Ray-Based Modeling of Gravitational Fields

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Abstract

A novel framework for modeling gravitational fields is developed using a ray-based approach. At any observation point, the field is defined as the direction-weighted aggregation of line-integrated mass contributions along all incoming directions. In the static case, the formulation reproduces the classical Newtonian gravitational field and shows potential to replicate also relativistic scenarios. The approach represents an initial step in ray-based gravitational modeling, open to further refinement and generalization.

1 Introduction

This paper presents a preliminary exploration of ray-based representation of the gravitational field. The viewpoint is strictly local at the observation point $\vec{p}$, which is assumed to access only the mass traversed by each incoming direction. With no other knowledge about global distribution, the gravitational field at $\vec{p}$ is defined to be the aggregation of these directional contributions multiplied by an absorption factor. For clarity, the amount of mass traversed by a ray is obtained as the line integral of the mass density over that ray. As a heuristic—and only as a possible explanatory device, not an ontological claim—one may picture non-interacting *tracers* of invariant speed c that traverse matter without perturbing it and carry a counter proportional to the traversed mass.

The static case is first formulated by defining the field as a direction-weighted integral of line-integrated density. The definition is illustrated with a simple example, for which the field reduces to the familiar inverse-square law. We then establish the equivalence between the ray-based expression and the classical Newtonian formulation.

Next, we incorporate finite propagation and uniform source motion. The kinematics is organized through special-relativistic aberration of directions between the observation frame and a co-moving integration frame, together with retarded time along each ray. This leads to a mixed-frame definition that can be reformulated entirely within the moving frame. A worked example ilustrates the relativistic case using both the original definition and the alternative formulation.

We conclude with a brief final section that merely points to possible avenues for further investigation. These pointers are deliberately high-level and speculative, offered as indications of where the framework might be extended rather than as claims or detailed developments.

2 Static case

Definition 1. Let $\rho(\vec{x})$ be a mass density supported in a domain $V \subset \mathbb{R}^N$, $N \geq 3$. Outside this domain the density is zero, $\rho(\vec{x}) = 0$ for $\vec{x} \notin V$.

We define the *ray-based gravitational field* by

$$\boxed{\vec{g}_{\text{ray}}(\vec{p}) = -G \int_{\Omega} \left(\int_{L(\vec{p},\hat{u})} \rho(\vec{x})\ \mathrm{d}\ell \right) \hat{u}\ \mathrm{d}\Omega} \tag{1}$$

where $\vec{p} \in \mathbb{R}^N$, $\Omega = S^{N-1}$ is the unit angular domain, $L(\vec{p}, \hat{u})$ is the ray $\{\, \vec{p} + c\, t\, \hat{u} :\ t \leq 0 \,\}$, $\mathrm{d}\ell$ is the arc length along the ray, and G is a coupling factor.

Example 1. Let V be a solid sphere of radius R and total mass M with constant density $\rho_0 = \frac{3M}{4\pi R^3}$. Compute the ray-based gravitational field at a point located at distance $D > R$ from the sphere's center.

We perform a translation to place the point $\vec{p}$ at the origin, followed by a rotation to place the sphere's center along the positive z-axis. These changes result in $\vec{p} = \vec{0}$, and the center of the sphere at $\vec{c} = D\,\hat{d}$, where $\hat{d} = \hat{z}$ is the unit vector from $\vec{p}$ to the sphere's center.

For each unit direction $\hat{u}$ with polar angle θ measured from $\hat{d}$, parameterize the ray by arc length $s = -c\, t$ with $s \geq 0$:

$$\vec{x}(\hat{u}, s) = \vec{p} - s\,\hat{u} = -s\,\hat{u}, \qquad \mathrm{d}\ell = \mathrm{d}s$$

The ray intersects the sphere when $\|\vec{x}(\hat{u}, s) - \vec{c}\| = R$, i.e.

$$\| - s\,\hat{u} - D\,\hat{d}\|^2 = s^2 + 2Ds\,(\hat{u}\cdot\hat{d}) + D^2 = R^2$$

