

# ASYMPTOTIC VALIDITY OF A FULLY SEQUENTIAL ELIMINATION PROCEDURE FOR INDIFFERENCE-ZONE RANKING AND SELECTION WITH TIGHT BOUNDS ON PROBABILITY OF CORRECT SELECTION

Saul Toscano

School of Operations Research and Information Engineering  
Cornell University  
206 Rhodes Hall  
Ithaca, NY 14853, USA

Peter I. Frazier

School of Operations Research and Information Engineering  
Cornell University  
206 Rhodes Hall  
Ithaca, NY 14853, USA

## ABSTRACT

We consider the indifference-zone (IZ) formulation of the ranking and selection problem. Conservatism leads classical IZ procedures to take too many samples in problems with many alternatives. The Bayes-inspired Indifference Zone (BIZ) procedure, proposed in Frazier (2014), is less conservative than previous procedures, but its proof of validity requires strong assumptions. In this paper, we present a new proof of asymptotic validity that relaxes these assumptions.

## 1 INTRODUCTION

One common problem in simulation is that of choosing the best among several simulated systems. The problem of deciding how many samples to use from each alternative to support our selection as the best is the ranking and selection problem. An efficient solution to this problem has to balance between the time spent simulating and the quality of the selection.

This paper will consider the indifference-zone (IZ) formulation of the ranking and selection problem, in which we choose the best system with probability larger than a pre-specified threshold, whenever the distance between the best system and the others is sufficiently large. We say that a sampling procedure having this property satisfies the IZ guarantee and the set of system configurations under which the best alternative is better than the second best by at least some given  $\delta > 0$  is called the preference zone (PZ). The seminal work dates back to Bechhofer (1954), with early work compiled in the monograph Bechhofer et al. (1968). The progress in the area has been summarized in Bechhofer et al. (1995), Swisher et al. (2003), Kim and Nelson (2006, 2007) and Frazier (2014).

The goal of an IZ algorithm is to take as few samples as possible while the IZ guarantee is satisfied. The first IZ procedures presented in Bechhofer (1954), Paulson (1964), Fabian (1974), Rinott (1978), Hartmann (1988, 1991), Paulson (1994) satisfy the IZ guarantee, but they usually take too many samples when there are many alternatives, in part because their probability of correct selection (PCS) is much larger than the probability specified by the user. A reason of this is the use of the Bonferroni's inequality which leads to

sample more than necessary. More recent algorithms in Kim and Nelson (2001), Goldsman et al. (2002), Hong (2006) improve the performance but they still use the Bonferroni's inequality, and so the methods are inefficient when there are several systems. Procedures in Kim and Dieker (2011), Dieker and Kim (2012) do not use the Bonferroni's inequality only when compare groups of three alternatives.

Since the classic IZ procedures take too many samples with many alternatives, these methods are unpopular when there are more than a few hundred alternatives. However, Frazier (2014) presented a new sequential elimination IZ procedure, called BIZ (Bayes-inspired Indifference Zone), whose lower bound on worst-case probability of correct selection in the preference zone is tight in continuous time, and almost tight in the discrete time. In numerical experiments, the number of samples required by BIZ is significantly smaller than that of the most popular IZ procedures, especially on problems with many alternatives. Unfortunately, the proof that the BIZ procedure satisfies the IZ guarantee for the discrete-time case assumes that variances are known and have an integer multiple structure which is not very realistic. In practice, variances are unknown. However, asymptotically we can use a central limit theorem that allows us to prove the asymptotic validity of the BIZ procedure for the discrete-time case. Moreover, we only need to assume that the systems are independent, identically distributed and have finite variance.

Kim et al. (2006) also proves the asymptotical validity of a IZ procedure. Our proof is larger because the BIZ procedure is more sophisticated and the bound for the PCS is tighter. To prove the case when the variances are known, we use a theorem for Ergodic processes that shows how to standardize the output data to make them behave like Brownian motion processes in the limit. We also use an extension of the Continuous Mapping Theorem (Theorem 5.5 of Billingsley 1968) to see that the algorithm behaves like a sequential elimination IZ procedure with a Brownian motion process instead of the standardized sum of the output data in the limit, and then we use the results of the paper of Frazier (?) to prove the validity of this algorithm in the limit. Finally, we use a random change of time argument to prove the case when the variances are unknown.

