最大公约数证明：

证明 $\gcd(m,n) = \gcd(n, m \bmod n)$

令 $k = \gcd(m,n)$

$\therefore k \mid m,\ k \mid n.$

令 $j = \gcd(n, m \bmod n)$

$\therefore j \mid n,\ j \mid (m \bmod n)$

m 可表示为 $m = pn + (m \bmod n)$

$\because \begin{cases} j \mid m \\ j \mid n \end{cases}$ $\therefore j$ 是 m,n 的公约数

k 是 m,n 的最大公约数 $\therefore k \geq j.$

$m \bmod n = m - pn$

$\because \begin{cases} k \mid (m \bmod n) \\ k \mid n \end{cases}$ $\therefore k$ 是 m,n 的公约数

j 是 $n, m \bmod n$ 最大公约数 $\therefore j \geq k$

$\therefore j = k$ $\therefore \gcd(m,n) = \gcd(n, m \bmod n)$

算法设计：

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牛顿迭代法.

令 $f(x) = x^2 - a = 0.$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^2 - a}{2x_k} = \frac{2x_k^2 - x_k^2 + a}{2x_k} = \frac{x_k^2 + a}{2x_k}$$

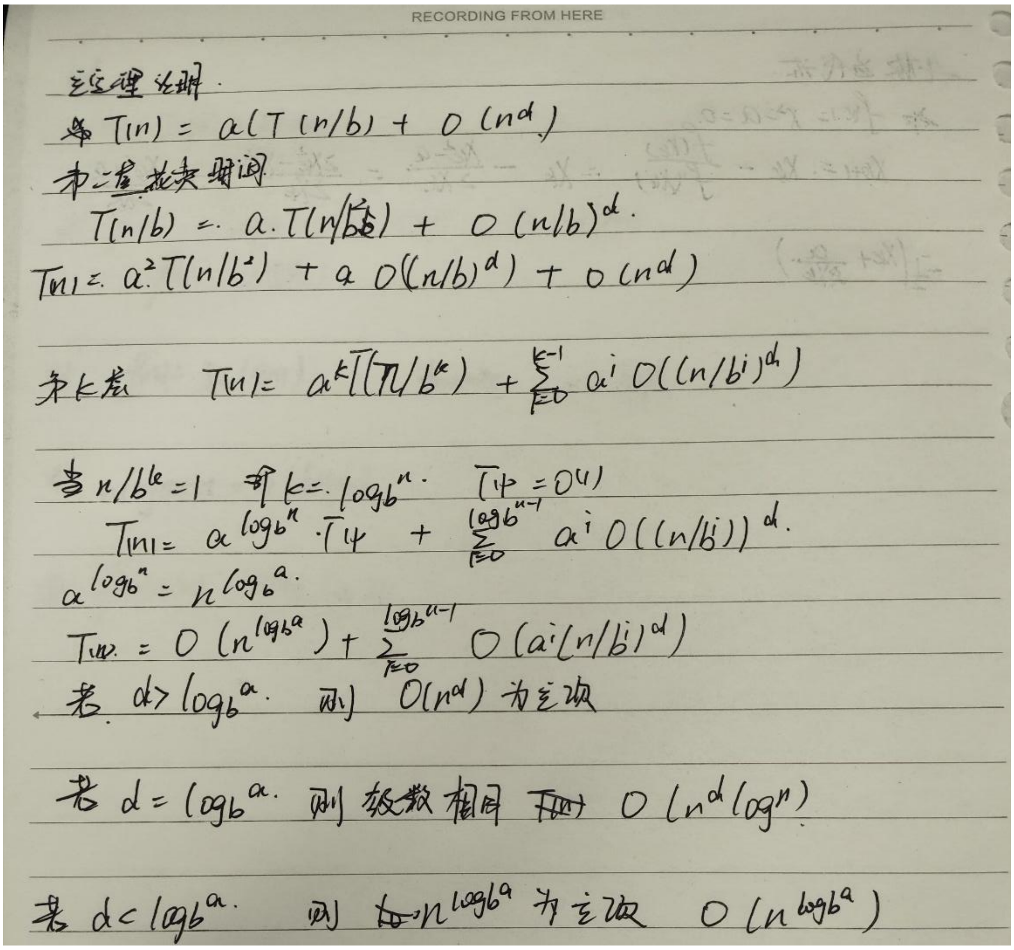
$$= \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)$$

代码如下

```python
def sqrt_newton(n):
    if n == 0:
        return 0
    x = n
    while True:
        root = 0.5 * (x + n / x)
        if abs(root - x) < 0.0001:
            break
```

```
    x = root
return int(root)
```

主定理证明：



主定理证明.

设 $T(n) = aT(n/b) + O(n^d)$

第二层拆开得到

$T(n/b) = a \cdot T(n/b^2) + O((n/b)^d)$.

$T(n) = a^2 T(n/b^2) + a\,O((n/b)^d) + O(n^d)$

第k层 $T(n) = a^k T(n/b^k) + \sum_{i=0}^{k-1} a^i O((n/b^i)^d)$

当 $n/b^k = 1$ 时 $k = \log_b n$. $T(1) = O(1)$

$T(n) = a^{\log_b n} \cdot T(1) + \sum_{i=0}^{\log_b n - 1} a^i O((n/b^i))^d$.

$a^{\log_b n} = n^{\log_b a}$.

$T(n) = O(n^{\log_b a}) + \sum_{i=0}^{\log_b n - 1} O(a^i (n/b^i)^d)$

若 $d > \log_b a$. 则 $O(n^d)$ 为主项

若 $d = \log_b a$. 则 级数相同 $O(n^d \log n)$

若 $d < \log_b a$. 则 $n^{\log_b a}$ 为主项 $O(n^{\log_b a})$

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(1) $f(n) = \log n^2 \quad g(n) = \log n + 5$

$f(n) = \Theta\, g(n)$

(2) $f(n) = \log n^2 \quad g(n) = \sqrt{n}$.

$f(n) = O(g(n))$

(3) $f(n) = n. \quad g(n) = \log^2 n$ 　　　　 $n = \sqrt{n^2}$ 　 $\lim\limits_{n \to +\infty} \sqrt{n} > \log n$.

$\therefore f(n) = \Omega\, g(n)$

(4) $f(n) = n\log n + n \quad g(n) = \log n$.

$f(n) = \Omega\, g(n)$

(5) $f(n) = 10 \quad g(n) = \log(10)$

$f(n) = \Theta\, g(n)$

(6) $f(n) = \log^2 n \quad g(n) = \log n$.

$f(n) = \Omega(g(n))$

(7) $f(n) = \cancel{\log}\, 2^n. \quad g(n) = 100n^2$ 　　 指数>幂

$f(n) = \Omega(g(n))$

(8) $f(n) = 2^n \quad g(n) = 3^n$.

$f(n) = O(g(n))$

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证明 $n! = o(n^n)$

$$\lim_{n\to\infty} \frac{n!}{n^n} = \frac{n\times(n-1)(n-2)(n-3)\cdots}{n\times n\times n\times n\cdots} = 0.$$

$\because$ ~~当n=1时~~ 当且仅当 $n=1$ 时 $n^n = n!$ 　当 $n\to\infty$ n^n 为 $n!$ 的上界

$\therefore n! = o(n^n)$