# Jell Kkaggi Development

Soft-body physics and RL model implementation for pseudo 3D games

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### Overview

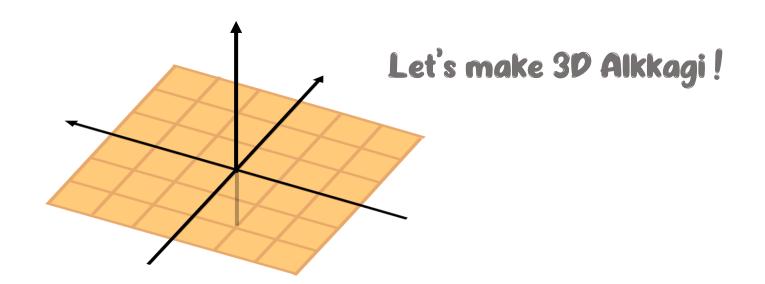
Pseudo 3D Implementation

Perspective of the player

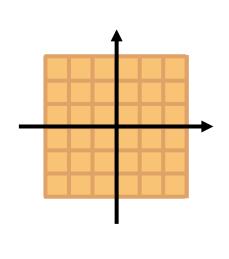
**Soft-body Simulation**Jelly & collision model

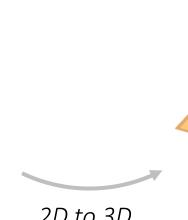
**Reinforcement Learning**Training Al player

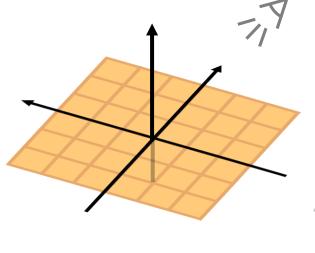
### 1 Pseudo 3D Implementation

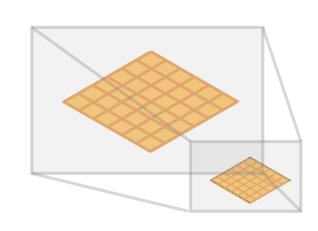


#### Pseudo 3D









2D to 3D

3D to 2D

2D plane

$$\mathcal{B}_{xy} = \{\widehat{e_1}, \widehat{e_2}\}$$

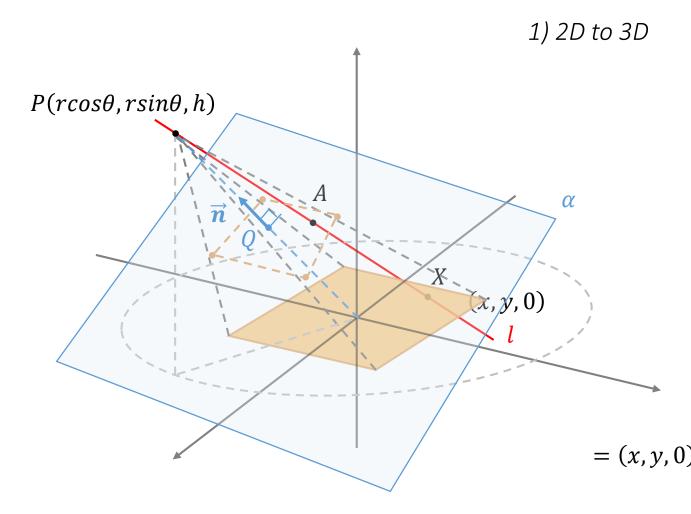
3D gameworld

$$\mathcal{B}_{xyz} = \{\widehat{e_1}, \widehat{e_2}, \widehat{e_3}\}$$

2D screen

$$\mathcal{B}_{\alpha} = \{\vec{u}, \vec{v}\}$$

### Perspective Projection



$$Q\left(\frac{r\cos\theta}{2}, \frac{r\sin\theta}{2}, \frac{h}{2}\right) \qquad \vec{n} = (r\cos\theta, r\sin\theta, h)$$