Writing $\cos\theta = \hat{u}\cdot\hat{d}$, the intersection parameters are the roots

$$s_{\pm}(\theta) = -D\cos\theta \pm \sqrt{R^2 - D^2\sin^2\theta}$$

which are real iff $D\sin\theta \leq R$, i.e. $\theta \in [\theta_0, \pi]$ with $\theta_0 = \pi - \arcsin(R/D)$. For such directions, the line-integrated density equals the chord length inside the sphere times ρ_0:

$$\int_{L(\vec{p},\hat{u})} \rho_0\ \mathrm{d}\ell = \int_{s_-}^{s_+} \rho_0\ \mathrm{d}s = \rho_0\, L(\theta), \qquad L(\theta) = s_+ - s_- = 2\sqrt{R^2 - D^2\sin^2\theta}$$

By symmetry, $\vec{g}_{\text{ray}}(\vec{p})$ is parallel to $\hat{d}$. Using spherical coordinates around $\hat{d}$, $\mathrm{d}\Omega = \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi$, the ray-based field is

$$\vec{g}_{\text{ray}}(\vec{p}) = -G\,\rho_0\,\hat{d} \int_0^{2\pi}\!\! \int_{\theta_0}^{\pi} L(\theta)\ \underbrace{\cos\theta}_{\hat{u}\cdot\hat{d}}\ \underbrace{\sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi}_{\mathrm{d}\Omega}$$

Substituting $L(\theta) = 2\sqrt{R^2 - D^2 \sin^2\theta}$,

$$\vec{g}_{\text{ray}}(\vec{p}) = -2G\,\rho_0\,\hat{d}\int_0^{2\pi}\int_{\theta_0}^{\pi}\sqrt{R^2 - D^2\sin^2\theta}\,\cos\theta\,\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Evaluate the ϕ-integral and set

$$I := \int_{\theta_0}^{\pi}\sqrt{R^2 - D^2\sin^2\theta}\,\cos\theta\,\sin\theta\,\mathrm{d}\theta$$

With $u = R^2 - D^2\sin^2\theta$ we have $\mathrm{d}u = -2D^2\sin\theta\cos\theta\,\mathrm{d}\theta$ and

$$I = \frac{-1}{2D^2}\int_{u(\theta_0)}^{u(\pi)} u^{1/2}\,\mathrm{d}u = \frac{-1}{2D^2}\cdot\frac{2}{3}\Big[u^{3/2}\Big]_{u=0}^{u=R^2} = -\frac{R^3}{3D^2}$$

Hence

$$\vec{g}_{\text{ray}}(\vec{p}) = -2G\,\rho_0\,\hat{d}\,2\pi\,I = \frac{4\pi G\rho_0 R^3}{3D^2}\,\hat{d} = G\,\frac{M}{D^2}\,\hat{d}$$

since $\rho_0 = 3M/(4\pi R^3)$. Therefore, the ray-based field reproduces the classical inverse-square law outside the sphere:

$$\vec{g}(\vec{p}) = G\,\frac{M}{D^2}\,\hat{d}$$

Proposition 1 (Equivalence)**.** Let $N \geq 3$ and let $\rho \in L^1(\mathbb{R}^N)$ have compact support V.

For any $\vec{p} \notin \operatorname{supp}\rho$, the ray-based field (1),

$$\vec{g}_{\text{ray}}(\vec{p}) = -G\int_{\Omega}\left(\int_{L(\vec{p},\hat{u})}\rho(\vec{x})\,\mathrm{d}\ell\right)\hat{u}\,\mathrm{d}\Omega$$

coincides with the Newtonian field [3],

$$\vec{g}(\vec{p}) = -G\int_V \frac{\vec{p} - \vec{x}}{\|\vec{p} - \vec{x}\|^N}\,\rho(\vec{x})\,\mathrm{d}V \tag{2}$$

Proof. To demonstrate the equivalence, we will expand the line integral in the ray-based field definition, then we will rewrite the classical Newtonian field in spherical coordinates, and finally we will compare the two expressions to show they are identical.

Step 1 – Expand the line integral in the ray-based field.

In spherical coordinates centered at $\vec{p}$, any point $\vec{x}$ over the ray is parameterized by the arc length by:

$$\vec{x} = \vec{p} - s\hat{u}$$

where $s = \|\vec{p} - \vec{x}\| \geq 0$ is the radial distance, and $\hat{u}$ is the unit vector indicating the direction from $\vec{x}$ to $\vec{p}$.

The differential arc length along the ray $L(\vec{p},\hat{u})$ is $\mathrm{d}\ell = \mathrm{d}s$. Substituting this into the ray-based field:

$$\vec{g}_{\text{ray}}(\vec{p}) = -G \int_{\Omega} \left(\int_{I(\hat{u})} \rho(\vec{p} - s\hat{u}) \, \mathrm{d}s \right) \hat{u} \, \mathrm{d}\Omega$$

where $I(\hat{u})$ is the interval of s such that $\vec{p} - s\hat{u} \in V$.

