linear algebra

Includes things like matrices and vectors to optimize our code.

vector is a 1-dimensional array of numbers. column vector: $\begin{bmatrix} -2 \\ 3 \\ 0.1 \end{bmatrix}$ row vector: $\begin{bmatrix} -9.5 & 4 & -2 \end{bmatrix}$

matrix is a 2-dimensional array of numbers. $M = \begin{bmatrix} 4 & 7.5 & -8.2 \\ -85 & -6 & 44 \end{bmatrix}$

$M_{i\,j} = the\ j^{th}\ number\ on\ the\ i^{th}\ row\ in\ matrix\ M$ $e.g. M_{1\,3} = -8.2$

initialization

Identity matrix (If you multiply a matrix with it, you get the original matrix):

```
M = eye(n);       % n is the size of the identity matrix
```

matrix of zeroes:

```
M = zeros(n, m);  % the matrix will be the size n * m
```

matrix of ones:

```
M = ones(n, m);   % the matrix will be the size n * m
```

random:

```
M = rand(n, m);   % the matrix will be the size n * m
```

custom:

```
M = [1, 2, 3; 4, 5, 6; 7, 8, 9];
```

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

*replace m with 1 if you want a vector.

matrix-vector multiplication

$$\begin{bmatrix} M_{1\,1} & \cdots & M_{1\,m} \\ \vdots & \ddots & \vdots \\ M_{n\,1} & \cdots & M_{n\,m} \end{bmatrix} \times \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} = \begin{bmatrix} M_{1\,1}v_1 + M_{1\,2}v_2 + \cdots + M_{1\,m}v_m \\ \vdots \\ M_{n\,1}v_1 + M_{n\,2}v_2 + \cdots + M_{n\,m}v_m \end{bmatrix}$$

n * m m * 1 n * 1

```
new_M = M * v;
```

matrix multiplication

$$\begin{bmatrix} A_{1\,1} & \cdots & A_{1\,m} \\ \vdots & \ddots & \vdots \\ A_{n\,1} & \cdots & A_{n\,m} \end{bmatrix} \times \begin{bmatrix} B_{1\,1} & \cdots & B_{1\,o} \\ \vdots & \ddots & \vdots \\ B_{m\,1} & \cdots & B_{m\,o} \end{bmatrix} = \begin{bmatrix} A_{1\,1}B_{1\,1} + \cdots + A_{1\,m}B_{m\,1} & \cdots & A_{1\,1}B_{1\,o} + \cdots + A_{1\,m}B_{m\,o} \\ \vdots & \ddots & \vdots \\ A_{n\,1}B_{1\,1} + \cdots + A_{n\,m}B_{m\,1} & \cdots & A_{n\,1}B_{1\,o} + \cdots + A_{n\,m}B_{m\,o} \end{bmatrix}$$

n * m m * o n * o

Just the concatenation of A times column vectors of B.

```
new_M = A * B;
```

element-wise operations

Add a dot in front of the operator.

```
A = [1, 2; 3, 4];

B = [5, 6; 7, 8];

A * B;            % matrix multiplication

A .* B;           % multiply each element with the same positions

C = 10;

A .+ C            % add all elements in A by 10
```

other useful functions

```
A = pinv(B);      % returns inverse of B
```

$$B \times B^{-1} = I \ \ (identity\ matrix)$$

```
A = B'            % returns transpose of B
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

```
reshape(A, n, m)  % reshapes the elements in A into a n * m matrix
```