$$\alpha : r\cos\theta + r\sin\theta y + hz - \frac{h^2 + z^2}{2} = 0$$

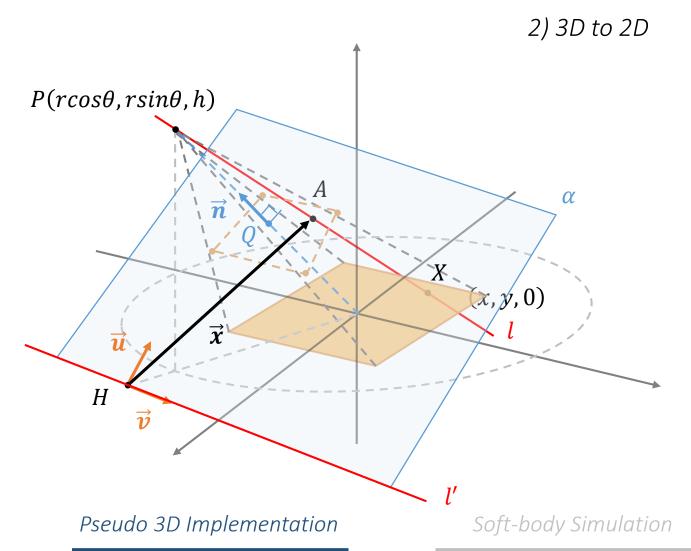
$$l: (x, y, 0) + t(rcos\theta - x, rsin\theta - y, h) = 0$$

$$A: \alpha \cap l$$

$$A = proj_{\alpha}(x, y, 0)$$

$$= (x, y, 0) + \frac{\frac{r^2 + h^2}{2} - xr\cos\theta - yr\sin\theta}{r^2 - xr\cos\theta - yr\sin\theta + h^2} (r\cos\theta - x, r\sin\theta - y, h)$$

### Perspective Projection



$$\mathcal{B}_{\alpha} = \{ \overrightarrow{\boldsymbol{u}}, \overrightarrow{\boldsymbol{v}} \}$$

$$\vec{u} = \overrightarrow{QP}$$
  $\vec{x} = \overrightarrow{PA}$   $\vec{v} = \frac{d\vec{r}}{dt} = (-\sin\theta, \cos\theta, 0)$ 

$$l': \alpha \cap \mathbb{R}^2 \qquad xrcos\theta + yrsin\theta - \frac{h^2 + r^2}{2} = 0$$

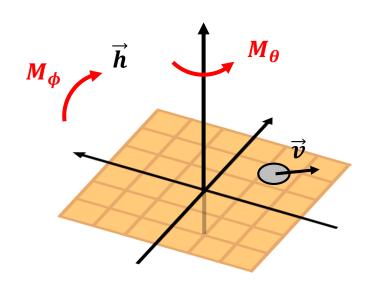
$$H: l' \cap dir \overrightarrow{r}$$
  $\left(\frac{r^2 + h^2}{4r} cos\theta, \frac{r^2 + h^2}{4r} sin\theta, 0\right)$ 

$$conv \vec{x} = [T]_{\mathcal{B}_{\alpha}}^{\mathcal{B}_{xyz}}$$

$$= \left(\frac{\vec{u} \cdot \vec{x}}{\vec{u} \cdot \vec{u}} \vec{u}, \frac{\vec{v} \cdot \vec{x}}{\vec{v} \cdot \vec{v}} \vec{v}\right)$$

Reinforcement Learning

### Perspective Rotation



$$\begin{aligned} \mathbf{M}_{\theta} &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{M}_{\phi} &= \begin{pmatrix} 1 & \tan\phi \\ 0 & 1 \end{pmatrix} \\ \vec{\mathbf{v}} &= \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix} \qquad \vec{\mathbf{h}} &= \begin{pmatrix} h \\ 1 \end{pmatrix} \end{aligned}$$

$$\begin{split} \mathbf{M}_{\theta} \vec{v} &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix} = \begin{pmatrix} v_x \cos\theta - v_y \sin\theta \\ v_x \sin\theta + v_y \cos\theta \\ 1 \end{pmatrix} \\ \mathbf{M}_{\phi} \vec{h} &= \begin{pmatrix} 1 & \tan\phi \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ 1 \end{pmatrix} = \begin{pmatrix} h + \tan\phi \\ 1 \end{pmatrix} \end{split}$$

