Multifidelity Example

Saul Toscano-Palmerin

We consider the following problem

$$\max_{x\in A} F(x)$$

where $A \subset \mathbb{R}^n$ is the feasible set.

We have two information sources of F, f and g. We suppose that f is an unbiased estimate of F, but it is noisy, and g is a biased estimate of F, but it is deterministic.

We place two independent Gaussian processes on F and $g-F$. This implies a Gaussian process on (F,g) with parameters μ_0 and Σ_0.

We define the value of information functions as

$$V_n(x,h) = \mathbb{E}\left[\max_z \mu_{n+1}(z,F) - \max_{z,} \mu_n(z,F) \mid x_{n+1} = x, h(x)\right]$$

where $h \in \{g, f\}$.

Algorithm.

1. (First stage of samples) Use maximum likelihood or maximum a posteriori estimation to fit the parameters $\alpha_0(\cdot), \beta_0(\cdot,\cdot)$ of the GP prior on F, and $\rho_0(\cdot), \Lambda_0(\cdot,\cdot)$ of the GP prior on $\delta = F - g$. We denote the parameters of the joint GP on F and g by μ_0, Σ_0

2. (Main stage of samples) For $n \leftarrow 1$ to N do

   (a) Update our joint Gaussian process posterior on F, g using all samples by time n.

   (b) Solve $(x_{n+1}, h_{n+1}) \in \arg\max_{x\in A, h\in\{f_{n+1}, g\}} V_n(x,h)$

   (c) Evaluate $h_{n+1}(x_{n+1})$

3. Return $x^* = \arg\max_x \mu_{N+1}(x,h)$ where $h \in \{F, g\}$.

Example. We draw $F \sim GP(0,\Sigma_0)$ where $\Sigma_0(x,y) = \exp\left(\sum_{i=1}^4 \frac{1}{2}(x_i - y_i)^2\right)$, and we define $g = F + GP(0,\Sigma^*)$ where $\Sigma^*(x,y) = 0.5\exp\left(\sum_{i=1}^4 \frac{1}{10}(x_i - y_i)^2\right)$. We also define $f = F + N(0,1)$.