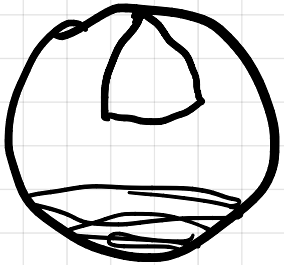
Visual and Scientific Computing

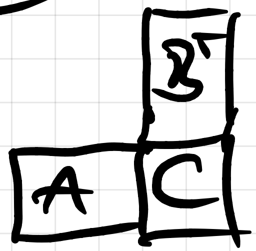
WS 2019/20

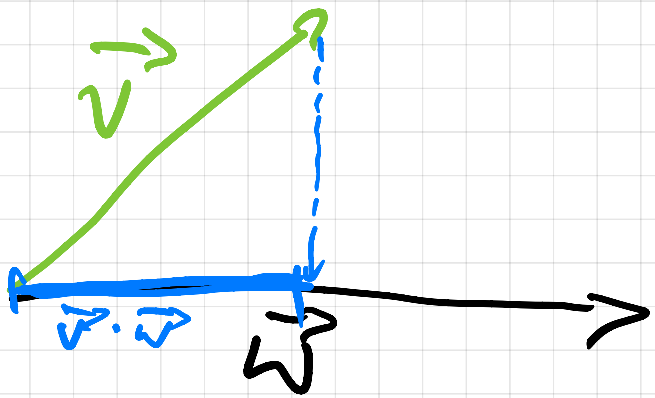


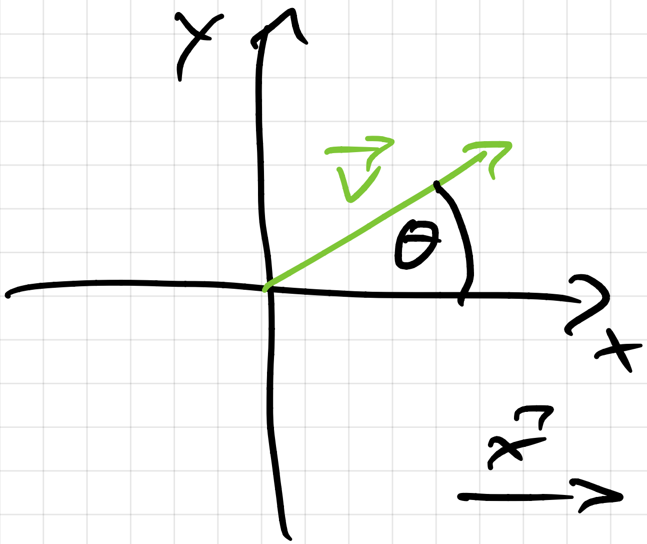
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

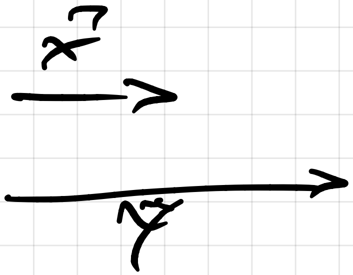
$$B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

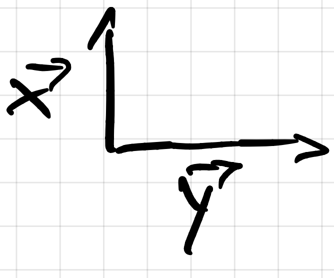
$\boxed{A} * \boxed{B}$



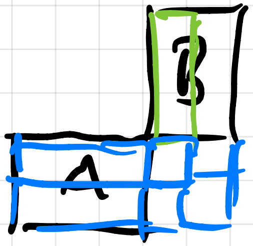


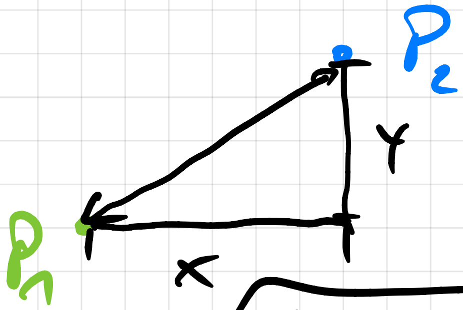






$x \cdot y = 0$





$x + y \Rightarrow L_1$-Norm

$\sqrt{(x^2 + y^2)}$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $\underbrace{\text{Rot, Sca, Shearing, Mirror}}_{\text{Lin. Trans.}}$
- Trans