This paper is organized as follows: In 2, we recall the indifference-zone ranking and selection problem. In 3, we recall the Bayes-inspired IZ (BIZ) procedure. In 4, we present the proof of the validity of the algorithm when the variances are known. In 5, we prove the case when the variances are unknown. In 6, we present some simulations showing the effectiveness of the algorithm. In 7, we conclude.

## 2 INDIFFERENCE-ZONE RANKING AND SELECTION

Ranking and Selection is a procedure for selecting the best system among a finite set of alternatives, i.e. the system with the largest mean. The method selects a system as the best based on the samples that are observed sequentially over the time. If the best system is selected, we say that the procedure has made the *correct selection* (CS). We define the *probability of correct selection* as

$$\text{PCS}(\mu, \lambda) = \mathbb{P}_{\mu, \lambda}(\hat{x} \in \arg \max_x \mu_x)$$

where  $\hat{x}$  is the alternative chosen by the procedure and  $\mathbb{P}_{\mu, \lambda}$  is the probability measure under which samples from system  $x$  have mean  $\mu_x$  and finite variance  $\lambda_x^2$ .

In the Indifference-Zone Ranking and Selection, the procedure is indifferent in the selection of a system whenever the means of the populations are nearly the same. Formally, let  $\mu = [\mu_k, \dots, \mu_1]$  be the vector of the true means, the *indifference zone* is defined as the set  $\{\mu \in \mathbb{R}^k : \mu_{[k]} - \mu_{[k-1]} < \delta\}$ . The complement of the indifference zone is called the *preference zone* (PZ) and  $\delta > 0$  is called the indifference zone parameter. We say that a procedure meets the *indifference-zone (IZ) guarantee* at  $P^* \in (1/k, 1)$  and  $\delta > 0$  if

$$\text{PCS}(\mu, \lambda) \geq P^* \text{ for all } \mu \in \text{PZ}(\delta).$$

We assume  $P^* > 1/k$  because IZ guarantees can be met by choosing  $\hat{x}$  uniformly at random from among  $\{1, \dots, k\}$ .

### 3 THE BAYES-INSPIRED IZ (BIZ) PROCEDURE

BIZ is an elimination procedure. This procedure maintains a set of alternatives that are in contention, and it takes samples from each alternative in this set at each point in time. At beginning, all alternatives are in contention, and over the time alternatives are eliminated. The procedure ends when there is only one alternative in the contention set and this remain alternative is chosen as the best.

Frazier (2014) showed that the BIZ procedure with known common variance satisfies the IZ guarantee when the systems follow the normal distribution, with tight bounds on worst-case preference-zone in continuous time. He also proved that this procedure retains the IZ guarantee when the systems follow the normal distribution, and the variances are known and are integer multiples of a common value. The continuous time version of this procedure also satisfies the IZ guarantee, with a tight worst-case preference-zone PCS bound.

The discrete-time BIZ procedure for unknown and/or heterogeneous sampling variances is given in Alg. 2. It takes a variable number of samples from each alternative, and  $n_{tx}$  is this number. This algorithm depends on a collection of integers  $B_1, \dots, B_k$ ,  $P^*$ ,  $c$ ,  $\delta$  and  $n_0$ .  $n_0$  is the number of samples to use in the first stage of samples, and 100 is the recommended value for  $n_0$ .  $B_x$  controls the number of samples taken from system  $x$  in each stage.

For each  $t$ ,  $x \in \{1, \dots, k\}$ , and subset  $A \subset \{1, \dots, k\}$ , we define a function

$$q_{tx}(A) = \exp\left(\delta \beta_t \frac{Z_{tx}}{n_{tx}}\right) / \sum_{x' \in A} \exp\left(\delta \beta_t \frac{Z_{tx'}}{n_{tx'}}\right), \quad \beta_t = \frac{\sum_{x' \in A} n_{tx'}}{\sum_{x' \in A} \hat{\lambda}_{tx'}^2}$$

where  $\hat{\lambda}_{tx}^2$  is the sample variance of all samples from alternative  $x$  thus far and  $Z_{tx} = Y_{n_{tx}, x}$ .

