Table of Contents

1a) 1.2;1.2

```
close all;
clear;
clc;
epsilon = 0.001;
x0 = [1.2; 1.2];

k = 1;
syms x1 x2 real;
x = [x1, x2];

f = @(x1, x2) 100*(x2-x1^2)^2 + (1-x1)^2;
xk = x0;
grad_f(x1, x2) = gradient(f, [x1,x2]);
hessian_f(x1,x2) = hessian(f, [x1, x2]);

p_k = newton_step(hessian_f, grad_f);
N = 20;

grad_value_k = double(subs(grad_f, [x1, x2], xk'));
num_alpha_eq_1_easy = 0;

while norm(grad_value_k,2) > epsilon
    x_easy(:,k) = xk;
    step = double(subs(p_k, [x1, x2], xk'));
    alpha_1 = backtracking_line_search(f, simplify(grad_f), xk,step);
    xk = xk + alpha_1*step;
    grad_value_k = double(subs(grad_f, [x1, x2], xk'));
    if alpha_1 == 1
        num_alpha_eq_1_easy = num_alpha_eq_1_easy +1;
    end
    k = k+1;
    alphas_easy(k-1) = alpha_1;
end
```

1a) -1.2;1

```
x0 = [-1.2; 1];
k = 1;

syms x1 x2 real;
x = [x1, x2];
```

```
f = @(x1, x2) 100*(x2-x1^2)^2 + (1-x1)^2;
xk = x0;
grad_f(x1, x2) = gradient(f, [x1,x2]);
hessian_f(x1,x2) = hessian(f, [x1, x2]);

p_k = newton_step(hessian_f, grad_f);
N = 20;

grad_value_k = double(subs(grad_f, [x1, x2], xk'));
num_alpha_eq_1_difficult = 0;
while norm(grad_value_k,2) > epsilon
    x_difficult(:,k) = xk;
    step = double(subs(p_k, [x1, x2], xk'));
    alpha_1 = backtracking_line_search(f, simplify(grad_f), xk,step);
    xk = xk + alpha_1*step;
    k = k+1;
    grad_value_k = double(subs(grad_f, [x1, x2], xk'));
    if alpha_1 == 1
        num_alpha_eq_1_difficult = num_alpha_eq_1_difficult + 1;
    end
    alphas_difficult(k-1) =  alpha_1;
end
```

Steepest descent

```
x0 = [-1.2; 1];
k = 1;

syms x1 x2 real;
x = [x1, x2];

f = @(x1, x2) 100*(x2-x1^2)^2 + (1-x1)^2;
xk = x0;
grad_f(x1, x2) = gradient(f, [x1,x2]);

N = 20;

grad_value_k = double(subs(grad_f, [x1, x2], xk'));
while norm(grad_value_k,2) > epsilon
    x_steepest_descent(:,k) = xk;
    step = -grad_value_k;
    alpha_1 = backtracking_line_search(f, simplify(grad_f), xk,step);
    xk = xk + alpha_1*step;
    k = k+1;
    grad_value_k = double(subs(grad_f, [x1, x2], xk'));
    if k == 30
        break;
    end
    k;
end
disp(k);
```

```
    30
```