Schulbsp.:

$$6a + 12b = 30$$
$$3a + 3b = 9$$

$$A = \begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$$

$$b = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$

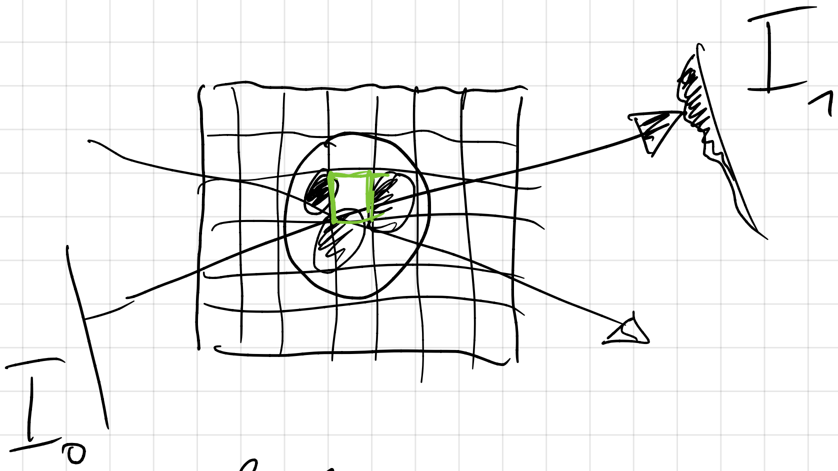
$$Ax = b$$

$$f(x) = x^2 \qquad f'(x) = 2x$$

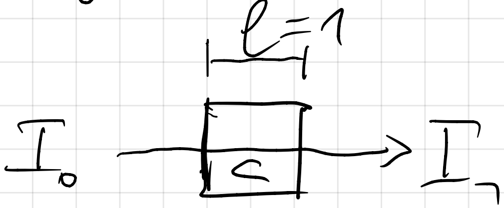
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Problem → math. Formulierung → Lösung

