Assignment 1

1. For the following distribution, is A ⊥ B (i.e., A and B are independent)? (33 points)

a	b	P(A=a,B=b)
0	0	0.5
0	1	0.0
1	0	0.0
1	1	0.5

If A & B are independent then any of the following equations hold:

$$P(A|B) = P(A) \equiv$$
$$P(B|A) = P(B) \equiv$$
$$P(A, B) = P(A)P(B)$$

First, find $P(A)$ and $P(B)$

$$P(A) = P(A = 1, B = 0) + P(A = 1, B = 1)$$
$$P(A) = 0.0 + 0.5$$
$$P(A) = 0.5$$

$$P(B) = P(A = 0, B = 1) + P(A = 1, B = 1)$$
$$P(B) = 0.0 + 0.5$$
$$P(B) = 0.5$$

So, to test independence we can take

$$P(A = 1, B = 1) = P(A = 1)P(B = 1)$$
$$0.5 = 0.5 * 0.5$$
$$0.5 \neq .25$$

Because the test for $A = 1$ and $B = 1$ failed, it is not necessary to test for other values of A and B and we can conclude that A & B are **not** independent.

2. For the following distribution, is A ⊥ B|C (i.e., A and B are conditionally independent given C)? (33 points)

a	b	c	P(A=a,B=b,C=c)
0	0	0	0.056
0	0	1	0.120
0	1	0	0.224
0	1	1	0.120
1	0	0	0.024
1	0	1	0.180
1	1	0	0.180
1	1	1	0.096

A and B are conditionally independent given C if any holds:

$$P(A|B,C) = P(A|C) \equiv$$
$$P(B|A,C) = P(B|C) \equiv$$
$$P(A,B|C) = P(A|C)P(B|C)$$

Using $P(A = 1|B = 1, C = 1) = P(A = 1|C = 1)$:

$P(A = 1|B = 1, C = 1) = .096)\ P(A|C$

First, find the probabilities of A, B, and C:

$$P(A) = P(A = 1, B = 0, C = 0) + P(A = 1, B = 0, C = 1) + P(A = 1, B = 1, C = 0) + P(A = 1, B = 1, C = 1)$$
$$P(A) = .024 + .180 + .180 + .096 = .48$$

$$P(B) = P(A = 0, B = 1, C = 0) + P(A = 0, B = 1, C = 1) + P(A = 1, B = 1, C = 0) + P(A = 1, B = 1, C = 1)$$
$$P(B) = .224 + .120 + .180 + .096 = .62$$

$$P(C) = P(A = 0, B = 0, C = 1) + P(A = 0, B = 1, C = 1) + P(A = 1, B = 0, C = 1) + P(A = 1, B = 1, C = 1)$$
$$P(C) = .12 + .120 + .180 + .096 = .516$$

Now, to test for independence we can test $P(A, B|C) = P(A|C)P(B|C)$

$$P(A,B|C) = P(A,B,C)/P(C)$$
$$= .096/.516$$
$$= .186$$

$$P(A|C) = P(A)P(C)/P(C)$$
$$= (.48 * .516)/.516$$
$$= .48$$
$$P(B|C) = P(B) * P(C)/P(C)$$
$$= (.62 * .516)/.516$$
$$= .62$$
$$P(A|C) * P(B|C) = .48 * .62 = .2976$$

$.186 \neq .2976$ so the statement A ⊥ B|C is **not** true. $

3. Consider two binary random variables A and B. If A ⊥ B (i.e., A and B are independent), and P(A = 0, B = 0) = 0.18 and P(A = 1, B = 0) = 0.28, what is the probability of P(A = 0, B = 1)? (34 points)

a	b	P(A=a,B=b)
0	0	0.18
0	1	n.a. (x)
1	0	0.28
1	1	n.a. (y)

From the given probabilities we know:

$$P(B=0) = P(A=0, B=0) + P(A=1, B=0)$$
$$P(B=0) = .18 + .28$$
$$P(B=0) = .46$$

Hence,

$$P(B=1) = 1 - P(B=0)$$
$$P(B=1) = 1 - .46$$
$$P(B=1) = .54$$

Also,

$$P(A) = .28 + y$$

$$P(A, B) = P(A)P(B)$$
$$y = P(A).54$$
$$y = (.28 + y).54$$
$$y = .15 * .54y$$
$$.46y = .15$$
$$y = .33$$

So,

$$P(A=1) = P(A=1, B=0) + P(A=1, B=1)$$
$$= .28 + .33$$
$$= .61$$

$$P(A=0) = 1 - P(A=1)$$
$$= 1 - .61$$
$$= .39$$

Now, find $P(A=0, B=1)$

$$P(A=0, B=1) = P(A=0) * P(B=1)$$
$$= .39 * .54$$
$$= .21$$