c. What is the gradient of this point?

$$\frac{\partial}{\partial w_0}\log_2 P(Y{=}0 \mid x_1 = -1, x_2 = 1) = ?$$

$$\frac{\partial}{\partial w_1}\log_2 P(Y{=}0 \mid x_1 = -1, x_2 = 1) = ?$$

$$\frac{\partial}{\partial w_2}\log_2 P(Y{=}0 \mid x_1 = -1, x_2 = 1) = ?$$

$$\frac{\partial}{\partial w_i} \log_2 P(Y{=}0 \mid x_1, x_2) = -\frac{\partial}{\partial w_i} \log_2\left(1+2^{w_2x_2+w_1x_1+w_0}\right)$$

$$= -\frac{1}{1+2^{w_2x_2+w_1x_1+w_0}} \cdot 2^{w_2x_2+w_1x_1+w_0} \frac{\partial}{\partial w_i}(w_2x_2+w_1x_1+w_0)$$

$$2^{w_2x_2+w_1x_1+w_0} = 2^{\cancel{(+1)+(-1\cdot 1)}+.5} = 2^{-.5} = .707$$

$$= -\frac{1}{1+.707} \cdot .707 = \frac{-.707}{1.707} = .414$$

$$w_0 = -.414 \cdot \frac{\partial}{\partial w_0} = -.414$$

$$w_1 = -.414 \cdot \frac{\partial}{\partial w_1} = .414$$

$$w_2 = -.414 \cdot \frac{\partial}{\partial w_2} = -.414$$

d.

$$P(y=1 \mid x_1, x_2) = \frac{2^{w_3(x_1x_2)+w_2x_2+w_1x_1+w_0}}{1+2^{w_3(x_1x_2)+w_2x_2+w_1x_1+w_0}}$$

With additional feature $x_1 x_2$ what weights provide good fit?

As far as the properties that need to be satisfied, the probability must be $>.5$ for positive examples, & that can be satisfied by $w_3(x_1x_2)+w_2x_2+w_1x_1+w_0 > 0$. For negative examples, it should be the opposite, summing to <0.

for each example:

(negative examples)	(positive examples)
$-(-1,1)$ $(-1)(1)w_3 + (1)w_2 + (-1)w_1 + w_0$	$+(-1,-1$ $(-1)(-1)w_3 + (-1)w_2 + (-1)w_1 + w_0 > 0$
$-(1,-1)$ $(1)(-1)w_3 + (-1)w_2 + (1)$ w_0	$+(1,1$ $(1)(1)w_3 + w_2 + w_1 + w_0 > 0$

if $w_0 = w_1 = w_2 = 0$, any positive value of w_3 will suffice such as $w_3 = 2$

e. the decision boundary is

- $x_1 = 0$, $x_2 = 0$

and on the 2-dimensional axis falls on the x_1, x_2 axis.

positive zone in yellow
negative zone pink