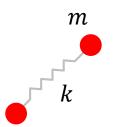
### Implementation

```
def conv3D(x,y):
   h_{-} = h[0]
                        → Get from global variable settings
   r_{-} = r[0]
   theta_ = theta[0]
    shift_y = (r_**2 + h_**2) / r_* 0.25 \longrightarrow Shift origin (visual)
   C = 1 - (h_{**2/2} + r_{**2/2}) / (r_{**2} + h_{**2} - x*r_{cos}(theta_) - y*r_{sin}(theta_))
   proj_x = x + C*(r_*cos(theta_) - x)
   proj_y = y + C*(r_*sin(theta_) - y)
   proj_z = C*h_
   P_x = (r_{*2} + h_{*2}) / (4*r_) * cos(theta_)
   P_y = (r_{*2} + h_{*2}) / (4*r_) * sin(theta_)
                                                                          Calculation
   Pz = 0
   u = [r_*\cos(theta_)/2-P_x, r_*\sin(theta_)/2-P_y, h_/2]
   v = [-sin(theta_), cos(theta_), 0]
   PA = [proj x - P x, proj y - P y, proj z - P z]
   conv x = np.dot(PA,v)
    conv y = np.dot(PA,u) / sqrt(np.dot(u,u))
   return (15*conv_x + WIDTH//2, HEIGHT//2 - 15*(conv_y - shift_y)) → Screen adjustment
```

2 | Soft-body Physics

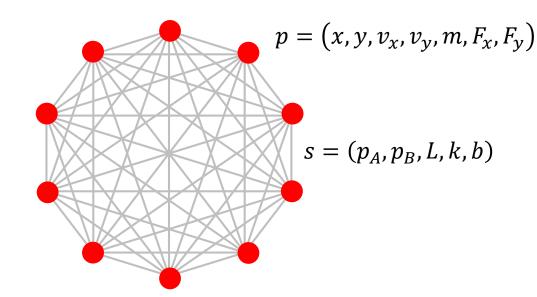


### Soft-body Physics



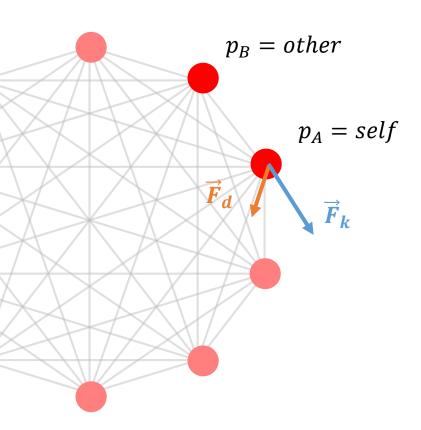
$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

→ Able to find analytic solution



N-body problem:
Numerical method needed

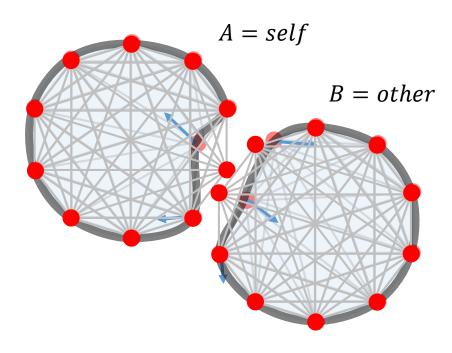
### Euler Integration



Pseudocode stone.py/class Stone/update()

```
self.connected = \{ (p_B, L), (p_C, L'), \dots \}
for (other, L) in self . connected:
        r = dist(self, other)
       F = k(r - L)
       \vec{F} = F \cdot (\vec{x}_{other} - \vec{x}_{self}) - d \cdot (\vec{v}_{other} - \vec{v}_{self})
       \vec{v}_{self} = \frac{dt}{m} \cdot \vec{F}
\vec{x}_{self} = dt \cdot \vec{v}
                                                 Euler Integration
```

#### Collision Model



$$\mathbf{B}(t) = (1-t)\mathbf{B}_{\mathbf{P}_0\mathbf{P}_1\cdots\mathbf{P}_{n-1}}(t) + t\mathbf{B}_{\mathbf{P}_0\mathbf{P}_1\cdots\mathbf{P}_n}(t)$$

#### Pseudocode stone.py/class Stone

```
def detectCollision( )
    stonesCollided = []
    if dist(cen_A, cen_B) < r_A + r_B: add
def update()
    for particles in self:
        if collided: change velocity
def draw()
    interpolate(particles)
                             → Bezier curve
    drawBorder()
```