**Algorithm: Discrete-time implementation of BIZ, for unknown and/or heterogeneous variances.**

**Require:**  $c \in [0, 1 - (P^*)^{\frac{1}{k-1}}]$ ,  $\delta > 0$ ,  $P^* \in (1/k, 1)$ ,  $n_0 \geq 0$  an integer,  $B_1, \dots, B_k$  strictly positive integers.

Recommended choices are  $c = 1 - (P^*)^{\frac{1}{k-1}}$ ,  $B_1 = \dots = B_k = 1$  and  $n_0$  between 10 and 30. If the sampling variances  $\lambda_x^2$  are known, replace the estimators  $\hat{\lambda}_{tx}^2$  with the true values  $\lambda_x^2$ , and set  $n_0 = 0$ .

- 1: For each  $x$ , sample alternative  $x$   $n_0$  times and set  $n_{0x} \leftarrow n_0$ . Let  $W_{0x}$  and  $\hat{\lambda}_{0x}^2$  be the sample mean and sample variance respectively of these samples. Let  $t \leftarrow 0$ .
- 2: Let  $A \leftarrow \{1, \dots, k\}$ ,  $P \leftarrow P^*$ .
- 3: **while**  $x \in \max_{x \in A} q_{tx}(A) < P$  **do**
- 4:   **while**  $\min_{x \in A} q_{tx}(A) \leq c$  **do**
- 5:     Let  $x \in \arg \min_{x \in A} q_{tx}(A)$ .
- 6:     Let  $P \leftarrow P / (1 - q_{tx}(A))$ .
- 7:     Remove  $x$  from  $A$ .
- 8:   **end while**
- 9:   Let  $z \in \arg \min_{x \in A} n_{tx} / \hat{\lambda}_{tx}^2$ .
- 10:   For each  $x \in A$ , let  $n_{t+1, x} = \text{ceil}\left(\hat{\lambda}_{tx}^2 (n_{tx} + B_z) / \hat{\lambda}_{tz}^2\right)$ .
- 11:   For each  $x \in A$ , if  $n_{t+1, x} > n_{tx}$ , take  $n_{t+1, x} - n_{tx}$  additional samples from alternative  $x$ . Let  $W_{t+1, x}$  and  $\hat{\lambda}_{t+1, x}^2$  be the sample mean and sample variance respectively of all samples from alternative  $x$  thus far.
- 12:   Increment  $t$ .
- 13: **end while**
- 14: Select  $\hat{x} \in \arg \max_{x \in A} Z_{tx} / n_{tx}$  as our estimate of the best.

This algorithm generalizes the BIZ procedure with known common variance. In that case, we have that  $B_1 = \dots = B_k = 1$  and  $n_{tx} = t$ . The algorithm 2 can be generalized to the continuous case (See appendix B and Frazier (2014)). This procedure is a slightly modification of the original BIZ procedure

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Creature	IQ	Diet
dog	70	anything
cat	75	almost nothing
human	60	ice cream
dolphin	120	fish fillet

Table 2: Counting in Maori.

English	Maori
one	tahi
two	rua
three	toru
four	wha

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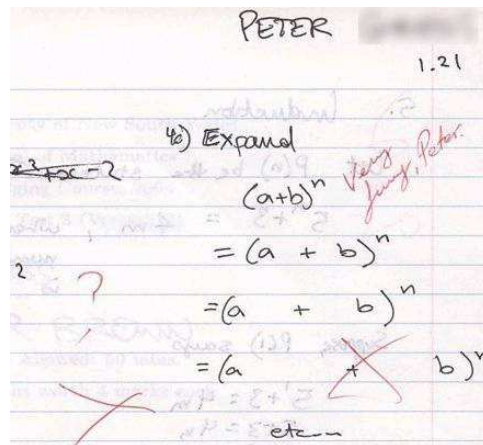


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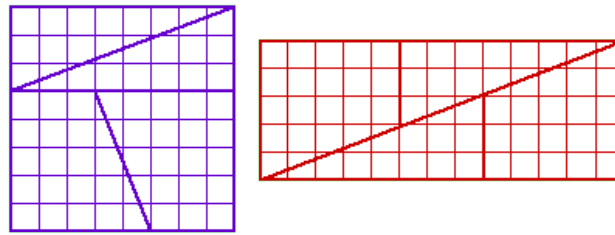


Figure 2: The area of the square is 64 squares, while that of the rectangle is 65 squares, yet they are made of the same pieces! How is this possible?

