

定理 1.14 対称差集合について，次の性質が成り立つ。

- (1) $A \oplus B = B \oplus A$ (2) $A \oplus f = A$ (3) $A \oplus U = \sim A$
 (4) $A \oplus A = f$ (5) $A \oplus B = (A \cup B) - (A \cap B)$
 (6) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

【証明】

(1) ~ (4): 対称差集合の定義により。

(5): $A \oplus B = (A - B) \cup (B - A)$

$$= (A \cap \sim B) \cup (B \cap \sim A)$$

$$= ((A \cap \sim B) \cup B) \cap ((A \cap \sim B) \cup \sim A)$$

$$= (A \cup B) \cap (\sim B \cup B) \cap (A \cap \sim A) \cap (\sim B \cup \sim A)$$

$$= (A \cup B) \cap (\sim (A \cap B))$$

$$= (A \cup B) - (A \cap B)$$

(6): $X \oplus Y = (X - Y) \cup (Y - X) = (X \cap \sim Y) \cup (\sim X \cap Y)$ であり，

$$\sim (X \oplus Y) = \sim ((X \cup Y) - (X \cap Y)) = \sim ((X \cup Y) \cap \sim (X \cap Y))$$

$$= \sim (X \cup Y) \cup (X \cap Y) = (\sim X \cap \sim Y) \cup (X \cap Y) \text{ であるので，}$$

$$(A \oplus B) \oplus C$$

$$= ((A \oplus B) \cap \sim C) \cup (\sim (A \oplus B) \cap C)$$

$$= (((A \cap \sim B) \cup (\sim A \cap B)) \cap \sim C) \cup (((\sim A \cap \sim B) \cup (A \cap B)) \cap C)$$

$$= (A \cap \sim B \cap \sim C) \cup (\sim A \cap B \cap \sim C) \cup (\sim A \cap \sim B \cap C) \cup (A \cap B \cap C)$$

$$= ((A \cap \sim B \cap \sim C) \cup (A \cap B \cap C)) \cup ((\sim A \cap B \cap \sim C) \cup (\sim A \cap \sim B \cap C))$$

$$= (A \cap ((\sim B \cap \sim C) \cup (B \cap C))) \cup (\sim A \cap ((B \cap \sim C) \cup (\sim B \cap C)))$$

$$= (A \cap \sim (B \oplus C)) \cup (\sim A \cap (B \oplus C))$$

$$= A \oplus (B \oplus C) \text{ が成り立つ。}$$