1b) BFGS

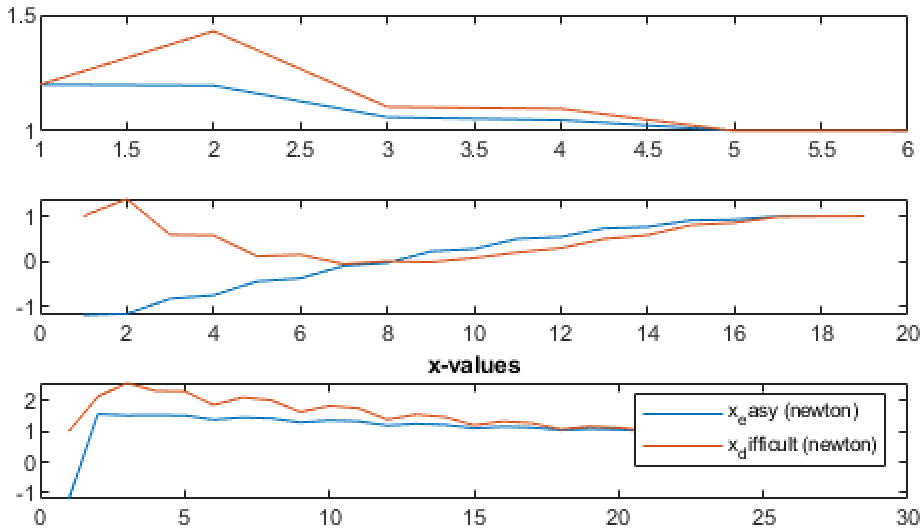
```
%Algorithm 6.1

x0 = [-1.2; 1];
rho = 0.9;
dim = size(x0, 1);
k = 1;
H0 = eye(dim); %inverse hessian approximation
inverse_hessian = H0;
alpha_k = 1;
xk = x0;
x_bfgs = x0;
grad_value_k = double(subs(grad_f, [x1, x2], xk'));
num_alpha_eq_1_bfgs = 0;
while norm(grad_value_k,2) > epsilon
    step = -inverse_hessian*grad_value_k;
    alpha_1 = backtracking_line_search(f, simplify(grad_f), xk,step);
    x_kp1 = xk + alpha_1*step;
    sk = x_kp1-xk;
    grad_value_k = double(subs(grad_f, [x1, x2], xk'));
    grad_value_kp1 = double(subs(grad_f, [x1, x2], x_kp1'));

    yk =  grad_value_kp1-grad_value_k;

    %compute next hessian
    inverse_hessian = inv_hessian(sk, yk, inverse_hessian);

    k = k+1
    xk = x_kp1
    grad_value_k = grad_value_kp1;
    x_bfgs = [x_bfgs, xk];
    if alpha_1 == 1
        num_alpha_eq_1_bfgs = num_alpha_eq_1_bfgs + 1;
    end
    alphas_bfgs(k-1) = alpha_1;
end

f1 = figure();
subplot(411);
T = 1:size(x_easy, 2);
plot(T, x_easy);
subplot(412);
T = 1:size(x_difficult, 2);
plot(T, x_difficult);
subplot(413);
T_bfgs = 1:size(x_bfgs, 2);
plot(T_bfgs, x_bfgs);
title('x-values');
legend('x_easy (newton)', 'x_difficult (newton)', 'x_bfgs');
```

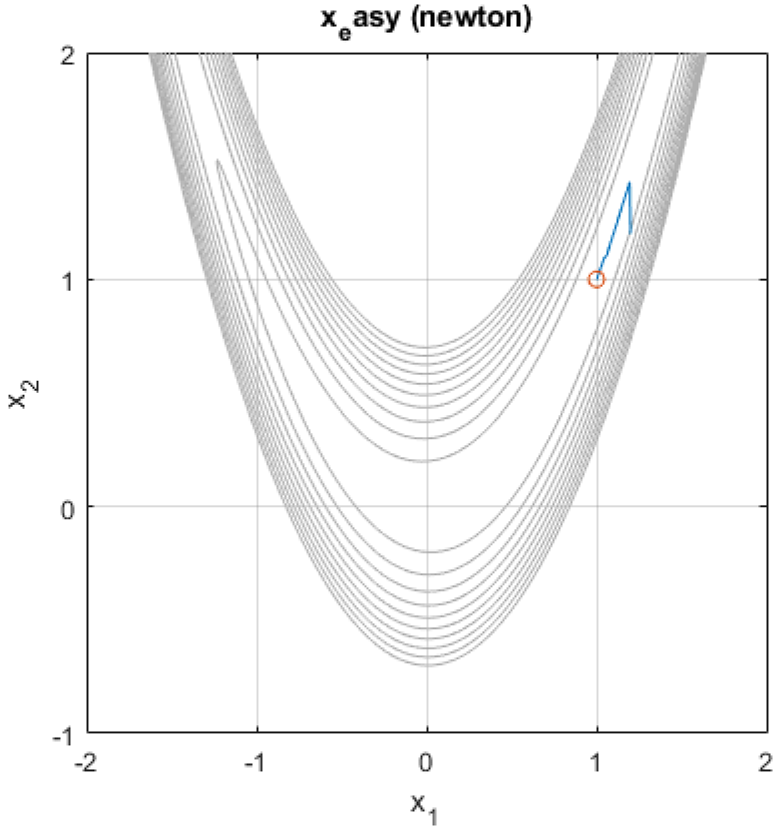
```
f2 = figure();
plot_iter_rosenbrock(x_easy);
title('x_easy (newton)');

f3 = figure();
plot_iter_rosenbrock(x_difficult);
title('x_difficult (newton)');

f4 = figure();
plot_iter_rosenbrock(x_bfgs);
title('x_bfgs');
f5 = figure();
plot_iter_rosenbrock(x_steepest_descent);
title('x_steepest_descent');

movegui(f1,'west');
movegui(f2,'north');
movegui(f3,'east');
movegui(f4,'south');
movegui(f5, 'center');
```

