Homotopical Topology
Notes

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1 Classical Spaces

1.1 Euclidean Spaces, Spheres, and Balls

$\mathbb{R}^n, \mathbb{C}^n$, sphere S^n, ball D^n; $\mathbb{R}^\infty$ is the inductive limit of the chain $\mathbb{R}^1 \subset \mathbb{R}^2 \subset \mathbb{R}^3 \subset ...$; thus $\mathbb{R}^\infty$ is the set of sequences of real numbers with only finitely many nonzero terms. The topology in $\mathbb{R}^\infty$ is introduced by the rule: A set $F \subset \mathbb{R}^\infty$ is closed $\iff$ all intersections $F \cap \mathbb{R}^n$ are closed in respective spaces. $\mathbb{C}^\infty, S^\infty, D^\infty$ have a similar sense.

Exercise 1. Show that sequence $(a_1, 0, 0, ...), (0, a_2, 0, ...), (0, 0, a_3, ...), ...$ has a limit $\iff$ it has finitely many nonzero terms.

Since in each dimension it is nonzero at most once the limit point if it exists must be $(0, 0, 0, ...)$. Consider open neighbourhood $((-|a_1/2|, |a_1/2|), ((-|a_2/2|, |a_2/2|), (-|a_3/2|, |a_3/2|), ...)$ (if $a_i = 0$ then take $(-1, 1)$): If a_i is nonzero then it is outside this neighbourhood. But there must $\exists N$ s.t. $\forall n > N$ the n-th point is in this neighbourhood, or in other words is zero. Therefore it only has finitely many nonzero terms.