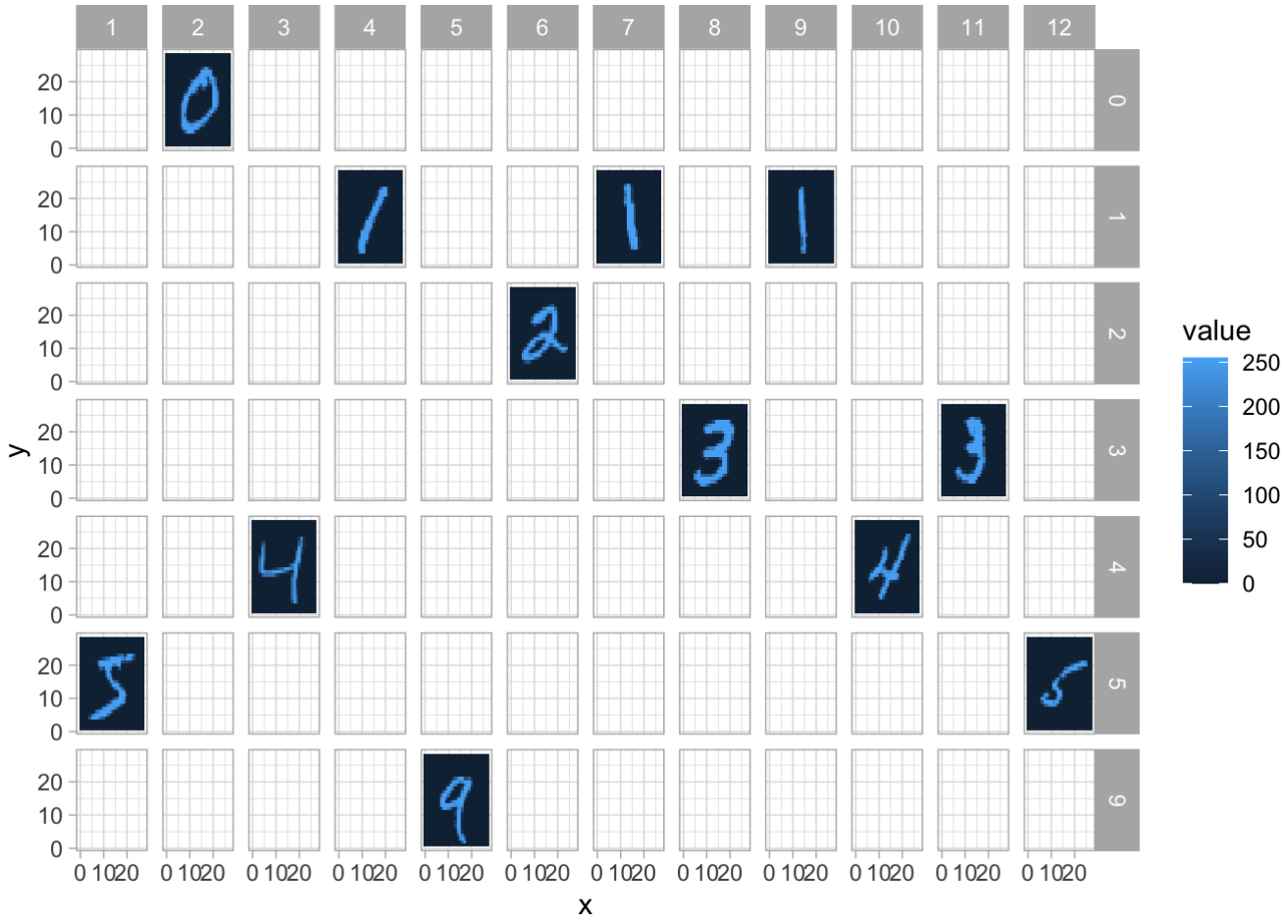
```
## See spec(...) for full column specifications.
```

```
# What is the dimension of the data set
dim(mnist_raw) # first column is the value, the rest are the pixels
```

```
## [1] 60000   785
```