Step 2 – Express the Newtonian field in spherical coordinates.

In the Newtonian field, substitute $\vec{x} = \vec{p} - s\hat{u}$ and $\|\vec{p} - \vec{x}\| = s$. Then:

$$\frac{\vec{p} - \vec{x}}{\|\vec{p} - \vec{x}\|^N} = \frac{s\hat{u}}{|s|^N} = \frac{\hat{u}}{|s|^{N-1}}$$

The volume element in spherical coordinates is $\mathrm{d}V = |s|^{N-1} \, \mathrm{d}s \, \mathrm{d}\Omega$. Substituting these into the Newtonian field:

$$\vec{g}(\vec{p}) = -G \int_V \frac{\hat{u}}{|s|^{N-1}} \, \rho(\vec{p} - s\hat{u}) \, |s|^{N-1} \, \mathrm{d}s \, \mathrm{d}\Omega$$

Simplifying:

$$\vec{g}(\vec{p}) = -G \int_{\Omega} \int_{I(\hat{u})} \hat{u} \, \rho(\vec{p} - s\hat{u}) \, \mathrm{d}s \, \mathrm{d}\Omega$$

Step 3 – Verify equivalence.

The ray-based field is:

$$\vec{g}_{\text{ray}}(\vec{p}) = -G \int_{\Omega} \left(\int_{I(\hat{u})} \rho(\vec{p} - s\hat{u}) \, \mathrm{d}s \right) \hat{u} \, \mathrm{d}\Omega$$

The Newtonian field is:

$$\vec{g}(\vec{p}) = -G \int_{\Omega} \int_{I(\hat{u})} \hat{u} \, \rho(\vec{p} - s\hat{u}) \, \mathrm{d}s \, \mathrm{d}\Omega$$

By the Tonelli and Fubini theorems ([2], [1]), the order of integration can be interchanged under the given assumptions (e.g., $\rho \in L^1(\mathbb{R}^N)$ with compact support or sufficient decay at infinity). Thus, the two expressions are identical.

□

3 Relativistic case

Building upon the static case, this section explores the relativistic extension of the ray-based gravitational field. In contrast to the static case, where closed-form solutions were derived and shown to be equivalent to the classical Newtonian gravitational field, the relativistic case does not yet yield results of this kind. Instead, we present preliminary indications and formulations that suggest the framework may be extended to more general scenarios.

We consider two reference frames:

- S - *Rest frame*: The reference frame in which the gravitational field is measured. The evaluation point $\vec{p}$ is at rest and receives isotropic radiation of rays with constant velocity c.
- S' - *Moving frame*: The reference frame in which the mass moves at velocity $\vec{v} = v\,\hat{v}$ relative to S.

Rays are parameterized based on their direction $\hat{u}$ and time t (negative values represent the past, and positive values represent the future).

In the S frame, the ray is parameterized as

$$r(\hat{u}, t) = \vec{p} + t\,c\,\hat{u}, \quad t \leq 0$$

In the S' frame, the same ray is parameterized as

$$r'(\hat{u}, t) = \vec{p} + t\,c\,h(\hat{u}; \vec{v}), \quad t \leq 0$$

where $h()$ is the angular aberration coefficient. When the mass moves with velocity $\vec{v}$ relative to the observation point, the angle of the ray observed at $\vec{p}$ differs from the angle of the ray that has traversed the mass. This discrepancy is called angular aberration and is modeled using the Lorentz transformation.

3.1 Angular aberration

The results presented in this section are well-established and can be found in [4]. These results will serve as a foundational basis for the relativistic formulations and derivations developed in subsequent sections.