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anchorcolor=black,linkcolor=black]{hyperref}
```

as the last `\usepackage` command in the preamble. (In general the `\usepackage` command above that works for MiKTeX running on a Windows system should also work for most implementations of L<sup>A</sup>T<sub>E</sub>X running on a Unix or Apple system.) Thus the hypertext link [conference website](#) (Winter Simulation Conference 2015) to the WSC website can be established by the command

```
\href{http://www.wintersim.org}{conference website}
```

This is especially important since WSC papers are filed in the IEEE Xplore digital library, which does not allow hyperlinks, so for that purpose the hyperlinks are removed. Therefore it is recommended to add all hypertext references to the `.bib` file and to refer to them from the text as it is done in the example above. All live hyperlinks still appear in the CD of the proceedings and in other repositories. If the authors use hyperlinked text in the main body of the paper, they must ensure that each hyperlink includes a citation (e.g., (WSC 2015) following the hyperlinked text “conference website” in Section 1), a corresponding entry is provided in the list of references, and the associated web address displayed for the hyperlink is complete and correct so that a reader who has only a hard copy of the paper can still access the cited material by

typing the relevant part of the displayed text of the hyperlink into the address bar of a web browser. If the authors opt for including the web address in the main body of the text itself, they must ensure that the hyperlink is complete and correct for the same reason.

If you use the package `hyperref` as suggested here, and if you use citation commands to handle references, then your citations will become hyperlinks (as in this document).

## 6.6 Citing a Reference

To cite a reference in the text, use the author-date method. Thus, Chien (1989) could also be cited parenthetically (Chien 1989). For a work with four or more authors, use an abbreviated form. For example, a work by Banks, Carson, Nelson, and Nicol would be cited in one of the following ways: Banks et al. (2000) or (Banks et al. 2000).

Parenthetical citations are enclosed in parentheses ( ), not square brackets [ ]. The items in a series of such citations are usually separated by commas. If an item in the series of parenthetical citations contains punctuation because (for example) it refers to a work with three or more coauthors, then all items should be separated by semicolons.

The following is a list of correct forms of citations:

- Brown and Edwards (1993),
- (Brown and Edwards 1993),
- (Brown and Edwards 1993, Smith 1997), and
- (Arnold, Brown, and Edwards 1992; Brown and Edwards 1993; Smith 1997; Brown et al. 1997).

The following is a list of incorrect forms of citations:

- Brown and Edwards [1993],
- (Brown and Edwards, 1993),
- (Brown and Edwards, 1993; Smith, 1997), and
- (Arnold Brown and Edwards 1992, Brown and Edwards 1993, Smith 1997)

For further details, please refer to *The Chicago Manual of Style* (The University of Chicago Press 2010). In Section 6 you can see how correct citations can easily be achieved by using `BIBTEX`.

## 6.7 List of References

Place the list of references after the appendices. The section heading is **REFERENCES**, and it is not numbered. List only references that are cited in the text. Arrange the references in alphabetical order (chronologically for a particular author or group of authors); do not number the references. Give complete references without abbreviations. To identify multiple references by the same authors and year, append a lower case letter to the year of publication; for example, 1984a and 1984b.

Use hanging indentation to distinguish individual entries. Do not insert additional space between references.

You can enter the references using (a) *BIBTEX* as discussed in Section 6, (b) using the environment `thebibliography` via the `\bibitem` and `\cite` commands, or (c) the `hangref` environment as shown below. Please note that neither (b) nor (c) are recommended. These alternatives may mean extra work for you and the editor during the editing process. Option (c) means in addition that the references will not be hyperlinks — as the proceedings are electronic proceedings this is not recommended at all.

*Whatever you do: the list of entries/items need to be included in your submission! And it needs to be included in such a way that the document can be compiled by the editor.*

To use `hangref` you would enter the following lines.

