MISO Problem: Two Information Sources with Coupling

1 Mathematical Formulation

We consider an optimization problem with two information sources

$$\min_{(x_1,x_2,x_3)\in A} f_2\left(f_1\left(x_1, x_3\right), x_2, x_3\right)$$

where A is the feasible compact set and f_1, f_2 are continuous functions (see Figure 1).

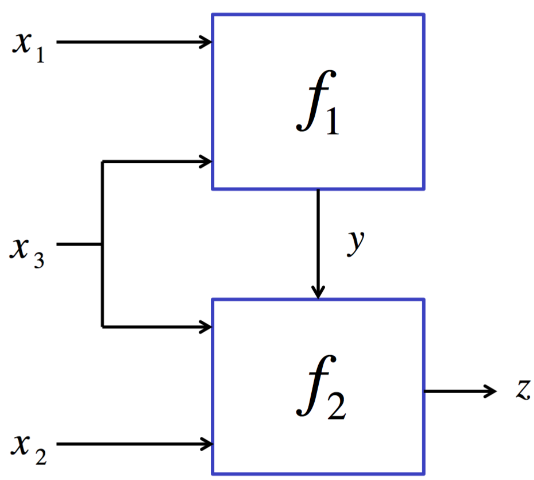

Figure 1: Diagram of the problem

This formulation can be used to solve different problems. For example, f_1 could be an aerodynamic simulation of a wing and f_2 could be a structural simulation of a wing, that includes aerodynamical forces.

2 Value of Information Functions and Gaussian Processes

We place two Gaussian processes on f_1 and f_2. Depending on the problem and the kernels of the Gaussian processes, we may have a Gaussian process on $f_2\left(f_1\left(x_1, x_3\right), x_2, x_3\right)$ with parameters μ_0 and Σ_0.

We define the value of information functions as

$$V_n\left(x, h\right) = \mathbb{E}\left[\max_z \mu_{n+1}\left(z\right) - \max_z \mu_n\left(z\right) \mid x_{n+1} = x, h\left(x\right)\right]$$

where $h \in \{f_1, f_2\}$ and $x \in A \bigcup B$ where B is the range of f_1, which is defined by $B = \{f\left(x_1, x_3\right) : \exists x_2 \text{ s.t } \left(x_1, x_2, x_3\right) \in A\}$.

3 Algorithm

1. (First stage of samples) Use maximum likelihood or maximum a posteriori estimation to fit the parameters $\alpha_0\left(\cdot\right), \beta_0\left(\cdot, \cdot\right)$ of the GP prior on f_1, and $\rho_0\left(\cdot\right), \Lambda_0\left(\cdot, \cdot\right)$ of the GP prior on f_2. We denote the parameters of GP on $f_2\left(f_1\left(x_1, x_3\right), x_2, x_3\right)$ by μ_0, Σ_0

2. (Main stage of samples) For $n \leftarrow 0$ to N do

   (a) Update our joint Gaussian process posterior on $f_2\left(f_1\left(x_1, x_3\right), x_2, x_3\right)$ using all samples by time n.

   (b) Solve $\left(x_{n+1}, h_{n+1}\right) \in \arg\max_{x, h \in \{f_1, f_2\}} V_n\left(x, h\right)$

   (c) Evaluate $h_{n+1}\left(x_{n+1}\right)$

3. Return $x^* = \arg\max_{\left(x_1, x_2, x_3\right)} \mu_{N+1}\left(x_1, x_2, x_3\right)$.