Given a velocity $\vec{v} = v\,\hat{v}$, we denote the decomposition of any vector $\vec{w}$ into its projections along $\vec{v}$ and its perpendicular plane as

$$\vec{w} = \vec{w}_{\perp} + \vec{w}_{||} = |\vec{w}|\sin(\theta)\;\hat{v}_{\perp} + |\vec{w}|\cos\theta\;\hat{v}, \quad \cos\theta = \hat{w}\cdot\hat{v}$$

The function $h()$ transforms a direction $\hat{u}$ in S (the rest frame) to the corresponding direction in S' (the moving frame).

$$h(\hat{u}; \vec{v}) = \frac{\sqrt{1-\beta^2}\;\hat{u}_{\perp} + (\cos\theta - \beta)\,\hat{v}}{1 - \beta\cos\theta}, \qquad \beta = \frac{v}{c}, \quad \cos\theta = \hat{u}\cdot\hat{v}$$

The inverse transformation ($S' \to S$) is

$$h^{-1}(\hat{u}'; \vec{v}) = \frac{\sqrt{1-\beta^2}\;\hat{u}'_{\perp} + (\cos\theta' + \beta)\,\hat{v}}{1 + \beta\cos\theta'}, \qquad \beta = \frac{v}{c}, \quad \cos\theta' = \hat{u}'\cdot\hat{v}$$

If the polar axis is aligned with $\vec{v}$, i.e., $\hat{v} \parallel \hat{z}$, then

$$\phi' = \phi, \qquad \cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}, \qquad \sin\theta' = \frac{\sqrt{1-\beta^2}\;\sin\theta}{1 - \beta\cos\theta}$$

$$\phi = \phi', \qquad \cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta'}, \qquad \sin\theta = \frac{\sqrt{1-\beta^2}\,\sin\theta'}{1 + \beta\cos\theta'}$$

The Jacobian of the solid angle transformation is

$$\frac{d\Omega'}{d\Omega} = \frac{1-\beta^2}{(1-\beta\cos\theta)^2} \quad \Longrightarrow \quad \frac{d\Omega}{d\Omega'} = \frac{(1+\beta\cos\theta')^2}{1-\beta^2}$$

3.2 Ray-based gravitational field

Definition 2. Let $\rho(\vec{x})$ be a mass density supported in a domain $V \subset \mathbb{R}^N$, $N \geq 3$. Outside this domain the density is zero, $\rho(\vec{x}) = 0$ for $\vec{x} \notin V$.

We define the *ray-based gravitational field* at observation time t_0 by

$$\boxed{\vec{g}_{\text{ray}}(\vec{p}, t_0) = -G \int_{\Omega} \left(\int_{L(\vec{p},\, h(\hat{u};\vec{v}))} \rho(\vec{x}', t_{\text{ret}})\ \mathrm{d}\ell' \right) \hat{u}\ \mathrm{d}\Omega, \quad t_{\text{ret}} = t_0 - \ell'/c} \tag{3}$$

where $\vec{p} \in \mathbb{R}^N$, $\Omega = S^{N-1}$ is the unit angular domain, $L(\vec{p}, \hat{u})$ is the ray $\{\, \vec{p} + c\,t\,\hat{u} :\ t \leq 0 \,\}$, $\mathrm{d}\ell$ is the arc length along the ray, G is a coupling factor, and $h(\hat{u};\vec{v})$ is the angular aberration function that transforms the direction $\hat{u}$ to the corresponding direction in the moving frame.

We observe that the calculation of the ray-based gravitational field is performed in two reference frames. The contribution of each ray is computed in the frame S', and the summation of contributions is carried out in the frame S. The following result reformulates the definition entirely in S'.

Proposition 2 (Alternative formulation)**.** Let $\rho(\vec{x})$ be a mass density supported in a domain $V \subset \mathbb{R}^N$, $N \geq 3$. Outside this domain, the density is zero, $\rho(\vec{x}) = 0$ for $\vec{x} \notin V$.

The ray-based gravitational field at observation time t_0 is:

$$\boxed{\vec{g}_{\text{ray}}(\vec{p}, t_0) = \frac{-G}{1-\beta^2} \int_{\Omega'} \left(\int_{L(\vec{p},\hat{u}')} \rho(\vec{x}', t_{\text{ret}})\ \mathrm{d}\ell' \right) \left(\sqrt{1-\beta^2}\, \hat{u}'_{\perp} + (\cos\theta' + \beta)\, \hat{v} \right) (1 + \beta\cos\theta')\ \mathrm{d}\Omega'} \tag{4}$$

Proof. We apply a change of variables to the ray-based gravitational field definition to obtain the proposition statement.