```
\begin{hangref}  
\item The first reference goes here, and if you happen to have enough  
information on the line you will be able to  
see how the second and if you really have lots of text to be displayed  
later lines of the reference are indented.  
\item The second reference goes here,  
and once again later lines are indented if you have a sufficient amount  
of words in the text block.  
\item Further references appear here.  
\end{hangref}
```

The output looks as follows.

The first reference goes here, and if you happen to have enough information on the line you will be able to see how the second and if you really have lots of text to be displayed later lines of the reference are indented.

The second reference goes here, and once again later lines are indented if you have a sufficient amount of words in the text block.

Further references appear here.

The bibliographic style for a journal article is:

<Surname of first author>, <First author initials>, <Initials and surnames of other authors>. <year>. <Capitalized article title in quotes>. <*Journal Name in Headline Italics*> <Volume number>: <page numbers>.

The format for other types of reference can be inferred from the examples in the references, which include:

- a technical report (Chien 1989),
- a proceedings article (Cheng 1994),
- a journal article (Gupta, Nagel, and Panchapakesan 1973),
- a book by 2 authors (Hammersley and Handscomb 1964),
- a chapter in a book (Schruben 1979),
- an unpublished thesis or dissertation (Steiger 1999),
- a book with no identified authors (The University of Chicago Press 2010), and
- a document available on the web (WSC Foundation 2011).

Again, please refer to *The Chicago Manual of Style* (The University of Chicago Press 2010) for further details and examples. Please note that the examples given in the reference section of this document are based on the 16th edition of *The Chicago Manual of Style*. Authors may use the style based on the 15th edition of the manual that they have been using in the papers for the past Winter Simulation Conferences at their discretion. However, the two styles should not be mixed. Clarity and consistency should be your primary concern.

Be sure that references to past WSC proceedings, such as Cheng (1994) include the necessary information such as *Proceedings of the xxxx Winter Simulation Conference*, following by the list of editors, then the page number range for the paper and finally the publisher information, Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.

Template for a bib entry of a (yyyy) Winter Simulation Conference proceedings paper:

```
@Inproceedings{(!Provide a unique key here!),  
author = {aaaa},
```

```

title   = {tttt},
year    = {yyyy},
pages   = {n-m},
booktitle = {Proceedings of the yyyy Winter Simulation Conference},
editors = {eeee},
address = {Piscataway, New Jersey},
publisher = {Institute of Electrical and Electronics Engineers, Inc.}
}

```

Please do not add any additional attributes.

## 7 USING BIB<sub>T</sub>E<sub>X</sub>

Using BIB<sub>T</sub>E<sub>X</sub> for referencing is the recommended way. Indeed, the references in this document were generated using BIB<sub>T</sub>E<sub>X</sub>, so the source for this document serves as an example of how to use BIB<sub>T</sub>E<sub>X</sub> to meet the WSC formatting requirements. One benefit of using BIB<sub>T</sub>E<sub>X</sub> is that bibliography formatting and referencing can be greatly simplified: the correct citation and reference list style is automatically created. We assume that you already know how to use BIB<sub>T</sub>E<sub>X</sub>. Software to manage BIB<sub>T</sub>E<sub>X</sub> files, for example JabRef (Java based), can support you on managing and creating valid bib files. *Please open your bib file with a software like JabRef BEFORE you submit your final version. Experience shows that almost all manually edited bib files contain duplicated bib keys (which means a random selection of references), broken entries which usually lead to missing bibliographic information, invalid keys, and last but not least invalid tokens in bib files. Bib files DO NOT support comments. BIB<sub>T</sub>E<sub>X</sub> should not report any error for your final submitted document.*

The BIB<sub>T</sub>E<sub>X</sub> input file `wsc.bst` and the L<sup>A</sup>T<sub>E</sub>X macros found in `wscbib.tex` are required, but are included in `wsc15papersty.tex`, so no other files (apart from your bibliography) are required. The macros in these files have been tested with L<sup>A</sup>T<sub>E</sub>X. They are not intended for use with L<sup>A</sup>T<sub>E</sub>X 2.09, which is obsolete. The file `wsc.bst` is essentially the same as `chicago.bst`, a file found on many L<sup>A</sup>T<sub>E</sub>X distributions, but is modified to be more compatible with WSC format requirements.

The simplest way to write a WSC article that uses BIB<sub>T</sub>E<sub>X</sub> is to take the source file for this document, and modify it to generate your article. The file `wsc15paper.tex` requires the file `wsc15papersty.tex`, which contains, among other things, `wsc.bst` and `wscbib.tex` that are needed for BIB<sub>T</sub>E<sub>X</sub>.

### 7.1 Set Up the BIB<sub>T</sub>E<sub>X</sub> Input Files

BIB<sub>T</sub>E<sub>X</sub> requires a bibliography style file (extension `.bst`) and a bibliography database file (extension `.bib`). This is achieved using

