Pareto distribution (type 1)

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The Pareto distribution (type 1) is defined via two parameters:

1. $a > 0$: the shape.
2. $m > 0$: the scale.

The probability density function $f_{a,m}$ of the Pareto distribution (type 1) with shape a and scale m is defined via, for all $x \geq m$,

$$f_{a,m}(x) = \frac{am^a}{x^{a+1}}$$

Given a sample $(x_i)_{1 \leq i \leq n}$, $n \in \mathbb{N}^*$ of independent and identically distributed variables following the Pareto distribution (type 1) with shape a and scale m, the maximum likelihood estimators of a and m are given by $\hat{a}$ and $\hat{m}$ defined by

$$\hat{m} = \min\{x_i, 1 \leq i \leq n\}$$
$$\hat{a} = \frac{n}{\sum_{i=1}^{n} \ln(x_i) - n \ln(\hat{m})}$$

It can be shown that $\hat{a}$ is a biased estimator of a, and that the adjusted estimator $\bar{a} = \frac{n-1}{n}\hat{a}$ is an unbiased estimator of a.