Assignment 1

1. For the following distribution, is A B (i.e., A and B are independent)? (33 points)

a	b	P(A=a,B=b)
0	0	0.5
0	1	0.0
1	0	0.0
1	1	0.5

If A & B are independent then any of the following equations hold:

$$P(A|B) = P(A) \equiv \qquad P(B|A) = P(B) \equiv \qquad P(A, B) = P(A)P(B)$$

First, find $P(A)$ and $P(B)$

$$P(A) = P(A = 1, B = 0) + P(A = 1, B = 1) \qquad P(A) = 0.0 + 0.5 \qquad P(A) = 0.5$$

$$P(B) = P(A = 0, B = 1) + P(A = 1, B = 1) \qquad P(B) = 0.0 + 0.5 \qquad P(B) = 0.5$$

So, to test independence we can take

$$P(A = 1, B = 1) = P(A = 1)P(B = 1) \qquad 0.5 = 0.5 * 0.5 \qquad 0.5 \neq .25$$

Because the test for $A = 1$ and $B = 1$ failed, it is not necessary to test for other values of A and B and we can conclude that A & B are **not** independent.

2. For the following distribution, is A   B|C (i.e., A and B are conditionally independent given C)? (33 points)

a	b	c	P(A=a,B=b,C=c)
0	0	0	0.056
0	0	1	0.120
0	1	0	0.224
0	1	1	0.120
1	0	0	0.024
1	0	1	0.180
1	1	0	0.180
1	1	1	0.096

A and B are conditionally independent given C if any holds:

$$P(A|B,C) = P(A|C) \equiv \qquad P(B|A,C) = P(B|C) \equiv \qquad P(A,B|C) = P(A|C)P(B|C)$$

Using $P(A=1|B=1,C=1) = P(A=1|C=1)$:

$P(A=1|B=1,C=1) = .096)\ P(A|C$

First, find the probabilities of A, B, and C:

$P(A) = P(A=1,B=0,C=0)+P(A=1,B=0,C=1)+P(A=1,B=1,C=0)+P(A=1,B=1,C=1)P($

$P(B) = P(A=0,B=1,C=0)+P(A=0,B=1,C=1)+P(A=1,B=1,C=0)+P(A=1,B=1,C=1)P($

$P(C) = P(A=0,B=0,C=1)+P(A=0,B=1,C=1)+P(A=1,B=0,C=1)+P(A=1,B=1,C=1)P($

Now, to test for independence we can test $P(A,B|C) = P(A|C)P(B|C)$

$$P(A,B|C) = P(A,B,C)/P(C) \qquad = .096/.516 \qquad = .186$$

$P(A|C) = P(A)P(C)/P(C) = (.516 * .516)/.516 = .516 P(B|C) = P(B) * P(C)/P(C) = (.62 * .516)/.516 = .62 P($

$.186 \neq .320$ so the statement A   B|C is **not** true.

$$

3. Consider two binary random variables A and B. If A B (i.e., A and B are independent), and P(A = 0, B = 0) = 0.18 and P(A = 1, B = 0) = 0.28, what is the probability of P(A = 0, B = 1)? (34 points)

a	b	P(A=a,B=b)
0	0	0.18
0	1	n.a. (x)
1	0	0.28
1	1	n.a. (y)

From the given probabilities we know:

$$P(B=0) = P(A=0,B=0) + P(A=1,B=0) \quad P(B=0) = .18 + .28 \quad P(B=0) = .46$$

Hence,

$$P(B=1) = 1 - P(B=0) \qquad P(B=1) = 1 - .46 \qquad P(B=1) = .54$$

Also,

$$P(A) = .28 + y$$

$$P(A,B) = P(A)P(B) \quad y = P(A).54 \quad y = (.28+y).54 \quad y = .15 * .54y \quad .46y = .15 \quad y = .33$$

So,

$$P(A=1) = P(A=1,B=0) + P(A=1,B=1) \qquad = .28 + .33 \qquad = .61$$

$$P(A=0) = 1 - P(A=1) \qquad = 1 - .61 \qquad = .39$$

Now, find $P(A=0,B=1)$

$$P(A=0,B=1) = P(A=0) * P(B=1) \qquad = .39 * .54 \qquad = .21$$