```

\bibliographystyle{wsc}
\bibliography{demobib}

```

just before the AUTHOR BIOGRAPHY section. The file `demobib` in the `\bibliography` command should be replaced with the base names of your BIB<sub>T</sub>E<sub>X</sub> `*.bib` files that you use for your bibliography. BIB<sub>T</sub>E<sub>X</sub> is then run as usual to create a bibliography file (`*.bbl`).

### 7.2 Use the Citation Macros

There are a number of macros available to cite references in the L<sup>A</sup>T<sub>E</sub>X source document. The `\cite` macro can be used to give a list of references in parentheses. For example,

```
\cite{law:simulationc,cheng:queuehetero}
```

results in the citation (Law and Kelton 2000, Cheng and Kleijnen 1999). A reference that functions as a noun is created with the `\citeN` macro. For example,

```
\citeN{law:simulationc} say \ldots
```

results in: Law and Kelton (2000) say ...

Citations within parentheses do not need the extra parentheses provided by the above citation commands. To suppress the inclusion of extra parentheses, use the `\citeNP` macro. To obtain (Cheng and Kleijnen 1999, Law and Kelton 2000), for example, use:

```
(\citeNP{cheng:queuehetero},  
\citeNP{law:simulationc}).
```

When there are four or more authors, the name of the first author should be given along with the text “et al.” This can be achieved with the `\shortcite` macro. To obtain (Banks et al. 2000), for example, use:

```
\shortcite{bcnn:simulation}
```

The macros `\shortciteN` and `\shortciteNP` are also available to obtain ‘et al.’ when a citation with many authors is used as a noun.

For further information on the available commands for citing, search for `\cite` in the file `wscbib.tex`, or consult the file `chicago.sty`. The commands for making `BIBTEX` work with `wsc.bst` are very similar to those used in the standard `LATEX` file `chicago.sty`.

### 7.3 Generate the Bibliography File

Run `PDFLATEX` (or `LATEX`), then `BIBTEX`, and then `PDFLATEX` two more times. Running `PDFLATEX` the first time creates the `.aux` file. Running `BIBTEX` creates the `.bbl` file. Running `PDFLATEX` again (twice) fixes the bibliography and citation references.

### 7.4 Include the Bibliography File in Your Submission

Be sure to include your `.bib` file(s) or your `.bbl` file as part of your submission. If you only include the `.bbl` file, then please verify that you include the most up-to-date version reflecting changes during the editing process by rerunning `BIBTEX` one last time before submission. Please be aware that submitting the `.bbl` file instead of the `.bib` file means extra work for the editing team and for you, as any changes to the reference list need to be done by you in this case. You can use, e.g., `JabRef` to create a minimal `.bib` file based on your bibliography and the document (see “Tools”, “New subdatabase based on aux file”). Please open and save the file before every submission with a software like `JabRef` — to see whether the file is correct and to check for duplicated entries and/or bib keys in the file.

## 8 AUTHOR CHECKLIST

We strive for a consistent appearance in all papers published in the proceedings. If you used the template and styles within this author’s kit, then almost all of the requirements in this checklist will be automatically satisfied, and there is very little to check.

Please **print a hard copy of your paper**, and go over your printed paper to make sure it adheres to the following requirements. *Thank you!*

1. Abstract
  - (a) 150 or fewer words.
  - (b) No list of keywords.
2. Paper Length
  - (a) At least 5, but no more than 12 pages (15 pages for papers in the introductory and advanced tutorial tracks, and for panels).
  - (b) Page size is letter size (8.5" × 11", or 216mm × 279mm).
3. All text is in 11-Point Times New Roman.
4. The paper has been spellchecked using US English.
5. Spacing and Margins
  - (a) Single spaced.
  - (b) Left and right margins are each 1 inch.
  - (c) Top and bottom margins are each 1 inch except first page.
  - (d) First page has 1.5 inch margin from the title to the top of the page, and a 1 inch bottom margin.
6. Section Headings
  - (a) Left justified and set in **BOLDFACE ALL CAPS**.
  - (b) Numbered, except for the abstract, acknowledgments, references and author biographies.
  - (c) Subsection headings are not set in all capitals.
7. No footnotes or page numbers.
8. The running head on the first page is as given in the template file, and the running head on subsequent pages is the surnames of the authors.
9. The title is in **11 POINT BOLDFACE ALL CAPS**
10. Multiple authors are formatted correctly, with email addresses and other information in the Author Biography section.
11. Equations are centered and any equation numbers are in parentheses and right-justified.
12. Figures and Tables
  - (a) All text in figures and tables is readable.
  - (b) Table captions appear above the table. Figure captions appear below the figure.
  - (c) Fonts are embedded in all non bitmap figures.
13. Citations and References (using BIB<sub>T</sub>E<sub>X</sub> is recommended)
  - (a) Citations are by author and year, and are enclosed in parentheses, not brackets.
  - (b) References are in the `hangref` style, and are listed alphabetically by the last name(s) of the author(s).
14. Author biographies are one paragraph per author.
15. Hyperlinks
  - (a) Hyperlinks will work as of the date of December 2015.
  - (b) Live hyperlinks are blue.
  - (c) URLs are given in the references section and are properly cited.
16. Include all files necessary to generate your paper, including
  - (a) Figures (either all in `.ps` or `.eps` format, or all in `.jpg`, `.png`, or `.pdf` format — see Section 5.4),
  - (b) Bibliography files, if used `.bib` files. Please avoid providing `.bbl` files — see Section 6.4),
  - (c) check whether all files included are correctly referenced by trying to compile the archived submission on your own, and
  - (d) any non-standard packages you use.

After verifying that your paper meets these requirements, please go to the final submission page linked on the [conference website](#) (Winter Simulation Conference 2015) and submit your paper. Be sure to complete the transfer of copyright and email a copy of your .pdf receipt to [the proceedings editors](#) in the process. Thank you for contributing to the WSC!

## **ACKNOWLEDGMENTS**

Place the acknowledgments section, if needed, after the main text, but before any appendices and the references. The section heading is not numbered. These instructions are adapted from instructions that have been updated and improved by proceedings editors and several other individuals, who are too numerous to name separately (our apologies, but it is necessary), since the first set of instructions were written by Barry Nelson for the 1991 WSC.

## **A APPENDICES**

Place any appendices after the acknowledgments and label them **A**, **B**, **C**, and so forth.

The solution to (1) has the form

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ if } a \neq 0. \quad (2)$$