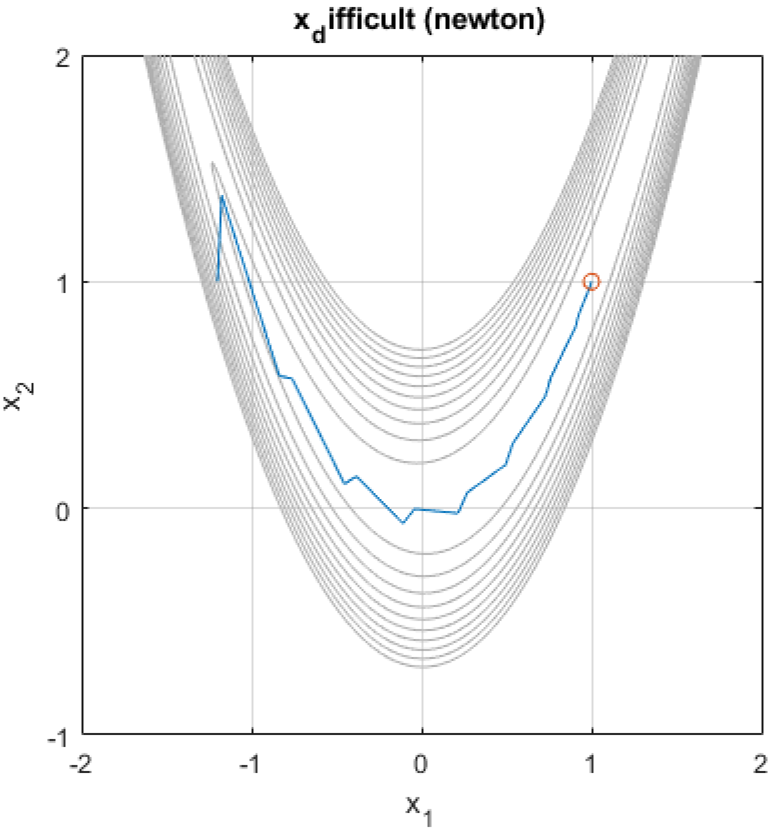
```
k =

     2


xk =

    1.5552
    2.1246


k =

     3


xk =

    1.5165
    2.5692


k =

     4


xk =

    1.5200
    2.3117
```

```
k =

     5


xk =

    1.5152
    2.2960


k =

     6


xk =

    1.3754
    1.8570


k =

     7


xk =

    1.4516
    2.1029


k =

     8


xk =

    1.4197
    2.0087


k =

     9


xk =

    1.2866
    1.6245


k =

    10


xk =

    1.3533
    1.8252


k =

    11


xk =

    1.3245
    1.7484


k =

    12


xk =

    1.1856
    1.3812


k =

    13


xk =

    1.2441
    1.5462


k =

    14


xk =
    1.2099
    1.4615


k =

    15


xk =

    1.1008
    1.1942


k =

    16


xk =

    1.1506
    1.3224


k =

    17


xk =

    1.1247
    1.2629


k =

    18


xk =

    1.0364
    1.0630


k =

    19


xk =
    1.0753
    1.1555


k =

    20


xk =

    1.0568
    1.1157


k =

    21


xk =

    1.0110
    1.0188


k =

    22


xk =

    1.0170
    1.0337


k =

    23


xk =

    1.0083
    1.0165


k =

    24
```

