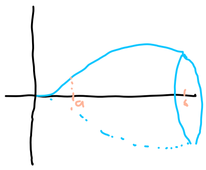
F: M6 22/02

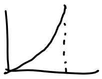
Wednesday, 22 February 2023 10:23

Rotations volym 1:

$$V = \int_a^b \pi (f(x))^2 \, dx$$

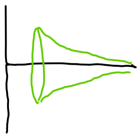


Ex) $f(x) = x^2$ $\quad 0 \le x \le 1$



$$V = \int_0^1 \pi x^4 \, dx = \pi \left[\frac{x^5}{5}\right]_0^1 = \frac{\pi}{5} \text{ v.e}$$

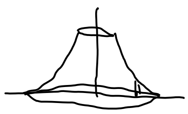
Ex) $f(x) = \frac{1}{x}$ $\quad x \ge 1$

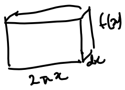


$$V = \int_1^\infty \pi x^{-2} \, dx = -\pi \lim_{r \to \infty} \left[\frac{1}{x}\right]_1^r = -\pi \text{ v.e.}$$

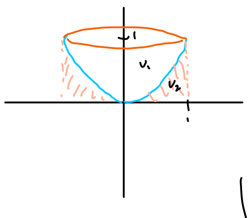
Skalmetod:

$$V = \int_a^b 2\pi x f(x) \, dx$$ kring y-axeln





$$dV = 2\pi x f(x) \, dx$$



Skalmetod:

$$V_2 = 2\pi \int_0^1 x \cdot x^2 \, dx = 2\pi \left[\frac{x^4}{4}\right]_0^1 = \frac{\pi}{2}$$

rotera:

$y = x^2 \Rightarrow x = \sqrt{y}$

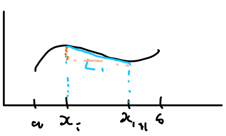
$$V_1 = \pi \int_0^1 y \, dy = \pi \left[\frac{y^2}{2}\right]_0^1 = \frac{\pi}{2} \text{ v.e}$$

Summan av cylindern ska bli: π

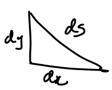
Kurvlängd:

$y = f(x)$, $\quad a \le x \le b$

$$L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx$$



Faktabevis



$$ds^2 = dx^2 + dy^2$$
$$= dx^2 \left(1 + \frac{dy^2}{dx^2}\right)$$

$$L_i = \sqrt{(x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2} = \frac{x_{i+1} - x_i}{x_{i+1} - x_i} \cdot \sqrt{(x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2}$$

$$= (x_{i+1} - x_i) \cdot \sqrt{\frac{(x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2}{(x_{i+1} - x_i)^2}} = (x_{i+1} - x_i) \cdot \sqrt{1 + \left(\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}\right)^2}$$

$$f'(x) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

$$= (x_{i+1} - x_i)\sqrt{1 + f'(x_i)^2}$$

Sedan summeras dessa

$$\sum_{i=1}^{k} (x_{i+1} - x_i)\sqrt{1 + f'(x_i)^2}$$

$ds^2 = dx^2 + dy^2$

$= dx^2\left(1+\frac{dy^2}{dx^2}\right)$

$= \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

sedan summeras dessa

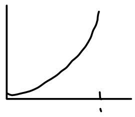
$\sum_{i=1}^{k}(x_{i+1}-x_i)\sqrt{1+f'(x_i)^2}$

Riemannsumman för flera $\sqrt{1+f'(x_i)^2}$

$\int_a^b \sqrt{1+(f'(x))^2}\,dx$

$f'(x) = \frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}$

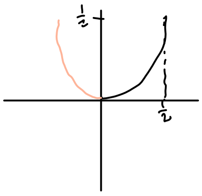
Ex) $f(x)=\frac{2}{3}x^{\frac{3}{2}}$ $\quad 0\le x\le 1$



$f'(x)=x^{\frac{1}{2}}$

$L=\int_0^1\sqrt{1+x}\,dx = \frac{2}{3}\left[(1+x)^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}\left(2^{3/2}-1\right)$ l.e.

Ex.) $f(x)=4x^3$ $\quad 0\le x\le \frac{1}{2}$



$V_y = 2\pi\int_0^{\frac{1}{2}} 4x^4\,dx = 8\pi\left[\frac{x^5}{5}\right]_0^{\frac{1}{2}} = 8\pi\cdot\frac{1}{\underset{4}{32\cdot 5}} = \frac{\pi}{20}$

$V_1 = \frac{\pi}{8}-\frac{\pi}{20} = \frac{3\pi}{40}$

$y=4x^3 \Rightarrow x = \frac{y^{\frac{1}{3}}}{4^{\frac{1}{3}}}$

$V = \frac{\pi}{4^{2/3}}\int_0^{\frac{1}{2}} y^{2/3}\,dy = \frac{\pi}{4^{2/3}}\left[\frac{3}{5}y^{5/3}\right]_0^{\frac{1}{2}} = \frac{\pi}{4^{2/3}}\frac{3}{5}\left(\frac{1}{2^{5/3}}\right) = \frac{3\pi}{5\cdot 16^{1/3}\cdot 32^{1/3}} = \frac{3\pi}{40}$

Rotationsarea:

$A = \int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx$

Ex). $y=\frac{1}{x}$ $\quad x\ge 1$ $\quad$ roteras kring x-axeln

$A = 2\pi\int_1^\infty \frac{1}{x}\sqrt{1+\left(\frac{1}{x^2}\right)^2}\,dx$

$= 2\pi\int_1^\infty \frac{1}{x}\left(1+\frac{1}{x^4}\right)^{\frac{1}{2}}\,dx$

Eftersom hela integranden $\frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}} \ge \frac{2\pi}{x}$ och $\int_1^\infty \frac{2\pi}{x}\,dx = \infty$

Alltså måste integranden divergera enligt jämförelse testet

$\boxed{\text{Slutsats: Arean är oändlig}}$

Om $f(x)=\sqrt{1-x^2}$ $\quad x\in[0,1]$ $\quad$ rot kring x-axeln.

får vi en halv sfär

Arean $= \int_0^1 2\pi f(x)\sqrt{1+f'(x)^2}\,dx$

$= 2\int_0^1\sqrt{1-x^2}$.