

Multifidelity Example

Saul Toscano-Palmerin

We consider the following problem

$$\max_{x \in A} F(x)$$

where $A \subset \mathbb{R}^n$ is the feasible set.

We have two information sources of F , f and g . We suppose that f is an unbiased estimate of F , but it is noisy, and g is a biased estimate of F , but it is deterministic.

We place two independent Gaussian processes on F and $g - F$. This implies a Gaussian process on (F, g) with parameters μ_0 and Σ_0 .

We define the value of information functions as

$$V_n(x, h) = \mathbb{E} [\max_z \mu_{n+1}(z, F) - \max_z \mu_n(z, F) \mid x_{n+1} = x, h(x)]$$

where $h \in \{g, f\}$.

Algorithm.

1. (First stage of samples) Use maximum likelihood or maximum a posteriori estimation to fit the parameters $\alpha_0(\cdot), \beta_0(\cdot, \cdot)$ of the GP prior on F , and $\rho_0(\cdot), \Lambda_0(\cdot, \cdot)$ of the GP prior on $\delta = F - g$. We denote the parameters of the joint GP on F and g by μ_0, Σ_0
2. (Main stage of samples) For $n \leftarrow 1$ to N do
 - (a) Update our joint Gaussian process posterior on F, g using all samples by time n .
 - (b) Solve $(x_{n+1}, h_{n+1}) \in \arg \max_{x \in A, h \in \{f_{n+1}, g\}} V_n(x, h)$
 - (c) Evaluate $f_{n+1}(x_{n+1})$
3. Return $x^* = \arg \max_x \mu_{N+1}(x, h)$ where $h \in \{F, g\}$.

Example. We draw $F \sim GP(0, \Sigma_0)$ where $\Sigma_0(x, y) = \exp(\sum_{i=1}^4 \frac{1}{2} (x_i - y_i)^2)$, and we define $g = F + GP(0, \Sigma^*)$ where $\Sigma^*(x, y) = 0.5 \exp(\sum_{i=1}^4 \frac{1}{10} (x_i - y_i)^2)$. We also define $f = F + N(0, 1)$.