Lin GS



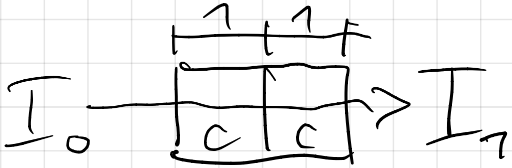
$c \in]0,1]$



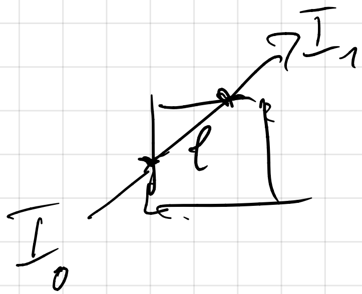
$$I_1 = I_0 \cdot c$$

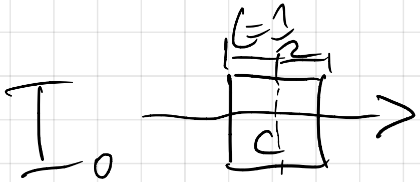
Weg des Strahl durch Element

$$I_1 = I_0 \cdot c^{\ell}$$

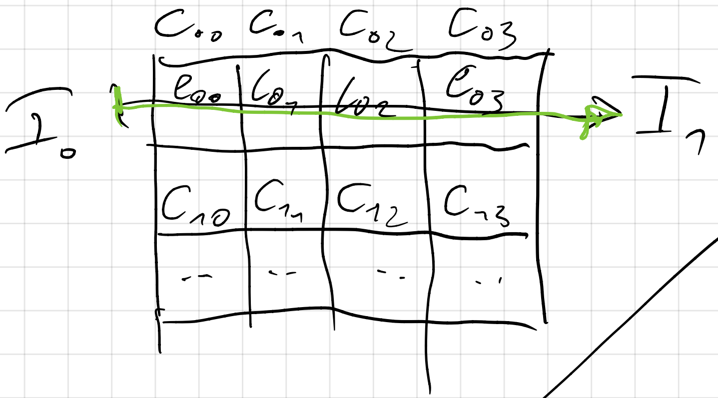


$$I_1 = I_0 \cdot c^2$$

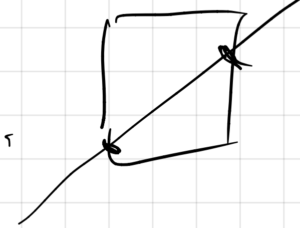




$$I_1 = I_0 \cdot c^{\frac{1}{2}}$$



$$I_1 = I_0 \cdot c_{00}^{l_{00}} \cdot c_{01}^{l_{01}} \cdot c_{02}^{l_{02}} \cdot c_{03}^{l_{03}} \cdot \ldots$$



$$Ax = b$$

$A x \qquad \vec{b}$

$$a_{00} x_1 + a_{01} x_2 + \ldots = b_0$$

$$= b_1$$

$I_0 = 1$

$$\underbrace{\log I_1}_{b} = \underbrace{\log I_0}_{0} + \underbrace{l_{00} \cdot \log c_{00} + l_{01} \cdot \log c_{01} + l_{01} \log c_{02} + \ldots}$$

$$x_{ij} = \log c_{ij} \rightarrow \text{Unbekannt}$$

$$l_{00} \ldots l_{nm} \rightarrow A$$

$$L \vec{x} = \vec{b}$$

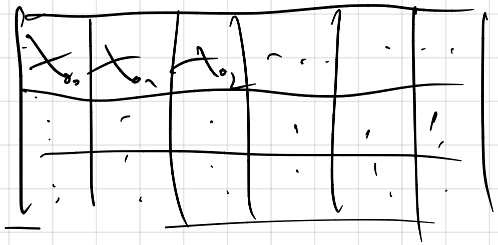
↗ Strahlindex

$$b_0 = l^0_{00} \cdot x_{00} + l^0_{01} \cdot x_{01} + l^0_{02} \cdot x_{02} \ldots$$

$$b_1 = l^1_{00} \cdot x_{00} + l^1_{01} x_{01} + l^1_{02} x_{02} \ldots$$

$L\vec{x} = \vec{b}$

$\vec{x} = [x_{00}, x_{01}, \dots x_{0m}, x_{10}, x_{11} \dots x_{nm}]$

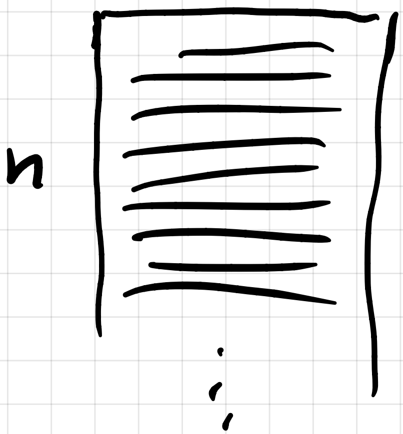


$[L][x] = [b]$

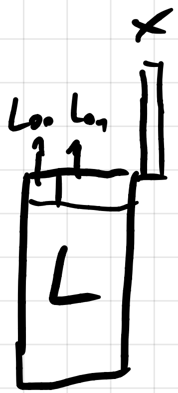
Raster $\neq$ Matrix
unbekannt

sondern Vektor $\vec{x}$

L_{100} $L_{n \times m}$



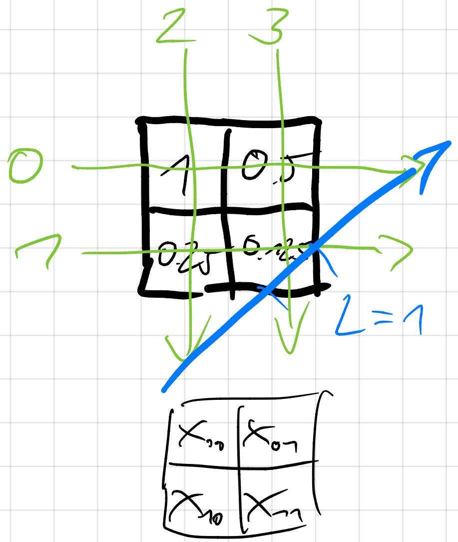
überbestimmtes LinGS



$l_{00}x_{00} + l_{01}x_{01} \dots$

$= [\ ]$ b

Bsp:



$I_0 = 1 \qquad L = 1$

$$\left.\begin{array}{l} 1 \cdot 0.5 = \frac{1}{2} \\ \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32} \\ 1 \cdot \frac{1}{4} = \frac{1}{4} \\ \cancel{\frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}} \end{array}\right\} I_1 \longrightarrow \vec{b}$$

$\frac{1}{8}$

Log zur Basis 2

$$\begin{bmatrix} b_0 = -1 \\ b_1 = -5 \\ b_2 = -2 \\ \cancel{b_3 = -4} \end{bmatrix}$$

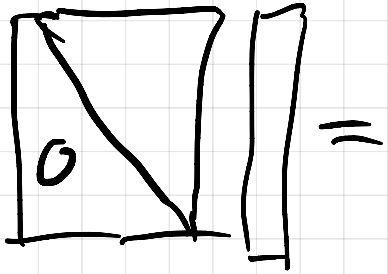
$b_4 = -3$

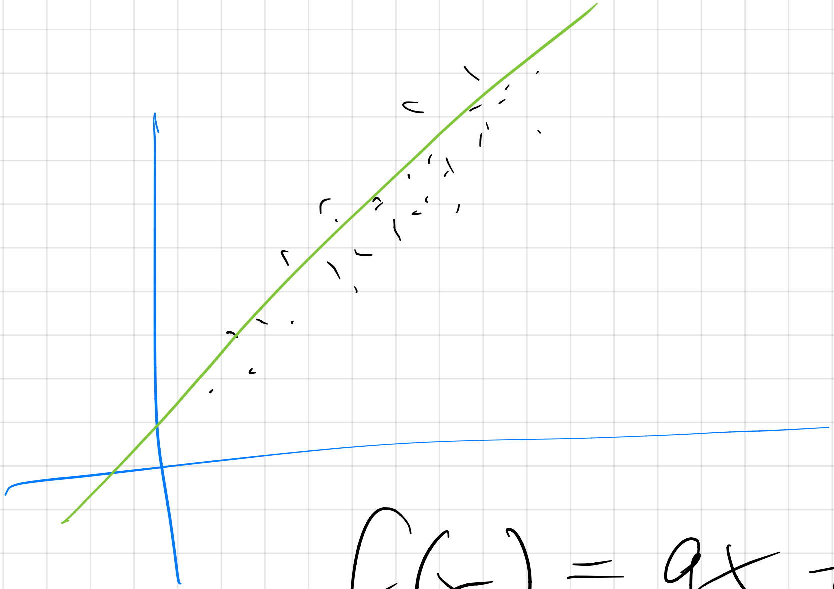
$$L = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} \end{bmatrix}$$

$0 \quad 0 \quad 0 \quad 1$

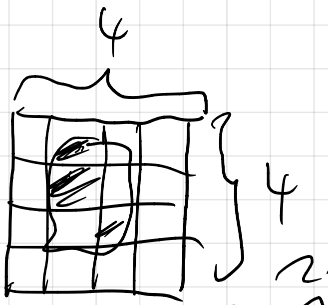
$$\begin{bmatrix} l^0_{00} & l^0_{01} & l^0_{10} & l^0_{11} \\ l^1_{00} & l^1_{01} & l^1_{10} & l^1_{11} \\ l^2_{00} & l^2_{01} & l^2_{10} & l^2_{11} \\ l^3 & l^3 & l^3 & l^3 \end{bmatrix} \begin{bmatrix} x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \end{bmatrix} = \begin{bmatrix} b^0 \\ b^1 \\ b^2 \\ b^3 \end{bmatrix}$$

