# Multifidelity Example

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We consider the following problem

$$\max_{x\in A} F(x)$$

where $A \subset \mathbb{R}^n$ is the feasible set.

We have two information sources of $F$, $f$ and $g$. We suppose that $f$ is an unbiased estimate of $F$, but it is noisy, and $g$ is a biased estimate of $F$, but it is deterministic.

We place two independent Gaussian processes on $F$ and $g-F$. This implies a Gaussian process on $(F,g)$ with parameters $\mu_0$ and $\Sigma_0$.

We define the value of information functions as

$$V_n(x,h) = \mathbb{E}\left[\max_z \mu_{n+1}(z,h_1) - \max_z \mu_n(z,h_1) \mid x_{n+1} = x, h(x)\right]$$

where $h, h_1 \in \{F, g\}$.

**Algorithm.**

1. (First stage of samples) Use maximum likelihood or maximum a posteriori estimation to fit the parameters $\alpha_0(\cdot), \beta_0(\cdot,\cdot)$ of the GP prior on $F$, and $\rho_0(\cdot), \Lambda_0(\cdot,\cdot)$ of the GP prior on $\delta = F - g$. We denote the parameters of the joint GP on $F$ and $g$ by $\mu_0, \Sigma_0$

2. (Main stage of samples) For $n \leftarrow 1$ to $N$ do

   (a) Update our joint Gaussian process posterior on $F, g$ using all samples by time $n$.

   (b) Solve $(x_{n+1}, h_{n+1}) \in \arg\max_{x\in A, h\in\{F,g\}} V_n(x,h)$

   (c) Evaluate $h_{n+1}(x_{n+1})$

3. Return $x^* = \arg\max_x \mu_{N+1}(x,h)$ where $h \in \{F,g\}$.

**Example.** We draw $F \sim GP(0,\Sigma_0)$ where $\Sigma_0(x,y) = \exp\left(\sum_{i=1}^4 \frac{1}{2}(x_i - y_i)^2\right)$, and we define $g = F + GP(0,\Sigma^*)$ where $\Sigma^*(x,y) = 0.5\exp\left(\sum_{i=1}^4 \frac{1}{10}(x_i-y_i)^2\right)$. We also define $f = F + N(0,1)$.