Starting from the original definition (3):

$$\vec{g}_{\text{ray}}(\vec{p}, t_0) = -G \int_{\Omega} \left(\int_{L(\vec{p},\, h(\hat{u};\vec{v}))} \rho(\vec{x}', t_{\text{ret}})\, d\ell' \right) \hat{u}\ \mathrm{d}\Omega$$

we perform the change of variables $\hat{u}' = h(\hat{u};\vec{v})$, i.e., $\hat{u} = h^{-1}(\hat{u}';\vec{v})$, and use the Jacobian $d\Omega = \frac{d\Omega}{d\Omega'}\, d\Omega'$ with

$$h^{-1}(\hat{u}';\vec{v}) = \frac{\sqrt{1-\beta^2}\;\hat{u}'_\perp + (\cos\theta' + \beta)\,\hat{v}}{1+\beta\cos\theta'}, \qquad \frac{d\Omega}{d\Omega'} = \frac{(1+\beta\cos\theta')^2}{1-\beta^2}.$$

Thus, we have:

$$\vec{g}_{\text{ray}}(\vec{p},t_0) = -G\int_{\Omega'}\left(\int_{L(\vec{p},\,\hat{u}')}\rho(\vec{x}',t_{\text{ret}})\;\mathrm{d}\ell'\right)\, h^{-1}(\hat{u}';\vec{v})\,\frac{d\Omega}{d\Omega'}\;\mathrm{d}\Omega'$$

and, simplifying the factors we obtain the final expression:

$$\vec{g}_{\text{ray}}(\vec{p},t_0) = \frac{-G}{1-\beta^2}\int_{\Omega'}\left(\int_{L(\vec{p},\,\hat{u}')}\rho(\vec{x}',t_{\text{ret}})\;\mathrm{d}\ell'\right)\left(\sqrt{1-\beta^2}\;\hat{u}'_\perp + (\cos\theta'+\beta)\,\hat{v}\right)(1+\beta\cos\theta')\;\mathrm{d}\Omega'$$

$\square$

3.3 Examples

We calculate the ray-based gravitational field at time t_0 generated by a sphere of radius R, mass M, and uniform density moving at a constant velocity $\vec{v}$, acting on a point $\vec{p}$ located at a distance D from the center of the sphere, with $D > R$. The sphere is moving towards the observation point.

We perform a translation to place the point $\vec{p}$ at the origin, followed by a rotation to align the z-axis with $\vec{v}$. These transformations result in:

- The point $\vec{p}$ remains at the origin, $\vec{p} = (0,0,0)$,
- The velocity vector of the sphere is $\vec{v} = -v\,\hat{z}$, with $v \geq 0$
- At t_0 the center of the sphere is located at $(0,0,D) = \vec{d} = D\hat{d} = D\hat{z}$.

Example 2 (mixed-frames)**.** We resolve the case using the ray-based field definition (3).

We parametrize the ray using the aberrated direction $h(\hat{u};\vec{v})$ while keeping the sphere worldline as in S (hence "mixed-frames"). We seek the intersection points between the sphere and the ray.

$$\left.\begin{array}{lrl} \text{Ray:} & \vec{x} & = \vec{p} + t\,c\,h(\hat{u},\vec{v}) \\ \text{Sphere:} & x^2+y^2+(z-D+v\,t)^2 & = R^2 \end{array}\right\}, \quad t \leq 0$$

We change coordinates using $\ell = -t\,c$ (arc length):

$$\left.\begin{array}{lrl} \text{Ray:} & \vec{x} & = \vec{p} - \ell\,h(\hat{u},\vec{v}) \\ \text{Sphere:} & x^2+y^2+(z-D-\beta\,\ell)^2 & = R^2 \end{array}\right\}, \quad \ell \geq 0$$

Aberration formulas apply for $\vec{v}$ aligned to the polar axis, but this is not the current case, $\vec{v} = -v\,\hat{z}$. However, we can handle this case by replacing $\beta \to -\beta$ everywhere in the aberration formulas. Thus,

$$h(\hat{u}; -v\hat{z}) = \frac{\sqrt{1-\beta^2}\,\hat{u}_\perp + (\cos\theta + \beta)\,\hat{z}}{1+\beta\cos\theta}.$$

$$\cos\theta' = \frac{\cos\theta + \beta}{1+\beta\cos\theta}, \quad \sin\theta' = \frac{\sqrt{1-\beta^2}\,\sin\theta}{1+\beta\cos\theta}$$

$$\frac{d\Omega'}{d\Omega} = \frac{1-\beta^2}{(1+\beta\cos\theta)^2} \quad \Longrightarrow \quad \frac{d\Omega}{d\Omega'} = \frac{(1-\beta\cos\theta')^2}{1-\beta^2}$$