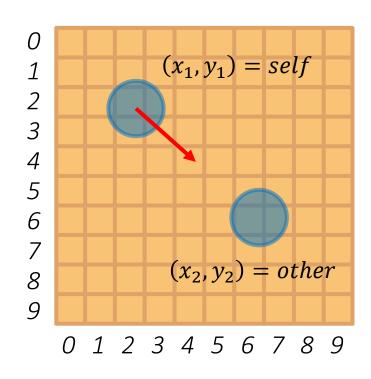
## Implementation



### Reinforcement Learning



### Training Model



State 
$$S = \{0, \dots, 9\}^2 \times \{0, \dots, 9\}^2 \longrightarrow Position$$

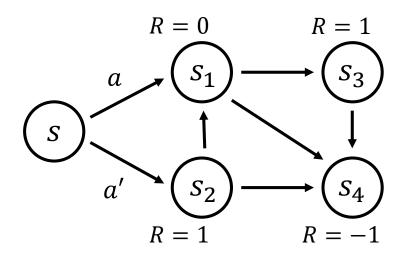
Action 
$$\mathcal{A} = \{1,2,3,4,5\}$$
  $\longrightarrow$  Shooting strength

Probability 
$$\mathcal{P}(s,a) = State after action a was done$$

Reward 
$$R(s,a) = \begin{cases} 1 & win \longrightarrow Other out of board \\ -1 & loose \longrightarrow Self out of board \\ 0 & tie \longrightarrow Otherwise \end{cases}$$

Discount 
$$\gamma = 0.9$$

### Q-Learning



Markov Decision Process

 $MDP(S, A, P, R, \gamma)$ 

 $\longrightarrow$  Under policy  $\pi: \mathcal{S} \times \mathcal{A} \rightarrow [0,1]$ 

State-value function

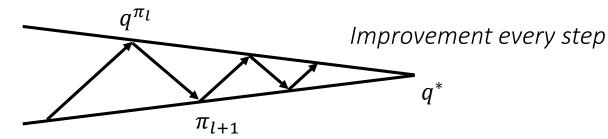
$$v^{\pi}(s) = \mathbb{E}\left[\sum_{k=0}^{\infty} \gamma^k R^{\pi}(N_k^{\pi}(s))\right]$$

Action-value function

$$q^{\pi}(s,a) = R(s,a) + \gamma \sum_{s' \in \mathcal{S}} (\mathcal{P}(s,a,s') \cdot v^{\pi}(s'))$$

Bellman optimality theorem

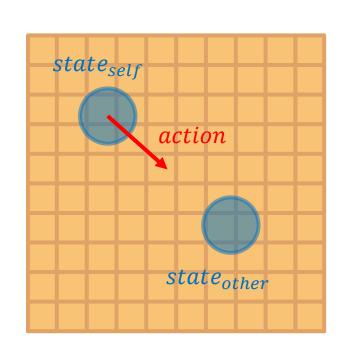
$$q^*(s,a) = R(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}(s,a,s') \cdot \left( \max_{a' \in \mathcal{A}} q^*(s',a') \right)$$



#### Al Model

Pseudocode MDP.py/Q\_learning()

```
q(s,a) = random(s \in S, a \in A) \longrightarrow Start by random position
for each episode:
      s = initial state
      for each step:
            a = \pi(s) in greedy \longrightarrow Predict the optimal action
            r, s' = takeAction(a)
            q(s,a) += (1-\alpha) \cdot q(s,a) + \alpha \cdot (r + \gamma \cdot \max_{a' \in \mathcal{A}} q(s',a'))
            s = s'
                        → Update the next state
```



## Summary

1

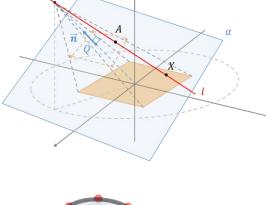
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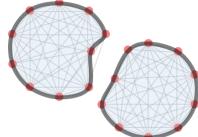
Perspective of the player

2

Soft-body Simulation

Jelly & collision model





3

Reinforcement Learning

Training AI player

$$q^*(s,a) = R(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}(s,a,s') \cdot \left( \max_{a' \in \mathcal{A}} q^*(s',a') \right)$$

#### Contact

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