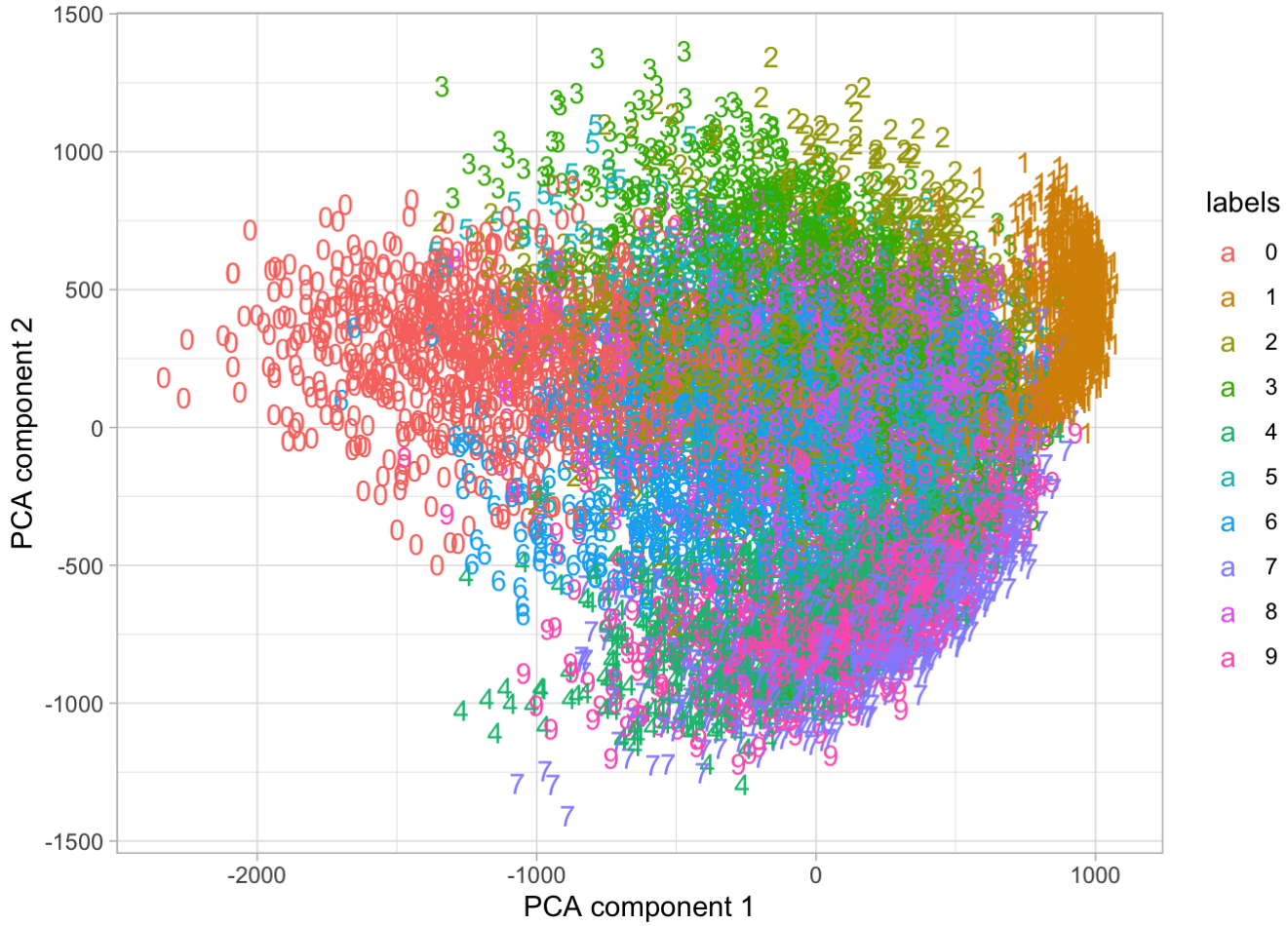
```
# Rearranging the data
pixels_gathered <- mnist_raw %>% head(10000) %>%
  rename(label = X1) %>%
  mutate(instance = row_number()) %>%
  gather(pixel, value, -label, -instance) %>%
  extract(pixel, "pixel", "(\\d+)", convert = TRUE) %>%
  mutate(pixel = pixel - 2,
         x = pixel %% 28,
         y = 28 - pixel %/% 28)

first_10k_samples =  mnist_raw[1:10000,-1] #%>% as.matrix()
first_10k_samples_labels =  mnist_raw[1:10000,1] %>% unlist(use.names=F)
colors = brewer.pal(10, 'Spectral')
```

```
# Visualizing the data
theme_set(theme_light())
pixels_gathered %>%
  filter(instance <= 12) %>%
  ggplot(aes(x, y, fill = value)) +
  geom_tile() +
  facet_grid(label~ instance )
```



a.

```
##############################################
##### Visualizing the PCA decomposition  #####
##############################################
pca = princomp(first_10k_samples)$scores[,1:2]
pca_plot = tibble(x = pca[,1], y =pca[,2], labels = as.character(first_10k_samples_label
s))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = pca_plot) + geom_text() +
  xlab('PCA component 1') +ylab('PCA component 2')
```



b.

```
##############################################
#####     Running the TSNE emebdding     #####
##############################################
itercost = c()
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 5,
                  theta = 0.5,
                  eta = 200,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 5.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 12.34 seconds (sparsity = 0.002104)!
## Learning embedding...
## Iteration 50: error is 118.296633 (50 iterations in 2.04 seconds)
## Iteration 100: error is 107.234275 (50 iterations in 2.49 seconds)
## Iteration 150: error is 99.561972 (50 iterations in 1.78 seconds)
## Iteration 200: error is 96.723478 (50 iterations in 1.71 seconds)
## Iteration 250: error is 95.000359 (50 iterations in 1.72 seconds)
## Iteration 300: error is 4.356011 (50 iterations in 1.62 seconds)
## Iteration 350: error is 3.798177 (50 iterations in 1.61 seconds)
## Iteration 400: error is 3.436557 (50 iterations in 1.66 seconds)
## Iteration 450: error is 3.173032 (50 iterations in 1.72 seconds)
## Iteration 500: error is 2.969045 (50 iterations in 1.75 seconds)
## Fitting performed in 18.10 seconds.
```

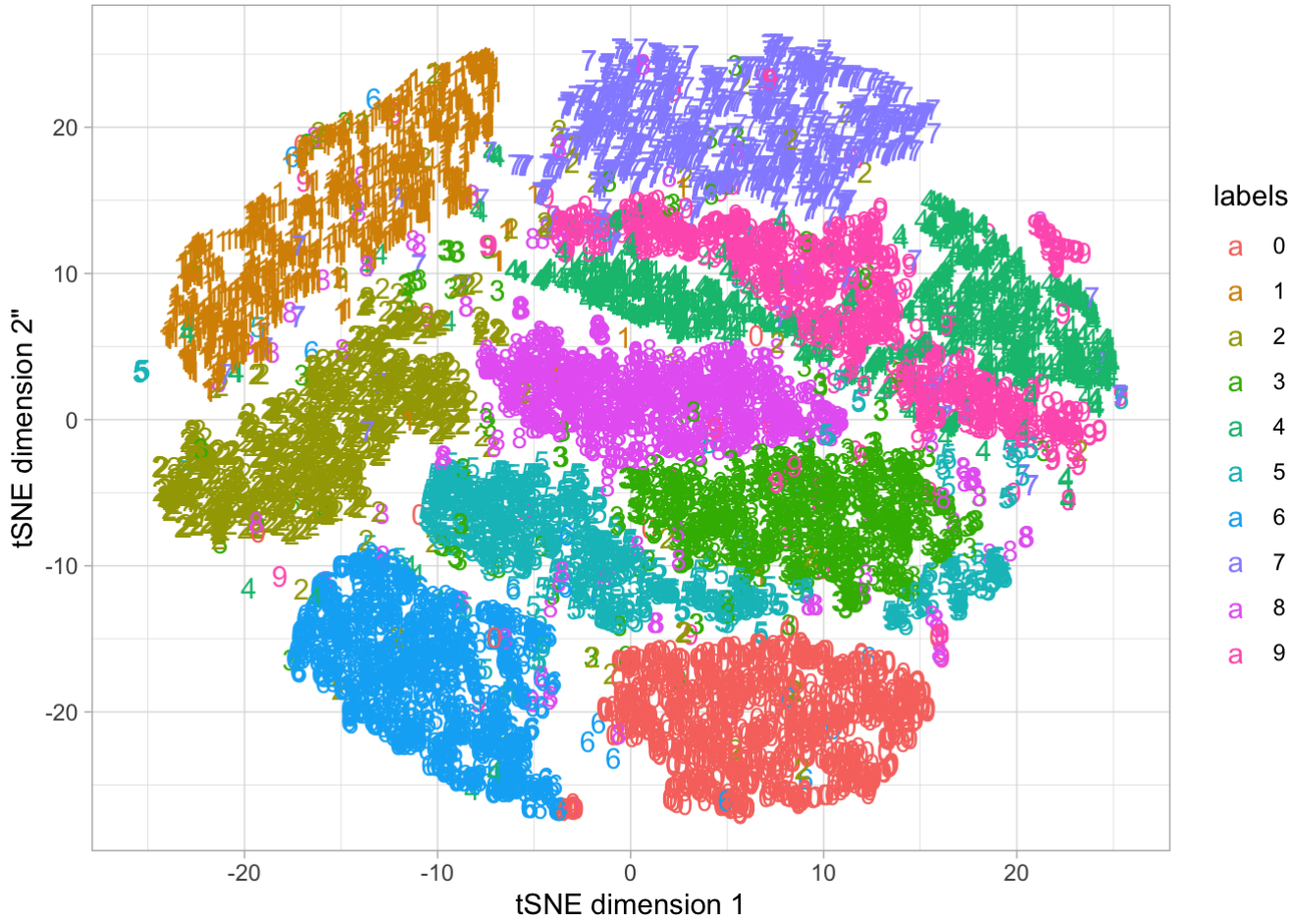
```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```

c. As the value of perplexity increases, I can see the distances between clusters also increase, which means the clusters become more dispersed.