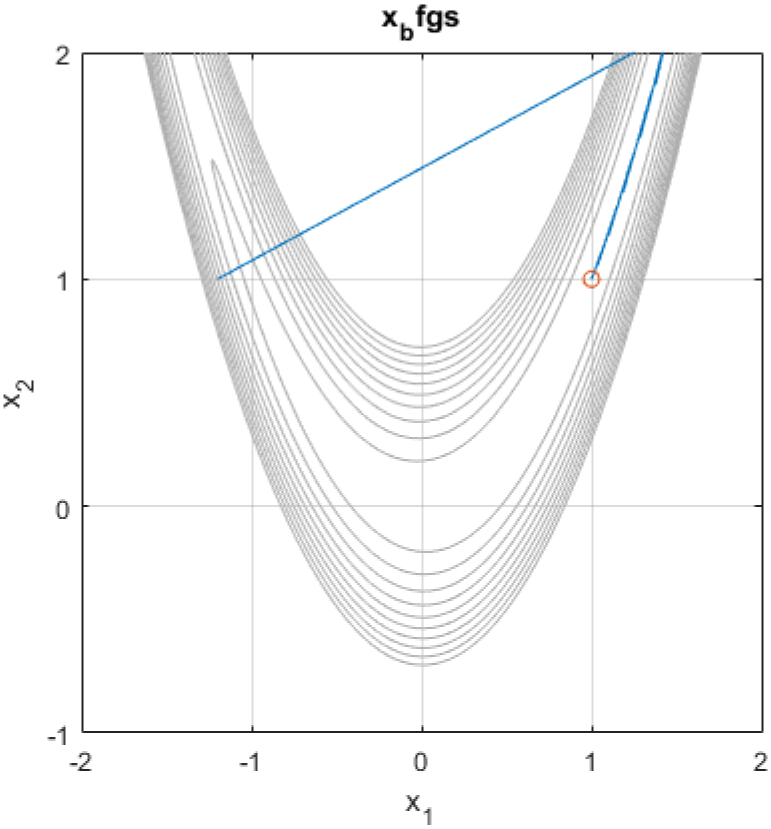
```
xk =

    1.0008
    1.0014


k =

    25


xk =

    1.0001
    1.0002


k =

    26


xk =

    1.0000
    1.0000

Warning: Ignoring extra legend entries.
```


1.5
1
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
1
0
-1
0
2
4
6
8
10
12
14
16
18
20
x-values
2
1
0
-1
0
5
10
15
20
25
30
$x_e$asy (newton)
$x_d$ifficult (newton)

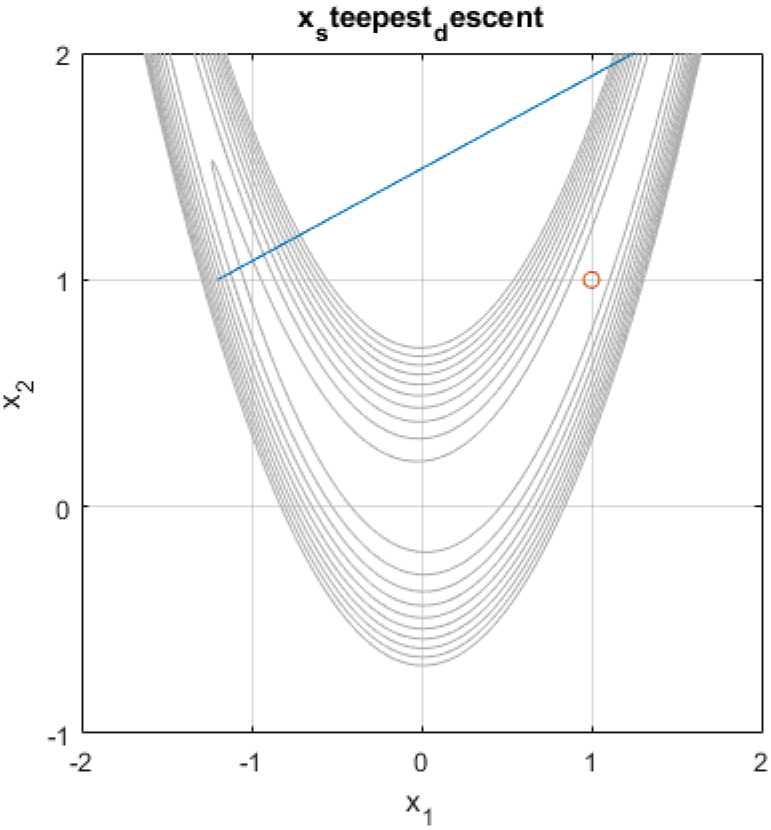

$x_e$asy (newton)
2
1
0
-1
-2
-1
0
1
2
x_2
x_1



$x_d$ifficult (newton)



$x_b$fgs



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