$$Ax = b$$

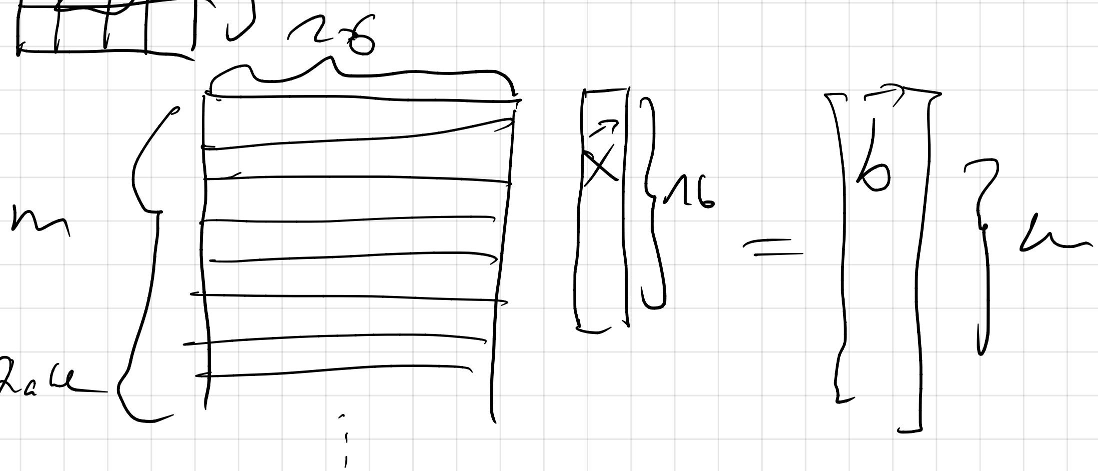




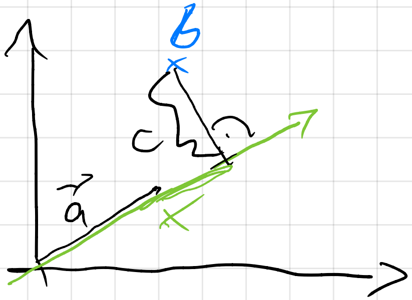
$$f(x) = ax + b$$



$$Ax = b$$



m = Anzahl der [illegible]



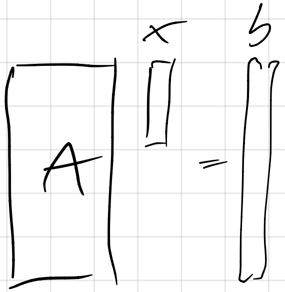
$$\vec{a} \cdot x \approx \vec{b} \qquad a \cdot x \neq b$$

$$c = \vec{a}x - \vec{b} \qquad \vec{a}^T(\vec{a}x - b) = 0$$

$$\iff \vec{a}^T\vec{a}x = \vec{a}^T b$$

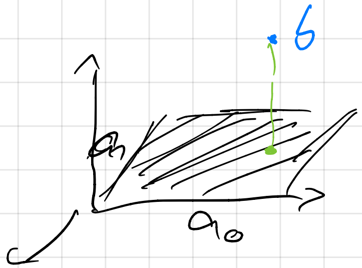
$$\vec{x} \in R^n \qquad A = [\vec{a}_0, \vec{a}_1, \dots, \vec{a}_n] \qquad \vec{a} \in R^d$$

$$n < d$$



$$Ax - b \approx 0 \qquad \vec{a}_0\vec{x}_0 + \vec{a}_1\vec{x}_1 + \dots a_n\vec{x}_n = \vec{b}$$