```
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 20,
                  theta = 0.5,
                  eta = 200,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 20.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 15.20 seconds (sparsity = 0.008217)!
## Learning embedding...
## Iteration 50: error is 102.343946 (50 iterations in 2.14 seconds)
## Iteration 100: error is 93.292482 (50 iterations in 2.06 seconds)
## Iteration 150: error is 88.798251 (50 iterations in 2.05 seconds)
## Iteration 200: error is 87.913978 (50 iterations in 2.20 seconds)
## Iteration 250: error is 87.654497 (50 iterations in 2.24 seconds)
## Iteration 300: error is 3.394537 (50 iterations in 1.86 seconds)
## Iteration 350: error is 2.951053 (50 iterations in 1.74 seconds)
## Iteration 400: error is 2.703117 (50 iterations in 1.83 seconds)
## Iteration 450: error is 2.536505 (50 iterations in 1.86 seconds)
## Iteration 500: error is 2.414508 (50 iterations in 1.85 seconds)
## Fitting performed in 19.83 seconds.
```

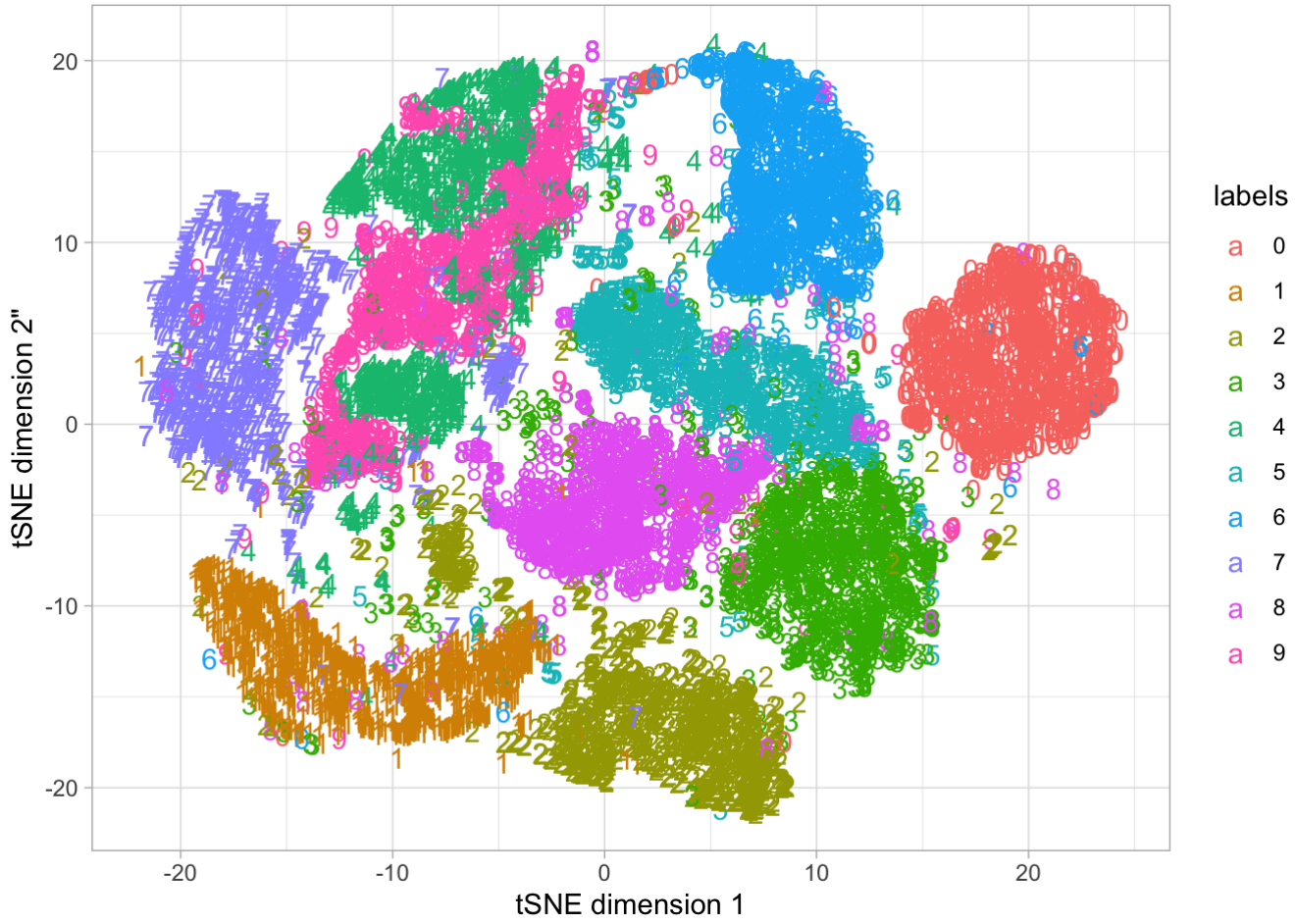
```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```




```
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 60,
                  theta = 0.5,
                  eta = 200,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 60.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 20.99 seconds (sparsity = 0.024328)!
## Learning embedding...
## Iteration 50: error is 89.428527 (50 iterations in 2.85 seconds)
## Iteration 100: error is 86.712359 (50 iterations in 3.73 seconds)
## Iteration 150: error is 81.658892 (50 iterations in 2.36 seconds)
## Iteration 200: error is 81.469172 (50 iterations in 2.57 seconds)
## Iteration 250: error is 81.450563 (50 iterations in 2.61 seconds)
## Iteration 300: error is 2.627544 (50 iterations in 2.39 seconds)
## Iteration 350: error is 2.282607 (50 iterations in 2.34 seconds)
## Iteration 400: error is 2.107304 (50 iterations in 2.27 seconds)
## Iteration 450: error is 1.997415 (50 iterations in 2.26 seconds)
## Iteration 500: error is 1.920060 (50 iterations in 2.30 seconds)
## Fitting performed in 25.69 seconds.
```

