

# Multifidelity Example

Saul Toscano-Palmerin

We consider the following problem

$$\max_{x \in A} F(x)$$

where  $A \subset \mathbb{R}^n$  is the feasible set.

We have two information sources of  $F$ ,  $f$  and  $g$ . We suppose that  $f$  is an unbiased estimate of  $F$ , but it is noisy, and  $g$  is a biased estimate of  $F$ , but it is deterministic.

We place two independent Gaussian processes on  $F$  and  $g - F$ . This implies a Gaussian process on  $(F, g)$  with parameters  $\mu_0$  and  $\Sigma_0$ .

We define the value of information functions as

$$V_n(x, h) = \mathbb{E} [\max_{z, h \in \{F, g\}} \mu_{n+1}(z, h) - \max_{z, h \in \{F, g\}} \mu_n(z, h) \mid x_{n+1} = x, h(x)]$$

where  $h, h_1 \in \{F, g\}$ .

## Algorithm.

1. (First stage of samples) Use maximum likelihood or maximum a posteriori estimation to fit the parameters  $\alpha_0(\cdot), \beta_0(\cdot, \cdot)$  of the GP prior on  $F$ , and  $\rho_0(\cdot), \Lambda_0(\cdot, \cdot)$  of the GP prior on  $\delta = F - g$ . We denote the parameters of the joint GP on  $F$  and  $g$  by  $\mu_0, \Sigma_0$
2. (Main stage of samples) For  $n \leftarrow 1$  to  $N$  do
  - (a) Update our joint Gaussian process posterior on  $F, g$  using all samples by time  $n$ .
  - (b) Solve  $(x_{n+1}, h_{n+1}) \in \arg \max_{x \in A, h \in \{F, g\}} V_n(x, h)$
  - (c) Evaluate  $h_{n+1}(x_{n+1})$
3. Return  $x^* = \arg \max_x \mu_{N+1}(x, h)$  where  $h \in \{F, g\}$ .

**Example.** We draw  $F \sim GP(0, \Sigma_0)$  where  $\Sigma_0(x, y) = \exp(\sum_{i=1}^4 \frac{1}{2} (x_i - y_i)^2)$ , and we define  $g = F + GP(0, \Sigma^*)$  where  $\Sigma^*(x, y) = 0.5 \exp(\sum_{i=1}^4 \frac{1}{10} (x_i - y_i)^2)$ . We also define  $f = F + N(0, 1)$ .