We write $\hat{u}$ in angular coordinates,

$$\hat{u} = \big(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta\big) \quad \theta \in [0,\pi],\ \phi \in [0, 2\pi)$$

We expand $\vec{x}$ in these coordinates, using the aberration formula for $\vec{v} = -v\,\hat{z}$ (see remark above):

$$\vec{x}(\hat{u}, \ell) = \Big(-\ell\,\frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta}\sin\theta\cos\phi,\ -\ell\,\frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta}\sin\theta\sin\phi,\ -\ell\,\frac{\cos\theta+\beta}{1+\beta\cos\theta}\Big)$$

Finally, we substitute $\vec{x}(\hat{u}, \ell)$ into the sphere's equation. This yields a quadratic polynomial in ℓ.

$$\ell^2 \frac{1-\beta^2}{(1+\beta\cos\theta)^2}\sin^2\theta + \left(-\frac{\cos\theta+\beta}{1+\beta\cos\theta}\ell - \beta\ell - D\right)^2 = R^2$$

Reorganizing terms by the coefficient of ℓ, we obtain:

$$A(\theta)\,\ell^2 + 2B(\theta)\,\ell + C(\theta) = 0, \quad \begin{cases} A(\theta) = (1-\beta^2)(1-\cos^2\theta) + (\cos\theta + \beta^2\cos\theta + 2\beta)^2 \\ B(\theta) = D\,(1+\beta\cos\theta)\,(\cos\theta + \beta^2\cos\theta + 2\beta) \\ C(\theta) = (1+\beta\cos\theta)^2(D^2 - R^2) \end{cases}$$

The chord length traversed by the ray is:

$$L(\theta) = 2\sqrt{\left(\frac{B(\theta)}{A(\theta)}\right)^2 - \frac{C(\theta)}{A(\theta)}}$$

The range of θ is determined by ensuring the discriminant is positive. Let θ_0 be the first root less than π such that the discriminant is zero.

The gravitational field is:

$$\vec{g}_{\text{ray}}(\vec{p}, 0) = -G\rho_0 \int_0^{2\pi}\int_{\theta_0}^{\pi} L(\theta)\left(\sin\theta\cos\phi\,\hat{\mathbf{i}} + \sin\theta\sin\phi\,\hat{\mathbf{j}} + \cos\theta\,\hat{\mathbf{k}}\right)\sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi$$

The components $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ cancel due to the ϕ factor.

Using the substitution $\mu = \cos\theta$ and denoting $s = \frac{D}{R}$:

$$L(\mu) = \frac{2R\sqrt{B^2(\mu) - A(\mu)C(\mu)}}{A(\mu)}, \qquad \begin{cases} A(\mu) = (1-\beta^2)(1-\mu^2) + (\mu + \beta^2\mu + 2\beta)^2 \\ B(\mu) = s\,(1+\beta\mu)(\mu + \beta^2\mu + 2\beta) \\ C(\mu) = (1+\beta\mu)^2(s^2-1) \end{cases}$$

We determine the range of μ by ensuring the discriminant is positive. This results in a degree-4 polynomial in μ. Let μ_0 be the first real root greater than -1.

The gravitational field simplifies to:

$$\vec{g}_{\text{ray}}(\vec{p}, 0) = -2\pi\hat{d}\,G\rho_0 \int_{-1}^{\mu_0} L(\mu)\,\mu\;\mathrm{d}\mu$$

Numerical integrations indicate qualitative agreement with the angular scaling of the linearized Liénard–Wiechert field:

$$\vec{g}_{\text{ray}}(\vec{p}) \;\propto\; -\frac{GM}{D^2}\,\frac{1-\beta^2}{\left(1+\beta\cos\theta\right)^3}, \qquad \beta = v/c$$

when interpreted as a shape/angle guide rather than an identity. Asymptotically (small β and/or far field $D \gg R$), the mixed-frames result approaches the Liénard–Wiechert formula.

Example 3 (single-frame)**.** We resolve the same problem as in the previous example using the alternative definition of the gravitational field (4).

