1 Healthcare Facility Optimization Assignment

A healthcare organization is planning to optimize its network of healthcare facilities. Your task is to assist in determining **optimal facility locations** based on different constraints and objectives. The organization has access to a grid-based map where each residential area is represented by coordinates, and each coordinate has a population size associated with it.

The healthcare organization is under pressure to provide equitable access to facilities while considering operational constraints. They need your help to design efficient and cost-effective solutions for different scenarios. Throughout this assignment, you will model, solve, and analyze these scenarios.

Problem description There are n key residential areas of interest, each located around coordinates (x_i, y_i) in kilometers (provided in `Residential_areas.txt`). Each of these n points is a potential site for a new healthcare facility.

The distance between two points i and j is given by

$$d_{ij}^E = |x_i - x_j| + |y_i - y_j|$$

(Manhattan distance).

Initially, the objective is to **minimize the number of facilities** while ensuring that **every residential area is within $d = 15$ km of at least one healthcare facility**. Note that the population sizes provided in `Residential_areas.txt` are not needed for this part of the assignment. Later, new constraints and objectives will be introduced.

Exercise 1. Write down a linear mathematical model for this problem. Cearly define any decision variables and/or parameters introduced. Make no assumptions on the values of the parameters: leave them as symbols. (1.5 points)

Exercise 2. Implement and solve the model on the supplied instance, report the objective value, and plot the optimal solution. Make sure to clearly identify the points that become healthcare facilities and also plot the corresponding 15-km Manhattan distance coverage area of each facility. (3 points)

Exercise 3. The Manhattan distance metric assumes that movement occurs along a grid. The restriction of 15 km is actually derived from a time constraint: all areas must be reachable within a maximum of 15 minutes. At an average speed of 60 km/h, this implies a maximum distance of 15 km. Discuss how the optimal facility placement could change if the grid is not isotropic—i.e., movement in one direction (e.g., north-south) is slower or costlier than in another (e.g., east-west). As part of your answer, explain what happens to the shape of the coverage area under this non-isotropic assumption. (1 point)

Actually, the organization has already built facilities at some of the locations. In particular, the following locations already have a facility:

- (45, 46)
- (7, 10)
- (35, 8)
- (18, 35)

Exercise 4. Considering that these 4 facilities have already been built, explain how you can change the model to find the minimal number of additional facilities. Explain how you can minimize the number of variables by ensuring that no unnecessary variables are used. Again, implement and solve the model on the supplied instance, report the new objective value, and plot the new optimal solution. Report on the differences in the optimal solution compared to Exercise 2. (2 points)

The organization is considering the possibility of **relocating existing facilities**. Relocation has a cost function that includes a fixed cost and an additional cost based on the Euclidean distance of the move. That is, in this context, the distance between two points i and j is calculated as

$$d_{ij}^A = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

Let $c_{build} = 5000$ be the cost of building a new facility, $c_{move} = 3500$ the fixed cost of moving a facility and $c = 150$ the additional cost per kilometer Euclidean distance of the move. The objective is now to minimize the total cost of building new facilities and possibly relocating old facilities. While the cost of existing facilities should not be considered, the organization prefers solutions with the maximum number of facilities. This preference applies when there are multiple optimal solutions with the same total cost but a different number of facilities.

Exercise 5. What is the maximum distance over which it could be optimal to relocate a facility compared to building a new facility? (0.5 points)

Exercise 6. Update your mathematical model to incorporate the relocation of existing facilities. Clearly define any new parameters, variables, and constraints. Explain under what circumstances multiple optimal solutions with a different number of facilities might arise and describe how you would ensure that the solution with fewer facilities is either infeasible or no longer optimal. Specifically, outline how you would adjust the model to guarantee that only solutions with the maximum number of facilities between two solutions with the same cost are considered optimal. Related to this, explain how you can minimize the number of variables by ensuring that no unnecessary variables are introduced. (2 points)

Exercise 7. Implement the extended model, solve it, and report the optimal solution. Plot the results, indicating:

- New facilities built.
- Existing facilities not relocated.
- Existing facilities relocated (clearly mark the movement paths and final locations).

Again, also plot the 15-km Manthattan distance coverage area for each of facilities in the optimal solution. Report on the differences in the optimal solution compared to Exercise 4. (2 points)

With increasing population sizes, the strain on healthcare facilities has become apparent and the organization has realized that it should introduce capacity constraints. Hence, each facility has a maximum capacity for the number of citizens it can serve. To simplify implementation, the organization has decided to **demolish all existing facilities**. Facilities must now be build such that:

- All residents are covered by a facility within the maximum Manhattan distance of $d = 15$ km. Note that residents from the same residential area may be served by different facilities.
- Each facility can only cover a population of at most its maximum capacity K.

Exercise 8. Write down the updated mathematical model to include the maximum capacity constraint. Clearly define any new parameters and decision variables used. (1 point)

Exercise 9. Solve the updated model for $K = 1000$ and $K = 2000$. Report the optimal objective values for both scenarios. Plot the solutions, i.e. the facilities that are built with their 15 km range. Also indicate which residents are covered by each facility (you may write this down in your answer instead of indicating it in the plot since this might be difficult). (2 points)

Bonus Exercise During summer, some areas experience significant population increases. This means that every residential area has one population size during normal season and another population size during peak season. The organization has decided to allow the construction of temporary facilities to handle the increased demand. Temporary facilities incur a lower building cost, can cover a smaller area and have a lower capacity. Moreover, these facilities can only be used during summer. Develop a new mathematical model that minimizes the total yearly cost while covering the entire population both during normal and peak season. Clearly define all decision variables, parameters, and constraints. (Note: Implementation is not required for this exercise.) (2 points)

2 Data

The file containing the coordinates of all the residential areas and their corresponding population is available in `Residential_areas.txt` on the Canvas website. Every row in this file corresponds to one residential area, presented in the form x_i, y_i, p_i with x_i the x-coordinate, y_i the y-coordinate and p_i the number of citizens associated with residential area i. All other data is provided in the exercise itself.

3 Instructions

The assignment can be carried out individually or in groups of two persons (note: the same individuals and groups should carry out Assignment 2).

- In case of a group of two, both members must participate in assignment preparation.

- You are not allowed to work together with other groups or individuals.
- Mention your names and student numbers clearly at the start of the report.
- Write your code in a way that it can be used with different sets of instances
- Document your code. Add comments in your code that makes them readable.
- You will need to hand in the assignment on Canvas, under "Assignments". Reports need to be in PDF format (LaTeX preferred), adhering to the naming scheme "*<studentnr>.pdf*" or "*<studentnr1><studentnr2>.pdf*". You will also need to hand in any source code combined archived in a .zip file. An example submission is
    1. "123456 789012.pdf"
    2. "123456 789012.zip"
- The assignment contains 15% of the final grade
- All models must be linear. Additionally, the efficiency of your models will be considered during grading. Introducing unnecessary variables may result in a deduction of points.
- The submission deadline is on **Tuesday 21th of January 2025** at 23:59 in the night. The submissions after this date and hour will not be corrected
- Achieving two bonus points is possible, and (part of) these points can be carried over to the second assignment if you score more than 13 regular points in the first assignment. However, there's a cap on the total points you can attain over the two assignments. To clarify, suppose you manage to secure 15 points in the first assignment and earn the two bonus points, such that you'll have a total of 17 points for that assignment.
    1. If you obtain 12 points for the second assignment, you total will be 29 out of 30 points.
    2. If you obtain a full score of 15 points for the second assignment, your overall score will be capped at 30 points.

  Hence, bonus points cannot be carried over to the final exam.