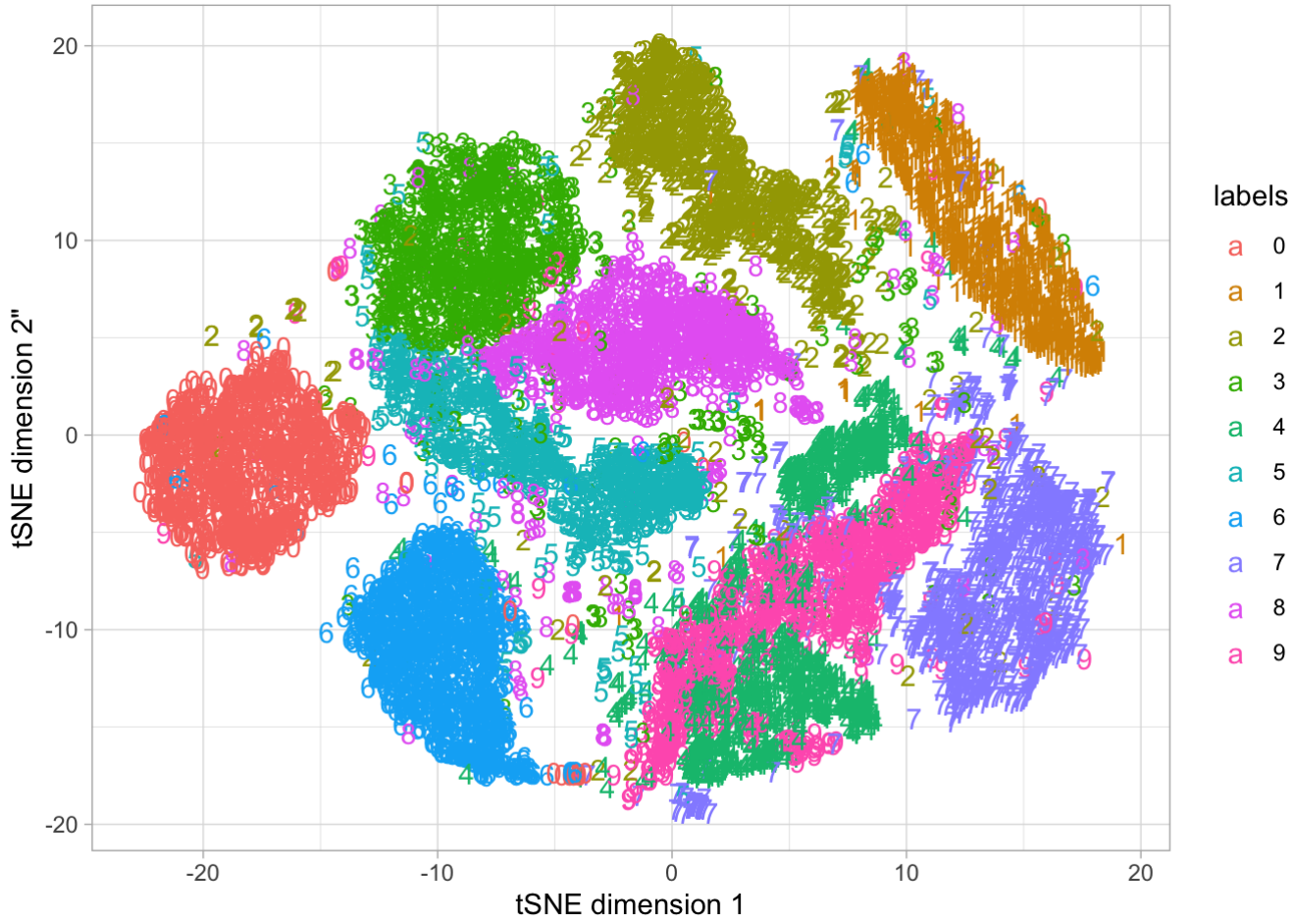
```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```