In the S' frame, we parametrize the rays and the sphere to determine the amount of mass traversed by each ray. We seek the intersection points between the sphere and the ray.

$$\left.\begin{array}{rcl} \text{Ray:} & \vec{x} & = \vec{p} + t\,c\,\hat{u}' \\ \text{Sphere:} & x^2 + y^2 + (z - D + v\,t)^2 & = R^2 \end{array}\right\}, \quad t \le 0$$

We change coordinates using $\ell = -t\,c$ (arc length):

$$\left.\begin{array}{rcl} \text{Ray:} & \vec{x} & = \vec{p} - \ell\,\hat{u}' \\ \text{Sphere:} & x^2 + y^2 + (z - D - \beta\,\ell)^2 & = R^2 \end{array}\right\}, \quad \ell \ge 0$$

Aberration formulas apply for $\vec{v}$ aligned to the polar axis, but this is not the current case, $\vec{v} = -v\,\hat{z}$. However, we can handle this case by replacing $\beta \to -\beta$ everywhere in the alternative definition (4).

We write $\hat{u}'$ in polar coordinates,

$$\hat{u}' = \big(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta\big), \quad \theta \in [0,\pi],\ \phi \in [0, 2\pi)$$

We expand $\vec{x}$ in these coordinates as,

$$\vec{x}(\hat{u}', \ell) = \Big(-\ell\,\sin\theta\cos\phi,\ -\ell\,\sin\theta\sin\phi,\ -\ell\,\cos\theta\Big)$$

Finally, we substitute $\vec{x}(\hat{u}', \ell)$ into the sphere equation. This yields in a quadratic polynomial in ℓ

$$\ell^2 \sin^2 \theta + (-\cos\theta\,\ell - \beta\ell - D)^2 = R^2$$

Rearranging terms by the coefficient of ℓ, we obtain:

$$A(\theta)\,\ell^2 + 2B(\theta)\,\ell + C(\theta) = 0, \quad \begin{cases} A(\theta) = 1 + 2\beta\cos\theta + \beta^2 \\ B(\theta) = D\,(\cos\theta + \beta) \\ C(\theta) = D^2 - R^2 \end{cases}$$

The chord length traversed by the ray is:

$$L(\theta) = 2\sqrt{\left(\frac{B(\theta)}{A(\theta)}\right)^2 - \frac{C(\theta)}{A(\theta)}}$$

Using the substitution $\mu = \cos\theta$ and defining $s = \frac{D}{R}$:

$$L(\mu) = \frac{2R\,\sqrt{A(\mu) - s^2(1-\mu^2)}}{A(\mu)}, \quad A(\mu) = 1 + 2\beta\mu + \beta^2$$

We determine the range of μ by ensuring the discriminant is positive and selecting the first root greater than -1:

$$\mu_0 = \frac{-\beta - \sqrt{(s^2-1)(s^2-\beta^2)}}{s^2}$$

Using the alternative definition of the gravitational field:

$$\vec{g}_{\text{ray}}(\vec{p}, t_0) = -\frac{G}{1-\beta^2}\,\hat{d}\,2\pi\,\rho_0 \int_{-1}^{\mu_0} \underbrace{L(\mu)}_{\text{line integral}}\ \underbrace{(\mu-\beta)}_{h^{-1}(\hat{u}')}\ \underbrace{(1-\beta\mu)\ \mathrm{d}\mu}_{\frac{d\Omega}{d\Omega'}d\Omega'}$$

The numerical results obtained are the same as those in Example 2.

Final Notes

We outlined a ray-transport construction that reproduces the Newtonian field in the static case and there are indications that it can be extended to the relativistic case. The following are aspects to consider in future research. These pointers are deliberately high-level and speculative, offered as indications of where the framework might be extended rather than as claims or detailed developments.

Inner case. The equivalence result in Proposition 1 is valid only outside the support of the mass density. Verify that the same result is obtained inside the support.

Cross-links to other areas of mathematics. The exploration of links with other mathematical frameworks remains open. In particular, it is natural to relate our line-integral operator to integral geometry and to the Radon/X-ray transform.

n**-dimensional manifold instead of a ray**. Consider generalizing the ray concept to an n-dimensional manifold. This generalization could be particularly useful in non-Euclidean geometries. For instance, in the one-dimensional case, the manifold could correspond to a geodesic. Such an approach would facilitate connections with geometric formulations, such as those found in General Relativity.

Equivalence with the standard model. The equivalence between the ray-based model and the standard formulation of gravity (GR) should be demonstrated, or at least the domain of validity of the ray-based model should be delimited. This could involve expanding the notion of a ray or reformulating or extending the definition of the ray-based field.

References

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