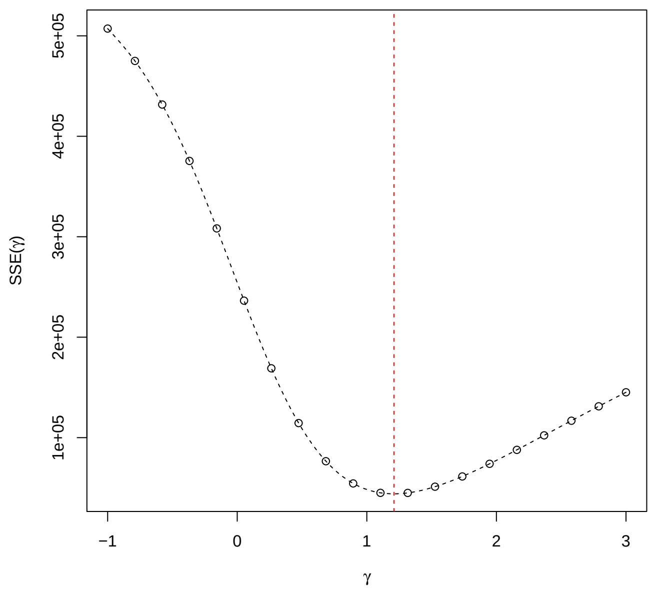

1 Fitting Non Linear Regression Models With the Parameter Cascade

Consider the following non-linear regression model where observed values y_i are values observed at times t_i :

$$y_i = \alpha + \beta e^{\gamma t_i} + \epsilon_i$$

All of the difficulty here comes from the $e^{\gamma t}$ term. If γ were known, α and β could be found through simple linear regression with the $e^{\gamma t_i}$ term acting as an independent variable predicting the y_i

This suggests the following regression strategy. Define a function $H(\gamma)$ to be the sum of squared errors from performing simple linear regression on the y_i against $e^{\gamma t_i}$. That is:

$$H(\gamma) = \min_{\alpha,\beta} \sum [y_i - \alpha - \beta e^{\gamma t_i}]^2$$

This defines a middle problem, with the inner problem being that of minimising the simple linear regression problem given γ. The non-linear model can be fitted by using Brent's Method to fit the middle problem.

This approach was applied to simulated data with $\alpha = 100, \beta = 4$, and $\gamma = 1$, and the results can be seen below.

1.1 Fitting Linear Homogenous ODEs Using the Parameter Cascade

Recall a linear homogenous ODE of order n is given by:

$$\frac{d^n y}{dt^n} = \sum_{k=0}^{n-1} a_k(t;\theta)\frac{d^k y}{dt^k}$$

Under some mild technical conditions, the set of solutions to such an ODE is an n dimensional vector space and has a unique solution for each set of intial conditions.

$$y''(t) = \alpha\sqrt{t}y(t) + \beta\sin(2t)y'(t)$$

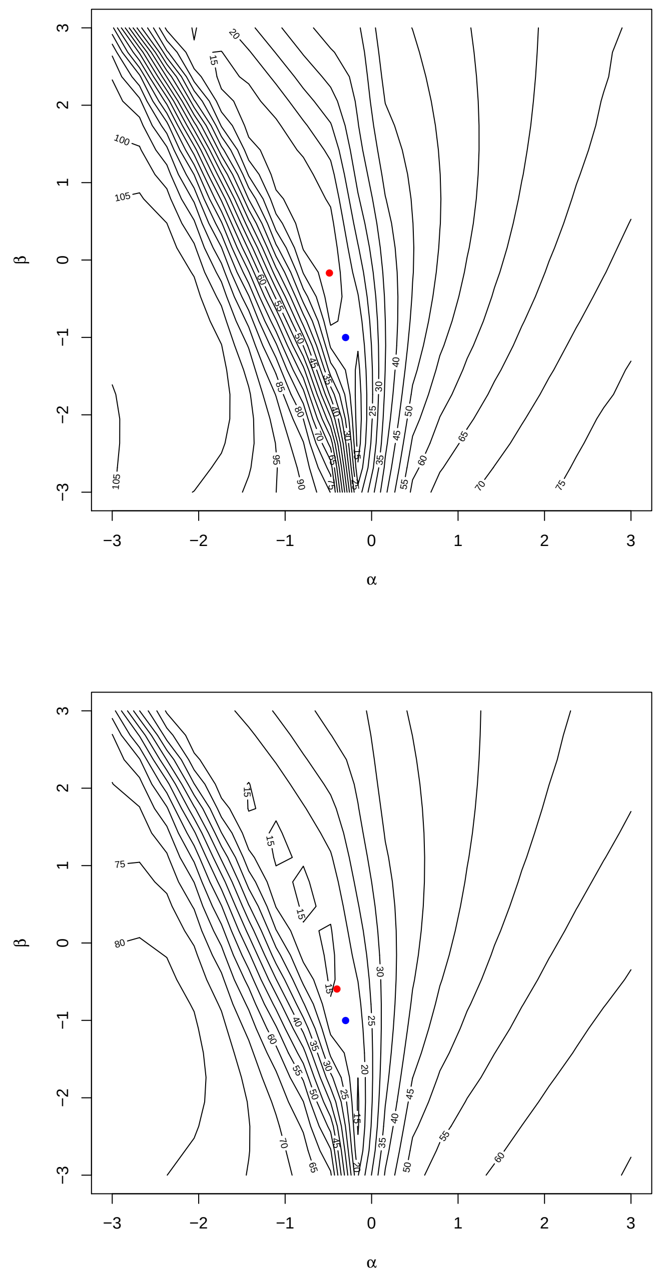

Figure 1: Plot of fit to simulated data, and contour plot of SSE against α and β. Blue dot is true values, red is estimated values.