```
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 100,
                  theta = 0.5,
                  eta = 200,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 100.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 23.58 seconds (sparsity = 0.040571)!
## Learning embedding...
## Iteration 50: error is 83.371128 (50 iterations in 3.23 seconds)
## Iteration 100: error is 81.959169 (50 iterations in 4.91 seconds)
## Iteration 150: error is 78.550870 (50 iterations in 3.95 seconds)
## Iteration 200: error is 78.344096 (50 iterations in 3.26 seconds)
## Iteration 250: error is 78.296656 (50 iterations in 3.20 seconds)
## Iteration 300: error is 2.261678 (50 iterations in 2.79 seconds)
## Iteration 350: error is 1.968606 (50 iterations in 2.70 seconds)
## Iteration 400: error is 1.825864 (50 iterations in 2.82 seconds)
## Iteration 450: error is 1.738897 (50 iterations in 2.77 seconds)
## Iteration 500: error is 1.680204 (50 iterations in 2.74 seconds)
## Fitting performed in 32.37 seconds.
```

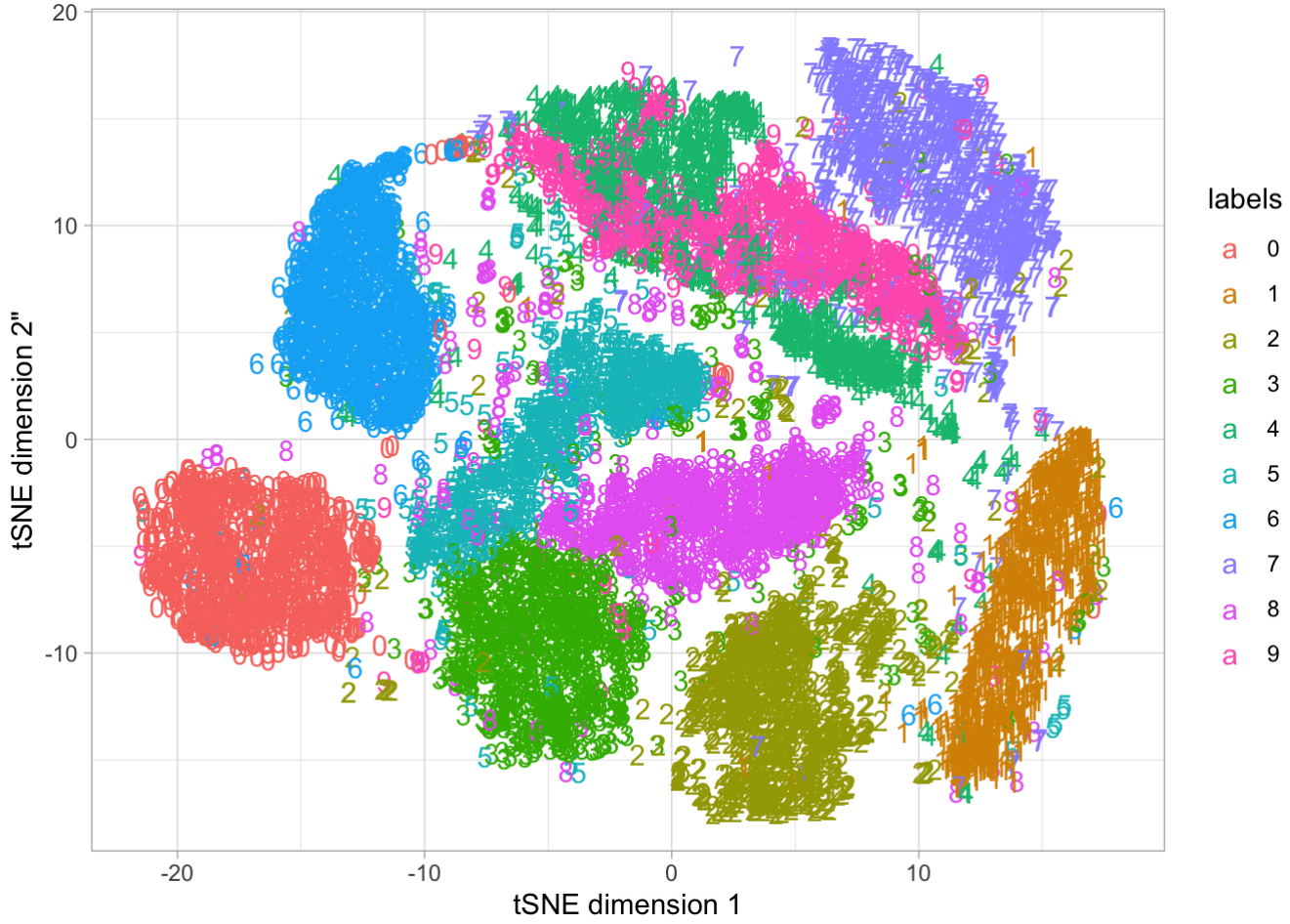
```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```




```
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 125,
                  theta = 0.5,
                  eta = 200,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 125.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 24.42 seconds (sparsity = 0.050806)!
## Learning embedding...
## Iteration 50: error is 80.714217 (50 iterations in 3.19 seconds)
## Iteration 100: error is 80.363285 (50 iterations in 3.20 seconds)
## Iteration 150: error is 76.794677 (50 iterations in 3.16 seconds)
## Iteration 200: error is 76.731696 (50 iterations in 3.11 seconds)
## Iteration 250: error is 76.727369 (50 iterations in 3.18 seconds)
## Iteration 300: error is 2.153940 (50 iterations in 2.95 seconds)
## Iteration 350: error is 1.865270 (50 iterations in 2.94 seconds)
## Iteration 400: error is 1.727065 (50 iterations in 2.94 seconds)
## Iteration 450: error is 1.645474 (50 iterations in 2.97 seconds)
## Iteration 500: error is 1.591008 (50 iterations in 2.96 seconds)
## Fitting performed in 30.59 seconds.
```

