Bayesian Inference:

Example 1 (Barber, BRML, 2011)

- 90% of people with McD syndrome are frequent hamburger eaters
- Probability of someone having McD syndrome: 1/10000
- Proportion of hamburger eaters is about 50%

What is the probability that a hamburger eater will have McD syndrome?

Bayesian Inference:

Example 1: Formalization

Let $McD \in \{0, 1\}$ be the variable denoting having the McD syndrome and $H \in \{0, 1\}$ be the variable denoting a hamburger eater. Therefore:

$$p(H = 1|McD = 1) = 9/10 \qquad p(McD = 1) = 10^{-4}$$
$$p(H = 1) = 1/2$$

We need to compute $p(McD = 1|H = 1)$, *the probability of a hamburger eater having McD syndrome*.

Any ballpark estimates of this probability?

Bayesian Inference:

Example 1: Solution

$$
\begin{aligned}
p(McD = 1 | H = 1) &= \frac{p(H = 1 | McD = 1) p(McD = 1)}{p(H = 1)} \\
&= 1.8 \times 10^{-4}
\end{aligned}
$$

Repeat the above computation if the proportion of hamburger eaters is rather small: (say in France) 0.001.

Example 2: Detecting Terrorists:

From understandinguncertainty.org

- Scanner detects true terrorists with 95% accuracy
- Scanner detects upstanding citizens with 95% accuracy
- There is 1 terrorist on your plane with 100 passengers aboard
- The shifty looking man sitting next to you tests positive (terrorist)

What are the chances of this man being a terrorist?

Example 2: Detecting Terrorists:

Simple Solution Using "Natural Frequencies" (David Spiegelhalter)

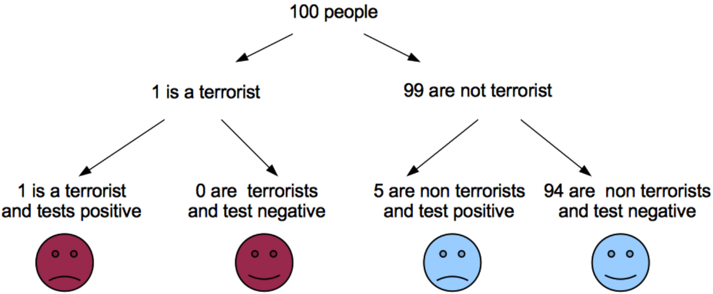

Figure: Figure reproduced from `understandinguncertainty.org`

The chances of the man being a terrorist are $\approx \frac{1}{6}$

- Relation to disease example
- Consequences when catching criminals

Example 2: Detecting Terrorists:

Formalization with Actual Probabilities

Let $T \in \{0, 1\}$ denote the variable regarding whether the person is a terrorist and $S \in \{0, 1\}$ denote the outcome of the scanner.

$$
\begin{aligned}
p(S = 1 | T = 1) &= 0.95 & \qquad p(S = 0 | T = 1) &= 0.05 \\
p(S = 0 | T = 0) &= 0.95 & \qquad p(S = 1 | T = 0) &= 0.05 \\
p(T = 1) &= 0.01 & \qquad p(T = 0) &= 0.99
\end{aligned}
$$

We want to compute $p(T = 1 | S = 1)$, *the probability of the man being a terrorist given that he has tested positive.*

Example 2: Detecting Terrorists:

Solution with Bayes' Rule

$$
\begin{aligned}
p(T = 1|S = 1) &= \frac{p(S = 1|T = 1)p(T = 1)}{p(S = 1|T = 1)p(T = 1) + p(S = 1|T = 0)p(T = 0)} \\
&= \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.05)(0.99)} \\
&\approx 0.16
\end{aligned}
$$

The probability of the man being a terrorist is $\approx \frac{1}{6}$

Example 2: Detecting Terrorists:

Posterior Versus Prior Belief

While the man has a low probability of being a terrorist, our belief has increased compared to our prior:

$$\frac{p(T = 1|S = 1)}{p(T = 1)} = \frac{0.16}{0.01} = 16$$

i.e. our belief in him being a terrorist has gone up by a factor of 16

Since terrorists are so rare, a factor of 16 does not result in a very high (absolute) probability or belief