Assignment 8
Due Nov. 18, 2023
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Q1. [100 pt.] Logistic regression model w/ parameters w_0, w_1, w_2

$$P(y=1 \mid x_1, x_2) = \frac{2^{w_2 x_2 + w_1 x_1 + w_0}}{1 + 2^{w_2 x_2 + w_1 x_1 + w_0}}$$

a. [20 pt.] for weights $w_0 = -.5$, $w_1 = 1$, $w_2 = 1$ draw the decision boundary

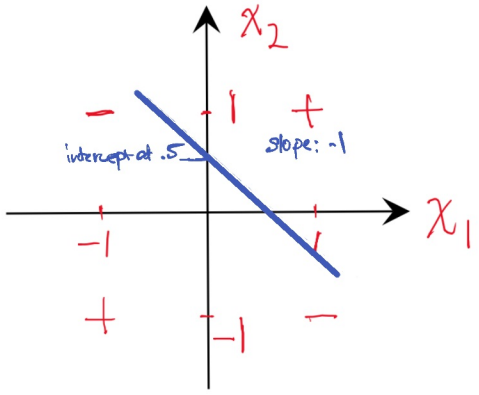
decision boundary is found where:

$0 = w_2 x_2 + w_1 x_1 + w_0$

$0 = x_1 + x_2 - .5$

$x_2 = -x_1 + .5$

line w/ slope 1 & intercept .5 on x_2 axis



b. [20 pt] What is the log likelihood of the negative data point $(x_1 = -1, x_2 = 1)$ i.e. The value of $\log_2 P(Y=0 \mid x_1=-1, x_2=1)$ from a.?

$$P(Y=0 \mid x_1, x_2) = 1 - P(Y=1 \mid x_1, x_2)$$

with weights and datapoints we get: $w_0 = -.5$, $w_1 = 1$, $w_2 = 1$, $x_1 = -1$, $x_2 = 1$

$$P(Y=0 \mid x_1=-1, x_2=1) = \log_2 \frac{1}{1 + 2^{(1-1)+(1\cdot -1)+(-.5)}}$$

* the $w_2 x_2 + w_1 x_1$ terms cancel, leaving $-.5$

$$= \log_2 \frac{1}{1 + 2^{(-.5)}} = -\log_2 (1 + 2^{(-.5)})$$

$$= -\log_2 (1.707) = -.771$$