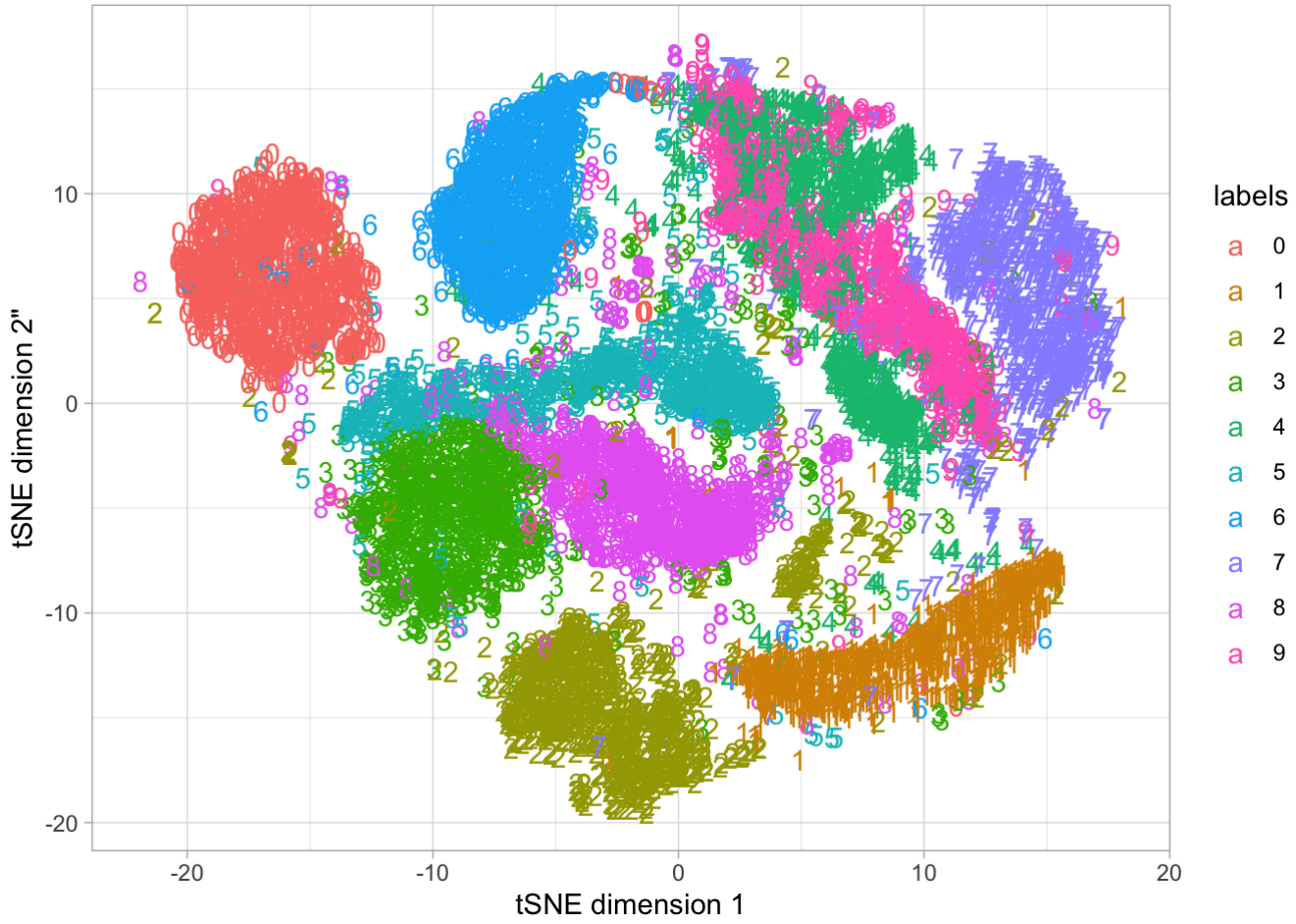
```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```




```
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 160,
                  theta = 0.5,
                  eta = 200,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 160.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 29.00 seconds (sparsity = 0.065215)!
## Learning embedding...
## Iteration 50: error is 77.767491 (50 iterations in 3.53 seconds)
## Iteration 100: error is 77.748853 (50 iterations in 4.52 seconds)
## Iteration 150: error is 75.081994 (50 iterations in 5.59 seconds)
## Iteration 200: error is 74.927615 (50 iterations in 3.91 seconds)
## Iteration 250: error is 74.924031 (50 iterations in 3.83 seconds)
## Iteration 300: error is 1.997740 (50 iterations in 3.40 seconds)
## Iteration 350: error is 1.739551 (50 iterations in 3.33 seconds)
## Iteration 400: error is 1.611347 (50 iterations in 3.35 seconds)
## Iteration 450: error is 1.537428 (50 iterations in 3.34 seconds)
## Iteration 500: error is 1.490170 (50 iterations in 3.34 seconds)
## Fitting performed in 38.14 seconds.
```

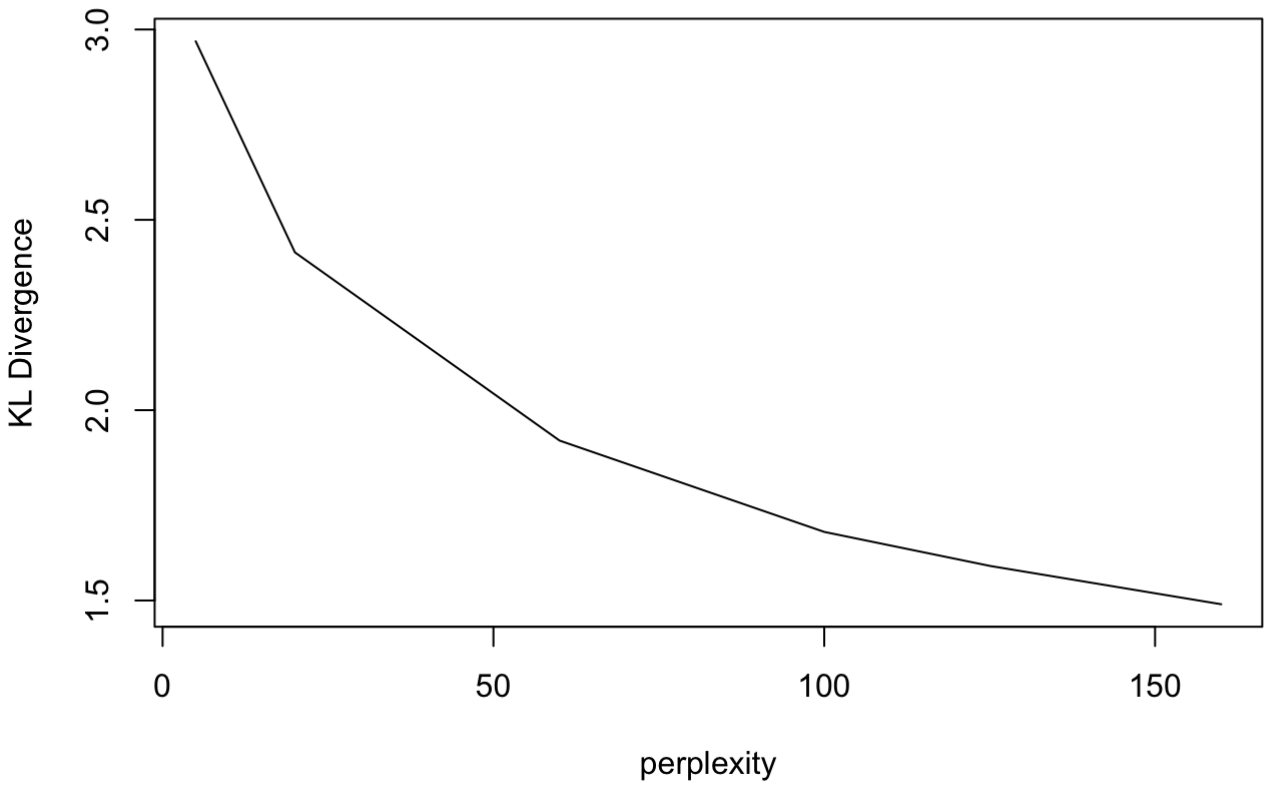
```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```