$$a_0^T(Ax - b) = 0$$
$$\vec{a}_1^T(Ax - b) = 0$$
$$\vdots$$



$$A^T(Ax-b)=0$$

$$\boxed{A^TAx = A^Tb} \rightarrow \text{Normalengleichung}$$

$$\min_{x\in\mathbb{R}^n} \|Ax-b\|^2 \qquad \sum_i^n r_i^2 = \|Ax_i - b\|^2$$

$$\sum_i^m r_i^2 = \|Ax_i - b\|^2$$

$$\hookrightarrow r^Tr = F$$

$$(b^TA)^T = A^Tb$$

$$\frac{\partial F}{\partial x_i}$$

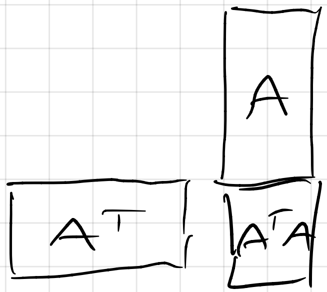
$$r^Tr = (Ax-b)^T(Ax-b)$$

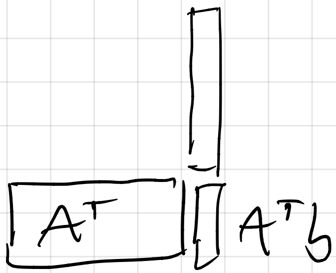
$$=(x^TA^T-b^T)(Ax-b)$$

$$=\underbrace{x^TA^TAx - x^TA^Tb - b^TAx + b^Tb}_{\text{Ableitung}}$$

$$\frac{\partial F}{\partial x} = 2A^TAx - A^Tb - b^TA = \cancel{2}A^TAx \overset{!}{=} \cancel{2}A^Tb$$

$$\frac{\partial F}{\partial x} = \boxed{A^TAx - A^Tb = 0}$$

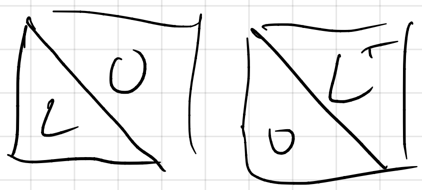


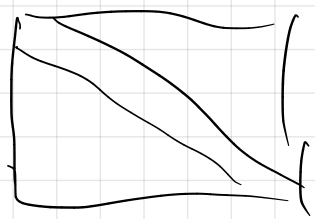


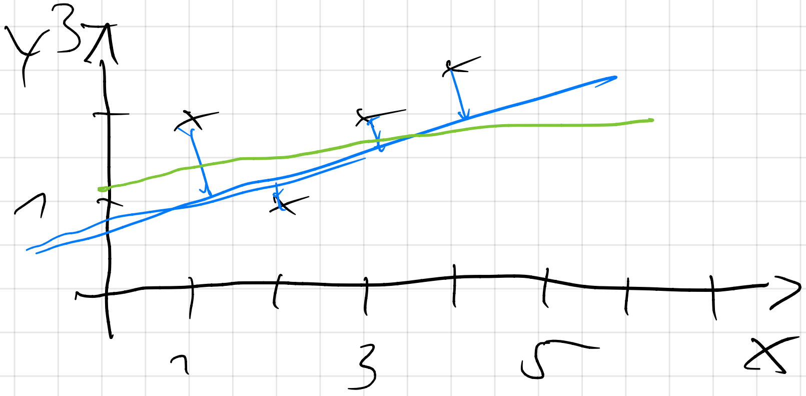
Cholesky $\longrightarrow$ Workhorse für $A^TAx = A^Tb$

$$\underbrace{A^TA}_{T}$$

$$T = LL^T$$







$$\begin{pmatrix} 1,2 \\ 2,1 \\ 3,2 \\ 4,2.5 \end{pmatrix}$$

$$f(x) = a_0 x + a_1$$

$$y = a_0 x^1 + a_1 x^0$$

$$\begin{aligned} 2 &= a_0 \cdot 1 + a_1 \cdot 1 \\ 1 &= a_0 \cdot 2 + a_1 \cdot 1 \\ 2 &= a_0 \cdot 3 + a_1 \cdot 1 \\ 2{,}5 &= a_0 \cdot 4 + a_1 \cdot 1 \end{aligned}$$

b

$$x = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$A \quad x \quad = \quad b$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2{,}5 \end{bmatrix}$$

$$\min_{x \in \mathbb{R}^n} \| Ax - b \|^2$$