

Representing Homology Classes by Locally Flat Surfaces of Minimum Genus*

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1 Introduction

A necessary and sufficient condition will be given for a nontrivial homology class of a simply connected 4-manifold to be represented by a simple, topologically locally flat embedding of a compact Riemann surface.

2 Splittings of Hermitian Modules

We begin with an algebraic result.

Theorem 1. *The following is a commutative diagram of pointed hermitian modules.*

$$\begin{array}{ccccc}
 (M, h, z) & \xrightarrow{\pi_1} & (M_1, h_1, 0) & & \\
 \downarrow \pi_0 & \searrow \alpha \cong & \downarrow \pi_{1d} & \searrow \alpha_1 \cong & \\
 & (M', h', z') \oplus H(\Lambda^k) & \xrightarrow{\pi_1} & (M'_1, h'_1, 0) \oplus H(\Lambda_1^k) & \\
 & \downarrow \pi_0 & & \downarrow \pi_{1d} & \\
 (M_0, h_0, z_0) & \xrightarrow{\pi_{0d}} & (M_d, h_d, 0) & & \\
 \downarrow \alpha_0 \cong & & \downarrow \alpha_d \cong & & \\
 & (M'_0, h'_0, z'_0) \oplus H(\Lambda_0^k) & \xrightarrow{\pi_{0d}} & (M'_d, h'_d, 0) \oplus H(\Lambda_d^k) & \\
 & \downarrow \beta'_0 \oplus id & & \downarrow \beta'_d \oplus id & \\
 (M_0, h_0, z_0) & \xrightarrow{\pi_{0d}} & (M_d, h_d, 0) & & \\
 \downarrow \beta_0 \cong & & \downarrow \beta_d \cong & & \\
 & (L, \lambda, x) \oplus H(\Lambda_0^k) & \xrightarrow{\pi_{0d}} & (L_d, \lambda_d, 0) \oplus H(\Lambda_d^k) &
 \end{array}$$

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