e. When perplexity is as large as 5000, the clusters will become very dispersed.

f. The optimal perplexity is 160.

```
plot(c(5,20,60,100,125,160),itercost,type='l',xlab='perplexity',ylab='KL Divergence')
```

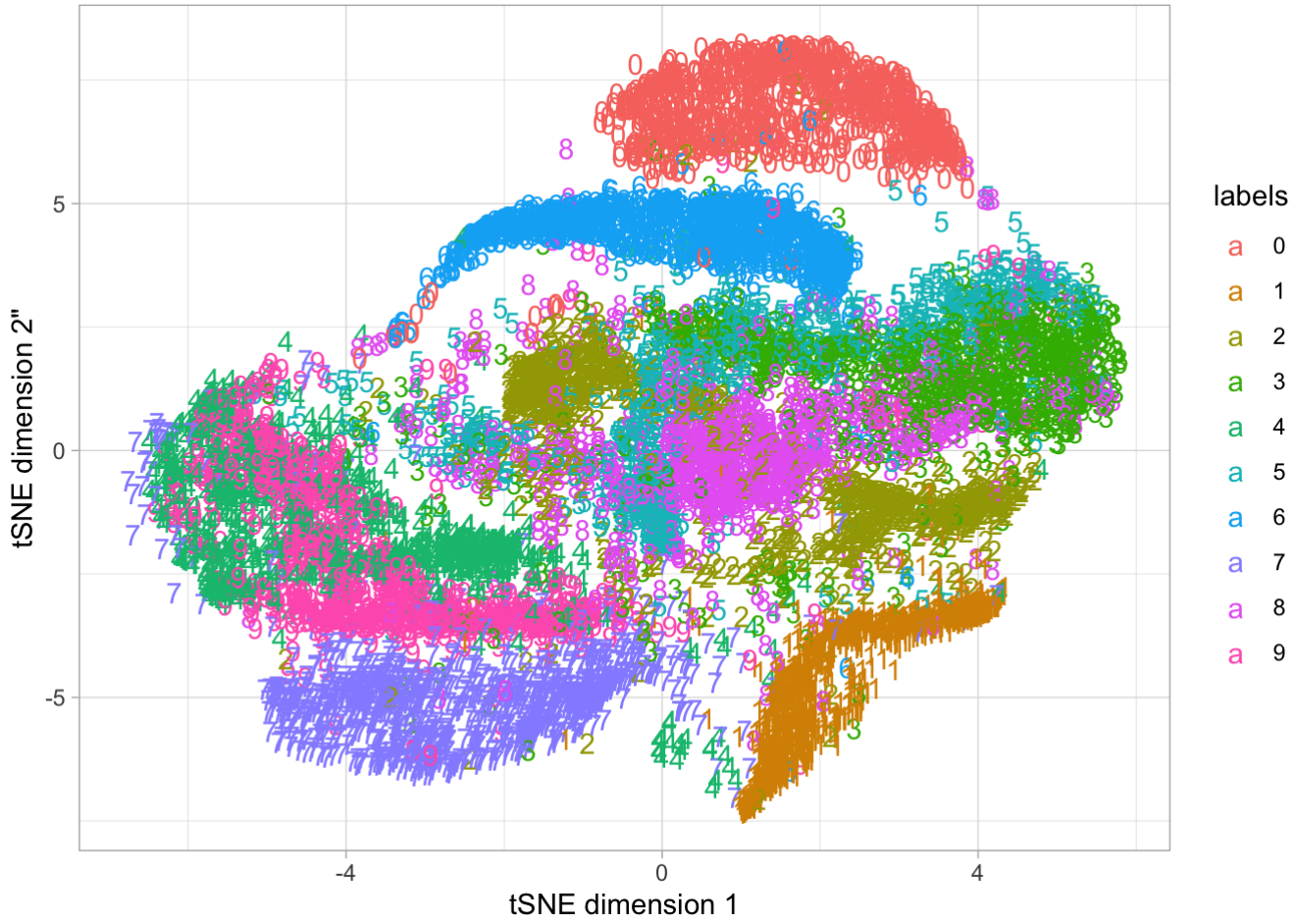


e. Here we run eta=10 and eta=100 at perplexity=160. (since eta=200 is run before). I notice that bigger learning rate will lead to more dispersed clustering result.

```
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 160,
                  theta = 0.5,
                  eta = 10,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 160.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 29.37 seconds (sparsity = 0.065215)!
## Learning embedding...
## Iteration 50: error is 77.767491 (50 iterations in 4.01 seconds)
## Iteration 100: error is 77.767491 (50 iterations in 3.69 seconds)
## Iteration 150: error is 77.767491 (50 iterations in 3.86 seconds)
## Iteration 200: error is 77.767491 (50 iterations in 4.18 seconds)
## Iteration 250: error is 77.767490 (50 iterations in 4.63 seconds)
## Iteration 300: error is 3.955333 (50 iterations in 5.48 seconds)
## Iteration 350: error is 2.846720 (50 iterations in 4.74 seconds)
## Iteration 400: error is 2.442388 (50 iterations in 3.54 seconds)
## Iteration 450: error is 2.236288 (50 iterations in 3.49 seconds)
## Iteration 500: error is 2.099385 (50 iterations in 3.43 seconds)
## Fitting performed in 41.05 seconds.
```

