Assignment 1

1. For the following distribution, is A B (i.e., A and B are independent)? (33 points)

a	b	P(A=a,B=b)
0	0	0.5
0	1	0.0
1	0	0.0
1	1	0.5

If A & B are independent then any of the following equations hold:

$$P(A|B) = P(A) \equiv \qquad P(B|A) = P(B) \equiv \qquad P(A,B) = P(A)P(B)$$

First, find $P(A)$ and $P(B)$

$$P(A) = P(A = 1, B = 0) + P(A = 1, B = 1) \qquad P(A) = 0.0 + 0.5 \qquad P(A) = 0.5$$

$$P(B) = P(A = 0, B = 1) + P(A = 1, B = 1) \qquad P(B) = 0.0 + 0.5 \qquad P(B) = 0.5$$

So, to test independence we can take

$$P(A,B) = P(A)P(B) \qquad 0.5 = 0.5 * 0.5 \qquad 0.5 \neq .25$$

So A & B are **not** independent.

2. For the following distribution, is A B|C (i.e., A and B are conditionally independent given C)? (33 points)

a	b	c	P(A=a,B=b,C=c)
0	0	0	0.056
0	0	1	0.120
0	1	0	0.224
0	1	1	0.120
1	0	0	0.024
1	0	1	0.180
1	1	0	0.180
1	1	1	0.096

A and B are conditionally independent given C if any holds:

$$P(A|B,C) = P(A|C) \equiv \qquad P(B|A,C) = P(B|C) \equiv \qquad P(A,B|C) = P(A|C)P(B|C)$$

First,

$P(A) = P(A=1, B=0, C=0) + P(A=1, B=0, C=1) + P(A=1, B=1, C=0) + P(A=1, B=1, C=1)P($

Look at : https://stats.libretexts.org/Bookshelves/Probability_Theory/Applied_Probability_(Pfeiffer)/05%3A

3. Consider two binary random variables A and B. If A B (i.e., A and B are independent), and P(A = 0, B = 0) = 0.18 and P(A = 1, B = 0) = 0.28, what is the probability of P(A = 0, B = 1)? (34 points)

a	b	P(A=a,B=b)
0	0	0.18
0	1	n.a. (x)
1	0	0.28
1	1	n.a. (y)

From the given probabilities we know:

$$P(B=0) = P(A=0, B=0) + P(A=1, B=0) \quad P(B=0) = .18 + .28 \quad P(B=0) = .46$$

Hence,

$$P(B=1) = 1 - P(B=0) \qquad P(B=1) = 1 - .46 \qquad P(B=1) = .54$$

**look back at lecture notes for bernoulli random variables