## **B GETTING HELP**

If you need help in preparing your paper, contact the proceedings editors. You can reach the entire team by writing to our unified point of contact at [wsc15proceedings@gmail.com](mailto:wsc15proceedings@gmail.com).

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Levent Yilmaz  
Auburn University  
Email: [wsc15yilmaz@gmail.com](mailto:wsc15yilmaz@gmail.com)

Wai Kin (Victor) Chan  
Rensselaer Polytechnic Institute  
Email: [wsc15chan@gmail.com](mailto:wsc15chan@gmail.com)

Il-Chul Moon  
KAIST  
Email: [wsc15moon@gmail.com](mailto:wsc15moon@gmail.com)

Theresa M. K. Roeder  
San Francisco State University  
Email: [wsc15roeder@gmail.com](mailto:wsc15roeder@gmail.com)

## **C OFTEN OBSERVED MISTAKES**

The following list comprises the most common sources of error that had to be corrected by previous editors. Please make sure to go through the following list and check that your paper is formatted correctly:

1. The paper can be *at most* 12 pages long (15 for tutorials and panel sessions). Longer papers cannot be published.
2. Paper title and section titles are in ALL CAPS, subsections capitalize the first letter of important words. Please use the templates to use correct indents and spaces.



3. Paper is letter format, not DinA 4 format. Please use the required margins (different on page 1 from the following pages).
4. Use the correct running heads! Use the proceedings editors and chairs on page one, and use the last names separated by commas for the other pages. Don't forget that the last Last Name is preceded by “, and ”
5. Double check the citation format!
6. Don't forget the “author biographies” section!
7. Double-check that figures and tables are referenced in the text and have the correct caption format!
8. Double check that the author section after the title is formatted correctly: the number of organizations defines the number of blocks, the number of blocks defines the layout.
9. In the heading on the title page, country names should be in ALL CAPS.
10. The first line of each paragraph is indented, with the exception of the first paragraph of a section or subsection.
11. There should be extra lines before and after enumerations, lists, etc.

## REFERENCES

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## AUTHOR BIOGRAPHIES

**LEVENT YILMAZ** is Professor of Computer Science and Software Engineering at Auburn University with a joint appointment in Industrial and Systems Engineering. He holds M.S. and Ph.D. degrees in Computer Science from Virginia Tech. He is the founding organizer and General Chair of the Agent-Directed Simulation Conference series and is currently serving as the Editor-in-Chief of *Simulation: Transactions*

of the SCS. His email address is [wsc15yilmaz@gmail.com](mailto:wsc15yilmaz@gmail.com).

**WAI KIN (VICTOR) CHAN** is an Associate Professor of the Department of Industrial and Systems Engineering at the Rensselaer Polytechnic Institute, Troy, NY. He holds a Ph.D. in industrial engineering and operations research from University of California, Berkeley. His research interests include discrete-event simulation, agent-based simulation, and their applications in social networks, service systems, transportation networks, energy markets, and manufacturing. He is a member of INFORMS, IIE, and IEEE. His e-mail address is [wsc15chan@gmail.com](mailto:wsc15chan@gmail.com).

**II-CHUL MOON** is an assistant professor at the Department of Industrial and Systems Engineering, KAIST, Daejeon, Republic of Korea. His theoretic research focuses on the overlapping area of computer science, management, sociology, and operations research. His practical research includes military command and control analysis, counterterrorism analysis, intelligence analysis, and disaster management; His email address is [wsc15moon@gmail.com](mailto:wsc15moon@gmail.com).

**THERESA M. K. ROEDER** is an Associate Professor of Decision Sciences at San Francisco State University. She holds a PhD in Industrial Engineering and Operations Research from UC Berkeley. Her research interests lie in O.R. education and simulation modeling, especially in healthcare and higher education. Her email address is [wsc15roeder@gmail.com](mailto:wsc15roeder@gmail.com).

*Toscano, Frazier*

First Name Last Name 1  
First Name Last Name 2

Institution  
Street Address 1  
Street Address 2  
City, ST Zip, COUNTRY

Figure 3: Example title page heading with 2 authors from the same institution.

First Name Last Name 1

First Name Last Name 2

Institution 1  
Street Address 1  
Street Address 2  
City, ST Zip, COUNTRY

Institution 2  
Street Address 1  
Street Address 2  
City, ST Zip, COUNTRY

Figure 4: Example title page heading with 2 authors from different institutions.

First Name Last Name 1

First Name Last Name 2

Institution 1  
Street Address Line 1  
Street Address Line 2  
City, ST Zip, COUNTRY

Institution 2  
Street Address Line 1  
Street Address Line 2  
City, ST Zip, COUNTRY

First Name Last Name 3

Institution 3  
Street Address 1  
Street Address 2  
City, ST Zip, COUNTRY

Figure 5: Alternate example title page heading with 3 authors from different institutions.

First Name Last Name 1

Institution 1  
Street Address Line 1  
Street Address Line 2  
City, ST Zip, COUNTRY

First Name Last Name 2

Institution 2  
Street Address Line 1  
Street Address Line 2  
City, ST Zip, COUNTRY

First Name Last Name 3

Institution 3  
Street Address Line 1  
Street Address Line 2  
City, ST Zip, COUNTRY

First Name Last Name 4

Institution 4  
Street Address Line 1  
Street Address Line 2  
City, ST Zip, COUNTRY

Figure 6: Example title page heading with 4 authors from different institutions.