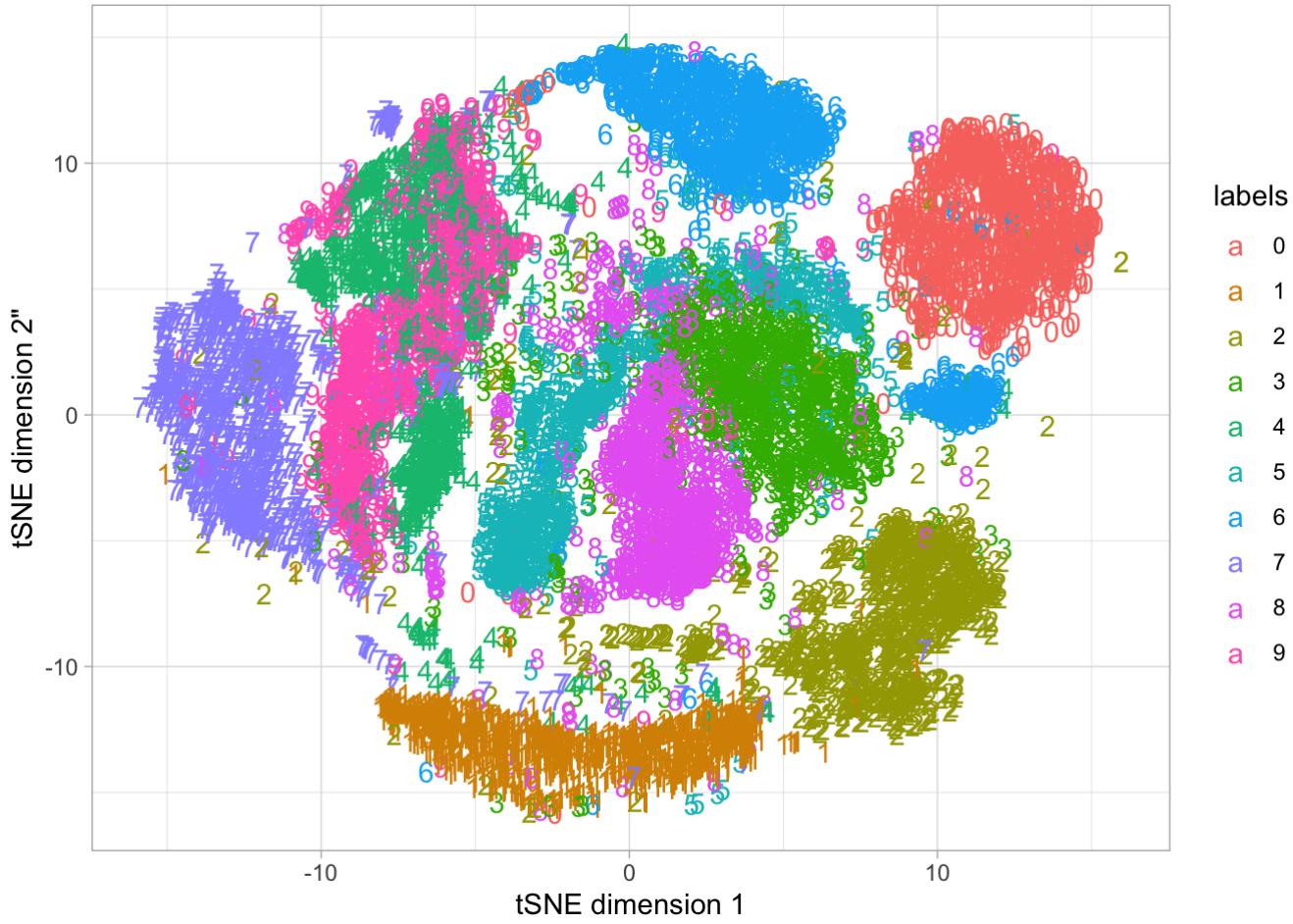
```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```




```
embedding = Rtsne(X = first_10k_samples, dims = 2,
                  perplexity = 160,
                  theta = 0.5,
                  eta = 100,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 500)
```

```
## Performing PCA
## Read the 10000 x 50 data matrix successfully!
## OpenMP is working. 1 threads.
## Using no_dims = 2, perplexity = 160.000000, and theta = 0.500000
## Computing input similarities...
## Building tree...
##  - point 10000 of 10000
## Done in 25.40 seconds (sparsity = 0.065215)!
## Learning embedding...
## Iteration 50: error is 77.767491 (50 iterations in 3.59 seconds)
## Iteration 100: error is 77.767490 (50 iterations in 3.84 seconds)
## Iteration 150: error is 76.618588 (50 iterations in 4.16 seconds)
## Iteration 200: error is 75.284082 (50 iterations in 4.21 seconds)
## Iteration 250: error is 74.980694 (50 iterations in 4.08 seconds)
## Iteration 300: error is 2.119916 (50 iterations in 3.41 seconds)
## Iteration 350: error is 1.849361 (50 iterations in 3.32 seconds)
## Iteration 400: error is 1.712394 (50 iterations in 3.28 seconds)
## Iteration 450: error is 1.629139 (50 iterations in 3.26 seconds)
## Iteration 500: error is 1.573809 (50 iterations in 3.25 seconds)
## Fitting performed in 36.41 seconds.
```

```
itercost = c(itercost,embedding$itercosts[10])
# Visualizing TSNE output
embedding_plot = tibble(x = embedding$Y[,1], y = embedding$Y[,2],
                        labels = as.character(first_10k_samples_labels))
ggplot(aes(x = x, y=y,label = labels, color = labels), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')
```


tSNE dimension 2"
10
0
-10
-10
0
10
tSNE dimension 1
labels
a 0
a 1
a 2
a 3
a 4
a 5
a 6
a 7
a 8
a 9